11th IFAC Symposium on Advanced Control of Chemical Processes (ADCHEM 2021)
June 13–16, 2021

Predictive control of activated sludge plants
to supply nitrogen for optimal crop growth

Otacílio B. L. Neto[1], **Antoine Haddon**[2], **Farouk Aichouche**[2], **Jérôme Harmand**[2], **Michela Mulas**[1], **Francesco Corona**[1,3]

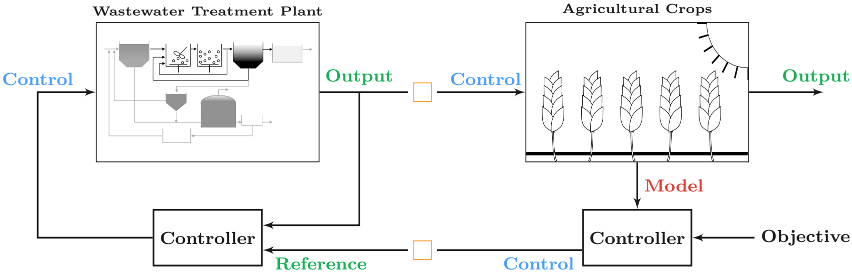
[1] Postgraduate Programme in Teleinformatics Engineering, Federal University of Ceará, Campus do Pici, Fortaleza-CE, Brazil

[2] INRAE, University of Montpellier, Narbonne, France.

[3] School of Chemical Engineering, Department of Chemical and Metallurgical Engineering, Aalto University, Finland.

Introduction and motivation*

- **Goal:** Water reuse for agricultural purposes.
  - ⇝ Nitrogen-based fertigation of crops using activated sludge plants.

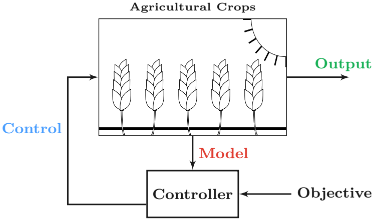


Motivation

We investigate the possibility to operate a common class of activated sludge plants to produce effluent wastewater of tailored quality for crop irrigation.

* This study has been done within the international project Control4Reuse. We acknowledge FUNCAP and the French Research National Agency for funding the Brazilian and French part of this project, respectively.

Dynamical systems, crop growth system



We consider the model of a continuously irrigated crop

- ⇝ Modern corn cultivar grown on a silty loam type soil
- ⇝ Dynamics are described by Pelak *et al* (2017)[1]

$$\dot{x}_C(t) = f_C(x_C(t), u_C(t)|\theta_C)$$

w/ $x_C(t) = [C \quad B \quad S \quad N]^\mathsf{T}$ and $u_C(t) = [I \quad F_N]^\mathsf{T}$

Optimal crop growth problem

$$\max_{u_C(\cdot)} \quad B(T)$$

$$\text{s.t.}_{\forall t \in [0,T]} \quad \dot{x}_C(t) = f_C(x_C(t), u_C(t)|\theta_C),$$

$$u_C(t) \in [0, I^{\max}] \times [0, F_N^{\max}],$$

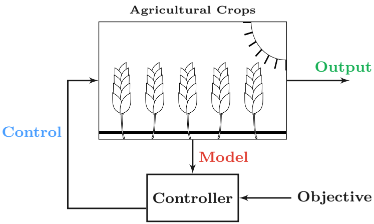
$$x_C(0) = x_0.$$

Nitrogen demand for optimal crop growth

- ▶ Maximisation of final biomass production
- ⇝ Solved using dynamic programming
- ▶ Analysed for three different Scenarios

[1] Pelak, N., Revelli, R., Porporato, A., 2017. A dynamical systems framework for crop models: Toward optimal fertilization and irrigation strategies under climatic variability. Ecological Modelling.

