

Master Thesis

April 27, 2017

# Workflow Optimization

Optimal Job Assignment in a Discrete Event  
Simulation Environment

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University of  
Zurich<sup>UZH</sup>





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**Project period:** November 15, 2016 - May 15, 2017

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# Acknowledgements

I would like to thank Prof. Dr. Daning Hu for giving me the marvelous opportunity of pursuing this topic in collaboration with immopac ag and supporting me all along the path with his invaluable flexibility.

I am deeply grateful and eternally indebted to Dr. Markus Uhr for his terrific knowledge, availability, patience, comprehension and guidance during the course of this thesis. His keen eye for details, both technical and not are the foundations on which this thesis is based. His tremendous patience and capability to convey difficult concepts in a pragmatic fashion were key factors for the successful completion of this work. Lastly, I am also grateful to Dr. Markus Uhr for proofreading this thesis.

I am enormously grateful to both Dr. Robert Höhener and Dr. Thomas Höhener for advocating for this collaborative project and for allowing me to conduct this research at immopac ag. Moreover I want to thank Dr. Robert Höhener for his invaluable feedback during the initial phase of this thesis and administrative consultation.

I am grateful to Dr. Andreas Horni for his technical suggestions and feedback on the L<sup>A</sup>T<sub>E</sub>X structure and for proofreading this thesis.

My deepest gratitude goes towards my parents, Dr. med. Tatjana Zafirovska and Ljupco Kočovski for believing in me as a person and for always motivating me to pursue my dreams and push my boundaries.

My warmest thankfulness goes to Martina Lardi for her academic feedback and for proofreading this thesis.

I want to thank Dustin Kerner for proofreading the German parts of this work and his respected grammatical insights.

Ultimately I want to thank the whole staff of immopac ag for their continuous support during the course of my work and for their interest in this thesis.



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# Abstract

Efficiently assigning human resources *i.e.*, users, in **Workflow Management Systems (WfMS)** is a vital aspect when implementing **WfMS** in corporate environments (Cheng, 2000; Mentzas et al., 2001). The traditional approach formulates a deterministic planning model using **Mixed Integer Linear Programming (MILP)** and solve to optimality which has lead to promising results (Zeng and Zhao, 2005). However this deterministic approach is a limiting factor when approaching the role resolution problem using **MILP** (Zeng and Zhao, 2005).

This thesis expands on existing work in the field of effective role resolution in **WfMS** by extending the already researched **MILP** methodologies and proposes a novel approach by introducing **Reinforcement Learning (RL)** based optimization approaches, which helps to overcome the deterministic limitations of **MILP** by approaching role resolution in a stochastic fashion (Sutton and Barto, 2017).

For evaluation a subset of **Business Process Model and Notation (BPMN)** elements have been implemented in a discrete event simulation framework, in which existing policies for role resolution in **WfMS**, such as **Shared Queue (SQ)**, **Least Loaded Qualified Person (LLQP)**, **K-Batch** and **1-Batch-1** are tested.

Both the extended **MILP** as well as the **RL** based methods outperform the traditional approaches up to a 1.3-fold speedup. Even though promising results have been obtained, precautions have to be taken when interpreting the results: on one hand the **MILP** based optimizations obtained are coupled with higher computational complexity which raise business trade-offs, on the other hand, **RL** based optimizations overcome the computational complexity problem of the former but require lengthy training sessions in order to assert optimal convergence.

**RL** based optimization methods lay the foundations for extensions by using alternative methods such as **Inverse RL (IRL)** (Ng and Russell, 2000) and **Apprenticeship Learning (AL)** which promise to overcome the limitations of **RL** methods by eliminating the requirement of internal reward functions and merely “observing” experts executing tasks and learning optimal behaviors from them (Abbeel and Ng, 2004). Future work could reconcile traditional **MILP** and **AL** based methods by using the former as the “expert” agent performing role resolution and the latter would observe its behavior in order to learn from it.





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# Zusammenfassung

**TODO** >Muss noch angepasst werden!<

Die Zuteilung von Ressourcen in Workflowprozessen zwecks Effizienz Maximierung, insbesondere menschliche Ressourcen *d.h.*, Nutzer, ist ein zentraler Aspekt bei der Umsetzung von Workflowinstallationen in Gesellschaftsumgebungen (Mentzas et al., 2001). Wie diese Zuteilung erfolgt oder im Allgemeinen die Lösbarkeit des Zuordnungsproblems wurden ausführlich erforscht (Zeng and Zhao, 2005). Übliche Methoden aus der kombinatorischen Optimierung mit numerischen Optimierungen haben zu vielversprechenden Resultate geführt (Zeng and Zhao, 2005). Allerdings spielt die rechnerische Komplexität aus der mathematischen Perspektive eine zentrale Rolle (Zeng and Zhao, 2005).

Diese Masterarbeit erweitert die prominente Arbeit im Bereich von der optimalen Aufgabenzuteilung in Workflowprozessen indem die mathematischen Optimierungsmethoden ausgebaut wurden und einen neuartigen Ansatz mit Reinforcement Learning (RL) vorgestellt wurde (Sutton and Barto, 2017).

Mittels einer diskreten Ereignissimulationsumgebung worin die bestehenden Policen für die optimalen Aufgabenzuteilung in Workflowprozessen z.B., Shared Queue (SQ), Least Loaded Qualified Person (LLQP), K-Batch und 1-Batch-1, unter Verwendung von mathematischen und RL basierten Methoden erweitert und getestet wurden. Die diskrete Ereignissimulationsumgebung aus dieser Masterarbeit ermöglicht die Steuerung von wesentlichen Variablen die die Effizienz einer Police beeinflussen können z.B., Anzahl Nutzer, Generationsintervall, Bearbeitungsintervall und Länge der Simulation.

Sowohl die mathematischen als auch die RL basierten Optimierungsmethoden übertreffen die bestehenden Methoden unter verschiedenen Konditionen um einen Beschleunigungsfaktor von 1.3. Obwohl vielversprechende Resultate erreicht wurden, muss man die Resultateinterpretation mit Vorsicht geniessen. Die mathematischen Optimierungsmethoden weisen einerseits höhere rechnerische Komplexität auf, welche Businesskompromisse aufwirft und andererseits überwinden die RL basierten Methoden die rechnerische Komplexität von den Optimierungsmethoden. Diese verlangen jedoch langwierige Trainingssessionen um die optimale Konvergenz sicherzustellen.

Die RL basierten Methoden legen die Grundlagen für Erweiterungen mittels alternativer Methoden z.B., Inverse RL (IRL) (Ng and Russell, 2000) und Apprenticeship Learning (AL) (Abbeel and Ng, 2004). AL verspricht die Einschränkungen von RL basierten Methoden durch die Auflösung von internen Entlohnungsfunktionen zu überwinden und lediglich optimale Verhalten durch die "Beobachtung" von Sachverständigen zu lernen (Abbeel and Ng, 2004).



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# Introduction

**Workflow Management Systems (WfMS)** empower business process automation (Zeng and Zhao, 2005). One crucial aspect of this empowerment consists in effectively assigning jobs to users in **WfMS** (Zeng and Zhao, 2005). This concept is termed role resolution in the literature (Cheng, 2000). Solving the role resolution problem in **WfMS** can lead to fully realize the potential gains such as: **I.** Cost savings **II.** Fairness in workload assignment **III.** Optimal resources usage.

Currently most **WfMS** implement elementary role resolution policies (Zeng and Zhao, 2005). Expanding on Zeng and Zhao (2005)'s work, this thesis focuses on further developing traditional **Mixed Integer Linear Programming (MILP)** based approaches and proposes novel **Reinforcement Learning (RL)** based methods for solving the role resolution problem in **WfMS**.

## 1.1 Objectives

The objectives of this thesis build upon the work of Zeng and Zhao (2005) in which they describe policies for optimal role resolution in **WfMS** and extends these capabilities from a twofold perspective: **I.** Further develops the **MILP** based approaches proposed by Zeng and Zhao (2005) **II.** Explores the capabilities offered by **RL** as in order to overcome the deterministic limitations of traditional **MILP** based methods by approaching role resolution in **WfMS** from a stochastic perspective.

Formally, this thesis tries to answer the following research questions:

- Q. I** Can current optimization methods for job assignment in **WfMS** be further developed?
- Q. II** Are there state of the art approaches that can complement job assignment with mathematical optimization methods?

## 1.2 Thesis Structure

This thesis is subdivided in seven main chapters:

- Chapter 2 gives an overview of the state of the art literature in the touched thematic topics of this work.
- Chapter 3 outlines the theoretical foundations in **Business Process Model and Notation (BPMN)**, **WfMS**, their limitations and role resolution.

- Chapter 4 maps the theoretical concepts outlined in Chapter 3 with the actual discrete event simulation framework used for evaluation. It explains in detail the governing logic of the subset of BPMN elements implemented and the role resolution approach used.
- Chapter 5 initially describes in detail the MILP based approach and how traditional methods have been extended, followed by a theoretical introduction in RL which is eventually used in the culminating part where the RL based approaches are explained.
- Chapter 6 describes the evaluation framework implemented, how both role resolution approaches are simulated and defines the Key Performance Indicators (KPIs) used to measure them.
- Chapter 7 initially exposes the results obtained by the MILP based methods and then compares them with the traditional methods of Zeng and Zhao (2005). Subsequently, the results of the RL based methods are also outlined.
- Chapter 8 is the culminating chapter in which the results obtained in Chapter 7 are critically discussed and interpreted, then a summary of the key findings is outlined and the research questions posed in Section 1.1 are answered. Finally consequences and outlooks about the future trends and how the empirical results can be extended by prospective researchers are given.

# Related Work

This chapter serves as an overview of the state of the art literature that exists and has been used as a foundation basis.

## 2.1 Queueing

Queueing is the act of how people, or more general agents, are to be served while waiting (*i.e.*, queueing) inside a system (Kendall, 1953). Starting with the seminal contribution of Kendall (1953) on Markov chains in queueing theory, where he formally defines different types of queues, upon which many researches have been based.

Pinedo (2008) outlines in his work the most prominent key metrics that can be used in order to assert and measure queues performance.

Adan and Resing (2015)'s statistical modeling techniques for randomized generation rates, such as the Erlang's distribution, are used in the discrete event simulation framework.

## 2.2 Workflow Management Systems

Baker (1974) formally defines the **Key Performance Indicator (KPI)** used by Zeng and Zhao (2005) for evaluating policies, called task flowtime.

Macintosh (1993) gives an overview of the five levels of process maturity: **I**. Initial, the process has to be set up **II**. Repeatable, the process has to be repeatable **III**. Defined, documentation standardization of processes **IV**. Managed, measurement and control of processes **V**. Optimizing, continuous process improvement.

Georgakopoulos et al. (1995) give a comprehensive business oriented overview of the different **Workflow Management Systems (WfMS)** technologies present on the market used as a sound foundation for analyzing the present technologies.

Cheng (2000) formally defines the act of assigning jobs to users *i.e.*, role resolution in **WfMS**.

Following Georgakopoulos et al. (1995)'s business oriented overview, Giaglis (2001) lays out four different process perspectives: **I**. Functional **II**. Behavioral **III**. Organizational **IV**. Informational. Giaglis (2001)'s framework focuses on three dimensions: **I**. Breadth, where modeling goals are typically addressed by technique **II**. Depth, where modeling perspectives are covered **III**. Fit, where typical project to which techniques can be fit. The presented framework is used to combine the three different dimensions in order to assert a possible best fit of a specific modeling technique based on which approach to be used under the constraints of a modeling perspective to cover (Giaglis, 2001).

Mentzas et al. (2001) focuses on a qualitative level on how WfMS can facilitate implementation of business processes by describing the pros and cons of adopting alternative Business Process Modeling (BPM) techniques. Moreover, Mentzas et al. (2001) formally define what a WfMS is and subdivide it in three main categories: I. Process modeling II. Process re-engineering III. WfMS implementation and automation. Each level of maturity as defined by Macintosh (1993) requires a different model, such as the first three levels might require more descriptive models whereas levels four and five require decision support keen models in order to monitor and control processes (Mentzas et al., 2001).

Aguilar-Savén (2004) reviews BPM literature and describes the main BPM techniques.

Interestingly enough, WfMS implementation in real world cases is not always only coupled with directly measurable effects, sometimes even unexpected results happen (Reijers and van der Aalst, 2005). What is called the “workflow paradox” according to Reijers and van der Aalst (2005) is the concept that the very fact of companies accepting requests for WfMS introduction might actually be the most promising way that leads to potentially better and more suitable alternatives.

Sörensen (2005) focuses on problematics when modeling WfMS with Business Process Model and Notation (BPMN) such as the presence of cycles and the consequences that these have in respect to termination and progress capability of WfMS.

Effective role resolution *i.e.*, the mechanism of assigning tasks to individual workers at run-time according to the role qualification defined in WfMS as defined by Zeng and Zhao (2005) is the main focus of this thesis. Zeng and Zhao (2005) differentiate between staffing decisions and role resolution, with the former being the assignment of one or more roles to each user and the latter being the assignment of a specific task to an appropriate worker at runtime. Staffing decisions are usually made off-line and periodically, thus being more of a strategic nature (Zeng and Zhao, 2005). If role resolution were to be made on-line it could translate to a major operational level decision *i.e.*, the differentiation between strategic vs operational playing role (Zeng and Zhao, 2005). They moreover define three roles a WfMS can fulfill (Zeng and Zhao, 2005): I. System built-in policies II. User customizable policies III. Rule based policies. Considering capacities of resources restrictions under the role resolution problem is NP-hard and Zeng and Zhao (2005) focus on how to solve the it when accounting for worker’s preferences. For this purpose they define five WfMS resolution policies: I. Least Loaded Qualified Person (LLQP) II. Shared Queue (SQ) III. K-Batch IV. K-Batch-1 V. 1-Batch-1 For all batch policies a simplified version of Dynamic Minimization of Maximum Task Flowtime (DMF) has to be solved (Zeng and Zhao, 2005). Zeng and Zhao (2005)’s key findings are outlined as follows: I. Batch policies are to be used when system load is medium to high II. Processing time variation has major impact on system performance *i.e.*, higher variation favors optimization based policies III. Average workload and workload variation can be significantly reduced by online optimization IV. 1-Batch-1 online optimization policy yields best results in operational conditions.

Data flow inside WfMS has to consider possible anomalies that might happen and this aspect has been extensively studies by Sun et al. (2006) where they formally define data flow methodologies for detecting such anomalies. Their framework is divided in two components (Sun et al., 2006): I. Data flow specification II. Data flow analysis. They moreover discuss aspects that data requirements have been analyzed but the required methodologies on discovering data flow errors have not been extensively researched (Sun et al., 2006).

A more recent taxonomy of different BPM applications is given by a collaboration between SAP and accenture (Evolved Technologist, 2009).

An analysis of the Critical Success Factors (CSFs) for BPM is required in order to assert product validity and this has been done by Trkman (2010) where he defines CSFs from three perspectives: I. Contingency theory II. Dynamic capabilities III. Task-technology fit theory.

The domain of WfMS is permeated by BPMN and Silver (2011)’s guidelines are excellent formal foundations.

Change management in WfMS is yet another interesting aspect that should be considered and this has been broadly studied by Wang and Zhao (2011) where they developed an analytical framework for WfMS change management through formal modeling of constraints.

In companies different types of WfMS can exist and Fan et al. (2012) focus on two of these, namely: I. Conceptual II. Logical. Conceptual models serve as documentation for generic process requirements whereas logical models are used as definitions for technology oriented requirements (Fan et al., 2012). One difficult aspect is the transition from the former to the latter and Fan et al. (2012) propose a formal approach to efficiently support such transitions.

Sun and Zhao (2013) cover the aspect of formal analysis for WfMS and they claim that it should help “alleviating the intellectual challenge faced by business analysts.” (Sun and Zhao, 2013, p. 2).

## 2.3 Reinforcement Learning

Reinforcement Learning (RL) is a branch of machine learning that promises to overcome the drawbacks posed by the latter by not requiring a training set for efficient machine decisions (Sutton and Barto, 2017).

One of the first Monte Carlo (MC) based Policy Gradient (PG) methods is the algorithm proposed by Williams (1992) called REINFORCE.

Vanishing Gradient Problem (VGP) states that even very large changes in partial derivatives on initial layers have imperceptible effects on subsequent layers, as outlined by Bengio et al. (1994), which is a problem that affects deep Artificial Neural Networks (ANNs).

Haykin (1998)’s comprehensive work on ANNs is an excellent theoretical foundation which serves as basis for critical concepts regarding the matter.

Theoretical definitions on deep ANNs are also given by Lecun et al. (1998) which were used to better understand more recent developments in this domain.

PG methods with Value Function Approximation (VFA) and their convergence is of vital importance and according to Sutton et al. (1999) this can be achieved by representing the policy by an own function approximation which is independent of the value function and it is updated according to gradient of the expected rewards with respect to the afore mentioned policy.

Another branch originated from RL is Inverse RL (IRL): Ng and Russell (2000) outline the required algorithms for this domain.

Discretization of the state action space is not always feasible and different techniques have to be used for tractability and Smith (2002) proposes such an approach which he calls “self-organizing map”.

Abbeel and Ng (2004) collaboration lays the basis for a bleeding edge branch of IRL called Apprenticeship Learning (AL), which trains policies without rewards function by merely observing “expert” agents performing a task in a specific domain.

Bengio (2009) further develops the foundations laid by Lecun et al. (1998) and expands the theoretical basis of deep ANNs.

Statistical identifiability in RL is a crucial aspect that has to be ensured for effective learning, as were it not the case RL update methods might remain stuck in suboptimal solutions and never converge (Zhang and Hyvärinen, 2011).

Yet another problem in which deep ANNs might incur in addition to VGP is the so called Exploding Gradient Problem (EGP) as defined by Pascanu et al. (2012).

Huge state space requirement is a clear limitation to lookup tables (Sutton and Barto, 2017). Even if memory would not be a constraint, the actual learning from such tables would be infeasible (Sutton and Barto, 2017). In order to pragmatically learn by reinforcement on such huge problems, VFA in the domain of RL proves to be a viable solution (Sutton and Barto, 2017). For

different types of **RL** approaches *i.e.*, **MC** or **Temporal Difference (TD)** methods exist different types of **VFA**, ranging from simple linear combinations of features for **MC** to **ANNs** for **TD** learning (Sutton and Barto, 2017). All these different methodologies are outlined in the tutorial by Geramifard et al. (2013) and complement the theoretical foundations laid by Sutton and Barto (2017).

Overfitting for deep **ANNs** is a common problem (Sutton and Barto, 2017, p. 218). Srivastava et al. (2014) propose the dropout method as solution which proves to be a domain standard in regards to this problem.

As **Markov Decision Processes (MDPs)** grow in size, so does the required computational memory to solve possible discrete lookup tables modeling the state-actions spaces that characterizes them (Sutton and Barto, 2017). Sutton and Barto (2017) show notable examples that show how large some of the most common problems can be: **I.** The game of backgammon has a total of  $10^{20}$  states **II.** The traditional Chinese abstract board game Go has an estimated total of  $10^{170}$  states **III.** Flying a helicopter or having a robot move in space all require a continuous state space.

When working with on-line algorithms such as **TD** it is important to choose correct parameters for an effective learning process, otherwise the learning algorithm put in place might never converge towards an optimal solution (Sutton and Barto, 2017). This aspect is being discussed by Korda and Prashanth (2014) in which they depict different non-asymptotic bounds for the **TD** learning algorithms.

There are two main fields in **RL**, one is using **VFA** for either the state value function or for using control mechanisms with the state **Action Value (AV)** function, while the other one is using **PG** methods for policy optimization (Sutton and Barto, 2017). The latter offers different methods such as the naive finite difference methods, **MC** based **PG** methods and finally **Actor Critic (AC)** **PG** methods as defined by Silver et al. (2014).

Clevert et al. (2015) discuss a special class of activation functions for **ANNs** called **Exponential Linear Units (ELUs)** which are improvements over traditional **Rectified Linear Units (ReLUs)**. **ELUs** activation functions are used for the modeling of the **ANNs** layers later on.

Gershman (2016) proposes further advancements to Zhang and Hyvärinen (2011)'s work on statistical identifiability by analyzing prior distributions of the parameters. He argues that his approach helps to some extent to overcome the identifiability problem (Gershman, 2016).

Notable works in the field of **RL** and its application include DeepMind Technologies Limited's work on novel algorithms for tackling fields previously barely scratched, as mentioned by Mnih et al. (2015) and Silver et al. (2016).

Sutton and Barto (2017) started working on the **RL** topic in the early nineties and are now planning their third edition of the famous book on **RL**, which is due in 2017.

## 2.4 Optimization

The NP-hardness of **DMF** is formally proved by Garey and Johnson (1990).

For role resolution in **WfMS** a **Mixed Integer Linear Programming (MILP)** problem must be solved in order to optimally assign jobs to users. The generalized assignment problem is a very well known problem in combinatorial mathematics and Cattrysse and Wassenhove (1992) give an overview of different algorithms for solving it. Heuristics are also a viable solution for solving such adaptation of the generalized assignment problem, as Racer and Amini (1994) state. Moreover a global perspective of optimization from a mathematical perspective is given in Boyd and Vandenberghe (2004)'s work on convex optimization.

According to the AIMMS guidelines, there are different linear programming maneuvers that can be used to shape such problems in solvable outlines (Bisschop, 2016). Zeng and Zhao (2005) propose that by adding auxiliary variables, their formulation of **DMF** can be transformed in a

MILP problem. For this purpose, the either-or constraints was used to transform DMF into Extended DMF (EDMF) (Bisschop, 2016, p. 77).

## 2.5 Simulation

Bahouth et al. (2007) focuses on algorithmic analysis of discrete event simulation supplemented with focus on factors such as compiler efficiency, code interpretation and caching memory issues. According to their findings, a significant speedup can be achieved if one addresses the aforementioned facets (Bahouth et al., 2007).

Simulating queues can prove to be arduous (Matloff, 2008). The main differentiation needed here is the one between continuous and step functions: the former is the result when the events being simulated yield values that if plotted against the simulation time give a continuous function (Matloff, 2008). On the other hand, if we simulate events that yield discrete values, such as inventory changes in a storage facility and plot the results against the simulation time we would get so called step functions (Matloff, 2008). According to Matloff (2008), different world views for discrete event programming exist, as he calls them paradigms (Matloff, 2008): I. Activity oriented II. Event oriented III. Process oriented. Activity oriented can be summarized as simulation events where time is being subdivided in tiny intervals at which the program checks the status for all simulated entities (Matloff, 2008). Since petite subdivisions of time are possible in such types of simulations, it is clear that the program might prove tedious, since most of the time there will not be any change in state for the simulated entities (Matloff, 2008). Event oriented circumnavigate this issue by advancing the simulation time directly to the next event to be simulated (Matloff, 2008). By filling these gaps, a dramatical increase in computation can be observed (Matloff, 2008). Last but not least, the process oriented simulation models each simulation activity as a process or thread (Matloff, 2008). Management of threads has steadily decreased in today's computation since many different packages for governing such tasks exist (Matloff, 2008).

