Introduction to Statistics

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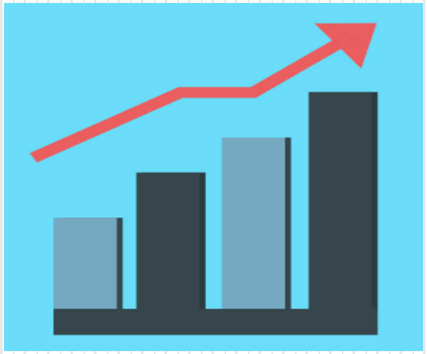
Lecture-5



Course Syllabus

- Introduction to the course
- Sampling and data presentation
- Basic of probability
- Distributions
- Confidence intervals
- Hypothesis testing
- Correlation and simple linear regression
- Multiple regression

Question 1: The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data. [points 50]

Monthly consumption (in units)	Number of consumers
65−85	4
85−105	5
105−125	13
125−145	20
145−165	14
165−185	8
185−205	4

Solution:

C.I	x_i	f_i	$f_i x_i$	c.f
65 − 85	75	4	300	4
85 − 105	95	5	475	9
105 − 125	115	13	1495	22
125 − 145	135	20	2700	42
145 − 165	155	14	2170	56
165 − 185	175	8	1400	64
185 − 205	195	4	780	68
		$\sum f_i = 68$	$\sum f_i x_i = 9320$	

$$\Rightarrow \text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{9320}{68} = 137.05$$

Now, $n = 68$ and $\frac{n}{2} = 34$

$\therefore$ Median class = 125 − 145

We have, $l = 125$, $c.f = 22$, $f = 20$ and $h = 20$

$$\Rightarrow \text{Median} = l + \frac{\frac{n}{2} - c.f}{f} \times h$$

$$= 125 + \frac{34 - 22}{20} \times 20$$

$$= 125 + 12$$

$\therefore$ Median = 137

Now, modal class = 125 − 145

Then we have $l = 125$, $f_0 = 13$, $f_1 = 20$, $f_2 = 14$ and $h = 20$

$$\Rightarrow \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 125 + \frac{20 - 13}{2 \times 20 - 13 - 14} \times 20$$

$$= 125 + 10.77$$

$\therefore$ Mode = 135.77

Question 2: The following are the test score of students in Statistics course. Find $Q_1, Q_2, Q_3, D_8, and\ P_{35}$ (use the class boundary) [points 50]

Solution:

Scores	Students	cf
10 – 14	2	2
15 – 19	5	7
20 – 24	8	15
25 – 29	9	24
30 – 34	3	27

