A Brief Study on How House Rules Change a Game of UNO (flip)

Fouad Kaadou

March 10, 2022

After playing (and giving up on) what seemed to be a never-ending game of UNO flip with family and friends, I decided to write an UNO flip simulator to study how our house rules were influencing the length of the games. After completing the UNO flip simulator, I went ahead repurposed the code to run regular UNO and tried our house rules as well.

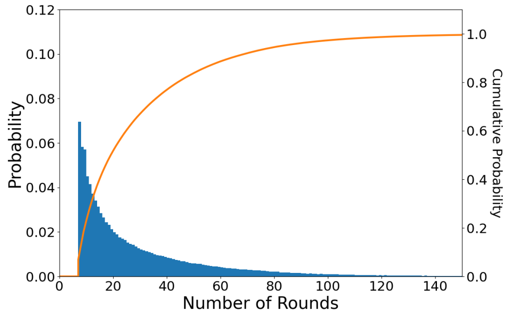
This document mainly consists of game length probability distributions measured in number of rounds, where one round is defined complete when all players have played a turn. All probability distributions were calculated by playing 1,000,000 games and keeping count of the number of rounds required to end a single game. Results are then tallied into histograms to create the probability distributions. This was done for 2-6 player games.

Code available on github.

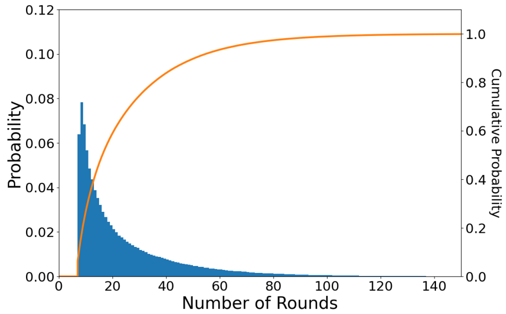
1 UNO

1.1 Official Rules

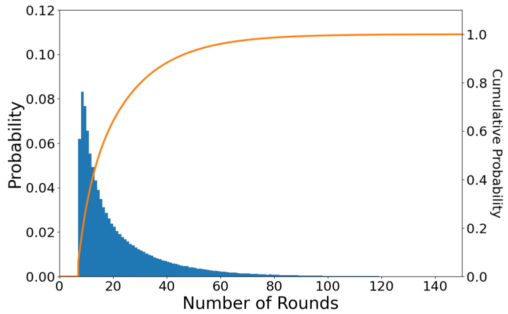
Simulation using official rules (UNO rules).



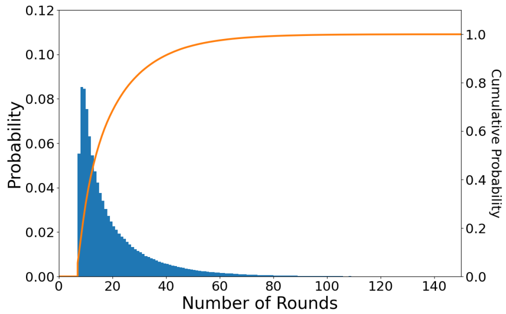
(a) 2 players.



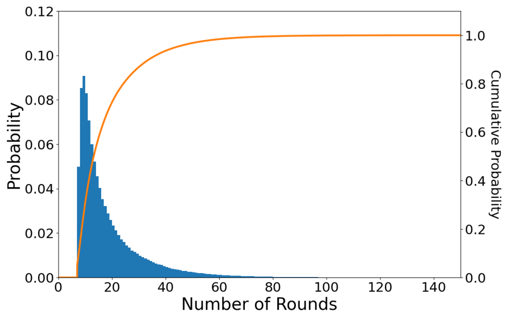
(b) 3 players.



(c) 4 players.



(d) 5 players.



(e) 6 players.

Figure 1: Probability distribution of UNO game length for 2-6 players using the official rules.

