数学实验作业第十二周

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目录

P111 2(1)

```
A=[1 -9 -10;-9 1 5;8 7 1];%录入方程式矩阵
b=[1;0;4];%录入常数项
x=[1 1 1]';%初值
m=x;%第二次迭代
```

范数的最小化

```
T=[1 -9 -10;-9 1 5;8 7 1];
E=zeros(3,3);
for i=1:3
    E(1,:)=T(i,:);
    for k=1:3
        if k~=i
           E(2,:)=T(k,:);
           l=6-k-i;
           E(3,:)=T(l,:);
           E
           L=-tril(E,-1);
           U=-triu(E,1);
           D=diag(diag(E));
           B1=D\(L+U);
           s1=norm(B1)
           s2=max(abs(eig(B1)))
        end
    end
end
```

```
E = 3×3
     1    -9   -10
    -9     1     5
     8     7     1
s1 = 14.8053
s2 = 8.2825
E = 3×3
     1    -9   -10
```

```
      8      7      1
     -9      1      5
s1 = 13.4547
s2 = 2.7844
E = 3×3
     -9      1      5
      1     -9    -10
      8      7      1
s1 = 10.6307
s2 = 1.9071
E = 3×3
     -9      1      5
      8      7      1
      1     -9    -10
s1 = 1.1637
s2 = 0.8520
E = 3×3
      8      7      1
      1     -9    -10
     -9      1      5
s1 = 1.8203
s2 = 1.2198
E = 3×3
      8      7      1
     -9      1      5
      1     -9    -10
s1 = 10.2962
s2 = 1.8852
```

可见，A= $\begin{bmatrix} -9 & 1 & 5 \\ 8 & 7 & 1 \\ 1 & -9 & -10 \end{bmatrix}$ 时的范数可知收敛

```
A=[-9 1 5;8 7 1;1 -9 -10];
b=[0;4;1];
```

Jacobi迭代

```
L=-tril(A,-1)%下三角
```

```
L = 3×3
      0      0      0
     -8      0      0
     -1      9      0
```

```
U=-triu(A,1)%上三角
```

```
U = 3×3
      0     -1     -5
      0      0     -1
      0      0      0
```

```
D=diag(diag(A))%提取并生成对角阵
```

```
D = 3×3
     -9      0      0
      0      7      0
      0      0    -10
```

```
B1=D\(L+U)%生成系数矩阵
```

```
B1 = 3×3
         0    0.1111    0.5556
   -1.1429         0   -0.1429
    0.1000   -0.9000         0
```

