Functions and branching

Foundation of programming (CK0030)

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Functions and branching
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FdP

- ☻ Intro to variables, objects, modules, and text formatting
- ☻ Programming with WHILE- and FOR-loops, and lists
- ☹ **Functions and IF-ELSE tests**

- ☹ Data reading/writing, error handling and making modules

- ☹ Arrays and array computing
- ☹ Plotting curves and surfaces

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FdP (cont)

Two fundamental and extremely useful programming concepts

- **Functions**, defined by the user
- **Branching**, of program flow

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Functions

Functions and branching

Functions

The term **function** has a wider meaning than a mathematical function

Definition

`Function`

A `function` is a collection of statements that can be run wherever and whenever needed in the program

The `function` may accept input variables

- *To influence what is computed inside*
- *(A function contains statements)*

The `function` may return new objects

`Functions` help avoid duplicating bits of code (puts all of them together)

- A strategy that saves typing and makes it easier to modify code

`Functions` are also used to split a long program into smaller pieces

Python has pre-defined `functions` (`math.sqrt`, `range`, `len`, `math.exp`, ...)

$\leadsto$ We discuss how to define own `functions`

Mathematical functions as Python functions

Functions

Math functions as Python functions

We construct a Python `function` that evaluates a mathematical function

Example

Consider a function $F(C)$ for converting degree Celsius C to Fahrenheit F

$$\leadsto \quad F(C) = \frac{9}{5}C + 32$$

The `function` (`F`) takes `C` (C) as its input argument

```python
def F(C):
 return (9.0/5)*C + 32
```

It returns value `(9.0/5)*C + 32` ($F(C)$) as output

Math functions as Python functions (cont.)

All Python `functions` begin with `def`, followed by the **function name**

$\leadsto$ Inside parentheses, a comma-separated list of **function arguments**

$\leadsto$ The argument acts as a standard variable inside the `function`

The statements to be performed inside the `function` must be indented

After the `function`, it is common (not necessary) to **return** a value

$\leadsto$ The **function output** value is sent out of the `function`

Math functions as Python functions (cont.)

Example

The `function name` is `F` (F)

$$F(C) = \frac{9}{5}C + 32$$

There is only one `input argument` `C` (C)

```
def F(C):
 return (9.0/5)*C + 32
```

The `return` value is computed as `(9.0/5)*C + 32` (it has no name)

- It is the evaluation of $F(C)$ (implicitly `F(C)`)

The `def` line (`function name` and `arguments`) is the **function header**

The indented statements are the **function body**

```
def F(C):                                              #      Function header
 return (9.0/5)*C + 32                                 # Function (mini) block
```

The `return` often (not necessarily) associates with the `function name`

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Math functions as Python functions (cont.)

To use a `function`, we must **call** or **invoke** it with `input arguments`

- ⇝ The `function` will process the `input arguments`
- ⇝ As a result, it will return an `output value`

We (may need to) store the result in a `variable`

Example

```
##########################################################################
def F(C):                                              # T conversion function #
 return (9.0/5)*C + 32                                 #                  F(C) #
##########################################################################
```

The value returned from `F(C)` is an object

- ⇝ Specifically, it is a `float object`

The call `F(C)` can be placed anywhere in a code

- A `float` must be valid

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Math functions as Python functions (cont.)

```
##########################################################################
def F(C):                                              # T conversion function #
 return (9.0/5)*C + 32                                 #                  F(C) #
##########################################################################

temp1 = F(15.5)              #    Store return value as variable (temp1)

a = 10                       #       Given input argument 'a' (value 10)
temp2 = F(a)                 #    Store return value as variable (temp2)


print F(a+1)                 # Given input argument 'a+1' (value 10 + 1)
                             # Print return value to screen (no storing)

sum_temp = F(10) + F(20)     #        Two calls to get two output values
                             #           Combine output values and store
```

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Math functions as Python functions (cont.)

Example

Consider the usual list `Cdegrees` of temperatures in degrees Celsius

- Interest in computing a list of corresponding Fahrenheits
- We want to use function `F`, in a list comprehension

```
##########################################################################
def F(C):                                              # T conversion function #
 return (9.0/5)*C + 32                                 #                  F(C) #
##########################################################################

Cdegrees = [-20, -15, -10, -5, 0, 5, 10, 15, 20, 25, 30, 35]

Fdegrees = [F(C) for C in Cdegrees]
```

Math functions as Python functions (cont.)

