Introduction to Statistics

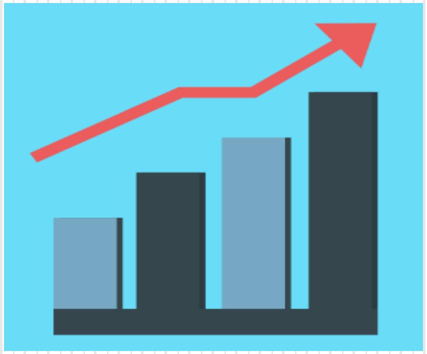
Dr. Farman Ali

Assistant Professor

DEPARTMENT OF SOFTWARE

SEJONG UNIVERSITY

Lecture-6



Course Syllabus

- Introduction to the course
- Sampling and data presentation
- Basic of probability
- Distributions
- Confidence intervals
- Hypothesis testing
- Correlation and simple linear regression
- Multiple regression

Outline

- **Laws of Sets**
- **Venn Diagram: Operation on Sets**
- **Probability Example**
- **Laws of Probability**
- **Conditional Probability**
- **Multiplication Law**

Laws of Sets

Let A, B and C be any subsets of the universal set S.

- **Commutative Law**

  $A \cup B = B \cup A$ $\qquad$ $A \cap B = B \cap A$

- **Associative Law**

  $A \cup (B \cup C) = (A \cup B) \cup C$ $\qquad$ $A \cap (B \cap C) = (A \cap B) \cap C$

- **Distributive Law**

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $\qquad$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- **Idempotent Laws**

  $A \cup A = A$ $\qquad$ $A \cap A = A$

- **Identity Laws**

  $A \cup S = S$ $\qquad$ $A \cap S = A$ $\qquad$ $A \cup \Phi = A$ $\qquad$ $A \cap \Phi = \Phi$

- **Complementation Laws**

$$AUA' = S, A \cap A' = \Phi, \left(A'\right)' = A, S' = \Phi, \Phi' = S$$

- **De-Morgan's Laws**

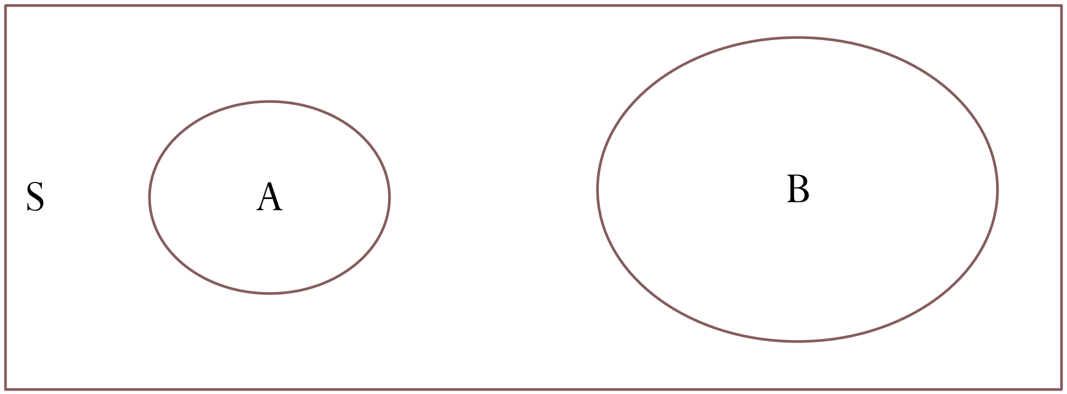
$$\left(AUB\right)' = A' \cap B' \qquad \left(A \cap B\right)' = A'UB'$$

Venn-Diagram

Venn-Diagrams are used to represent sets and subsets in a pictorial way and to verify the relationship among sets and subsets.

In venn-diagram, a rectangle is used to represent the universal set or space S, whereas the sets are represented by circular regions.

