W1–W2

## PART 1 Numerical Optimization

- Root finding / Newton's type methods
- Nonlinear Optimisation ($1^{st}$ + $2^{nd}$ ORDER OPTIMALITY CONDITIONS)
- Newton's type methods

Convergence rates
Contraction
Invariance
Conditions for convergence

Equality constrained
Inequality constrained

- (Automatic Differentiation)
- (Parameter estimation)

0

W3–W4

## PART 2 Discrete time Optimal Control

- Formulation, Analysis and Structure
- Dynamic Programming (Discrete state space, LQ, H∞, LQR)
  - → Gradient of the value function
  - → Minimum principle in DT
  - → Iterative DP
  - → Differential Dynamic Programming.

W5–W6

## PART 3 Optimal Control in Continuous-time

- Formulation, overview
- Numerical Integration (Explicit One Step, Orthogonal Collocation)
- Hamilton-Jacobi-Bellman (LQR and Ricatti Eqn etc)
- Pontryagin and Indirect Approach (Solution, Regular + Singular Arcs)
- Direct Approaches (Single / Multiple Shooting, Direct Collocation)

W6

## (PART 4 MPC and MOVING PREDICTION HORIZON)

- NMPC
- Parametric NLP
- MHE (State and Parameter estimation)
- Feedback Linearization

APC 01

CHEM-E7165

- 24h LECTURES
- 24h EXERCISES

} HYBRID, AS ALWAYS

THE OBJECTIVE OF THE COURSE IS TO PROVIDE AN INTRODUCTION TO OPTIMAL CONTROL (MATHEMATICS + NUMERICS)

THERE WILL BE SOME CODING

- PYTHON
- MATLAB

- INTUITIVE ROUTINES FOR OPTIMAL CONTROL

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TUESDAYS — 2/1 SESSIONS (LECTURE + EXERCISES)

WEDNESDAYS — 1 SESSION (LECTURE)

FRIDAYS — 1/2 SESSION (EXERCISES + CLASSES)

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EXAM ON FEB 14 — WE WILL NOT HAVE AN EXAM

→ EXERCISES → PROJECT WORK

## OUTCOMES

- UNDERSTAND THE BASIC FORMULATION OF NONLINEAR PROGRAMMING (GENERAL CLASS OF NUMERICAL OPTIMIZATION PROBLEMS)

- UNDERSTAND THE BASIC METHODS OF NUMERICAL INTEGRATION OF DYNAMICAL MODELS (ODE SIMULATION)

- UNDERSTAND HOW TO COMBINE THE ABOVE TO FORMULATE AND SOLVE OPTIMAL CONTROL PROBLEMS
  - DISCRETE TIME
  - CONTINUOUS TIME

OPTIMAL CONTROL CONSIDERS THE OPTIMIZATION OF DYNAMICAL SYSTEMS

IT BRIDGES TWO LARGE FIELDS OF APPLIED MATHEMATICS

* NUMERICAL OPTIMIZATION
* SYSTEM THEORY ( NUMERICAL SIMULATION + ANALYSIS )

IN THIS COURSE WE REFRESH OR INTRODUCE BASIC CONCEPTS FROM BOTH FIELDS WITH SOME MORE WEIGHT TO NUMERICAL OPTIMIZATION

WHAT IS REQUIRED ?

- LINEAR ALGEBRA
- NUMERICAL ANALYSIS
- CODING

THERE ARE ~~(FOUR)~~ (THREE) MAJOR PARTS

- NUMERICAL OPTIMIZATION
- DISCRETE-TIME OPTIMAL CONTROL
- CONTINUOUS-TIME OPTIMAL CONTROL
- ( ONLINE OPTIMAL CONTROL )

