Numerical Interpretation

Numerical Integration

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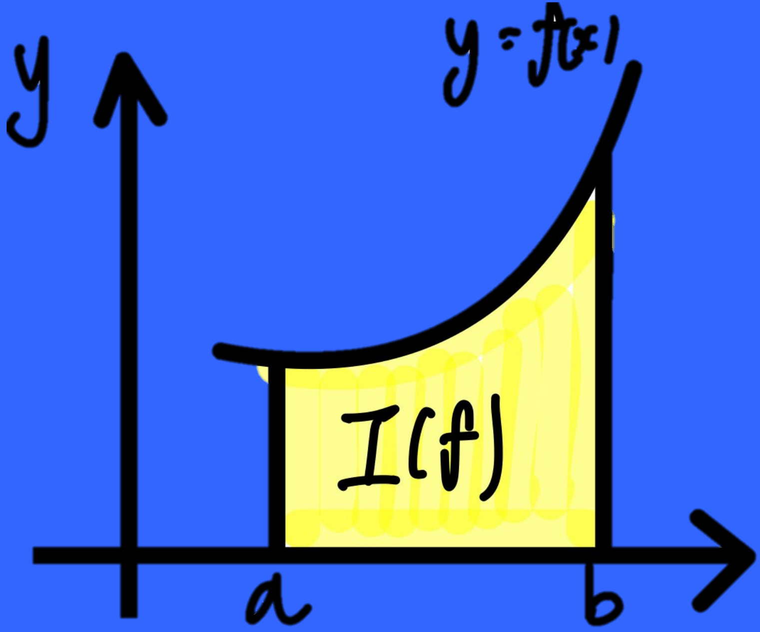
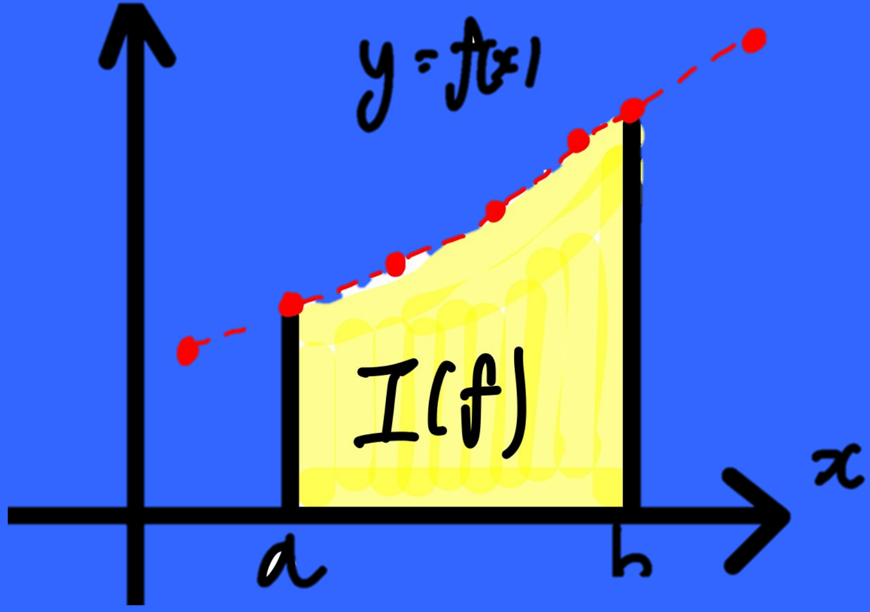
Why use numerical integration?

Discrete real-world data obtained through experiments or observations is often challenging to analytically integrate due to its complexity.

As a result, numerical integration is used to approximate the integral and obtain an estimation, providing a practical solution despite not yielding exact values.