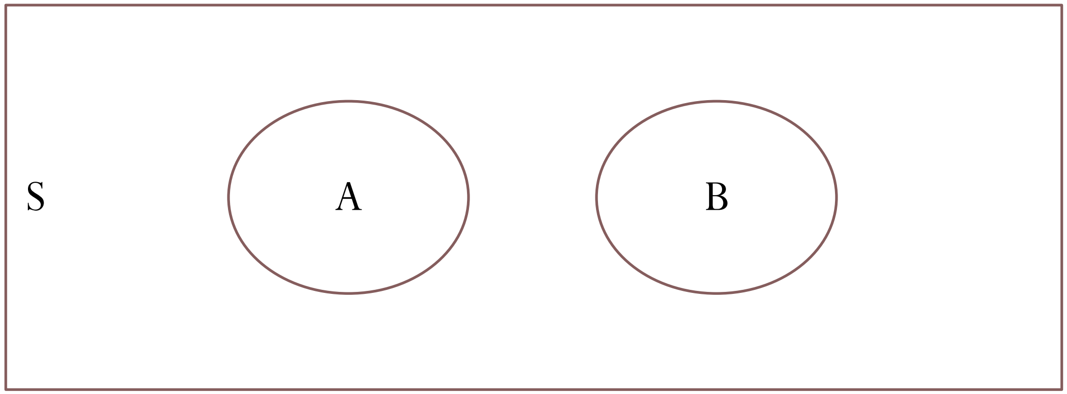
Example:



A simple venn-diagram

Venn-Diagram: Operations on Sets

AUB



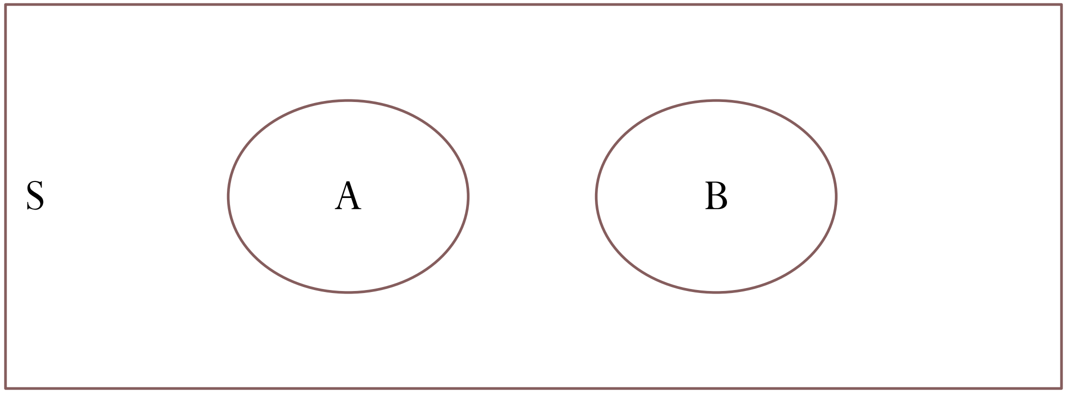
To find the area which is occurred in A and B

A∩B

To find the area which is common in A and B

Venn-Diagram: Operations on Sets

A’



A’= S-A

All outcomes that are not in event A

A-B or A difference B

Properties of Probability

➢ Probability of an event A:

- Let S be a sample space and A be an event in the sample space. Then the probability of occurrence of event A is defined as:
- P(A)=Number of sample points in A/ Total number of sample points
- Symbolically, $P(A)=n(A)/n(S)$

➢ Properties of Probability of an event:

- $P(S)=1$ for the sure event S
- For any event A, $0 \leq P(A) \leq 1$
- If A and B are mutually exclusive events, then $P(A \cup B)=P(A)+P(B)$

Probability: Example

Example 1: A fair coin is tossed once, Find the probabilities of the following events:

a) An head occurs

b) A tail occurs

Tossing a coin

Solution:

Probability: Example

Example 1: A fair coin is tossed once, Find the probabilities of the following events:

a) An head occurs

b) A tail occurs

Tossing a coin

Solution: Here S={H,T}, so, n(S)=2

Let A be an event representing the occurrence of an Head, i.e. A={H}, n(A)=1

$$P(A)=n(A)/n(S)=1/2=0.5 \text{ or } 50\%$$

Let B be an event representing the occurrence of a Tail, i.e. B={T}, n(B)=1

$$P(B)=n(B)/n(S)=1/2=0.5 \text{ or } 50\%.$$

Probability: Example

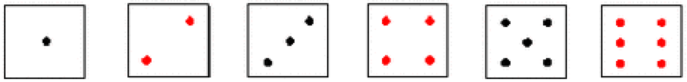
Example 2: A fair die is rolled once, Find the probabilities of the following events:

a) An even number occurs

b) A number greater than 4 occurs

c) A number greater than 6 occurs

Rolling a die



Solution:

Probability: Example

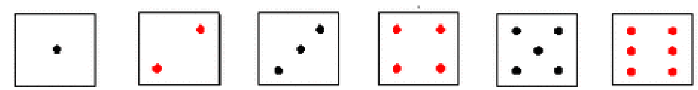
Example 2: A fair die is rolled once, Find the probabilities of the following events:

a) An even number occurs

b) A number greater than 4 occurs

c) A number greater than 6 occurs

Rolling a die



Solution: Here S={1,2,3,4,5,6}, n(S)=6

a). An even number occurs

Let A=An even number occurs={2,4,6}, n(A)=3

P(A)=n(A)/n(S)=3/6=1/2=0.5 or 50%

b). A number greater than 4 occurs

Let B=A number greater than 4 occurs={5,6}, n(B)=2

P(B)=n(B)/n(S)=2/6=1/3=0.3333 or 33.33%

c). A number greater than 6 occurs

Let C=A number greater than 6 occurs={}, n(C)=0

P(C)=n(C)/n(S)=0/6=0 or 0%

Probability: Example

Example 3: If two fair dice are thrown, what is the probability of getting (i) a double six? (ii). A sum of 11 or more dots?

Solution:

Die 1	Die 2					
	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Probability: Example

Example 3: If two fair dice are thrown, what is the probability of getting (i) a double six? (ii). A sum of 11 or more dots?

Solution: Here

n(S)=36

Die 1	Die 2					
	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Let A=a double six={(6,6)}

n(A)=1

P(A)=1/36

Let B= a sum of 11 or more dots

B={(5,6), (6,5), (6,6)}, n(B)=3

P(B)=3/36

Probability: Example

Example 4: A fair coin is tossed three times. What is the probability that:

Tossing a coin

a) At-least one head appears

b) More heads than tails appear

c) Exactly two tails appear

Solution: Here S={HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}, n(S)=8

a). At-least one head appears

Let A=At-least one head appears={HHH, HHT, HTH, THH, HTT, THT, TTH}, n(A)=7

P(A)=n(A)/n(S)=7/8

b). More heads than tails appear

Let B= More heads than tails appear ={HHH, HHT, HTH, THH}, n(B)=4

P(B)=n(B)/n(S)=4/8=1/2=0.5 or 50%

c). Exactly two tails appear

Let C=Exactly two tails appear={HTT, THT, TTH}, n(C)=3

P(C)=n(C)/n(S)=3/8

Probability: Example

Example 5: An employer wishes to hire three people from a group of 15 applicants, 8 men and 7 women, all of whom are equally qualified to fill the position. If he selects the three at random. What is the probability that:

- All three will be men

Solution: Total number of ways in which three people can be selected out of 15 are: $\binom{15}{3} = 455$. so n(S)=455

a). All three will be men

Let A= All three will be men, so

$$n(A) = \binom{8}{3} = 56$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{\binom{8}{3}}{\binom{15}{3}} = \frac{56}{455}$$

Probability: Example

Example 5: If a card is drawn from an ordinary deck of 52 playing cards, find the probability that:

a) It is a red card

b) Card is a diamond

c) Card is a 10

d) Card is a king

e) A face card

Solution:

Probability: Example

Example 5: If a card is drawn from an ordinary deck of 52 playing cards, find the probability that:

a) It is a red card

b) Card is a diamond

c) Card is a 10

d) Card is a king

e) A face card

Solution:

Solution: Since total playing cards are 52, So, n(S)=52

a). A red Card

Let A=A red card, n(A)=26, P(A)=n(A)/n(S)=26/52=1/2

b). Card is a diamond

Let B= Card is a diamond, n(B)=13, P(B)=n(B)/n(S)=13/52=1/4

c). Card is a ten

Let C=Card is a ten, n(C)=4, P(C)=n(C)/n(S)=4/52=1/13

d). Card is a King

Let D=Card is a King, n(D)=4, P(D)=n(D)/n(S)=4/52=1/13

e). A face card

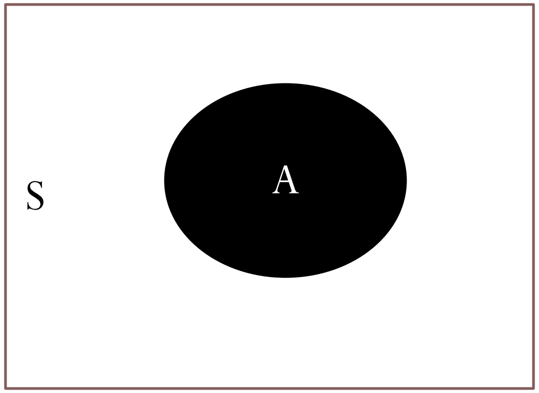
Let E=A face card, n(E)=12, P(E)=n(E)/n(S)=12/52=3/13

Laws of Probability

- If A is an impossible event then P(A)=0

- If A’ is complement of an event A relative

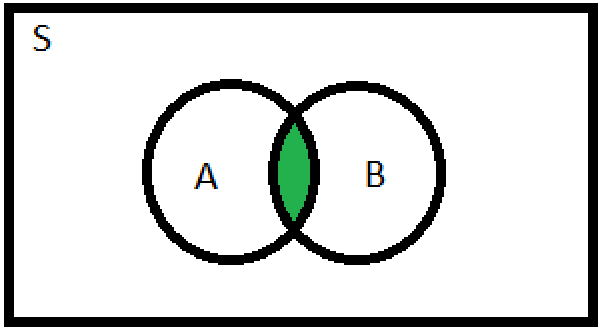
to Sample space S then P(A’)=1-P(A)



Laws of Probability

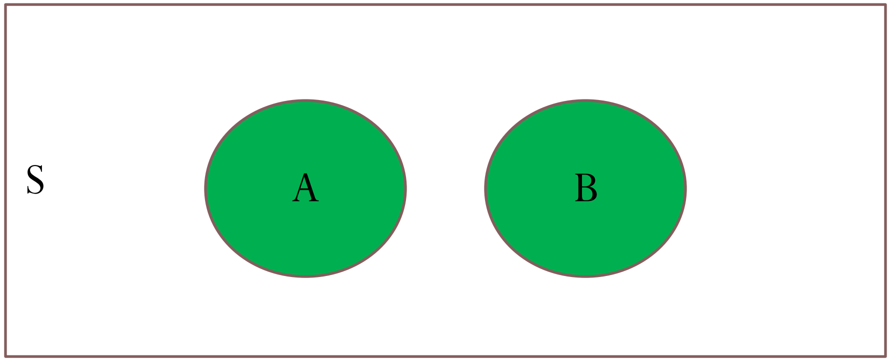
Addition Law: If A and B are any two events defined in a sample space S then:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



If A and B are two Mutually Exclusive events defined in a sample space S then:

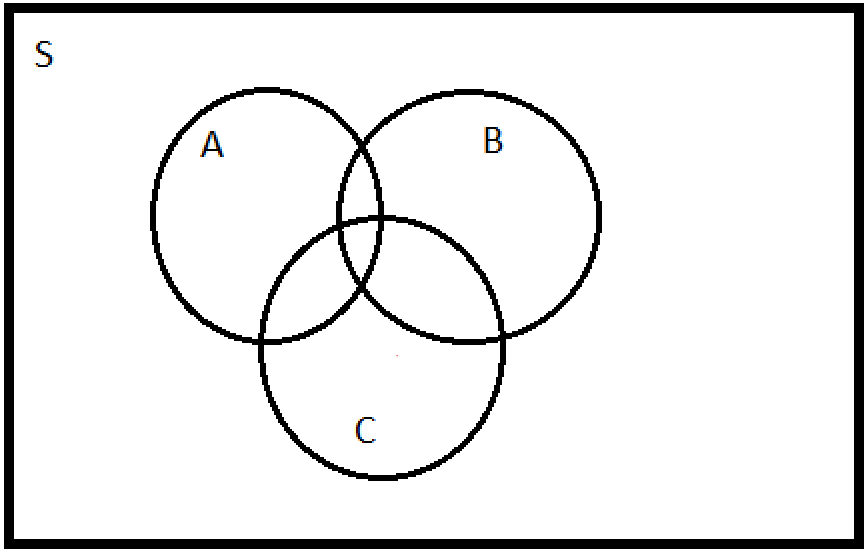
$P(A \cup B) = P(A) + P(B)$



Laws of Probability

Addition Law:

If A, B and C are any three events defined in a sample space S then:



$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

If A and B are two Mutually Exclusive events defined in a sample space S then:

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

Structure of a Deck of Playing Cards

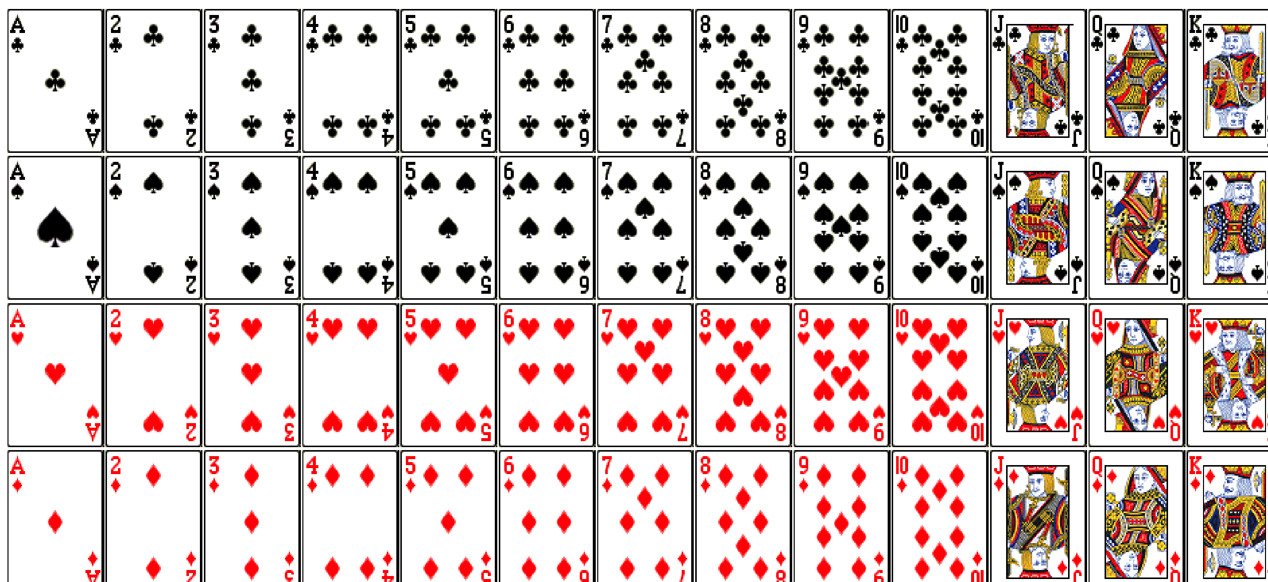
- Total Cards in an ordinary deck: 52
- Total Suits: 4 **Spades (♠), Hearts (♥), Diamonds (♦), Clubs (♣)**
- Cards in each suit: 13
- Face values of 13 cards in each suit are:
- Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King
- Honor Cards are: Ace, 10, Jack, Queen and King
- Face Cards are: Jack, Queen, King
- Popular Games of Cards are: Bridge and Poker

Clubs (♣)

Spades (♠)

Hearts (♥)

Diamonds (♦)



Probability: Card Example

Example: If a card is drawn from an ordinary deck of 52 playing cards, what is the probability that the card is a club or a face card?