Example

Consider a slight variation of the `F(C)` function

⇝ `F2(C)`

We define `F2(C)` to return a formatted string

⇝ (Instead of a real number)

```python
##########################################################################
def F2(C):                                                               #
    F_value = (9.0/5)*C + 32                                             #
    return '%.1f degrees Celsius correspond to '\                        #
           '%.1f degrees Fahrenheit' % (C, F_value)                      #
##########################################################################
```

How to use this new function?

```python
>>> s1 = F2(21)

>>> print s1
    21.0 degrees Celsius correspond to 69.8 Fahrenheits
```

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Math functions as Python functions (cont.)

```python
##########################################################################
def F2(C):                                                               #
    F_value = (9.0/5)*C + 32                                             #
    return '%.1f degrees Celsius correspond to '\                        #
           '%.1f degrees Fahrenheit' % (C, F_value)                      #
##########################################################################
```

Note the `F_value` assignment inside the function

- We can create variables inside a function
- We can perform operations with them

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Math functions as Python functions (cont.)

Example

Consider the construction of a temperature-conversion program `c2f.py`

```python
##########################################################################
def F(C):                                                                #
    F = 9./5*C + 32                                                      #
    return F                                                             #
##########################################################################

dC = 10
C = -30

while C <= 50:
    print '%5.1f %5.1f' % (C, F(C))
    C += dC
```

The code contains a function `F(C)` and a `while` loop

⇝ Print a table of temperatures

⇝ Both Celsius and Fahrenheit

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Math functions as Python functions (cont.)

Programmers must understand the sequence of statements in a program

- There are excellent tools that help build such understanding
- A **debugger** and/or the **Online Python Tutor**

A debugger should be used for all sorts of programs, large and small

- **Online Python Tutor** is an educational tool (small programs)

Go to **Online Python Tutor** (link/click me), copy and paste your code

Use the 'forward' button to advance, one statement at a time

- Observe the sequence of operations
- Observe the evolution of variables
- Observe, observe, observe, ...

Local and global variables

Functions

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Local and global variables

Definition

Local variables are `variables` that are defined within a `function`

`Local variables` are invisible outside `functions`

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Local and global variables

Example

Consider the following function

```
########################################################################
def F2(C):                                                             #
 F_value = (9.0/5)*C + 32                                              #
 return '%.1f degrees Celsius correspond to '\                         #
        '%.1f degrees Fahrenheit' % (C, F_value)                       #
########################################################################
```

Consider a simple function call

```
>>> s1 = F2(21)

>>> s1
    '21.0 degrees Celsius correspond to 69.8 Fahrenheits'
```

In `function` `F2(C)`, `variable` `F_value` is a `local variable`

- It is inside a `function`

A `local variable` does not ‘exist’ outside the `function`

- (It cannot be accessed and used for computations)

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Local and global variables (cont.)

```
########################################################################
def F2(C):                                                             #
 F_value = (9.0/5)*C + 32                                              #
 return '%.1f degrees Celsius correspond to '\                         #
        '%.1f degrees Fahrenheit' % (C, F_value)                       #
########################################################################
```

The (main) program around function `F2(C)` is not aware of variable `F_value`

⇝ If invoked, an error message is returned

```
>>> c1 = 37.5

>>> s2 = F2(c1)

>>> F_value
 ...
 NameError: name 'F_value' is not defined
```

Local and global variables (cont.)

Remark

`Local variables` are created inside a `function`

⤳ They are destroyed when leaving the `function`

Also `input arguments` are `local variables`

⤳ They cannot be accessed outside the `function`

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Local and global variables (cont.)

Example

Consider the `input argument` to `function F2`, variable `C`

⤳ Variable `C` is a `local variable`

```
##########################################################################
def F2(C):                                                               #
 F_value = (9.0/5)*C + 32                                                #
 return '%.1f degrees Celsius correspond to '\                           #
        '%.1f degrees Fahrenheit' % (C, F_value)                         #
##########################################################################
```

We cannot access variable `C` outside the `function`

```
>>> c1 = 37.5
>>> s2 = F2(c1)
>>> F_value
 ...
 NameError: name 'F_value' is not defined

>>> C
 ...
 NameError: name 'C' is not defined
```

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Local and global variables (cont

Definition

`Variables` defined outside the `function` are `global variables`

`Global variables` are accessible everywhere in a program

⤳ *Also from inside a `function`*

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Local and global variables (cont.)