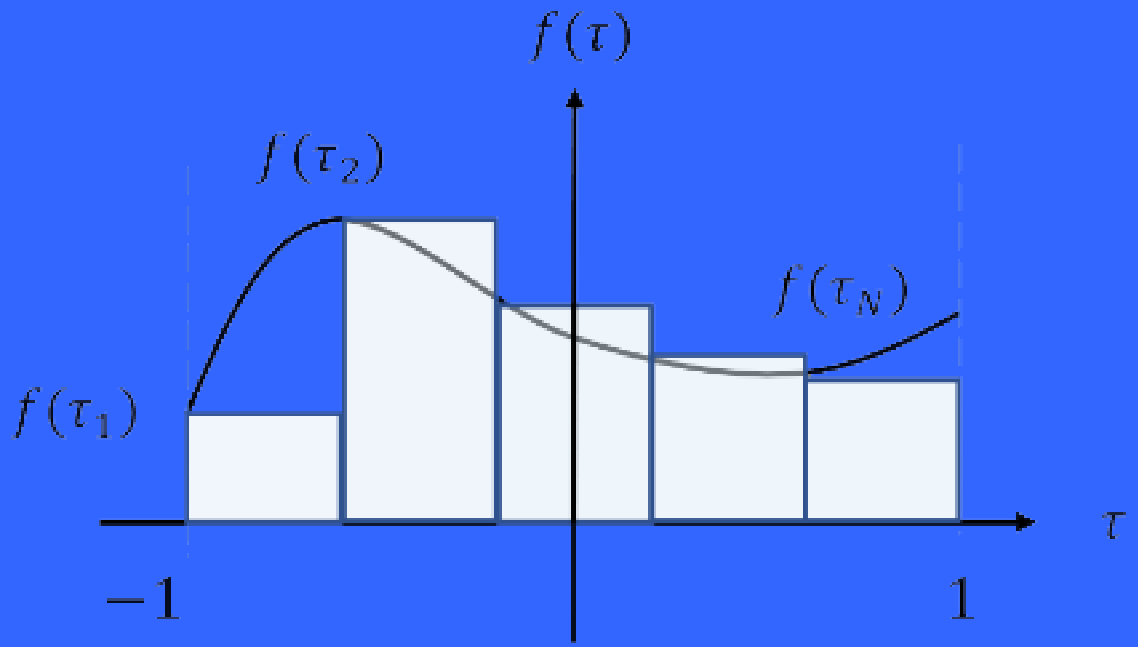
There are two parts of numerical integration

Newton-cotes formulas

Gaussian Quadrature

Gaussian Quadrature

It is a numerical integration method that calculates the weighted sum of function values over a specific interval



$$\int_{-1}^{1} f(\tau)\, d\tau \approx \sum_{i=1}^{N} w_i f(\tau_i)$$

Newton-cotes formulas

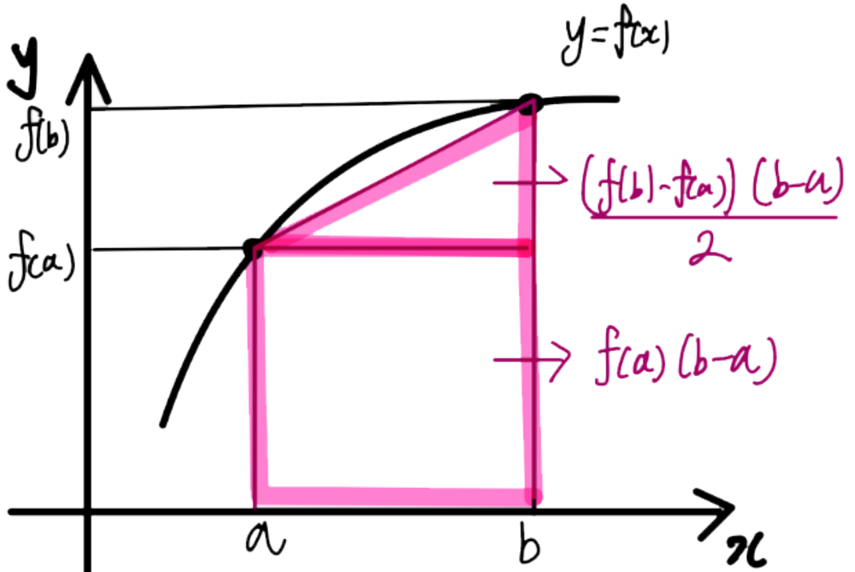
Trapezoid Rule	Newton-cotes closed formulas
Simpson Rule	Newton-cotes closed formulas
Mid-point Rule	Newton-cotes open formulas