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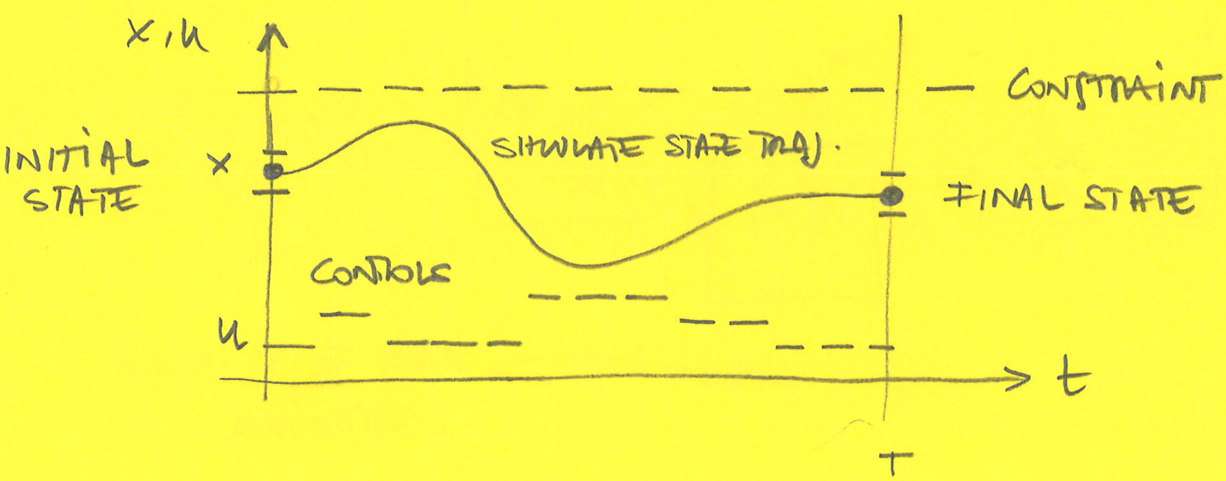
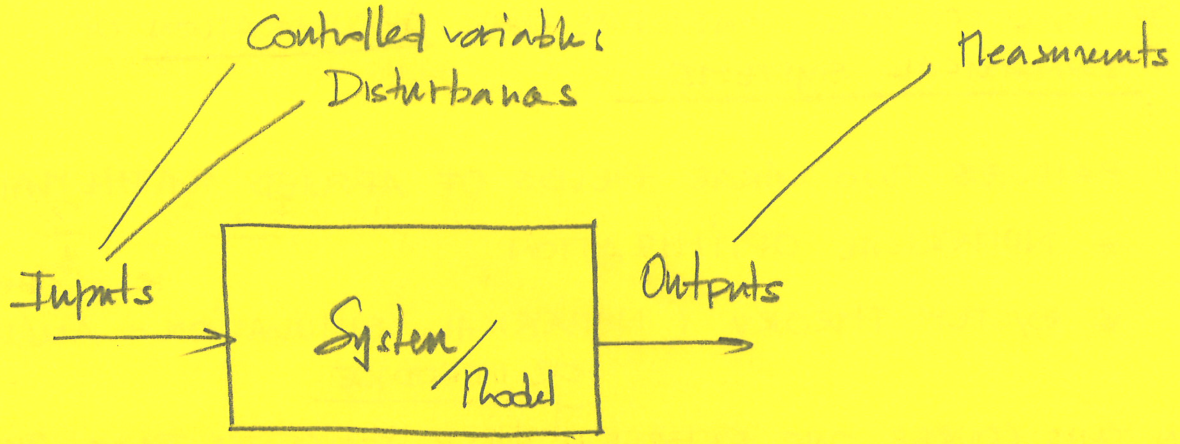
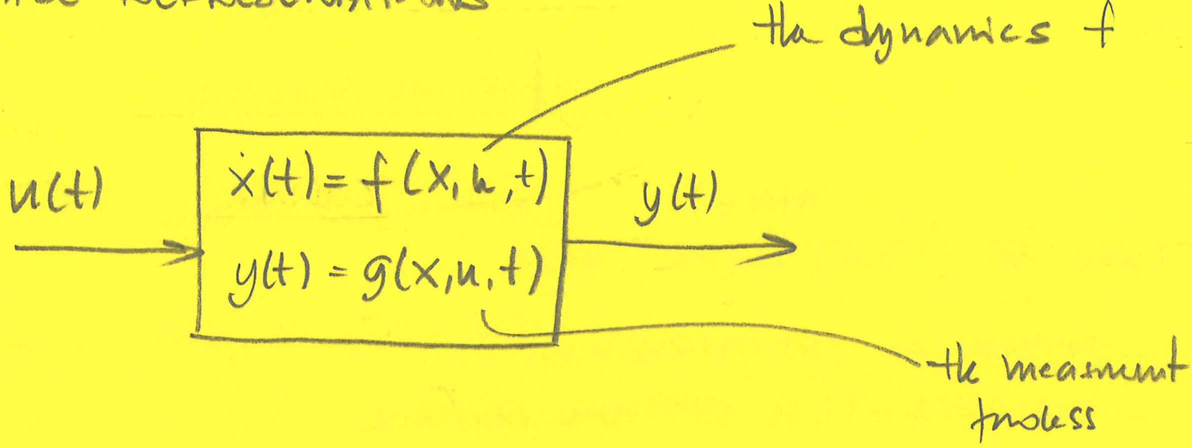
FOR A GIVEN SYSTEM STATE $x$ , WHICH CONTROLS $u$ LEAD TO THE BEST (IN SOME SENSE) OBJECTIVE, WITHOUT VIOLATING THE CONSTRAINTS ?