Dynamical systems, crop growth system



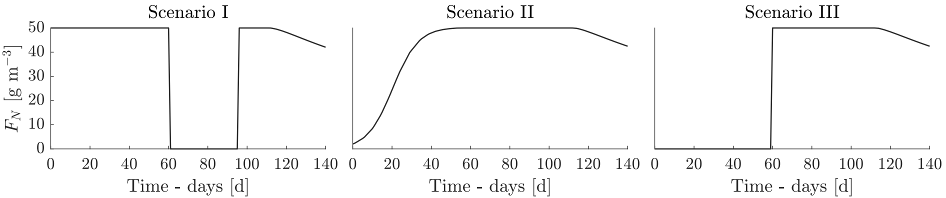
We consider the model of a continuously irrigated crop

- Modern corn cultivar grown on a silty loam type soil
- Dynamics are described by Pelak *et al* (2017)[1]

$$\dot{x}_C(t) = f_C(x_C(t), u_C(t)|\theta_C)$$

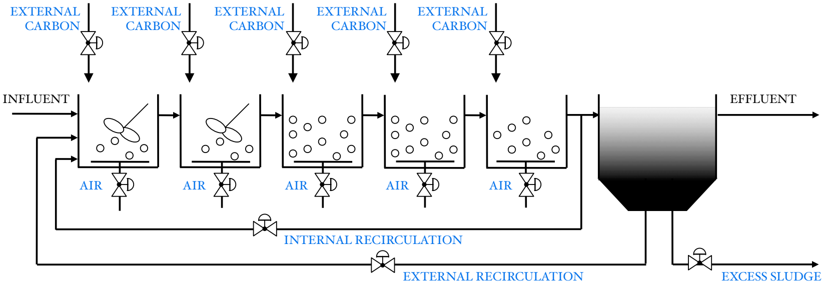
$$\text{w/ } x_C(t) = [C \quad B \quad S \quad N]^\mathsf{T} \text{ and } u_C(t) = [I \quad F_N]^\mathsf{T}$$

- The nitrogen concentrations of the irrigation water are considered as reference trajectories



[1] Pelak, N., Revelli, R., Porporato, A., 2017. A dynamical systems framework for crop models: Toward optimal fertilization and irrigation strategies under climatic variability. Ecological Modelling.

Dynamical systems, activated sludge plants



We consider a conventional Activated Sludge Process (ASP)

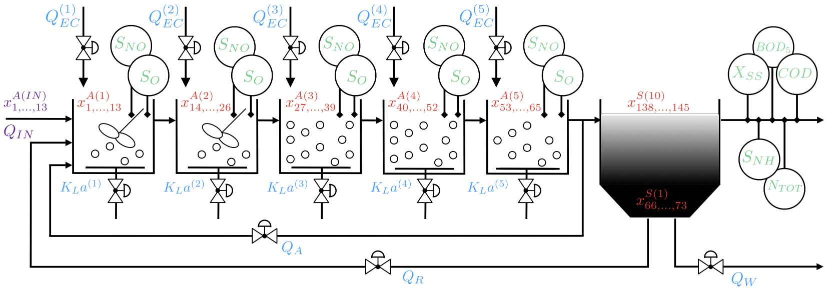
- The **Benchmark Simulation Model no. 1 (BSM1)**[2]

Plant layout

- 5 sequential bio-reactors (Activated Sludge Model no. 1)
- 1 non-reactive settler (10-layers double-exponential settling model)

[2] Gernaey, K., Jeppsson, U., Vanrolleghem, P., Copp, J., 2014. Benchmarking of Control Strategies for Wastewater Treatment Plants. IWA.

Dynamical systems, activated sludge plants (cont.)



$$\dot{x}(t) = f(x(t), u(t), w(t)|\theta_x)$$
$$y(t) = g(x(t)|\theta_y)$$

- $\rightsquigarrow$ $x(t) = [x^{A(1)} \ \cdots \ x^{A(5)} \ x^{S(1)} \ \cdots \ x^{S(10)}]^\mathsf{T}$
- $\rightsquigarrow$ $u(t) = [Q_A \ Q_R \ Q_W \ u^{A(1)} \ \cdots \ u^{A(5)}]^\mathsf{T}$
- $\rightsquigarrow$ $y(t) = [y^{A(1)} \ \cdots \ y^{A(5)} \ y^{S(10)}]^\mathsf{T}$
- $\rightsquigarrow$ $w(t) = [Q_{IN} \ x^{A(IN)}]^\mathsf{T}$
- $\rightsquigarrow$ $\{\theta_x, \theta_y\}$: Model parameters