Solution: Since total playing cards are 52, So, $n(S)=52$

Let A=Card is a club, and let B=A face card

$P(A \text{ or } B)=P(AUB)=?$

By addition, law, we have, $P(AUB)=P(A)+P(B)-P(A\cap B)$

Note, $P(A\cap B)=n(A\cap B)/n(S)=3/52$ (As we have three face cards in the club suit)

$n(A)=13$, $P(A)=13/52$ $n(B)=12$, $P(B)=12/52$

So, $P(AUB)=P(A)+P(B)-P(A\cap B)=13/52+12/52-3/52=22/52$

Probability: Example

Example: An integer is chosen at random from the first 10 positive integers. What is the probability that the integer chosen is divisible by 2 or 3?

Solution: Since there are a total of 10 integers, So, n(S)=10

Let A=Integer is divisible by 2={2,4,6,8,10}, n(A)=5, P(A)=5/10

Let B=Integer is divisible by 3={3,6,9}, n(B)=3, P(B)=3/10

By addition, law, we have, P(A or B)=P(AUB)=P(A)+P(B)-P(A∩B)

(A∩B)={6}, n(A∩B)=1, P(A∩B)=n(A∩B)/n(S)=1/10

So, P(AUB)=P(A)+P(B)-P(A∩B)=5/10+3/10-1/10=7/10=0.7 or 70%

Conditional Probability

Unconditional: Without restriction to any part of the sample space, then the probability would be **unconditional**

Conditional Probability: Let S be the sample space of any Random experiment. And A and B are two associated events with S. The conditional probability of Event A if B has occurred is denoted by p(A|B) and given by **P(A|B) = P(A ∩ B) / P(B)**

TABLE 2.1 Sample space containing 1000 aluminum rods

	Diameter		
Length	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

Unconditional Probability

P(Diameter OK)

= 928/1000

= 0.928

TABLE 2.2 Reduced sample space containing 942 aluminum rods that meet the length specification

	Diameter		
Length	Too Thin	OK	Too Thick
Too Short	—	—	—
OK	38	900	4
Too Long	—	—	—

Conditional Probability

P(Diameter OK|Length OK)

= 900/942

= 0.955

Conditional Probability

Example: If a family has two children, what is the probability that both children are girls, given that

1) at least one is a girl
2) The youngest one is a girl

Assume that each born child is equally to be a boy or a girl.

G or B → elder
g or b → younger

S = {G g, G b, B g, B b}

P(both are girls) = $\frac{1}{4}$

P(both are girls|at least one is a girl) = $\frac{1}{3}$

E → both the children are girls

F → at least one is a girl

$P(E \mid F) = ?$

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

$E \cap F = \{G\ g\}$

$F = \{G\ g,\ B\ g,\ G\ b\}$

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{4}$$

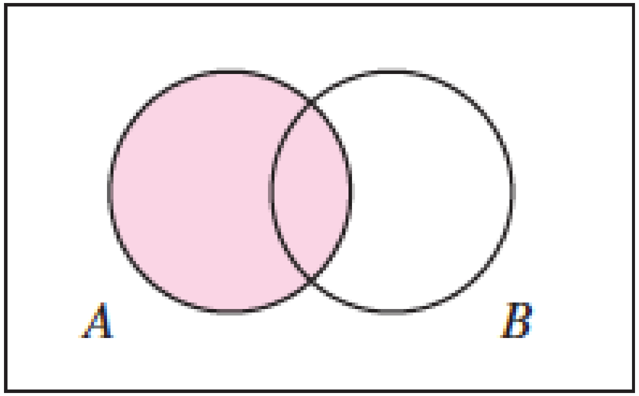
$$P(F) = \frac{3}{4}$$

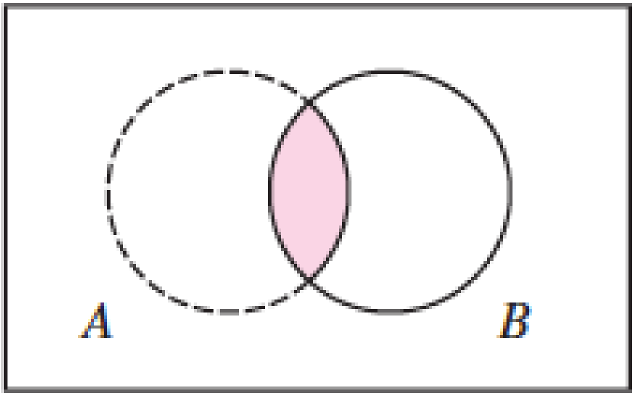
$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Conditional Probability