Trapezoid Rule

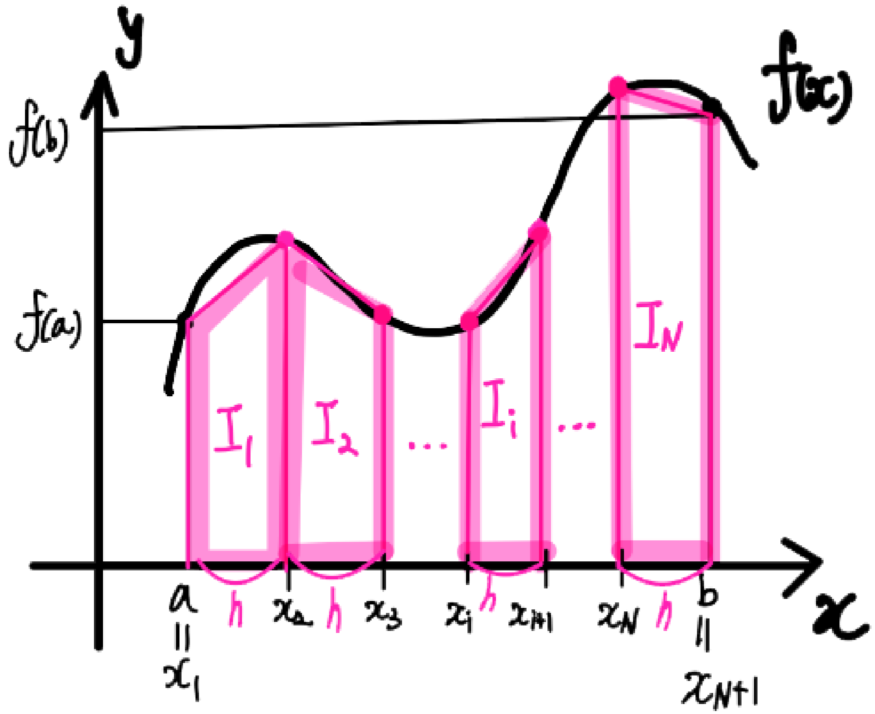
Explanation

The method involves integrating the area under the function that is formed by connecting two points with a linear interpolating polynomial, which is a first-degree polynomial.

To reduce errors, the integration interval is divided into smaller segments for calculation. This is known as the composite trapezoidal rule.



