A short introduction to AmpCalc

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In the following I give a short description of some functions defined in the Mathematica package **AmpCalc**.

1 Basic commands in AmpCalc

- `TreeAmplitude[lag, particles, momenta]`
  This is the most important or the final function in the package. It calculates the tree-level amplitudes (Feynman rules) given a Lagrangian `lag`. The convention is such that the resulting amplitude is the matrix element of $-\mathcal{H}$ or $\mathcal{L}$, so a factor of i should be multiplied to get the S-matrix element. Partial derivative is translated into $-ip$ with p being momentum. Hence, this corresponds to that all the fields are incoming.

  `particles` should be given as a List. `momenta` should also be given as a List. For instance, the amplitude for $B^{*+}(l) \to \pi^+(q)B^0(l-q)$ is given by
  `TreeAmplitude[Lag, {Bstarp, pim, B0dag}, {l, -q, q-l}]`
  Notice that one should use `pim` (pi-minus) for the outgoing π^+, which is relativistic and can be created by annihilating its anti-particle. B^0 is nonrelativistic, so its creation is not related to the annihilation of $\bar{B}^0$, and `B0dag` ($B^{0\dagger}$) is used here. Notice also the sign of the momenta.

  All the particles appeared in `particles` should have been declared as fields. This can be done using the following functions.

- `MoreScalar[s1,s2,...]`, `MoreVector[v1,v2,...]`, `MoreTensor[t1,t2,...]`
  These three function declare fields `s1,...`, `v1,...`, and `t1,...` as they are fields with spin 0, 1 and 2, respectively. The arguments of these three functions can also be lists.

- `LagrangianPart[lag, particles]`
  This function gives the relevant part containing fields `particles` (as a list) in the Lagrangian `lag`.

- `Levicivitaϵ[i,j,...]`
  The anti-symmetric Levi-Civita tensor.

- `$NonRelativistic`
  `$NonRelativistic=True` for the nonrelativistic case. Momenta and polarization vectors in the output are in bold face. More importantly, the product of two Levi-Civita tensors is positive. For instance, $\epsilon_{ijk}\epsilon_{ijk} = 6$.

  `$NonRelativistic=False` for the relativistic case. A factor of -1 is included in the product of two Levi-Civita tensors so that $\epsilon_{\mu\nu\alpha\beta}\epsilon^{\mu\nu\alpha\beta} = -24$.

  By default, `$NonRelativistic=False`, so in the Z_b calculations using the NREFT, one must set `$NonRelativistic=True` before calculating amplitudes!

- `LorentzIndexed[f,i]`
  This function attaches Lorentz index `i` to field `f`. `f` can be just one field, or a list or matrix of fields. `LorentzIndexed[vec,i]` can also be simply written as `vec[i]` once `vec` is declared as a vector field using `MoreVector`.

- `pd[f,i]`
  This gives $\partial^{\mathtt{i}}\mathtt{f}$. `pd[f,i,j]` is equivalent to `pd[pd[f,i],j]`, and gives $\partial^{\mathtt{i}}\partial^{\mathtt{j}}\mathtt{f}$.

- `momentum[p,i]`
  This gives a momentum $\mathtt{p}^i$.

- `polarization[p,i]`
  This gives the polarization vector for a particle with momentum `p`, $\epsilon^{\mathtt{i}}(\mathtt{p})$.

- Notations of the meson fields:
  π^+(`pip`), π^-(`pim`), π^0(`pi0`),
  K^+(`Kp`), K^-(`Km`), K^0(`K0`), $\bar{K}^0$(`K0bar`),
  η(`eta`), η'(`etap`), η_0(`eta0`), η_8(`eta8`).
  F_π is written as `Fpi`.

- `AntiParticle`
  This gives the antiparticle of a (list of) Goldstone bosons. For instance, `AntiParticle[pip]` gives π^-, and `AntiParticle[{pip,Km}]` gives `{`π^-, K^+`}`.

- `sp[v1,v2]`
  This gives the scalar product of two vectors `v1,v2`, which can be either a momentum or polarization vector. This function is orderless.

