Efficient Construction of Large Search Spaces for GPU autotuning

ICS 2025 Submission #**NaN** – Confidential Draft – Do NOT Distribute!!

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Abstract

Graphics Processing Units (GPUs) have become indispensable as a computing resource due to their exceptional computational performance for data- and compute-intensive tasks. Auto-tuning is used to optimize the performance, accuracy, and energy efficiency of GPU programs to select the best configurations from a vast search space. However, as GPU architectures and applications become more complex, the demands on auto-tuners have increased significantly. In particular, the construction of the search space at the start of the tuning process has become a bottleneck in the tuning process due to the large number of possible combinations — often exceeding millions — and the complex constraints applied on each of these. Real-world applications have been encountered where the search space construction takes minutes to hours or even days.

This work addresses this challenge by leveraging Constraint Satisfaction Problem (CSP) solvers to construct and represent GPU kernel search spaces. We introduce several key optimizations to enhance solver efficiency, substantially reducing the overhead of search space construction while maintaining flexibility and scalability, and provide the implementations to the CSP and auto-tuning communities. Evaluation on real-world applications demonstrates that our optimized solver improves search space construction performance by four orders of magnitude over naive solving and is one to two orders of magnitude faster than the current state-of-the-art, enabling the exploration of previously unattainable problem scales in auto-tuning and related domains.

CCS Concepts

• **Computing methodologies** → *Discrete space search*; Parallel programming languages.

Keywords

Search spaces, Constraint satisfaction, Constraint solving, Discrete space, Autotuning, GPU applications, Optimization problems

ACM Reference Format:

Anonymous Author(s). 2025. Efficient Construction of Large Search Spaces for GPU autotuning: ICS 2025 Submission #**NaN** – Confidential Draft – Do NOT Distribute!!. In *Proceedings of The 39th ACM International Conference on Supercomputing ('ICS 2025')*. ACM, New York, NY, USA, 13 pages. https://doi.org/XXXXXXX.XXXXXXX

1 Introduction

Graphics Processing Units (GPUs) have revolutionized the computing landscape in the past decade, providing previously unattainable computational performance for compute-intensive tasks such as artificial intelligence and climate simulation [23, 29]. Nine out of the top ten supercomputers in the TOP 500 listing of November 2024 use GPUs as the main source of compute power, and systems with accelerators account for 82.6% of the combined TOP 500 RMax performance [21]. GPUs excel in terms of compute performance and energy efficiency for tasks that involve large data sets and dense computation, making them increasingly vital in various scientific domains [54]. In the past decade, GPUs have become increasingly complex computing devices with larger register files, more specialized cores, and larger and more complex streaming multiprocessors (SMs), while also dramatically increasing the number of SMs per chip [27]. In addition, energy efficiency and accuracy play an increasingly important role in the auto-tuning of GPU applications [24, 49].

GPU programming models, such as CUDA, HIP, and OpenCL, allow developers to create highly parallel functions, called *kernels*, that run on the GPU. Developers are confronted with a myriad of implementation choices and optimization techniques related to thread organization, memory usage, and computation strategies to achieve optimal compute performance [27]. Many different design choices have a substantial and hard-to-predict impact on the performance, energy efficiency, and accuracy of GPU kernels, as the optimal kernel configuration depends on a complex interplay of hardware, device software, and the program itself. This optimization problem leads to an overwhelming number of code variants if done manually, spurring the creation of frameworks that facilitate automatic performance tuning, or *auto-tuning*, to automatically tune GPU kernels and related software [2, 18, 39, 42, 53].

As a consequence of the widespread adoption of GPUs for computation, increased complexity of GPUs, and improvements in auto-tuning, the number of parameters to be tuned is increasing, as well as the range of values per parameter. This leads to a large number of possible combinations (the *Cartesian* product) and is reflected in auto-tuned GPU applications, with the number of valid kernel code variants, or *configurations*, per search space at millions and approaching billions in recent work [24, 25, 28].

In auto-tuning, the collection of all possible combinations of all parameter values tends to contain many configurations that are not valid. For example, because the product of the thread block sizes can not be larger than some hardware limitation, or because some combination of parameter values would lead to incorrect results in the program. To filter out such configurations, constraints are specified on combinations of tunable parameter values. The dramatically increased search space size creates new challenges for auto-tuning frameworks, as with possibly billions of code variants to enumerate in a high-dimensional space, where each constraint must be checked on each variant to resolve the validity, constructing the search space can become a bottleneck at the start of the tuning process. This is observed for several real-world tunable applications, where the search space construction time can take several minutes, or even days (measured per the experimental setup described in Section 4). This is time that could have been spent on tuning, but is instead lost to the overhead of constructing the search space.

'ICS 2025', June 9–11, 2025, Salt Lake City, USA
2025. ACM ISBN 000-0-0000-0000-0/00/00
https://doi.org/XXXXXXX.XXXXXXX

To address this, we examine and evaluate various solver techniques and implementations to decide which approach is best suited for auto-tuning, and greatly optimize the best approach, dramatically improving over the state-of-the-art in search space construction for auto-tuning. In particular, we:

- Introduce the use of Constraint Satisfaction Problem (CSP) solvers for constructing auto-tuning search spaces.
- Optimize an algorithm for the auto-tuning domain.
- Extend built-in constraints for cases common in auto-tuning.
- Optimize built-in constraints using preprocessing and dynamic runtime compilation.
- Parse general user inputs to subsets of built-in constraints.
- Create C-extensions to improve performance.
- Provide various output formats to avoid rearrangement.
- Represent and index the resulting search space for efficient exploration and navigation during tuning.

Our contributions have been implemented in python-constraint [1] and Kernel Tuner [53] [2], both available as open-source packages, enabling straightforward adoption by the auto-tuning community and related fields.

The remainder of this work is structured as follows. Section 2 discusses related work. Section 3 describes the context, selection, optimization, and implementation of search space solvers for constructing and representing auto-tuning search spaces. In Section 4, we evaluate the efficiency and scalability of our optimized CSP-based approach against various other state-of-the-art solutions on a wide variety of synthetic and real-world search spaces, demonstrating the orders-of-magnitude improvements in performance. Section 5 concludes this work.

2 Related Work

In this section, we discuss related works to provide context on the developments and current state of auto-tuning, gradually focusing on search spaces. There are many different automated approaches to improving the performance of software that are collectively referred to as auto-tuning. For a survey of different uses of auto-tuning in high-performance computing, see Balaprakash et al. [6]. As described in this survey, at the heart of every auto-tuning approach is a *search space* of *code variants* that affect code organization, data structures, high-level algorithms, or low-level implementation details, while remaining functionally equivalent to some original implementation [6].

There are several different axes along which auto-tuning approaches can be compared. For example, an auto-tuner can be application-specific, e.g., FFTW [19], ATLAS [57], or *generic*, meaning it can be used to optimize any application. Auto-tuners may use different approaches to score code variants, relying either on some performance model [46] or on empirical measurements using the targeted hardware. Some auto-tuners optimize applications at compile-time, while others aim to optimize application performance at runtime, creating a distinction between auto-tuning during development (*"offline"*) or execution (*"online"*). Some auto-tuning frameworks focus on minimizing the execution time of whole applications [2, 31, 36, 59], whereas others focus on the optimization of individual functions [18, 42, 53]. As a comprehensive overview of the field of auto-tuning research is beyond the scope of this work, we focus our discussion to works that auto-tune individual kernels for GPUs at compile-time.