$$\int_a^b f(x)dx \approx f(a)(b-a) + \frac{1}{2}(f(b)-f(a))(b-a)$$

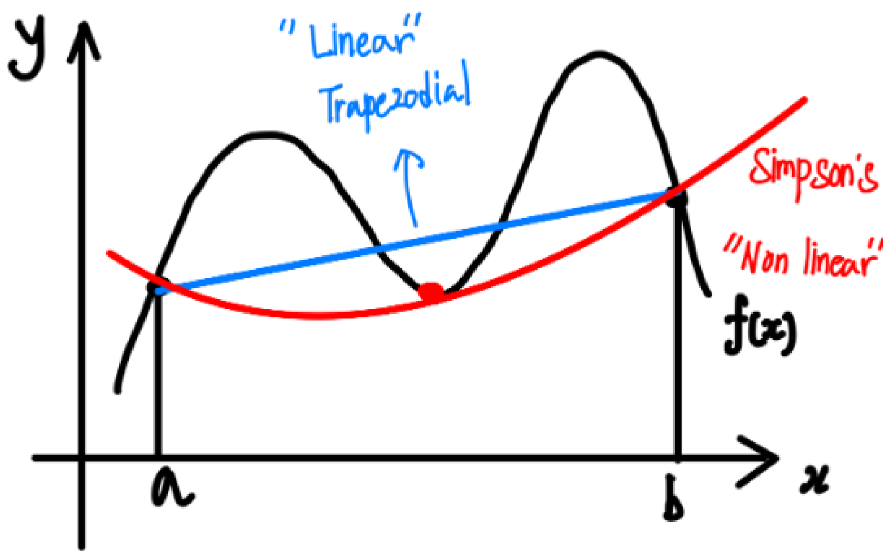


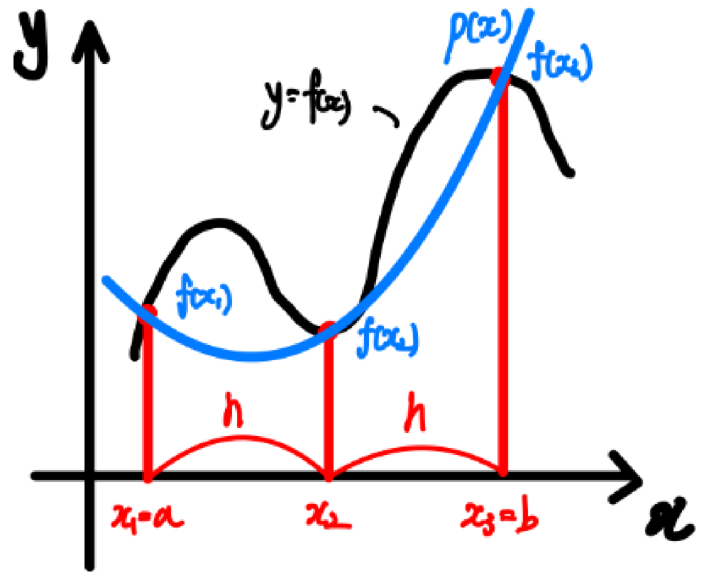
Simpson Rule

Explanation

The method involves integrating the area under the function that is formed by connecting points with a quadratic or cubic interpolating polynomial, which are second-degree or third-degree polynomials, respectively.

Numerical integration based on second-degree interpolating polynomials is called Simpson's 1/3 rule, while numerical integration based on third-degree interpolating polynomials is referred to as Simpson's 3/8 rule.