- An "expanded model" when compared to common representations

$\rightsquigarrow$ $N_x = 5 \times 13 + 10 \times 8 = 145$ state variables

$\rightsquigarrow$ $N_u = 3 + 5 \times 2 = 13$ controls

$\rightsquigarrow$ $N_w = 1 + 13 = 14$ disturbances

$\rightsquigarrow$ $N_y = 5 \times 2 + 5 = 15$ sensors

Model predictive control, formulation

Model predictive control problem

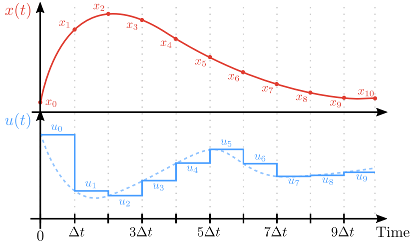
$$\min_{\substack{x_k \\ u_k,\cdots,u_{k+N-1}}} \sum_{n=k}^{k+N-1} L(x_n, u_n) + L_f(x_{k+N})$$

$$\text{s.t.}\ \forall n \in [k, k+N-1] \quad x_{n+1} = f_{\Delta t}(x_n, u_n, \hat{w}_n | \theta_x)$$
$$x_n \in \mathcal{X}, \quad u_n \in \mathcal{U},$$
$$\Phi(x_k, x_{k+N}) = 0$$

- Finite horizon of size $N > 0$
- Stage and terminal cost functions
  $$L(\cdot) : \mathbb{R}^{N_x} \times \mathbb{R}^{N_u} \to \mathbb{R}$$
  $$L_f(\cdot) : \mathbb{R}^{N_x} \to \mathbb{R}$$
- Path constraint sets
  $$\mathcal{X} \subseteq \mathbb{R}^{N_x}, \quad \mathcal{U} \subseteq \mathbb{R}^{N_u}$$
- Initial and terminal conditions
  $$\Phi(\cdot) : \mathbb{R}^{N_x} \times \mathbb{R}^{N_x} \to \mathbb{R}^{N_\Phi}$$

Discretise-then-optimise: The problem is transcribed into a NLP then solved numerically

- Zero-order hold of controls (given $\Delta t > 0$):
  $$u(t) = u(t_n) \quad (t \in [t_n, t_{n+1})), \quad t_n = n\Delta t$$
- Evolution given by transition function
  $$x_{n+1} = f_{\Delta t}(x_n, u_n, \hat{w}_n) = x_n + \int_{t_n}^{t_{n+1}} f(x(t), u_n, \hat{w}_n) dt$$
- Fixed disturbances at each horizon (given $\Delta t_w > 0$)
  $$\hat{w}_n = w_{\lfloor \frac{k\Delta t}{\Delta t_w} \rfloor} = w(\lfloor \frac{k\Delta t}{\Delta t_w} \rfloor \Delta t_w)$$



Model predictive control, formulation (cont.)

Constrained affine quadratic regulator (c-AQR)

$$\min_{u_k,\dots,u_{k+N-1}} \sum_{n=k}^{k+N-1} \left(\|x_n - x_n^{sp}\|_Q^2 + \|u_n - u_n^{sp}\|_R^2 \right) + \|x_{k+N} - x_{k+N}^{sp}\|_Q^2$$

$$\begin{aligned} \text{s.t.} \quad & x_{n+1} = z_{\Delta t}^{(k)} + A_{\Delta t}^{(k)} x_n + B_{\Delta t}^{(k)} u_n + G_{\Delta t}^{(k)} \hat{w}_n \\ \forall n \in [k, k+N-1] \quad & H_x x_n \leq h_x, \quad H_u u_n \leq h_u, \\ & x_k = \hat{x}_k \end{aligned}$$

c-AQR: We consider affine dynamical constraints and quadratic cost functions

- Quadratic cost functions

$$L(\cdot) = \|x_n - x_n^{sp}\|_Q^2 + \|u_n - u_n^{sp}\|_R^2$$

$$L_f(\cdot) = \|x_{k+N} - x_{k+N}^{sp}\|_Q^2$$

- Linearisation of $f(\cdot)$ around $P_k := (x_k^{sp}, u_k^{sp}, w_k^{sp})$

$$x_{n+1} = z_{\Delta t}^{(k)} + A_{\Delta t}^{(k)} x_n + B_{\Delta t}^{(k)} u_n + G_{\Delta t}^{(k)} \hat{w}_n$$

