Numerical explorations in a modified potential of the TBP

F. J. Muñoz-Almaraz*, J. Galán-Vioque†, E. Freire†, A. Vanderbawhede‡

November 25, 2018

Abstract

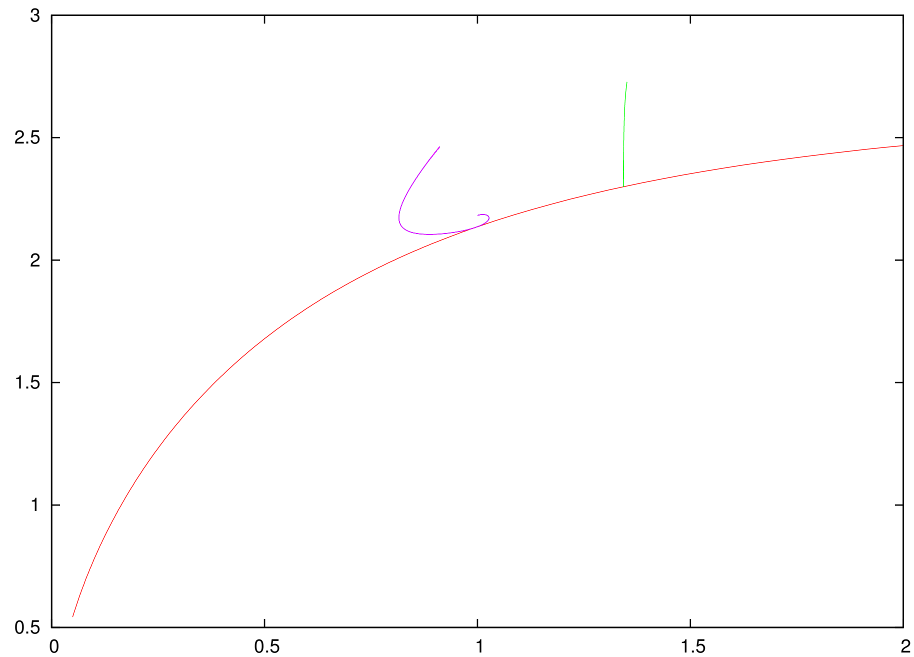
This is a working document distributed in 2005 among our group and other researchers interested about bifurcation for numerical continuation of modified potential of the three body problem (TBP) starting from the figure-8[1]. In 2018, Dr. Toshiaki Fujiwara told us that he was going to cite our private communication about this topic. Therefore, this document is making publicly available that communication as well as the code for numerical continuation with AUTO. The body of this document consists in the working document of 2005, adding some remarks as footnotes and a bibliography with the papers where the algoritm are described.

We are going to study central forces

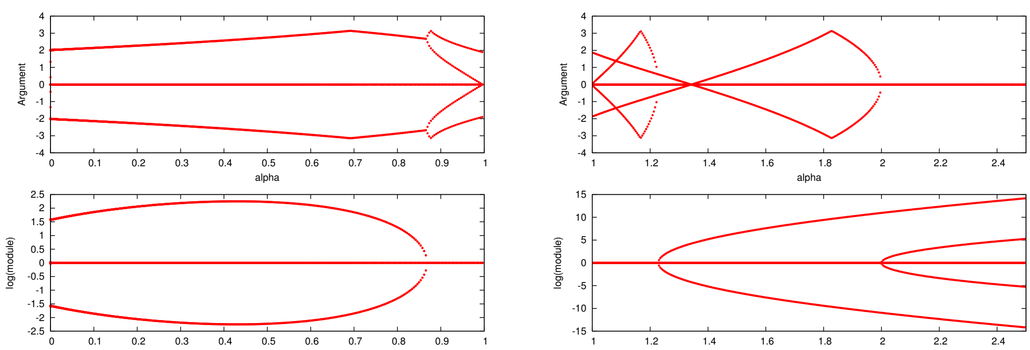
$$H = \frac{1}{2}(\|\mathbf{p}_1\|^2 + \|\mathbf{p}_2\|^2 + \|\mathbf{p}_3\|^2) - \frac{1}{\|\mathbf{q}_1 - \mathbf{q}_2\|^\alpha} - \frac{1}{\|\mathbf{q}_1 - \mathbf{q}_3\|^\alpha} - \frac{1}{\|\mathbf{q}_2 - \mathbf{q}_3\|^\alpha} \tag{1}$$

where $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{q}_1$, $\mathbf{q}_2$ and $\mathbf{q}_3$ are vectors on the plane.

