Equations for Dynamic Analysis of a Multi-machine Power System by Simplification to SMIB Models

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Abstract
In this note, it is shown that a three-machine power system with 2 PV and 1 slack busses can be modelled by using two Single Machine Infinite Bus models. It was found that a relation between the active powers of the two models at final resulted equations maintains interaction between two SMIB models. It provides a theoretical background to understand dynamic behavior of such power networks.

In this note, IEEE 9-bus system is selected to dynamical study by using the basic SMIB (Single Machine Infinite Bus) model. First, the loads were changed into proper impedance models and dissipative loads transferred to the busses 1, 2, and 3. The relation between dispatched active powers P_2 and P_3 and the busses voltage angles θ_2 and θ_3 relative to the slack bus 1 are assumed similar to various SMIB models [1] and can be shown as

$$\begin{cases} P_2 = G_2 V_{t2}^2 + \dfrac{V_{t2} V_B}{X_2} \sin(\theta_2) + \dfrac{V_{t2} V_{t3}}{X_m} \sin(\theta_2 - \theta_3) \\ P_3 = \dfrac{V_{t3} V_{t2}}{X_m} \sin(\theta_3 - \theta_2) + \dfrac{V_{t3} V_B}{X_3} \sin(\theta_3) + G_3 V_{t3}^2 \end{cases} \tag{1}$$

Also for the provided reactive powers of busses we can write

$$\begin{cases} Q_2 = \dfrac{V_{t2}^2}{X_2} - \dfrac{V_{t2} V_B}{X_2} \cos(\theta_2) + \dfrac{V_{t2}^2}{X_m} - \dfrac{V_{t2} V_{t3}}{X_m} \cos(\theta_2 - \theta_3) \\ Q_3 = \dfrac{V_{t3}^2}{X_m} - \dfrac{V_{t3} V_{t2}}{X_m} \cos(\theta_3 - \theta_2) + \dfrac{V_{t3}^2}{X_3} - \dfrac{V_{t3} V_B}{X_3} \cos(\theta_3) \end{cases} \tag{2}$$

If the 3rd order type II model is used for the synchronous generators connected to busses 2 and 3

$$P_2 = \frac{V_{t2} e'_{q2}}{X'_{d2}} \sin \delta_2 \;, P_3 = \frac{V_{t3} e'_{q3}}{X'_{d3}} \sin \delta_3 \tag{3}$$

$$Q_2 = \frac{V_{t2} e'_{q2}}{X'_{d2}} \cos \delta_2 - V_{t2}^2 \left(\frac{1}{X'_{d2}} + \frac{1}{X_{q2}} \right) , Q_3 = \frac{V_{t3} e'_{q3}}{X'_{d3}} \cos \delta_3 - V_{t3}^2 \left(\frac{1}{X'_{d3}} + \frac{1}{X_{q3}} \right) \tag{4}$$

Now, the relations (1-4) are shown in the small signal form around an operating point as

$$\begin{cases} \Delta P_2 = \dfrac{V_{t2}}{X'_{d2}} \left(e'_{q20} \cos \delta_{20} \, \Delta\delta_2 + \sin \delta_{20} \, \Delta e'_{q2} \right) \\ \Delta P_3 = \dfrac{V_{t3}}{X'_{d3}} \left(e'_{q30} \cos \delta_{30} \, \Delta\delta_3 + \sin \delta_{30} \, \Delta e'_{q3} \right) \end{cases} \tag{5}$$

$$\begin{cases} \Delta Q_2 = -\left(\dfrac{V_{t2}e'_{q20}}{X'_{d2}}\sin\delta_{20}\right)\Delta\delta_2 + \left(\dfrac{V_{t2}}{X'_{d2}}\cos\delta_{20}\right)\Delta e'_{q2} \\ \Delta Q_3 = -\left(\dfrac{V_{t3}e'_{q30}}{X'_{d3}}\sin\delta_{30}\right)\Delta\delta_3 + \left(\dfrac{V_{t3}}{X'_{d3}}\cos\delta_{30}\right)\Delta e'_{q3} \end{cases} \tag{6}$$