Q_1

$$Q_1 = L + \frac{\left(\frac{n}{4} - cfb\right)i}{f}$$

$$\frac{n}{4} = \frac{27}{4} = 6.75$$

$$L = 15 - 0.5 = 14.5$$

$$cfb = 2$$

$$i = 5$$

$$f = 5$$

$$Q_1 = L + \frac{\left(\frac{n}{4} - cfb\right)i}{f}$$

$$Q_1 = 14.5 + \frac{(6.75 - 2)5}{5}$$

$$Q_1 = 14.5 + \frac{(4.75)5}{5}$$

$$Q_1 = 14.5 + 4.75$$

$$\boxed{Q_1 = 19.25}$$

Q_3

$$Q_3 = L + \frac{\left(\frac{3n}{4} - cfb\right)i}{f}$$

$$\frac{3n}{4} = \frac{3(27)}{4} = \frac{81}{4} = 20.25$$

$$L = 25 - 0.5 = 24.5$$

$$cfb = 15$$

$$i = 5$$

$$f = 9$$

$$Q_3 = L + \frac{\left(\frac{3n}{4} - cfb\right)i}{f}$$

$$Q_3 = 24.5 + \frac{(20.25 - 15)5}{9}$$

$$Q_3 = 24.5 + \frac{(5.25)5}{9}$$

$$Q_3 = 24.5 + \frac{26.25}{9}$$

$$Q_3 = 24.5 + 2.92$$

$$\boxed{Q_3 = 27.42}$$

Q_2

$$Q_2 = L + \frac{\left(\frac{n}{2} - cfb\right)i}{f}$$

$$\frac{n}{2} = \frac{27}{2} = 13.5$$

$$L = 20 - 0.5 = 19.5$$

$$cfb = 7$$

$$i = 5$$

$$f = 8$$

$$Q_2 = L + \frac{\left(\frac{n}{2} - cfb\right)i}{f}$$

$$Q_2 = 19.5 + \frac{(13.5 - 7)5}{8}$$

$$Q_2 = 19.5 + \frac{(6.5)5}{8}$$

$$Q_2 = 19.5 + \frac{32.5}{8}$$

$$Q_2 = 19.5 + 4.06$$

$$\boxed{Q_2 = 23.56}$$

Question 2: The following are the test score of students in Statistics course. Find $Q_1, Q_2, Q_3, D_8, and\ P_{35}$ (use the class boundary) [points 50]

Solution:

Scores	Students	cf
10 – 14	2	2
15 – 19	5	7
20 – 24	8	15
25 – 29	9	24
30 – 34	3	27

D_8

$$D_8 = L + \frac{\left(\frac{8n}{10} - cfb\right)i}{f}$$

$$\frac{8n}{10} = \frac{8(27)}{10} = \frac{216}{10} = 21.6$$

$L = 25 - 0.5 = 24.5$

$cfb = 15$

$i = 5$

$f = 9$

$$D_8 = L + \frac{\left(\frac{8n}{10} - cfb\right)i}{f}$$

$$D_8 = 24.5 + \frac{(21.6 - 15)5}{9}$$

$$D_8 = 24.5 + \frac{(6.6)5}{9}$$

$$D_8 = 24.5 + \frac{33}{9}$$

$$D_8 = 24.5 + 3.67$$

$$D_8 = 28.17$$

P_{35}

$$P_{35} = L + \frac{\left(\frac{35n}{100} - cfb\right)i}{f}$$

$$\frac{35n}{100} = \frac{35(27)}{100} = 9.45$$

$L = 20 - 0.5 = 19.5$

$cfb = 7$

$i = 5$

$f = 8$

$$P_{35} = L + \frac{\left(\frac{35n}{100} - cfb\right)i}{f}$$

$$P_{35} = 19.5 + \frac{(9.45 - 7)5}{8}$$

$$P_{35} = 19.5 + \frac{(2.45)5}{8}$$

$$P_{35} = 19.5 + \frac{12.25}{8}$$

$$P_{35} = 19.5 + 1.53$$

$$P_{35} = 21.03$$

Outline

- Probability Experiment
- Sample Space
- Events
  - Complement Events
  - Mutually Exclusive Events
  - Independent and Dependent Event
- Classical Probability
- Empirical Probability
- Counting techniques
  - Fundamental Counting rule.
  - Permutation
  - Combination

Probability

- Probability (or likelihood) is a measure or estimation of how likely it is that something will happen or that a statement is true.

For example

- It is very likely to rain today, or I have a fair chance of passing annual examination or I will probably win a prize etc.

In each of these statements the natural state of likelihood is expressed.

- Probabilities are given a value between 0 (0% chance or *will not happen*) and 1 (100% chance or *will happen*).
- Probability is used widely in different fields such as: mathematics, statistics, economics, management, physics, engineering, gambling and artificial intelligence/machine learning ....

Random Experiment

Experiment: Experiment means a planned activity or process whose results yield a set of data.

Trial: A single performance of an experiment is called a trial.

Outcome: The result obtained from an experiment, or a trial is called an outcome.

Random Experiment: An experiment which produces different results even though it is repeated a large number of times under essentially similar conditions, is called a random experiment.

Examples:

- The tossing of a fair coin
- The throwing of a balanced die
- Drawing a card from a well shuffled deck of 52 playing cards etc.