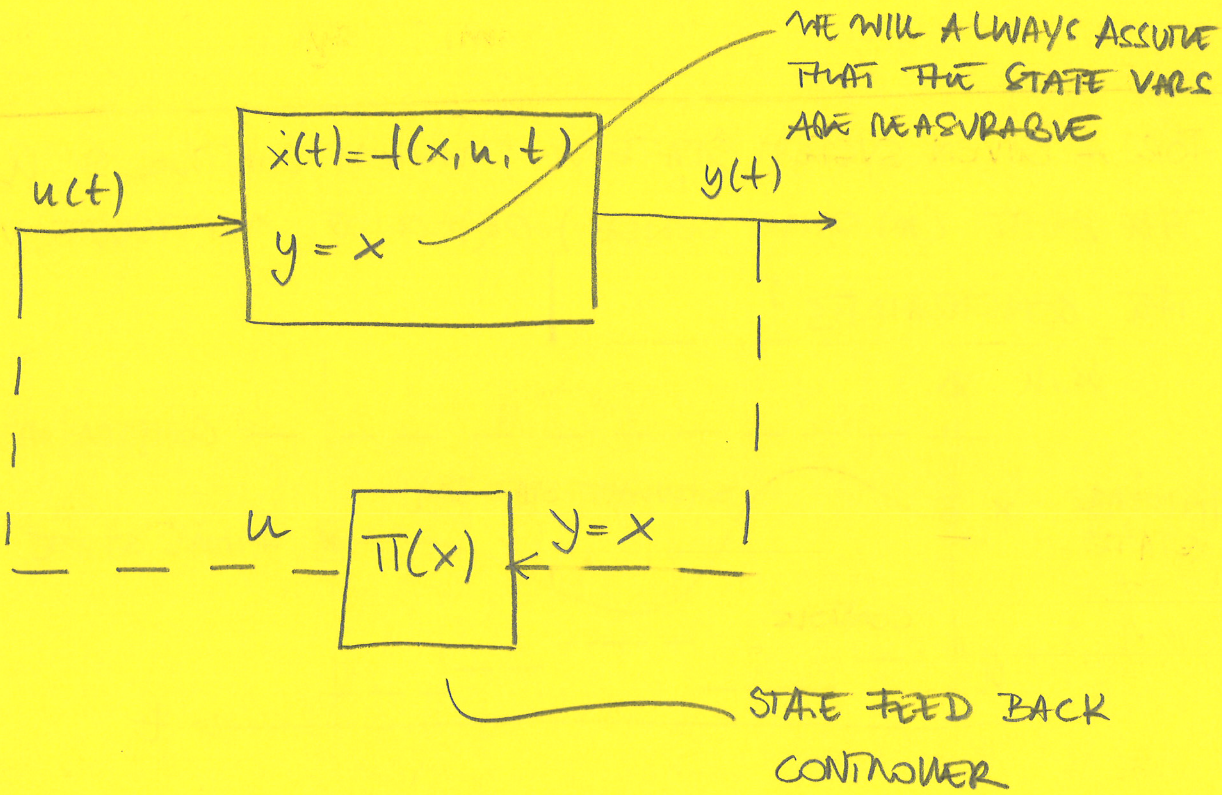





STATE SPACE REPRESENTATIONS




$x$ are the state variables $\longrightarrow$ they evolve in time.