- Convex constraint sets

$$\mathcal{X} = \{x \in \mathbb{R}^{N_x} \mid H_x x \leq h_x\}$$

$$\mathcal{U} = \{u \in \mathbb{R}^{N_u} \mid H_u u \leq h_u\}$$

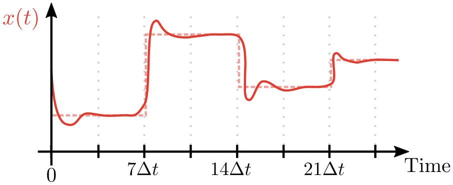
- Fixed initial state (given $\hat{x}_k$)

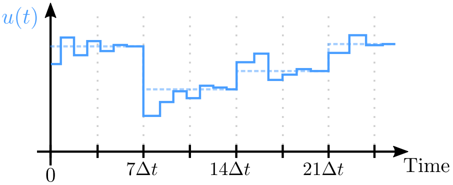
$$\Phi(x_k, x_{k+N}) = x_k - \hat{x}_k$$

Model predictive control, zero-offset set-point tracking

- Linearisation-based MPCs cannot ensure zero-offset with a single linearisation
- **Approach:** Linearise the process around nonconstant references changing every $\Delta t_{sp} > 0$
  (With each w_m^{sp} fixed to the available disturbance measurement at the k-th horizon)

$$P = \left\{ (x_m^{sp}, u_m^{sp}, w_m^{sp}) : x_m^{sp} = x^{sp}(m\Delta t_{sp}),\ u_m^{sp} = u^{sp}(m\Delta t_{sp}),\ w_m^{sp} = w\left(\left\lfloor \frac{k\Delta t}{\Delta t_w} \right\rfloor \Delta t_w\right) \right\}_{m=0}^{M}$$





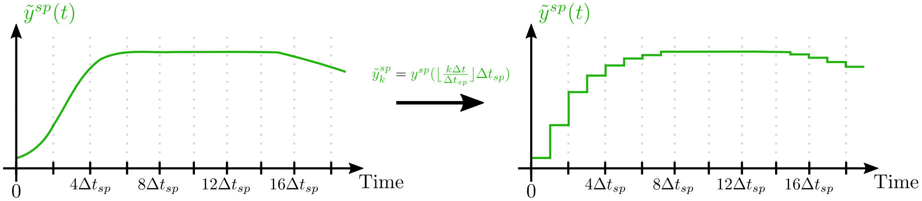
- **Affine switching system representation:**
  (Only the $\lfloor \frac{n\Delta t}{\Delta t_{sp}} \rfloor$-th linearisation is "active" at each n-th instant)

$$x_{n+1} = \sum_{m=0}^{M} \mathbb{I}_{m=\lfloor \frac{n\Delta t}{\Delta t_{sp}} \rfloor} \left(z_{\Delta t}^{(m)} + A_{\Delta t}^{(m)} x_n + B_{\Delta t}^{(m)} u_n + G_{\Delta t}^{(m)} \hat{w}_n \right), \qquad \mathbb{I}_S = \begin{cases} 1 & \text{if S true} \\ 0 & \text{if S false} \end{cases}$$

Model predictive control, zero-offset set-point tracking (cont.)