Basic Ideas

Definition:

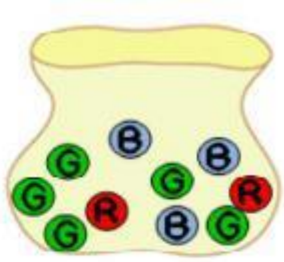
- A **probability experiment** is a chance process that leads to well-defined results called outcomes.
- OR, an **experiment** is a process that results in an outcome that cannot be predicted in advance with certainty.
- **Example**

Tossing a coin

Rolling a die

Drawing a card from deck

Drawing a marble from a bag



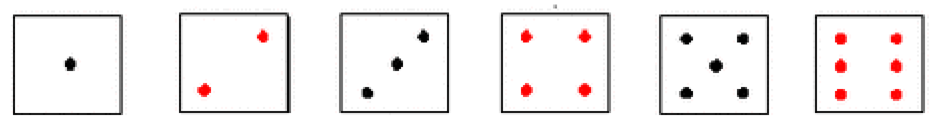
Probability

Sample space

Definition: The set of all possible outcomes of an experiment is called the **sample space** for the experiment.

Examples:

For rolling a fair die, the sample space is $\{1, 2, 3, 4, 5, 6\}$.



For a coin toss, the sample space is $\{\text{heads, tails}\}$.

Experiment	Sample Space
Students Attendance	Present, Absent/leave
Toss one coin	Head, Tail
Toss 2 coins	H-H, H-T, T-T, T-H
True, False Questions	T, F

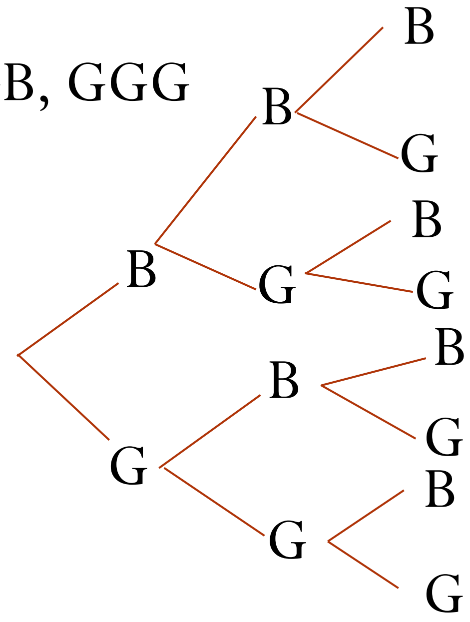
Sample space

Sample space for rolling 2 dice

Die 1	Die 2					
	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Sample space

- **Example:** Sample space using a tree diagram of three children in a family
- A tree diagram display all the possible outcomes of an event. Each branch in a tree diagram denotes a likely outcome.
- There are eight possibilities.
- BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG



Event

Definition: A subset of a sample space is called an **event**.

Event

- S={heads, tails}. ➡ Subset={heads}, {tails}.

- An event that cannot be decomposed is called a **simple event**.

- The set of all simple events of an experiment is called the sample space, S.

- For any sample space, the empty set φ is an event, as is the entire sample space.

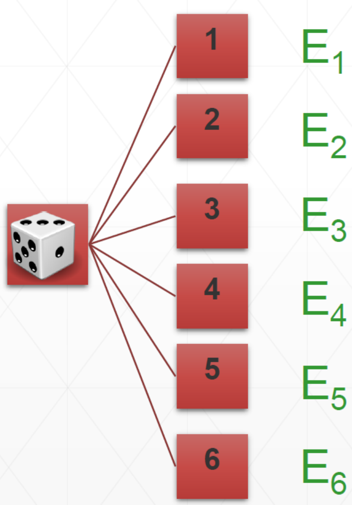
Event

- A given event is said to have occurred if the outcome of the experiment is one of the outcomes in the event.

- For example, if a die comes up 2, the events {2, 4, 6} and {1, 2, 3} have both occurred, along with every other event that contains the outcome "2."