Milo (2012) defines speedup as a metric for evaluating comparisons in computer architecture which is used when analyzing the results of this thesis.





# Theoretical Foundations

## 3.1 Business Process Model and Notation

**Business Process Modeling (BPM)** is the act of representing a business process (Silver, 2011). Modeling requires a standardized notation that allows for consistent model representation and this is achieved in the BPM field by the **Business Process Model and Notation (BPMN)** which is a graphical representation of a semantic process (Silver, 2011). BPMN is thus to be understood as a language used to draw diagrams *i.e.*, a diagramming language (Silver, 2011). Its most favorable point is that it is a standard maintained by the **Object Management Group (OMG)** and that it is widely adopted worldwide (Silver, 2011). Silver (2011) outlines two fundamental principles that BPMN should adhere to:

- P. I** “A given BPMN diagram should have one and only one interpretation. The process logic should be completely and unambiguously described by the diagram alone.” (Silver, 2011, p. v).
- P. II** “A given BPMN diagram should have one and only one **Extensible Markup Language (XML)** serialization. Otherwise model interchange between tools cannot be achieved.” (Silver, 2011, p. v).

**Principle I** is more modelers-oriented with no necessary implementation knowledge that should be able to fully understand a diagram by looking at it, while **Principle II** applies more to implementers and process interpreters (Silver, 2011).

Silver (2011) differentiates between two important concepts: **I. Activities II. Processes**.

Activities are to be understood as units of work performed, or as Silver (2011) defines them: discrete actions with well-defined start and end that are performed repeatedly in the course of business, whose instances can be arbitrarily repeated (Silver, 2011, p. 10). More informally one can see each activity instance as a token that is being generated at a starting state and during its lifetime it will follow a path following the process flow. On the other hand, processes are sequence of actions that lead from a start state to an end state (Silver, 2011, p. 11).

Modeling such processes is done by relying as previously mentioned on BPMN in which different building blocks are defined (Silver, 2011). A traditional model designed following the BPMN guidelines can be found in Figure 3.1.

In the following paragraphs the different elements outlined are explained and Figure 3.2 gives an overview of the graphical representation in BPMN of them.

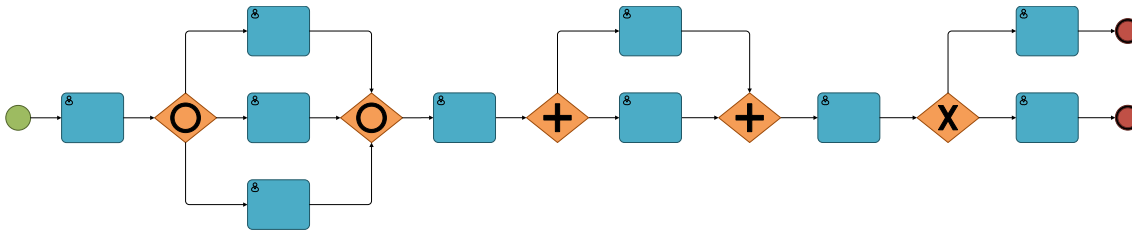


Figure 3.1: BPMN process which shows the most important shapes used

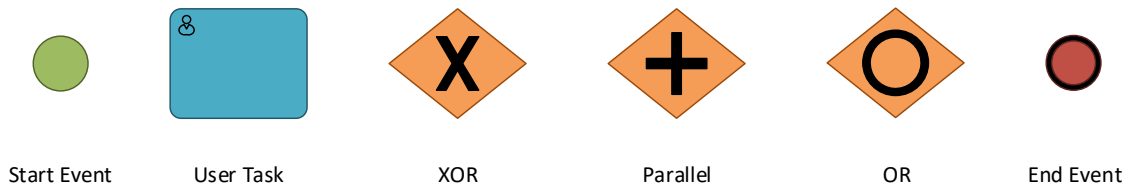


Figure 3.2: BPMN elements

### 3.1.1 Start Event

The main purpose of start events is to indicate where a process starts and it is a strict requirement set by **BPMN** that each process must have such one and at most one (Silver, 2011). Technically, start events are those process elements that generate tokens that flow through a process which when they reach a user task get worked by the assigned user.

### 3.1.2 User Task

Activities are defined by Silver (2011) as atomic units of work that are performed in a process. They are the only elements that assign a performer to them (Silver, 2011). For the purpose of this thesis, the only type of activities considered is the user task. They get a user assigned which will work the token that has reached this task.

### 3.1.3 Gateway

Diamond shaped gateways are elements that control the tokens flow inside a process by effectively splitting its path into alternative ones (Silver, 2011). There are three main types of gateways: I. **Exclusive OR (XOR)** II. **Parallel** III. **Inclusive OR (OR)**.

**XOR** gateways define exclusivity among the alternative paths a token can flow through, meaning that only one path can be pursued per instance (Silver, 2011). **XOR** gateways do not make decisions, they merely test conditions to activate a specific path (Silver, 2011). The decision making process is left to tasks (Silver, 2011).

Parallel gateways, in contrast to **XOR** gateways, unconditionally split and duplicate the flow of a token inside a process (Silver, 2011).

Last but not least, **OR** gateways are in a way similar to **XOR** gateways since they also have conditional flows but each condition is independent *i.e.*, more than one can be true and if that is the case, then the paths are split following the parallel gateway logic (Silver, 2011). Normally, **OR** gateways always have a default path to be followed (Silver, 2011).

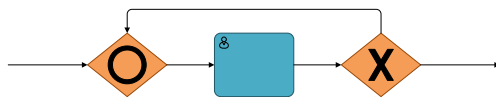
### 3.1.4 End Event

End events indicate the end of the path for a token inside the process (Silver, 2011). In contrast to start events, it is possible to have multiple end events (Silver, 2011). As soon as a token reaches an end event, the corresponding activity instance is completed.

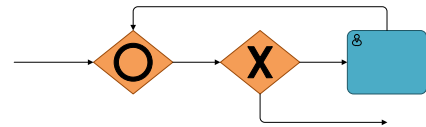
## 3.2 Workflow Management Systems Limitations

Even though BPMN advocates for a formal semantic definition, there are formal gaps in more complex semantic constructs which are under-specified that should be accounted for when modeling processes with BPMN (Sörensen, 2005). Sörensen (2005) mentions the most critical specification areas lacking formal semantics: I. OR gateways with arbitrary number of cycles II. Cyclic processes in general III. Processes presenting deadlocks that might not ensure termination of progress.

Both Figure 3.3 and Figure 3.4 show the representation of two different types of cyclic diagrams that can cause problems, as outlined by Sörensen (2005).



**Figure 3.3:** Cyclic diagram with repeat loop (own plot based on Sörensen (2005, p. 12))



**Figure 3.4:** Cyclic diagram with while loop (own plot based on Sörensen (2005, p. 12))

As it is out of scope for this thesis to reiterate over Sörensen (2005)'s work on cyclic processes, we only model and subsequently simulate acyclic processes.

In a real world adoption of Workflow Management Systems (WfMS) bottlenecks can be encountered on each stage along a process. In the simulation environment proposed in this thesis, however, the focus is put only on user tasks and all other process elements are considered to be executed instantly.

## 3.3 Role Resolution

As mentioned in Section 3.1, user task elements get users assigned which then work tokens that reach this specific task. The concept of assigning a user to a token (or task) is referred to in the literature as role resolution (Cheng, 2000; Zeng and Zhao, 2005). When referring to Figure 3.1, we see that tokens are being generated by the left most start event and immediately reach a user task. Let us moreover assume that we have a pool of different users that all qualify to process the process token at this specific user task, however the all exhibit different skills level and at the time of arrival they are also variously occupied. The act of choosing the best user under consideration of different process environment factors is that of role resolution (Zeng and Zhao, 2005). Formally, applying role resolution is done by following predefined policies which govern how users are being effectively assigned to work tokens (Zeng and Zhao, 2005). As already mentioned, this thesis focuses on extending the foundations laid by Zeng and Zhao (2005) in which they define five different types of policies for effective role resolution.

In order to effectively test different policies governing role resolution in **WfMS**, in this thesis a simulation environment has been prepared. By means of simulating processes with different policies acting as supervisors, different metrics can be evaluated to assess one policy's efficiency. Role resolution in this simulation environment happens on a per user task basis *i.e.*, when a token reaches a user task, a policy follows its internal definitions on how to optimally assign available users to this token. A simulation environment has been implemented since on one hand it allows to generate the required amount of data relatively fast (which in a real world situation not only could prove to be hard to obtain, but extremely costly as well) and on the other hand, it allows to equally test all role resolution policies by ensuring that they all undergo the exact same processes.

# Discrete Event Simulation

As previously outlined in Section 3.3, a discrete event simulation has been implemented in which role resolution policies have been tested. What follows is the descriptions of how this environment has been implemented, then in Section 4.1 the **Workflow Management System (WfMS)** environment's implementation based on the outlined foundations in Section 3.1 is described and eventually in Section 4.2 the role resolution's implementation based on the outlined foundations in Section 3.3 is also explained.

`SimPy` is a `Python` process-based discrete-event simulation framework. It exploits `Python`'s generators according to which it models its processes.

Active components such as agents in a **WfMS** are modeled as processes which live inside an environment and the interaction between them happens via events.

As previously mentioned, processes in `SimPy` are described by `Python` generators. During their lifetime they create events, `yield` (note that with the term `yield` here it is to be understood as `Python`'s `yield` statements)<sup>1</sup> them to the environment, which then wait until they are triggered. The important logic to understand here is how `SimPy` treats yielded events: when a process yields an event it gets suspended. From the suspended state a process gets resumed when the event actually occurs (or in `SimPy`'s notation when it gets triggered).

`SimPy` offers a built-in event type called `Timeout`: events of this type are automatically triggered after a determined simulation time step. Consistency is asserted since timeout events are created and called by the appropriate method of the passed `Environment`.

## 4.1 Workflow Management System Implementation

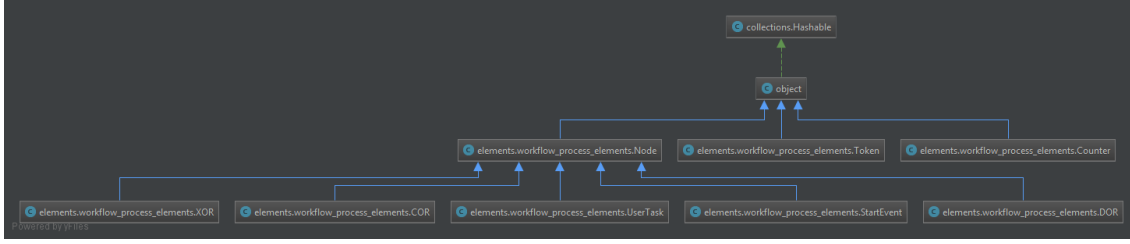
The analysis environment consists in an object-oriented implementations of **WfMS** elements such as start events, user tasks, gateways and end events which have been developed to allow the simulation framework to effectively run and are built upon the formal foundations outlined in Section 3.1. This object-oriented exoskeleton implementation of the **WfMS** elements can be seen depicted in Figure 4.1.

### 4.1.1 Start Event

Start event objects require a simulation environment, a generation interval, an actions to follow array and its corresponding weights. The generation interval is a plain scalar value in contrast to [Zeng and Zhao \(2005\)](#)'s work, where the generation  $\lambda$  interval follows a Poisson distribution and

---

<sup>1</sup><http://stackoverflow.com/questions/231767/what-does-the-yield-keyword-do-in-python> (accessed 26.04.2017)



**Figure 4.1:** WfMS elements

is defined as shown in Equation 4.1 with a fixed service interval time unit  $s$ , number of users  $n$  and an average system load  $l$ .

$$\lambda = \frac{ln}{s} \quad (4.1)$$

The path flow to be followed by the tokens generated is defined in a two step process: **I.** A per process action pool is defined a priori in order to assert that tokens navigate the process in a “semantically correct” fashion. This is achieved by creating an array containing an arbitrary number of dictionaries *i.e.*, hash-maps, where the key is the gateway node id and the value is which path *i.e.*, child, should be chosen next (see Listing 4.1) **II.** A weights vector is defined which assigns a probability to each possible path flow to the actions pool (see Listing 4.2).

```
actions_pool = [{xor.node_id: 0, xor_a.node_id: 0, dor_c.node_id: (1, 2),
                xor_d.node_id: 1, xor_f.node_id: 0, cor_g.node_id: 2, xor_h.node_id: 0}]
```

**Listing 4.1:** Actions pool

```
weights = [1 / len(actions_pool) for _ in range(len(actions_pool))]
```

**Listing 4.2:** Probability weights vector for each path flow

Such an approach allows to fine tune how often tokens will follow a predefined path flow along the process in order to efficiently simulate and put under stress specific path flows of the process.

The length of the simulation can either run indefinitely or be constrained to an upper bound in time steps  $t$  as it can be seen from Listing 4.3. This means that the simulation will persist for 100 time steps and it will then stop when the internal clock reaches 100<sup>2</sup>. The logic is similar to a new environment where the clock is zero and no events have been processed yet.

```
# "global" variables
SIM_TIME = 100
# runs simulation
env.run(until=SIM_TIME)
```

**Listing 4.3:** Starting the simulation with discrete time steps

## Master Random State

In order to assert fairness among all simulation runs a master random state is assigned to the start event. This master random state is generated from the PCG<sup>3</sup> family of random generators which

<sup>2</sup>Note that events that have been scheduled for time step 100 will not be processed.

<sup>3</sup><http://www.pcg-random.org/> (accessed 25.04.2017)

exhibit peculiar characteristics, one amongst all is the possibility of “jumping ahead” in the state. The novelty of this approach allows to assign a fixed number of random yet consistent choices among all runs, since each generated token receives from the start event a “jumped” copy from the master state. The implementation of this recursive ahead jumps in random states is depicted in Listing 4.4.

```
def generate_tokens(self):
    while True:
        random_state = self.advance_master_state()
        token = Token(random_state)

def advance_master_state(self):
    current_state = self.master_state.get_state()
    random_state = pcg.RandomState()
    random_state.set_state(current_state)
    self.master_state.advance(int(1e9))
    return random_state
```

**Listing 4.4:** Master random state jump ahead

This approach greatly simplifies the analysis of the results since no matter the simulation configuration, it permits to ensure consistency and reproducibility among all configurations. Specifically, since each token gets a “jumped” copy of the master random state, it is ensured that under any circumstances the generation time during the simulation is consistent.

job	arrival	assigned	started	finished
0	5.311109689	5.311109689	5.311109689	5.932433185
4	9.712642076	9.712642076	9.712642076	9.830042582
6	19.58479207	19.58479207	19.58479207	20.27429297
11	22.38095224	22.38095224	22.38095224	22.77909589
15	26.638781	26.638781	26.638781	26.69025169
18	33.52744551	33.52744551	33.52744551	33.55946552
23	36.31998313	36.31998313	36.31998313	37.47852927
24	36.50819015	36.50819015	36.50819015	37.38701089
25	36.94657375	36.94657375	36.94657375	37.00416155
31	37.6882068	37.6882068	37.6882068	39.14904432
38	39.38223763	39.38223763	40.84913047	42.13016635
39	40.19008378	40.19008378	42.13016635	42.15046548
41	41.75883248	41.75883248	41.75883248	42.75968576
52	48.83897106	48.83897106	48.83897106	48.91014329

**Figure 4.2:** Simulation run with MSA

job	arrival	assigned	started	finished
0	5.311109689	5.311109689	5.311109689	5.932433185
4	9.712642076	9.712642076	9.712642076	9.830042582
6	19.58479207	19.58479207	19.58479207	20.27429297
11	22.38095224	22.38095224	22.38095224	22.77909589
15	26.638781	26.638781	26.638781	26.69025169
18	33.52744551	33.52744551	33.52744551	33.55946552
23	36.31998313	36.31998313	36.31998313	37.47852927
24	36.50819015	36.50819015	36.50819015	36.76911785
27	36.94657375	36.94657375	36.94657375	37.30895996
33	37.6882068	38.2358957	38.2358957	39.69673321
37	39.38223763	39.38223763	39.38223763	40.12551887
41	40.19008378	40.19008378	40.19008378	40.21038291
47	41.75883248	41.75883248	41.75883248	43.45140794
52	48.83897106	48.83897106	48.83897106	48.91014329

**Figure 4.3:** Simulation run with ST

Figure 4.2 and Figure 4.3 are snippets of the statistical data dumps for two arbitrary simulation runs that clearly display the randomness consistency concept outlined before. When comparing the arrival column on both figures, it is possible to see that even though different number of jobs are being generated, they are consistently generated across different simulation, each arriving at the same simulation time  $t^4$ .

<sup>4</sup>Note that both figures only show data filtered corresponding to the first direct user task connected to the start event.

### 4.1.2 User Task

User task objects also require a simulation environment, a policy, a descriptive name, a service interval and task variability. Each user task has a unique `child` field which is being set prior to starting the simulation.

Each user task object has a claim token method, which takes tokens as input parameters and finally makes a call to its designed policy, passing the token. On this top level, without stepping into the single policies implementations, the logic is straightforward: start events generate tokens, user tasks that are direct children of start events claim the newly generated tokens, ask the designated policies to assign a user to the token and finally, after a service interval timeout which corresponds to the user's specific service interval has passed, they release the token. The logic can be seen in Listing 4.5.

```
def claim_token(self, token):
    token.worked_by(self)
    policy_job = self.policy.request(self, token)
    service_time = yield policy_job.request_event
    yield self.env.timeout(service_time)
    self.policy.release(policy_job)
```

Listing 4.5: User task claim method

### 4.1.3 Gateway

Gateways are those process nodes where conditions are tested (see Section 3.1). As previously mentioned, in the discrete event simulation environment each newly generated token holds an array with all conditions assigned to it, which are then tested at decision nodes in order to correctly forward or split the token along the path flow. For the purpose of this thesis two types of gateways have been implemented: I. **Exclusive OR (XOR)** II. **Inclusive OR (OR)** whose logic has been split between its converging and diverging part.

#### XOR

**XOR**'s logic is straightforward: each gateway has a forward method which is used to correctly move the token along the process. As soon as a token reaches a **XOR** gateway, the token's condition is tested and then forwarded to the next element. The implementation can be seen in Listing 4.6.

```
class XOR(Node):
    def forward(self, token):
        token.worked_by(self)
        action = token.get_action(self)
        child = self.children[action]
        self.child_forward(child, self.env, token)
```

Listing 4.6: XOR's forward method

#### OR

**OR** gateways, in contrast to **XOR** gateways, require a synchronization between number of split path flows being fired at their diverging state which eventually reach a converging state. In other



words, the number of parallel tokens fired at the diverging node must all be accounted for at the converging node. This is achieved by a counter object that each token holds: when a token reaches a divergent **OR** gateway, the token's conditions are read and for each path flow that the token has to follow its counter is incremented by 1. The logic is depicted in Listing 4.7.

```
class DOR(Node):
    def choose_child(self, action, token):
        child = self.children[action]
        token.counter.increment()
        self.child_forward(child, self.env, token)
```

**Listing 4.7:** Token's counter increment logic at a divergent OR gateway

Each diverging **OR** gateway has a corresponding converging **OR** gateway which is used to catch all parallel tokens fired by the former. When a token reaches a converging **OR** gateway, its counter is decremented each time by 1 and only when the counter reaches 0 the token is actually forwarded along. This logic can be seen in Listing 4.8.

```
class COR(Node):
    def forward(self, token):
        token.worked_by(self)
        token.counter.decrement()
        if token.counter.count == 0:
            action = token.get_action(self)
            child = self.children[action]
            self.child_forward(child, self.env, token)
```

**Listing 4.8:** Token's counter decrement logic at a convergent OR gateway

### 4.1.4 End Event

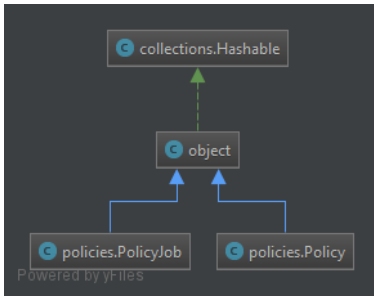
In Python one does not have to preallocate or deallocate memory by hand as when using pointer based programming languages such as C or C++ since this is being taken care of internally. According to Silver (2011)'s definition of activities, processes and end events (refer to Section 3.1), when a token reaches an end event, the corresponding process' activity instance is deleted and to be never again used. This is exactly what can be implicitly achieved by Python's internal garbage collection strategies: tokens are objects that are being generated by start events, when they reach the last element in the process, all references to the token object are extinguished thus effectively engaging the internal garbage collection strategies which deallocate any memory previously allocated for it.

## 4.2 Policy Based Role Resolution Implementation

Role resolution according to policies acting as supervisors (as explained in Section 3.3) in the discrete event simulation environment is achieved by means of a particular class of objects *i.e.*, policies and policy jobs.

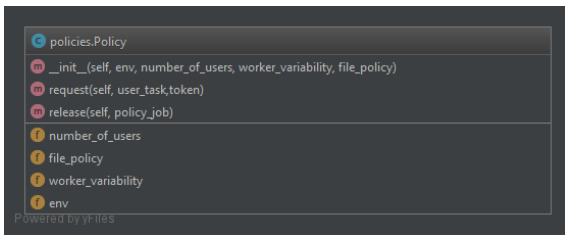
Policies are a particular object that does not directly participate in the WfMS, it serves a role as a general supervisor that has the whole overview of the process allowing it to operate on an abstract level. The implementation of the policy objects can be seen in Figure 4.4.

Each policy is a blueprint for the actual implementation of the policy itself. It holds minimal information such as a simulation environment, number of users and worker variability. As a



**Figure 4.4:** Policies abstract implementation

blueprint, each policy object defines two abstract methods for requesting an optimal assignment for a specific token and for later releasing that token and effectively freeing the user that was busy working on it. Refer to Figure 4.5 for its implementation overview.