*Universidad CEU-Cardenal Herrera
†Universidad de Sevilla
‡University of Gent



The bifurcation diagram displayed above has as horizontal axis the exponent in the potential of Hamiltonian, α, and the vertical axis is the norm L^2 given by AUTO as output. I do not know why it could not be the pink curve and the red curve tangent. The red curve is the family where the figure-eight is. This red family goes from $\alpha = 1$ up to very high values for α, in the other direction is going to go from $\alpha = 1$ up to close to $\alpha = 0$. Every orbit in the family is the three bodies following a curve with figure eight. Characteristic multipliers are shown in the following figure

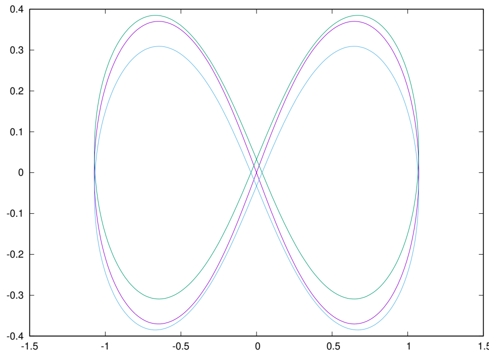


The red family is symmetric with respect to the two time-reversal symmetries of the figure eight[1]. The continuation with both of them allows us to find two bifurcation points at $\alpha =$

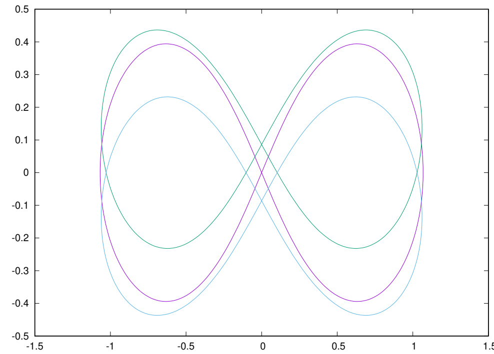
[1] The general procedure and its theoretical explanation of continuation with time–reversal symmetries is described in [4]. These techniques are applied for the TBP in [3] to calculate numerical continuation starting from the eight orbits. The two time–reversal symmetries are described there depeding on the matrix we use $S_{M_1} = \left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right]$

0.996562 and $\alpha = 1.34242$. However, the other two bifurcation are not detected with the symmetric continuation and with the method for period orbit. [2] The two detected bifurcation have a “passing” behauvior (the characteristic multiplier pass through the one and stays on the unit circle) and the undetected are “splitting” (there exists a change in the linear stability). The undetected are more or less close to $\alpha = 1.2283$ and $\alpha = 2$. Is $\alpha = 2$ a special point?

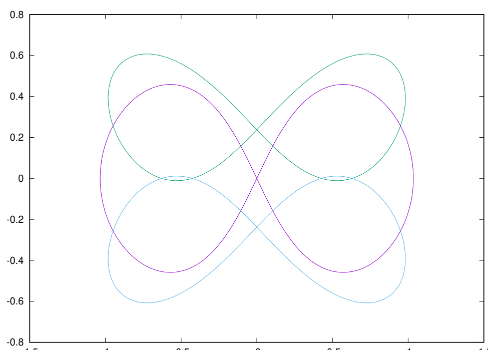
From the bifurcation point at $\alpha = 0.996562$, a family of doubly symmetric periodic orbits appears. In the diagram is the pink curve. This family looks like tangent to the red curve, I should check it. I do not understand which is the reason because of these curves can not be tangent. I would thank an explanation. Here some orbits of the family in a direction:



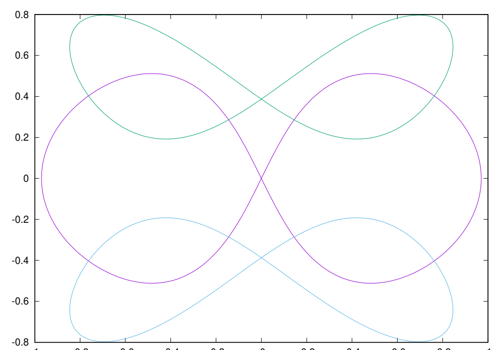
$\alpha = 0.98974558119$



$\alpha = 0.97657871845$



$\alpha = 0.92520305647$

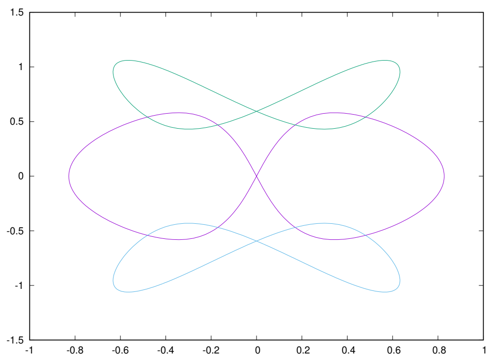


$\alpha = 0.86299535233$

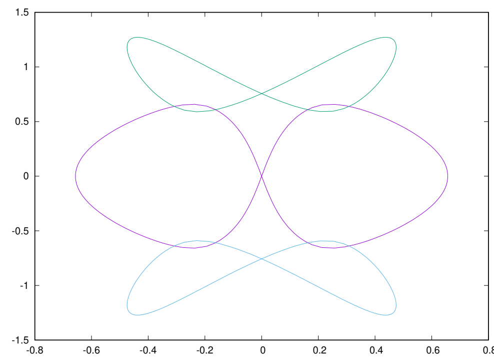
---

or $S_{E_1} = \left[\begin{array}{cc} -1 & 0 \\ 0 & -1 \end{array} \right]$

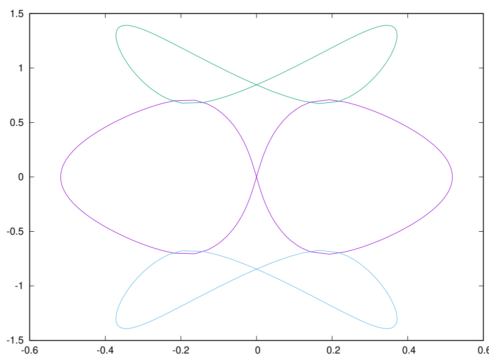
[2] Detection of bifurcation with the method of continuation explained in [2] is very sensitive and not always is detecting bifurcations. There are there values where a change of stability is undergoing according to the characteristics multipliers: $\alpha = 0.8\ldots$, $\alpha = 1.2283$ and $\alpha = 2$, they are undetected with the time-reversal continuation schemes. We have studied in [5] a situation with change of stability without branching. We have not studied whether or not this is the situation for these values.