Example

```
##########################################################################
def F2(C):                                                               #
 F_value = (9.0/5)*C + 32                                                #
 return '%.1f degrees Celsius correspond to '\                           #
        '%.1f degrees Fahrenheit' % (C, F_value)                         #
##########################################################################
```

⤳ `C` and `F_value` are `local variables`

```
>>> c1 = 37.5
>>> s2 = F2(c1)
```

⤳ `c1` and `s2` (and `s1`) are `global variables`

Local and global variables (cont.)

Example

```
###########################################################################
def F2(C):                                                                #
 F_value = (9.0/5)*C + 32                                                 #
 return '%.1f degrees Celsius correspond to '\                            #
        '%.1f degrees Fahrenheit' % (C, F_value)                          #
###########################################################################
```

```
>>> c1 = 37.5
>>> s2 = F2(c1)

>>> F_value
 ...
 NameError: name 'F_value' is not defined
>>> C
 ...
NameError: name 'C' is not defined

>>> c1
 ... 37.5
>>> s2
 ... '37.5 degrees Celsius correspond to 99.5 Fahrenheits'
```

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Local and global variables (cont.)

Example

Consider a slight modification of our original function

```
###########################################################################
def F3(C):                                                                #
 F_value = (9.0/5)*C + 32                                                 #
 print  'In F3: C=%s F_value=%s r=%s' % (C,F_value,r)                     #
 return '%.1f Celsius correspond to '\                                    #
        '%.1f Fahrenheit' % (C,F_value)                                   #
###########################################################################
```

We ask the function to write out its variables

- Two `local variables` `F_value`, `C`
- A `global variable` `r`

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Local and global variables (cont.)

```
###########################################################################
def F3(C):                                                                #
 F_value = (9.0/5)*C + 32                                                 #
 print  'In F3: C=%s F_value=%s r=%s' % (C,F_value,r)                     #
 return '%.1f Celsius correspond to '\                                    #
        '%.1f Fahrenheit' % (C,F_value)                                   #
###########################################################################
```

```
>>> C = 60                                      #        Make a global variable, C
>>> r = 21                                      #  Make another global variable, r

>>> s3 = F3(r)                                  # Execute F3 with input argument r
    In F3: C=21 F_value=69.8 r=21               #          Print internal variables
                                                # Internal variable C has value 21
                                                #     Assign output to variable s3

>>> s3                                          #                Output variable s3
    '21.0 Celsius correspond to 69.8 Fahrenheit' # It is a string object

>>> C                                           # External variable C has value 60
    60
```

The example illustrates also that there are two different variables `C`

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Local and global variables (cont.)

The two variables `C`

- `C` `local variable` exists only when the program flow is inside `F3`
- `C` `global variable` is defined outside in the main (an `int object`)

```
###########################################################################
def F3(C):                                                                #
 F_value = (9.0/5)*C + 32                                                 #
 print  'In F3: C=%s F_value=%s r=%s' % (C,F_value,r)                     #
 return '%.1f Celsius correspond to '\                                    #
        '%.1f Fahrenheit' % (C,F_value)                                   #
###########################################################################
```

```
>>> C = 60
>>> r = 21
```

The value of the latter (`local`) `C` is given in the call to `function F3`

- When we refer to `C` in `F3`, we access the `local variable`
- Inside `F3`, `local variable C` shades `global variable C`

`Local variables` hide/shade `global variables`

⇝ This is important

Local and global variables (cont.)

Remark

Technically, `global variable C` can (still) be accessed as `globals()['C']`

- This practice is deprecated

Avoid `local` and `global variables` with the same name at the same time!

The general rule, when there are `variables` with the same name

1. Python first looks up the name among `local variables`
2. Then, it searches among `global variables`
3. And, then among `built-in functions`

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Local and global variables (cont.)

Example

Consider the single-line piece of code

```
print sum                                # sum is a built-in Python function
```

There are no `local variables` in the first line of code

Python then searches for a `global variable`, `sum`

⇝ It cannot find any

Python then checks among all `built-in functions`

⇝ It finds a `built-in function` with name `sum`

⇝ `print sum` returns `<built-in function sum>`

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Local and global variables (cont.)