An important distinction in auto-tuning is how code variants are created. There are generally two approaches, known as *compiler-based* or *software-level* auto-tuning. In compiler-based auto-tuning, the user implements a single version of the code and a compiler is responsible for generating different, functionally equivalent, code variants that exhibit performance differences when executed. Software-level auto-tuning on the other hand generally leaves the responsibility of specifying different code variants with the programmer, using for example metaprogramming approaches, such as code generators, macros, and templates. We discuss related work from both approaches in Section 2.1 and Section 2.2 respectively.

2.1 Compiler-based auto-tuning

Several compiler-based auto-tuning approaches for GPU kernels have been presented in the past two decades.

Orio [22] is a framework that transforms annotated kernels to target languages, incorporating an auto-tuning phase to select optimal compiler optimization parameters. BOAST [56] is a metaprogramming framework built on top of Orio that targets high-performance computing by simplifying application optimization through a high-level interface language. BOAST selects compiler optimizations based on user-specified kernels and options. The Adaptive Sampling Kit (ASK) [15] employs active learning to efficiently sample large search spaces, providing various methods for sampling. Coding Ants [40] uses ant colony optimization for auto-tuning. The framework is built as an extension of the Polyhedral Parallel Code Generator (PPCG) [55] for generating CUDA code from C code. Ashouri et al. [3] wrote a comprehensive survey on machine-learning methods for compiler-based auto-tuning.

Compiler-based auto-tuning generates and tunes code variants as part of the compilation process. As there is no direct user input on the code variants and the order in which optimization passes are applied matters, the search space construction process of compiler-based auto-tuning is substantially different from the software-level auto-tuners we will now focus on.

2.2 Software-level auto-tuning

Over the last decade, several software-level auto-tuning frameworks for GPU kernels have been introduced, with various search space construction techniques and a wide variety in search spaces of the benchmarks evaluated on.

AUMA [16] utilizes neural network models for auto-tuning the performance of OpenCL kernels on Intel CPUs, as well as GPUs from Nvidia and AMD, intending to enable performance portability across different hardware architectures. It is evaluated using three benchmark kernels: *convolution*, *raycasting*, and *stereo*. Respectively, their search spaces consist of 131072, 655360, and 2359296 configurations.

Dao and Lee [13] present an auto-tuner for tuning the work-group size of OpenCL kernels with an extensive evaluation of 54 OpenCL kernels on 4 different GPUs. Given that they only tune the

[1] https://github.com/python-constraint/python-constraint
[2] https://github.com/KernelTuner/kernel_tuner

workgroup sizes in at most two dimensions, the resulting search spaces are relatively small.

CLTune [39] is another open-source framework for auto-tuning for OpenCL kernels. It is the first framework to employ parameter insertion using preprocessor macros and supports validation by a reference kernel. CLTune implements several optimization algorithms to accelerate the auto-tuning process, including simulated annealing and particle swarm optimization. The CLTune framework is evaluated on two kernels [39], a 2D convolution and matrix multiplication. The convolution kernel has a Cartesian product size of 12288 parameter combinations, of which 3424 are valid configurations (28%). The matrix multiplication kernel has a Cartesian size of 2654208, of which 995328 are valid configurations (37.5%). Source code inspection reveals CLTune uses brute-force search space construction by recursively iterating over all permutations of the user-defined parameters.

OpenTuner [2], an open-source framework introduced in 2014, supports multiple languages but does not specifically target GPUs and requires manual host code implementation for each kernel. OpenTuner optimizes the search space using different techniques simultaneously. Source code inspection confirmed OpenTuner uses brute-force search space construction by applying constraints in mapping over all permutations of the user-defined parameters. OpenTuner is evaluated on applications such as High-Performance Linpack, Halide, and PetaBricks. While the true number of configurations is not specified, the Cartesian sizes listed range from $10^{6.5}$ to 10^{6328}.

Kernel Tuning Toolkit (KTT) [18] is an open-source auto-tuning framework that supports both compile-time and *online* auto-tuning, where code variants are tested while the application is running in production. KTT supports auto-tuning of Vulkan, CUDA, and OpenCL kernels. Filipovič et al. [17] extended KTT with a machine-learning approach that incorporates performance counter data collected by a profiler to accelerate the *online* auto-tuning process. KTT constructs the search space by using a tree-based resolution, which can be resolved in parallel when creating independent subspaces (called groups) that do not share constraints, which are used when tuning composite kernels and must be labeled as separate groups by the user. By default a single group is used, resulting in sequential recursive resolution. In the evaluation section [18], three kernels are used: a 2D and 3D Coulomb summation, and a reduction kernel. Both Coulomb summations use the same parameters, resulting in a Cartesian size of 16128 and 14784 valid configurations (91.7%), and the reduction kernel has a Cartesian size of 10080 parameter combinations and 2640 valid configurations (26.2%). It must be noted that the parameters used in the publication differ from those in the source code [3]; the source definition with widest parameter values is used here as the constraints are not specified in the publication.

The most closely related work is on Auto-Tuning Framework (ATF) [42, 44], as this is the state-of-the-art in search space construction. ATF can do efficient search space construction for large optimization spaces with interdependent parameters using chain-of-trees [44]. ATF supports auto-tuning of GPU kernels through a domain-specific language and is available as an open-source library in both C++ and Python implementations (referred to as *ATF* and *pyATF* respectively). ATF [42] evaluates on the *XgemmDirect* kernel of [38] with four different input sizes, resulting in four search spaces. The parameters used in the publication have substantially more values than those in the source code at the reported version 0.11.0, but do not report enough detail to determine the search space sizes, which we therefore omit.

The Bayesian Compiler Optimization framework (BaCO) is an auto-tuning framework for GPU, CPU, and FPGA applications. It supports a wide variety of parameter types and introduces the concept of "hidden" constraints to auto-tuning, which are detected during optimization. For search space construction, it uses the chain-of-trees approach introduced in [42] and improves upon the sampling approaches introduced by ATF. BaCO is evaluated on a total of fifteen search spaces from three real-world applications: *TACO*, *RISE & ELEVATE*, and *HPVM2FPGA*. The average Cartesian size for these search spaces is 208006300000, 16821461143, and 285085, respectively. The average number of configurations and the percentage of the Cartesian size are 1961000 (21%), 23471314 (9.7%), and 285085 (100%), respectively.

Kernel Tuner [53], an open-source Python-based software auto-tuner for GPU applications, supports mainstream GPU programming languages such as OpenCL, HIP [33], and CUDA, as well as OpenMP and OpenACC in both C and Fortran. Kernel Tuner is capable of optimizing GPU kernels for energy efficiency, accuracy, and other custom objectives besides minimizing kernel execution time [49], and provides a wide variety of optimization algorithms [48, 58]. Kernel Launcher [24] is a library that builds on top of Kernel Tuner to facilitate the integration of tuned kernels in C++ applications. Source code inspection reveals that Kernel Tuner uses brute-force search space construction. The search spaces used in these publications illustrate the general trend of larger search spaces; where in 2019 the number of valid configurations is in the order of thousands, in the tens of thousands in 2022a, 2022b, 2021, and has gotten to millions in 2023.

3 Design & Implementation

This section discusses the design and implementation details of our novel method for efficiently constructing large search spaces for GPU auto-tuning. We first provide the background and possible approaches to solutions within the context of auto-tuning frameworks in Section 3.1. Following this, Section 3.2 examines various constraint-solving techniques to find a basic approach best suited within the problem context. Next, the selected basic approach is optimized in Section 3.3, resulting in a substantially improved search space construction process. Finally the representation and application of the resulting search space in auto-tuning frameworks is detailed in Section 3.4.