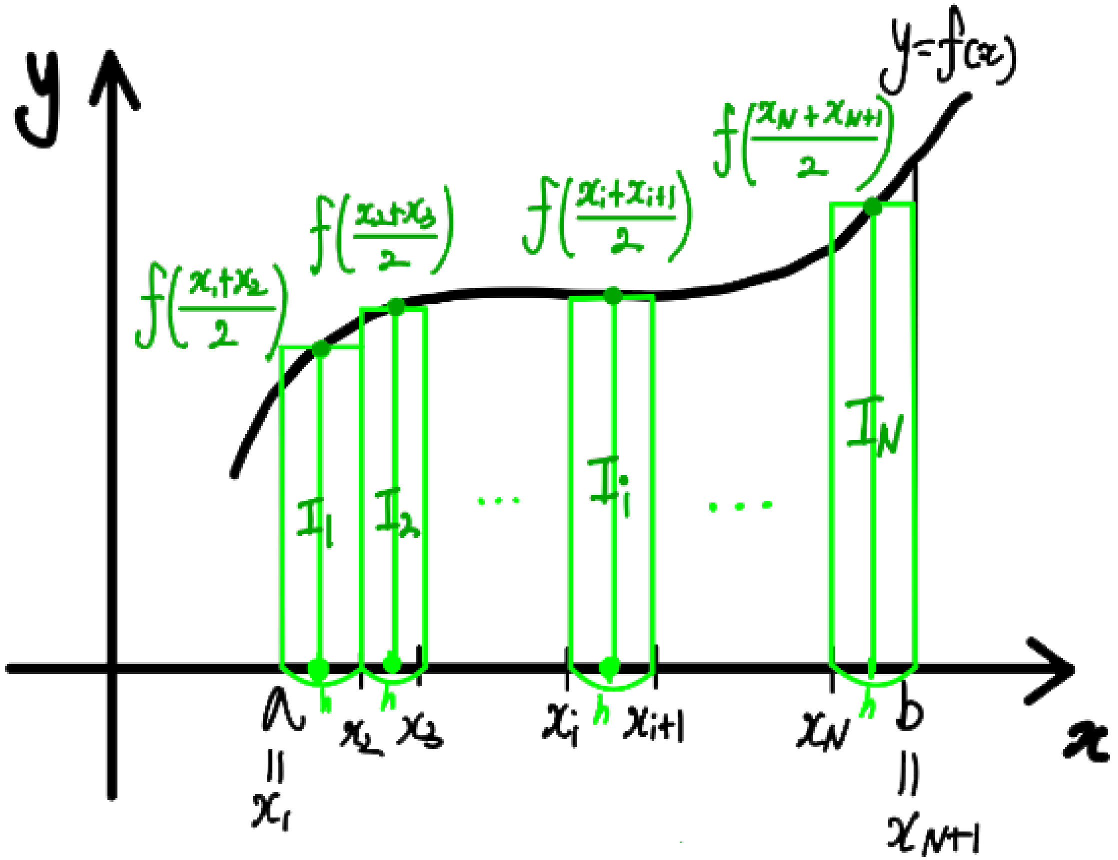


Mid-point Rule

Explanation

The method involves calculating the area of rectangles formed by using the function values at the midpoints of each interval as the height, without considering the function values at the left and right endpoints.

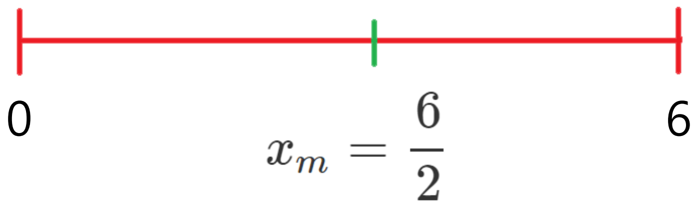
$$I(f)=\int_a^b f(x)\,dx \approx h\sum_{i=1}^{N} f\left(\frac{x_i+x_{i+1}}{2}\right)$$



Question: Using the mid-point rule to approximate the integral

Example

$$\int_0^6 x^2\,dx$$



Solution: by mid-point rule, we have

$$a = 0 \qquad x_m = \frac{0+6}{2} \qquad b = 6$$

$$\int_0^6 x^2\,dx = (6-0)f(x_m) = 6f(\frac{6}{2})$$

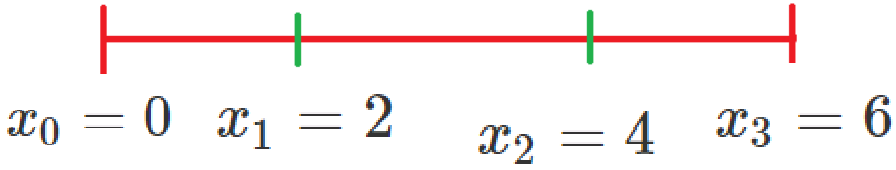
$$= 6 * 3^2 = 54$$

Exact value = 72

Question: Using the mid-point rule to approximate the integral

Example
Two point formula

$$\int_0^6 x^2\,dx$$



Solution: by mid-point rule, we have

$$\int_0^6 x^2\,dx = \frac{6-0}{2}[f(2)+f(4)] = 3[2^2+4^2]$$
$$= 60$$

END