Usually, reference trajectories are available only for (a subset of) output variables

- For continuous-time trajectories, we consider a discretisation with $\Delta t_{sp} > 0$



⇝ Each pair (x_k^{sp}, u_k^{sp}) satisfying $\tilde{y}_k^{sp} \in \mathbb{R}^{N_{\tilde{y}}}$ is the solution of an optimisation:

Steady-state optimisation

$$\min_{x_k^{sp}, u_k^{sp}} \quad \| H g(x_k^{sp}) - \tilde{y}_k^{sp} \|^2_{W_{\tilde{y}}} + \| u_k^{sp} - \tilde{u}_k^{sp} \|^2_{W_u}$$

$$\text{s.t.} \quad f(x_k^{sp}, u_k^{sp}, w_k^{sp} | \theta_x) = 0,$$

$$x_k^{sp} \in \mathcal{X}^{sp}, \quad u_k^{sp} \in \mathcal{U}^{sp}$$

(We consider fixed $w_k^{sp} = w^{SS}$ and $\tilde{u}_k^{sp} = 0$)

⇝ Search for stationary point $(x_k^{sp}, u_k^{sp}, w_k^{sp})$

⇝ The $N_{\tilde{y}} \leq N_y$ outputs of interest are selected by matrix $H \in \{0,1\}^{N_{\tilde{y}} \times N_y}$

⇝ $W_{\tilde{y}}, W_u \succeq 0$ are tuning parameters

Results, MPC simulation (Case I, Scenario I)

Model predictive control parameters

General parameters:

$$T = 140 \text{ days}, \quad N = 21 \text{ (7 days)}, \quad Q = C_{\Delta t}^{(k)\mathsf{T}} C_{\Delta t}^{(k)}$$

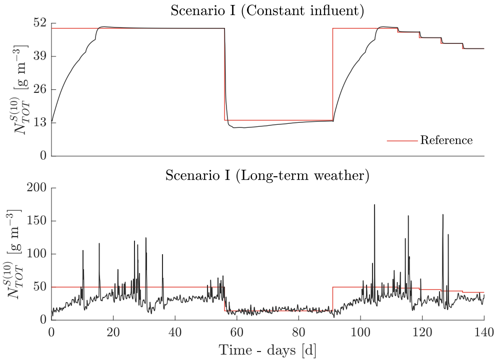
$$R = \text{diag}[\underbrace{10^{-4}}_{Q_A}\ \underbrace{0.01}_{Q_R}\ \underbrace{0.01}_{Q_W}\ \underbrace{10^{-3} I_5}_{K_La^{(k)}}\ \underbrace{10^{4} I_5}_{Q_{EC}^{(k)}}]$$

Sampling periods:

$$\Delta t = (1/3)d, \quad \Delta t_w = 1d, \quad \Delta t_{sp} = 7d$$

Influent conditions:

$$w = \begin{cases} \texttt{CONSTANT INFLUENT} \\ \texttt{LONG-TERM WEATHER} \end{cases}$$



Case I: ($W_{\tilde{y}} = 100$, $W_u = 0$)

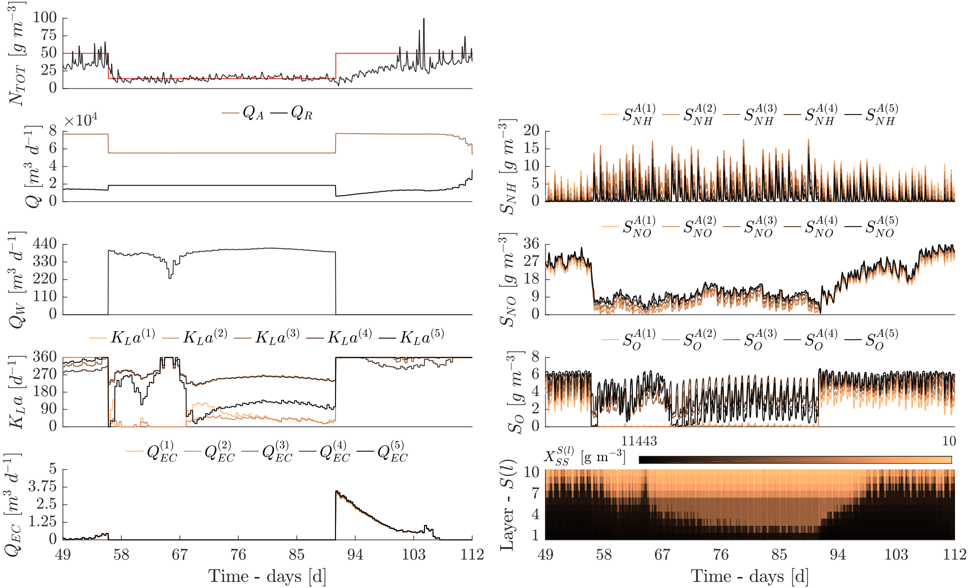
- Constant influent: zero-offset is achieved
- Dynamic influent: performance decreases