Fig. 1 shows the probability distributions for game length as a function of the number of rounds required to end the game. All distributions have the same general shape, a sharp rise around 7 or 8 rounds that decays exponentially with increasing number of rounds. The shortest playable games end in 7 rounds. That is to be expected since all players start the game with 7 cards in hand. The shortest possible game would require

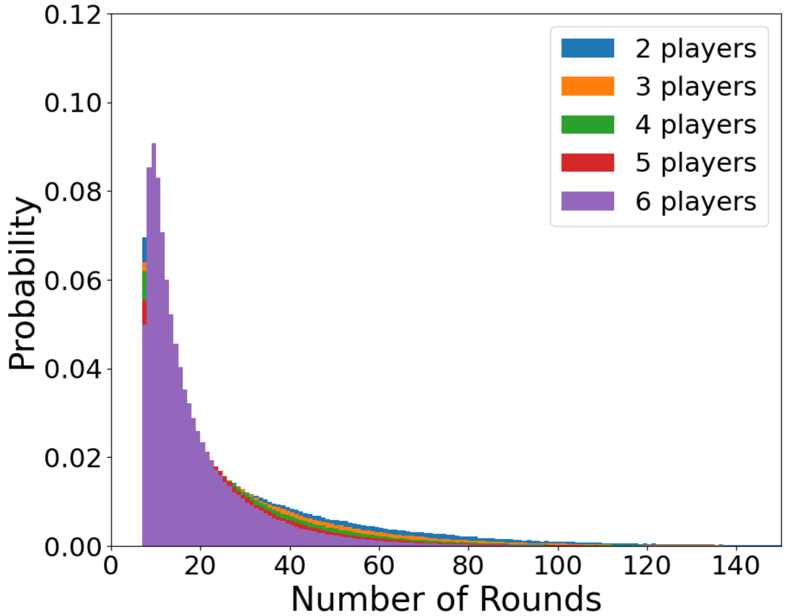


Figure 2: Overlayed probability distributions of UNO game length for 2-6 players using the official rules.

The above results predict, what to me seems likes, a counter-intuitive result: the length of the games decreases with increasing number of players. This can be clearly seen in Fig. 2 where the distributions shrink in width. To gauarantee normalization, this compression in width in naturally accompanied by an increase in the height of the distribution.

Results are quantified in Table. 1. The increase in distrubution max height or peak is seen to increase with number of players as recorded in the first row of Table. 1. The peak also shifts from 7 to 9 turns as the number of players goes from 2 to 6. A possible explanation to this would be that with a higher number of players, it is more likely that any player holds game lengthening cards such as a plus or skip card.

Here, percentile is analogous to the cumulative probability where the n^{th} percentile is defined to be the number of rounds up to which the probabilities are aggregated to

	2 players	3 players	4 players	5 players	6 players
Most likely number of rounds	7 (6.95%)	8 (7.84%)	8 (8.32%)	8 (8.54%)	9 (9.08%)
$\approx 25^{th}$ percentile	10	10	9	9	9
$\approx 50^{th}$ percentile	19	16	15	14	13
75^{th} percentile	38	30	26	23	21
90^{th} percentile	63	50	43	37	34
95^{th} percentile	81	64	54	48	43
99^{th} percentile	117	93	79	69	62

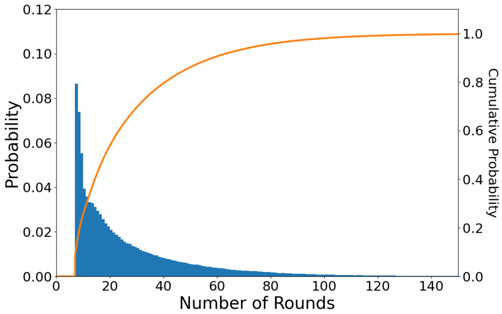
Table 1: Various percentiles (equivalent to cumulative probabilities) and most likely number of rounds required to end a game with corresponding probabilities given in parentheses.

have a cumulative probability of $n\%$. This measure is useful to study in the change in width of the distributions. The percentiles tend to decrease as the number of players increases indicating a shrinkage in the distribution's width with more players. We see that the first half (up to the $\approx 50^{th}$ percentile) of the distribution is not strongly affected by the increase in number of players. Most of the width contraction occurs in the later half, most clearly seen in the significant drop of the 99^{th} percentile. Although this indicates that games with more players require less rounds to finish, they do not take less time as the time required to play a single round increases proportionally with the number of players.

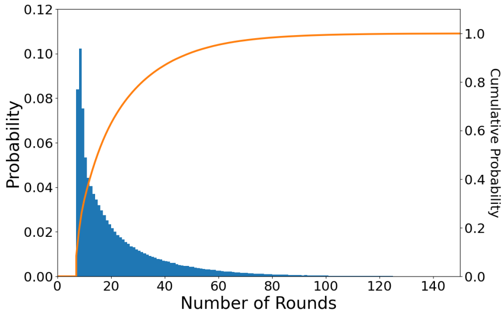
Note that the 25^{th} and 50^{th} percentiles in all tables are presented to only an approximate value. Due to the discrete number of rounds, integration of the curve is not continuous and is not guaranteed to result in specific values (example: 0.25 or 0.50). This becomes less of an issue for higher percentiles since adjacent values in the distribution are much more closely spaced.