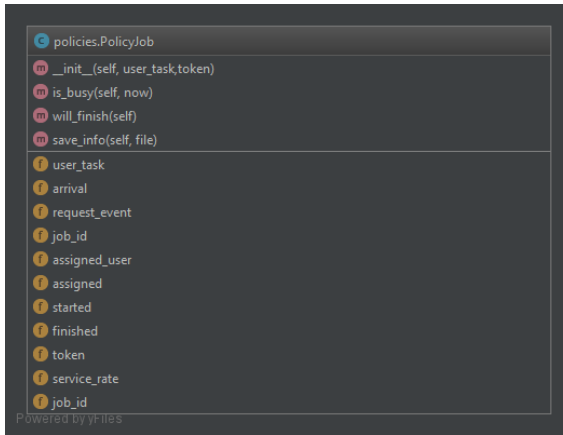


**Figure 4.5:** Policy methods and attributes

In its request method, each policy generates a policy job object, which is again an abstract implementation of a job that the policy will work in order to return an optimized assignment to a user task. Each policy job requires a user task and a token as initialization parameters in order to be uniquely identifiable inside the whole process. Moreover, each policy job object serves as a bookkeeping agent by storing and dumping useful information every time its status changes, such as arrival, assigned, started and finished times, assigned user and a list of service times for all available users. Refer to Figure 4.6 for its implementation overview.

In regards to parameters service interval and task variability defined in Subsection 4.1.2 a detailed explanation is required. Both are used to randomly sample service rate intervals for each user active during the simulation. Zeng and Zhao (2005, p. 8) follow a two way process to generate such intervals. However a refined version of this process is used in this case: **I.** At initialization time, each user task receives a service rate  $s$  and a task variability  $t$  value **II.** Inside the policy request method, for each user task a sample of an average processing time following an Erlang distribution (a special case of the gamma distribution as defined by Adan and Resing (2015)) which takes as input parameters a shape  $k$  and a scale  $\theta$  is made. In this case both values  $k$  and  $\theta$  are dynamically evaluated at runtime as  $k = s/t$  and  $\theta = t$ . This concept is depicted in Listing 4.9 **III.** The average processing time becomes a unique value of each user task and is used by each policy to sample each user's service time, again from an Erlang sampled pool as depicted in Listing 4.9 which we shall call  $p_j$ .

For each user eligible to work the assigned token, its service rate is sampled following the Erlang distribution. This time, the Erlang distribution takes as parameters the unique average processing time  $p_j$  of user task  $j$  and a worker variability  $w$ , which is a unique property of each



**Figure 4.6:** Policy job methods and attributes

policy.

In order to sample a service rate  $p_{ij}$  following the Erlang distribution for each user  $i$ , shape  $k$  is evaluated as  $k = p_j/w$  and scale  $\theta$  as  $\theta = w$  as it can be seen in Listing 4.9

```
def request(self, user_task, token):
    average_processing_time = token.random_state.gamma(
        user_task.service_interval ** 2 / user_task.task_variability,
        user_task.task_variability / user_task.service_interval)

    policy_job.service_rate = [token.random_state.gamma(
        average_processing_time ** 2 / self.worker_variability,
        self.worker_variability /
        average_processing_time) for
        _ in range(self.number_of_users)]
```

**Listing 4.9:** User service rate sampling following an Erlang distribution

The Erlang distribution is better suited to model service rates since with an appropriate  $k$  one can approximate a normal distribution without incurring in the aspect of having to manually reset negative values to one. This is asserted by the formal definition of Erlang range  $x \in [0, \infty)$ .

NumPy's implementation of its Erlang distribution is used<sup>5</sup>. Equation 4.2 defines the probability density function of the Erlang's distribution with the alternative parametrization that uses  $\mu$  instead of  $\lambda$  as scale parameter, which is its reciprocal.

$$f(x; k, \mu) = \frac{x^{k-1} e^{-\frac{x}{\mu}}}{\mu^k (k-1)!} \text{ for } x, \mu \geq 0 \quad (4.2)$$

<sup>5</sup><https://docs.scipy.org/doc/numpy/reference/generated/numpy.random.gamma.html> (accessed 06.01.2017)



# Policy Based Role Resolution

Zeng and Zhao (2005, p. 7) investigate five “role-resolution” policies used for optimal role resolution. Following a brief description of the five aforementioned policies: **I.** A load balancing policy consists in assigning a task as soon as it arrives to a qualified worker with the shortest task queue at that moment. In this policy workers execute tasks assigned to them on a **First In First Out (FIFO)** fashion. The authors call this policy the “**Least Loaded Qualified Person (LLQP)**” **II.** A policy that maintains a single queue being shared among all users is referred to the authors as “**Shared Queue (SQ)**” **III.** Another policy that maintains both a **SQ** among all users and each user having an own queue and transfers tasks from the former to the latter is called “**K-Batch**” policy. Transfer of tasks from the **SQ** to users is done using an optimal task assignment procedure as soon as the **SQ** reaches a critical batch size  $K$  **IV.** The next policy takes the “**K-Batch**” policy but reduces the individual queue size of each user to one. This means that the optimization problem is still being solved as soon as the **SQ** reaches the critical size  $K$ , however actual movement of tasks from the **SQ** to the individual user queue happens only when user  $i$  is not busy *i.e.*, his individual queue is empty at simulation time  $t$ . This policy is called according to the authors as “**K-Batch-1**” **V.** The last policy further simplifies the fourth by weakening the batch size constraint and reduces it to one. This means that the optimal task assignment procedure is executed immediately. This policy is referred by the authors as “**1-Batch-1**”.

## 5.1 Mixed Integer Linear Programming Policies

All batch policies require the solution of an optimization problem which Zeng and Zhao (2005) define as “minimizing the maximum flowtime given the dynamic availability of the workers” and call it “**Minimizing Sequential Assignment (MSA)**”(Zeng and Zhao, 2005, p. 7). Zeng and Zhao (2005) define the task flowtime as the elapsed simulation time between task generation and its completion (Baker, 1974; Zeng and Zhao, 2005).

Table 5.1 gives an overview of all variables used in this section for all **Mixed Integer Linear Programming (MILP)** formulations.

Formally **MSA** is formulated as follows:

Variable	Description
$i$	User
$j$	Job
$k$	Job assignment order
$x$	Job to user assignment
$p$	User service time for job
$a$	User busy time
$w$	Job waiting time in queue
$z$	Maximum flowtime

**Table 5.1:** Variables description used for all MILP formulations

$$\begin{aligned}
& \min \quad z \\
& \text{subject to:} \\
& \sum_{i \in W} x_{ij} = 1 \quad \forall j \in T \\
& a_i + \sum_{j \in T} x_{ij} p_{ij} \leq z \quad \forall i \in W \\
& x_{ij} = 0 \quad \text{or} \quad x_{ij} = 1 \quad \forall i \in W, \forall j \in T
\end{aligned} \tag{5.1}$$

Zeng and Zhao (2005)' definition of **MSA** is however a simplified version of “minimizing the maximum task flowtime with consideration of the dynamic arrival of tasks”, defined as **Dynamic Minimization of Maximum Task Flowtime (DMF)** (Baker, 1974; Zeng and Zhao, 2005). **DMF** is formally defined by Zeng and Zhao (2005) as follows:

$$\begin{aligned}
& \min \quad z \\
& \text{subject to:} \\
& \sum_{i \in W} \sum_{k \in T} x_{ijk} = 1 \quad \forall j \in T \\
& s_j \geq r_j \quad \forall j \in T \\
& (x_{ijk} + x_{ij'(k+1)} - 1)(s_j + p_{ij}) \leq s_{j'} \quad \forall i \in W, \forall k \in T, \forall j \in T, \forall j' \in T \\
& s_j + \sum_{i \in W} \sum_{k \in T} p_{ij} x_{ijk} - r_j \leq z \quad \forall j \in T \\
& x_{ijk} = 0 \quad \text{or} \quad x_{ijk} = 1 \quad \forall i \in W, \forall j \in T, \forall k \in T \\
& s_j \geq 0
\end{aligned} \tag{5.2}$$

As Zeng and Zhao (2005) note in their work, Equation 5.2 contains nonlinear constraints and mention, that by adding auxiliary variables the aforementioned **DMF** formulation can be effectively converted into a **MILP** problem and thus solve it (Zeng and Zhao, 2005, p. 6). On this note they argue that the application of **DMF** in practice poses some problems (Zeng and Zhao, 2005). In this thesis a conversion of the **DMF** formulation proposed by Zeng and Zhao (2005) by adding the required auxiliary variables is introduced in order to adequately solve the optimization problem. The formulation is called **Extended DMF (EDMF)** and is devised as follows:

$$\begin{aligned}
& \min \quad z_{\max} \\
& \text{subject to:} \\
& \sum_{i \in W} \sum_{k \in T} x_{ijk} = 1 \quad \forall j \in T \\
& a_i + \sum_{j \in T} p_{ij} x_{ijk} \leq z_{i*k} \quad \forall i \in W, \forall k \in T \quad \text{for } k = 0 \\
& z_{i*k-1} + \sum_{j \in T} p_{ij} x_{ijk} \leq z_{i*k} \quad \forall i \in W, \forall k \in T \quad \text{for } k > 0 \\
& z_{i*k} + \sum_{j \in T} w_j x_{ijk} \leq z_{\max} \quad \forall i \in W, \forall k \in T \\
& \sum_{j \in T} x_{ijk} \leq 1 \quad \forall i \in W, \forall k \in T \quad \text{for } k = 0 \\
& \sum_{j \in T} x_{ijk} \leq \sum_{j \in T} x_{ijk-1} \quad \forall i \in W, \forall k \in T \quad \text{for } k > 0 \\
& z_{i*k} \geq 0 \quad \forall i \in W, \forall k \in T
\end{aligned} \tag{5.3}$$

This formulation clearly gets rid of the nonlinear constraints while still accounting for dynamical arrival of tasks, making thus **DMF** as defined by **Zeng and Zhao (2005)** effectively solvable.

When considering the minimization of the maximum flowtime of a task inside a process, **EDMF** can be further simplified by adopting some assumptions about the order and sequence of tasks. Based on how batch policies are implemented, queued policy jobs are implicitly stored in a sorted fashion. This means that the  $z$  helper variable defined for **EDMF** is not strictly necessary and thus can be compressed by Equation 5.4:

$$a_i + \sum_{t=1}^k \sum_j (p_{ij} + w_j I(t=k)) x_{ijt} \tag{5.4}$$

The whole concept consists in the introduction of an identity variable  $I$  which is true if and only if task  $j$  is currently being assigned as the  $k$ th task to user  $i$ , meaning that for this specific case also the waiting time for task  $j$  has to be accounted for. For all other cases *i.e.*,  $j < k$ , the identity variable  $I$  will not hold thus effectively zeroing the  $w_j$  variable.

In order to calculate  $z_{ijk}$ , one has to consider when user  $i$  will actually be available to process his first task as well. This is depicted by the variable  $a_i$ , which summed together with the respective service times of user  $i$  for task  $j$  gives the complete work time user  $i$  will require in order to process all tasks assigned to him.

Without further ado, the simplified formulation of **EDMF** (called **Simplified DMF (SDMF)**) is the following:

$$\begin{aligned}
& \min \quad z_{\max} \\
& \text{subject to:} \\
& \sum_{i \in W} \sum_{k \in T} x_{ijk} = 1 \quad \forall j \in T \\
& a_i + \sum_{t=1}^k \sum_j (p_{ij} + w_j I(t=k)) x_{ijt} \leq z_{\max} \\
& \sum_{j \in T} x_{ijk} \leq 1 \quad \forall i \in W, \forall k \in T \quad \text{for } k=0 \\
& \sum_{j \in T} x_{ijk} \leq \sum_{j \in T} x_{ijk-1} \quad \forall i \in W, \forall k \in T \quad \text{for } k > 0
\end{aligned} \tag{5.5}$$

By comparing both formulation it is clear that **SDMF** manages to simplify the mathematical formulation and relaxing the required amount of constraints while still attaining the same level of effectiveness<sup>1</sup>.

Based on this approach and by further exploiting the implicit order implementation of task arrival, it is possible to argue that the  $k$  sequence indexing can be relaxed as well.

The formulation of **DMF** by relaxing both the  $z$  variables and  $k$  indexes is the following:

$$\begin{aligned}
& \min \quad z_{\max} \\
& \text{subject to:} \\
& \sum_{i \in W} x_{ij} = 1 \quad \forall j \in T \\
& a_i + \sum_{k=1}^j (p_{ik} + w_k I(k=j)) x_{ik} \leq z_{\max}
\end{aligned} \tag{5.6}$$

and is colloquially called **Extremely Simplified DMF (ESDMF)**.

This version is however only possible by the nature of its implementation. Since both the global as well as the local queues are implemented as **FIFO** queues, it is possible to relax the ordering constraint from the mathematical formulation since it is already implicitly defined by the implementation.

The culminating “flagship” improvement of the optimization method proposed by **Zeng and Zhao (2005)** done in this thesis is a method that aims to optimally solve the assignment problem by changing its goal: minimize the service times by setting an upper bound on the maximum flowtime and is called **Service Time Minimization with ESDMF as Upper Bound (ST)**. This method uses a two step process in order to optimally apply role resolution: **I**. Optimally solve by means of using one of the **DMF** optimization methods previously outlined. This yields an upper bound for the maximum flowtime **II**. Use this upper bound as a constraint for the actual optimization in order to effectively optimize the problem for the minimal service time amongst users, jobs and their corresponding service time.

---

<sup>1</sup>Note, however, that this simplification is only possible because of the nature of the implementation.



$$\begin{aligned}
& \min \quad \sum_{i \in W} \sum_{k \in T} z_{ik} \\
& \text{subject to:} \\
& \quad \sum_{i \in W} \sum_{k \in T} x_{ijk} = 1 \quad \forall j \in T \\
& \quad a_i + \sum_{j \in T} p_{ij} x_{ijk} - M(1 - \sum_{j \in T} x_{ijk}) \leq z_{i*k} \quad \forall i \in W, \forall k \in T \quad \text{for } k = 0 \\
& \quad z_{i*k-1} + \sum_{j \in T} p_{ij} x_{ijk} - M(1 - \sum_{j \in T} x_{ijk}) \leq z_{i*k} \quad \forall i \in W, \forall k \in T \quad \text{for } k > 0 \\
& \quad z_{i*k} + \sum_{j \in T} w_j x_{ijk} \leq z_{\max} + \epsilon \quad \forall i \in W, \forall k \in T \\
& \quad \sum_{j \in T} x_{ijk} \leq 1 \quad \forall i \in W, \forall k \in T \quad \text{for } k = 0 \\
& \quad \sum_{j \in T} x_{ijk} \leq \sum_{j \in T} x_{ijk-1} \quad \forall i \in W, \forall k \in T \quad \text{for } k > 0 \\
& \quad z_{i*k} \geq 0 \quad \forall i \in W, \forall k \in T \\
& \quad M = \max_a a_i + \max_p \sum_{i \in W} \sum_{j \in T} p_{ij} \\
& \quad \epsilon = 1 \times 10^{-4}
\end{aligned} \tag{5.7}$$

Table 5.2 shows the formulation complexity for the methods outlined in this chapter where  $m$  is the number of users and  $n$  is the batch size. The formulation complexity is expressed in number of binary variables that each problem formulation has. The **MSA** method is the simplest formulation and exhibits a linear complexity compared to the **DMF** method proposed by Zeng and Zhao (2005). As it can be seen the methods implemented in this thesis, specifically the **ESDMF** method, all solve the **DMF** method and do it by keeping the same linear complexity as the **MSA** method. The **ST** method proposed in this thesis exhibits a higher formulation complexity as shown in Equation 5.8

$$\mathcal{O}(mn + mn^2) \subseteq \mathcal{O}(mn^2) \tag{5.8}$$

but achieves a better optimization. This trade-off however requires a more in depth explanation which will follow in Section 8.1.

Problem Formulation	Formulation Complexity
MSA	$\mathcal{O}(mn)$
DMF	$\mathcal{O}(mn^2)$
SDMF	$\mathcal{O}(mn^2)$
ESDMF	$\mathcal{O}(mn)$
ST	$\mathcal{O}(mn^2)$

**Table 5.2:** Comparison of formulation complexities expressed in number of binary variables for different problem formulations where  $m$  is the number of users and  $n$  is the batch size

## 5.2 Reinforcement Learning Theory

In this section the **Reinforcement Learning (RL)** approach used to solve the different role resolution problems is depicted. Initially Subsection 5.2.1 outlines the key definitions.

### 5.2.1 Reinforcement Learning Definition

**RL** is a novel approach originated as a branch from the broader field of machine learning (Sutton and Barto, 2017). It is an automated approach to understanding and automating learning and decision-making (Sutton and Barto, 2017, p. 15). It distinguishes itself from other approaches by its novel focus on learning thanks to an agent which directly interacts with its environment, without the necessity of relying on training sets (Sutton and Barto, 2017, p. 15).

The formal framework used by **RL** defines the interaction between the so called learning agent and its environment by means of states, actions and rewards (Sutton and Barto, 2017, p. 15).

Key concepts in the field of **RL** are those of values and value functions which help distinguish **RL** methods from evolutionary methods which have to undergo scalar evaluations of entire policies (Sutton and Barto, 2017, p. 15).

### 5.2.2 Finite Markov Decision Processes

**RL** approaches learn by interacting with the environment in order to achieve a goal (Sutton and Barto, 2017). The agent interacting with the environment does this in a sequence of discrete time steps, it performs actions<sup>2</sup>, reaches then states<sup>3</sup> and eventually receives rewards<sup>4</sup> (Sutton and Barto, 2017, p. 73). Moreover, a policy is a stochastic rule that the agent relies upon to choose actions as a function of states (Sutton and Barto, 2017, p. 73). Ultimately, the sole goal of the agent is to maximize the reward that it receives over time (Sutton and Barto, 2017, p. 73).

Returns are modeled as functions of future rewards that an agents must maximize (Sutton and Barto, 2017, p. 73). There exist two types of return functions which depend on the nature of the tasks and a discounting preference (Sutton and Barto, 2017, p. 73): **I.** For episodic tasks a non discontinued approach is preferred **II.** For continuous tasks a discounted approach is better suited.

Equation 5.9 defines the sum of the rewards received over time step  $t$  (Sutton and Barto, 2017, p. 73):

$$G_t \doteq R_{t+1} + R_{t+2} + R_{t+3} + \cdots R_T \quad (5.9)$$

If we account for discounting, Equation 5.9 has to be slightly adapted by introducing a discounting factor  $\gamma$  and can be found in Equation 5.10:

$$G_t \doteq R_{t+1} + R_{t+2} + R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \quad (5.10)$$

where  $0 \leq \gamma \leq 1$  (Sutton and Barto, 2017, p. 73).

An environment with which one agent interacts, can satisfy a Markov property if the information contained at present summarizes the past without affecting the capability of effectively predicting the future (Sutton and Barto, 2017, p. 73). If the Markov property is satisfied, then this environment is called a **Markov Decision Process (MDP)** (Sutton and Barto, 2017, p. 73).

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<sup>2</sup>Choices made by the agent.

<sup>3</sup>Basis for making decisions.

<sup>4</sup>Basis for evaluating the choices.

Last but not least, value functions are used to assign each state or state-action pair an expected return based on the policy used by the agent (Sutton and Barto, 2017, p. 74). Optimal value functions assign the highest achievable return by any policy to a state or state-action pair and such policies, whose values are optimal, are called optimal policies (Sutton and Barto, 2017, p. 74).

Optimal **State Value (SV)** functions  $v_*$  are formally defined as follows (Sutton and Barto, 2017, p. 74):

$$v_*(s) \doteq \max_{\pi} v_{\pi}(s) \quad (5.11)$$

whereas optimal **Action Value (AV)** functions  $q_*$  are formally defined as follows (Sutton and Barto, 2017, p. 74):

$$q_*(s, a) \doteq \max_{\pi} q_{\pi}(s, a) \quad (5.12)$$

### 5.2.3 Dynamic Programming

**Dynamic Programming (DP)** is a set of ideas and algorithms that can be used to solve **MDPs** (Sutton and Barto, 2017, p. 95). There are two approaches in **DP** for solving **MDPs** (Sutton and Barto, 2017, p. 95): **I.** Policy evaluations is the iterative computation of value functions of a given policy **II.** Policy improvement is the idea of computing an improved policy under the conditions of its given value functions.

By combining these two approaches we obtain the two most notable **DP** methods *i.e.*, policy and value iteration (Sutton and Barto, 2017, p. 95).

One captivating property of **DP** methods is the concept of bootstrapping: updating estimates of values of states by approximating the values of future states (Sutton and Barto, 2017, p. 96).

### 5.2.4 Monte Carlo Methods

**Monte Carlo (MC)** methods use experience in form of sample episodes in order to learn value functions and optimal policies (Sutton and Barto, 2017, p. 123). This approach yields different advantages over the **DP** methods seen in Subsection 5.2.3 (Sutton and Barto, 2017, p. 123): **I.** They do not need a model of the environment's dynamics as they learn the optimal solutions by merely interacting with the environment **II.** Since they learn from sample episodes, they are very well suited for simulation environments **III.** It is efficient and surprisingly easy to use **MC** methods to focus on smaller regions or subsets of a problem **IV.** **MC** methods are more robust when it comes to violations of the Markov property since they do not bootstrap for updating their values.

One of the drawbacks that **MC** methods bring along is the concept of maintaining sufficient exploration: by always acting greedily, alternative states will never yield their returns thus potentially never learning that they might prove to be better (Sutton and Barto, 2017, p. 123).

A **MC** simplified method can be formally defined as follows:

$$V(S_t) \leftarrow V(S_t) + \alpha[G_t - V(S_t)] \quad (5.13)$$

where  $G_t$  is the discounted return function defined by Equation 5.10 and  $\alpha$  is a constant step-size parameter (Sutton and Barto, 2017, p. 127). **MC** methods must wait until the end of one episode in order to evaluate the incremental value of  $V(S_t)$  since only at that point in time  $G_t$  is known (Sutton and Barto, 2017, p. 128).

### 5.2.5 Temporal Difference Learning

**Temporal Difference (TD)** are yet another set of learning methods for **RL** (Sutton and Barto, 2017). Compared to the **MC** methods explained in Subsection 5.2.4, **TD** methods do not need to wait all the way up to the end of an episode to actually learn, they only must wait until the next step *i.e.*, they can bootstrap (Sutton and Barto, 2017, p. 128). When they reach time step  $t + 1$ , they observe a reward  $R_{t+1}$  which then use to estimate  $V(S_{t+1})$  (Sutton and Barto, 2017, p. 128). The simplest **TD** method is defined as follows:

$$V(S_t) \leftarrow V(S_t) + \alpha[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)] \quad (5.14)$$

**TD** methods, same as **MC** methods, are not exempt from sufficient exploration (Sutton and Barto, 2017, p. 147). **TD** methods deal with this complication in two different ways (Sutton and Barto, 2017, p. 128): **I. On Policy (ONP)** by using an algorithm called **State-Action-Reward-State-Action (SARSA)** **II. Off Policy (OP)** by using an algorithm called Q-learning.

