# Wave Interference

A Conceptual Introduction to Superposition of Harmonic Waves

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# Conceptual Framework

The study of wave interference offers an accessible starting point for introducing the dynamics governed by the Navier-Stokes equations. In geophysical fluid dynamics, we often begin with linear approximations to explore the behavior of waves under simplified conditions.

In this exercise, we explore the superposition principle, a foundational concept in wave theory. It describes how two or more waveforms traveling in the same medium combine algebraically to form a resultant wave. When two sinusoidal waves of slightly different wavelengths and periods travel in the same direction, the result is a phenomenon known as *beat interference*, where constructive and destructive interferences lead to modulated amplitude patterns.

Mathematically, if two traveling waves are given by:

$$f_1(x,t) = A_1 \sin\left(2\pi\left(\frac{x}{\lambda_1} - \frac{t}{T_1}\right)\right), \quad f_2(x,t) = A_2 \sin\left(2\pi\left(\frac{x}{\lambda_2} - \frac{t}{T_2}\right)\right),$$

their superposition becomes:

$$f(x,t) = f_1(x,t) + f_2(x,t),$$

which may exhibit amplitude modulation depending on the differences in wavelength and period.

Here, $A_1$ and $A_2$ represent the amplitudes of each wave (not necessarily equal), $\lambda_1$ and $\lambda_2$ are the wavelengths, and $T_1$ and $T_2$ the periods of wave 1 and wave 2, respectively. The variable $x$ denotes the position along the propagation direction, and $t$ is time.

While the full Navier-Stokes equations are nonlinear and generally unsolvable analytically, examining solutions where linear wave propagation dominates allows us to develop intuition about the nature of fluid motion and energy dispersion. This sets the stage for more complex interactions and introduces essential terminology such as phase speed, wavelength, and frequency.

This example comes from Exercise 2 of Jochen Kaempf's *Ocean Modelling for Beginners*, which provides the theoretical framework for the code.

# Code and Animation

- **Code available at:** https://bit.ly/OOM_WaveInterference
- **Animation available at:** https://www.youtube.com/watch?v=57q1dusKc2Q

# Description

This Python exercise simulates the interference of two traveling sinusoidal waves. It displays the individual waves and their superposition using matplotlib animations. The wave parameters can be customized to explore different behaviors, such as phase shifts, varying amplitudes, and more extreme beat patterns. The visualization helps students grasp the

dynamic nature of wave interactions, which is often difficult to conceptualize from static plots.

**Acknowledgment:** A huge thanks to Jochen Kaempf for his incredible contribution to ocean modeling. If you are interested in his book, contact him via ResearchGate for a copy.