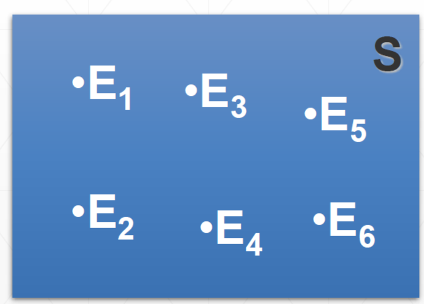
The die toss:

Simple events:



Sample space:

$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$



Probability

Probability of an event A:

- Let S be a sample space and A be an event in the sample space. Then the probability of occurrence of event A is defined as:

- P(A)=Number of sample points in A/ Total number of sample points

Symbolically, $P(A)=n(A)/n(S)$

Properties of Probability of an event:

- $P(S)=1$ for the sure event S
- For any event A,
- If A and B are mutually exclusive events, then $P(A \cup B)=P(A)+P(B)$

Example: A fair coin is tossed once, Find the probabilities of the following events:

a) An head occurs

b) A tail occurs

Solution: Here $S=\{H,T\}$, so, $n(S)=2$

Let A be an event representing the occurrence of an Head, i.e. $A=\{H\}$, $n(A)=1$

$$P(A)=n(A)/n(S)=1/2=0.5 \text{ or } 50\%$$

Let B be an event representing the occurrence of a Tail, i.e. $B=\{T\}$, $n(B)=1$

$$P(B)=n(B)/n(S)=1/2=0.5 \text{ or } 50\%.$$

Complement Event

- The complement of an event is all outcomes that are not the event.

- Example: For dice, when the event is (1, 5) the complement is (2, 3, 4, 6). Together the event and its complement make all possible outcomes, or sample space.

$$\underset{(1,\,5)}{P(E)} + \underset{(2,\,3,\,4,\,6)}{P(\bar{E})}$$

- The complement of an event E, denote by $\bar{E}$, is the set of outcomes in the sample space that are not included in the outcomes of event E.

Find the complement of the following

Event	Complement of the events
Rolling a die and getting a 5.	Getting a 1, 2, 3, 4, or 6.
Selecting a month that has 28 days	Jan, & Mar to Dec

Complement Event

- Together the event and its complement make all possible outcomes, or sample space. The sum of the probability of the event and the probability of its complement will equal to 1.

$$P(E) + P(\bar{E}) = 1$$

- If 2 coins are tossed,, so their sample space is HH, TT, TH, HT

  If event E is all TT, which will be ¼.

  Its complement will be ¾.

  Put these values in the above formula.

$$\frac{1}{4} + \frac{3}{4} = 1$$

Rules for Complement Event

- $P(E) + P(\bar{E}) = 1$

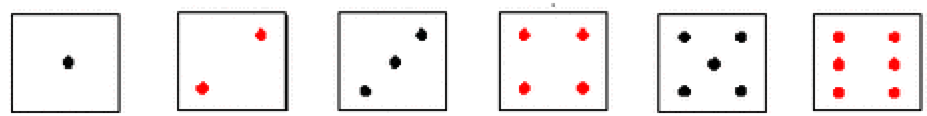
- $P(\bar{E}) = 1 - P(E)$

- $P(E) = 1 - P(\bar{E})$

- **Note:** If the probability of an event or the probability of its complement is known, then the other can be found by subtracting the probability from 1.

Mutually and non-Mutually exclusive Event

- When two events cannot occur at same time.
- For rolling a fair die, the sample space is {1, 2, 3, 4, 5, 6}.



- For a coin toss, the sample space is {heads, tails}.

- A={2,4,6}, B={1, 3, 5} (these two events cannot occur at the same time), called mutually exclusive event.
- When two event occur at same time is called non-mutually exclusive event.
- Outcome set S= {1, 2, 3, 4, 5, 6}.

event A= {1,3,5}

B= {4,5,6}

Formula for non-Mutually exclusive Event

- If two events suppose A and B are not mutually exclusive, then:

$$P(A\ or\ B) = P\ (A) + P\ (B) - P\ (A\ and\ B)$$

- **Example:** A single card is drawn from an ordinary deck of cards. Find the probability that it is either an ace (A) or a red card.