3.1 Problem and Solution Context

To understand the background of the search space construction problem and explore various approaches to solutions in this subsection in an auto-tuning context, we focus on a specific auto-tuning framework. In Section 2, we discussed three open-source auto-tuning frameworks that use brute-force search space construction, which suffices for auto-tuning problems that could be manually

[3] https://github.com/HiPerCoRe/KTT/blob/master/Examples/CoulombSum2d/CoulombSum2d.cpp, https://github.com/HiPerCoRe/KTT/blob/master/Examples/Reduction/Reduction.cpp

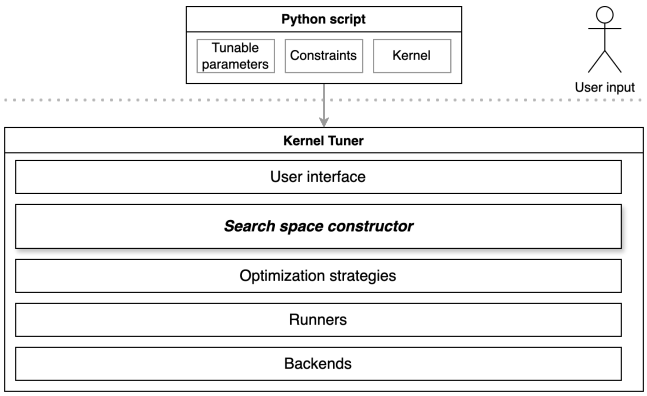

Figure 1: Abstract Kernel Tuner software architecture.

explored, but can take a substantial amount of time with the large search spaces currently encountered. As in this work we aim to provide a generic efficient solution for the construction of large search spaces for auto-tuning, we implement this in the most actively developed of these three open-source auto-tuning frameworks, Kernel Tuner, to demonstrate our novel method.

Kernel Tuner is an external framework for developers to benchmark and optimize GPU kernels in isolation, which can be used with applications in any host programming language. The abstract software architecture of Kernel Tuner is shown in Figure 1. Users of Kernel Tuner create a small Python script that points to the kernel and describes both the tunable parameters and any constraints (referred to as *restrictions* by Kernel Tuner) to filter out invalid combinations of parameter values. There are also various optional settings that users can specify, such as derived metrics to be computed, the optimization objective to use, which optimization algorithm to use, and hyperparameters to the optimization algorithm of choice.

Search space construction is the first step towards auto-tuning any function or application, preceding the search through the myriad of configurations or code variants. This is apparent from Figure 1, where the modular structure of Kernel Tuner is shown, with processing flowing generally from the top (user input), through the search space constructor, to the strategies (optimization algorithms) that determine the next configuration to evaluate, to the runners that prepare the evaluation, and the backends that compile and execute the kernel. After execution, the process flows back up in the diagram, either to the strategies to determine the next configuration based on the new result and repeat the process, or by reporting the best configuration found to the user.

In this work, we will focus on the *search space constructor* part of Figure 1, as with possibly billions of code variants to enumerate in a high-dimensional space, where user-defined constraints cut out parts of the space that are considered invalid, constructing the search space can become a bottleneck at the start of the tuning process. In GPU auto-tuning, the Cartesian product of the tunable parameters, that is, the collection of all possible combinations of all parameter values, tends to contain many configurations that are not valid. For example, because the product of the thread block sizes can not be larger than some hardware limitation, or because some combination of parameter values would lead to incorrect results in the kernel. To filter out such configurations, the user can specify constraints on certain combinations of tunable parameter values.

The most straight-forward solution is brute force: generate all the combinations of parameter values and filter out combinations based on the user-specified constraints, leaving only valid configurations. This is reasonable for small search spaces but becomes increasingly time-consuming as the search space size and number of constraints increase. Hence, to improve performance on large search spaces, the search space must be constructed more efficiently.

One approach could be to resolve the constraints dynamically, either by only checking the constraints of combinations suggested by the optimization strategy before executing kernel configurations, or by resolving the search space in parallel while executing. However, these dynamic approaches pose problems. If we take the approach of only checking combinations suggested by the strategy, in tuning problems with sparse search spaces (where the vast majority of combinations of parameter values are not valid configurations), a substantial amount of time would be spent on finding any valid configuration as for each combination the optimization strategy needs to be consulted and the combination checked against the constraints. This problem is exacerbated by the fact that during the tuning process, the number of valid non-executed configurations decreases. To illustrate this, in the case of a random search on a search space of 100 valid configurations in a search space where 99% is invalid (10000 combinations) and configurations are taken from the search space once executed, the expected number of attempts required to find a valid first configuration is 100. However, the expected number of attempts required to find the fiftyfirst valid configuration is 199, and the sum of expected attempts required from the first to the fiftyfirst valid configuration is over 7000 - over two-thirds of the total Cartesian size of the space. While this can be mitigated by keeping track of all attempted combinations, this by itself produces a large memory footprint. Moreover, dynamic resolution can skew initial sampling and strategies, as the knowledge of the constraints is not fully incorporated in the search.

An example of a dynamic approach to search space resolution in auto-tuning is the chain-of-tree approach used by ATF [42] as described in Section 2.2. While this is efficient for search spaces where the vast majority of possible combinations are invalid, individual constraints may only use a small subset of all the parameters to achieve this efficiency. In addition, search space characteristics such as the true parameter bounds, which can help optimization algorithms navigate the space more efficiently and enable fairly distributed sampling methods such as Latin Hypercube Sampling [58], are not resolved with the dynamic approach. Furthermore, randomized sampling is inherently biased to the sparser parts of the tree, although this has been addressed by BaCO [25]. Moreover, selecting neighbors of configurations as extensively used by various optimization algorithms in Kernel Tuner is potentially expensive.

Instead, we aim to fully resolve the search space before starting the tuning process, with a minimal impact on the total execution time, to incorporate the full information of the search space in the initial sampling and search strategies.

3.2 Using Constraint Solvers in Auto-tuning

In general, this type of problem, where parameter values and constraints are resolved to valid combinations, can be encoded as a Boolean Satisfiability Problem (SAT) [10], Satisfiability Modulo Theories (SMT) [8], or Constraint-Satisfaction Problem (CSP) [12]. At the time of writing, several frameworks are readily available as Python packages that have implemented solvers for these types of problems: *CSP-solver* [35], *Google ORTOOLS* [32], *PicoSAT* [47], *CPMpy* [20], *PyChoco* [41], *SATisPy* [34], *PySMT* [51], *python-constraint* [37]. Nevertheless, not all of these frameworks can be applied to auto-tuning; they have their limitations in how expressive and efficient they are. In general, the SAT, SMT, and CSP problem types mentioned serve different purposes, owing to their different origins. SAT solvers are generally most efficient, at the cost of expressivity, as they are highly optimized for propositional logic problems. SMT solvers allow for non-integer finite domains such as floating-point numbers, strings, and lists, and as such are more expressive, although not as optimized as SAT solvers. CSP solvers offer high-level abstractions suitable for modeling complex constraints, providing special types such as "all-different" [50], which are otherwise difficult to efficiently express, making them ideal for complex combinatorial constraints.