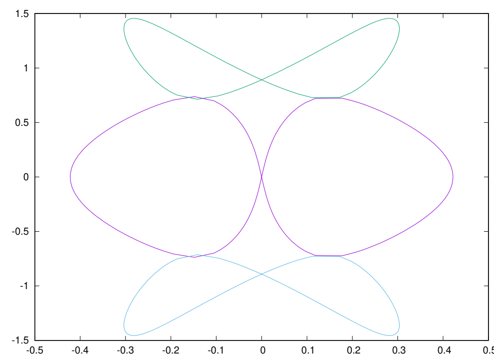
$\alpha = 0.81555487427$



$\alpha = 0.84462774170$

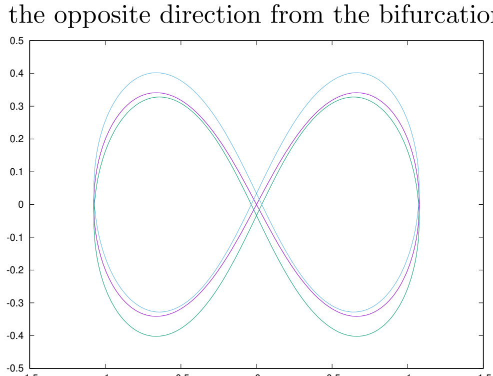


$\alpha = 0.88342919391$

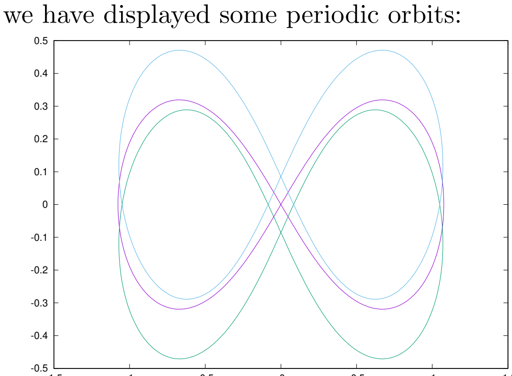


$\alpha = 0.90984042548$

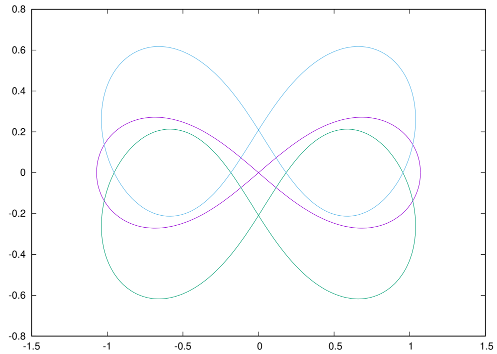
In the opposite direction from the bifurcation, we have displayed some periodic orbits:



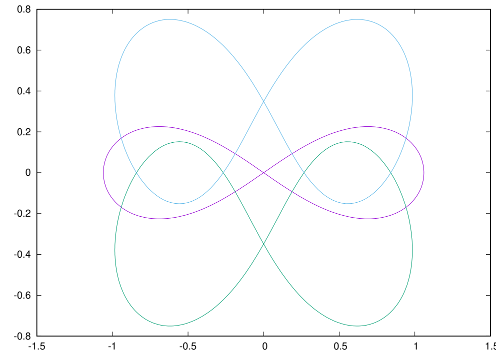
$\alpha = 1.0025843346$



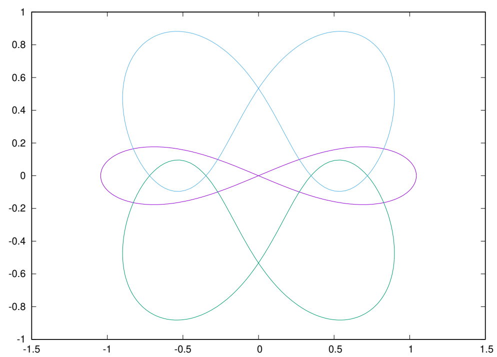
$\alpha = 1.0101864584$



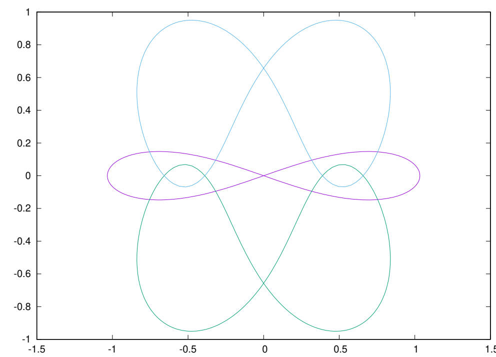
$\alpha = 1.0215325651$



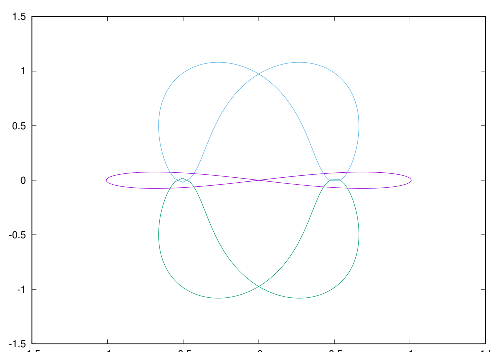
$\alpha = 1.0265396023$



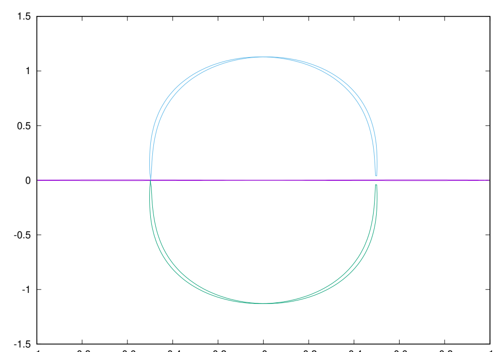
$\alpha = 1.0265820623$



$\alpha = 1.0244124526$



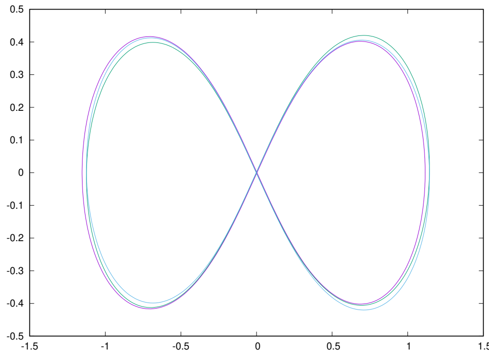
$\alpha = 1.0137412091$



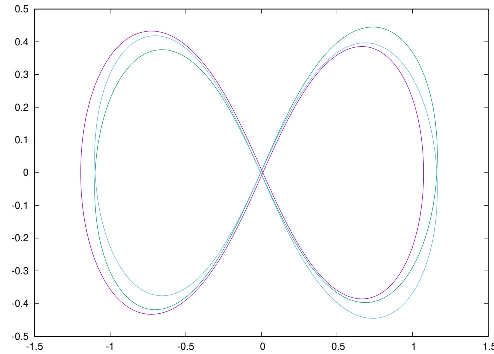
$\alpha = 1.0005848879$

About the above family, I am surprised because they look like the orbits in the family which belongs the figure eight when one of the masses is varied.

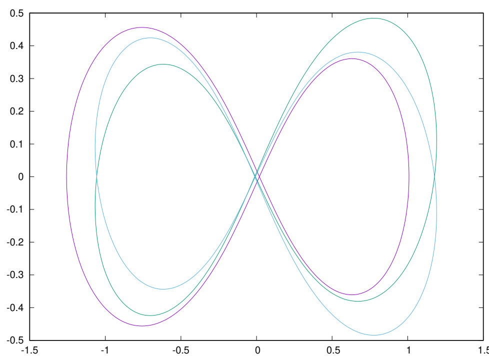
From the other bifurcation at $\alpha = 1.34242$ appears the family which correspond to the green curve in the above bifurcation diagram. This family is also symmetric with respect to the two time-reversal symmetries. Again, there are some orbits from this family.