- **Solution:** $P(A\ or\ B) = P\ (A) + P\ (B) - P\ (A\ and\ B)$

$$= 4/52 \quad + \ 26/52 \ - \quad 2/52$$
$$= 28/52 = 7/13 \ = 0.538$$

Explanation: Since there are 4 aces and 26 red cards (13 hearts and 13 Dimond), 2 of aces are red cards (hearts and Dimond). So the probability of two outcomes must be subtracted since they have been counted twice.

Formula for non-Mutually exclusive Event

- **Example:** In a school there are 10 teachers and 6 educational assistant. Out of them, there are 7 female teachers, and 4 female educational assistants. If a staff person is selected, what is the probability that the person selected is a teacher or male.
- **Solution:** $P(A\ or\ B) = P\ (A) + P\ (B) - P\ (A\ and\ B)$

$P(teacher\ or\ male) = P\ (teacher) + P\ (male) - P\ (teacher\ male)$

$= 10/16 \quad + \ 5/16 - \quad 3/16$

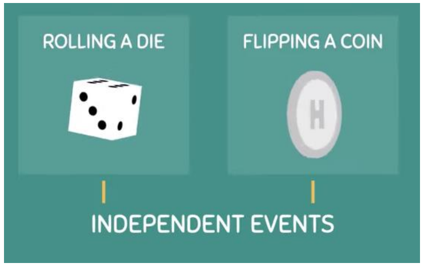
$= 12/16$

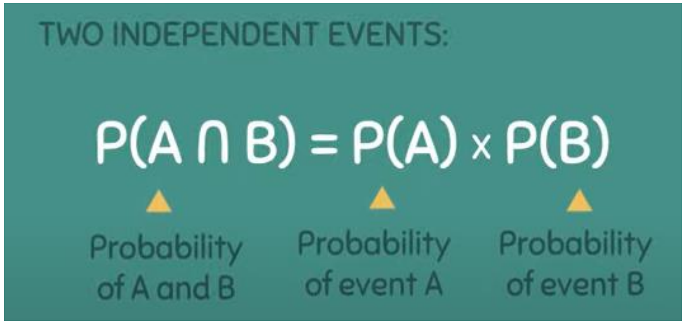
$P(teacher\ or\ male) = 0.75$

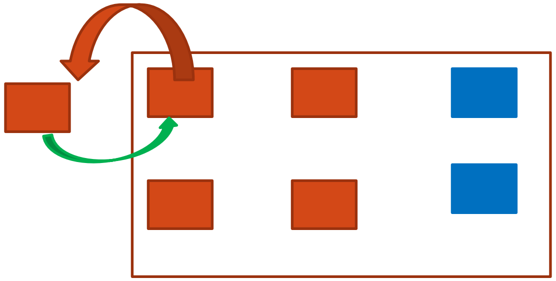
Staff	Females	Males	Total
Teachers	7	3	10
Educational Assistants	4	2	6
Total	11	5	16

Independent and dependent Event

- **Independent event:** When we have two event and event (one) can replace with other.
- If sample space doesn't affect called independent event.
- Two events are independent if the outcome of the first event does not affect the outcome of the second event.







Box

- A= red chocolate
- B= red chocolate
- **P(A)= 4/6**
- **P(B)=4/6**

Independent and dependent Event

EXAMPLE | If you roll a six-sided die and flip a coin, what is the probability of rolling a five and getting heads?

$$P(\text{Event}) = \frac{\text{total \# of favourable outcomes}}{\text{total \# of possible outcomes}}$$

$$P(\text{Rolling a 5}) = \frac{1}{6}$$

$$P(\text{Getting heads}) = \frac{1}{2}$$

$$P(A \cap B) = P(A) \times P(B)$$

$$= \frac{1}{6} \times \frac{1}{2}$$

$$= \frac{1}{12}$$

$$P\left(\begin{matrix}\text{Rolling a 5 and}\\ \text{Getting heads}\end{matrix}\right) = \frac{1}{12}$$

$$= 0.0833$$

Independent and dependent Event

- **Example:** Robert picks a number at random, puts it back, and then picks another number at random