Consider now this three-line piece of code

```
print sum                                # sum is a built-in Python function

sum = 500                                #  rebind name sum to an int object
                                         #          sum is a global variable

print sum
```

The second line binds global name `sum` to an `int object`

At accessing `sum` in `print` statement, Python searches `global variables`

- Still no `local variables` are present
- It finds the one just defined

The printout becomes 500

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Local and global variables (cont.)

```
print sum                                     # sum is a built-in Python function
sum = 500                                     #  rebind name sum to an int object
                                              #          sum is a global variable
print sum

########################################################################
def myfunc(n):                                                         #
 sum = n + 1                                        # sum is a local variable #
 print sum                                                             #
 return sum                                                            #
########################################################################

sum = myfunc(2) + 1                             # new value in global variable sum
print sum
```

Call `myfunc(2)` invokes a `function` where `sum` is a `local variable`

`print sum` makes Python first search among `local variables`

⇝ `sum` is found there, the printout is 3

⇝ (The printout is not 500, the value of `global variable sum`)

Value of `local variable sum` is returned, added to 1, to form an `int object`

- The `int object` is then bound to `global variable sum` (value 4)

Final `print sum` searches `global variables`, it finds one (value 4)

Local and global variables (cont.)

Remark

The values of `global variables` can be accessed inside `functions`

- Though their values cannot be changed
- Unless the variable is declared as global

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Local and global variables (cont.)

Example

Consider the following piece of code

```
a = 20; b = -2.5                                    # global variables

#########################################################################
def f1(x):                                                              #
 a = 21                                   # this is a new local variable #
 return a*x + b                                                         #
#########################################################################

print a                                                        # shows 20

#########################################################################
def f2(x):                                                              #
 global a                                          #    a is declared global #
 a = 21                                          # the global a is changed #
 return a*x + b                                                         #
#########################################################################

f1(3); print a                                              # 20 is printed
f2(3); print a                                              # 21 is printed
```

Note that within `function f1`, `a = 21` creates a `local variable a`

- This does not change the `global variable a`

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Multiple arguments

Functions

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Multiple arguments

`Functions F(C)` and `F2(C)` are `functions` of one single variable `C`

- Both `functions` take one `input argument` (`C`)

Yet, `functions` can have as many `input arguments` as needed

- Need to separate the `input arguments` by commas (`,`)

Multiple arguments (cont.)

Example

Consider the mathematical function

$$\rightsquigarrow \quad y(t) = v_0 t - \frac{1}{2} g t^2$$

g is a fixed constant and v_0 is a physical parameter that can vary

Mathematically, function y is a function of one variable, t

- The function values also depend on the value v_0
- To evaluate y, we need values for both t and v_0

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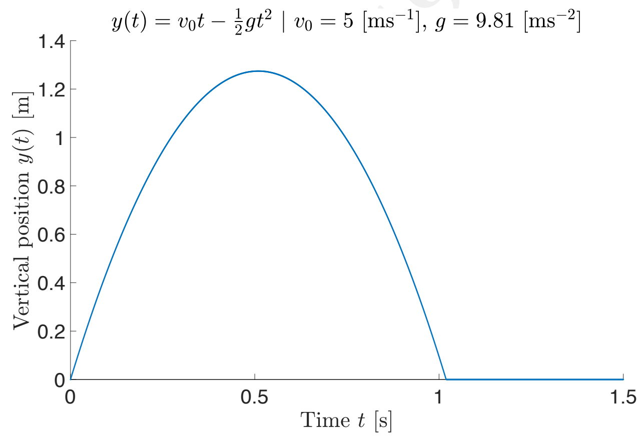
Multiple arguments (cont.)



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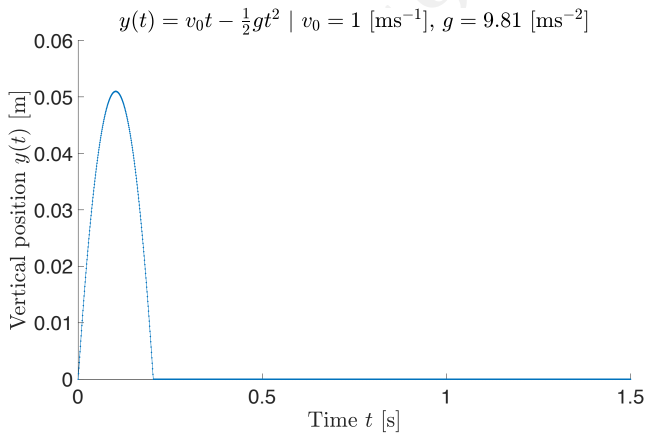
Multiple arguments (cont.)