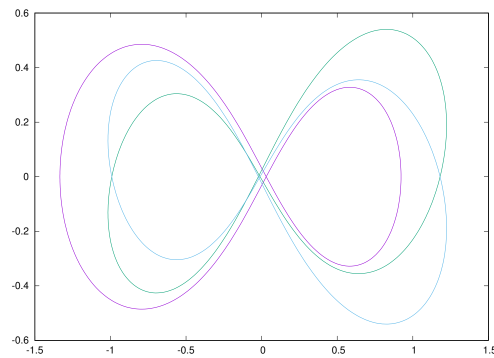
$\alpha = 1.3424242750$



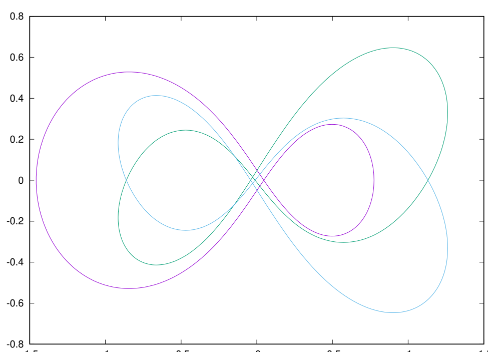
$\alpha = 1.3424432064$



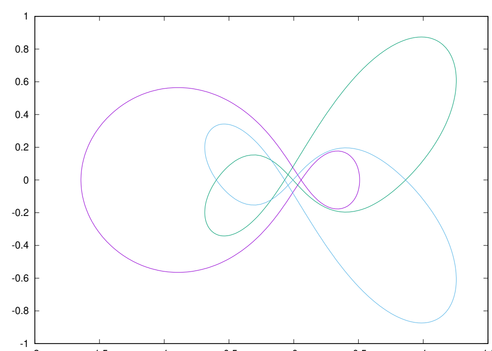
$\alpha = 1.3425063709$



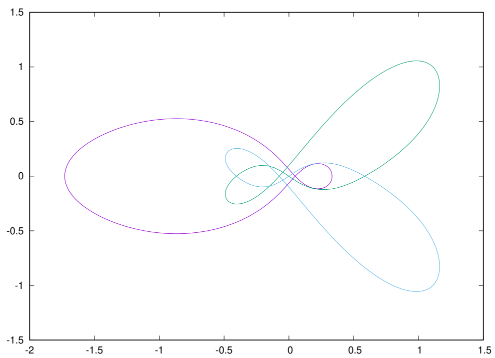
$\alpha = 1.3426429099$



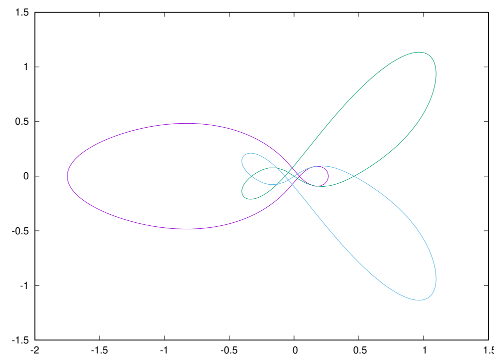
$\alpha = 1.3429612329$



$\alpha = 1.3439693118$



$\alpha = 1.3470762333$



$\alpha = 1.3504602980$

References

[1] Chenciner, A. & Montgomery, R. (2000). *A remarkable periodic solution of the three-body problem in the case of equal masses.* Annals of Mathematics-Second Series, 152(3), 881-902.`http://emis.ams.org/journals/Annals/152_3/chencine.pdf`

[2] Muñoz-Almaraz, F. J., Freire, E., Galán, J., Doedel, E., & Vanderbauwhede, A. (2003). *Continuation of periodic orbits in conservative and Hamiltonian systems.* Physica D: Nonlinear Phenomena, 181(1–2), 1–38. `https://doi.org/10.1016/S0167-2789(03)00097-6`

[3] Muñoz–Almaraz, F. J., Galán-Vioque, J., & Freire, E. (2004). *Families of symmetric periodic orbits in the three body problem and the figure eight.* Monografías de La Real Academia de Ciencias de Zaragoza, 25, 229–240.

[4] Muñoz-Almaraz, F. J., Freire, E., Gal'an-Vioque, J., & Vanderbauwhede, A. (2007). Continuation of normal doubly symmetric orbits in conservative reversible systems. Celestial Mechanics and Dynamical Astronomy, 97(1), 17–47. `https://doi.org/10.1007/s10569-006-9048-3`

[5] Muñoz–Almaraz, F. J., Freire, E., Galán-Vioque, J., & Vanderbauwhede, A. (2009). *Change of Stability withour Bifurcation: An Example.* In Proceedings of the 14th International Conference on Difference Equations and Applications (pp. 3–18). Istambul. Turkey: Ugur-Bahcesehir University Publishing.