- The control strategy tracks $N_{TOT}^{S(10)}$ mainly by manipulating $NO_2^- + NO_3^-$ nitrogen

The predictive controller only partially achieves zero-offset under typical influent conditions

Neto O., Haddon A., Aichouche F., Harmand J., Mulas M., Corona F.

Results, MPC simulation (Case I, Scenario I, $t \in [49,\ 112]$)



Results, MPC simulation (Case II, Scenario I)

Model predictive control parameters

General parameters:

$$T = 140 \text{ days}, \quad N = 21 \ (7 \text{ days}), \quad Q = C_{\Delta t}^{(k)\mathsf{T}} C_{\Delta t}^{(k)}$$

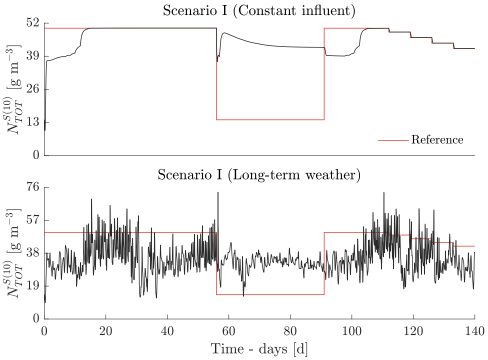
$$R = \mathrm{diag}[\underbrace{10^{-4}}_{Q_A} \ \underbrace{0.01}_{Q_R} \ \underbrace{0.01}_{Q_W} \ \underbrace{10^{-3} I_5}_{K_L a^{(k)}} \ \underbrace{10^{4} I_5}_{Q_{EC}^{(k)}}]$$

Sampling periods:

$$\Delta t = (1/3)d, \quad \Delta t_w = 1d, \quad \Delta t_{sp} = 7d$$

Influent conditions:

$$w = \begin{cases} \texttt{CONSTANT INFLUENT} \\ \texttt{LONG-TERM WEATHER} \end{cases}$$



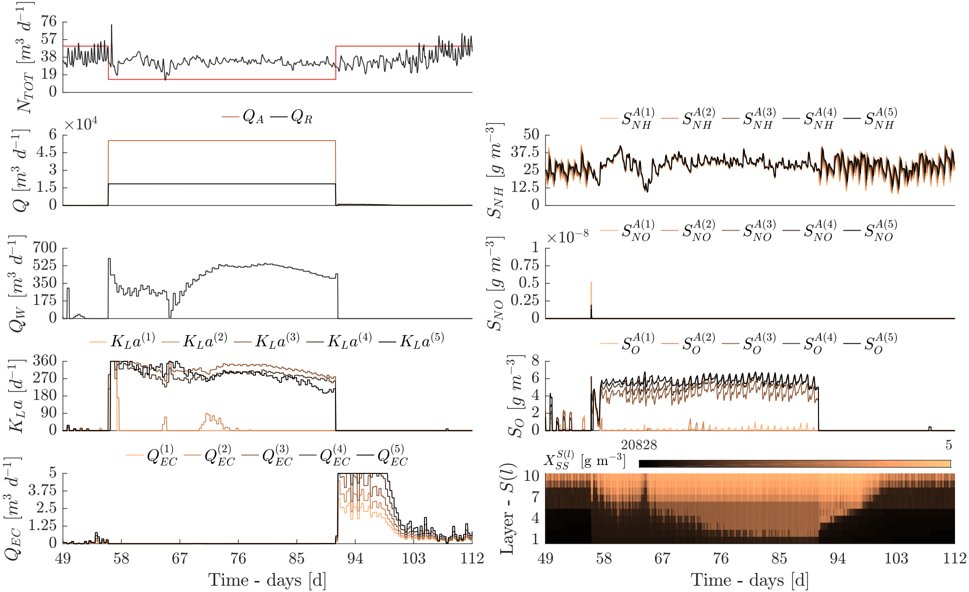
Case II: ($W_{\tilde{y}} = 100$, $W_u = 0.01 I_{N_u}$)