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Multiple arguments (cont.)

$$y(t) = v_0 t - 1/2 g t^2$$

A natural implementation would be a `function` with two `arguments`

```
#######################################################################
def yfunc(t, v0):                                                     #
 g = 9.81                                                             #
 return v0*t - 0.5*g*t**2                                             #
#######################################################################
```

Within the `function` `yfunc`, `arguments` `t` and `v0` are `local variables`

- `g` is also a `local variable`

Suppose that we are interested in the function $y(t) = v_0 t - 1/2 g t^2$

- $v_0 = 6$ [ms^{-1}], $t = 0.1$ [s]

```
#######################################################################
def yfunc(t, v0):                                                     #
 g = 9.81                                                             #
 return v0*t - 0.5*g*t**2                                             #
#######################################################################

y1 = yfunc(0.1  , 6.)               #                   value1 , value2
y2 = yfunc(0.1  , v0=6.)            #           value1 , argument2=value2
y3 = yfunc(t=0.1, v0=6.)            # argument1=value1 , argument2=value2
y4 = yfunc(v0=6 , t=0.1)            # argument2=value2 , argument1=value1
```

Advantages deriving from writing `argument=value` in the call

- Reading and understanding the statement is easier

Multiple arguments (cont.)

Suppose that the `argument=value` syntax is given for all `arguments`

- The sequence of the `arguments` is no longer important
- (We can place `v0` before `t`)

Suppose that we omit the `argument=` part

- Then, it is important to remember that the sequence of `arguments` in the call must match (exactly) the sequence of `arguments` in the header

Remark

Consider `argument=value` `arguments`

They must appear AFTER all the `arguments` where only value is provided

```
###########################################################################
def yfunc(t, v0):                                                         #
 g = 9.81                                                                 #
 return v0*t - 0.5*g*t**2                                                 #
###########################################################################
```

⇝ `yfunc(0.1, v0=6)` is correct

⇝ `yfunc(t=0.1, 6)` is illegal

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Multiple arguments (cont.)

Consider the case in which `yfunc(0.1, 6)` or `yfunc(v0=6, t=0.1)` is used

The `arguments` are automatically initialised as `local variables`

- The 'exist' within the `function`

Initialisation is the same as assigning values to `variables`

```
t = 0.1
v0 = 6.

###########################################################################
def yfunc(t, v0):                                                         #
 g = 9.81                                                                 #
 return v0*t - 0.5*g*t**2                                                 #
###########################################################################
```

Such `statements` are not visible in the code

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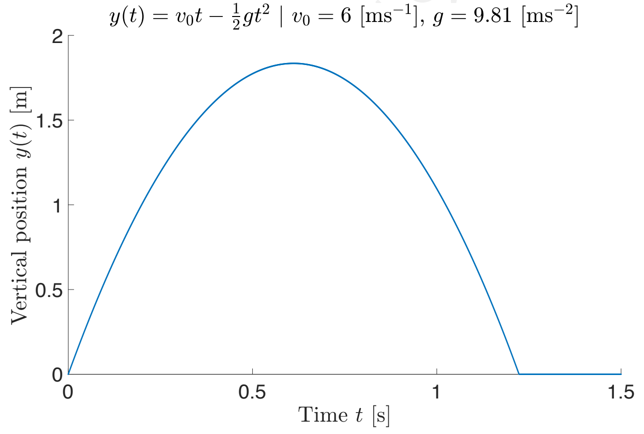
Multiple arguments (cont.)



