Introduction to Statistics

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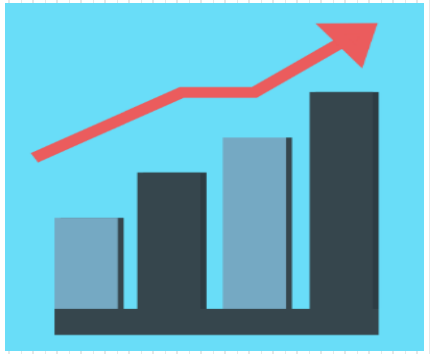
DEPARTMENT OF SOFTWARE

SEJONG UNIVERSITY

Lecture-9



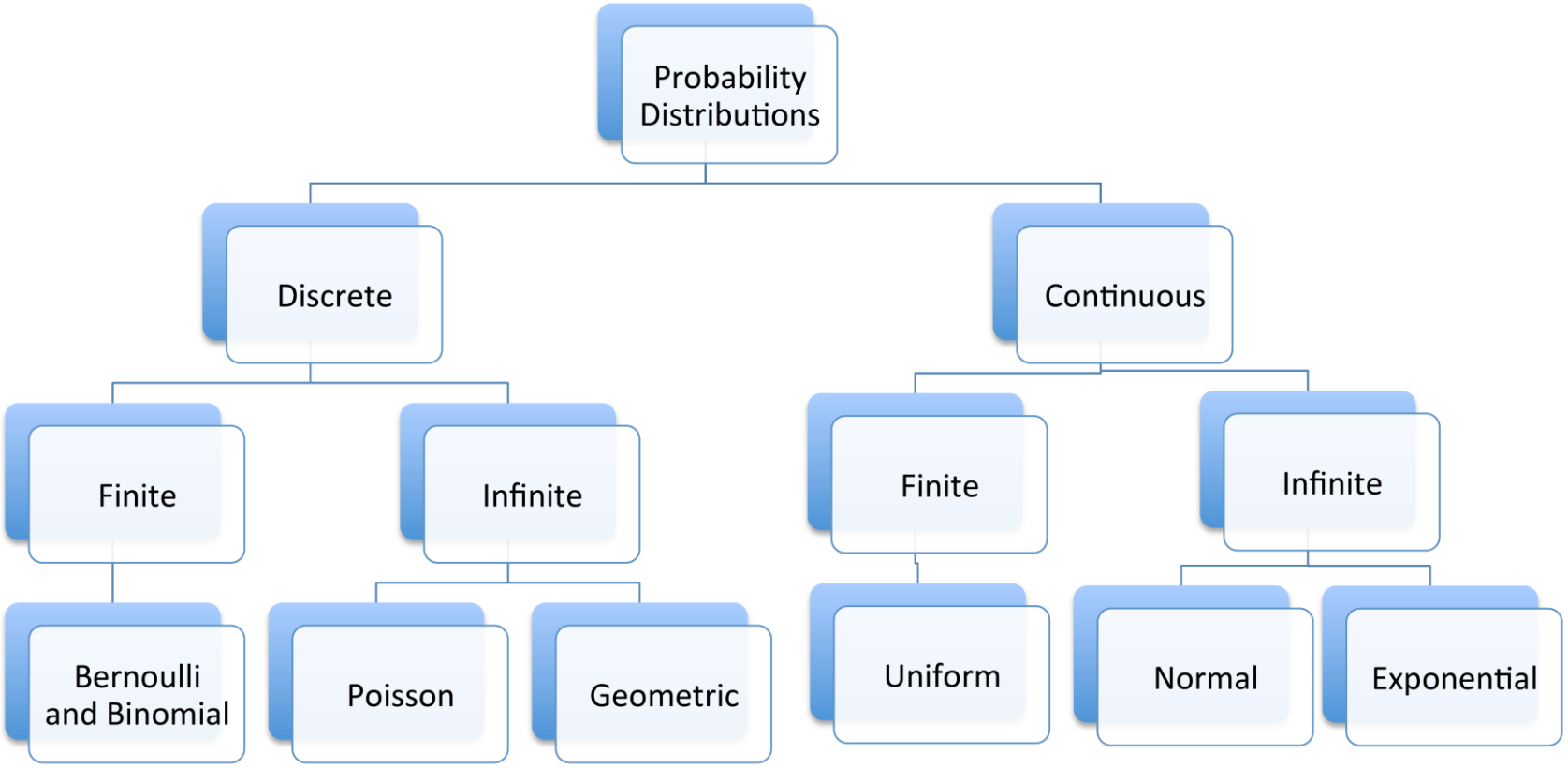
Course Syllabus

- Introduction to the course
- Sampling and data presentation
- Basic of probability
- Distributions
- Confidence intervals
- Hypothesis testing
- Correlation and simple linear regression
- Multiple regression

Outline

- Uniform Distribution
- Normal Distribution

Some Common Distributions



Uniform Distribution

- A uniform distribution is a type of continuous random variable such that each possible value of X has exactly the same probability of occurring.

- As a result the graph of the function is a horizontal line and forms a rectangle with the X axis. Hence, its secondary name the rectangular distribution.

- In common with all continuous random variables the area under the function between all the possible values of X is equal to 1 and as a result it is possible to work out the probability density function of X, for all uniform distributions using a simple formula.