- Constant influent: zero-offset partly achieved
- Dynamic influent: performance decreases

- The control strategy tracks $N_{TOT}^{S(10)}$ mainly by manipulating $NH_4^+ + NH_3$ nitrogen

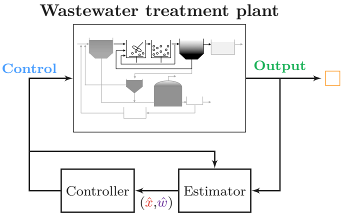
The predictive controller partially achieves zero-offset, but is unable to recover to treatment control

Results, MPC simulation (Case II, Scenario I, $t \in [49,\ 112]$)



Output MPC, formulation and estimation problem

- **Output MPC:** Initial state and disturbance $(\hat{x}_k, \hat{w}_k)$ are not measured but rather estimated



Moving horizon estimation problem

$$\min_{\substack{\hat{x}_{k-N_e+1} \\ \hat{w}_{k-N_e+1}, \cdots, \hat{w}_k}} \quad L_0(\hat{x}_{k-N_e+1}) + \sum_{n=k-N_e+1}^{k} L(\hat{x}_n, \hat{w}_n | y_n)$$

$$\underset{\forall n \in [k-N_e+1, k]}{\text{s.t.}} \quad \hat{x}_{n+1} = f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n | \theta_x)$$

$$\hat{x}_n \in \mathcal{X}, \quad \hat{w}_n \in \mathcal{W}$$

The optimal estimation problem derives from a *maximum a posteriori* estimate solution

Stochastic state-space

$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t), \hat{w}(t) | \theta_x)$$
$$y(t) = g(\hat{x}(t) | \theta_y) + v(t)$$

with $\hat{x}(0) \sim e^{-L_0(\hat{x}(0) | Q_{x_0})}$

$$\hat{w}(t) \overset{i.i.d}{\sim} e^{-L_w(\hat{w}(t) | R_w)}$$

$$v(t) \overset{i.i.d}{\sim} e^{-L_v(v(t) | Q_v)}$$

- Zero-order hold of disturbances:
  (Given the rate of measurement Δt_e)

$$\hat{w}(t) = \hat{w}(t_n) \ \ (t \in [t_n, t_{n+1})), \quad t_n = n\Delta t_e$$

- Evolution given by *transition function*

$$\hat{x}_{n+1} = f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n) = \hat{x}_n + \int_{t_n}^{t_{n+1}} f(\hat{x}(t), u_n, \hat{w}_n) dt$$

Output MPC, formulation and estimation problem (cont.)

Constrained Affine Gauss-Markov (c-AGM) estimator

$$\min_{\substack{\hat{x}_{k-N_e+1} \\ \hat{w}_{k-N_e+1},\dots,\hat{w}_k}} \|\hat{x}_{k-N_e+1} - \bar{x}_{k-N_e+1}\|^2_{Q_{x_0}^{-1}} + \sum_{n=k-N_e+1}^{k} \left(\|y_n - C^{(n)}_{\Delta t_e}\hat{x}_n\|^2_{Q_v^{-1}} + \|\hat{w}_n - \bar{w}_n\|^2_{R_w^{-1}} \right)$$

$$\underset{\forall n \in [k-N_e+1,k]}{\text{s.t.}} \quad \hat{x}_{n+1} = z^{(n)}_{\Delta t_e} + A^{(n)}_{\Delta t_e}\hat{x}_n + B^{(n)}_{\Delta t_e}u_n + G^{(n)}_{\Delta t_e}\hat{w}_n$$

$$H_x\hat{x}_n \le h_x, \quad H_w\hat{w}_n \le h_w$$

- **c-AGM:** We assume Gaussian distributions and affine dynamical models
  (Given means $(\bar{x}_{k-N_e+1}, \{\bar{w}_n\}^k_{k-N_e+1})$ as the solutions from the previous horizon)

$$\hat{x}_{k-N_e} \sim \mathcal{N}(\bar{x}_{k+N_e}, Q_{x_0}), \quad \hat{w}_n \overset{i.i.d}{\sim} \mathcal{N}(\bar{w}_n, R_w), \quad v_n \overset{i.i.d}{\sim} \mathcal{N}(0, Q_v)$$