  5, 6, 7, 8, 9

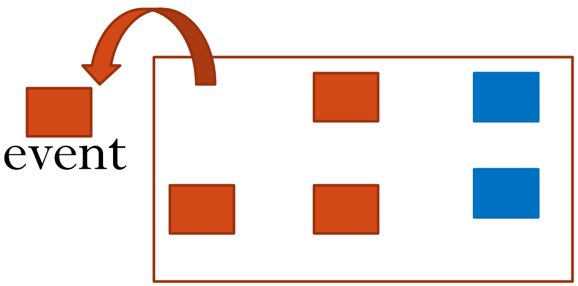
- Are these two events dependent or independent?
- **Explanation:** The Two events are **independent** as Robert puts the first number back, so his first pick does not affect which number are left for his second pick.

- **Example:** On the patio of pizza hut, the Sarah's family decides on a type of pizza to order. In the back of the restaurant, another family also chooses a pizza to order.
- ?
- **Explanation:** The Two events are **independent.** The pizza that the Sarah’s family orders does not affect which pizza another family orders.

- **Example:** Ali took a coin out of his pocket and flipped it. Then he flipped it again.
- ?
- **Explanation:** The Two events are **independent.** The first flip does not affect the second flip.

Independent and dependent Event

- **Dependent event:** When we have two event and event (one) cannot replace with other.
- If sample space effect/ change called dependent event.
- Two events are dependent if the outcome of the first event affect the outcome of the second event.



Box

- A= red chocolate
- B= red chocolate
- **P(A)= 4/6**
- **P(B)=3/5**

- **Example:** Ashley Purchases a new car from a dealership in New York. An hour later, another customer buys a car at that same dealership.
- ?
- **Explanation:** The Two events are **dependent.** Ashley's choice of car affects which ones are left at the dealership for the second customer to buy.

Independent and dependent Event

- **Example:** In Mrs. Jane class, the students pick unique numbers from a hat to decide the order in which they will present their group projects. William picks a number first, and David picks second.
- ?
- **Explanation:** The Two events are **dependent.** The number William picks affects which numbers are left for David to pick.

- **Example:** Kathrine has a variety of scares in her laundry basket. Without looking, she selects one scarf, then a second.
- ?
- **Explanation:** The Two events are **dependent.** The scarf Katherine picks first affects which scarves are left for her to pick second.

Classical Probability

- **Definition:** Classical probability is the statistical model that measures the possibility of something happening, but in a classic sense. It also means that every statistical experiment will contain elements that are equally likely to happen.
- All outcomes in the sample space be equally likely to occur.

- **Example:** Classical probability is a simple form of probability that has equal chances of something happening. For example: Rolling a die. It's equally likely would get a 1, 2, 3, 4, 5, or 6. or each outcome has a probability of 1/6.
- When a card is selected from a deck, each card has a same probability of being selected i.e. 1/52.

Formula for classical probability

- The probability of any event E is

$$\frac{\text{Number of outcomes in E}}{\text{Total number of outcomes in the sample space}}$$

- This probability is denoted by
- $P(E) = \dfrac{n\ (E)}{n\ (S)}$
- n (E) number of outcomes in event
- n (S) number of outcomes in sample space

Note: Probability can be expressed as a fraction, decimal, or percentages. Probability of getting head can be expressed ½, or 0.5, or 50%.

Formula for classical probability

- **Example:** Find the probability of getting a black face card, (jack, queen, or king)
- **Solution:** Total cards are 52, and 6 are black (jack, queen, or king) diamond and hearts.

$$\frac{\text{Number of outcomes in E}}{\text{Total number of outcomes in the sample space}}$$

- This probability is denoted by

$$P(E) = \frac{n\ (E)}{n\ (S)}$$

$$P(E) = \frac{6}{52}$$

$$P(E) = \frac{3}{26}$$

Empirical Probability

- **Definition:** Empirical probability is probability based on data collected through an experiment or observation. These probabilities are found by dividing the number of times an event occurred in an experiment by the total number of trails or observations.

- Empirical probability relies on actual experience to determine the likelihood of outcomes.