- Let A and B be events with $P(B) \neq 0$. The conditional probability of A given B is

$$\mathbf{P(A \mid B) = P(A \cap B) \,/\, P(B)}$$





Unconditional probability of A. Ignores any information about B

Conditional probability of A **given** B occurred. Only consider the part of A that overlaps with B

Conditional Probability

Example: Let us consider the die throwing experiment with sample space=S={1,2,3,4,5,6}

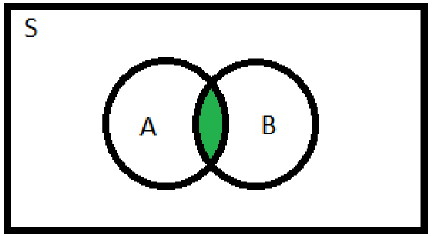
Suppose we wish to know the probability of the outcome that the die shows 6, say event A. So, P(A)=1/6=0.166

If before seeing the outcome, we are told that the die shows an even number of dots, say event B. Then this additional information that the die shows an even number excludes the outcomes 1,3 and 5 and thereby reduces the original sample space to only three numbers {2,4,6}. So P(6)=1/3=0.333

We call 1/3 or 0.333 as the conditional probability of event A because it is computed under the condition that the die has shown even number of dots.

P(Die shows 6/die shows even numbers)=P(A/B)=1/3=0.333

$$P(A/B)=\frac{n(A\cap B)}{n(B)}=\frac{\dfrac{n(A\cap B)}{n(S)}}{\dfrac{n(B)}{n(S)}}=\frac{P(A\cap B)}{P(B)},\left(P(B)\neq 0\right)$$



Conditional Probability: Examples

Example: Two coins are tossed . What the probability that two heads result, given that there is at-least one head?

Solution: S={HH,HT,TH,TT} , n(S)=4

Let A=Two Heads appear={HH}

Let B=at-least one head={HH,HT,TH}

P(A/B)=?

We have, $P(A/B)=P(A \cap B)/P(B)$

$P(A)=1/4$, $P(B)=3/4$

$(A \cap B)=\{HH\}$, $P(A \cap B)=1/4$

$P(A/B)=P(A \cap B)/P(B)=(1/4)/(3/4)=1/3=0.33$

Conditional Probability: Examples

Example: Three coins are tossed . What the probability that two tails result, given that there is at-least one head?

Solution: S={HHH,HHT,HTH, THH, HTT, THT, TTH, TTT} , n(S)=8

Let A=Two tails appear={HTT, THT, TTH }

Let B=at-least one head={HHH,HHT,HTH, THH, HTT, THT, TTH}

$P(A/B)=?$

We have, $P(A/B)=P(A\cap B)/P(B)$

$P(A)=3/8$, $P(B)=7/8$

$(A\cap B)=\{HTT, THT, TTH\}$, $P(A\cap B)=3/8$

$P(A/B)=P(A\cap B)/P(B)=(3/8)/(7/8)=3/7$

Conditional Probability: Examples

- **Example:** A lot of 1000 components contains 300 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective.

a. Find P(A).

b. Find P(B|A).

c. Find P(A ∩ B).

Conditional Probability: Examples

- **Example:** A lot of 1000 components contains 300 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective.

a. Find P(A).
b. Find P(B|A).
c. Find P(A ∩ B).

(a) $P(A) = 300/1000 = 3/10$

(b) Given that A occurs, there are 999 components remaining, of which 299 are defective. Therefore $P(B|A) = 299/999$.