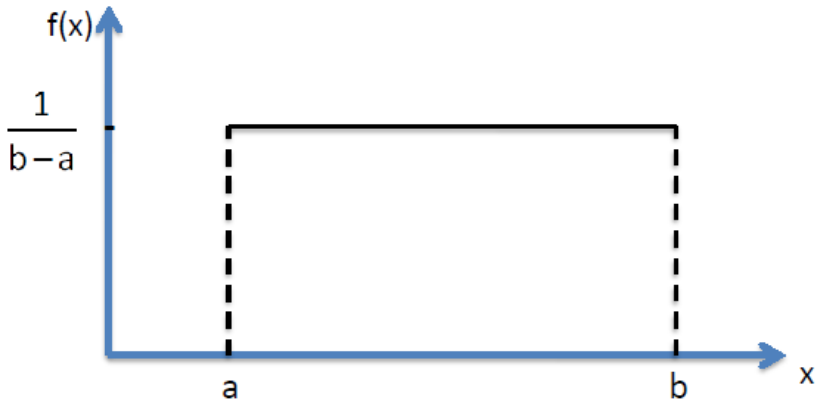
Uniform Distribution

Definition: Given that a continuous random variable X has possible values from $a \leq X \leq b$ such that all possible values are equally likely, it is said to be uniformly distributed. i.e. X~U(a,b).

Hence the probability distribution function (pdf) of X is:

$$f(x) = \frac{1}{b-a} \qquad \text{for } a \leq x \leq b$$

$$= 0 \qquad \text{elsewhere}$$



Note: Uniform Distribution has TWO parameters: ‘a’ and ‘b’.

Properties of Uniform Distribution

Properties:

Let X~U(a,b):

- Mean of X is: (a+b)/2

Properties of Uniform Distribution

Properties:

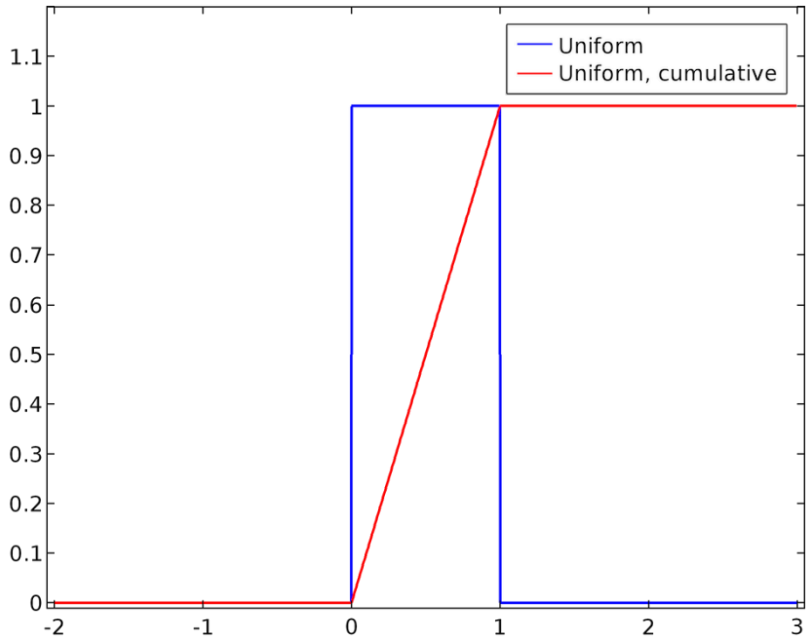
Let X~U(a,b):

- Variance of X is: $(b-a)^2/12$

Uniform Distribution

Cumulative Distribution Function

$$f(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$$



Uniform Example

Example: Consider the data on 55 smiling times in seconds of an eight-week-old baby.

10.4	19.6	18.8	13.9	17.8	16.8	21.6	17.9	12.5	11.1	4.9
12.8	14.8	22.8	20.0	15.9	16.3	13.4	17.1	14.5	19.0	22.8
1.3	0.7	8.9	11.9	10.9	7.3	5.9	3.7	17.9	19.2	9.8
5.8	6.9	2.6	5.8	21.7	11.8	3.4	2.1	4.5	6.3	10.7
8.9	9.4	9.4	7.6	10.0	3.3	6.7	7.8	11.6	13.8	18.6

sample mean $=$ 11.49 and sample standard deviation $=$ 6.23

We assume that smiling times, in seconds, follow a uniform distribution between 0 and 23 seconds, inclusive.

This means that any smiling time from 0 to and including 23 seconds is Equally Likely.

Uniform Example

Example: Consider the data on 55 smiling times in seconds of an eight-week-old baby.

10.4	19.6	18.8	13.9	17.8	16.8	21.6	17.9	12.5	11.1	4.9
12.8	14.8	22.8	20.0	15.9	16.3	13.4	17.1	14.5	19.0	22.8
1.3	0.7	8.9	11.9	10.9	7.3	5.9	3.7	17.9	19.2	9.8
5.8	6.9	2.6	5.8	21.7	11.8	3.4	2.1	4.5	6.3	10.7
8.9	9.4	9.4	7.6	10.0	3.3	6.7	7.8	11.6	13.8	18.6

sample mean = 11.49 and sample standard deviation = 6.23

Let X = length, in seconds, of an eight-week old baby's smile.
The notation for the uniform distribution is
$X \sim U$ (a,b) where a = the lowest value of X and b = the highest value of X.
The probability density function is $f(X) = \frac{1}{b-a}$ for $a \leq X \leq b$.
For this example, $X \sim U(0, 23)$ and $f(X) = \frac{1}{23-0}$ for $0 \leq X \leq 23$.

Uniform Example

Example: Consider the data on 55 smiling times in seconds of an eight-week-old baby.

10.4	19.6	18.8	13.9	17.8	16.8	21.6	17.9	12.5	11.1	4.9
12.8	14.8	22.8	20.0	15.9	16.3	13.4	17.1	14.5	19.0	22.8
1.3	0.7	8.9	11.9	10.9	7.3	5.9	3.7	17.9	19.2	9.8
5.8	6.9	2.6	5.8	21.7	11.8	3.4	2.1	4.5	6.3	10.7
8.9	9.4	9.4	7.6	10.0	3.3	6.7	7.8	11.6	13.8	18.6

sample mean $=$ 11.49 and sample standard deviation $=$ 6.23

Formulas for the theoretical mean and standard deviation are

$\mu = \frac{a+b}{2}$ and $\sigma = \sqrt{\frac{(b-a)^2}{12}}$

For this problem, the theoretical mean and standard deviation are

$\mu = \frac{0+23}{2} = 11.50$ seconds and $\sigma = \sqrt{\frac{(23-0)^2}{12}} = 6.64$ seconds

Notice that the theoretical mean and standard deviation are close to the sample mean and standard deviation.