Formula for empirical probability

$$P(E) = \frac{\text{Frequency for the class}}{\text{Total frequences in the distribution}} = \frac{f}{n}$$

Empirical Probability

Example: Kidney transplant patients stayed in hospital for the following number of days.

$$\mathrm{P(E)} = \frac{f}{n}$$

1. $\mathrm{P}(5) = \frac{12}{83}$
2. $\mathrm{P}(8) = \frac{20}{83}$
3. $\mathrm{P}(12) = \frac{15}{83}$
4. $\mathrm{P}(7) = \frac{7}{83}$
5. $\mathrm{P}(6) = \frac{13}{83}$
6. $\mathrm{P}(9) = \frac{16}{83}$

Hospital Days	Frequency
5	12
8	20
12	15
7	7
6	13
9	16
	83

Empirical Probability

Example: Kidney transplant patients stayed in hospital for the following number of days.

$$P(E) = \frac{f}{n}$$

1. Patient stayed less than 9 days.
=12/83+20/83+7/83+13/83

2. Patient stayed at most 6 days.
= 12/83+13/83

3. Patients stayed at least 12 days.
= 15/83

Hospital Days	Frequency
5	12
8	20
12	15
7	7
6	13
9	16
	83

Empirical Probability

Example: Blood type of people

$$P(E) = \frac{f}{n}$$

1. People have AB.
=12/54
2. People have O.
= 20/54
3. People have A.
= 15/ 54
4. People have B.
= 7/54

Blood Type	Frequency
AB	12
O	20
A	15
B	7
	54

Counting Techniques

In order to find out the all possible outcomes for the sequence of events, three rules can be used.

- **Fundamental counting rule.**
- **Permutation rule.**
- **Combination rule.**

Counting Techniques

Fundamental counting rule:

- If one event can occur in *m* ways and a second event can occur in *n* ways, the number of ways the two events can occur in sequence is $\boldsymbol{m \cdot n}$. This rule can be extended for any number of events occurring in a sequence.

OR

- In a sequence of n events in which the first one has k_1, and so forth, then the total number of possibilities of the sequence will be

$$\boldsymbol{k_1 * k_2 * k_3 \ldots . k_n}$$

Note: The fundamental counting rule is also called the multiplication of choices.

Counting Techniques

- **Example**: A meal consists of a main dish, a side dish, and a dessert. How many different meals can be selected if there are 4 main dishes, 2 side dishes and 5 desserts available?

# of main dishes	# of side dishes	# of desserts	
4	**2**	**5**	**= 40**

There are 40 meals available.

➢**Example:** Pick two letters suppose A and B, and rolling a die. Find the total number of outcomes for the sequence of an events.

Solution: As there are 2 events A and B, and Six (6) outcomes of rolling a die, i.e., 1,2,3,4,5 and 6.

2*6=12 there are 12 possibilities.

A tree diagram can also be drawn for the sequence of an events.

Counting Techniques

- **Example**: The access code to a house's security system consists of 5 digits. Each digit can be 0 through 9. How many different codes are available if
- a.) each digit can be repeated?
- b.) each digit can only be used once and not repeated?

a.) Because each digit can be repeated, there are 10 choices for each of the 5 digits.

$$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100{,}000 \text{ codes}$$

b.) Because each digit cannot be repeated, there are 10 choices for the first digit, 9 choices left for the second digit, 8 for the third, 7 for the fourth and 6 for the fifth.

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30{,}240 \text{ codes}$$

Permutation

- **Permutation rule:** A **permutation** is an ordered arrangement of objects. The number of different permutations of n distinct objects is $n!$.
- Permutation uses factorial notation; the factorial notation uses exclamation mark.
- $$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

$$4! = 4 * 3 * 2 * 1$$

- Factorial is the product of all the positive numbers from 1 to a number

Example: How many different surveys are required to cover all possible question arrangements if there are 7 questions in a survey?

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040 \text{ Survey}$$

Permutation

- **Example:** A home buyer has choice of 6 houses to buy. He decides to rank each house according to certain criteria such as price of the house and location. How many different ways can he rank the 6 houses?