1.2 House Rules

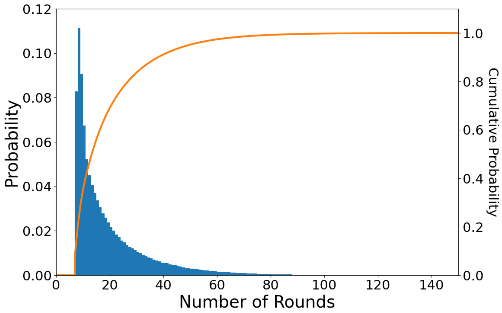
These simulations were run with the official UNO rules with the addition of a single house rule: you are allowed to stack plus cards so that the player that is unable to play one must draw the total amount of pluses accumulated up to that point.



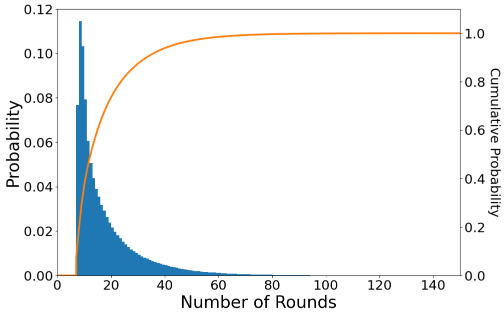
(a) 2 players.



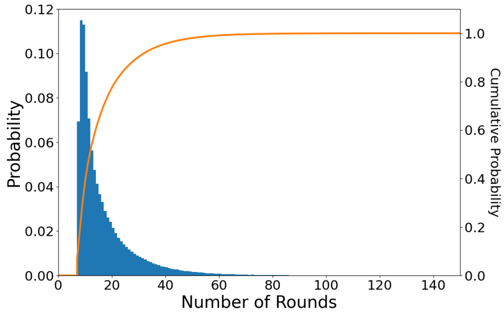
(b) 3 players.



(c) 4 players.



(d) 5 players.



(e) 6 players.

Figure 3: Probability distribution of UNO game length for 2-6 players with house rules.

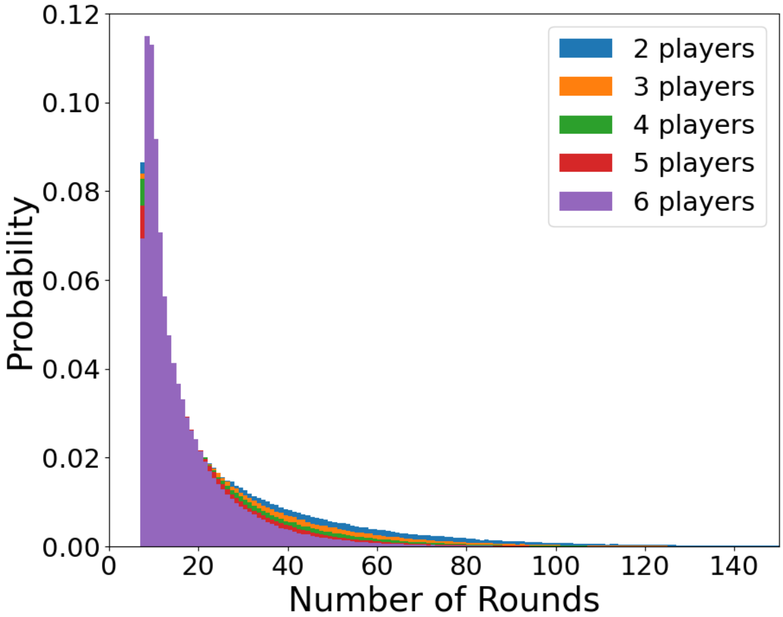


Figure 4: Overlayed probability distributions of UNO game length for 2-6 players using house rules.

Intuitively, I would have thought that this would lengthen the games. Although this extra rule makes it possible for a single player to draw many cards from the deck making the game more difficult for them, the added benefit of allowing players to get rid of their plus cards without having to pick up negates this effect. This ends up making the games slightly shorter. Comparing Fig. 2 and Fig. 4 we see that the distributions are compressed and their peaks elongated with the addition of this rule. Table. 2 backs this up as we see the percentile values are slightly less than those presented in Table. 1 for games without the house rule.