In auto-tuning, the constraint problem consists of a set of named parameters, each parameter having a finite list of possible values, usually numeric but also strings or other types. As such, SMT and CSP solvers are the best fit in this case, leaving *CSP-solver*, *PyChoco*, *PySMT*, and *python-constraint*. In addition, there are practical considerations when it comes to choosing a solver; *PyChoco* is in beta at the time of writing and requires building from source, and *PySMT* requires manual steps to install actual solvers, making it cumbersome to deploy as a dependency within a framework. Various solvers, such as *CSP-solver* and the Microsoft Z3 solver in *PySMT*, aim to find any solution, rather than all solutions, as required in the case of auto-tuning. To obtain all solutions, such solvers must iteratively find a solution, add this solution as an additional constraint, and look for the next solution until there are no solutions left [11]. If there are many solutions, as is commonly the case with auto-tuning problems, this can have a substantial impact on performance.

Hence, we focus on *python-constraint*, as this is a CSP-based Python package with built-in support to find all solutions. Initially developed by Gustavo Niemeyer and afterwards maintained by Sébastien Celles, the *python-constraint* package was first released in 2005. The more than 22000 weekly downloads[4] at the time of writing indicate that the package has a substantial user base.

3.3 Implementation of Optimizations

Based on Section 3.2, we use the *python-constraint* package as a basis for our implementation. To obtain the level of performance required to construct auto-tuning search spaces efficiently, we implement several key improvements in various areas: algorithmically (Section 3.3.1), by extending and improving constraints (Section 3.3.2), by introducing parsing (Section 3.3.3), by providing tailored output formats (Section 3.3.4), and finally engineering (Section 3.3.5).

[4] https://pypistats.org/packages/python-constraint

3.3.1 Algorithm. We select and optimize a backtracking solver optimized for finding all solutions rather than any solution. This algorithm maintains a dictionary of variable assignments and uses a stack-based approach to implement iterative backtracking, avoiding recursive function calls. Variables are selected dynamically using a combination of the Minimum Remaining Values (MRV) and Degree heuristics, prioritizing those with fewer remaining values and higher connectivity. For each selected variable, domain values are checked against the constraints. If a constraint is violated, the algorithm backtracks by restoring previous states from a queue until all possibilities are explored. We have optimized this algorithm by sorting the variables on the number of internal constraints, making it faster to find unassigned variables, and by reducing the number of sorts required. By systematically iterating through variable assignments and leveraging heuristics, the algorithm efficiently constructs all valid solutions while minimizing unnecessary exploration.

3.3.2 Constraints. We expand and improve built-in specific constraints to optimize operations that are commonly used. Commonly used operations allow for increased efficiency over generic functions by applying knowledge of the operation. For example, given a constraint where $p \cdot q > 0$, we know to ignore all cases where $(p \leq 0) \veebar (q \leq 0)$. We added *MaxProduct* and *MinProduct* constraints as they are commonly used in auto-tuning constraints (e.g. a maximum product of block sizes due to hardware limits). We also rewrote and added preprocessing steps to the *Function*, *MaxSum* and *MaxProduct* constraints. The *Function* constraint is particularly interesting for optimization, as it is both a computationally expensive constraint due to its generality and a common occurrence as it is used wherever the more specific constraints do not apply. We have optimized this by employing function rewriting and dynamic runtime compilation, as the one-off expense of compilation to bytecode is offset by the potentially many times a *Function* constraint is executed.

3.3.3 Parser. We introduce a parser for constraints written in string format, which has three important benefits: to apply the more efficient specific constraints instead of generic functions where possible, to break down constraints into the smallest subsets of variables, thereby aiding the constraints solver, and to provide all of this without requiring users to write their constraints in a complex format that requires understanding how the solvers work.

To address the latter benefit first, most solvers and tuners require specific function calls or a form of domain-specific language when defining the constraints (as will be encountered in Section 4.1). However, as opposed to the users of CSP-solvers, auto-tuning users are generally not aware of the search space construction process and the specific constraints available that result in efficient resolution of the search space. Instead, we provide users the option to write their constraints in Python-evaluable string format, which is then automatically optimized by parsing. This usage of Python-evaluable strings has various benefits, as they are both familiar to the user as Python is already the interface language, and rewritable by our parser, allowing the application of specific constraints instead of generic functions and the decomposition of constraints into subsets.

In particular, the automatic reduction of constraints can be important in scalability and efficiency in practice, as users unfamiliar with the intrinsics of constraint solving, such as the users of

auto-tuning frameworks, might write sub-optimal constraints in practice. For example, consider the constraint $3 \leq X \cdot Y < 9 \leq Z$, where X, Y, and Z are tunable parameters with numerical values. Constraints can not be evaluated until values for the involved parameters are at least partially resolved, resulting in subpar performance in the case of compound statements like the given example, as it depends on three parameters as-is. This can be improved by automatically breaking down the constraint into multiple constraints with fewer involved variables where possible. For the given example this can be $[3 \leq X \cdot Y, X \cdot Y < 9, 9 \leq Z]$, which allows partially resolved values for either X, Y, or Z to be enough to discard configurations not meeting the constraint earlier in the construction process. In addition, this automatic reduction enables the application of specific constraints, as is the case with the example, which can be represented with specific constraints as `[(MinProd(3), [x,y]), (MaxProd(9-1), [x,y]), (MinProd(9), [z])]`. As discussed in Section 3.3.2, application of specific constraints can preemptively exclude values through preprocessing, resulting in an even more efficient construction.

3.3.4 Output Formats. We implement various output formats to avoid expensive rearrangements to different formats. Expensive rearrangement of the structure in which solutions are output by the solver is mitigated by providing output formats that are close to the internal representation, further described in Section 3.4.

3.3.5 Employing C-extensions. In general, C and similar languages outperform Python in terms of execution speed [1, 60]. To attain this level of performance without losing the flexibility and user-friendliness of Python [4], we employ C-extensions. We transpile the codebase from Python to C-code using Cython [9], which is then compiled into Python-importable C-extensions. We added type hints where possible to aid in compilation. Binaries are precompiled for Linux, macOS, and Windows on the supported Python versions (3.9 through 3.13 as of this writing).

While the improvements detailed in Sections 3.3.1 to 3.3.5 are specifically intended to obtain the level of performance required to construct auto-tuning search spaces efficiently, they are generally applicable to any CSP problem. Our optimizations have been approved in the main branch of *python-constraint*, benefitting all users of the package and the community.

3.4 Search space Representation

With the efficient construction of search spaces implemented in *python-constraint*, we consider how this is represented and applied in auto-tuning frameworks in practice to achieve a comprehensive approach.

As per Section 3.1, after the search space construction, the optimization algorithms use the information obtained in the construction step to select configurations. Instances of this are obtaining the true bounds of the search space to use balanced initial sampling methods or the selection of valid neighbors that have not been evaluated yet. As these type of operations are commonly used in auto-tuning, it can be useful to provide an abstract representation of the search space that implements these operations, providing various views and mappings on the configurations in the search space.

We have implemented this in Kernel Tuner as the *SearchSpace* class, which takes the tunable parameters and constraints based on the the user specification, constructs the search space using *python-constraint*, and provides various representations and operations on the resulting search space. The *SearchSpace* class has multiple internal representations for varying purposes, such as hash- and index-based for efficient lookups. Externally it provides a single interface for all search space-related operations, which in contrast to the initial situation where strategies would implement these operations individually, enables reuse in a modular architecture. For example, the mutation step in the *genetic algorithms* optimization strategy requires selecting only valid neighbors within a certain Hamming distance. This, along with other neighbor selection algorithms, is implemented in the *SearchSpace* class and can be indexed before running the algorithm, improving overall performance.

While we focus on an optimized method for search space construction in auto-tuning, resulting in a generically applicable search space constructor, considering the wider context results in a comprehensive method for search space representations and operations.