  As there are 6 choices $6! = 6*5*4*3*2*1 = 720$

  There are 720 different ways.

- Suppose he wishes to rank only the top 3 of the houses. How many different ways can he rank them?

  $6* 5*4=120$

Permutation

- **Permutation rule :** Find the number of ways that "r" objects can be selected from "n" objects.
- The arrangement of an n objects in a specific order using r objects at a time is called Permutation of n objects taking r objects at a time.
- The number of **permutations of *n* elements taken *r* at a time** is ${}_nP_r$ and formula is

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

in the group

taken from the group

Permutation

- **Example:** The arrangement of 6 objects in a specific order using 6 objects at a time is called Permutation of 6 objects taking 6 objects at a time. It is written as ${}_nP_r$ and formula is

$0! = 1$

$$ {}_nP_r = \frac{n!}{(n-r)!} \qquad {}_6P_6 = \frac{6!}{(6-6)!} = \frac{6*5*4*3*2*1}{0!} $$

$$ = 720\ different\ ways $$

Example: You are required to read 5 books from a list of 8. In how many different orders can you do so?

$$ {}_nP_r = {}_8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8*7*6*5*4*3*2*1}{3*2*1} $$

$$ = 6720\ ways $$

Combination

- **Combination rule:** A selection of distinct objects without regard to order is called a combination.
- Order of selection is not important. Suppose 10 students are to be selected from 100 students, as a member of student council. 10 students represent a combination, as it does not matter who is selected first, second, third and so on.
- A combination is a selection of r objects from a group of n things when order does not matter. The number of combinations of r objects selected from a group of n objects is

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

in the collection

taken from the collection

Combination

- The number of combinations of r objects selected from n objects is denoted by ${}_{n}C_{r}$ and is given by the formula $${}_{n}C_{r} = \frac{n!}{(n-r)!r!}$$
- **Example:** How many combination of 5 objects are there, taken 3 at a time?
- **Solution:** $${}_{n}C_{r} = \frac{n!}{(n-r)!r!} \qquad {}_{5}C_{3} = \frac{5!}{(5-3)!\,3!} \quad = \frac{5*4*3*2*1}{2*3*2*1}$$

$$= 10\ combinations$$

Example: You are required to read 5 books from a list of 8. In how many different ways can you do so if the order doesn't matter?

$${}_{8}C_{5} = \frac{8!}{3!\,5!} \qquad = \frac{8*7*6*5!}{3*2*1*5!} = 56\ Combinations$$

Combination

➢ **Example:** In a state lottery, you must correctly select 6 numbers (in any order) out of 44 to win the grand prize.

a.) How many ways can 6 numbers be chosen from the 44 numbers?

b.) If you purchase one lottery ticket, what is the probability of winning the top prize?

Solution:

a.) $${}_{44}C_6 = \frac{44!}{(44-6)!\,6!} = \frac{44!}{38!\,6!} = 7{,}059{,}052 \; combinations$$

b.) There is only one winning ticket, therefore,

$$P(win) = \frac{1}{7059052} = 0.00000014$$

Difference between permutation and combination

- The difference are shown using the letters A,B,C, and D. The permutations for the letters A,B, C, D are

AB	BA	CA	DA
AC	BC	CB	DB
AD	BD	CD	DC

- In permutations, AB is different from BA. But in combination, AB is the same as BA since the order does not matter in combinations. Therefore, if duplicates are removed from a list of permutations, what is left is a list of combinations, as shown

AB	BA	CA	DA
AC	BC	CB	DB
AD	BD	CD	DC

- Hence, the combinations of A, B, C, and D are AB, AC, AD, BC, BD, and CD. (Alternatively, BA could be listed and AB crossed out, etc.) The combinations have been listed alphabetically for convenience but is not a requirement.

Summary

- Probability Experiments
- Sample Space
- Events
  - Complement Events
  - Mutually Exclusive Events
  - Independent and Dependent Event
- Classical Probability
- Empirical Probability
- Counting techniques
  - Fundamental Counting rule.
  - Permutation
  - Combination

Introduction to Statistics

Thank You !