深度滤波

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第一章 深度滤波

1.1 问题引出

在沿着极线搜索，计算特征点（块）的相似性过程中，通常会得到一个非凸函数：如图 1.1.1 左图所示，纵坐标表示相似度量，横坐标表示深度值，图中函数存在多个极大值，且真实的深度值只有一个，然而匹配度量容易受到纹理，光照等因素影响，因此对于取哪个极大值作为深度值存在不确定性。

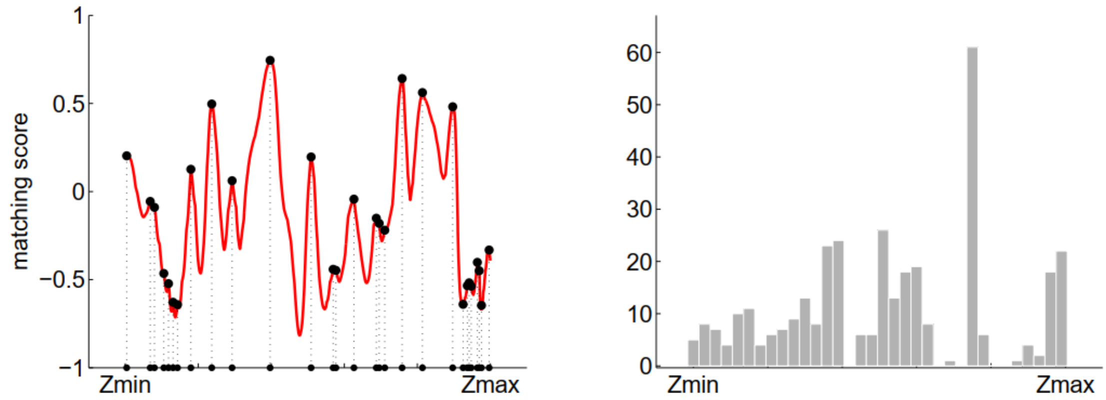

图 1.1.1 匹配得分沿距离分布（左图），60 幅图像极大值累加（右图），图来自文献[1]

对于多幅图像而言，那就能计算出这一点的多个匹配得分沿距离分布，把这些分布的极大值点累加，构成了极大值直方图，如图 1.1.1 的右图所示。即便知道了最高的那个直方图对应的深度值，但也不能确定是该点为外点的概率。

因此，需要一个方法不仅能推算出深度值，而且能知晓特征点对应的外点的概率。

1.2 问题解决

1.2.1 问题分析

我们知道某个像素在两幅图中的位置，就能利用三角测量法确定它的深度。进而可以利用多幅图的测量（约束），使得深度估计从一个不确定的量，逐渐收敛到一个稳定值[2]。如图 1.2.1 所示，我们假设深度值符合高斯分布，一开始深度估计的不确定性较大（浅绿色部分），通过三角化得到一个深度估计值以后，能够极大的缩小这个不确定性（墨绿色部分）[3]。

对像素点深度的估计，本身也可建模为一个状态估计问题，于是就自然存在滤波器与非线性优化两种求解思路。虽然非线性优化效果较好，但是在实时性要求场合，通常采用计算量较少的滤波器方式[2]。文献[1]提到的滤波模型很好解决

了深度值估计以及内点概率不确定得问题——深度滤波，它可以通过多次观测，计算出一个地图点是假的地图点还是真实地图点的概率，并且计算出其作为真实地图点的最有可能的位置，其模型公式如式 1.1 所示。

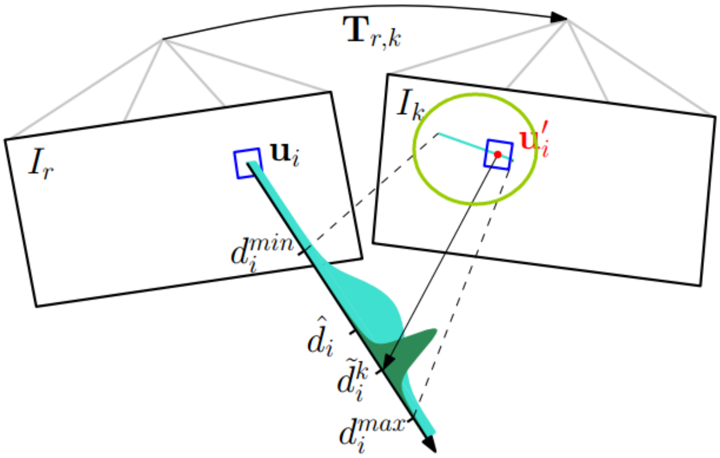

图 1.2.1 三角化计算深度值，图来自文献[4]