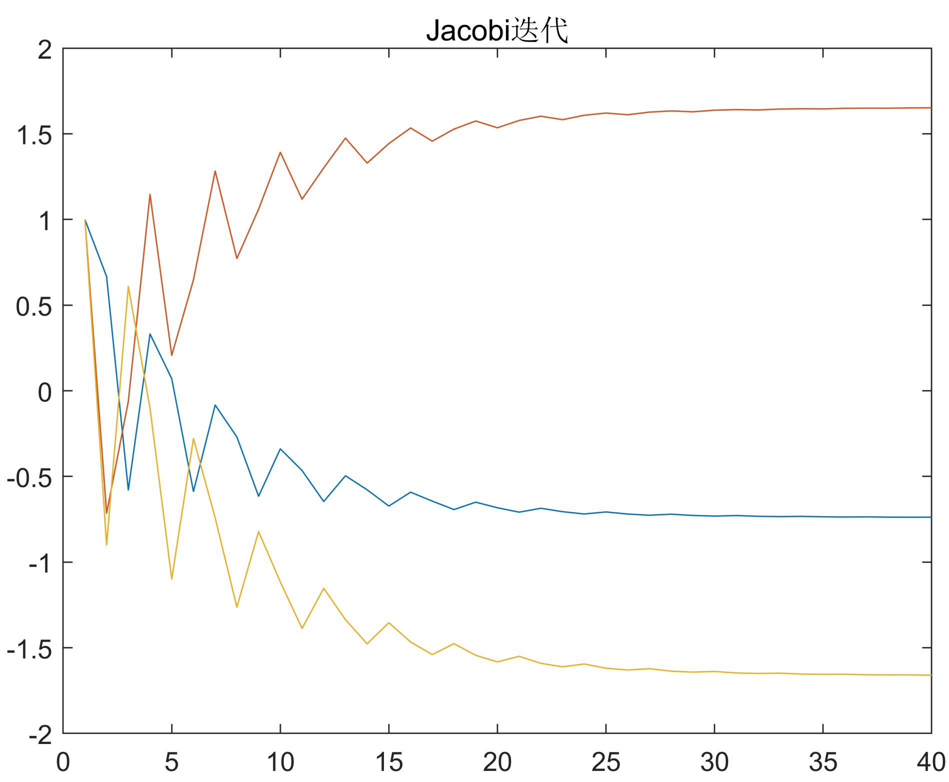
```
s1=norm(B1)%范数>1, 迭代法不收敛
```

```
s1 = 1.1637
```

```
s2=max(abs(eig(B1)))%特征根的模的最大值<1，迭代法收敛
```

```
s2 = 0.8520
```

```
k=40;%迭代次数
x0=1:k;
y1=zeros(1,k);
y2=zeros(1,k);
y3=zeros(1,k);
for i=1:k
    y1(i)=x(1);
    y2(i)=x(2);
    y3(i)=x(3);
    x=B1*x+D\b;
end
plot(x0,y1,x0,y2,x0,y3);
title('Jacobi迭代');
```



可见，jacobi在此运算下收敛,结果趋向于[-0.7377 1.6517 -1.6599]，接近于真实结果