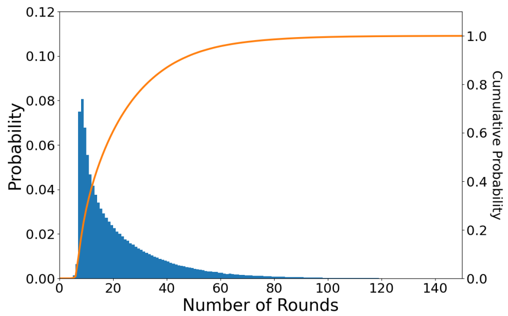
	2 players	3 players	4 players	5 players	6 players
Most likely number of rounds	7 (8.65%)	8 (10.23%)	8 (11.14%)	8 (11.45%)	8 (11.50%)
$\approx 25^{th}$ percentile	10	9	9	9	9
$\approx 50^{th}$ percentile	18	15	13	12	12
75^{th} percentile	35	27	23	20	19
90^{th} percentile	58	45	38	33	30
95^{th} percentile	74	57	48	42	37
99^{th} percentile	108	83	70	60	54

Table 2: Various percentiles (equivalent to cumulative probabilities) and most likely number of rounds required to end a game with corresponding probabilities given in parentheses.

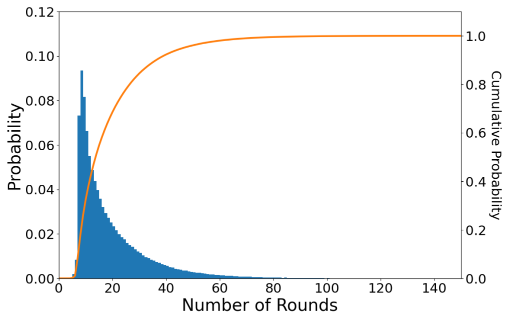
2 UNO Flip

2.1 Official Rules

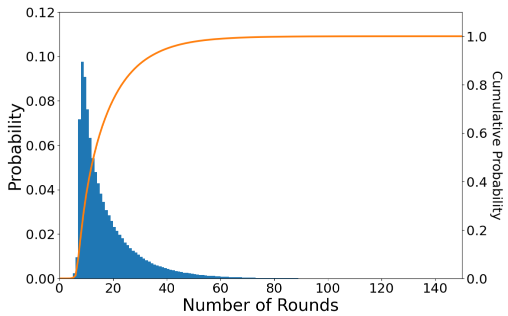
Simulations using official UNO Flip rules (UNO Flip rules).



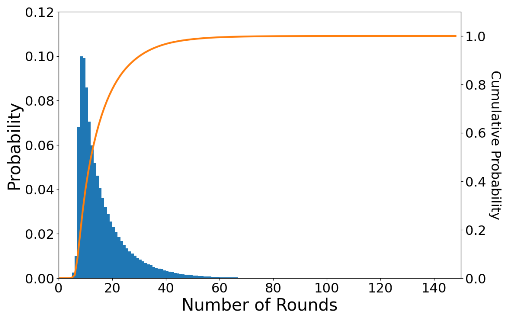
(a) 2 players.



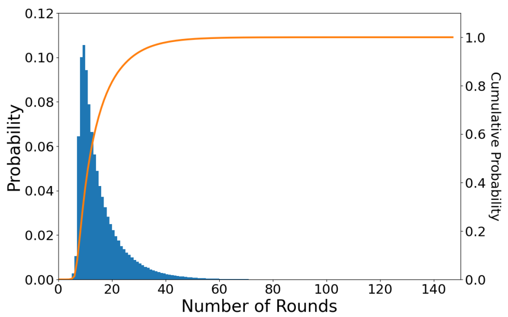
(b) 3 players.



(c) 4 players.



(d) 5 players.



(e) 6 players.

Figure 5: Probability distribution of UNO Flip game length for 2-6 players using the official rules.