1.2.2 深度滤波器

给定已知相对位姿的两个视角下的图像 ref，cur。由两幅图像中的对应点及位姿开源计算得到一个深度值d，又因重建误差和误匹配的存在，考察实际情况中d的直方图分布，[1]认为 d 的分布可以用高斯分布和均匀分布来联合表示[5]，其模型如式 1.1 所示：

$$p(d_k|\hat{d},\rho) = \rho N(d_k|\hat{d},\tau_n^2) + (1-\rho)U(d_k|d_{min},d_{max}) \tag{1.1}$$

其中 $\hat{d}$ 为真实的深度值，ρ是内点概率，d_k 服从均匀-高斯分布，其均值为$\hat{d}$，方差为τ_n^2。根据贝叶斯定理，其最大后验概率为：

$$p(\hat{d},\rho|d_{r+1},\dots,d_k) \propto p(\hat{d},\rho)\prod_l p(d_l|\hat{d},\rho) \tag{1.2}$$

其中 $p(\hat{d},\rho)$ 为先验概率，为了更好的迭代求解，使用 Gaussian × Beta 分布来近似右端项。

$$q(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2) = Beta(\rho|a_k,b_k)N(\hat{d}|,\mu_k,\sigma_k^2) \tag{1.3}$$

上式中 $\hat{d}$ 服从高斯分布，ρ 服从 Beta 分布。更新方程如下：

$$p(\hat{d},\rho|d_{r+1},\dots,d_k) = q(\hat{d},\rho|a_{k-1},b_{k-1},\mu_{k-1},\sigma_{k-1}^2)\,p(d_k|\hat{d},\rho) \tag{1.4}$$

虽然式 1.4 不在符合 Gaussian × Beta 分布，但是我们依然把它近似为该分布，同样使用$q(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2)$来表示。将式 1.1 代入 1.4 有：

$$\begin{aligned}
&q(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2)\\
&= Beta(\rho|a_k,b_k)N(\hat{d}|,\mu_k,\sigma_k^2)\\
&= [\rho N(d_k|\hat{d},\tau_n^2) + (1-\rho)U(d_k|d_{min},d_{max})]N(\hat{d}|,\mu_{k-1},\sigma_{k-1}^2)Beta(\rho|a_{k-1},b_{k-1})
\end{aligned} \tag{1.5}$$

根据 Beta 分布性质：

$$\begin{cases} Beta(\rho|a,b)=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\rho^{a-1}(1-\rho)^{b-1} \\ Beta(\rho|a+1,b)=\rho\frac{a+b}{a}Beta(\rho|a,b) \\ Beta(\rho|a,b+1)=(1-\rho)\frac{a+b}{b}Beta(\rho|a,b) \end{cases} \tag{1.6}$$

将式 1.6 代入式 1.5：

$$\begin{aligned} & q\left(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2\right) \\ & =Beta(\rho|a_k,b_k)N\left(\hat{d}|,\mu_k,\sigma_k^2\right) \\ & =\rho N\left(d_k|\hat{d},\tau_k^2\right)N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1},b_{k-1}) \\ & \qquad +(1-\rho)U(d_k|d_{min},d_{max})N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1},b_{k-1}) \\ & =\frac{a_{k-1}}{a_{k-1}+b_{k-1}}N\left(d_k|\hat{d},\tau_n^2\right)N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1}+1,b_{k-1})+ \\ & \frac{b_{k-1}}{a_{k-1}+b_{k-1}}U(d_k|d_{min},d_{max})N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1},b_{k-1}+1) \end{aligned} \tag{1.7}$$

根据附录公式 A.1，式 1.7 可以转换为：

$$\begin{aligned} & q\left(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2\right) \\ & =\frac{a_{k-1}}{a_{k-1}+b_{k-1}}N\left(d_k|\mu_{k-1},\sigma_{k-1}^2+\tau_n^2\right)N\left(\hat{d}|m,s^2\right)Beta(\rho|a_{k-1}+1,b_{k-1})+ \\ & \frac{b_{k-1}}{a_{k-1}+b_{k-1}}U(d_k|d_{min},d_{max})N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1},b_{k-1}+1) \end{aligned} \tag{1.8}$$