Gauss-Seideil迭代

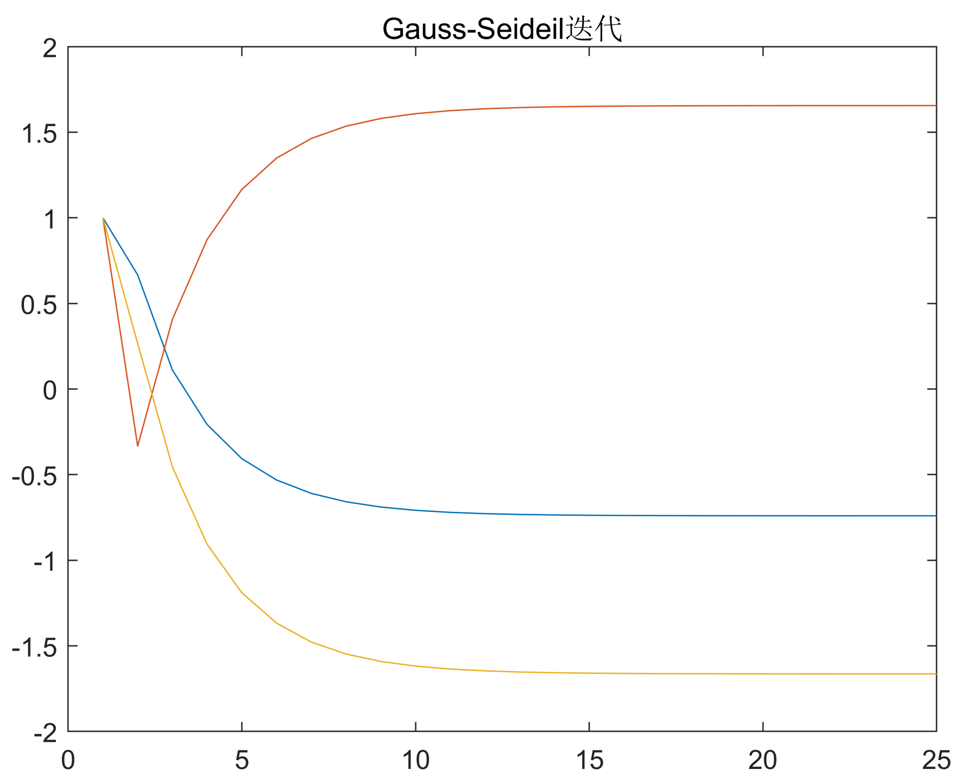
```
B2=(D-L)\U;%生成系数矩阵
s3=norm(B2)%范数>1, 迭代法不收敛
```

```
s3 = 1.2363
```

```
s4=max(abs(eig(B1)))%特征根的模的最大值<1，迭代法收敛
```

```
s4 = 0.8520
```

```
p=25;%迭代次数
m0=1:p;
n1=zeros(1,p);
n2=zeros(1,p);
n3=zeros(1,p);
for i=1:p
    n1(i)=m(1);
    n2(i)=m(2);
    n3(i)=m(3);
    m=B2*m+(D-L)\b;
end
plot(m0,n1,m0,n2,m0,n3);
title('Gauss-Seideil迭代');
```