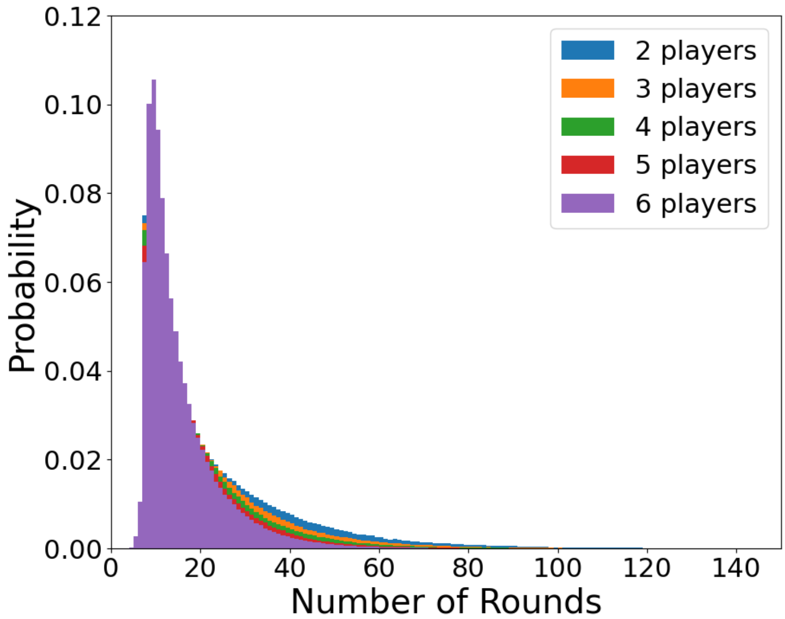


Figure 6: Overlayed probability distributions of UNO Flip game length for 2-6 players using the official rules.

The probability distributions remain qualitatively the same as those generated from playing regular UNO. The main difference is that games can now end in less than 7 rounds, down to only 3 rounds (very unlikely). This is due to the addition of the "again" or "skip everyone" card which allows a player to play multiple turns in one go. Games also follow the same trend with increasing number of players as they did in regular UNO i.e. the distribution width is compressed.

	2 players	3 players	4 players	5 players	6 players
Most likely number of rounds	8 (8.07%)	8 (9.36%)	8 (9.75%)	8 (10.00%)	9 (10.55%)
$\approx 25^{th}$ percentile	9	9	9	9	9
$\approx 50^{th}$ percentile	16	14	13	12	12
75^{th} percentile	28	23	20	19	17
90^{th} percentile	44	36	31	28	26
95^{th} percentile	56	45	40	35	32
99^{th} percentile	80	65	56	49	44

Table 3: Various percentiles (equivalent to cumulative probabilities) and most likely number of rounds required to end a game with corresponding probabilities given in parentheses.

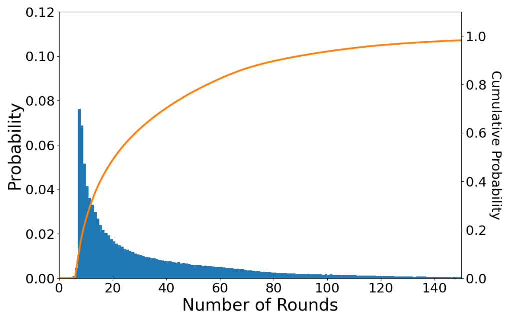
2.2 House Rules

Simulations using official UNO Flip rules with 2 additional house rules:

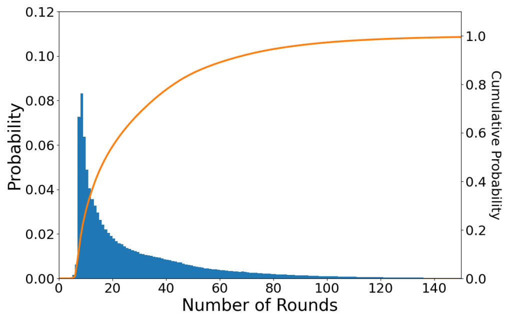
- Allowed to stack plus cards so that the player that is unable to play one must draw the total amount of pluses accumulated up to that point.
- Player using "attack" or "wild draw color" card is free to choose any player to attack.

Note: I have coded players to always attack the opponent with the least number of cards in hand.

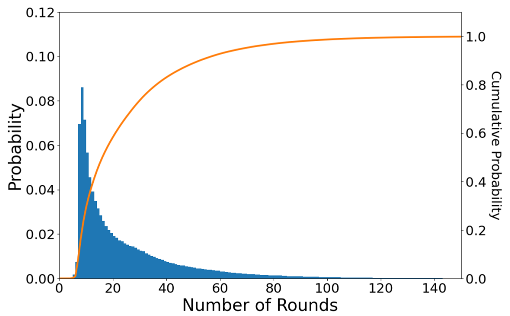
Unlike the case with regular UNO, the inclusion of these house rules in UNO Flip increase the length of the games. This can be seen in the wider distributions (Fig. 7–8). Since our house rules include plus card stacking for both versions of UNO, the game lengthening seen in UNO Flip can be attributed to the second rule that allows players to "attack" any player of their choosing. Attacking a player, i.e.using the "attack" card, allows players to target those that are closest to winning. The "attack" card forces them to keep drawing from the deck until they draw a card with a color of the user's choice.