令$C_1=\frac{a_{k-1}}{a_{k-1}+b_{k-1}}N\left(d_k|\mu_{k-1},\sigma_{k-1}^2+\tau_n^2\right)$，$C_2=\frac{b_{k-1}}{a_{k-1}+b_{k-1}}U(d_k|d_{min},d_{max})$，代入公式 1.8：

$$\begin{aligned} & q\left(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2\right) \\ & =C_1N\left(\hat{d}|m,s^2\right)Beta(\rho|a_{k-1}+1,b_{k-1})+C_2N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1},b_{k-1}+1) \end{aligned} \tag{1.9}$$

计算公式 1.9 关于 $\hat{d}$ 的一阶矩和二阶矩，具体形式见附录 A.2, A.3, A.4：

$$\begin{cases} C=C_1+C_2,C_1'=\frac{C_1}{C},C_2'=\frac{C_2}{C} \\ \mu_k=C_1'm+C_2'\mu_{k-1} \\ \mu_k^2+\sigma_k^2=C_1'(m^2+s^2)+C_2'\left(\mu_{k-1}^2+\sigma_{k-1}^2\right) \end{cases} \tag{1.10}$$

同理，计算公式 1.9 关于 ρ 的一阶矩和二阶矩，具体形式见附录 A.5, A.6, A.7：

$$\begin{cases} \frac{a_k}{a_k+b_k}=C_1'\frac{a_{k-1}+1}{a_{k-1}+b_{k-1}+1}+C_2'\frac{a_{k-1}}{a_{k-1}+b_{k-1}+1} \\ \frac{a_k(a_k+1)}{(a_k+b_k)(a_k+b_k+1)}=C_1'\frac{(a_{k-1}+1)(a_{k-1}+2)}{(a_{k-1}+b_{k-1}+1)(a_{k-1}+b_{k-1}+2)} \\ \qquad +C_2'\frac{a_{k-1}(a_{k-1}+1)}{a_{k-1}+b_{k-1}+1(a_{k-1}+b_{k-1}+1)(a_{k-1}+b_{k-1}+2)} \end{cases} \tag{1.11}$$

附录

附录公式 A.1

$$N\left(d_k|\hat{d},\tau_k^2\right)\cdot N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)$$

$$=\frac{1}{\sqrt{2\pi}\tau_k}e^{-\frac{\left(d_k-\hat{d}\right)^2}{2\tau_k^2}}\cdot\frac{1}{\sqrt{2\pi}\sigma_{k-1}}e^{-\frac{\left(\hat{d}-\mu_{k-1}\right)^2}{2\sigma_{k-1}^2}}$$

$$=\frac{1}{2\pi\tau_k\sigma_{k-1}}e^{-\left\{\frac{\left[\left(\tau_k^2+\sigma_{k-1}^2\right)\hat{d}-\left(\sigma_{k-1}^2d_k+\tau_k^2\mu_{k-1}\right)\right]^2+\left(\tau_k^2\sigma_{k-1}^2\right)\left(d_k-\mu_{k-1}\right)^2}{2\tau_k^2\sigma_{k-1}^2\left(\tau_k^2+\sigma_{k-1}^2\right)}\right\}}$$

$$=\frac{1}{2\pi\tau_k\sigma_{k-1}}e^{-\left\{\frac{\left[\hat{d}-\frac{\left(\sigma_{k-1}^2d_k+\tau_k^2\mu_{k-1}\right)}{\left(\tau_k^2+\sigma_{k-1}^2\right)}\right]^2}{\frac{2\tau_k^2\sigma_{k-1}^2}{\left(\tau_k^2+\sigma_{k-1}^2\right)}}+\frac{(d_k-\mu_{k-1})^2}{2\left(\tau_k^2+\sigma_{k-1}^2\right)}\right\}}$$