### 5.2.6 On Policy Prediction with Approximation

Up until now, the different methods presented are not suited for arbitrarily large state spaces (Sutton and Barto, 2017). There exist solutions to tackle such large state spaces: approximate solution methods (Sutton and Barto, 2017). Under the assumption that one must always account for finite and limited computational resources, it is not feasible to find an optimal policy or value function, instead we have to settle for a good approximation of the solution (Sutton and Barto, 2017, p. 189).

An essential characteristic for **RL** algorithms venturing in the area of approximation is being able of generalization *i.e.*, using experience from a limited subset of the state space to effectively generalize and produce a valid approximation of a much larger subset (Sutton and Barto, 2017, p. 189). **RL** methods are capable of achieving this by relying on supervised-learning function approximation which essentially uses backups as training examples (Sutton and Barto, 2017, p. 222). Specifically, one brilliant set of methods are those using parametrized function approximation *i.e.*, the policy is parametrized by a weight vector  $\theta$  (Sutton and Barto, 2017).

The parametrized functional form with weight vector  $\theta$  can be used to write  $\hat{v}(s, \theta) \approx v_\pi(s)$ , which is the approximated value of state  $s$  given weight vector  $\theta$  (Sutton and Barto, 2017, p. 191).

It is then clear that the weight vector  $\theta$  has to be chosen wisely: this can be done by using variations of **Stochastic Gradient Descent (SGD)** methods (Sutton and Barto, 2017, p. 223). **SGD** methods adjust the weight vector after each step by a tiny amount following the direction that would reduce the error the most:

$$\begin{aligned} \theta_{t+1} &\doteq \theta_t - \frac{1}{2} \alpha \nabla [v_\pi(S_t) - \hat{v}(S_t, \theta_t)]^2 \\ &= \theta_t + \alpha [v_\pi(S_t) - \hat{v}(S_t, \theta_t)] \nabla \hat{v}(S_t, \theta_t) \end{aligned} \quad (5.15)$$

where  $\alpha$  is a positive step size parameter and  $\nabla f(\theta)$ :

$$\nabla f(\theta) \doteq \left( \frac{\partial f(\theta)}{\partial \theta_1}, \frac{\partial f(\theta)}{\partial \theta_2}, \dots, \frac{\partial f(\theta)}{\partial \theta_n} \right)^\top \quad (5.16)$$

is the vector of partial derivatives with respect to  $\theta$  (Sutton and Barto, 2017, p. 195).

An exceptional case is linear methods for function approximation, where the approximate function  $\hat{v}(\cdot, \theta)$  is a linear function of the weight vector  $\theta$  (Sutton and Barto, 2017, p. 198). This means that for each state  $s$  there is a corresponding vector of features

$$\phi(s) \doteq (\phi_1(s), \phi_2(s), \dots, \phi_n(s))^\top \quad (5.17)$$

which has the same number of components as  $\theta$  (Sutton and Barto, 2017, p. 198). With this definition in mind, we can now formally define the **State VFA (SVFA)** as the inner product between  $\theta$  and  $\phi(s)$  (Sutton and Barto, 2017, p. 198):

$$\hat{v}(s, \theta) \doteq \theta^\top \phi(s) \doteq \sum_{i=1}^n \theta_i \phi_i(s) \quad (5.18)$$

This simplified case of linear function approximation for **SV** functions finally brings us to how we can use the **SGD**:

$$\nabla \hat{v}(s, \theta) = \phi(s) \quad (5.19)$$

Equation 5.19 tells us that for the simple linear case the **SGD** is nothing more than the corresponding features value (Sutton and Barto, 2017, p. 199).

## Artificial Neural Networks

**Artificial Neural Networks (ANNs)** can be used for nonlinear function approximation (Sutton and Barto, 2017, p. 199). The simplest case of an **ANN** is a single feedforward perceptron, meaning that it has only one hidden layer *i.e.*, a layer that is neither an input nor an output layer and that the **ANN** at hand has no loops between its neurons, meaning that the output cannot influence the input<sup>5</sup> (Sutton and Barto, 2017, p. 216).

The connections between neurons inside an **ANN** are called weights and the analogy to its human counterpart is how strong synaptic connections are (Sutton and Barto, 2017, p. 216). Refer to Figure 5.1 for a depiction of a single layer **ANN**.

The key about solving nonlinearity with **ANNs** is how they apply nonlinear functions to the sum of their weights and this process is done by means of activation functions (Sutton and Barto, 2017, p. 216). There are different types of activation functions that can be used but each one of them usually exhibits an “S” shape (*i.e.*, sigmoid), such as the sigmoid  $\sigma(x) = \frac{1}{(1+e^{-x})}$ , the  $\tanh(x) = 2\sigma(2x) - 1$  or the different classes of **Rectified Linear Units (ReLUs)**, which have become captivating in the last few years due to their peculiar characteristics such as  $f(x) = \max(0, x)$  (Sutton and Barto, 2017, p. 216).

Even though single layer perceptrons are powerful enough to approximate nonlinearity, in the past years a development towards more complex **ANNs** with multiple hidden layers *i.e.*, multi-layer perceptrons, has been on the rise (Sutton and Barto, 2017, p. 217). These complex **ANNs** allow to solve many artificial intelligence tasks in a much more efficient way (Bengio, 2009). This area is called deep **RL** and it has shed light on solutions that were never though possible before (Bengio, 2009). Refer to Figure 5.2 for a depiction of a multi layer **ANNs**.

Despite appearing more complex, **ANNs** rely on a similar approach for learning *i.e.*, updating their internal parameters, or in this case the whole network’s synaptic connections, based on the **SGD** method outlined in Subsection 5.2.6 (Sutton and Barto, 2017, p. 217). This algorithm is called backpropagation and consists of doing a forward pass in which the activation function of each neuron is computed and then a backward pass computes the partial derivatives for each synaptic connection (Sutton and Barto, 2017, p. 218).

As any other function approximation method, overfitting can be a problem for **ANNs** as well and it is particularly present for deep **ANNs** (Sutton and Barto, 2017, p. 218). There are different techniques that can be used in order to mitigate this effect, with the most prominent one being the dropout method outlined by Srivastava et al. (2014).

<sup>5</sup>Compared to recurrent **ANNs**, in which the output indeed can influence the input.

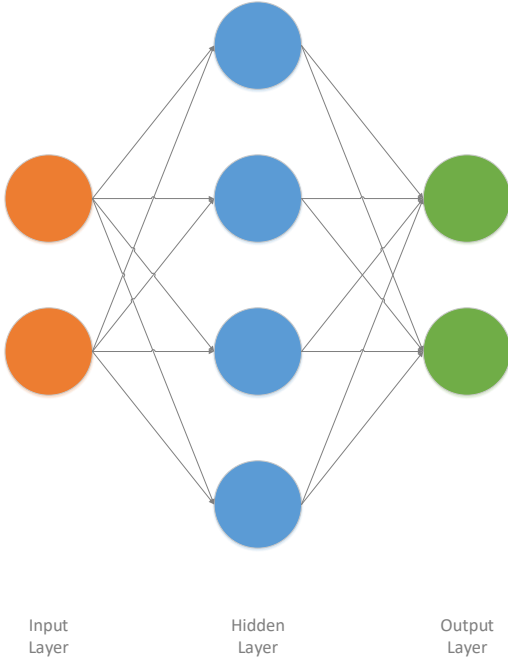


Figure 5.1: Single layer ANN

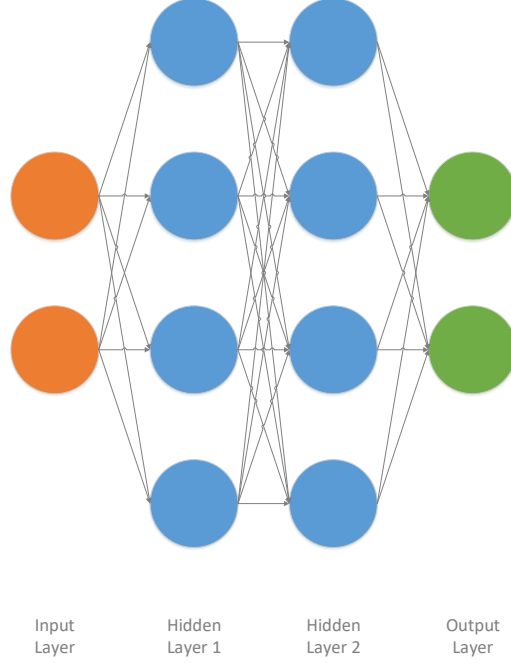


Figure 5.2: Multi layer ANN

### 5.2.7 On Policy Control with Approximation

Moving towards control with **Value Function Approximation (VFA)**, we now focus on the approximation of the **AV** function  $\hat{q}(s, a, \theta) \approx q_*(s, a)$  (Sutton and Barto, 2017, p. 229).

For the special case of the so called one-step **SARSA** method, its **SGD** update for the **AV** function is defined as follows:

$$\theta_{t+1} \doteq \theta_t + \alpha [R_{t+1} + \gamma \hat{q}(S_{t+1}, A_{t+1}, \theta_t) - \hat{q}(S_t, A_t, \theta_t)] \nabla \hat{q}(S_t, A_t, \theta_t) \quad (5.20)$$

and this method has excellent convergence properties towards optimality (Sutton and Barto, 2017, p. 230).

### 5.2.8 Off Policy Methods with Approximation

When moving towards the field of **OP** learning, one of the biggest problems that one might incur in is the convergence problem: **OP** learning with approximation is considerably harder compared to its tabular counterpart (Sutton and Barto, 2017, p. 243). **OP** learning defines two policies,  $\pi$  and  $\mu$ , where the former is the value function we seek to learn based on the latter (Sutton and Barto, 2017, p. 243).

A new aspect being introduced in **OP** learning is the importance sampling concept, formally defined as follow:

$$\rho_t \doteq \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} \quad (5.21)$$

which can be used to “warp the update distribution back to the **ONP** distribution, so that semi-gradient methods are guaranteed to converge.” (Sutton and Barto, 2017, p. 243).

During **OP** learning  $\pi$  is defined as full greedy and  $\mu$  is somewhat more exploratory  **$\epsilon$ -Greedy (EP)** (Sutton and Barto, 2017, p. 243).

For the purpose of this thesis, the focus has been put upon the episodic **AV** update algorithm, defined as follows:

$$\theta_{t+1} \doteq \theta_t + \alpha \delta_t \nabla \hat{q}(S_t, A_t, \theta_t) \quad (5.22)$$

where  $\delta_t$  is defined as:

$$\delta_t \doteq R_{t+1} + \gamma \underbrace{\sum_a \pi(a|S_{t+1}) \hat{q}(S_{t+1}, a, \theta_t)}_{\max_a \hat{q}(S_{t+1}, a, \theta_t)} - \hat{q}(S_t, A_t, \theta_t) \quad (5.23)$$

what is important to note here, is that the episodic **OP** algorithm does not use importance sampling as defined by Equation 5.21 (Sutton and Barto, 2017, p. 244). Sutton and Barto (2017, p. 244) state that this approach is clear for tabular methods but it is rather a “judgment call” for methods using approximation functions and deeper understanding of the theory of function approximation is needed.

## 5.2.9 Policy Gradient Methods

Up until now all methods were based on the concept of learning values of actions and subsequently choosing the correct actions based on estimates, however, we now move our focus towards methods that actually learn a parametrized policy without needing value functions at all<sup>6</sup> (Sutton and Barto, 2017, p. 265). Parametrized policies work with probabilities that a specific action  $a$  will be chosen at time  $t$  if the agent finds itself in state  $s$  at time  $t$  with a weight vector  $\theta$  (Sutton and Barto, 2017, p. 265). For **PG** methods it is crucial to learn the weight vector based on a performance measure  $\eta(\theta)$  by trying to maximize and thus approximating the **Stochastic Gradient Ascent (SGA)** of  $\eta$  as follows:

$$\theta_{t+1} \doteq \theta_t + \alpha \widehat{\nabla \eta(\theta_t)} \quad (5.24)$$

where  $\widehat{\nabla \eta(\theta_t)}$  is nothing else than a stochastic estimate that approximates the gradient of  $\eta(\theta)$  (Sutton and Barto, 2017, p. 265).

For discrete action spaces, a suitable solution consists in forming parametrized numerical preferences  $h(s, a, \theta) \in \mathbb{R}$  (Sutton and Barto, 2017, p. 266). This means that the best action is given the highest probability according to a softmax distribution:

$$\pi(a|s, \theta) \doteq \frac{e^{h(s, a, \theta)}}{\sum_b e^{h(s, b, \theta)}} \quad (5.25)$$

where  $e \approx 2.71828$  (Sutton and Barto, 2017, p. 266). Moreover, the preferences can be, as previously mentioned:

$$h(s, a, \theta) \doteq \theta^\top \phi(s, a) \quad (5.26)$$

simply linear in features (Sutton and Barto, 2017, p. 266).

With these definitions in mind, one can formally define one of the very first **MC** based **PG** methods: **REINFORCE** (Williams, 1992).

Williams (1992) defines his **REINFORCE** algorithm by the following update function:

<sup>6</sup>Actor Critic (AC) methods are an exception, where a learned value function is used in combination with Policy Gradient (PG) as a baseline in order to lower variance (Sutton and Barto, 2017).



$$\theta_{t+1} \doteq \theta_t + \alpha \gamma^t G_t \frac{\nabla_{\theta} \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \quad (5.27)$$

note that the vector  $\frac{\nabla_{\theta} \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)}$  is called eligibility vector and it is usually written in a more compact form of  $\nabla \log \pi(A_t|S_t, \theta)$  by relying on the mathematical identity  $\nabla \log x = \frac{\nabla x}{x}$  (Sutton and Barto, 2017, p. 271). For the REINFORCE algorithm, its eligibility vector is defined as follows:

$$\nabla_{\theta} \log \pi(a|s, \theta) = \phi(s, a) - \sum_b \pi(b|s, \theta) \phi(s, b) \quad (5.28)$$

and this method has solid convergence properties (Sutton and Barto, 2017, p. 271).

### Policy Gradient with Baseline

REINFORCE, however, being a method based on MC it might exhibit high variance and prove relatively slow in its learning rate (Sutton and Barto, 2017, p. 271). By introducing a baseline  $b(s)$  to compare the AV:

$$\theta_{t+1} \doteq \theta_t + \alpha (G_t - \overbrace{b(S_t)}^{\text{baseline}}) \frac{\nabla_{\theta} \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \quad (5.29)$$

one can achieve a positive effect towards diminishing variance of the update rule (Sutton and Barto, 2017, p. 271).

### Actor Critic Policy Gradient

By introducing a base line, we have seen that variance can be lowered, however, the REINFORCE algorithm with a baseline is not a proper AC method as its SV function is only used as baseline and not as a critic *i.e.*, it is not used for bootstrapping (Sutton and Barto, 2017, p. 273). By introducing bootstrapping we introduce bias and dependence of the quality of the approximated function, which in turn help to reduce variance and learn faster (Sutton and Barto, 2017, p. 273).

The only negative aspect still remaining is that PG methods are still based on a full MC update trajectory: this can be also mitigated by replacing the update function by TD learning approaches, such as those defined in Subsection 5.2.5 (Sutton and Barto, 2017, p. 273). The formal definition of a one-step AC update method is depicted as follows:

$$\theta_{t+1} \doteq \theta_t + \alpha (R_{t+1} + \overbrace{\gamma \hat{v}(S_{t+1}, w) - \hat{v}(S_t, w)}^{\text{TD update}}) \frac{\nabla_{\theta} \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \quad (5.30)$$

and this is now a fully online implementation that executes updates after each newly visited state (Sutton and Barto, 2017, p. 274).

## 5.3 Reinforcement Learning Policies

Analog to Section 5.1, the same policies are considered but now the RL methods and techniques outlined in Section 5.2 are used to solve role resolution.

Different subsections are used in order to separate better the different approaches used for each type of policy: **I.** Batch policy methods **II.** LLQP policy methods **III.** Other policy methods that do not fit in any of the previous categories.



The two key concepts required in order to effectively apply **RL** techniques for role resolution in **Workflow Management Systems (WfMS)** are: **I.** Correctly defined states and actions spaces **II.** Precise rewards definition **III.** Effective update method for the policy's internal parameters.

In the next subsections a distinction between prediction and update methods is outlined.

### 5.3.1 Prediction and Control Methods

As previously outlined in Subsection 5.2.6, Subsection 5.2.7, Subsection 5.2.8 and Subsection 5.2.9 there are different prediction and control methods that can be applied.

#### Value Function Approximation

As mentioned in Section 5.3, it is crucial to correctly define the states and actions space for each problem. Each request that the policy receives generates a policy job which is then passed in the internal evaluate method of the policy. Inside this method, for each policy job the state space  $S$  is defined as a  $m \times n + 1$  matrix which contains all busy times of the potential candidates (*i.e.*, users) concatenated to the user's current service time for the job. Formally the state space is defined as depicted in Equation 5.31:

$$S_{m,n+1} = \begin{bmatrix} a_1 & \cdots & a_1 \\ a_2 & \cdots & a_2 \\ \vdots & \ddots & \vdots \\ p_{1,j} & \cdots & p_{i,j} \end{bmatrix} \quad (5.31)$$

Since the possible actions are represented by the number of users, the state space is modeled such that for each possible actions a 1-D vector containing all busy times plus the service time of the user are present.

During the evaluation phase of a job the policy has to choose an action *i.e.*, a user, by taking into account the current state space and its internal  $\theta$  parameters. By using a **SVFA** as defined in Equation 5.18, the policy evaluates the highest score for each possible user.

As an example, let us consider a snippet of how a K-Batch policy using a linear **SVFA** performs its choices during its greedy phase: it iterates over all possible actions and performs the dot product between the state space and the corresponding  $\theta$  vector and then maximizes the returned  $Q$  value. This approach can be seen in Listing 5.1

```
if self.greedy:
    action = max(range(self.number_of_users), key=lambda action: self.q(
        state_space, action))
else:
    rnd = self.EPSILON_GREEDY_RANDOM_STATE.rand()
    if rnd < self.epsilon:
        action = self.EPSILON_GREEDY_RANDOM_STATE.randint(0, self.
            number_of_users)
    else:
        action = max(range(self.number_of_users), key=lambda action: self.q(
            state_space, action))
```

**Listing 5.1:**  $\epsilon$ -Greedy approach

and its respective **SVFA** in Listing 5.2.

```
def q(self, states, action):
    features = self.features(states, action)
    q = np.dot(features[action], self.theta[action])
    return q
```

Listing 5.2: SVFA

$Q$  values are however only one part of the requirements set by **RL** methods, the next crucial aspect is defining the reward function. Since **RL** agents are able to back-propagate what they have learned from one episode and thus update their internal factors, correctly defining a reward is a must. Since the goal for our domain is minimizing the maximum flowtime (from now on this metric will be referred to as lateness) of a job, the reward itself corresponds to the lateness of a job during a specific task. This can be evaluated a priori since for each policy job we know its internal parameters required to calculate the lateness *i.e.*, busy time of user  $i$  plus the service time of user  $i$  for job  $j$ , or formally  $a_i + p_{ij}$ .

