Coastal Kelvin Waves

A Conceptual Introduction to Coastal Wave Propagation under Rotation

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Conceptual Framework

Kelvin waves are a class of gravity waves influenced by the Coriolis effect. When confined to boundaries, such as coastlines, they become *coastal Kelvin waves*. These waves travel with the coast on their right in the Northern Hemisphere (and on their left in the Southern Hemisphere), maintaining maximum amplitude at the coast and decaying exponentially offshore.

Originally described by Lord Kelvin in the late 19th century, coastal Kelvin waves are solutions to the linearized shallow water equations under geostrophic balance in the across-shore direction and wave propagation in the alongshore direction.

The sea surface elevation η satisfies a solution of the form:

$$\eta(x, y, t) = Ae^{-y/R} \cos(kx - \omega t),$$

where $R = c/f$ is the Rossby radius of deformation, $c = \sqrt{gH}$ is the wave phase speed, f is the Coriolis parameter, A is the amplitude at the coast ($y = 0$), and k is the zonal wavenumber.

This example draws inspiration from Exercise 15 of Jochen Kaempf's *Ocean Modelling for Beginners*, which lays out the theory behind our model.

Code and Animation

- **Code available at:** https://bit.ly/coastal_kelvin_waves
- **Animation available at:** https://www.youtube.com/watch?v=DedkjGfDtpw

Description

This Python simulation implements a two-dimensional nonlinear shallow water model to simulate a coastal Kelvin wave. The coastline is represented as an impermeable boundary, and sea level is periodically perturbed at a single location, initiating a wave that propagates alongshore.

The Coriolis effect is treated semi-implicitly, and nonlinear advection is incorporated through a high-resolution scheme. The animation reveals the asymmetric structure of Kelvin waves, their confinement to the coast, and the absence of westward propagation.

Acknowledgment: A huge thanks to Jochen Kaempf for his incredible contribution to ocean modeling. If you are interested in his book, contact him via ResearchGate for a copy.