$$=\frac{1}{\sqrt{2\pi}\frac{\tau_k\sigma_{k-1}}{\sqrt{\tau_k^2+\sigma_{k-1}^2}}}e^{-\frac{\left[\hat{d}-\frac{\left(\sigma_{k-1}^2d_k+\tau_k^2\mu_{k-1}\right)}{\left(\tau_k^2+\sigma_{k-1}^2\right)}\right]^2}{\frac{2\tau_k^2\sigma_{k-1}^2}{\left(\tau_k^2+\sigma_{k-1}^2\right)}}}\cdot\frac{1}{\sqrt{2\pi}\sqrt{\tau_k^2+\sigma_{k-1}^2}}e^{-\frac{(d_k-\mu_{k-1})^2}{2\left(\tau_k^2+\sigma_{k-1}^2\right)}}$$

$$=N\left(\hat{d}|\frac{\sigma_{k-1}^2d_k+\tau_k^2\mu_{k-1}}{\tau_k^2+\sigma_{k-1}^2},\frac{\tau_k^2\sigma_{k-1}^2}{\tau_k^2+\sigma_{k-1}^2}\right)N\left(d_k|\mu_{k-1},\tau_k^2+\sigma_{k-1}^2\right)$$

$$=N\left(\hat{d}|m,s^2\right)N\left(d_k|\mu_{k-1},\tau_k^2+\sigma_{k-1}^2\right)$$

其中，$\frac{1}{s^2}=\frac{1}{\tau_k^2}+\frac{1}{\sigma_{k-1}^2}$，$m=s^2\left(\frac{d_k}{\tau_k^2}+\frac{\mu_{k-1}}{\sigma_{k-1}^2}\right)$

附录公式 A.2

$$\int_{\hat{d},\rho} q\left(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2\right)\mathrm{d}\hat{d}\,\mathrm{d}\rho$$

$$=\int_{\hat{d},\rho} C_1 N\left(\hat{d}|m,s^2\right)Beta(\rho|a_{k-1}+1,b_{k-1})\mathrm{d}\hat{d}\,\mathrm{d}\rho+\int_{\hat{d},\rho} C_2 N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1},b_{k-1}+1)\mathrm{d}\hat{d}\,\mathrm{d}\rho$$

$$=C_1\int_{\hat{d}} C_1 N\left(\hat{d}|m,s^2\right)\mathrm{d}\hat{d}\ \underbrace{\int_{\rho} Beta(\rho|a_{k-1}+1,b_{k-1})\mathrm{d}\rho}_{1}+\int_{\hat{d}} C_2 N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)\mathrm{d}\hat{d}\ \underbrace{\int_{\rho} Beta(\rho|a_{k-1},b_{k-1}+1)\mathrm{d}\rho}_{1}$$

$$=C_1\underbrace{\int_{\hat{d}} N\left(\hat{d}|m,s^2\right)\mathrm{d}\hat{d}}_{1}+C_2\underbrace{\int_{\hat{d}} N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)\mathrm{d}\hat{d}}_{1}$$

$$=C_1+C_2=C$$

附录公式 A.3

$q(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2)$

$= Beta(\rho|a_k,b_k)N(\hat{d}|,\mu_k,\sigma_k^2)$

$= C_1N(\hat{d}|m,s^2)Beta(\rho|a_{k-1}+1,b_{k-1}) + C_2N(\hat{d}|\mu_{k-1},\sigma_{k-1}^2)Beta(\rho|a_{k-1},b_{k-1}+1)$

对$\hat{d}$的一阶矩：

$$\int_{\hat{d},\rho}\hat{d}N(\hat{d}|,\mu_k,\sigma_k^2)Beta(\rho|a_k,b_k)\mathrm{d}\hat{d}\,\mathrm{d}\rho$$

$$=\int_{\hat{d}}\hat{d}N\left(\hat{d}|,\mu_k,\sigma_k^2\right)\mathrm{d}\hat{d}\underbrace{\int_{\rho}Beta\left(\rho|a_k,b_k\right)\mathrm{d}\rho}_{1}$$

$$=\mu_k$$

$$\frac{1}{C}\int_{\hat{d},\rho}\hat{d}\left[C_1N(\hat{d}|m,s^2)Beta(\rho|a_{k-1}+1,b_{k-1}) + C_2N(\hat{d}|\mu_{k-1},\sigma_{k-1}^2)Beta(\rho|a_{k-1},b_{k-1}+1)\right]\mathrm{d}\hat{d}\,\mathrm{d}\rho$$