- `spExpand[exp]`
  When there is a sum of several vectors in `sp`, such as `sp[(p1+p2),p3]`, then one needs `spExpand` to expand an expression involving such terms.

2 CHPT Langrangians

- `LMchpt[`ϕ`,Option][2]` and `LMchpt[`ϕ`,Option][4]`
  They give the Lagrangians for meson CHPT at $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$, respectively. External fields such as photons have not been included. ϕ is the matrix for Goldstone bosons. `Option` is used as `n`$\rightarrow$`4` means expanding U up to the forth power of ϕ.

- `LWZW[`ϕ`]`
  It gives the five-pseudoscalar part of the Wess-Zumino-Witten terms.

- Φ`, Φnonet, Φnonet08`
  They are 3×3 matrices, the standard representations of the SU(3) (nonet for U(3)) GBs.

- `$IsospinSymmetry`
  The default value is `$IsospinSymmetry=True`, which means the isospin symmetric case, i.e., $m_u = m_d = \hat{m}$, $M_{\pi^{\pm,0}} = M_\pi$, and $M_{K^\pm,K^0} = M_K$. It can be switched off by `$IsospinSymmetry=False`.

3 Loop functions

3.1 Definitions and conventions

The convention I used is such that multiplying the loop functions by a factor of $-16\pi^2$ gives the ones in `LoopTools`.

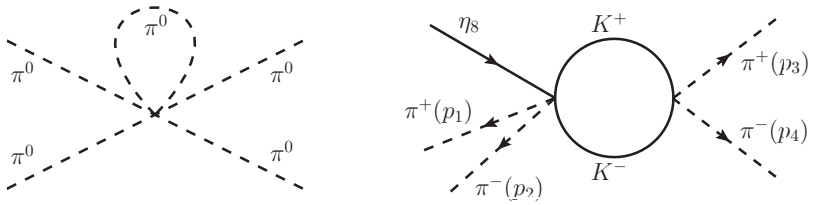

Figure 1: Examples for tadpole (left) and two-point (right) one-loop diagrams.

Tadpole (one-point) loop functions:

$$\begin{aligned} i\mu^{4-D}\int\frac{d^Dl}{(2\pi)^D}\frac{1}{l^2-m^2} &= A_0(m^2), \\ i\mu^{4-D}\int\frac{d^Dl}{(2\pi)^D}\frac{l^\mu l^\nu}{l^2-m^2} &= g^{\mu\nu}A_{00}(m^2). \end{aligned} \tag{1}$$

$A_{00}(m^2)$ is related to $A_0(m^2)$: $A_00(m^2) = m_1^2 A_0(m^2)/D$.

Two-point functions:

$$\begin{aligned} i\mu^{4-D}\int\frac{d^Dl}{(2\pi)^D}\frac{1}{(l^2-m_1^2)[(l+q)^2-m_2^2]} &= B_0(q^2,m_1^2,m_2^2), \\ i\mu^{4-D}\int\frac{d^Dl}{(2\pi)^D}\frac{l^\mu}{(l^2-m_1^2)[(l+q)^2-m_2^2]} &= q^\mu B_1(q^2,m_1^2,m_2^2), \\ i\mu^{4-D}\int\frac{d^Dl}{(2\pi)^D}\frac{l^\mu l^\nu}{(l^2-m_1^2)[(l+q)^2-m_2^2]} &= g^{\mu\nu}B_{00}(q^2,m_1^2,m_2^2) + q^\mu q^\nu B_{11}(q^2,m_1^2,m_2^2), \\ i\mu^{4-D}\int\frac{d^Dl}{(2\pi)^D}\frac{l^\mu l^\nu l^\rho}{(l^2-m_1^2)[(l+q)^2-m_2^2]} &= (g^{\mu\nu}q^\rho + g^{\mu\rho}q^\nu + g^{\nu\rho}q^\mu)B_{001}(q^2,m_1^2,m_2^2) \\ &\quad + q^\mu q^\nu q^\rho B_{111}(q^2,m_1^2,m_2^2), \\ i\mu^{4-D}\int\frac{d^Dl}{(2\pi)^D}\frac{l^\mu l^\nu l^\rho l^\sigma}{(l^2-m_1^2)[(l+q)^2-m_2^2]} &= (g^{\mu\nu}g^{\rho\sigma} + g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})B_{0000}(q^2,m_1^2,m_2^2) + [g^{\mu\nu}q^\rho q^\sigma \\ &\quad + g^{\mu\rho}q^\nu q^\sigma + g^{\nu\rho}q^\mu q^\sigma + (\rho\leftrightarrow\sigma)]\,B_{0011}(q^2,m_1^2,m_2^2) \\ &\quad + q^\mu q^\nu q^\rho q^\sigma B_{1111}(q^2,m_1^2,m_2^2) \end{aligned} \tag{2}$$