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Function argument v global variable

Functions

Function argument v global variable

$$y(t) = v_0 t - \frac{1}{2} g t^2$$

Mathematically, function y is understood as a function of one variable, t

A Python implementation as `function yfunc` should reflect this fact

- `yfunc` should be a `function` of `t` only

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Function argument v global variable

Example

Consider the following construction

```
########################################################################
def yfunc(t):                                                          #
 g = 9.81                                                              #
 return v0*t - 0.5*g*t**2                                              #
########################################################################
```

Variable `v0` is interpreted as a `global variable`

It needs be initialised outside `function yfunc`

- Before we attempt to call `yfunc`

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Function argument v global variable (cont.)

```
########################################################################
def yfunc(t):                                                          #
 g = 9.81                                                              #
 return v0*t - 0.5*g*t**2                                              #
########################################################################
```

Failing to initialise a `global variable` leads to an error message

```
>>> yfunc(0.6)
    ...
    NameError: global name 'v0' is not defined
```

We need to define `v0` as a `global variable` prior to calling `yfunc`

```
>>> v0 = 5.
>>> yfunc(0.6)
    1.2342
```

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Beyond math functions

Functions

Beyond math functions

So far, Python `functions` have typically computed some mathematical expression, but their usefulness goes beyond mathematical functions

- Any set of statements to be repeatedly executed under slightly different circumstances is a candidate for a Python `function`

Example

We want to make a list of numbers

Starting from some value (`start`) and stop at some other value (`stop`)

- We have given increments (`inc`)

Consider using variables `start=2`, `stop=8`, and `inc=2`

- This would produce numbers 2, 4, 6, and 8

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Beyond math functions (cont.)

```
##########################################################################
def makelist(start, stop, inc):                                          #
 value = start                                                           #
 result = []                                                             #
                                                                         #
 while value <= stop:                                                    #
  result.append(value)                                                   #
  value = value + inc                                                    #
                                                                         #
 return result                                                           #
##########################################################################
```

```
>>> mylist = makelist(0, 100, 0.2)

>>> print mylist                        #      It will print the sequence
                                        # 0, 0.2, 0.4, 0.6, ... 99.8, 100
```

- Function `makelist` has three arguments: `start`, `stop`, and `inc`
- Inside the `function`, the `arguments` become `local variables`
- Also `value` and `result` are `local variables`

In the surrounding program (`main`), we define one variable, `mylist`

- Variable `mylist` is a `global variable`

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Beyond math functions (cont.)

`range(start, stop, inc)` does not make the `makelist` function redundant

⇝ `makelist` can generate real numbers

⇝ `range` can only generate integers

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Multiple returns

Functions

Multiple returns

Example

Suppose that we are interested in a function $y(t)$ and its derivative $y'(t)$

$$y(t) = v_0 t - \frac{1}{2}gt^2$$
$$y'(t) = v_0 - gt$$

Suppose that we want to get both $y(t)$ and $y'(t)$ from function `yfunc`

```
###########################################################################
def yfunc(t, v0):                                                         #
 g = 9.81                                                                 #
                                                                          #
 y = v0*t - 0.5*g*t**2                                                    #
 dydt = v0 - g*t                                                          #
                                                                          #
 return y, dydt                                                           #
###########################################################################
```

We included both calculations, then we separated variables in the `return`

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Multiple returns (cont.)

```
###########################################################################
def yfunc(t, v0):                                                         #
 g = 9.81                                                                 #
 y = v0*t - 0.5*g*t**2                                                    #
 dydt = v0 - g*t                                                          #
 return y, dydt                                                           #
###########################################################################
```

In the main, `yfunc` needs two names on LHS of the assignment operator

$\rightsquigarrow$ (Intuitively, as the function now returns two values)

```
>>> position, velocity = yfunc(0.6, 3)
```

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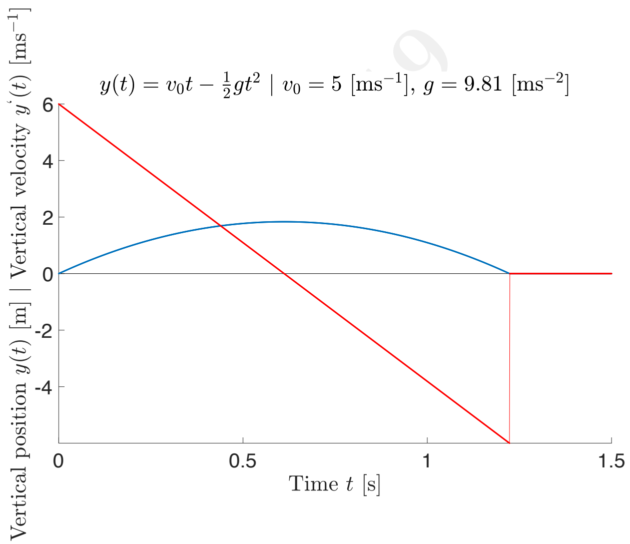
Multiple returns (cont.)



