Lab 3

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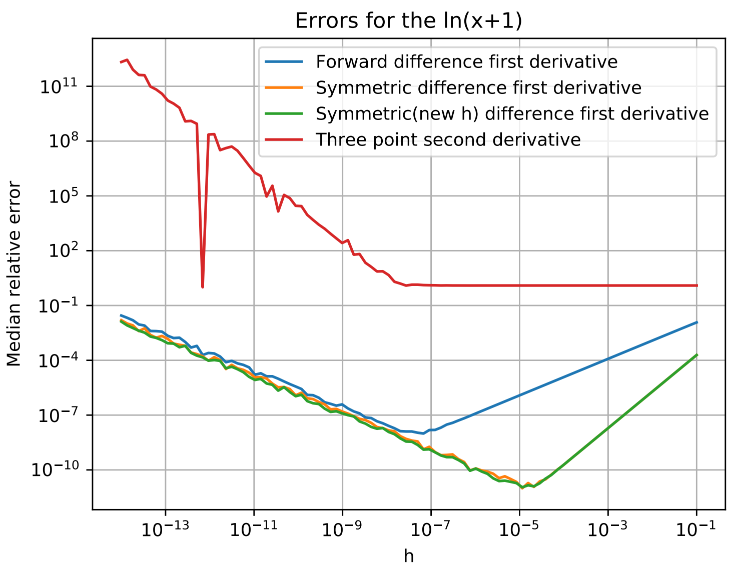
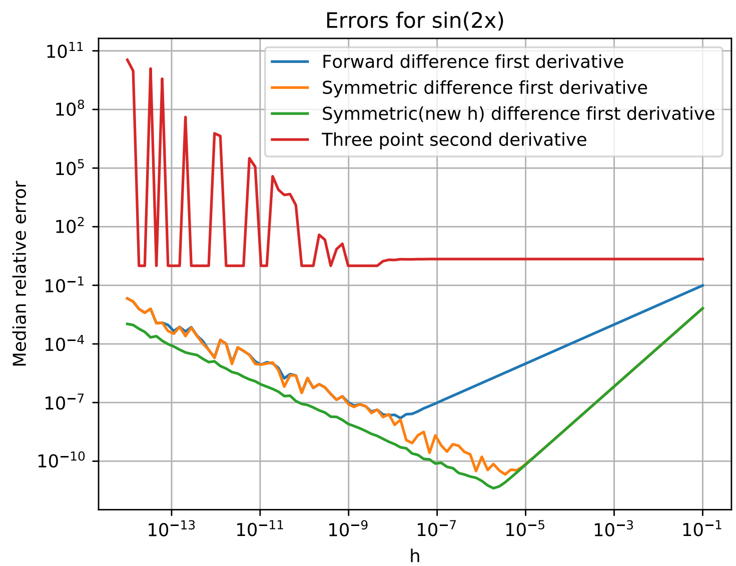
1 Part 1. Precision of Numerical Differentiation

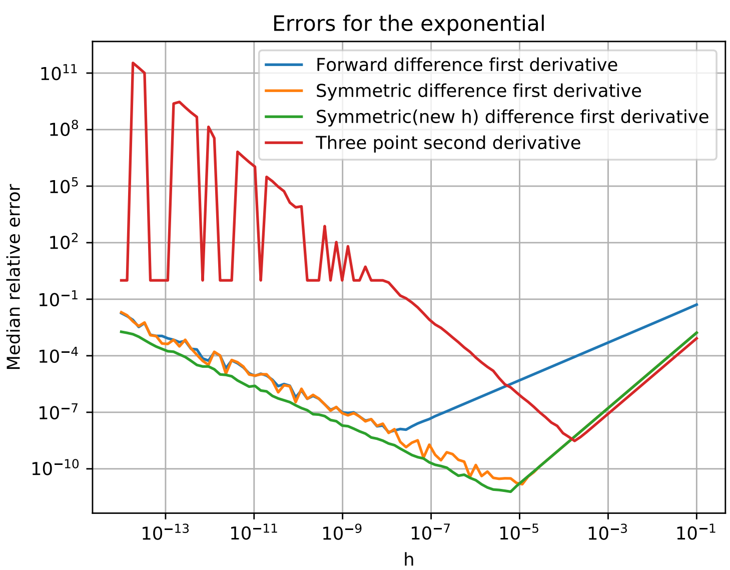
The slope of the plots inside each region agrees with my expectations. The region of decreasing slope is where the round-off error dominates. The region of increasing slope is where the approximation error dominates. At first the slope did not agree but after correcting the spacing of the range of h the slopes began to look like $\mathcal{O}(h)$ and $\mathcal{O}(h^2)$ respectively. Section 2 depicts the median relative error plots from attempting to take three first derivatives and one second derivative of the functions e^x, $ln(x+1)$, and $sin(2x)$.

2 Plots

Please find the plots on page 2.


Errors for the ln(x+1)
Forward difference first derivative
Symmetric difference first derivative
Symmetric(new h) difference first derivative
Three point second derivative
Median relative error
10^{11}
10^{8}
10^{5}
10^{2}
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-11}
10^{-9}
10^{-7}
10^{-5}
10^{-3}
10^{-1}
h


Errors for sin(2x)
Forward difference first derivative
Symmetric difference first derivative
Symmetric(new h) difference first derivative
Three point second derivative
Median relative error
10^{11}
10^{8}
10^{5}
10^{2}
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-11}
10^{-9}
10^{-7}
10^{-5}
10^{-3}
10^{-1}
h


Errors for the exponential
Forward difference first derivative
Symmetric difference first derivative
Symmetric(new h) difference first derivative
Three point second derivative
Median relative error
10^{11}
10^{8}
10^{5}
10^{2}
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-11}
10^{-9}
10^{-7}
10^{-5}
10^{-3}
10^{-1}
h

3 Part 2. Optimizing Projectile Firing Angle

4 Choices

In this section, I will describe the choices for certain parameters and reasons for those choices.
I chose the Runge-Kutta RK45 algorithm since it allowed me to ensure that the flight distance as a function of the firing angle has a continouous second derivative. Parabolic interpolations, i.e. Brent's method, are not useful unless that second derivative is constant hence the choice of Runge-Kutta RK45.