Uniform Example

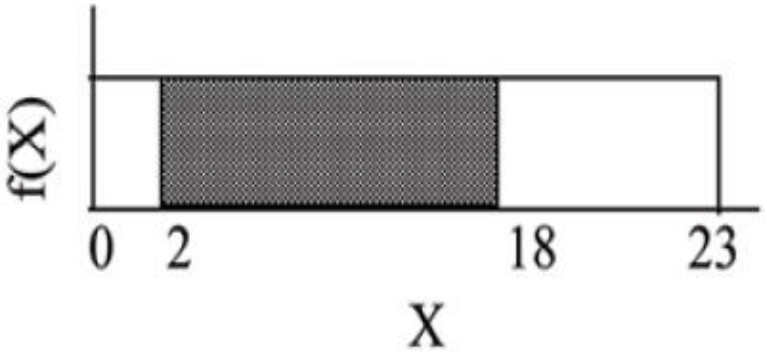
Problem 1

What is the probability that a randomly chosen eight-week old baby smiles between 2 and 18 seconds?

Solution

Find $P(2 < X < 18)$.

$$P(2 < X < 18) = (\text{base})\,(\text{height}) = (18 - 2) \cdot \tfrac{1}{23} = \tfrac{16}{23}.$$



Uniform Example

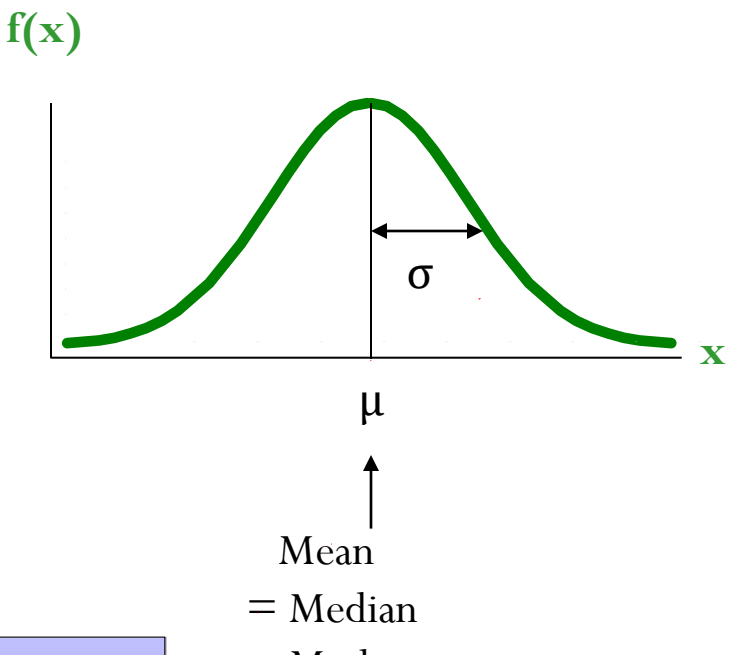
Example: If X is uniformly distributed over (0,10), calculate the probability that

a) $X<3$

b) $1<X<6$

The Normal Distribution

- Bell Shaped
- Symmetrical
- Mean, Median and Mode are Equal

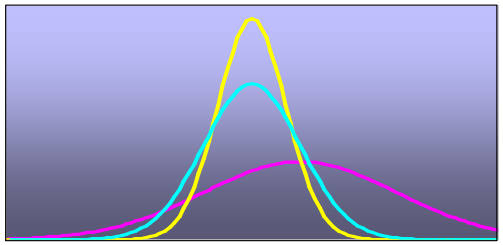


Location is determined by the mean, μ

Spread is determined by the standard deviation, σ

The random variable has an infinite theoretical range:

- ∞ to + ∞



The Normal Probability Density Function

The formula for the normal probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Where e = the mathematical constant approximated by 2.71828

π = the mathematical constant approximated by 3.14159

μ = the population mean

σ = the population standard deviation

x = any value of the continuous variable, $-\infty < x < \infty$

X is r.v. which follows Normal Distribution with **mean 'μ'** and **variance 'σ²'**, we use the notation:

$$X \sim N(\mu, \sigma^2)$$

Properties of Normal Distribution

- The function f(x) defining the normal distribution is a proper p.d.f. i.e.

$$a). \quad f(x) \geq 0$$

$$b). \quad \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

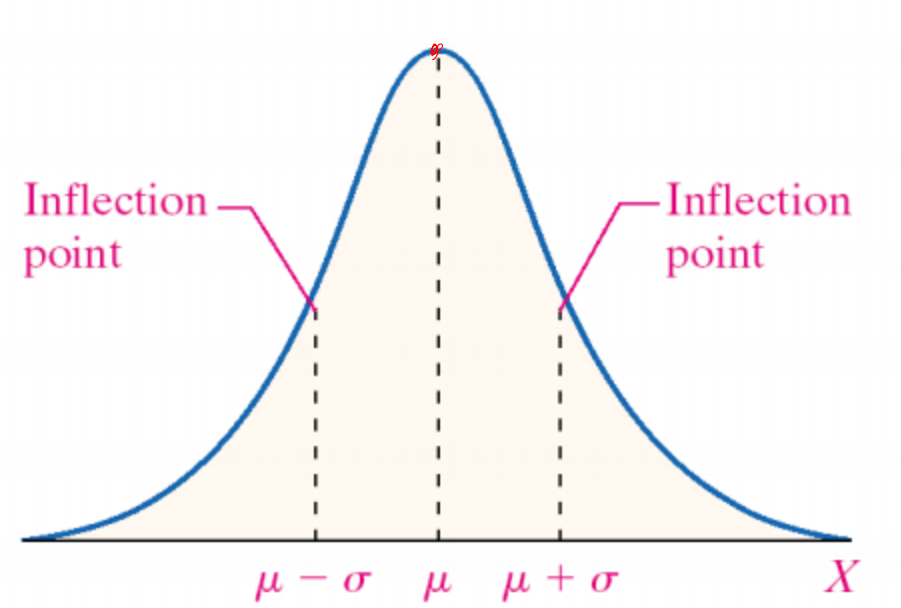
- Mean and variance of Normal Distribution are $\boldsymbol{\mu}$ and $\boldsymbol{\sigma^2}$ respectively.