(c) $P(A \cap B) = P(A)P(B|A) = (3/10)(299/999) = 299/3330$

Multiplication Law

If A and B are any two events defined in a sample space S, then:

- $P(A \text{ and } B)=P(A \cap B)=P(A/B).P(B)$, provided $P(B) \neq 0$
- $P(A \text{ and } B)=P(A \cap B)=P(B/A).P(A)$, provided $P(A) \neq 0$

Independent Events: Two events A and B defined in a sample space S are said to be independent if the probability that one event occurs, is not affected by whether the other event has or has not occurred,

$P(A/B)=P(A)$ and $P(B/A)=P(B)$

So, the above laws simplifies to:

- $P(A \text{ and } B)=P(A \cap B)=P(A/B).P(B)=P(A).P(B)$

Similarly, in case of three events, A,B and C, we have:

$P(A \text{ and } B \text{ and } C)=P(A \cap B \cap C)=P(A).P(B).P(C)$

Note: Two events A and B defined in a sample space S are said to be dependent if: $P(A \cap B) \neq P(A).P(B)$

Multiplication Law: Examples

Example: A box contains 15 items, 4 of which are defective and 11 are good. Two items are selected. What is the probability that the first is good and the second is defective?

Solution:

Multiplication Law: Examples

Example: A box contains 15 items, 4 of which are defective and 11 are good. Two items are selected. What is the probability that the first is good and the second is defective?

Solution: Let A=First item is good and B=Second item is defective

P(First is good and second is defective)=P(A and B)=P(A∩B)=?

We have, P(A∩B)=P(B/A)*P(A)

P(A)=11/15, P(Second is defective/fir st is good)=P(B/A)=4/14

So, P(A∩B)=P(B/A)*P(A)=(4/14)*(11/15)=44/210=0.16

Multiplication Law: Examples

Example: A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a green and then a yellow marble?

Solution:

Multiplication Law: Examples

Example: A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a green and then a yellow marble?

Solution:

Total marbles=16

Let A=Green marble

Let B=Yellow marble

$P(A)=5/16$, $P(B)=6/16$

P(A Green and then a yellow marble)=P(A and B)=$P(A\cap B)$=?

Since A and B are independent events, so,

$P(A\cap B)=P(A).P(B) =(5/16).(6/16)=30/256=15/128$

Multiplication Law: Examples

Example: Two cards are drawn from a well-shuffled ordinary deck of 52 cards. Find the probability that they are both aces if the first card is (i) replaced, (ii) not replaced.

Solution:

Multiplication Law: Examples

Example: Two cards are drawn from a well-shuffled ordinary deck of 52 cards. Find the probability that they are both aces if the first card is (i) replaced, (ii) not replaced.

Solution: Let A=an Ace on first card and B=an Ace on second card

P(Both are Aces)=P(Ace on first and Ace on second)=P(A and B)=$P(A\cap B)$ =?

i). In case of replacement, events A and B are independent

So, $P(A\cap B)=P(A).P(B)=4/52.\ 4/52=1/13.\ 1/13=1/169$

ii). If the first card is not replaced, then, events A and B are dependent

P(both are Aces)=P(Ace on first and Ace on second given that first card is an Ace)=$P(A\cap B)=P(A).P(B/A)=4/52.\ 3/51=1/13.\ 1/17=1/221$

So, $P(A\cap B)=P(B/A)/P(A)=(4/14)/(11/15)=44/210=0.16$

Multiplication Law: Examples

Example: A nationwide survey found that 50% of the young people in a country like pizza. If 3 people are selected at random, what is the probability that all three like pizza?

Solution:

Multiplication Law: Examples

Example: A nationwide survey found that 50% of the young people in a country like pizza. If 3 people are selected at random, what is the probability that all three like pizza?

Solution:

Let A=First person likes pizza

Let B=Second person likes pizza

Let C=Third person likes pizza

P(all three like pizza)=$P(A \cap B \cap C)$=?

Since A, B and C are independent events, so,

P(all three like pizza)=$P(A \cap B \cap C) = P(A).P(B).P(C) = (0.5)(0.5)(0.5) = 0.125$

Summary

- Laws of Sets
- Venn Diagram: Operation on Sets
- Properties of Probability
- Probability Example
- Laws of Probability
- Conditional Probability
- Multiplication Law

Thank You !