- WE IDENTIFY DYNAMICAL SYSTEMS AS PROCESSES THAT EVOLVE IN TIME

⇝ They can be characterized by STATES $x$ that allow us to predict the future behaviour of the system

⇝ Often the dynamics can be controlled by a suitable choice of inputs, we denote them as CONTROLS $u$

TYPICALLY THE CONTROLS SHOULD BE OPTIMALLY CHOSEN

⇝ In such a way that an OBJECTIVE FUNCTION is optimized subjected to some CONSTRAINTS

| |
|---|
| TECHNIQUES FOR NUMERICALLY CHOSING OPTIMAL CONTROLS IS THE MAIN TOPIC OF THIS COURSE |

An important property of a dynamical system is the INITIAL VALUE OF THE STATE VARIABLES, $x_0$ and the CONTROL INPUT TRAJECTORY $u(t)$ for all $t \in [0, T]$

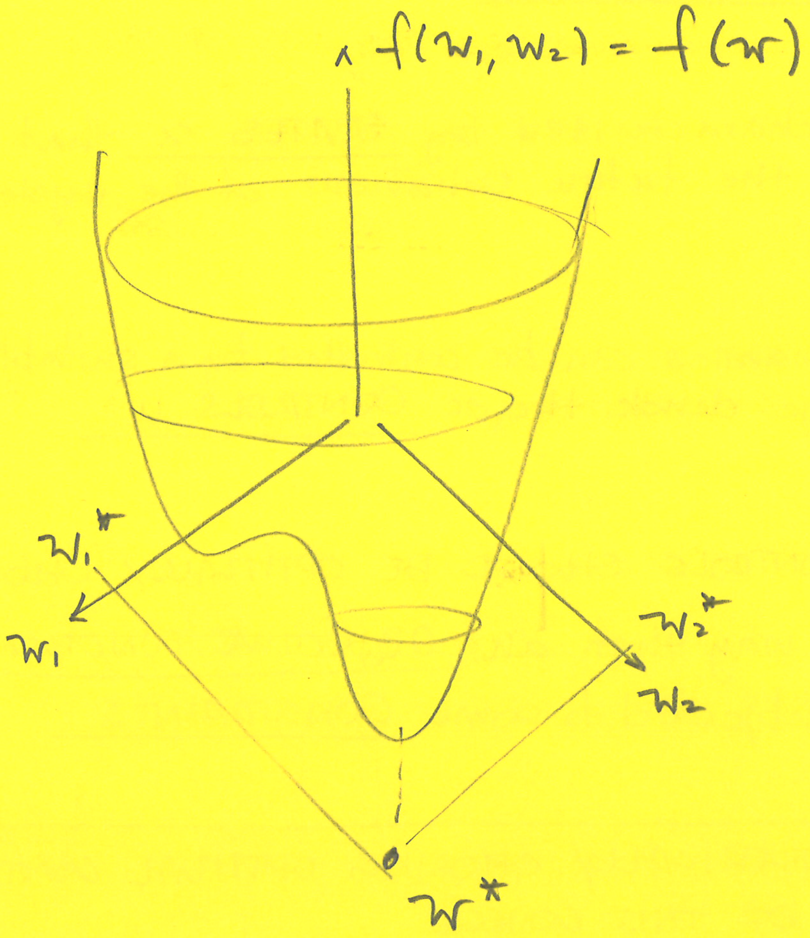
Their knowledge allow us to determine the state trajectory $x(t)$, for all $t \in [0, T]$