The last definition required in order to effectively apply the update on the policy's internal parameters  $\theta$  is defining the **SGD** method as outlined by Equation 5.19. This method will give us the direction in which we have to update our internal  $\theta$  parameters during our chosen update method and it is nothing more than the features themselves. As an example, refer to Listing 5.3 for the concrete implementation.

```
def features(self, states, action):
    features = np.zeros((self.number_of_users, self.number_of_users + 1))
    features[action] = states[action]
    return features
```

Listing 5.3: Features definition

The features method outputs a matrix which has its values populated only for the actual chosen action. Let us assume our policy has chosen user 1 out of two possible users, then the state space looks as defined by Equation 5.32:

$$S_{2,3} = \begin{bmatrix} a_1 & a_1 \\ a_2 & a_2 \\ p_{1,j} & p_{2,j} \end{bmatrix} \quad (5.32)$$

and its features vector looks as defined by Equation 5.33:

$$\phi_{2,3} = \begin{bmatrix} a_1 & 0 \\ a_2 & 0 \\ p_{1,j} & 0 \end{bmatrix} \quad (5.33)$$

## Policy Gradient

With **PG** methods the approach on how an action is chosen is shifted. Instead of maximizing a  $Q$  value through internal  $\theta$  parameters in order to choose the “best greedy” action, we now have probabilistic choices. As already outlined in Subsection 5.2.9, having a probabilistic policy  $\pi$  means that the best action is now chosen according to the highest probability which follows a softmax distribution as defined in Equation 5.25 and its implementation can be seen in Listing 5.4.

```
def policy_probabilities(self, busy_times):
    probabilities = [None] * self.number_of_users
    for action in range(self.number_of_users):
```

```

probabilities[action] = np.exp(np.dot(self.features(busy_times, action
), self.theta)) / sum(
    np.exp(np.dot(self.features(busy_times, a), self.theta)) for a in
    range(self.number_of_users))
return probabilities

```

**Listing 5.4:** Softmax distribution of preferences probabilities

The policy probabilities method takes as input parameter the current state space and computes for each user its probability according to the current internal  $\theta$  parameter as defined in Equation 5.26. The result of this method is a 1-D probabilities vector corresponding to a preference to assign a job to a specific user, where the index of the vector corresponds to the user and the value to its preference. Based on this preferences vector, the policy then computes a weighted random choice among all users, as can be seen in Listing 5.5.

```

chosen_action = self.RANDOM_STATE_PROBABILITIES.choice(self.number_of_users,
p=probabilities)

```

**Listing 5.5:** Probabilistic user choice

## Artificial Neural Networks as Function Approximation

Up to this point we have used linear functions for the approximation of the  $Q$  value for the different policies. As mentioned in Subsection 5.2.6, ANNs can be used for nonlinear function approximation. The assignment problem poses itself very well for this kind of application, in which we model our input layer as a 1-D vector containing all required information such as waiting time  $w$  of job  $j$ , service time  $p_{ij}$  of user  $i$  for job  $j$  and busy time  $a_i$  of user  $i$ . By following a PG approach, we can categorize the output layer of our ANNs using a softmax categorization function, mapping the preferences of job  $j$  to user  $i$  assignment as probabilities. Listing 5.6 show the modeling of a single layer perceptron in Tensorflow (TF): one hidden layer connects the state space (i.e., input to the ANNs) together with its weights and biases, creates an activation function and maps the prediction layer (i.e., output) with a softmax classification.

```

with tf.name_scope("neural_network"):
    layer_1 = tf.add(tf.matmul(state_space_input, weights['h1']), biases['b1'])
    layer_1 = tf.nn.elu(layer_1)
    pred = [tf.add(tf.matmul(layer_1, weights['out'][b]), biases['out'][b])
            for b in range(batch_input)]
    probabilities = [tf.nn.softmax(pred[b]) for b in range(batch_input)]

```

**Listing 5.6:** Modeling of a single perceptron in TF

In order to update the ANNs, a backpropagation has to take place. Such an update can both be made following a MC or TD approach (refer to Subsection 5.3.2 for a detailed distinction between these two update methods). As outlined in Subsection 5.2.6, we follow a SGD approach in which we update all the synaptic connections by computing the partial derivatives of all the weights. Listing 5.7 shows how the backpropagation for an ANNs following a MC update method is done.

```

def train(self):
    for t, (state, output, choices) in enumerate(self.history):
        disc_rewards = self.discount_rewards(t)
        tmp_choices = [choice for choice in choices if choice is not None]

```

```

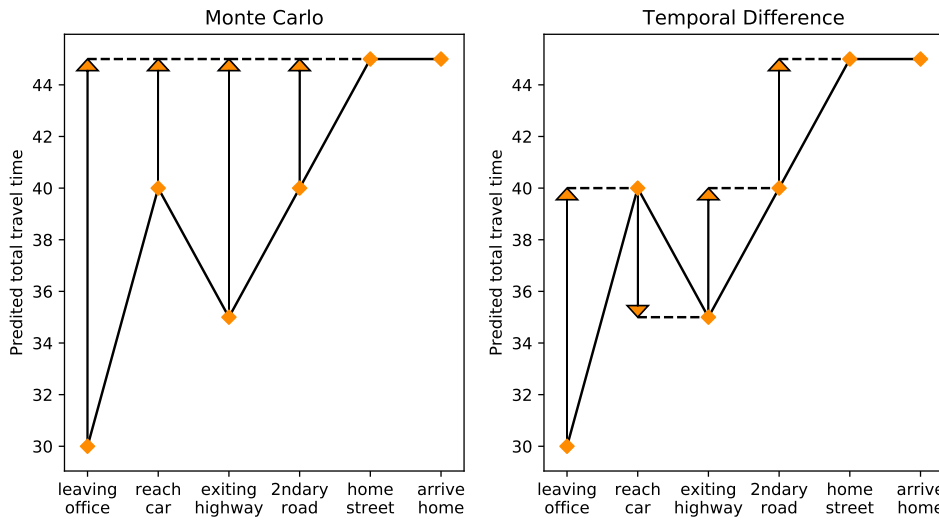
for job_index, chosen_user in enumerate(tmp_choices):
    prob_value = output[job_index].flatten()[chosen_user]
    reward = disc_rewards[job_index]
    factor = reward / prob_value
    grad_input = np.zeros((self.number_of_users, 1))
    grad_input[chosen_user] = 1.0
    self.sess.run(self.apply[job_index], {self.state_space_input: state
        , self.gradient_input: grad_input, self.factor_input: factor})

```

**Listing 5.7:** Backpropagation algorithm following a MC update approach

### 5.3.2 Update Methods

As outlined in Subsection 5.2.4 and Subsection 5.2.5, there are mainly two different methods to update the policy's internal  $\theta$  parameters *i.e.*, **MC** and **TD**. Let us take the example outlined by Sutton and Barto (2017, p. 130) of leaving the office and getting home and the respective updates proposed by the two update methods. Figure 5.3 shows the graphical updates proposed by the two update methods.



**Figure 5.3:** MC and TD proposed updates comparison (own plot based on Sutton and Barto (2017, p. 130))

As it can be clearly seen, the main difference lays in when the actual updating takes place. On one hand, the **MC** method needs to reach the end of an entire episode (*i.e.*, here it consists of actually arriving home) in order to fully back-propagate its learned value and update the  $\theta$  parameters. On the other hand, **TD** is much more flexible and robust since it executes its updates at each time step, hence its name: **TD**. For a formal overview of the difference between the two update methods, refer to Equation 5.13 for the **MC** update and to Equation 5.14 for the **TD** update. For the case at hand, this means that training the policies has to be done in a different fashion for the two update methods: while **TD** based policies can be updated “on-the-fly”, **MC** methods require batch training sessions (*i.e.*, episodes) at the end of which they can effectively learn and

update their internal  $\theta$  parameters to be used for the next episode. Not only the training approach is different, but the logic of the policy itself is also different: for **TD** based policies, the update method is being called internally since the policy knows its temporal steps, while for **MC** based policies the policy itself cannot know a priori when an episode will finish and thus must relay on an “artificial” definition of such. The overall overhead is also different since **MC** based policies have to keep track of their whole episode history which usually is composed of the state space, chosen action and reward at time step  $t$ .

[Sutton and Barto \(2017\)](#) outline a qualitative comparison between both update methods which can be found summarized in table Table 5.3.

Characteristic	MC	TD
Bootstrap	No	Yes
Update Time	End of episode	Each time step
Discount	Required	Not required
Convergence	Good	Very Good
Learning Rate	Slow for long episodes	Very fast even for long episodes

**Table 5.3:** Qualitative comparison between MC and TD update methods ([Sutton and Barto, 2017](#), p. 130)

### 5.3.3 Batch Size Emulation

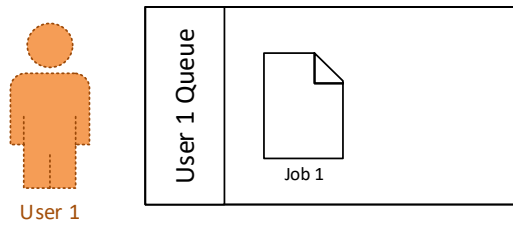
Correctly defined state spaces is a crucial requirement for effective **RL** methods. **LLQP** policies are relatively easy to be modeled, on the other hand policies with batch sizes require a more meticulous consideration. By introducing a batch size that retains jobs in its global queue *i.e.*, all batch sizes  $K > 1$ , not only the role resolution plays a role, but the ordering of the assignment influences greatly the final outcome as well. Let us consider a simple case with number of jobs  $m = 3$ , number of users  $n = 2$  and at time step  $t$ . Table 5.4 summarizes the service times  $p_{ij}$  in time units  $t$  of both users for all three jobs.

User	Job 1	Job 2	Job 3
User 1	1	2	3
User 2	4	5	6

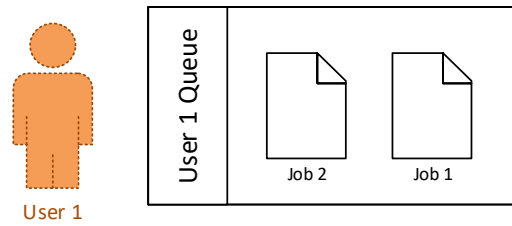
**Table 5.4:** Service times of both users for all three jobs

It is clear that the ordering of the jobs assigned has an impact on the final outcome. Figure 5.4 outlines a possible conformation where user 1 receives job 1 while user 2 gets assigned to jobs 2 and 3 respectively. In this case, job 1 is started at  $t$  and is finished at  $t + 1$ , job 2 is started at  $t$  and is finished at  $t + 5$  and job 3 is started at  $t + 5$  and is finished at  $t + 11$ . The respective lateness per job is: 1 for job 1, 5 for job 2 and 6 for job 3.

By changing the assignment order (refer to Figure 5.5 for a graphical representation), the final outcome changes as well: In this case, job 1 is started at  $t + 2$  and is finished at  $t + 3$ , job 2 is started at  $t$  and is finished at  $t + 2$  and job 3 is started at  $t$  and is finished at  $t + 6$ . The respective lateness per job is: 1 for job 1, 2 for job 2 and 6 for job 3. By merely changing the assignment order a 2.5 speedup factor in lateness is observed for job 2.



**Figure 5.4:** User 1 receives job 1 while user 2 receives jobs 2 and 3



**Figure 5.5:** User 1 receives jobs 2 and 1 while user 2 receives job 3

Subsection 7.2.3 outlines three different types of policies (refer to Table 7.9 for a detailed explanation of the policies) that take into account the previously outlined job assignment ordering principle and exploit it in their state space modeling. This is done by means of integrating an additional parameter  $B$  that defines how many jobs have to be considered when trying to optimally assign jobs queuing in the global queue to users. This is done by listing all possible combinations by accounting for the job order as well. Refer to Listing 5.8 for the actual implementation.

```
combinations = list(itertools.product(range(self.number_of_users), repeat=
    self.wait_size))
for i, combination in enumerate(combinations):
    state_space[i] = a + [p[user_index][job_index] for job_index, user_index
        in enumerate(combination)]
```

**Listing 5.8:** State space modeling by considering  $B$  jobs from the global queue and integrating all possible combinations

This approach effectively simulates batch policies with batch sizes  $K > 1$ .

# Empirical Analysis

In order to consistently and fairly evaluate all policies with the methods defined in the previous chapters, the following methodology was put in place: **I.** Each policy has its own simulation script that initializes a process that uses the predefined policy as means to solve role resolution **II.** Parameters are centrally defined **III.** Different **Key Performance Indicators (KPIs)** have been defined which are used to assert the efficiency of one policy against one another.

## 6.1 Simulation Script

Each simulation script is the abstract element that imports all required dependencies, initializes the `SimPy` simulation environment, the statistics file into which the policy dumps all data during runtime, the policy object itself to be used for the assignment and the process to be used.

The script initializes the chosen process and then calls the tokens generation method of the start event. Eventually the whole simulation is started by calling the `run` method of the `SimPy` environment. A snippet of a simulation script can be found in Listing 6.1.

```
import simpy
from evaluation.statistics import calculate_statistics
from evaluation.subplot_evolution import evolution
from policies.optimization.batch.k_batch import K_BATCH
from simulations import *
from solvers.dmf_solver import dmf

policy_name = "{}_BATCH_DMF_NU{}_GI{}_SIM{}".format(BATCH_SIZE,
    NUMBER_OF_USERS, GENERATION_INTERVAL, SEED, SIM_TIME)

env = simpy.Environment()

file_policy = create_files("{}_csv".format(policy_name))

policy = K_BATCH(env, NUMBER_OF_USERS, WORKER_VARIABILITY, file_policy,
    BATCH_SIZE, dmf)

start_event = acquisition_process(env, policy, 1, GENERATION_INTERVAL, False,
    None, None, None)
```

```

env.process(start_event.generate_tokens())

env.run(until=SIM_TIME)

file_policy.close()

calculate_statistics(file_policy.name, outfile=True)

evolution(file_policy.name, outfile=True)

```

**Listing 6.1:** Example of the structure of a simulation script. Here for the K-Batch policy using the DMF formulation

## 6.2 Business Process Modeling

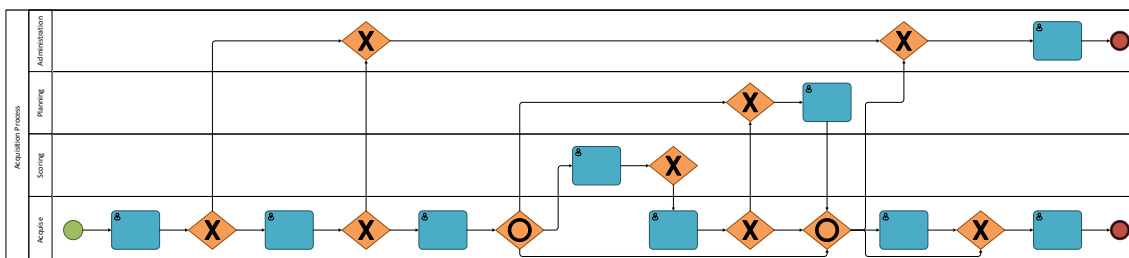
Two different types of processes have been defined: **I.** Consisting of only one user task **II.** A complex process that is modeled against an acquisition process used in the real estate field for the acquisition of real estate properties.

Figure 6.1 illustrates the simple process.



**Figure 6.1:** Simple process consisting of only one user task

Figure 6.2 illustrates the complex acquisition process.



**Figure 6.2:** Acquisition process consisting of multiple user tasks and decision nodes



## 6.3 Global Simulation and Process Parameters Definition

One key aspect in order to assert comparability across policies while simulated is to centrally define all parameters. Listing 6.2 shows the key central parameters defined as global variables.

```
NUMBER_OF_USERS = 3
SERVICE_INTERVAL = 1
GENERATION_INTERVAL = 3
SIM_TIME = 1000
BATCH_SIZE = 5
TASK_VARIABILITY = 0.2 * SERVICE_INTERVAL
WORKER_VARIABILITY = 0.2 * SERVICE_INTERVAL
SEED = 2
```

**Listing 6.2:** Global parameters definition that ensures comparability across simulation runs

## 6.4 KPIs for Asserting Policy's Efficacy and Data Visualization

Based on Pinedo (2008)'s and Zeng and Zhao (2005)'s definitions, different KPIs have been defined to assert the efficacy of a policy, such as lateness, waiting time, service time, number of tokens completed, user loads and system load. Following the formal definitions of the per token  $j$  KPIs in respect to lateness  $L_j$ , wait time  $w_j$ , service time  $p_{ij}$  of assigned user  $i$  to token  $j$ , arrival time  $A_j$ , assignment time  $a_j$ , start time  $S_j$  and finish time  $F_j$ .

$$L_j = F_j - A_j \quad (6.1)$$

$$w_j = S_j - A_j \quad (6.2)$$

$$p_{ij} = F_j - S_j \quad (6.3)$$

Moreover, if we account for simulation time  $T$ , load  $l_i$  of user  $i$  is defined as the sum of all service times of tokens that have been assigned to him during the simulation divided by the total simulation time  $T$ , or formally:

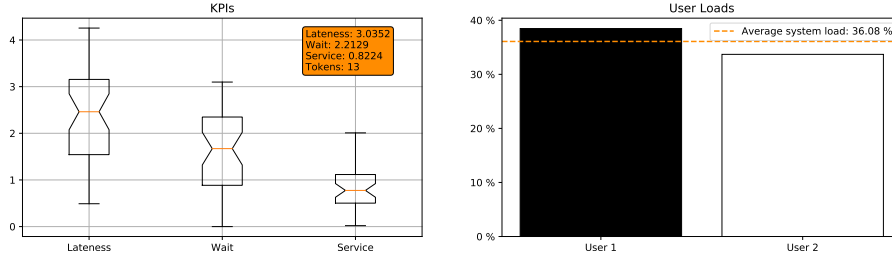
$$l_i = \frac{\sum_j p_{ij}}{T} \quad (6.4)$$

and thus the average system load  $\bar{l}$  over all  $n$  users participating is defined as the average across all user's loads *i.e.*,

$$\bar{l} = \frac{\sum_i l_i}{n} \quad (6.5)$$

A summary plot with all KPIs is done for each simulation script. Figure 6.3 shows an example of how this summary looks like.

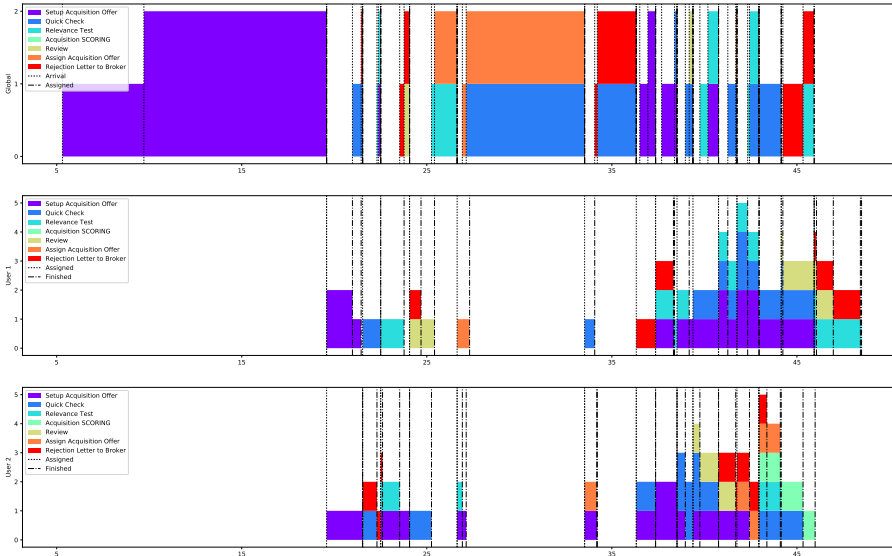
Additionally, for a more in depth visualization of a policy's performance, an evolution plot is also necessary. All types of policies share a common queues configuration, with a single global



**Figure 6.3:** KPIs summary plot for a 3-Batch policy using MSA, two users, generation interval set to 3 and simulation time set to 50

queue and a user specific queue. Each policy defines the maximal threshold a specific queue can reach. For a detailed explanation of the queues conformation refer to Chapter 5.

The evolution plot shows the state change for the policy being analyzed by plotting the flow of a token across different user tasks. Figure 6.4 shows such an example.



**Figure 6.4:** Evolution plot for a 3-Batch policy using MSA, two users, generation interval set to 3 and simulation time set to 50

Finally, when comparing different policies between each other, Milo (2012)'s definition of speedup  $S$  between two quantities  $q_1$  and  $q_2$  is adopted, which he formally defines as the quotient  $S = \frac{q_1}{q_2}$ .

# Results

This chapter outlines the results obtained by the **Mixed Integer Linear Programming (MILP)** approach in Section 7.1 respectively the **Reinforcement Learning (RL)** approach in Section 7.2 following the guidelines defined in Chapter 6.

All simulations have been tested with different combination of global variables *i.e.*, number of users, service interval, generation interval, length of simulation time, batch size (where it applies), task variability, worker variability and random state seed (where it applies). For ease of reading purposes, the global variables have been set to the following parameters according to the default column in Table 7.1.

Variable	Default	Valid Range
Number of Users	5	$1 - \infty$
Service Interval	1	$1 - \infty$
Generation Interval	3	$1 - \infty$
Simulation Time	1000	$1 - \infty$
Batch Size	5 (1 for 1-Batch-1)	$1 - \infty$
Task Variability	20%	$0\% - 100\%$
Worker Variability	20%	$0\% - 100\%$
Random State Seed	2	$\emptyset - \infty$
Process	Acquisition	Acquisition, Simple

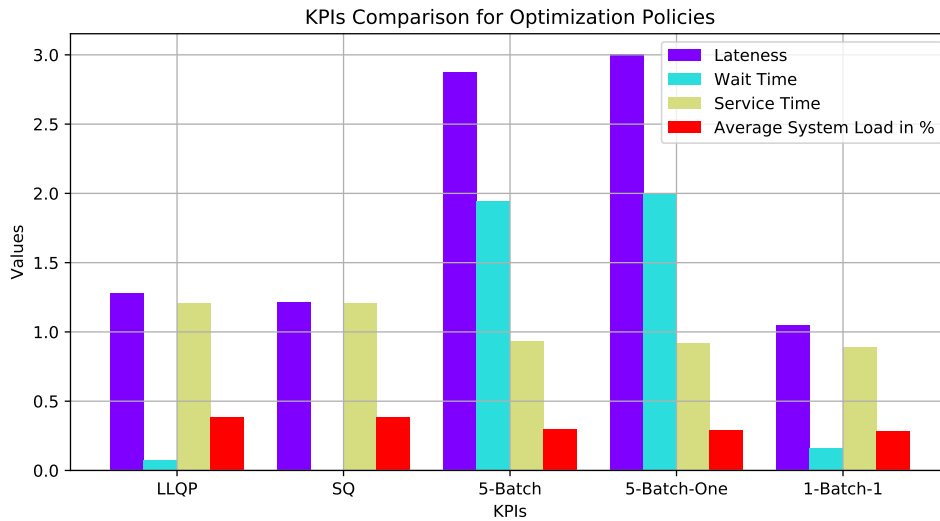
**Table 7.1:** Global parameters for simulation

## 7.1 Mixed Integer Linear Programming Results

Zeng and Zhao (2005, pp. 18–22) outline how different global parameters configurations and policy usage can affect **Key Performance Indicators (KPIs)**. They summarize their key findings as follows: **I.** Usage of batch optimization should be done only with medium to high system load (Zeng and Zhao, 2005, p. 24) **II.** Batch optimization policies without a fixed batch size, such as 1-Batch-1 yield best results (Zeng and Zhao, 2005, p. 24).

In order to assert the validity of the interpretation of Zeng and Zhao (2005)’s works and all subsequent derivative policies a comparison with similar configurations has been made for all five optimization policies. Zeng and Zhao (2005)’s main efficiency parameter is defined as the maximum flowtime or in their own words: “In business terms, maximum flowtime represents

the guaranteed response time across tasks, indicating the quality of services” (Zeng and Zhao, 2005, p. 17). In this study, the comparable parameter used to evaluate a policy’s efficiency is called lateness and has been previously defined in Equation 6.1. In regards to lateness, Figure 7.1 shows that akin results to Zeng and Zhao (2005) are obtained.



**Figure 7.1:** KPIs comparison for different optimization policies using MSA

The simulations have been run with the parameters outlined in Table 7.1 by using the same formulation used by Zeng and Zhao (2005): **Minimizing Sequential Assignment (MSA)**.

By running the same simulations with the optimization formulation implemented for this thesis *i.e.*, **Service Time Minimization with ESDMF as Upper Bound (ST)**, refer to Section 5.1, *ceteris paribus*, the summarized KPIs amongst all optimization policies can be seen in Figure 7.2.

The speedup for both batch policies with a higher batch size is tiny, but when considering the speedup between the 1-Batch-1 policy with MSA and ST, a wealthy speedup is present for all KPIs. For a detailed overview of the overall speedups of ST against MSA refer to Figure 7.3.

Astonishing speedups have been observed for the 1-Batch-1 policy<sup>1</sup>, which is indeed the most efficient policy as mentioned by Zeng and Zhao (2005, p. 24). Table 7.2 summarizes these values.

KPI	Speedup
Lateness	1.23
Wait Time	4.0
Service Time	1.1
Average System Load	1.1

**Table 7.2:** Speedup across all KPIs for ST vs MSA

<sup>1</sup>For a detailed comparison of how different batch sizes affect the policy’s KPIs refer to Subsection B.3.2 and Subsection B.4.2.

