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Take Home Exam 2
Convex Optimization

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Problem 1
We have worked out the elementary vision of Lagrange multipliers, assuming that , from $g(x, y) = 0$, we can find a function $y = h(x)$ such that $g(x, h(x)) = 0$.

But sometimes, what we get is that there is an h such that $g(x, h(x)) = 0$. Rewrite the Lagrange multiplier analysis in the lecture slides under this assumption.

Consider $f, g : \mathbb{R}^2 \to \mathbb{R}$, and the following minimization problem:

$$\min f(x, y) \ \text{ s.t. } g(x, y) = 0 \tag{1}$$

Now, we can use the **implicit function theorem** to find a dependence between the variables of the restriction. This theorem (not completely formally) **states** the following: let $g : \mathbb{R}^{n+m} \to \mathbb{R}^m$ be a continuously differentiable function, $(x, y) \in \mathbb{R}^{n+m}$ such that $g(x, y) = 0$. If the jacobian with respect to the variables in y is invertible, then there exists an open subset U such that $h(x) = y$ and $f(x, h(x)) = 0$ for all $x \in U$.

Assuming that the **conditions for this theorem are matched**, we can apply it to the **jacobian with respect to the variables in y** to obtain an U' where $h(y) = x$ and $f(h(y), y) = 0$ for all $y \in U$. Thus, we can write:

$$f(x, y) = f(h(y), y) = \psi(y)$$

The, we can keep the procedure as it is done in the slides. Let us see this:
Consider that y^* is a minimum with $x^* = h(y^*)$. Then, we have:

$$0 = \psi'(y^*) = \frac{\partial f}{\partial x}(x^*, y^*)h'(y^*) + \frac{\partial f}{\partial y}(x^*, y^*).$$

Using that (x^*, y^*) is a minimum and that $g(h(y), y) = 0$, we have that:

$$0 = \frac{\partial g}{\partial x}(x^*, y^*)h'(y^*) + \frac{\partial g}{\partial y}(x^*, y^*) \implies h'(y^*) = \frac{a}{b}$$