WE DISCUSS IMPORTANT CLASSES OF DYNAMICAL MODELS

2 DISCRETE-TIME

1 CONTINUOUS-TIME

THEN WE OVERVIEW IMPORTANT CLASSES OF OPTIMIZATION PROBLEMS



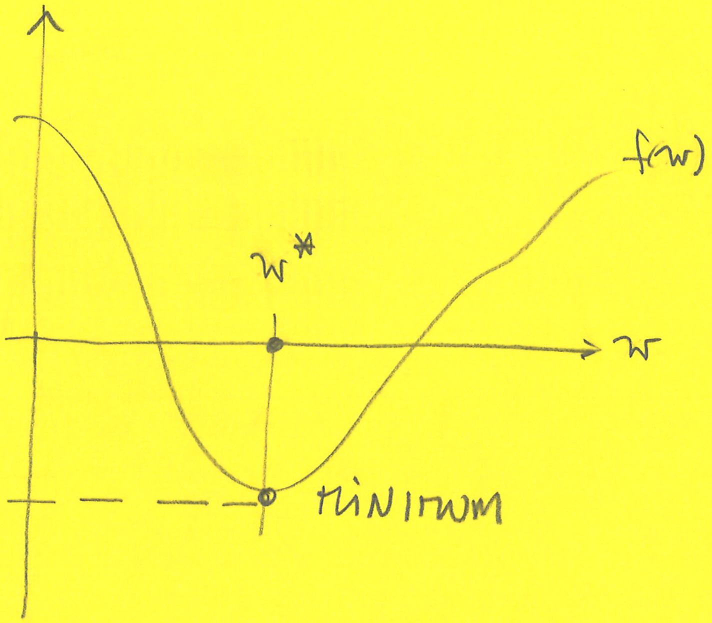
$f(w_1, w_2) = f(w)$
$w_1^*$
$w_1$
$w_2^*$
$w_2$
$w^*$

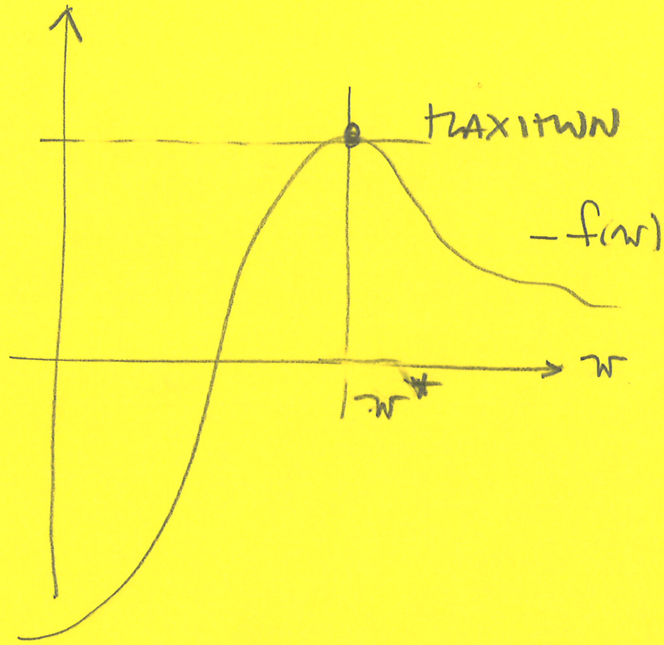
# WHAT IS OPTIMISATION?

OPTIMISATION MEANS 'SEARCH FOR THE $\underbrace{\underbrace{\text{BEST}}_{\text{OPTIMAL}}}_{\text{IN SOME SENSE}}$ SOLUTION'

MATHEMATICALLY, THIS MEANS 'MINIMISATION' OR 'MAXIMISATION' OF SOME OBJECTIVE FUNCTION

- $f(w)$ IS USED TO DENOTE THE OBJECTIVE FUNCTION
- $w$ ARE THE 'DECISION VARIABLES'



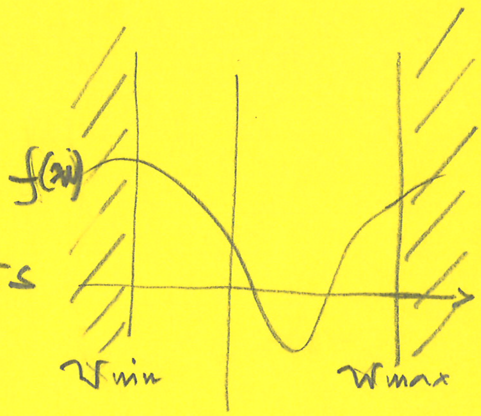




SOLVING MAXIMISATION PROBLEMS IS EQUIVALENT TO SOLVING MINIMISATION PROBLEM

→ SAME $w^*$

FROM NOW ON, MINIMISATION

VARIABLES X ARE SUBJECTED TO CONSTRAINTS




APC 04

PYTHON (or MATLAB) + CASADI

WE WILL HAVE A QUICK INTRO

WE ALREADY HAD A QUICK DEMO (GIVE SLIDES)

WE WILL HAVE A QUICK INTRO

APC 05