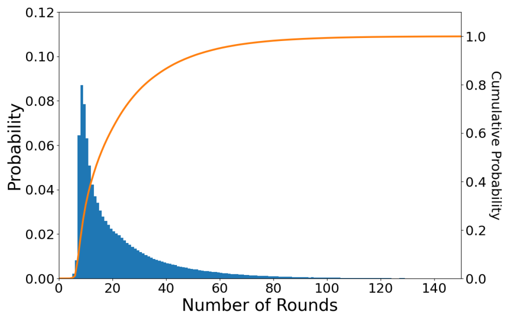
(a) 2 players.



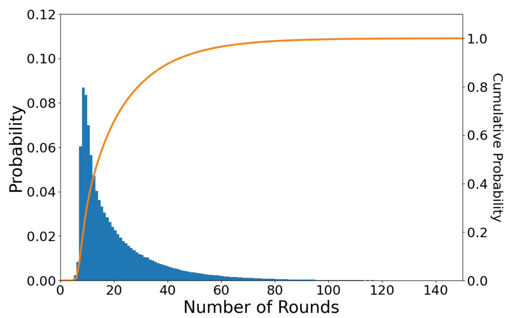
(b) 3 players.



(c) 4 players.



(d) 5 players.



(e) 6 players.

Figure 7: Probability distribution of UNO Flip game length for 2-6 players with house rules.

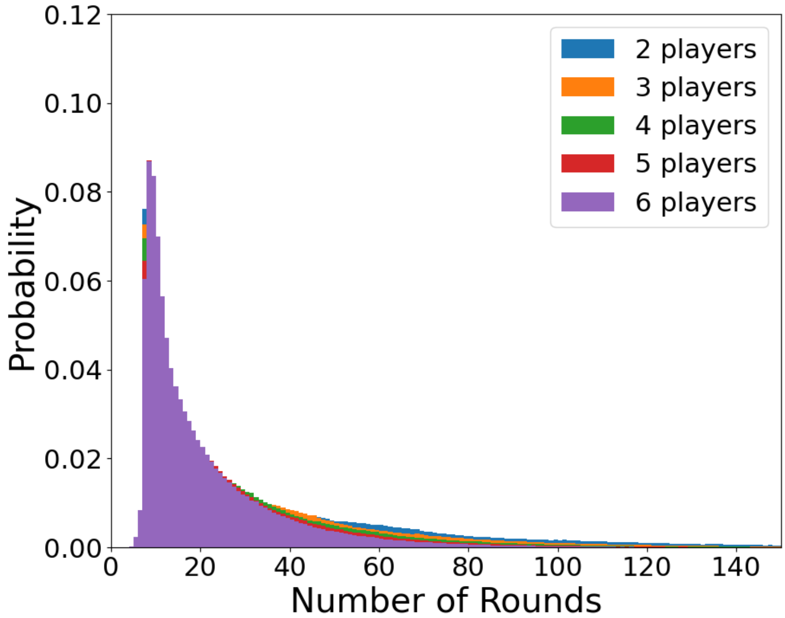


Figure 8: Overlayed probability distributions of UNO game length for 2-6 players using house rules.

Comparing Table. 3 and 4 we see an increase in almost all percentile values. Again, the later half of the distribution is mostly affected. This widening leads to longer games (on average) across the board; however, this effect is mostly pronounced for 2 player games. Fig. 7a indicated that the probability distribution for 2 player UNO flip games with house rules has qualitatively changed. The tail end now has a hump (around 60 rounds on the x-axis) that breaks the regular exponential decay. This hump is still present for 3-4 players but is gradually smoothed out with increasing number of players. The probability distribution percentiles for 2 player games also see the largest relative change(Table. 4)

	2 players	3 players	4 players	5 players	6 players
Most likely number of rounds	7 (7.62%)	8 (8.31%)	8 (8.61%)	8 (8.71%)	8 (8.69%)
$\approx 25^{th}$ percentile	10	9	9	9	9
$\approx 50^{th}$ percentile	21	17	16	15	14
75^{th} percentile	46	37	31	28	25
90^{th} percentile	80	62	52	45	41
95^{th} percentile	106	80	67	58	52
99^{th} percentile	156	118	98	85	75

Table 4: Various percentiles (equivalent to cumulative probabilities) and most likely number of rounds required to end a game with corresponding probabilities given in parentheses.