- The Median and the Mode of the Normal Distribution are each equal to the Mean of the distribution. i.e. Mean=Median=Mode

Properties of Normal Distribution

- The Normal Distribution has points of inflection which are equidistant from the mean. i.e. **$\mu - \sigma$** and **$\mu + \sigma$.**

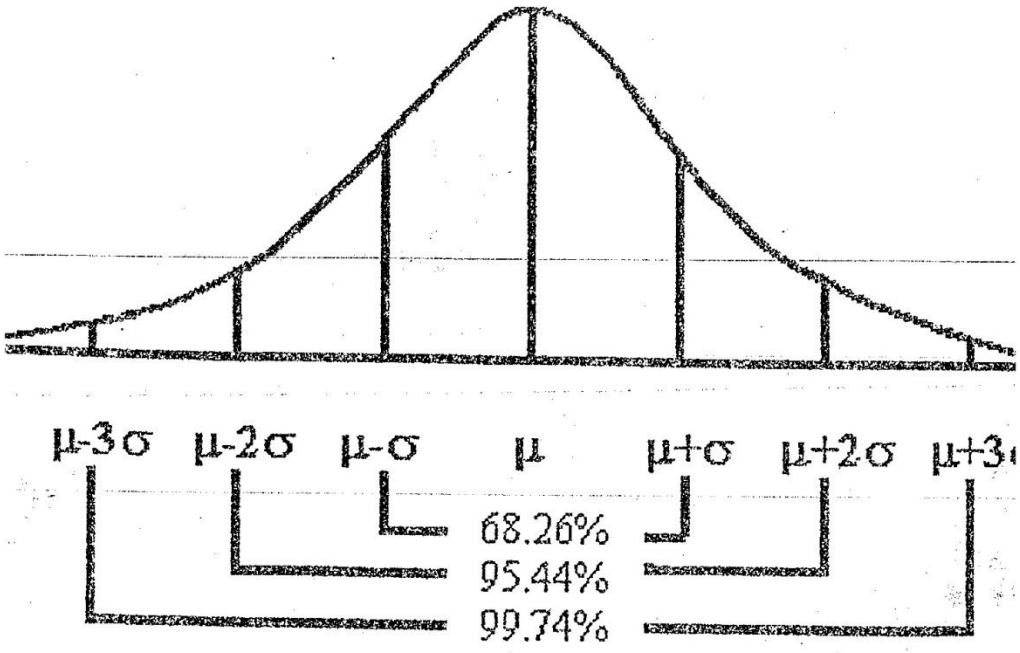
Definition: Point of Inflection: A point at which the concavity of the function changes.



Properties of Normal Distribution

Empirical Rule:

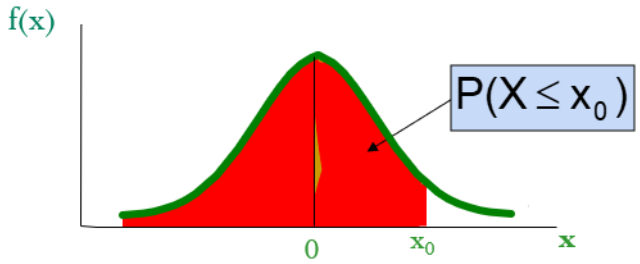
- Approximately 68% of the area under the curve is between $\mu-\sigma$ and $\mu+\sigma$.
- Approximately 95% of the area under the curve is between $\mu-2\sigma$ and $\mu+2\sigma$.
- Approximately 99.7% of the area under the curve is between $\mu-3\sigma$ and $\mu+3\sigma$



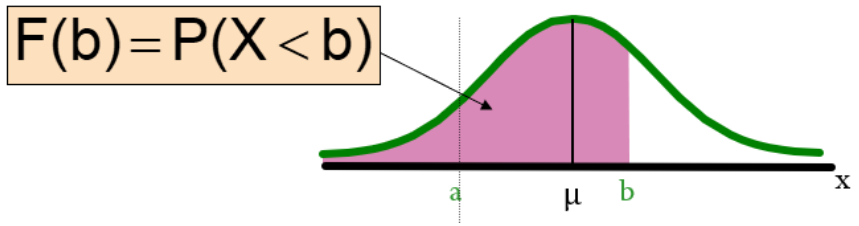
Cumulative Normal Distribution

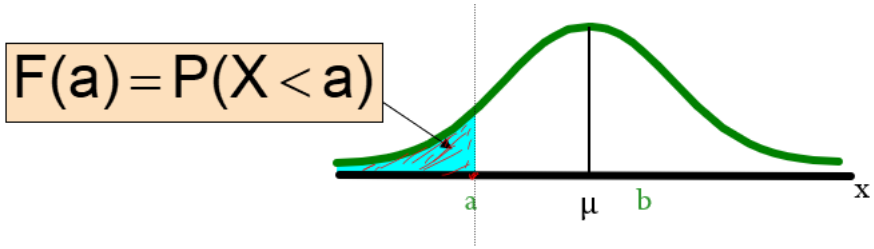
- For a normal random variable X with mean μ and variance σ^2 , i.e., $X \sim N(\mu, \sigma^2)$, the **cumulative distribution function** is