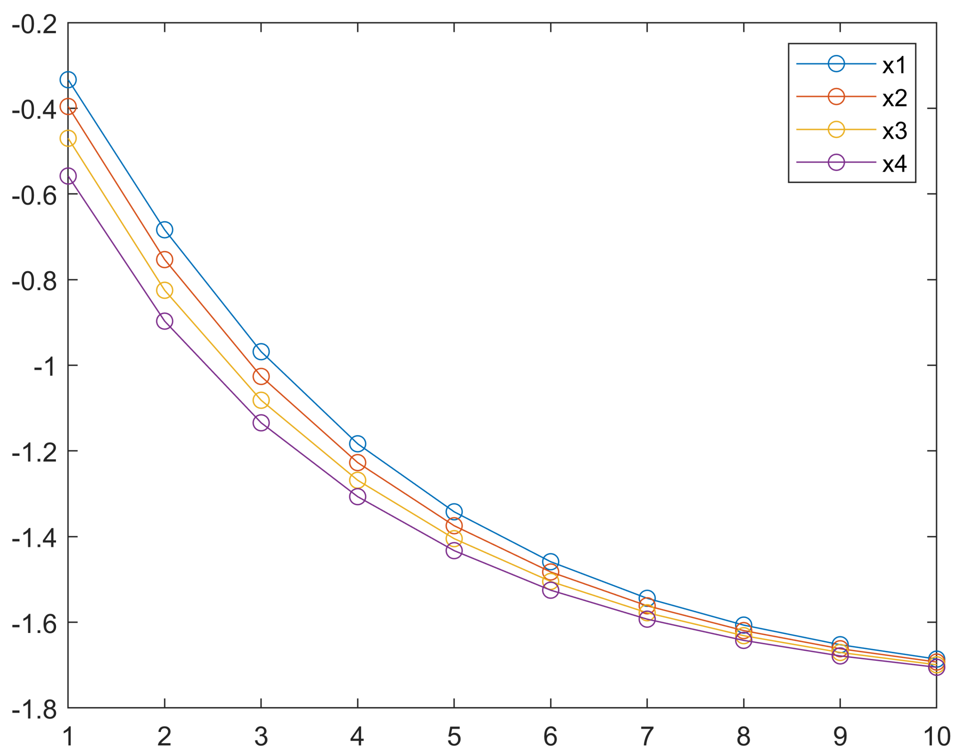


可见，Gauss-Seideil在此运算下收敛,结果趋向于[-0.7404 1.6553 -1.6638],接近于真实结果

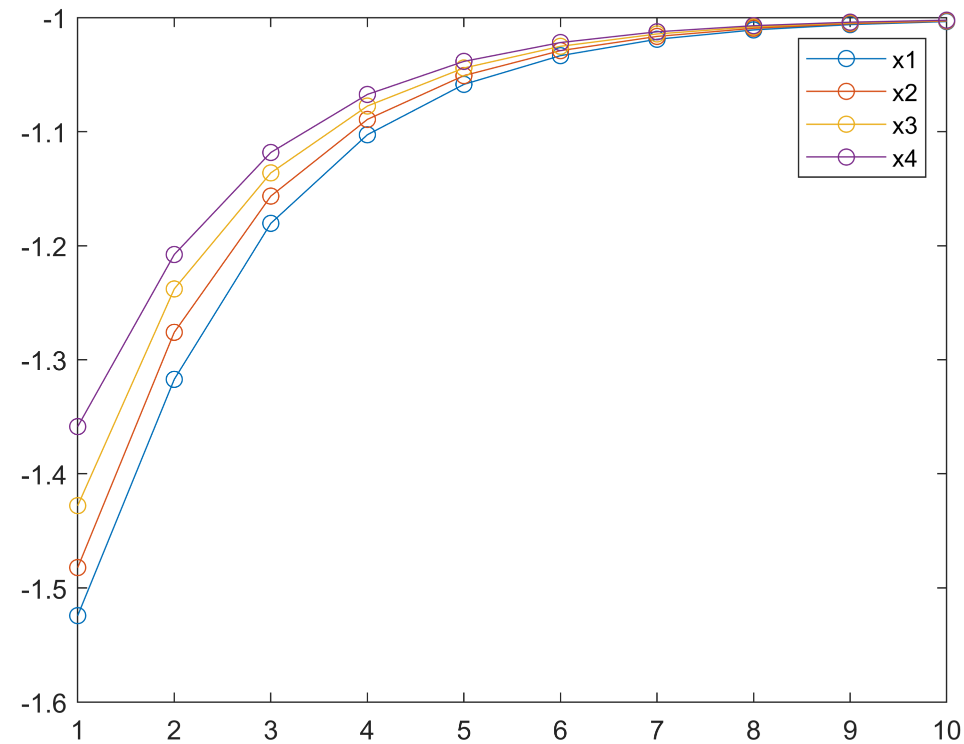
P113 10

```
A=[-4 1 1 1;1 -4 1 1;1 1 -4 1;1 1 1 -4];%录入方程式矩阵
b=[1;1;1;1];%录入常数项
x=[0 0 0 0]';%初值
L=-tril(A,-1);%下三角
U=-triu(A,1);%上三角
D=diag(diag(A));%提取并生成对角阵
p=10;%迭代次数
w0=[0.75 1.0 1.25 1.5];
k=length(w0);
for j=1:k
    y=[];
    w=w0(j)
    B=(D-w*L)\(w*U+(1-w)*D);
    f=w*(D-w*L)\b;
    x0=1:p;
    for i=1:p
        x=B*x+f;
        y=[y x];
    end
    figure;
    plot(x0,y(1,:),'-o',x0,y(2,:),'-o',x0,y(3,:),'-o',x0,y(4,:),'-o');
    legend('x1','x2','x3','x4')
end
```