- Linearisations of $f(\cdot)$ around each $P_n \coloneqq (\hat{x}_n, u_n, \hat{w}_n)$
  (Given $(\bar{x}_n, \bar{w}_n)$ as the estimates from the previous horizon)

$$\hat{x}_{n+1} = z^{(n)}_{\Delta t_e} + A^{(n)}_{\Delta t_e}\hat{x}_n + B^{(n)}_{\Delta t_e}u_n + G^{(n)}_{\Delta t_e}\hat{w}_n$$
$$y_n = C^{(n)}_{\Delta t_e}\hat{x}_n + v_n$$

- Convex support sets

$$\mathcal{X} = \{x \in \mathbb{R}^{N_x} \mid H_x x \le h_x\}$$
$$\mathcal{W} = \{w \in \mathbb{R}^{N_w} \mid H_w w \le h_w\}$$

Results, Output MPC simulation (Case I, Scenario I)

Output model predictive control parameters*

General parameters:

$T = 140$ days, $N_e = 12$ (3 hours),

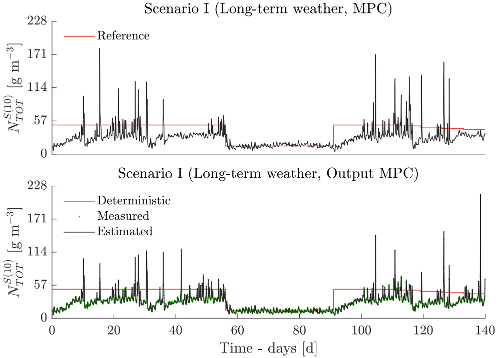
$Q_{x_0} = \mathtt{diag}[(0.01y^{SS})^2]$, Q_v, $R_w = \mathtt{diag}[\cdots]$

Sampling periods:

$\Delta t_e = (1/96)d$

Influent conditions:

$w = \mathtt{LONG\text{-}TERM\ WEATHER}$



Case I: $(W_{\tilde{y}} = 100, W_u = 0)$

- The internal state is accurately estimated
- Slight performance improvement

The Output MPC provides similar results to the MPC, implying accuracy in the state and disturbance estimation

* The control-related parameters are the same as in the previous section. Steady-state $SS := (x^{SS}, u^{SS}, w^{SS}, y^{SS})$ is the operating point suggested by the benchmark.

Neto O., Haddon A., Aichouche F., Harmand J., Mulas M., Corona F.

Final remarks

The predictive control of activated sludge plants for water reuse was investigated

Our results show that

- ✓ Zero-offset is achieved under constant influent conditions
- × The nitrogen demand is only partially met under typical influent conditions
- ▶ An *ad hoc* tuning of the predictive controller leads to alternative control policies
  I. **($W_u = 0$):** The controller favours manipulating $NO_2^- + NO_3^-$ nitrogen
  II. **($W_u = 0.01 I_{N_u}$):** The controller favours manipulating $NH_4^+ + NH_3$ nitrogen

---

The MPC tracking accuracy is summarised by the normalized mean-squared error (NMSE)

$$J_{\mathrm{NMSE}}(\tilde{y}, \tilde{y}^{sp}) = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \|\tilde{y}(t) - \tilde{y}^{sp}(t)\|^2 / \|\tilde{y}(t)\|^2 dt$$

	Case I		Case II	
	J_{NMSE} (Constant influent)	J_{NMSE} (Dynamic influent)	J_{NMSE} (Constant influent)	J_{NMSE} (Dynamic influent)
Scenario I	0.1612	0.9797	0.1573	0.4117
Scenario II	0.0500	0.4309	0.0159	0.3255
Scenario III	0.0820	0.6611	0.0366	0.2531

Thank you!

Questions?