Numerical Analysis Final exam

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Q1. Euler's method

step size = 0.3

$\frac{dy}{dx} + x^2$, $y_0 = 1$ $\Rightarrow$ init $x_0 = 0$, $y_0 = 1$, find $y(0.9)$

i) $x_1 = 0 + 0.3 = 0.3$

$y_1 = 1 + 0.3 \cdot (0^2) = 1$

ii) $x_2 = 0.3 + 0.3 = 0.6$

$y_2 = 1 + 0.3 \cdot (0.3)^2 = 1.027$

iii) $x_3 = 0.6 + 0.3 = 0.9$

$y_3 = 1.027 + 0.3 \cdot (0.6)^2 = 1.135$

$\hookrightarrow y(0.9) = \boxed{1.135}$

Gauss-Seidel iteration method

Q2 $x_{i+1} = \frac{1}{83}(95 - 11y_i + 4z_i)$

$y_{i+1} = \frac{1}{52}(104 - 7x_i - 13z_i)$

$z_{i+1} = \frac{1}{29}(71 - 3x_i - 8y_i)$

$(x_0, y_0, z_0) = (0, 0, 0)$

i) $x_1 = \frac{95}{83} = 1.1446$

$y_1 = \frac{1}{52}(104 - 7 \cdot \frac{95}{83}) = 1.8459$

$z_1 = \frac{1}{29}(71 - 3 \cdot \frac{95}{83} - 8 \cdot 1.8459) = 1.8207$

ii) $x_2 = \frac{1}{83}(95 - 11 \cdot 1.8459 + 4 \cdot 1.8207) = 0.9877$

~~$\frac{1}{83}(8.$~~

$y_2 = \frac{1}{52}(104 - 7 \cdot 0.9877 - 13 \cdot 1.8207) = 1.4117$

$z_2 = \frac{1}{29}(71 - 3 \cdot 0.9877 - 8 \cdot 1.4117) = 1.9567$

iii) $x_3 = \frac{1}{83}(95 - 11 \cdot 1.4117 + 4 \cdot 1.3567) = 1.0518$

$y_3 = \frac{1}{52}(104 - 7 \cdot 1.0518 - 13 \cdot 1.9567) = 1.3692$

$z_3 = \frac{1}{29}(71 - 3 \cdot 1.0518 - 8 \cdot 1.3692) = 1.9618$

iv) $x_4 = \frac{1}{83}(95 - 11 \cdot 1.3692 + 4 \cdot 1.9618) = \boxed{1.0577}$

$y_4 = \frac{1}{52}(104 - 7 \cdot 1.0577 - 13 \cdot 1.9618) = \boxed{1.3672}$

$z_4 = \frac{1}{29}(71 - 3 \cdot 1.0577 - 8 \cdot 1.3672) = \boxed{1.9617}$

Q3.

x	0	1	2	5
$f(x)$	2	3	12	147

Lagrange polynomial

$$f(x) = f_0 l_0 + f_1 l_1 + f_2 l_2 + f_3 l_3$$

$$= 2 \cdot \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$+ 3 \cdot \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x-x_3)}$$

$$+ 12 \cdot \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$$+ 147 \cdot \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= 2 \cdot \frac{(x-1)(x-2)(x-5)}{-1 \cdot (-2) \cdot (-5)}$$

$$+ 3 \cdot \frac{(x-0)(x-2)(x-5)}{1 \cdot (-1) \cdot (-4)}$$

$$+ 12 \cdot \frac{(x-0)(x-1)(x-5)}{2 \cdot 1 \cdot (-3)}$$

$$+ 147 \cdot \frac{(x-0)(x-1)(x-2)}{5 \cdot 4 \cdot 3}$$

$$= -\frac{1}{5}(x^3 - 8x^2 + 17x - 10)$$
$$+ \frac{3}{4}(x^3 - 7x^2 + 10x)$$
$$- 2(x^3 - 6x^2 + 5x)$$
$$+ \frac{49}{20}(x^3 - 3x^2 + 2x)$$

$$= x^3 + x^2 - x + 2$$

↳ $f(3) = 27 + 9 - 3 + 2 = \boxed{35}$

$$f'(x) = 3x^2 + 2x - 1$$

$$f'(3) = 27 + 6 - 1 = \boxed{32}$$

Q4 Newton's divided difference formula

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

↳

x	$f(x)$	1st	2nd	3rd	4th	5th
4	48					
		52				
5	100		15			
		97		1		
7	294		21		0	
		202		1		0
10	900		27		0	
		310		1		
11	1210		33			
		409				
13	2028					

$$f(x) = f_0 + \Delta f_0 (x - x_0) + \Delta^2 f_0 (x - x_0)(x - x_1)$$
$$+ \Delta^3 f_0 (x - x_0)(x - x_1)(x - x_2)$$
$$+ \Delta^4 f_0 (x - x_0)(x - x_1)(x - x_2)(x - x_3)$$
$$\vdots$$

$$\Rightarrow (x-4)(x-5)(x-7) + 15(x-4)(x-5) + 52(x-4) + 48$$

$$= x^3 - x^2 - 48,$$

$f(8) = 400,\ f(9) = 600,\ f(15) = 3102$

Q5. Richardson extrapolation

x	1	2	3	4	5
$f(x)$	2	4	8	16	32

$h = 2$

$$D(h) = \frac{f(x_3+h) - f(x_3-h)}{2h} = \frac{f(5) - f(1)}{4}$$

$$= \frac{32 - 2}{4} = 7.5$$

$$D\left(\frac{h}{2}\right) = \frac{f(x_3+\frac{h}{2}) - f(x_3-\frac{h}{2})}{h} = \frac{f(4) - f(2)}{2}$$

$$= \frac{16 - 4}{2} = 6$$

$$D = \frac{4 \cdot D(\frac{h}{2}) - D(h)}{3} = \frac{4 \cdot 6 - 7.5}{3} = \frac{16.5}{3}$$

$$= \boxed{5.5}$$