4 Evaluation

In this section, we evaluate the advancements presented in Section 3 to determine their performance and scalability impact using a case study with various applications. First, we discuss how we compare against the current state-of-the-art solvers in Section 4.1. Following this, we evaluate the solvers on a collection of synthetically generated search spaces to assess scalability differences between solvers under various search space characteristics in Section 4.2. Finally, we evaluate the solvers on a variety of real-world applications to indicate actual performance in Section 4.3.

The evaluations in this work are performed on the sixth generation DAS VU-cluster [5] using an NVIDIA A4000 GPU node. The GPU is paired with a 24-core AMD EPYC-2 7402P CPU, 128 GB of memory, and running Rocky Linux 4.18. While none of the tested solvers use the GPU, we use a GPU node to obtain an environment as similar as possible to real-world GPU auto-tuning. For all tests performed, the results of each solver were validated against a brute-forced solution of the search space.

4.1 Comparison against state-of-the-art

To provide additional reference on the performance in this evaluation, we compare the results to the state-of-the-art in auto-tuning search space construction: Auto-Tuning Framework (ATF) [43], which specifically focuses on large optimization spaces with inter-dependent parameters. ATF has two independent implementations, in C++ and Python, both of which we use in this evaluation to compare our method to. The C++ version available as of August 2024 with Python bindings is used and denoted as *ATF* in the results. The Python version, called *pyATF*, is used at version 0.0.9, the latest version at the time of writing.

For the majority of the discussed solvers, the notation of tunable parameters and constraints is largely separated (e.g. a user first defines the tunable parameters and values, then defines the constraints to apply). However, both implementations of ATF have a

notation that combines the definition of tunable parameters, values, and constraints into one statement. As a result of this, constraints can only reference tunable parameters that have been previously defined. Due to the large number of search spaces used in this evaluation, it is not feasible to write each of these search space definition files by hand for both ATF implementations, and we have instead written parsers that define the ATF search space files from an abstract definition of the search spaces. These parsers take the aforementioned parameter-constraint order relation into account and convert to built-in ATF types such as intervals where applicable to provide search space definitions that are as closely possible to what is expected by the authors. To reflect the user experience as accurately as possible, the search space file compilation time is included in the total construction time. The C++ version of ATF and search space files is compiled with GCC 9.4.0 using the optimization commands recommended by the ATF documentation.

In addition, we compare against PySMT at version 0.9.6 using the Microsoft Z3 solver to evaluate differences in scalability for solvers without support for resolving all solutions, as described in Section 3.2. The Z3 theorem prover is developed by Microsoft for software verification and analysis [14], and is the winner of the 3rd Annual Satisfiability Modulo Theories Competition (SMT-COMP) [7]. Similarly to ATF, we have written a parser to use PySMT-specific operations where applicable.

We have published the implementation of this evaluation in a repository for further reference.

4.2 Synthetic Tests

To understand how search space characteristics influence the construction time of the evaluated solvers, we use synthetic tests. We have generated a set of search spaces with a varying number of dimensions (between 2 and 5), target Cartesian sizes (with $\{1 \times 10^4, 2 \times 10^4, 5 \times 10^4, 1 \times 10^5, 2 \times 10^5, 5 \times 10^5, 1 \times 10^6\}$), and number of constraints (between 1 and 6). While these arbitrary parameters result in search spaces that are not as large and do not have as many tunable parameters as the real-world search spaces evaluated on in Section 4.3, the goal of these in total 112 synthetic search spaces is to gain insight into which of these factors has the greatest effect on performance, and which solution provides good scalability across the variations in these factors.

Given a Cartesian size, a number of dimensions, and a number of constraints, we want to generate a synthetic search space. To prevent an unfair advantage to solvers optimized for a limited number of dominant dimensions, this number of values per dimension v is kept approximately uniform. This is done by first determining the number of values per dimension as $v = s^{\frac{1}{d}}$, where s is the desired Cartesian size and d is the desired number of dimensions. For each of the dimensions, a linear space with v number of elements is instantiated. Given a non-integer value of v, this is rounded to an integer for all but the last dimension, where v is rounded contradictory (e.g. $5.8 \rightarrow 5, 5.2 \rightarrow 6$) to be closer to the desired Cartesian size. A list of constraints involving a variety of operations is generated for each combination of dimensions, which are randomly chosen up to the desired number of constraints.

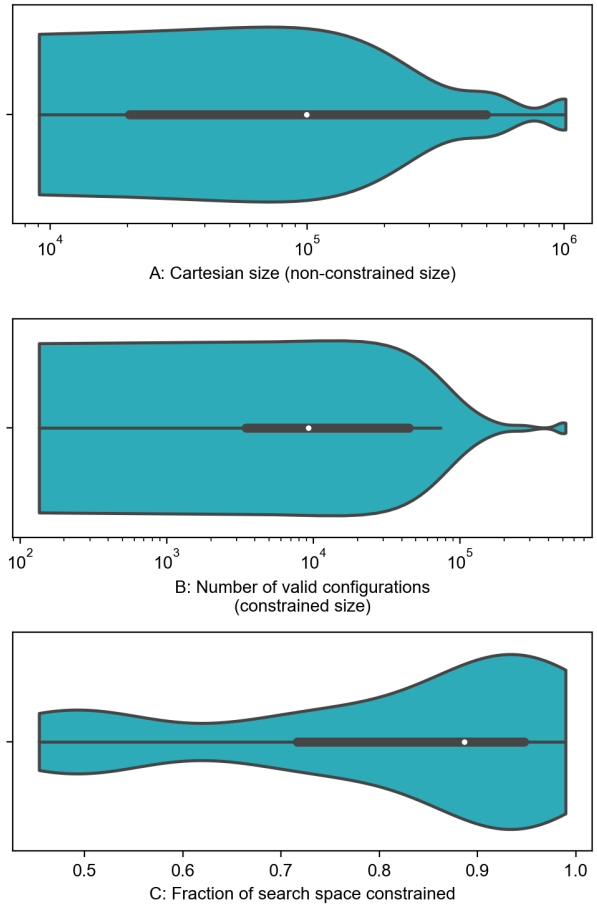

Figure 2: Characteristics of the 112 synthetic search spaces

Figure 2 shows the dispersion of the resulting 112 search spaces in violin plots for three characteristics: the Cartesian size, the number of valid configurations, and the fraction of the search space constrained (the number of valid configurations relative to the total Cartesian size). As the application of the constraints does not affect the already uniformly distributed number of parameters, this characteristic is not included in the figure. These violin plots should be interpreted by examining both the y-axis width and central tendencies of the distributions. The wider sections indicate a higher density of search spaces with the associated x-axis values, while the white dot represents the median, and the thick bar around it marks the interquartile range. Figure 2A shows the actual Cartesian size, representing the total number of possible configurations before constraints are applied, revealed to be in line with the set of target values used. Figure 2B depicts the number of valid configurations remaining after constraints are enforced, which follows a distribution similar to the Cartesian size of Figure 2A, yet covering a wider range on the lower end, also demonstrating the general effect of constraints on the difference between the Cartesian size and actual number of configurations. Finally, Figure 2C displays the fraction of sparsity of the search space, i.e. the fraction of non-valid configurations relative to the Cartesian size. Though the fraction