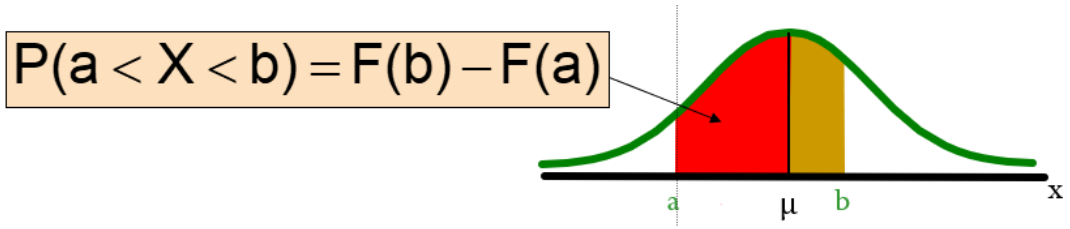
$$F(x_0) = P(X \le x_0)$$



Finding Normal Probabilities



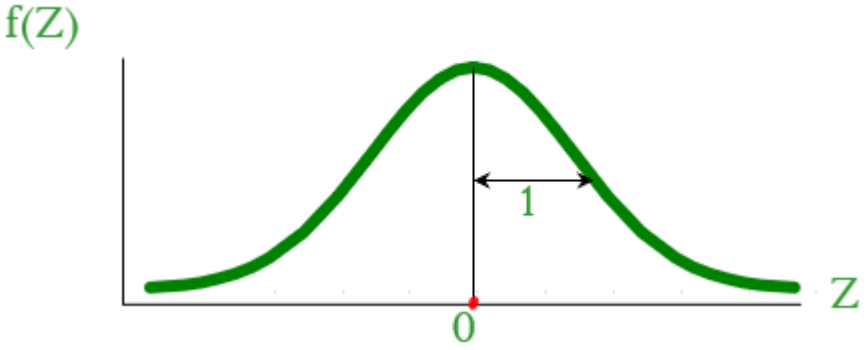




The Standardized Normal

- Any **normal distribution (with any mean and variance combination) can be transformed into the** standardized normal distribution (Z), with mean 0 and variance 1.

$$Z \sim N(0,1)$$



- Need to transform X units into Z units by subtracting the mean of X and dividing by its standard deviation

$$Z = \frac{X - \mu}{\sigma}$$

Example

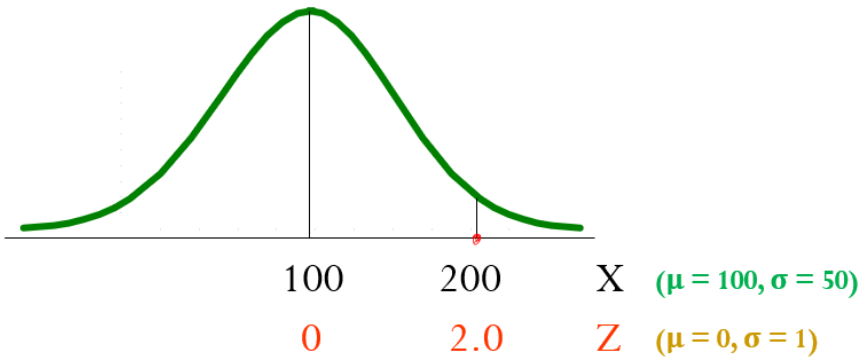
- If X is distributed normally with mean of 100 and standard deviation of 50, the Z value for X = 200 is

$$Z = \frac{X - \mu}{\sigma} = \frac{200 - 100}{50} = 2.0$$

- This says that X = 200 is two standard deviations (2 increments of 50 units) above the mean of 100.

Note that the distribution is the same, only the scale has changed.

We can express the problem in original units (X) or in standardized units (Z)



The Standardized Normal Probability Density Function

- The formula for the Standardized Normal Probability Density Function can be obtained by replacing $\mu=0$ and $\sigma=1$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