3.2 Mathematica commands

The following commands evaluate a given one-loop Feynman diagrams, and the result is expressed in terms of the A and B loop functions defined in the Sec. 3.1.

- `FeynmanParameterization[{{den1,n1},{den2,n2},...},{x1,x2,...},loopmomentum]`
  This gives the general formula for Feynman parameterization. `den1`, `den2`,... are the denominators of the propagators. `n1`, `n2`,... are their powers with default value 1. `{x1,x2,...}` is a list of Feynman parameters, and `loopmomentum` is the momentum to be integrated. Example:
  `FeynmanParameterization[{{l^2 - m[1]^2}, {(l-p)^2 - m[2]^2, 2}}, {x, 1-x}, l]` gives

$$\frac{2(1-x)}{\left[(l+p(x-1))^2 + x\left(-m_1^2+m_2^2+p^2\right) - m_2^2 - p^2x^2\right]^3}$$

- `Tadpole[lag,ext,mom,int]`
  The Lagrangian `lag` should be provided. External particles and their momenta are included in lists `ext` and `mom`. The intermediate particle can be given either as a list or not. Example:
  `Op26 = NGB[ LMchpt[Φ, n→6][2], 6 ];`

`Tadpole[ Op26, {{pi0, pi0}, {pi0, pi0}}, {{p1, p2}, {−p3, −p4}}, pi0 ]`
gives the tadpole diagram shown in Fig. 1 from the $\mathcal{O}(p^2)$ Lagrangian, and the result is

$$-\frac{B_0\hat{m}A_0\left(M_\pi^2\right)}{F_\pi^4}.$$

The second line can also be written as
`Tadpole[ Op26, {pi0, pi0, pi0, pi0}, {p1, p2, −p3, −p4}, pi0 ]`.
If the tadpole is for wave function renormalization, then either `ext` or `mom` should only contain one element.

`OnePoint` is defined to be `Tadpole`.

- `TwoPoint[lag,ext,mom,int,Option]`
  Lagrangians for both vertices should be included in `lag`. External particles and their momenta are included in lists `ext` and `mom`. Either `ext` or `mom` is a list of two lists which refer to the two vertices in the two-point one-loop diagram. For a $2 \to 2$ process, `MandelstamRules` is used. For a process with five external particles, `MandelstamRulesij` is used. One can use `Option` to define additional replacing rules. It should be written as `Rules→ user defined rules`. p_i^2 have been replaced by the on-shell masses in the last step. Example: the diagram shown in the right panel of Fig. 1 can be evaluated using
  `Op2 = NGB[ LMchpt[Φ,n→4][2],4 ];`
  `lag = LWZW[Φnonet08] + Op2;`
  `TwoPoint[ lag, {{eta8, pim, pip}, {pim, pip}}, {{p1+p2+p3+p4, −p1, −p2}, {−p3, −p4}}, {Kp, Km} ]`
  It takes about 1 second in a laptop, and the result is

$$\frac{\sqrt{3}B_{00}\left(s_{34}, M_K^2, M_K^2\right)}{4\pi^2 F_\pi^7}\epsilon_{\mu\nu\alpha\beta}p_1^\mu p_2^\nu p_3^\alpha p_4^\beta.$$

- `LoopFunction[exp,L,`μ`]`
  This gives the explicit expression of loop integrals for expression `exp`. The divergence and regularization scale are `L` and μ. The abbreviation is `LF`.