$$=C_1\int_{\hat{d}}\hat{d}N(\hat{d}|m,s^2)\mathrm{d}\hat{d}+C_2\int_{\hat{d}}\hat{d}N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)\mathrm{d}\hat{d}$$

$$=\frac{1}{C}\left(C_1m+C_2\mu_{k-1}\right)$$

综上：$\mu_k=\frac{1}{C}(C_1m+C_2\mu_{k-1})$

附录公式 A.4

$$q\left(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2\right)$$
$$= Beta(\rho|a_k,b_k)N\left(\hat{d}|,\mu_k,\sigma_k^2\right)$$
$$= C_1N\left(\hat{d}|m,s^2\right)Beta(\rho|a_{k-1}+1,b_{k-1}) + C_2N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1},b_{k-1}+1)$$

对$\hat{d}$的二阶矩：

$$\int_{\hat{d},\rho}\hat{d}^2N\left(\hat{d}|,\mu_k,\sigma_k^2\right)Beta(\rho|a_k,b_k)\mathrm{d}\hat{d}\,\mathrm{d}\rho$$

$$= \int_{\hat{d}}\hat{d}^2N\left(\hat{d}|,\mu_k,\sigma_k^2\right)\mathrm{d}\hat{d}\underbrace{\int_{\rho}Beta(\rho|a_k,b_k)\mathrm{d}\rho}_{1}$$

$$= \mu_k^2 + \sigma_k^2$$

$$\frac{1}{C}\int_{\hat{d},\rho}\hat{d}^2\left[C_1N\left(\hat{d}|m,s^2\right)Beta(\rho|a_{k-1}+1,b_{k-1}) + C_2N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)Beta(\rho|a_{k-1},b_{k-1}+1)\right]\mathrm{d}\hat{d}\,\mathrm{d}\rho$$
$$= \frac{1}{C}C_1\int_{\hat{d}}\hat{d}^2N\left(\hat{d}|m,s^2\right)\mathrm{d}\hat{d} + C_2\int_{\hat{d}}\hat{d}^2N\left(\hat{d}|\mu_{k-1},\sigma_{k-1}^2\right)\mathrm{d}\hat{d}$$
$$= \frac{1}{C}\left[C_1\left(m^2+s^2\right) + C_2\left(\mu_{k-1}^2+\sigma_{k-1}^2\right)\right]$$

综上：$\mu_k^2 + \sigma_k^2 = \frac{1}{C}\left[C_1(m^2+s^2) + C_2\left(\mu_{k-1}^2+\sigma_{k-1}^2\right)\right]$

附录公式 A.5

$$Beta(\rho|a,b)=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\rho^{a-1}(1-\rho)^{b-1}$$

$$E(\rho)=\int_0^1\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\rho^{a}(1-\rho)^{b-1}d\rho$$

$$=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^1\rho^{a}(1-\rho)^{b-1}d\rho$$

$$=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\cdot\frac{\Gamma(a+1)\Gamma(b)}{\Gamma(a+b+1)}$$

$$=\frac{a}{a+b}$$

$$E(\rho^2)=\int_0^1\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\rho^{a+1}(1-\rho)^{b-1}d\rho$$

$$=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^1\rho^{a+1}(1-\rho)^{b-1}d\rho$$

$$=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\cdot\frac{\Gamma(a+2)\Gamma(b)}{\Gamma(a+b+2)}$$

$$=\frac{a(a+1)}{(a+b+1)(a+b)}$$

伽玛函数

$$\Gamma(x)=\int_0^{+\infty}t^{x-1}e^{-t}dt\quad t>0$$

附录公式 A.6

$$q(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2)$$
$$= Beta(\rho|a_k,b_k)N(\hat{d}|,\mu_k,\sigma_k^2)$$
$$= C_1N(\hat{d}|m,s^2)Beta(\rho|a_{k-1}+1,b_{k-1}) + C_2N(\hat{d}|\mu_{k-1},\sigma_{k-1}^2)Beta(\rho|a_{k-1},b_{k-1}+1)$$