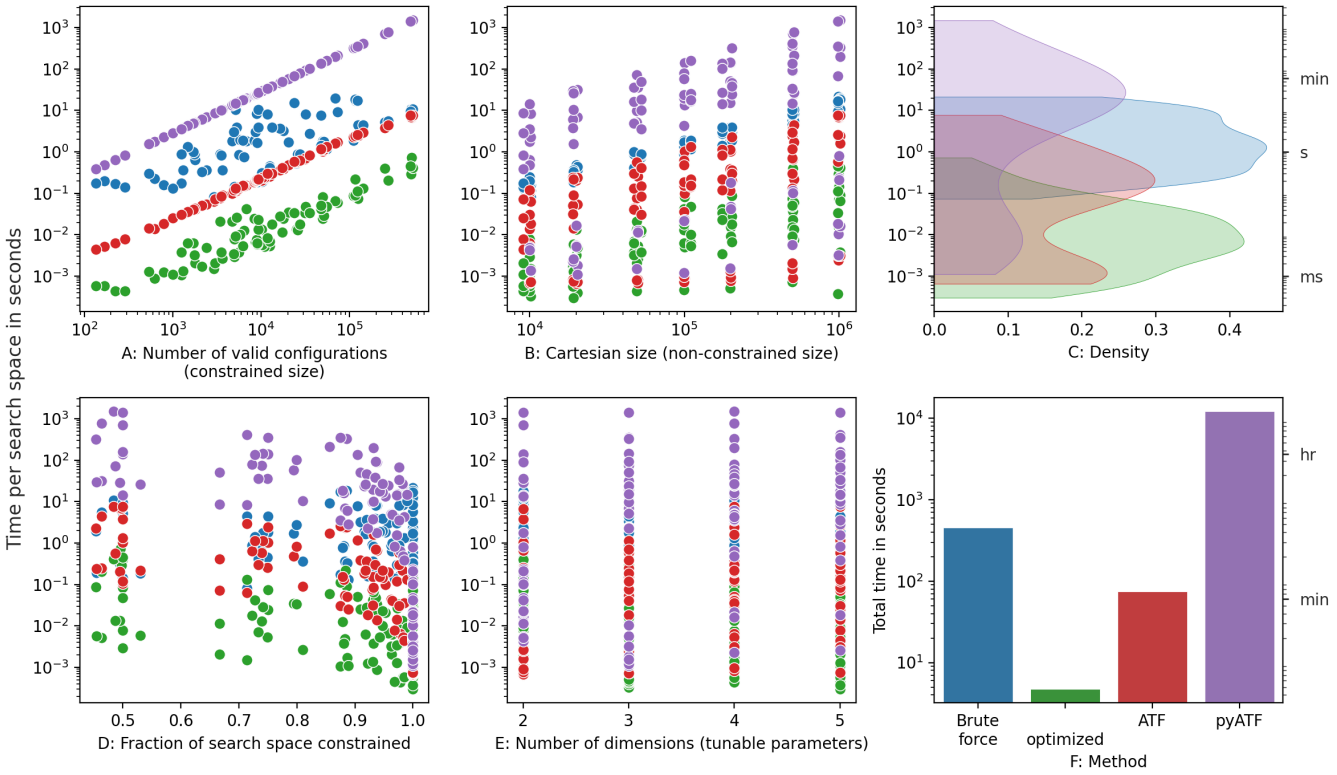

Figure 3: Search space construction performance on synthetic tests. Lower times are better. Colors correspond to Figure 3F barplot methods.

of constrained configurations is skewed toward higher values, indicating a propensity towards sparsity, a wide range of variations in sparsity is present.

The performance on the synthetic search spaces is displayed in various plots in Figure 3, where the colors used correspond to the colors of the methods in the Figure 3F barplot.

It is noteworthy how in Figure 3A there appears to be an approximately linear correlation between the time required to construct a search space and the number of valid configurations, which is not as evident in the other characteristics in Figure 3B, Figure 3D, and Figure 3E. Our *optimized* solver consistently achieves the lowest execution times, with several orders of magnitude better performance compared to the *brute force*, *ATF*, and *pyATF* solvers. The *pyATF* solver exhibits the highest execution times, being generally outperformed even by brute-force solving, an interesting result which we will examine using the other characteristics.

Figure 3B examines the effect of the Cartesian size on execution time, where a general trend of increasing computation time with larger Cartesian sizes can be seen. Interestingly, the *ATF* and *pyATF* solvers in some cases do not conform to this trend, creating a large variance in their performances.

The reason for this appears to be found in Figure 3D, which shows the relationship between the fraction of the sparsity of a search space and execution time. Both *ATF* and *pyATF* appear to be severely optimized for very sparse search spaces, in contrast to the other methods.

Figure 3C presents the execution time distributions of the solvers as a continuous probability density curve using a kernel density estimate (KDE). Particularly noteworthy is the bimodality demonstrated by *ATF* and *pyATF*, which appears to be caused by the sensitivity to search space sparsity seen in Figure 3D.

Observing Figure 3E, it appears that there is no strong correlation between solver performance and the number of tunable parameters.

Figure 3F summarizes the overall performance of each solver in a bar chart. It is remarkable that pyATF takes considerably longer than the brute-force method on these search spaces, which might be due to how optimized the ATF approach is to highly sparse search spaces. Our optimized method achieves a 96x speedup over the brute-force method (4.75 seconds versus 455.3 seconds), a 16x speedup over ATF, and a 2547x speedup over pyATF.

As described in Section 3.2, a traditional solver without support for finding all solutions requires adding the previous solution as a constraint and iterating over the solutions until all solutions have been found. To demonstrate the lack of scalability of such a solver, Figure 4 compares PySMT using the Microsoft Z3 solver to the brute-force method and the optimized solver. To make executing this experiment feasible, we had to reduce the size of the generated synthetic search spaces by one order of magnitude in this instance. As seen in Figure 4, PySMT performs poorly relative to

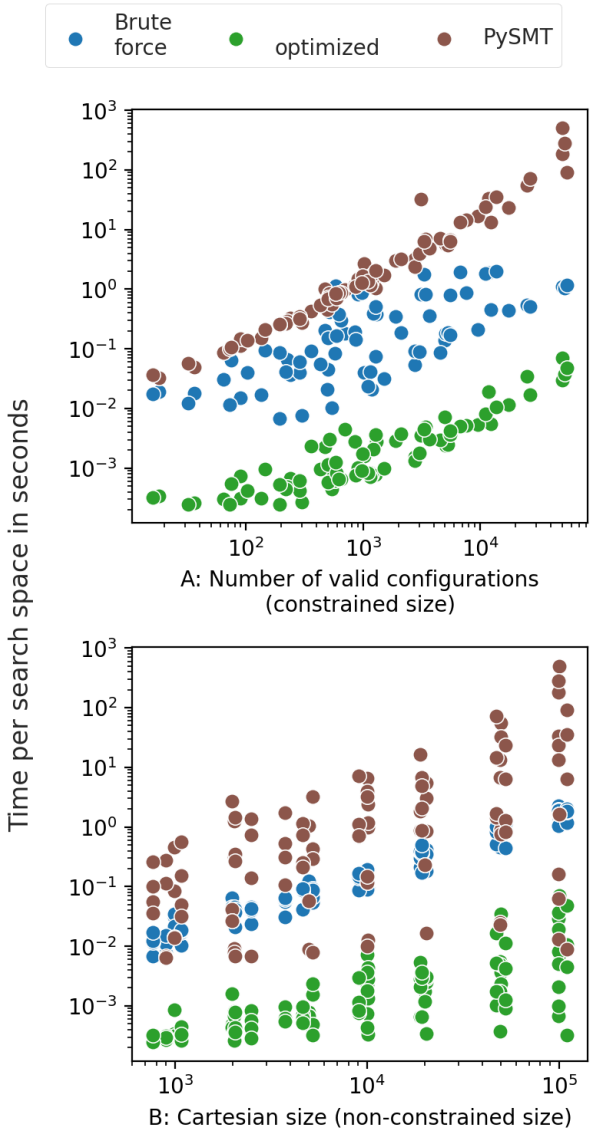

Figure 4: Search space construction performance of PySMT on synthetic tests (reduced search spaces size).

both brute force and our optimized method. As expected, this difference increases as the number of valid configurations increases, demonstrating the infeasibility of this approach when many valid configurations are present. Despite the reduced search space sizes, PySMT with the Z3 solver still takes nearly a thousand seconds on the largest search spaces, whereas the brute-force solver takes about ten seconds. Our optimized solver takes about as long to solve the largest search spaces as PySMT with the Z3 solver takes to solve the smallest search spaces. PySMT with the Z3 solver will not be included in the remainder of the evaluation as it is infeasible to evaluate the large search spaces of the real-world applications.