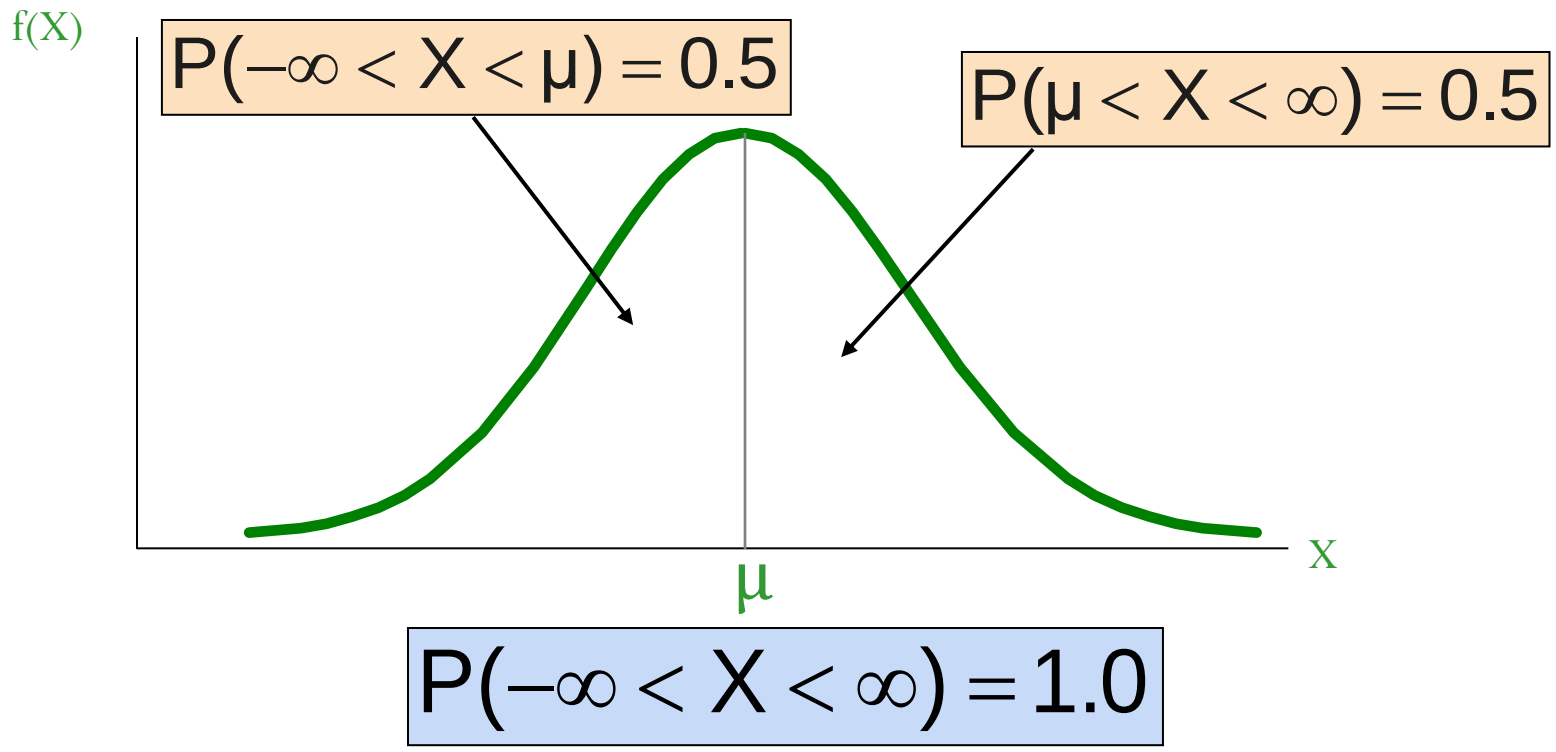
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x)^2}$$

OR

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2}$$

Probability as Area Under the Curve

The total area under the curve is 1.0, and the curve is symmetric, so half is above the mean, half is below



Standardized Normal Area Table

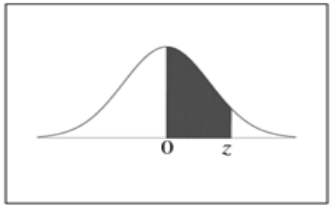
- It gives the probability from 0 to Z,

i.e. P(0<Z<2)=0.4772

Since the distribution is symmetric, so

P(-2<Z<0)=0.4772

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Continuous Probability Distributions

P(Z>2)=?

$P(Z>2)=0.5-P(0<Z<2)$

$=0.5-0.4772$

$=0.0228$

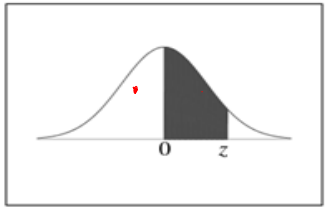
P(Z<-2)=?

$P(Z<-2)=0.5-P(-2<Z<0)$

$=0.5-0.4772$

$=0.0228$

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
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0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Continuous Probability Distributions

P(-2<Z<+2)=?

P(-2<Z<+2)

= P(-2<Z<0)+ P(0<Z<+2)

= 0.4772 + 0.4772

=0.9544

P(Z<-2.15)=?

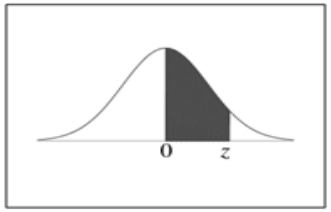
P(Z<-2.15)

= 0.5 - P(-2.15<Z<0)

= 0.5 - 0.4842

=0.0158

Standard Normal Distribution Table

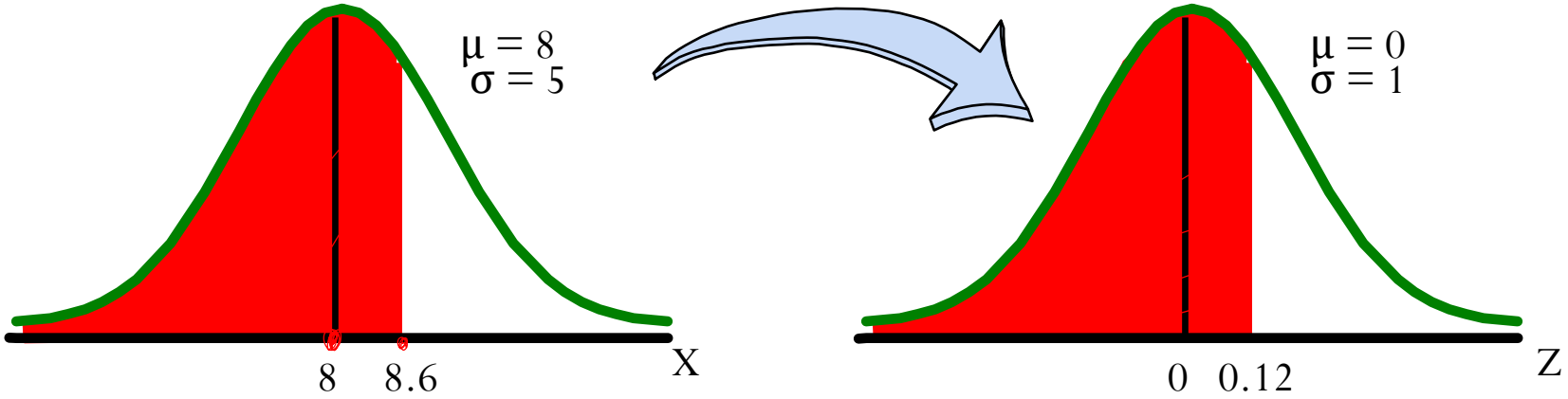


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Finding Normal Probabilities