$$\begin{cases} \Delta P_2 = \left(\dfrac{V_{t2}V_B}{X_2}\cos\theta_{20} + \dfrac{V_{t2}V_{t3}}{X_m}\cos(\theta_{20}-\theta_{30})\right)\Delta\theta_2 - \dfrac{V_{t2}V_{t3}}{X_m}\cos(\theta_{20}-\theta_{30})\,\Delta\theta_3 \\ \Delta P_3 = -\dfrac{V_{t2}V_{t3}}{X_m}\cos(\theta_{20}-\theta_{30})\,\Delta\theta_2 + \left(\dfrac{V_{t3}V_B}{X_3}\cos\theta_{30} + \dfrac{V_{t2}V_{t3}}{X_m}\cos(\theta_{20}-\theta_{30})\right)\Delta\theta_3 \end{cases} \tag{7}$$

$$\begin{cases} \Delta Q_2 = \left(\dfrac{V_{t2}V_B}{X_2}\sin\theta_{20} + \dfrac{V_{t2}V_{t3}}{X_m}\sin(\theta_{20}-\theta_{30})\right)\Delta\theta_2 - \left(\dfrac{V_{t2}V_{t3}}{X_m}\sin(\theta_{20}-\theta_{30})\right)\Delta\theta_3 \\ \Delta Q_3 = \left(\dfrac{V_{t2}V_{t3}}{X_m}\sin(\theta_{20}-\theta_{30})\right)\Delta\theta_2 + \left(\dfrac{V_{t3}V_B}{X_3}\sin\theta_{30} - \dfrac{V_{t2}V_{t3}}{X_m}\sin(\theta_{20}-\theta_{30})\right)\Delta\theta_3 \end{cases} \tag{8}$$

Eliminating $\Delta\theta_2$ and $\Delta\theta_3$ in (7) and (8) leads to general form of equations as

$$\begin{bmatrix}\Delta Q_2\\ \Delta Q_3\end{bmatrix} = \begin{bmatrix} a & b \\ c & d\end{bmatrix}\begin{bmatrix}\Delta P_2\\ \Delta P_3\end{bmatrix} \tag{9}$$

Also by using (5) and (6) we found

$$\begin{bmatrix}\Delta P_2\\ \Delta Q_2\end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1\end{bmatrix}\begin{bmatrix}\Delta\delta_2\\ \Delta e'_{q2}\end{bmatrix} \tag{10}$$

$$\begin{bmatrix}\Delta P_3\\ \Delta Q_3\end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2\end{bmatrix}\begin{bmatrix}\Delta\delta_3\\ \Delta e'_{q3}\end{bmatrix} \tag{11}$$

By using 6 equations for 8 variables of (9-11), if $\Delta\delta_2$ and $\Delta\delta_3$ were known, $\Delta e'_{q2}$ and $\Delta e'_{q3}$ can be found by a general form of

$$\begin{bmatrix}\Delta e'_{q2}\\ \Delta e'_{q3}\end{bmatrix} = \begin{bmatrix} k_{22} & k_{23} \\ k_{32} & k_{33}\end{bmatrix}\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\end{bmatrix} \tag{12}$$

By replacing (12) in (5) it can be obtained

$$\begin{bmatrix}\Delta P_2\\ \Delta P_3\end{bmatrix} = \begin{bmatrix} l_{22} & l_{23} \\ l_{32} & l_{33}\end{bmatrix}\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\end{bmatrix} \tag{13}$$

Eq. (13) together with mechanical dynamics of rotors of generators as follows can be used in dynamical analysis:

$$\begin{cases} J_2\dfrac{d^2\Delta\delta_2}{dt^2} + D_2\dfrac{d\Delta\delta_2}{dt} + \omega_0(l_{22}\Delta\delta_2 + l_{23}\Delta\delta_3) = 0 \\ J_3\dfrac{d^2\Delta\delta_3}{dt^2} + D_3\dfrac{d\Delta\delta_3}{dt} + \omega_0(l_{32}\Delta\delta_2 + l_{33}\Delta\delta_3) = 0 \end{cases} \tag{14}$$

References

[1] Kundur P, Balu NJ, Lauby MG (1994) Power system stability and control. McGraw-hill, New York.