对ρ的一阶矩：

$$\int_{\hat{d},\rho}\rho Beta(\rho|a_k,b_k)N(\hat{d}|,\mu_k,\sigma_k^2)\mathrm{d}\hat{d}\,\mathrm{d}\rho$$
$$= \underbrace{\int_{\hat{d}}\hat{d}N(\hat{d}|,\mu_k,\sigma_k^2)\mathrm{d}\hat{d}}_{1}\int_{\rho}\rho Beta(\rho|a_k,b_k)\mathrm{d}\rho$$
$$= \frac{a_k}{a_k+b_k}$$

$$\frac{1}{C}\int_{\hat{d},\rho}\rho[C_1N(\hat{d}|m,s^2)Beta(\rho|a_{k-1}+1,b_{k-1}) + C_2N(\hat{d}|\mu_{k-1},\sigma_{k-1}^2)Beta(\rho|a_{k-1},b_{k-1}+1)]\mathrm{d}\hat{d}\,\mathrm{d}\rho$$
$$= \frac{1}{C}C_1\int_{\rho}\rho Beta(\rho|a_{k-1}+1,b_{k-1})\mathrm{d}\rho + C_2\int_{\rho}\rho Beta(\rho|a_{k-1},b_{k-1}+1)\mathrm{d}\rho$$
$$= \frac{1}{C}\left[C_1\frac{a_{k-1}+1}{a_{k-1}+b_{k-1}+1} + C_2\frac{a_{k-1}}{a_{k-1}+b_{k-1}+1}\right]$$

综上：$\frac{a_k}{a_k+b_k} = \frac{1}{C}\left[C_1\frac{a_{k-1}+1}{a_{k-1}+b_{k-1}+1} + C_2\frac{a_{k-1}}{a_{k-1}+b_{k-1}+1}\right]$

附录公式 A.7

$$q(\hat{d},\rho|a_k,b_k,\mu_k,\sigma_k^2)$$
$$= Beta(\rho|a_k,b_k)N(\hat{d}|,\mu_k,\sigma_k^2)$$
$$= C_1N(\hat{d}|m,s^2)Beta(\rho|a_{k-1}+1,b_{k-1}) + C_2N(\hat{d}|\mu_{k-1},\sigma_{k-1}^2)Beta(\rho|a_{k-1},b_{k-1}+1)$$

对ρ的二阶矩：

$$\int_{\hat{d},\rho}\rho^2 Beta(\rho|a_k,b_k)N(\hat{d}|,\mu_k,\sigma_k^2)\mathrm{d}\hat{d}\,\mathrm{d}\rho$$
$$= \underbrace{\int_{\hat{d}}\hat{d}N(\hat{d}|,\mu_k,\sigma_k^2)\mathrm{d}\hat{d}}_{1}\int_{\rho}\rho^2 Beta(\rho|a_k,b_k)\mathrm{d}\rho$$
$$= \frac{a_k(a_k+1)}{(a_k+b_k)(a_k+b_k+1)}$$

$$\frac{1}{C}\int_{\hat{d},\rho}\rho^2[C_1N(\hat{d}|m,s^2)Beta(\rho|a_{k-1}+1,b_{k-1}) + C_2N(\hat{d}|\mu_{k-1},\sigma_{k-1}^2)Beta(\rho|a_{k-1},b_{k-1}+1)]\mathrm{d}\hat{d}\,\mathrm{d}\rho$$
$$= \frac{1}{C}C_1\int_{\rho}\rho^2 Beta(\rho|a_{k-1}+1,b_{k-1})\mathrm{d}\rho + C_2\int_{\rho}\rho Beta(\rho|a_{k-1},b_{k-1}+1)\mathrm{d}\rho$$
$$= \frac{1}{C}\left[C_1\frac{(a_{k-1}+1)(a_{k-1}+2)}{(a_{k-1}+b_{k-1}+1)(a_{k-1}+b_{k-1}+2)} + C_2\frac{a_{k-1}(a_{k-1}+1)}{a_{k-1}+b_{k-1}+1(a_{k-1}+b_{k-1}+1)(a_{k-1}+b_{k-1}+2)}\right]$$

综上： $\frac{a_k(a_k+1)}{(a_k+b_k)(a_k+b_k+1)} = \frac{1}{C}\left[C_1\frac{(a_{k-1}+1)(a_{k-1}+2)}{(a_{k-1}+b_{k-1}+1)(a_{k-1}+b_{k-1}+2)} + C_2\frac{a_{k-1}(a_{k-1}+1)}{a_{k-1}+b_{k-1}+1(a_{k-1}+b_{k-1}+1)(a_{k-1}+b_{k-1}+2)}\right]$

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