- Suppose X is normal with mean 8.0 and standard deviation 5.0. **Find P(X < 8.6)**

$$Z = \frac{X - \mu}{\sigma} = \frac{8.6 - 8.0}{5.0} = 0.12$$

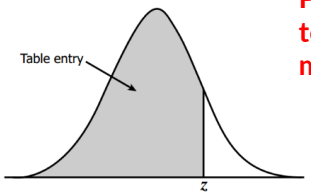


P(X < 8.6) P(Z < 0.12)

$$P(X < 8.6) = P(Z < 0.12) = 0.5 + P(0 < Z < 0.12) = 0.5 + 0.0478 = 0.5478$$

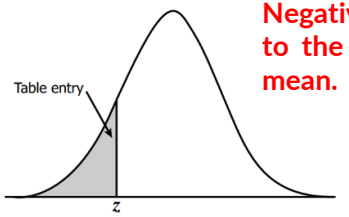
Continuous Probability Distributions

Separate tables for greater than and less than the mean



Positive scores in the Z-table correspond to the values which are greater than the mean.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998



Negative scores in the z-table correspond to the values which are less than the mean.

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

Finding Normal Probabilities

Example: Marks in an exam are normally distributed with mean 60 and standard deviation 0f 4. Find the percent of marks **a) less than 55 b) more than 70 c) Between 65 and 75.**

$$Z = \frac{X - \mu}{\sigma}$$

Solution: Here we have $\mu = 60$, $\sigma = 4$

a) less than 55

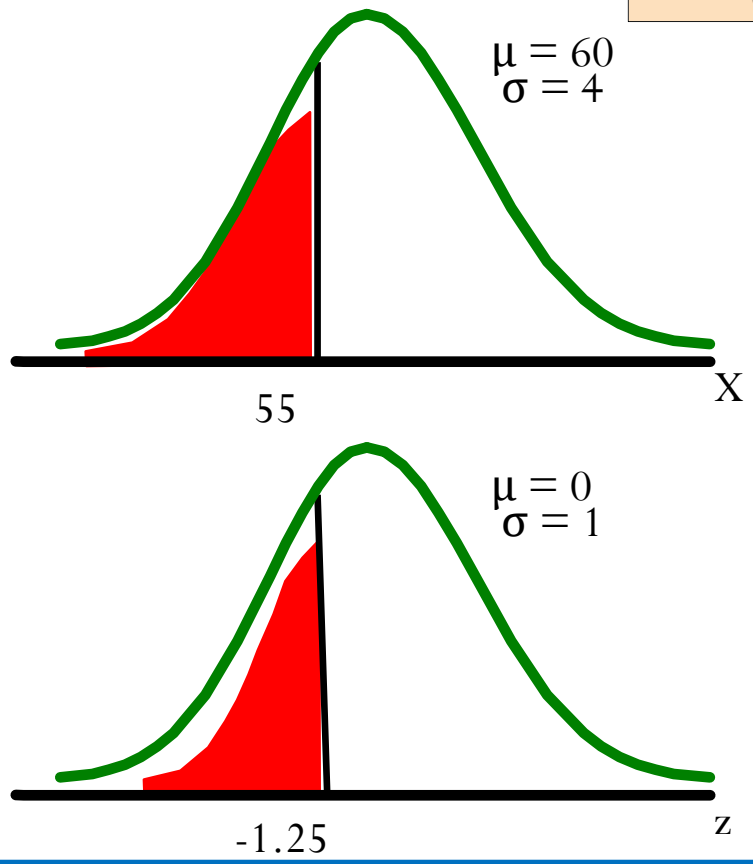
$P(X < 55)$

$$Z = \frac{55 - 60}{4} = \frac{-5}{4} = -1.25$$

$P(X < 55) = P(Z < -1.25)$

The Z value in table is .1056

$P(Z < -1.25) = 0.1056$



Finding Normal Probabilities

- Marks in an exam are normally distributed with mean 60 and standard deviation 0f 4. Find the percent of marks **a) less than 55 b) more than 70 c) Between 65 and 75.**