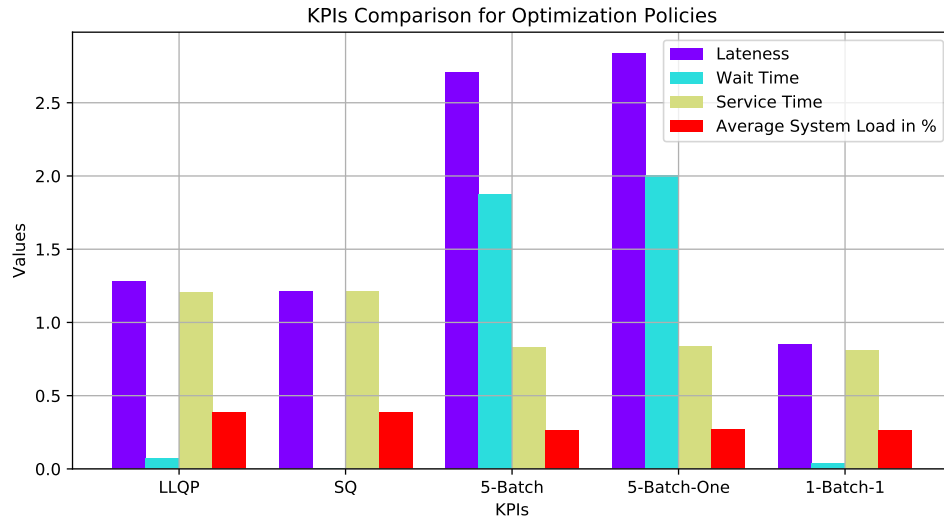


Figure 7.2: KPIs comparison for different optimization policies using ST

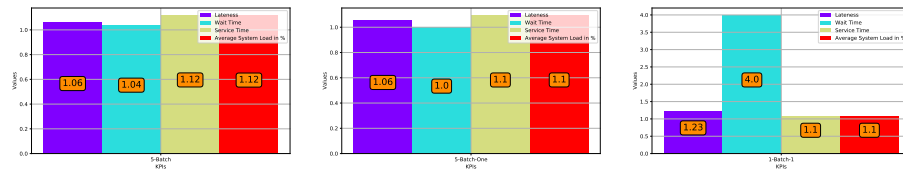


Figure 7.3: KPIs speedup comparison between MSA and ST for 5-Batch, 5-Batch-1 and 1-Batch-1

## 7.2 Reinforcement Learning

This section focuses on the results obtained with the RL methods outlined in Section 5.3. A more in-depth review of the different policies is required, thus a finer subdivision has been made in different subsections per policy type: Subsection 7.2.1 focuses on batch policies, Subsection 7.2.2 focuses on **Least Loaded Qualified Person (LLQP)** policies and Subsection 7.2.3 focuses on all remaining policies that do not fit in either of the previous categories.

In order to maintain fairness amongst RL training methods, all required parameters are globally set and equal across all simulation scripts and can be found summarized in Table 7.3 which complement the global simulation parameters depicted in Table 7.1.

Comparisons will be made, where not otherwise stated, with the corresponding MILP policy under the same conditions.

### 7.2.1 Batch

Five different batch policies with RL have been developed. Table 7.4 gives an overview.

Table 7.5 summarizes the results. For the detailed results refer to Subsection C.1.2 for 1-Batch respectively to Subsection C.2.2 for 1-Batch-1.

Parameter	Value
Discount factor $\gamma$	0.5
Step size parameter $\alpha$	0.0001
$\epsilon$ -Greedy threshold $\epsilon$	0.1
MC epochs	1000
ANNs MC epochs	10000
MC and ANNs MC epoch training time	100
TD training time	100000

**Table 7.3:** Global RL parameters

Technical Name	Policy Type	Update Method	$Q$ Value Method	Other Characteristics
k_batch_mc_vfa	1-Batch	MC	VFA	None
k_batch_mc_vfa_op	1-Batch	MC	VFA	OP
k_batch_mc_vfa_opep	1-Batch	MC	VFA	$\epsilon$ -Greedy, OP
k_batch_td_vfa_op	1-Batch	TD	VFA	OP
k_batchone_td_vfa_op	1-Batch-1	TD	VFA	OP

**Table 7.4:** Overview of developed batch policies with RL

KPI	k_batch_mc_vfa	k_batch_mc_vfa_op	k_batch_mc_vfa_opep	k_batch_td_vfa_op	k_batchone_td_vfa_op
Lateness	1.22	1.24	1.22	1.23	1.23
Wait Time	2.63	3.08	2.39	3.62	3.51
Service Time	1.11	1.11	1.12	1.1	1.1
Average System Load	1.11	1.11	1.12	1.1	1.1

**Table 7.5:** Speedup across all KPIs for batch policies with RL vs MSA

## 7.2.2 Least Loaded Qualified Person

Three different **LLQP** policies with **RL** have been developed. Table 7.6 gives an overview. Other policies have been implemented for evaluating different **RL** methods, however they are not considered for the final evaluation. These policies can be seen in Table C.1.

Technical Name	Policy Type	Update Method	$Q$ Value Method	Other Characteristics
llqp_mc_vfa_op	LLQP	MC	VFA	OP
llqp_td_vfa_op	LLQP	TD	VFA	OP
llqp_td_tf_op	LLQP	TD	ANNs	OP, 1L

**Table 7.6:** Overview of developed LLQP policies with RL

Table 7.7 shows the summarized results. For the detailed results refer to Subsection C.3.2.

Table 7.8 shows the comparison between **Off Policy (OP)** and **On Policy (ONP)** approaches.

KPI	llqp_mc_vfa_op	llqp_td_vfa_op	llqp_td_tf_op
Lateness	1.0	1.0	1.0
Wait Time	0.92	0.99	1.02
Service Time	1.01	1.0	1.0
Average System Load	1.01	1.0	1.0

**Table 7.7:** Speedup across all KPIs of LLQP policies with RL vs MSA

KPI	llqp_mc_vfa_op	llqp_mc_vfa	Speedup
Lateness	1.2756	1.2914	0.99
Wait Time	0.0796	0.0711	1.12
Service Time	1.1960	1.2203	0.98
Average System Load	38.22%	39.00%	0.98

**Table 7.8:** KPIs comparison between OP and ONP approaches

### 7.2.3 Others

Three different additional policies with **RL** have been developed which have been used to fully emulate the behavior of K-Batch and 1-Batch-1 (as explained in Subsection 5.3.3). Table 7.9 gives an overview.

Technical Name	Policy Type	Update Method	$Q$ Value Method	Other Characteristics
wz_td_vfa_op	WZ	TD	VFA	OP
wz_one_td_vfa_op	WZO	TD	VFA	OP
bi_one_mc_tf	BI	MC	ANNs	PG

**Table 7.9:** Overview of additional developed policies with RL

Table 7.10 shows the summarized results. For the detailed results refer to Subsection C.4.2.

KPI	wz_td_vfa_op	wz_one_td_vfa_op	bi_one_mc_tf_1l	bi_one_mc_tf_2l	bi_one_mc_tf_3l	bi_one_mc_tf_4l
Lateness	0.97	1.2	1.07	0.9	0.84	0.84
Wait Time	0.89	2.33	2.46	1.11	0.85	0.86
Service Time	1.21	1.1	0.98	0.87	0.84	0.84
Average System Load	1.22	1.1	0.98	0.87	0.84	0.84

**Table 7.10:** Speedup across all KPIs of the additional policies with RL against the MSA formulation



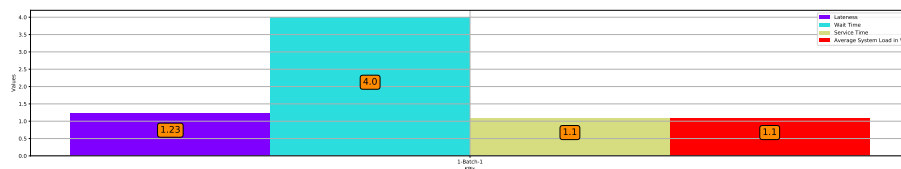


# Conclusion

This chapter interprets the results obtained in Chapter 7 by discussing the **Mixed Integer Linear Programming (MILP)** results in Section 8.1, the **Reinforcement Learning (RL)** results in Section 8.2, summarizing them in Section 8.3, exposing the consequences in Section 8.4 and eventually outlining future work in Section 8.5.

## 8.1 Mixed Integer Linear Programming Results Discussion

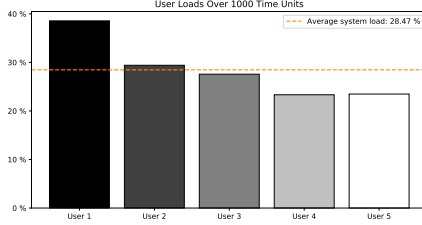
Using **MILP** to solve role resolution proves to be an efficient measure, however different formulations yield different solutions and formulation complexities. Zeng and Zhao (2005, p. 15) mention that **Minimizing Sequential Assignment (MSA)** greatly simplifies **Dynamic Minimization of Maximum Task Flowtime (DMF)** since only those tasks that are immediately available are considered. This consideration is done since the **DMF** problem proves to be computationally expensive to solve (Zeng and Zhao, 2005, p. 13), (Garey and Johnson, 1990). Zeng and Zhao (2005, p. 13) propose however that by introducing auxiliary variables one can reduce the complexity of **DMF** and effectively solving it. This is what has been done in this thesis, as outlined in Section 5.1 by introducing new types of formulations. The **Service Time Minimization with ESDMF as Upper Bound (ST)** “flagship” formulation significantly outperforms the **MSA** formulation (refer to Figure 8.1). Having said that, the higher formulation complexity of **ST** compared to **MSA** (see Table 5.2) questions the practical use of this formulation over the other methods. A 1.23-fold speedup in respect to lateness having quadratic higher formulation complexity poses a dubious trade-off from a business perspective.



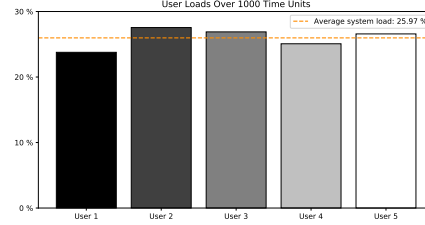
**Figure 8.1:** KPIs speedup comparison between MSA and ST for 1-Batch-1

Yet another aspect mentioned by Zeng and Zhao (2005, pp. 17–18) is a more “social” aspect: the fairness of a policy *i.e.*, how fairly are single users treated by a policy during job assignment.

Figure 8.2 and Figure 8.3 both show how fairly are users treated in the same scenario by the two formulations. It is clear that **ST** is “fairer” at balancing loads across users compared to **MSA**.



**Figure 8.2:** User loads distribution for 1-Batch-1 using MSA



**Figure 8.3:** User loads distribution for 1-Batch-1 using ST

Lastly, **MILP** based approaches solve role resolution in **Workflow Management Systems (WfMS)** in a deterministic way *i.e.*, the “blindly” optimize only at a specific point in time without accounting for the process dynamics. This kind of solution is done by the **RL** methods outlined in Section 8.2.

## 8.2 Reinforcement Learning Results Discussion

Let us first focus on **Least Loaded Qualified Person (LLQP)**, which, as explained in Section 5.1, focuses on assigning a job to the least loaded qualified person. By comparing the results obtained with **RL** against the optimization method, we clearly see that speedups across all **Key Performance Indicators (KPIs)** are imperceptible (refer to Subsection C.3.2 for more details). This sheds light on two key aspects: **I. LLQP** policies are intrinsically optimized by their nature of implementation. **II. RL** methods converge really well (for a 1000 time steps **LLQP** simulation, **LLQP** with **Temporal Difference (TD)** and **Tensorflow (TF)** only needs 20 times the simulation time as training in order to perfectly converge to actual **LLQP**) and, even if only slightly, exploit internal mechanisms of **LLQP** policies to extract better results.

On the other hand, batch policies exhibit a much bigger optimization potential for which **RL** methods do really adapt well. 1.24-fold speedup for 1-Batch (see Subsection C.1.2) respectively 1.23 for 1-Batch-1 (see Subsection C.2.2) confirm the previous claim.

Lastly, when accounting for job order during assignment (refer to Subsection 5.3.3 for the detailed explanation), improvements are observed only under specific conditions: **I. By using Waiting Zone One for K-Batch-1 Emulation (WZO)** (see Figure C.25) **II. Artificial Neural Networks (ANNs) with One Hidden Layer for ANN (1L)** (see Figure C.26 for KPIs comparison against **MSA** and Figure C.17 for the graphical representation of the **ANN** implemented with **TF**).

Having said that, deep **ANNs** exhibits worse results compared to their equivalent emulated optimization methods. This can be explained from a twofold perspective: **I. Vanishing Gradient Problem (VGP)** and **II. Exploding Gradient Problem (EGP)**.

In brief, **VGP** states that even very large changes in partial derivatives on initial layers have imperceptible effects on subsequent layers (Bengio et al., 1994) and **EGP** states that huge spikes in the norm of changes in partial derivatives which could potentially grow exponentially can happen under training, thus influencing internal parameters (Bengio et al., 1994; Pascanu et al., 2012).

Yet another crucial aspect to consider when undergoing **RL** methods is usage between **On Pol-**

icy (ONP) or Off Policy (OP) learning (refer to Subsection 5.2.6 for ONP respectively to Subsection 5.2.8 for OP Subsection 5.2.7). As shown in Table 7.8 (and Figure 8.4), OP methods converge slightly better compared to ONP, however the change is imperceptible and can be found in different factors *e.g.*, random state seed choice or length of simulation. Having said that, it is important to note that these different approaches are being currently heavily studied by pioneers and the debate is still open, as Sutton and Barto (2017, pp. 245–249) explain. The current key takeaways from this outgoing debate can be summarized as follows (Sutton and Barto, 2017): **I.** Learning **OP** **II.** Usage of scalable function approximation methods like linear semi-gradient **III.** Usage of bootstrapping which is used in **TD** methods.

The combination of all these three factors is referred to as “the deadly triad” (Sutton and Barto, 2017, p. 249). Sutton and Barto (2017, p. 249) argue that dangers can arise only when all three aspects are present, if used singularly convergence properties are safe.

On a more general note, different comparisons have shown huge spikes in speedup of waiting time *e.g.*, as seen in Subsection C.1.2. Even though compelling, the practical usage is limited: when considering WfMS with fixed available resources *i.e.*, users participating, huge speedup in job waiting times does not always correlate with better system performance, since it might be the case that even if a job is ready to be assigned there are no available resources to complete such job.

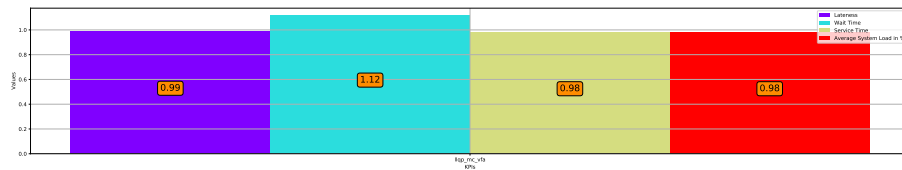


Figure 8.4: KPIs comparison between OP and ONP approaches

## 8.3 Summary

The focus of this thesis was analyzing different approaches for optimal role resolution in WfMS by means of MILP and RL based methods.

Initially, a thorough literature review has been made in order to establish the existing optimization solution *i.e.*, foundations set by Zeng and Zhao (2005), then a roadmap on how to further develop such methods has been laid out. The roadmap encloses a twofold approach: **I.** Further develop MILP based methods **II.** Use RL methods as novel solution for role resolution in WfMS.

Subsequently, a discrete event simulation environment as outlined in Chapter 4 has been implemented which served as foundation for evaluating all future optimization policies.

For Approach I five existing optimization policies as outlined in Section 5.1 have been implemented and evaluated against the existing policies defined by Zeng and Zhao (2005, pp. 13–14). Furthermore, for Approach II a profound literature review about RL had to be made and following on the theoretical foundations laid (see Section 5.2), novel policies for solving the assignment problem based on RL have been implemented (see Section 5.3). Lastly, all RL policies have been also evaluated and a comparison between them and the corresponding MILP methods has been made.

## 8.4 Consequences

**MILP** methods are viable solutions for role resolution in **WfMS**, as already argued by **Zeng and Zhao (2005)**. Further developed policies relying on **MILP** done for this thesis proved indeed to yield better results (see Section 7.1), however the higher complexity required (refer to Section 8.1) poses a critical trade-off. **Research Question I** can be affirmatively answer: there is definitely potential for further development and optimization of existing methods but further questions arise: **I**. Does a slight increase in performance justify the higher formulation complexity requirements? **II**. Can only one **KPI** be chosen as central parameter to evaluate a policy's performance or is a combination of different **KPIs** more suitable? **III**. Is it always the best solution to use only one type of optimizer or is it better to follow a more flexible approach and chose different policies on a per case basis?

All these questions must be accounted for when considering policy based optimization in **WfMS**.

On the other hand, **RL** methods demonstrate to overcome some of the problematics laid by optimization very well: **I**. Generally less computationally expensive **II**. Can be adopted relatively fast **III**. Are generally more "dynamic" and can adapt and exploit case specific characteristics *i.e.*, overcome determinism of **MILP** based approached in favor of stochastic approaches.

Then again **RL** methods are not exempt from disadvantages: **I**. They require long training sessions in order to equal (or even outperform) optimization methods **II**. When using **ANNs**, overfitting might lead to suboptimal solution in which policies get stuck (for viable solutions refer to **Srivastava et al. (2014)**) **III**. Multilayer **ANNs** (*i.e.*, deep **ANNs**) are very sensitive to **VGP** and **EGP** (**Bengio et al., 1994; Pascanu et al., 2012**).

In general **RL** based approaches are a refreshing and novel methodology for solving optimization problems but require further development and sound domain knowledge.

## 8.5 Outlook

This thesis sets the foundations for viable alternatives to existing **MILP** based techniques for role resolution in **WfMS**. A direct follow up consists in testing in operative environments the introduction feasibility and efficiency measured in the simulation environment outlined in this thesis. This includes testing the robustness, efficacy and viability of the proposed policies by putting them under "real-world" stress situation in order to assert the speedup claims. For **RL** methods executing lengthy training sessions might prove impractical: thousands of training sessions required by **RL** methods in order to comply with convergence properties can prove infeasible for companies that are not able to generate such amounts of data, thus directly limiting applicability of these methods.

**RL** is a novel field that is currently still being actively researched and pursued, as it has yield promising results (**Mnih et al., 2015; Silver et al., 2016**). By using **ANNs** one can effectively approximate nonlinear functions as it has been outlined in Subsection 5.2.6. Even though in this thesis only feedforward **ANNs** have been used, the problem domain is very well suited for the recurrent variant as well.

Moreover, bleeding edge domains have originated from **RL** such as **Inverse RL (IRL)** (**Ng and Russell, 2000**) and **Apprenticeship Learning (AL)** which is based on the former (**Abbeel and Ng, 2004**). **AL** could prove to be yet another captivating approach to solve the assignment problem in which the reward function is not explicitly modeled, instead an "expert" of the domain *i.e.*, **MILP** based agents such as **MSA**, demonstrates a task and by means of **AL** the policy is trained (**Abbeel and Ng, 2004**).

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Having said that, **Research Question II** can be answered positively as well: there are indeed state of the art approaches that can be used as alternatives or complements to the job assignment mathematical optimization.



## Appendix A

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# Tools Used

Different tools were used in the analysis environment in order to efficiently simulate and analyze the work of this thesis:

- I. The simulation environment is based on Python 3.5.2<sup>1</sup> using the Anaconda<sup>2</sup> platform.
- II. The discrete event simulation environment is implemented by using the SimPy 3.0.10<sup>3</sup> package.
- III. The resulting data is plotted using Matplotlib 2.0.0<sup>4</sup>.
- IV. Tensorflow (TF) 1.0<sup>5</sup> is the library used for the Artificial Neural Networks (ANNs) modeling.
- V. Coding was done using PyCharm 2017.1<sup>6</sup> as Integrated Development Environment (IDE) for Python.
- VI. For solving the Mixed Integer Linear Programming (MILP) problems for batch policies Gurobi 7.0.1<sup>7</sup> was used.