4.3 Real-world Applications

To evaluate solver performance on the search spaces of real-world applications, we select the three largest search spaces in the Benchmark suite for Auto-Tuners (BAT) [28]. These are *Dedispersion*, *Hotspot*, and *ExpDist*. In addition, we use the relatively large search spaces of the commonly used General Matrix Multiplication kernel *(GEMM)* [38] and *MicroHH* computational fluid dynamics kernel [52]. To provide a fair comparison to ATF, the Probabilistic Record Linkage (*PRL*) kernel used by the ATF paper [45] is used as well, resulting in three additional search spaces for a total of eight real-world search spaces. The characteristics of the real-world search spaces are displayed in Table 1, where the rightmost column denotes the average number of constraint evaluations that are required to brute-force solve a search space. For each combination in the Cartesian product, all constraints need to be evaluated until the combination is rejected or all constraints have been evaluated. Hence the average number of constraint evaluations can be calculated by taking the average of the best case (the first constraint rejects the combination) and worst case (the last constraint rejects the combination), and adding all valid combinations which are never rejected. Given a search space S, let S_i be the set of non-valid combinations, S_v the set of valid combinations, and S_c the set of constraints, the average number of constraint evaluations can be calculated as $\frac{|S_i|+|S_i|\cdot|S_c|}{2} + |S_v|$. Descriptions of each of the kernels and their search spaces are given in Sections 4.3.1 to 4.3.6, before the results are discussed in Section 4.3.7.

4.3.1 Dedispersion. The Dedispersion kernel presented in [28] is designed to compensate for the time delay experienced by radio waves as they propagate through space. This delay occurs due to the frequency-dependent dispersion of the signal. By applying a specific dispersion measure (DM) and reversing the dispersion effect, the kernel reconstructs the original signal. During the iteration over frequency channels, threads process multiple time samples and dispersion measures in parallel. Comparing the Dedispersion search space to the other real-world search spaces tested on in Table 1, the resulting search space is the smallest in Cartesian size, but as it has the highest percentage of valid configurations at nearly 50 %, it is not the smallest in number of valid configurations.

4.3.2 ExpDist. The ExpDist kernel described in [28] is utilized in a localization microscopy application that performs template-free particle fusion by integrating multiple observations into a single super-resolution reconstruction [26]. During the registration process, the ExpDist kernel is repeatedly invoked to evaluate the alignment of two particles. The algorithm exhibits quadratic complexity with respect to the number of localizations per particle, making it highly computationally intensive. The resulting search space is the second-most sparse of the real-world search spaces in Table 1.

4.3.3 Hotspot. The Hotspot kernel in [28] is part of a thermal simulation application used for estimating the temperature of a processor by considering its architecture and simulating power currents. Through an iterative process, the kernel solves a set of differential equations. The inputs to the kernel consist of power and initial temperature values, while the output is a grid displaying average temperature values across the entire chip. It is interesting to note that the Hotspot search space is the largest in number of

Name	Cartesian size	Constraint size	Number of parameters (dimensions)	Number of constraints	Avg. unique parameters per constraints	Range of number of values per parameter	% of configurations in Cartesian size	Avg. number of constraint evaluations required
Dedispersion	22272	11130	8	3	2	1 - 29	49.973	33414
ExpDist	9732096	294000	10	4	2	1 - 11	3.021	23889240
Hotspot	22200000	349853	11	5	3.8	1 - 37	1.576	65900294
GEMM	663552	116928	17	8	3.25	1 - 4	17.622	2576736
MicroHH	1166400	138600	13	8	2.375	1 - 10	11.883	4763700
ATF PRL 2x2	36864	1200	20	14	2.429	1 - 3	3.255	268680
ATF PRL 4x4	9437184	10800	20	14	2.429	1 - 4	0.114	70708680
ATF PRL 8x8	2415919104	48720	20	14	2.429	1 - 8	0.002	18119076600
Mean	*307322534*	*121403*	*14.875*	*8.75*	*2.589*	*1 - 13.25*	*10.93*	*2285902168*

Table 1: Overview of the basic characteristics of the real-world search spaces and the mean values for each of the columns.

valid configurations, second-largest in Cartesian size, and has the highest number of values for a single parameter.

4.3.4 GEMM. Generalized dense matrix-matrix multiplication is a fundamental operation in the BLAS linear algebra library and is widely used across various application domains. GEMM is known for its high performance on GPU hardware and has frequently served as a benchmark in studies of GPU code optimization [30, 39, 46]. In this evaluation, we utilize the GEMM kernel from CLBlast [38], a tunable OpenCL BLAS library. GEMM is implemented as the multiplication of two matrices (A and B); $C = \alpha A \cdot B + \beta C$, where α and β are constants, and C is the output matrix. The dimensions of all three matrices are set to 2048×2048, resulting in a relatively dense search space.

4.3.5 MicroHH. The computational fluid dynamics kernel of [52] is used for weather and climate modeling, specifically for the simulation of turbulent flows in the atmospheric boundary layer. In this case, we use the search space resulting from the auto-tunable GPU implementation of the *advec_u* kernel with extended parameter values as specified in the source of [24]. Looking at Table 1, it is notable that the MicroHH search space is the closest to the mean values of all search spaces in the number of parameters, number of constraints, and percentage of configurations. It is also second-closest in constraint size and number of values per parameter, making it perhaps the most average search space in our set of tests.

4.3.6 ATF PRL. The Probabilistic Record Linkage (PRL) kernel used in [45] is a parallelized implementation of an algorithm that is commonly used in data mining to identify data records referring to the same real-world entity. In this kernel, the input sizes determine the size of the search space. As shown in Table 1, the brute-force resolution of this search space with input sizes $8x8$ requires 1.8119×10^{10} constraint evaluations on average, which took ~27 hours to execute. As an input size of $16x16$ would require 4.639×10^{12} constraint evaluations on average, it is not feasible to brute force beyond the $8x8$ input size. Because the brute-forced solution is used for validation and serves as a reference point in the performance comparisons, we use the search spaces resulting from the ATF PRL kernel with input sizes $2x2$, $4x4$, and $8x8$. It is notable that while the $8x8$ search space results in the largest Cartesian size of the set, the ATF PRL search spaces are very sparse.

4.3.7 Results. Figure 5 presents the search space construction performance across the eight real-world benchmarks for five different constraint solver methods: *brute force*, *original*, *optimized*, *ATF*, and *pyATF*. To determine the impact of the optimizations described in Section 3.3, the *original* method denotes the use of vanilla *python-constraint* before the optimizations, whereas our *optimized* method includes the optimizations of Section 3.3 as before in Section 4.2.