$$Z = \frac{X - \mu}{\sigma}$$

Solution: Here we have $\mu = 60$, $\sigma = 4$

b) More than 70

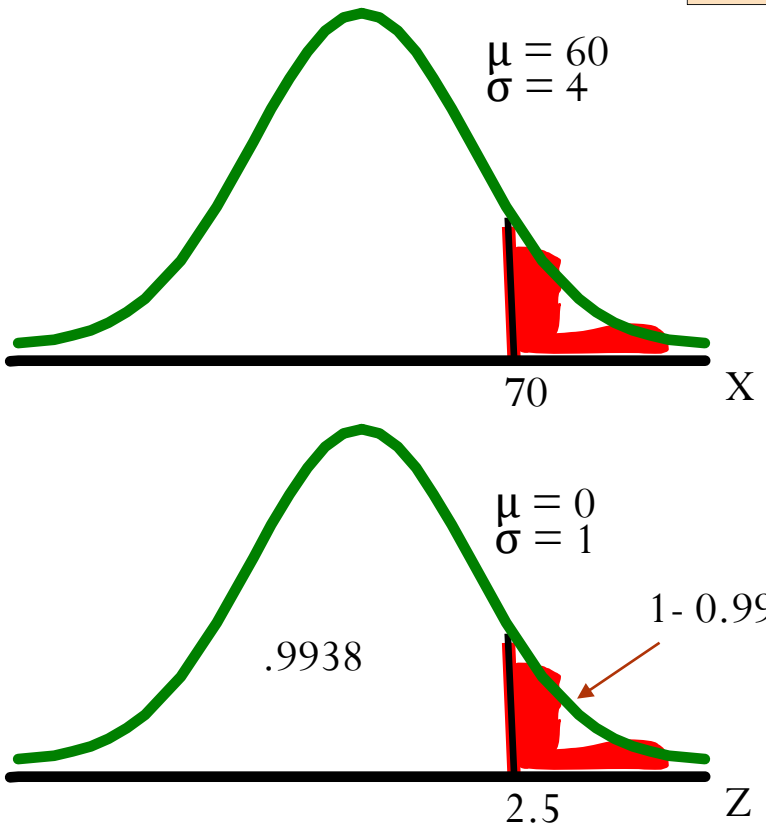
$P(X > 70)$

$$Z = \frac{70 - 60}{4} = \frac{10}{4} = 2.5$$

$P(X > 70) = P(Z > 2.5)$

The Z value in table is .9938

$P(Z > 2.5) = 1 - 0.9938 = 0.0062$



Finding Normal Probabilities

- Marks in an exam are normally distributed with mean 60 and standard deviation 0f 4. Find the percent of marks **a) less than 55 b) more than 70 c) Between 65 and 75.**

$$Z = \frac{X - \mu}{\sigma}$$

Solution: Here we have $\mu = 60$, $\sigma = 4$

b) Between 65 and 75

P(65<X <75)

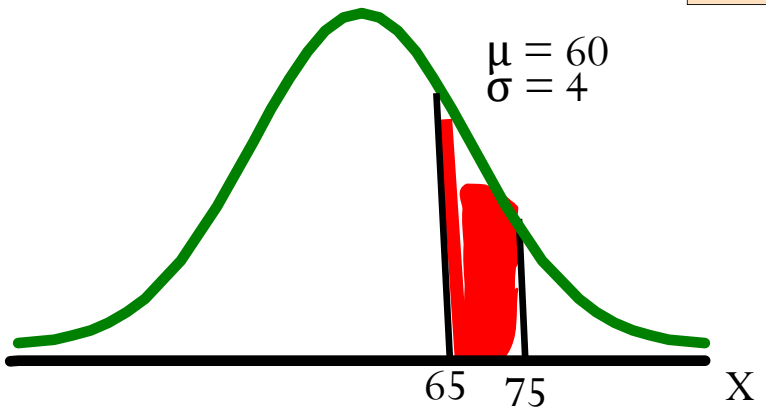
$$Z = \frac{65 - 60}{4} = \frac{5}{4} = 1.25$$

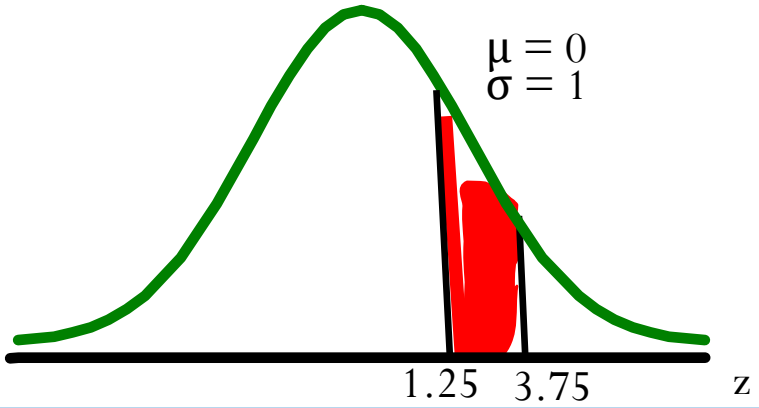
$$Z = \frac{75 - 60}{4} = \frac{15}{4} = 3.75$$

P(65<X <75) = P(1.25<Z < 3.75)

The Z value in table for 1.25 is 0.8944 and for 3.75 is 0.9999

P(1.25<X < 3.75) = 0.9999 - 0.8944 = 0.1055





Finding Normal Probabilities

Example: A 100-watt light bulb has an average brightness of 1640 lumens, with a standard deviation of 62 lumens.

a) What is the probability that a 100-watt light bulb will have a brightness more than 1800 lumens?

$$Z = \frac{X - \mu}{\sigma}$$

Solution: Here $\mu = 1640$, $\sigma = 62$, $X = 1800$

$$Z = \frac{1800 - 1640}{62} = 2.58$$

$P(X > 1800) = P(Z > 2.58)$

$= 1 - 0.9951 = 0.0049$

Finding Normal Probabilities

Example: A 100-watt light bulb has an average brightness of 1640 lumens, with a standard deviation of 62 lumens.

b) What is the probability that a 100-watt light bulb will have a brightness less than 1550 lumens?

$$Z = \frac{X - \mu}{\sigma}$$

Solution: Here $\mu = 1640$, $\sigma = 62$, $X = 1550$

$$Z = \frac{1550 - 1640}{62} = -1.45$$

$P(X<1550) = P(Z<-1.45)$
$= 0.0735$

Finding Normal Probabilities

Example: A 100-watt light bulb has an average brightness of 1640 lumens, with a standard deviation of 62 lumens.

b) What is the probability that a 100-watt light bulb will have a brightness between 1600 and 1700 lumens?

$$Z = \frac{X - \mu}{\sigma}$$

Solution: Here $\mu = 1640$, $\sigma = 62$,

P(1600<X <1700)

$$Z = \frac{1600 - 1640}{62} = -0.65$$

$$Z = \frac{1700 - 1640}{62} = 0.97$$

P(1600<X <1700)= P(-0.65<Z < 0.97)

The Z value in table for -0.65 is 0.2578 and for 0.97 is 0.8340

P(-0.65<X < 0.97) = 0.8340-0.2578= 0.5762

Finding Normal Probabilities

Example: The average height of a students in Sejong University is 65 inches with a standard deviation of 3 inches. Find

A) $P\,(X \leq 60)$

$$Z = \frac{X - \mu}{\sigma}$$

B) $P\,(X \geq 70)$

C) $P\,(60 \leq X \leq 70)$

Summary

- Uniform Distribution
- Normal Distribution


Thank
You !