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<sup>1</sup><https://www.python.org> (accessed 06.01.2017)

<sup>2</sup><https://www.continuum.io/anaconda-overview> (accessed 03.04.2017)

<sup>3</sup><https://simpy.readthedocs.io/en/latest/> (accessed 06.01.2017)

<sup>4</sup><http://matplotlib.org/> (accessed 03.04.2017)

<sup>5</sup><https://www.tensorflow.org/> (accessed 03.04.2017)

<sup>6</sup><https://www.jetbrains.com/pycharm/> (accessed 03.04.2017)

<sup>7</sup><http://www.gurobi.com> (accessed 06.01.2017)





# Mixed Integer Linear Programming Results

## B.1 Least Loaded Qualified Person

### B.1.1 KPIs

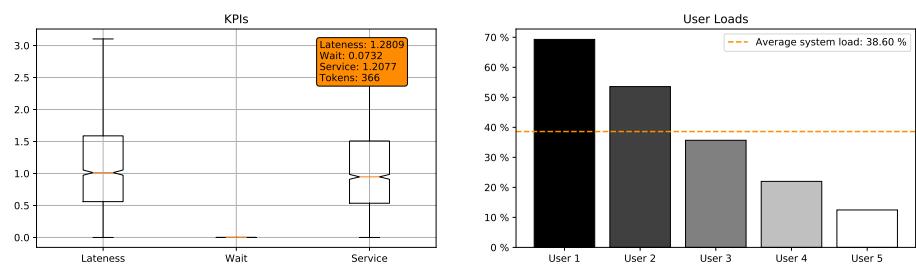


Figure B.1: LLQP KPIs

## B.2 Shared Queue

### B.2.1 KPIs

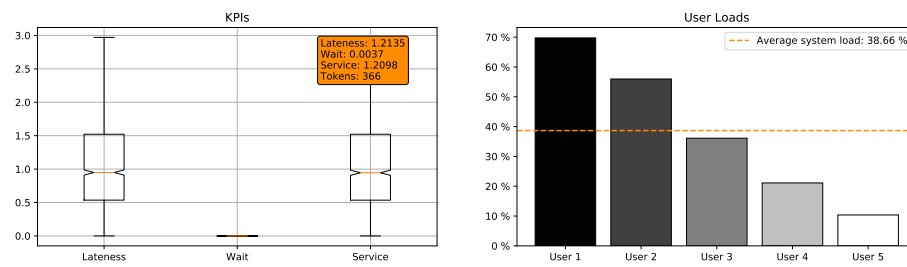


Figure B.2: SQ KPIs

## B.3 K-Batch

### B.3.1 KPIs

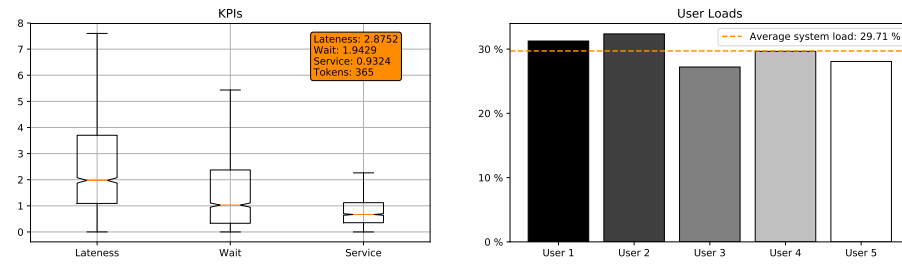


Figure B.3: K-Batch with MSA KPIs

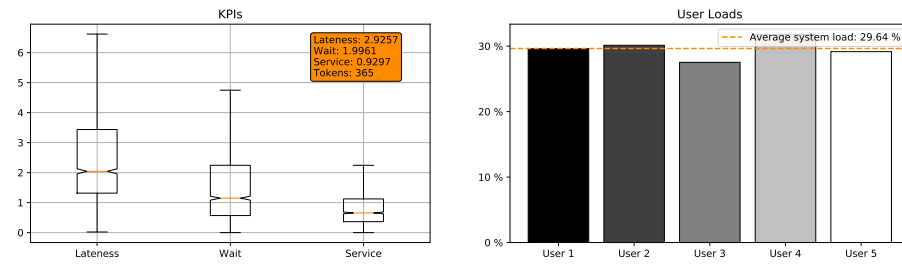


Figure B.4: K-Batch with DMF KPIs

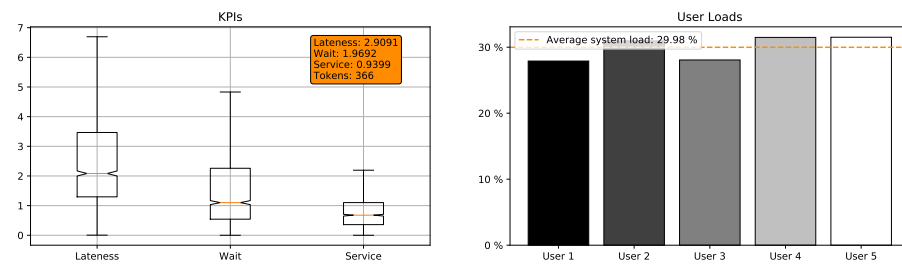
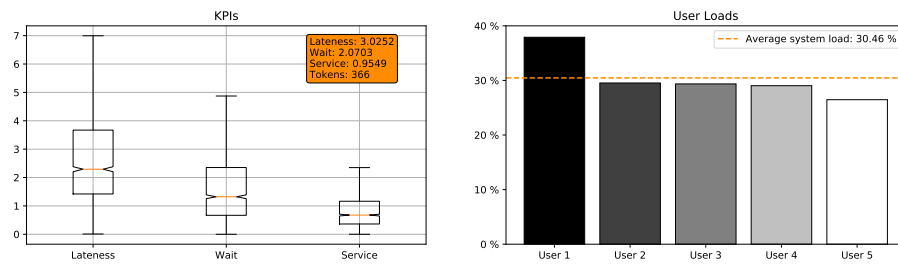
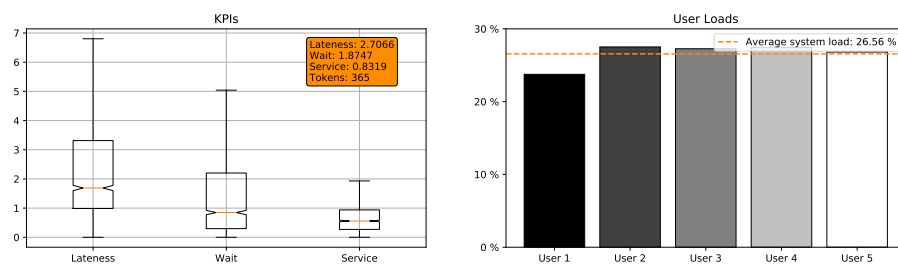


Figure B.5: K-Batch with SDMF KPIs

**Figure B.6:** K-Batch with ESDMF KPIs**Figure B.7:** K-Batch with ST KPIs

### B.3.2 Batch Sizes Comparison

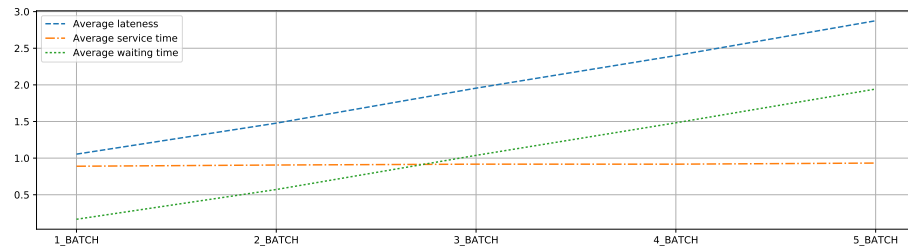


Figure B.8: K-Batch with MSA batch size comparison

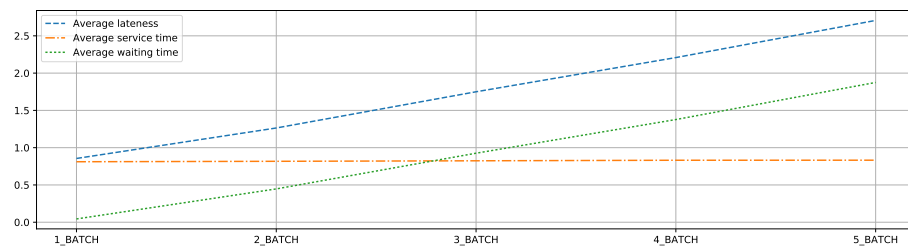


Figure B.9: K-Batch with ST batch size comparison

## B.4 K-Batch-1

### B.4.1 KPIs

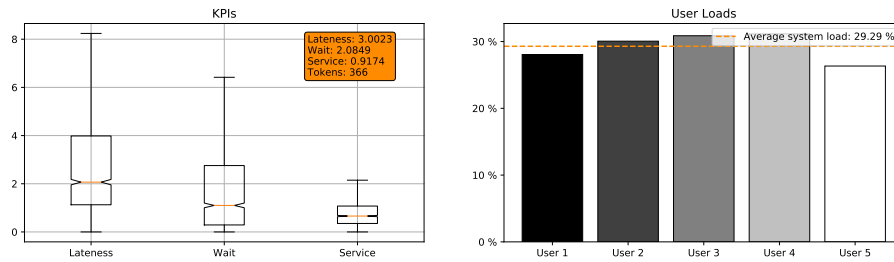


Figure B.10: K-Batch-1 with MSA KPIs

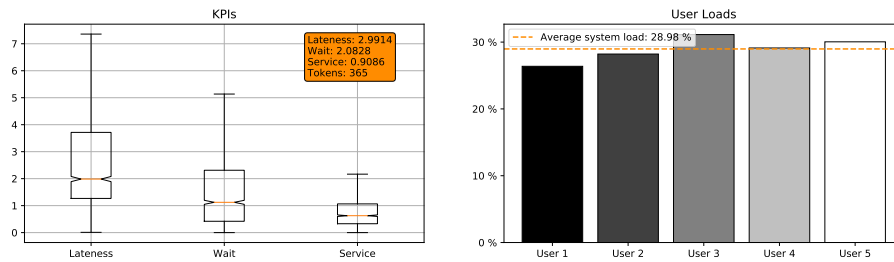


Figure B.11: K-Batch-1 with DMF KPIs

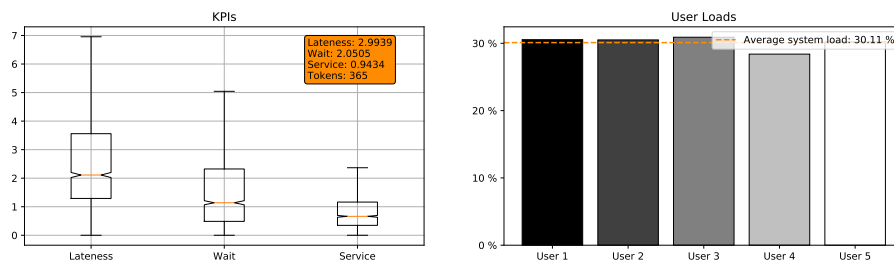
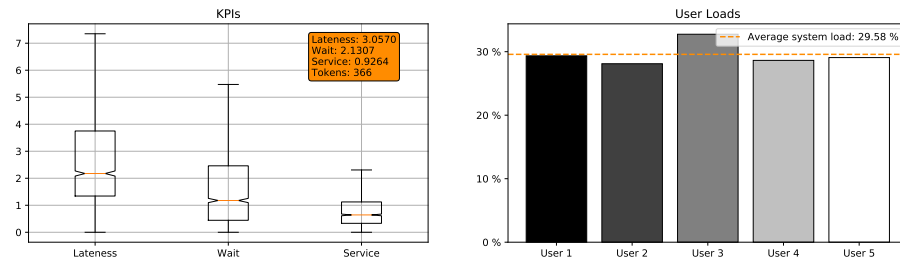
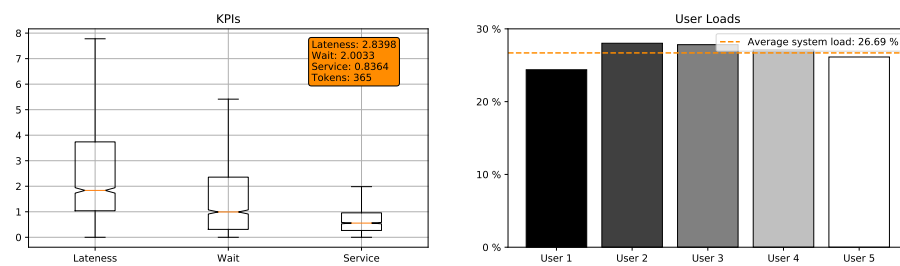


Figure B.12: K-Batch-1 with SDMF KPIs

**Figure B.13:** K-Batch-1 with ESDMF KPIs**Figure B.14:** K-Batch-1 with ST KPIs

## B.4.2 Batch Sizes Comparison

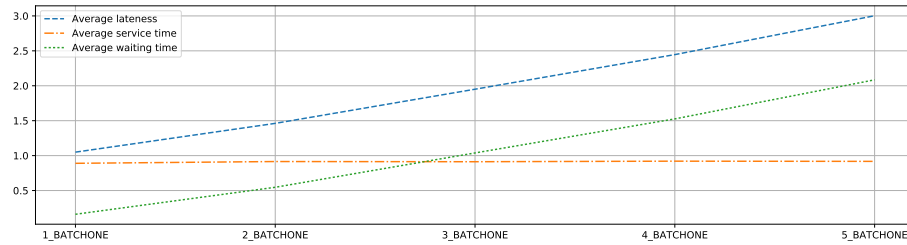


Figure B.15: K-Batch-1 with MSA batch size comparison

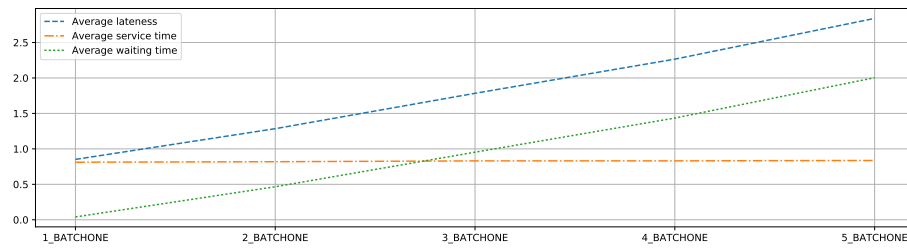


Figure B.16: K-Batch-1 with ST batch size comparison



## B.5 1-Batch-1

### B.5.1 KPIs

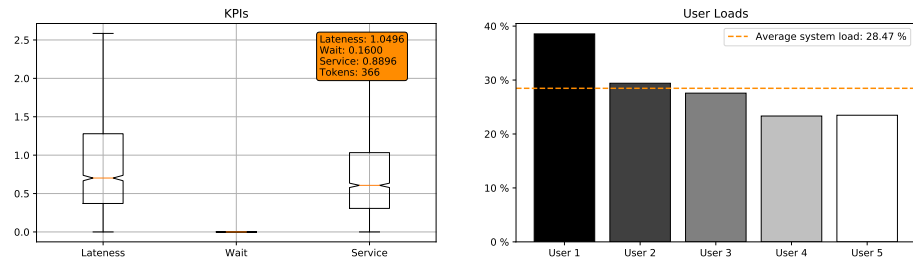


Figure B.17: 1-Batch-1 with MSA KPIs

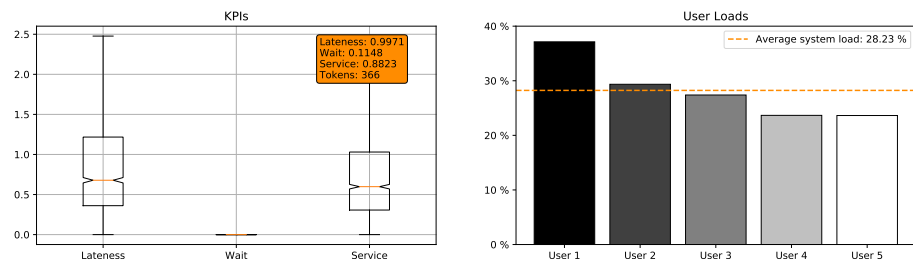


Figure B.18: 1-Batch-1 with DMF KPIs

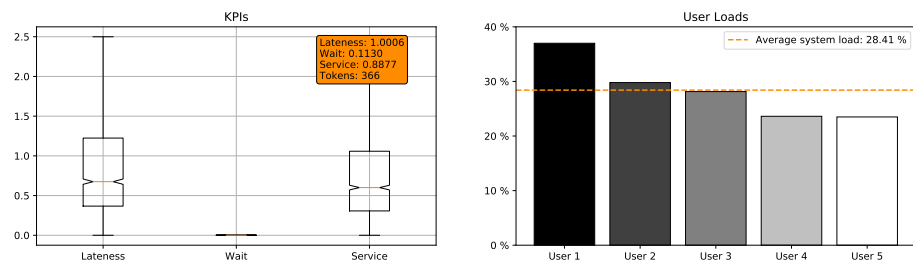


Figure B.19: 1-Batch-1 with SDMF KPIs

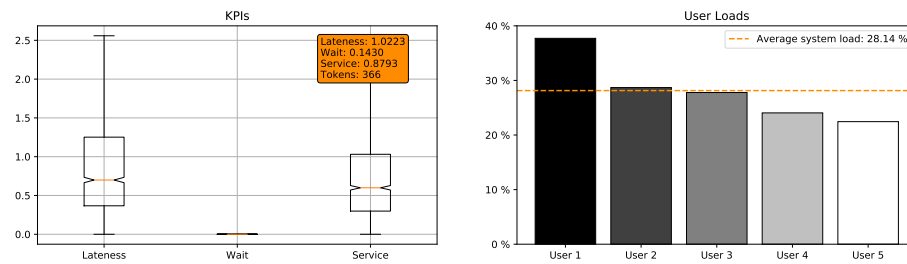


Figure B.20: 1-Batch-1 with ESDMF KPIs

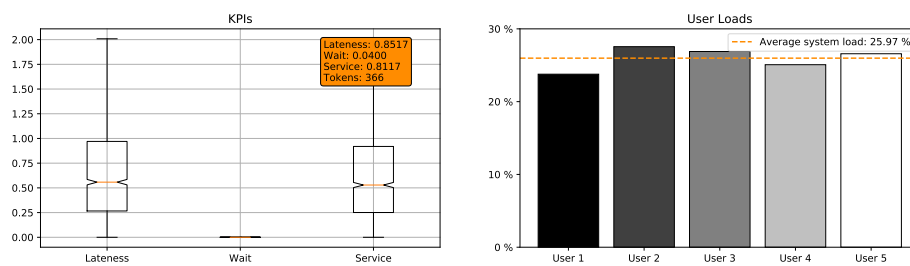
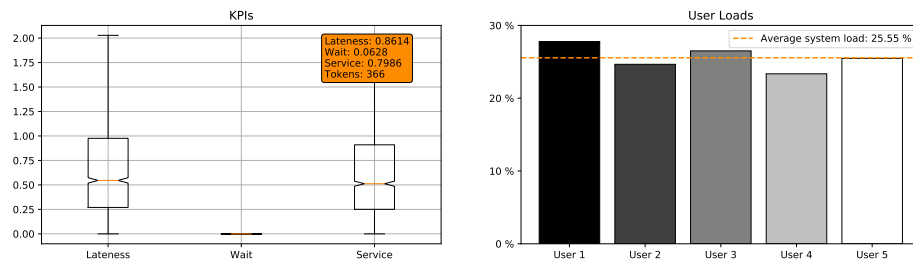


Figure B.21: 1-Batch-1 with ST KPIs

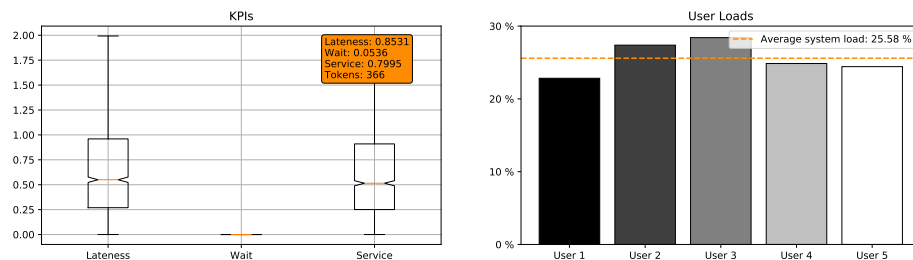
## Reinforcement Learning Results

### C.1 1-Batch

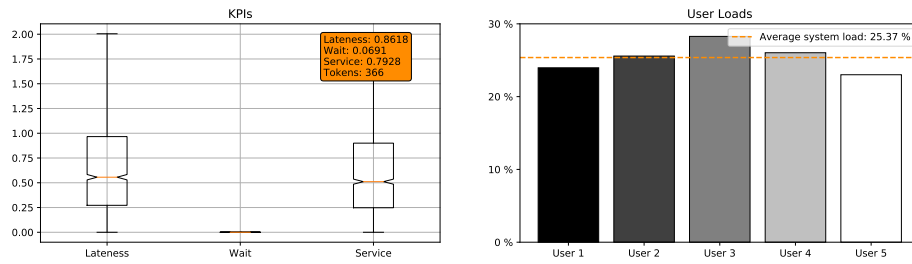
#### C.1.1 KPIs



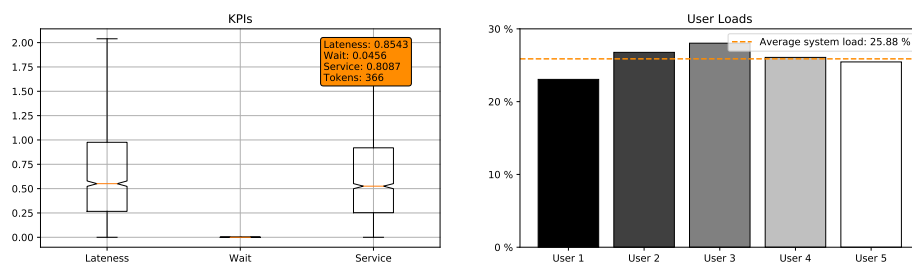
**Figure C.1:** 1-Batch with MC and VFA KPIs



**Figure C.2:** 1-Batch with MC, VFA and OP KPIs

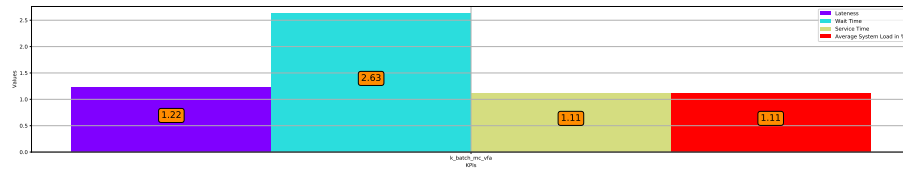


**Figure C.3:** 1-Batch with MC, VFA, OP and  $\epsilon$ -Greedy KPIs

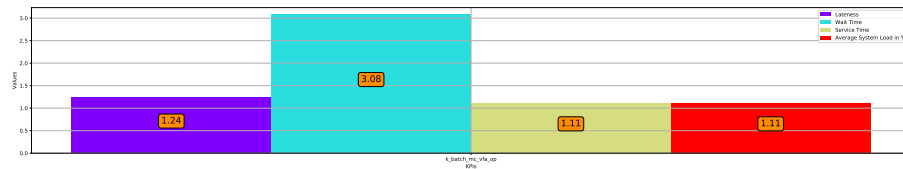


**Figure C.4:** 1-Batch with TD, VFA and OP KPIs

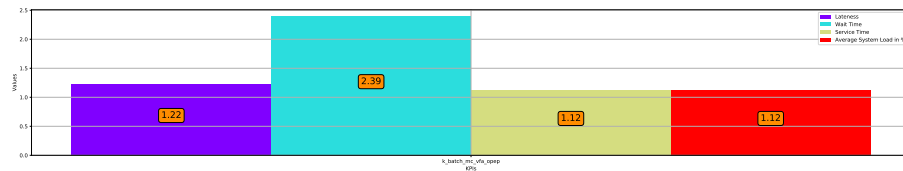
## C.1.2 Comparison with MSA



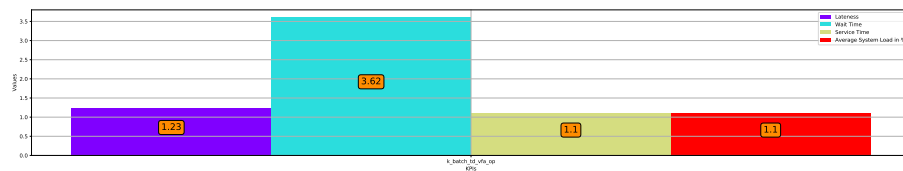
**Figure C.5:** 1-Batch with MC and VFA MSA comparison



