Computer Architecture and Programming Languages
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Homework 2

Problem 2.1

Solution:

a) $777_8 + 1_8 = \mathbf{1000_8}$

b) $888_16 + 1_{16} = \mathbf{889_{16}}$

c) $32007_8 + 1_8 = \mathbf{32010_8}$

d) $32108_{16} + 1_{16} = \mathbf{32109_{16}}$

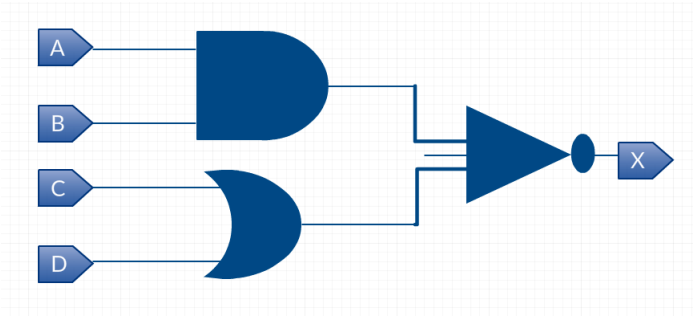
e) $8BFF_{16} + 1_{16} = \mathbf{8C00_{16}}$

f) $1219_{16} + 1_{16} = \mathbf{121A_{16}}$

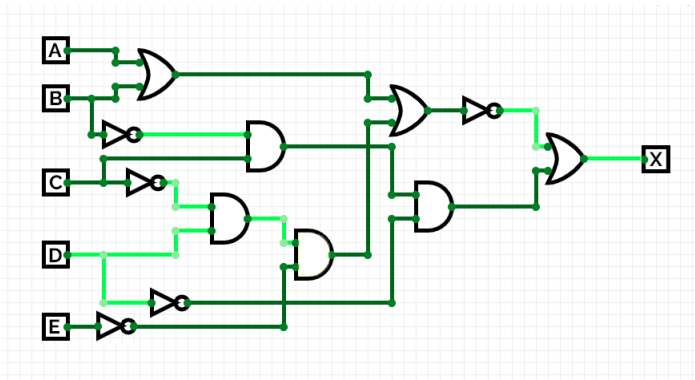
Problem 2.2

Solution:

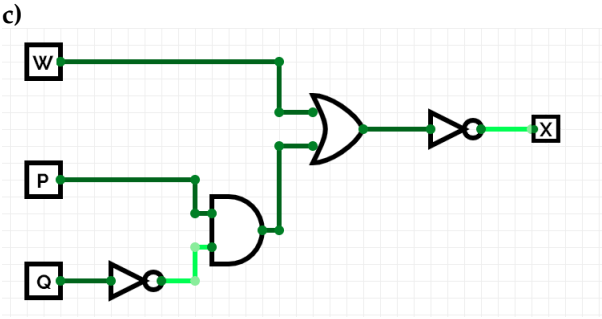
a) $X = \overline{A * B * (C + D)}$



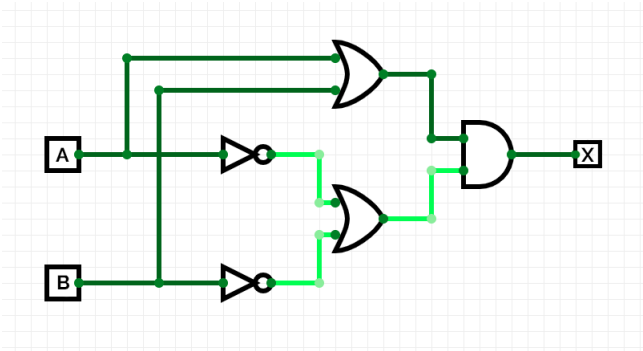
b) $X = \overline{A + B + \overline{C} * D * \overline{E}} + \overline{B} + C + \overline{D}$



c)



d)



Problem 2.3

Solution:

$R1 = \overline{M * N * Q}$
$R2 = \overline{M * \overline{N} * Q}$
$R3 = \overline{\overline{M} * N * Q}$

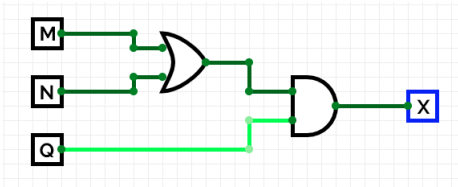
The truth table of the circuit:

M	N	Q	R1	R2	R3	X
0	0	0	1	1	1	0
0	0	1	1	1	1	0
0	1	0	1	1	1	0
0	1	1	1	1	0	1
1	0	0	1	1	1	0
1	0	1	1	0	1	1
1	1	0	1	1	1	0
1	1	1	0	1	1	1

The sum of products: $X = \overline{M} * N * Q + M * \overline{N} * Q + M * N * Q$

Simplify:
$X = \overline{M} * N * Q + M * \overline{N} * Q + M * N * Q =$
$Distributive\ Law := Q(\overline{M} * N + M * \overline{N} + M * N) =$
$Distributive\ Law := Q(\overline{M} * N + M * \overline{N} + M * N + M * N) =$
$Distributive\ Law := Q(N(\overline{M} + M) + M(\overline{N} + N)) =$
$Identity\ Law := Q(N * 1 + M * 1) =$
$Distributive\ Law := Q(N + M) = Q * N + Q * M$

Circuit Representation after simplification:



Problem 2.4

Solution:

a)
$X + \overline{X} * Y = X + Y$

$X + \overline{X} * Y$:

X	Y	$\overline{X}$	$\overline{X} \wedge Y$	$X \vee (\overline{X} \wedge Y)$
0	0	1	0	0
0	1	1	1	1
1	0	0	0	1
1	1	0	0	1

$X + Y$:

X	Y	$X \vee Y$
0	0	0
0	1	1
1	0	1
1	1	1

As it is evident, both sides of the equation result in the same output.

b)
$\overline{X} + X * Y = \overline{X} + Y$

$\overline{X} + X * Y$:

X	Y	$\overline{X}$	$X \wedge Y$	$\overline{X} \vee (X \wedge Y)$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	0
1	1	0	1	1

$\overline{X} + Y$:

X	Y	$\overline{X}$	$\overline{X} \vee Y$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

As it is evident, both sides of the equation result in the same output.

Problem 2.5

Solution:

a) $A + 1 = 1$

b) $A * A = A$

c) $B * \overline{B} = 0$

d) $C + C = C$

e) $X * 0 = 0$

f) $D * 1 = D$

g) $D + 0 = D$

h) $C + \overline{C} = 1$

i) $G + G * F = G$

j) $Y + \overline{W} * Y = Y$

Problem 2.6

Solution:

1. $\overline{W+Y} = \overline{W} * \overline{Y}$

W	Y	$\overline{W}$	$\overline{Y}$	$W+Y$	$\overline{W+Y}$	$\overline{W} \wedge \overline{Y}$
0	0	1	1	0	**1**	**1**
0	1	1	0	1	**0**	**0**
1	0	0	1	1	**0**	**0**
1	1	0	0	1	**0**	**0**

2. $\overline{W*Y} = \overline{W} + \overline{Y}$

W	Y	$\overline{W}$	$\overline{Y}$	$W*Y$	$\overline{W*Y}$	$\overline{W} + \overline{Y}$
0	0	1	1	0	**1**	**1**
0	1	1	0	0	**1**	**1**
1	0	0	1	0	**1**	**1**
1	1	0	0	1	**0**	**0**

3. $1 + W = 1$

1	W	$1 \vee W$
1	0	1
1	0	1
1	1	1
1	1	1

4. $0 + W = W$

0	W	$1 \vee W$
0	0	0
0	0	0
0	1	1
0	1	1

5. $0 * W = 0$

0	W	$0 \wedge W$
0	0	0
0	0	0
0	1	0
0	1	0

6. $1 * W = W$

1	W	$1 \wedge W$
1	0	0
1	0	0
1	1	1
1	1	1

Problem 2.7

DL = Distributive Law
CL = Complement Law
IL = Identity Law

The sum-of-product expression:

$X = \overline{A}*\overline{B}*\overline{C}*D+\overline{A}*\overline{B}*C*D+\overline{A}*B*\overline{C}*D+\overline{A}*B*C*D+A*\overline{B}*\overline{C}*\overline{D}+A*B*\overline{C}*D+A*B*C*D =$

$DL := A*\overline{B}*\overline{C}*\overline{D}+D(\overline{A}*\overline{B}*\overline{C}+\overline{A}*\overline{B}*C+\overline{A}*B*\overline{C}+\overline{A}*B*C+A*B*\overline{C}+A*B*C) =$

$DL := A*\overline{B}*\overline{C}*\overline{D}+D(\overline{A}*\overline{B}*(\overline{C}*C)+\overline{A}*B*(\overline{C}+C)+A*B*(\overline{C}+C)) =$

$CL := A*\overline{B}*\overline{C}*\overline{D}+D(\overline{A}*\overline{B}*1+\overline{A}*B*1+A*B*1) =$

$IL := A*\overline{B}*\overline{C}*\overline{D}+D(\overline{A}*\overline{B}+\overline{A}*B+A*B)$

Since $A+A=A$, write $\overline{A}*B = \overline{A}*B+\overline{A}*B$:

$X = A*\overline{B}*\overline{C}*\overline{D}+D(\overline{A}*\overline{B}+\overline{A}*B+\overline{A}*B+A*B) =$

$DL := A*\overline{B}*\overline{C}*\overline{D}+D(\overline{A}*(\overline{B}+B)+B*(\overline{A}+A)) =$

$CL := A*\overline{B}*\overline{C}*\overline{D}+D(\overline{A}*1+B*1) =$

$IL := A*\overline{B}*\overline{C}*\overline{D}+D(\overline{A}+B) =$

$DL := A*\overline{B}*\overline{C}*\overline{D}+\overline{A}*D+B*D$

Problem 2.8

Solution:

	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\overline{B}$	0	1	1	0
$\overline{A}B$	0	$1 \wedge 1$	$1 \wedge 1$	0
AB	0	1	1	0
$A\overline{B}$	1	0	0	0

$Note$: the 1s in the second row belong to both groups (red 1s and blue 1s) in order to give the minimalistic expression.

After building the Karnaugh-map using gray coding (00, 01, 11, 10) from the truth table given in **Problem 2.7**, we first try to create groups of 1s. We have 3 groups in total, distinguished by their colors (red, blue and green).
Green group (single 1): all terms are included $=> A*\overline{B}*\overline{C}*\overline{D}$.
Blue group: $\overline{A}*D =>$ These terms do not change regardless of the output.
Red group: similarly $=> B*D$
As a result, we obtain: $\mathbf{X = A*\overline{B}*\overline{C}*\overline{D}+\overline{A}*D+B*D}$