Figure 5A and Figure 5B illustrate the relationship between search space size and solver performance. In general, larger constrained search spaces (A) and Cartesian sizes (B) result in increased search times, particularly for the brute-force method. Our optimized solver consistently achieves the lowest execution times across all problem sizes, demonstrating its efficiency. ATF and pyATF show a similar trend but with higher execution times compared to our optimized solver, particularly for larger spaces. The original solver exhibits significantly higher execution times than our optimized version, though it performs better than the brute-force method.

Figure 5C visualizes the distribution of execution times, providing an indication of the average performance and variability. Due to the substantially lower number of real-world search spaces compared to the synthetic search spaces, some observations made for Figure 3C are obscured in Figure 5C, in particular the bimodality of the pyATF and ATF distributions. Nevertheless, it is interesting to observe that while the *original* python-constraint method is one order of magnitude faster than the *brute-force* method, both methods have very similar distributions. A clear trend emerges from this plot, where our optimized solver has the best performance and the least variability.

In Figure 5D, the relation between how constrained a search space is and solver performance is displayed. In contrast to the synthetic tests in Section 4.2, the correlation between solver performance and the sparsity of the search space is not as clear. Nevertheless, *ATF* and *pyATF* performance again appears influenced by the sparsity, as for fraction > 0.9 *ATF* performance is better than the *original* solver, in contrast to ≤ 0.9, where at fraction $\simeq 0.5$ even the unoptimized *original* python-constraint outperforms *ATF*.

Similar to what is observed in Section 4.2, the number of tunable parameters displayed in Figure 5E do not appear to have as much of an impact on performance as the other plots discussed. Nevertheless, Figure 5E is useful to discern the individual search spaces based on the number of parameters. For instance, it can be noted that the performance difference between our optimized method and all other methods appears to be relatively stable, even for the ATF PRL search spaces detailed in Table 1, as can be discerned by the number of tunable parameters, where the three ATF search spaces have 20 tunable parameters.

Finally, Figure 5F summarizes the total time taken by each solver. The brute-force approach is the least performant, taking almost a full day to resolve the eight search spaces. Although the *original*

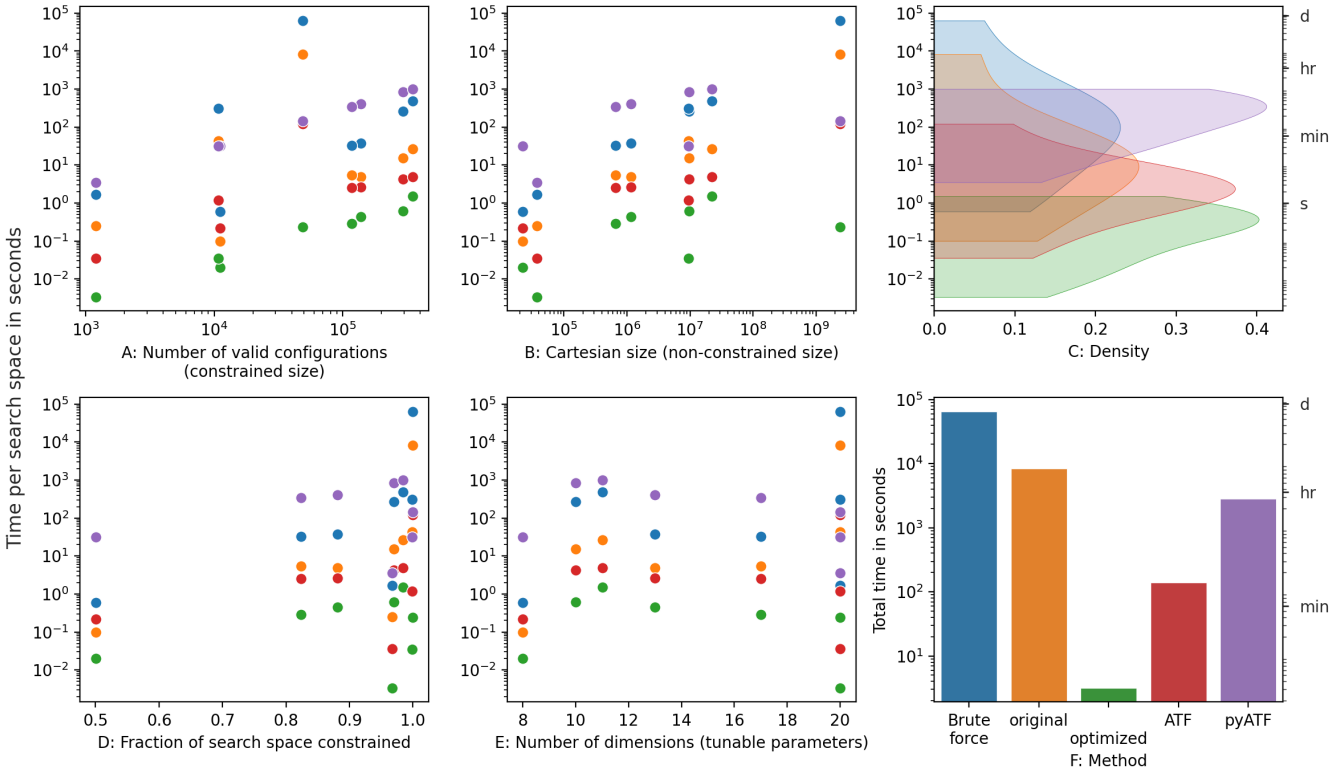

Figure 5: Search space construction performance on real-world tests. Lower times are better. Colors correspond to Figure 5F barplot methods.

python-constraint solver is faster than brute force, our *optimized* solver achieves a ~2643x speedup over it, demonstrating the efficiency of our optimizations. While ATF and pyATF achieve intermediate performance levels, the optimized solver considerably outperforms all others: our optimized method achieves a ~20611x overall speedup over the brute-force method (3.16 seconds versus 65230.47 seconds), ~44x over ATF, and ~891x over pyATF.

Overall, it is noteworthy that our optimized solver consistently outperforms any alternative on all of the search spaces by a wide margin. These findings emphasize the advantages of our optimized solver in efficiently handling large and complex search spaces.

5 Conclusions

We introduced a novel approach to constructing auto-tuning search spaces for GPU kernels using an optimized Constraint Satisfaction Problem (CSP) solver, addressing the specific challenges posed by the complexity of auto-tuning and large search spaces. Our contributions, available to the CSP-solving and auto-tuning community in the open-source python-constraint and Kernel Tuner packages, substantially outperform state-of-the-art methods in search space construction performance, enabling the exploration of previously unattainable problem scales in auto-tuning and related domains.

Through rigorous evaluation, we demonstrated that our optimized CSP-based approach reduces construction time by several orders of magnitude, even for search spaces with billions of possible combinations. On average over the evaluated real-world applications, our optimized method is four orders of magnitude faster than brute force, three orders of magnitude faster than the unoptimized CSP solver, and one to two orders of magnitude faster than the state-of-the-art in search space construction. Our optimized search space construction method reduces the construction time of real-world applications to sub-second levels, eliminating it as a substantial factor in the overall tuning process overhead. This breakthrough allows researchers and developers to more effectively harness the performance potential of modern GPUs and provides an efficient generic solver for similar problem domains.

Availability: The methods presented in this work are available as user-friendly software packages, enabling straightforward adoption by the auto-tuning community and related fields. They can be installed with `pip install python-constraint2` and `pip install kernel-tuner`. Both python-constraint and Kernel Tuner are open-source software welcoming contributions. For more information, visit the Kernel Tuner[5] and python-constraint[6] repositories.

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