**Figure C.6:** 1-Batch with MC, VFA and OP MSA comparison



**Figure C.7:** 1-Batch with MC, VFA, OP and  $\epsilon$ -Greedy MSA comparison



**Figure C.8:** 1-Batch with TD, VFA and OP MSA comparison

## C.2 1-Batch-1

### C.2.1 KPIs

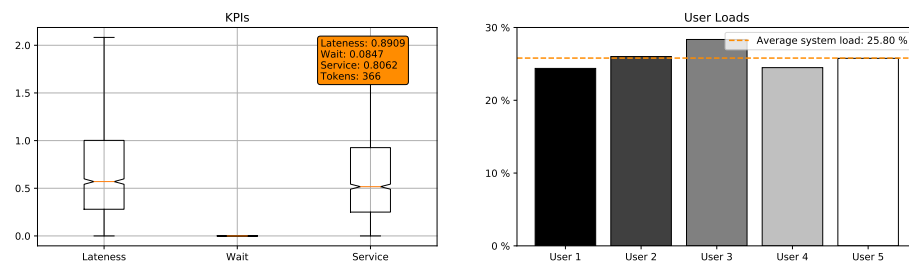
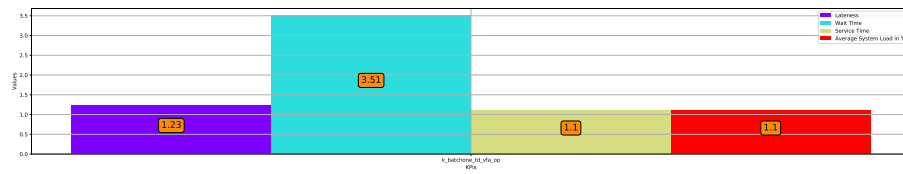


Figure C.9: 1-Batch-1 with TD, VFA and OP KPIs

## C.2.2 Comparison with MSA



**Figure C.10:** 1-Batch-1 with TD, VFA and OP MSA comparison

## C.3 Least Loaded Qualified Person

### C.3.1 KPIs

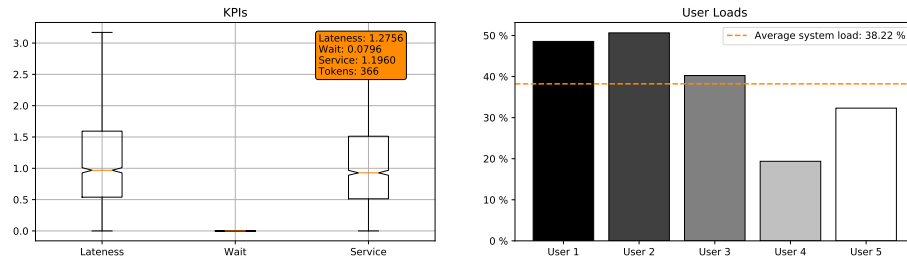


Figure C.11: LLQP with MC, VFA and OP KPIs

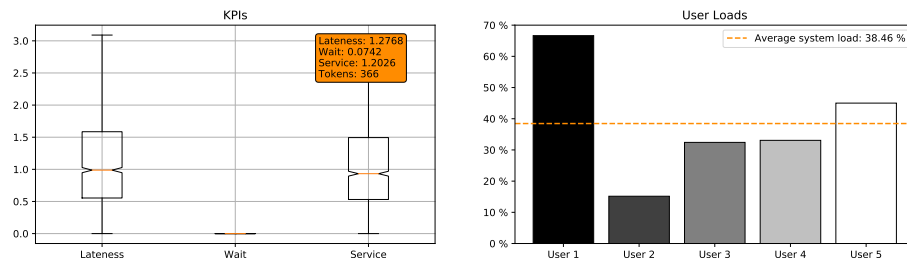


Figure C.12: LLQP with TD, VFA and OP KPIs

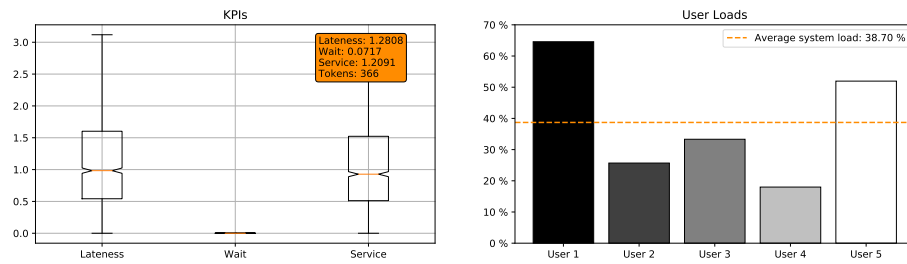
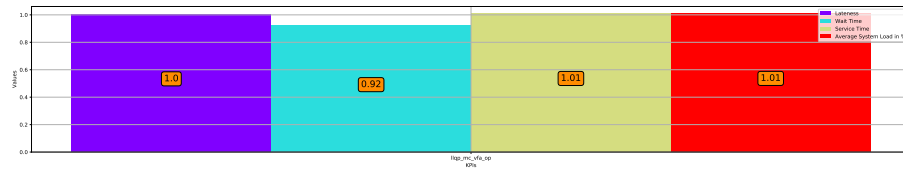


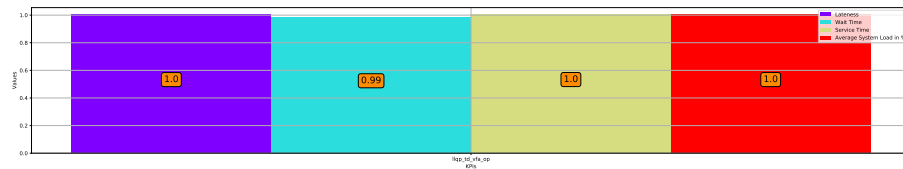
Figure C.13: LLQP with TD, TF and OP KPIs



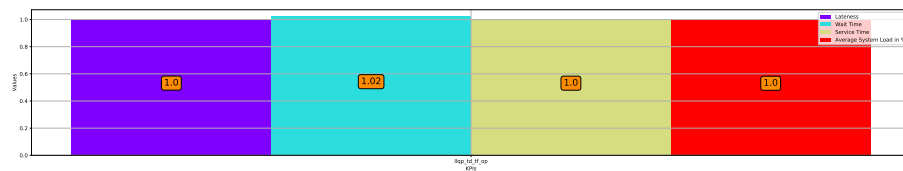
### C.3.2 Comparison with MSA



**Figure C.14:** LLQP with MC, VFA and OP MSA comparison



**Figure C.15:** LLQP with TD, VFA and OP MSA comparison



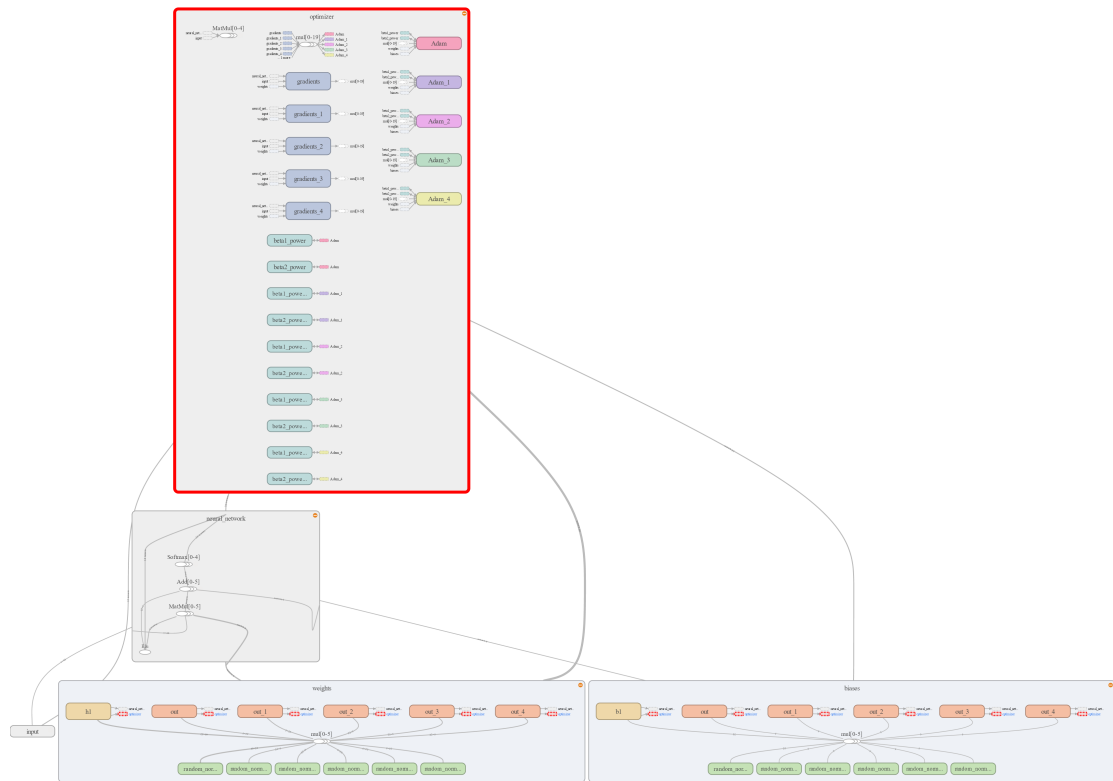
**Figure C.16:** LLQP with TD, TF and OP MSA comparison

### C.3.3 Additional LLQP Policies

Technical Name	Policy Type	Update Method	$Q$ Value Method	Other Characteristics
llqp_mc_vfa	LLQP	MC	VFA	None
llqp_td_vfa	LLQP	TD	VFA	None
llqp_mc_pg	LLQP	MC	PG	None
llqp_mc_pg_wb	LLQP	MC	PG	With Baseline
llqp_td_pg_ac	LLQP	TD	PG	AC
llqp_td_pg_avac	LLQP	TD	PG	AV, AC

**Table C.1:** Overview of additional LLQP policies with RL

### C.3.4 ANN Implementation in TF for LLQP



**Figure C.17:** 1L implementation in TF for LLQP policy

## C.4 Others

### C.4.1 KPIs

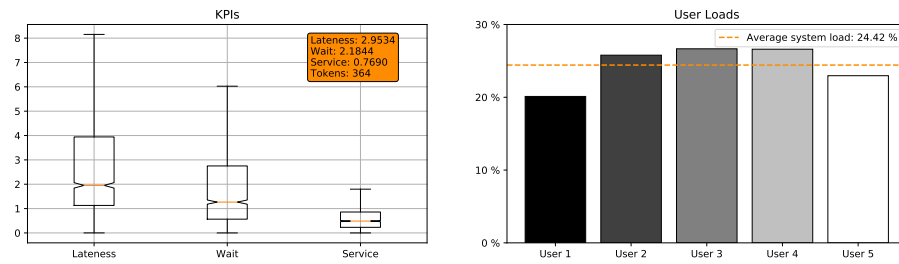


Figure C.18: WZ with TD, VFA and OP KPIs

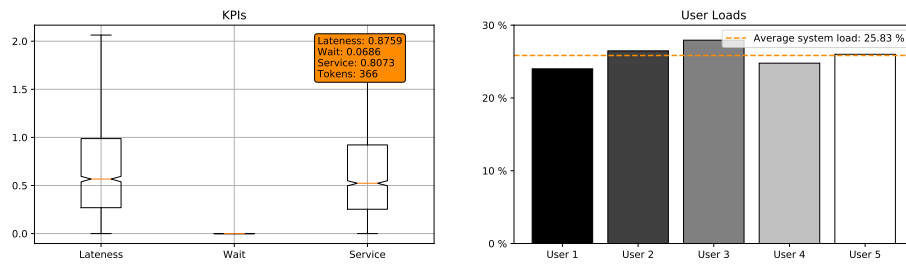


Figure C.19: WZO with TD, VFA and OP KPIs

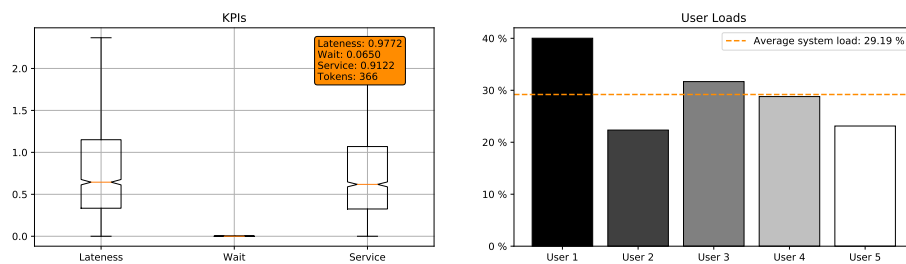


Figure C.20: BI with MC, TF and 1L KPIs

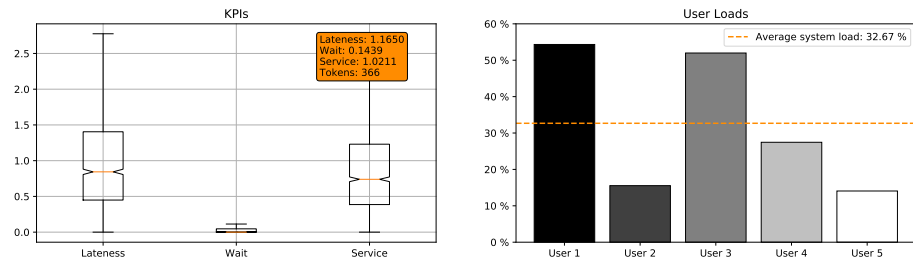


Figure C.21: BI with MC, TF and 2L KPIs

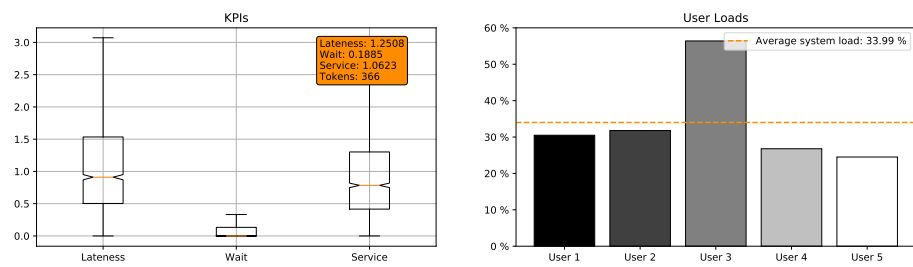


Figure C.22: BI with MC, TF and 3L KPIs

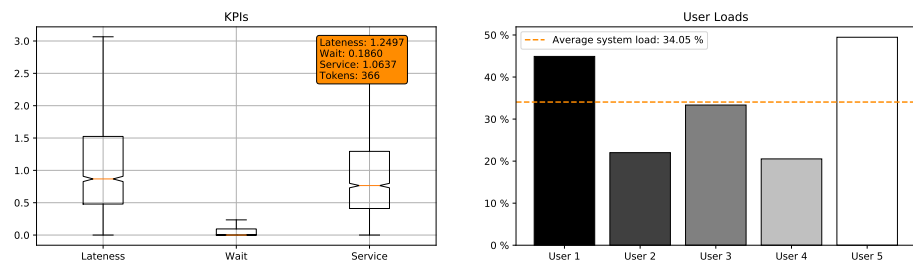


Figure C.23: BI with MC, TF and 4L KPIs

## C.4.2 Comparison with MSA

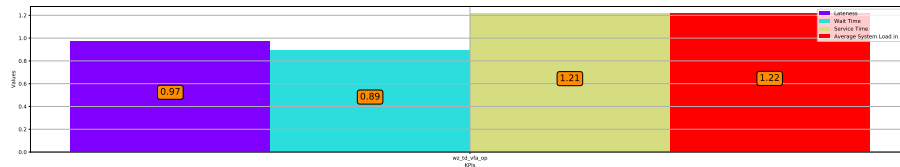


Figure C.24: WZ with TD, VFA and OP MSA comparison

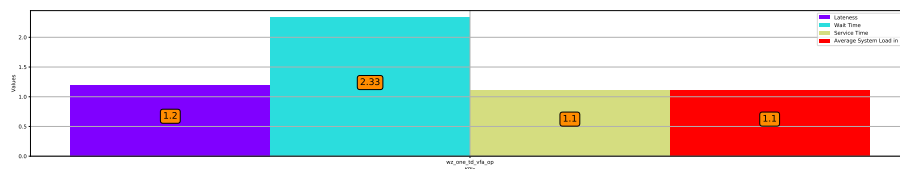


Figure C.25: WZO with TD, VFA and OP MSA comparison

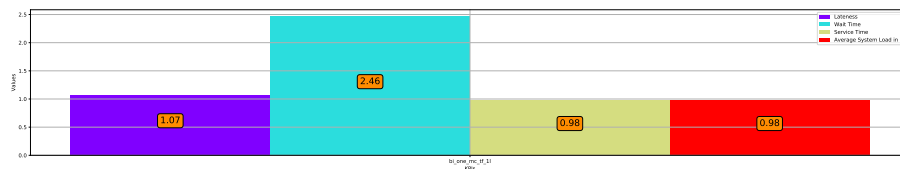


Figure C.26: BI with MC, TF and 1L MSA comparison

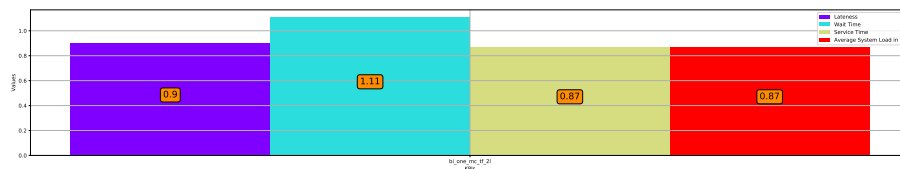
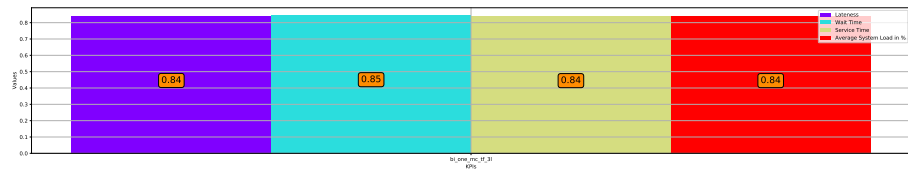
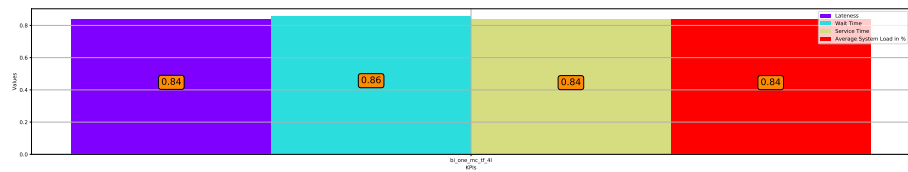


Figure C.27: BI with MC, TF and 2L MSA comparison



**Figure C.28:** BI with MC, TF and 3L MSA comparison



**Figure C.29:** BI with MC, TF and 4L MSA comparison





## Appendix D

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# CD Content

- A `Zusfsg.txt` file which contains a German summary of the thesis
- An `Abstract.txt` file which contains an English summary of the thesis
- A `Masterarbeit.pdf` file which corresponds to this report
- A `SourceCode.zip` file which contains the whole source code of this thesis
- A `Latex.zip` file which contains the whole source code of this report
- A `Presentation.pptx` file which corresponds to the presentation of this report



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# Acronyms

- 1L** One Hidden Layer for ANN. 50
- AC** Actor Critic. 6, 31, 32
- AL** Apprenticeship Learning. iii, v, 5, 52
- ANN** Artificial Neural Network. 5, 6, 29, 35, 50, 52, 55
- AV** Action Value. 6, 27, 30–32
- BPM** Business Process Modeling. 4, 9
- BPMN** Business Process Model and Notation. iii, 1, 2, 4, 9–11
- CSF** Critical Success Factor. 4
- DMF** Dynamic Minimization of Maximum Task Flowtime. 4, 6, 7, 22–25, 40, 49
- DP** Dynamic Programming. 27
- EDMF** Extended DMF. 7, 22, 23
- EGP** Exploding Gradient Problem. 5, 50, 52
- ELU** Exponential Linear Unit. 6
- EP**  $\epsilon$ -Greedy. 31
- ESDMF** Extremely Simplified DMF. 24, 25
- FIFO** First In First Out. 21, 24
- IDE** Integrated Development Environment. 55
- IRL** Inverse RL. iii, v, 5, 52
- KPI** Key Performance Indicator. 2, 3, 39, 41, 43, 44, 50, 52
- LLQP** Least Loaded Qualified Person. iii, v, 4, 21, 32, 37, 45, 46, 50

- MC** Monte Carlo. 5, 6, 27, 28, 31, 32, 35–37
- MDP** Markov Decision Process. 6, 26, 27
- MILP** Mixed Integer Linear Programming. iii, 1, 2, 6, 7, 21, 22, 43, 45, 49–52, 55
- MSA** Minimizing Sequential Assignment. 21, 22, 25, 44, 49, 50, 52
- OMG** Object Management Group. 9
- ONP** On Policy. 28, 30, 46, 50, 51
- OP** Off Policy. 28, 30, 31, 46, 51
- OR** Inclusive OR. 10, 11, 16, 17
- PG** Policy Gradient. 5, 6, 31, 32, 34, 35
- ReLU** Rectified Linear Unit. 6, 29
- RL** Reinforcement Learning. iii, v, 1, 2, 5, 6, 26, 28, 29, 32–34, 37, 43, 45–47, 49–52
- SARSA** State-Action-Reward-State-Action. 28, 30
- SDMF** Simplified DMF. 23, 24
- SGA** Stochastic Gradient Ascent. 31
- SGD** Stochastic Gradient Descent. 28–30, 34, 35
- SQ** Shared Queue. iii, v, 4, 21
- ST** Service Time Minimization with ESDMF as Upper Bound. 24, 25, 44, 49, 50
- SV** State Value. 27, 29, 32
- SVFA** State VFA. 29, 33
- TD** Temporal Difference. 6, 28, 32, 35–37, 50, 51
- TF** Tensorflow. 35, 50, 55
- VFA** Value Function Approximation. 5, 6, 30
- VGP** Vanishing Gradient Problem. 5, 50, 52
- WfMS** Workflow Management System. iii, 1, 3–6, 11–13, 17, 33, 50–52
- WZO** Waiting Zone One for K-Batch-1 Emulation. 50
- XML** Extensible Markup Language. 9
- XOR** Exclusive OR. 10, 16