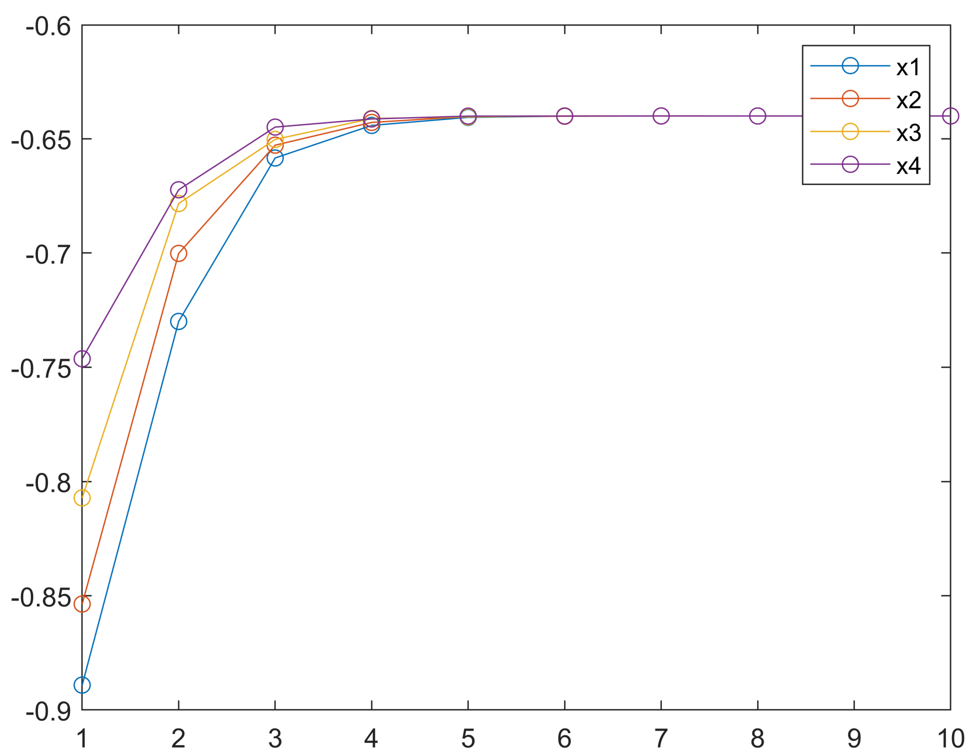
```
w = 0.7500
```



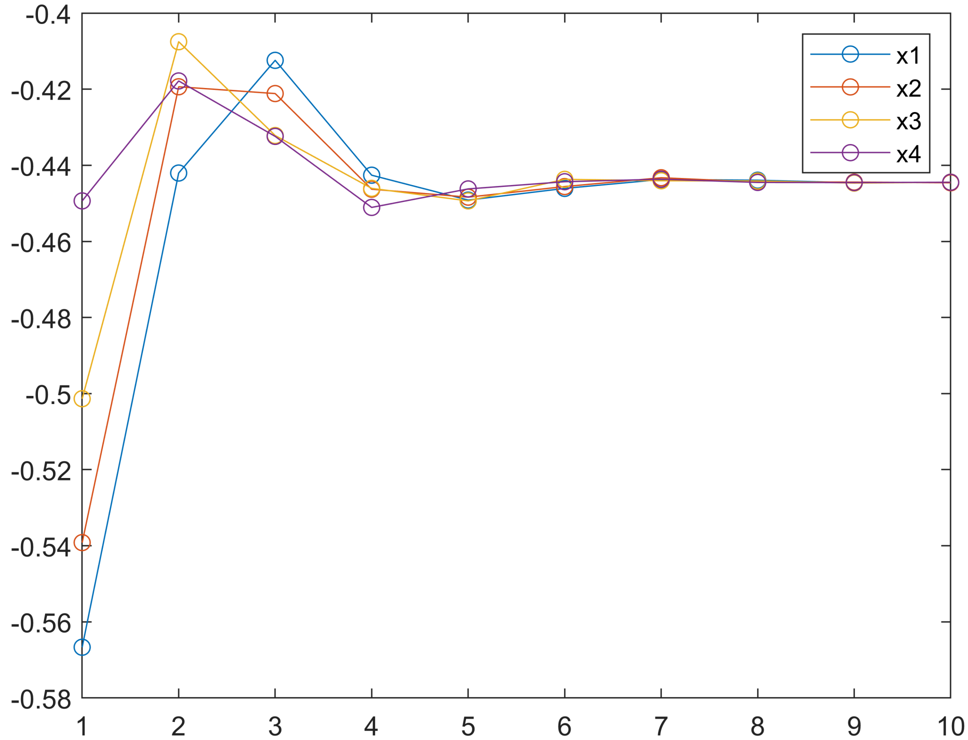

```
w = 1
```



w = 1.2500



w = 1.5000



其精确值为[-1 -1 -1 -1]，由上式可知，只有w=1时比较精确