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Multiple returns (cont.)

We can use the function `yfunc` in the production of a formatted table

- Values of t, $y(t)$ and $y'(t)$

```
###########################################################################
def yfunc(t, v0):                                                         #
 g = 9.81                                                                 #
 y = v0*t - 0.5*g*t**2                                                    #
 dydt = v0 - g*t                                                          #
 return y, dydt                                                           #
###########################################################################

t_values = [0.05*i for i in range(10)]

for t in t_values:
 position, velocity = yfunc(t, v0=5)
 print 't=%-10g position=%-10g velocity=%-10g' % \
        (t, position, velocity)
```

Format `%-10g` prints a real number as compactly as possible

- Whether in decimal or scientific notation)
- Within a field of width 10 characters

The minus sign (`-`) after the percentage sign (`%`)

- Prints a number that is left-adjusted

(Important for creating nice-looking columns)

Multiple returns (cont.)

```
t=0          position=0          velocity=5
t=0.05       position=0.237737   velocity=4.5095
t=0.1        position=0.45095    velocity=4.019
t=0.15       position=0.639638   velocity=3.5285
t=0.2        position=0.8038     velocity=3.038
t=0.25       position=0.943437   velocity=2.5475
t=0.3        position=1.05855    velocity=2.057
t=0.35       position=1.14914    velocity=1.5665
t=0.4        position=1.2152     velocity=1.076
t=0.45       position=1.25674    velocity=0.5855
```


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Multiple returns (cont.)

Remark

`Functions` returning multiple (comma-separated) values returns a `tuple`

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Multiple returns (cont.)

Example

Consider the following function

```
########################################################################
def f(x):                                                              #
 return x, x**2, x**4                                                  #
########################################################################
```

Three objects are returned as output arguments

```
>>> s = f(2)
>>> s                                         #             Stored as a tuple
    (2, 4, 16)

>>> type(s)
    <type 'tuple'>

>>> x, x2, x4 = f(2)                          #  Stored as separate variables
```


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Summation

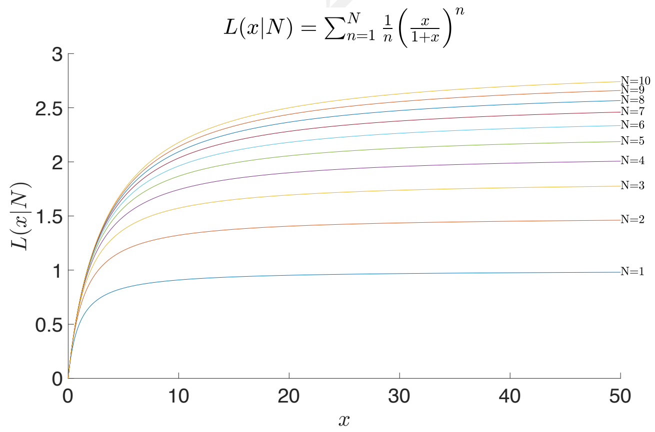
Functions

Summation

Example

Suppose we are interested in creating a function to calculate the sum

$$L(x;n) = \sum_{i=1}^{n} \frac{1}{i}\Big(\frac{x}{1+x}\Big)^{i}$$



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Summation

To compute the sum, a loop and add terms to an accumulation variable

- We performed a similar task with a while loop

Summations with integer counters (like i) are normally (often) implemented by a for-loop over the i counter (we performed also this task)

$$\sum_{i=1}^{n} i^2$$

```
s = 0
for i in range(1, n+1):
 s += i**2
```

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Summation (cont.)

$$L(x;n) = \sum_{i=1}^{n} \frac{1}{i}\Big(\frac{x}{1+x}\Big)^{i}$$

```
s = 0
for i in range(1, n+1):
 s += (1.0/i)*(x/(1.0+x))**i
```

Observe the terms 1.0 used to avoid integer division

$\rightsquigarrow$ i is an int object and x may also be an int

We want to embed the computation of the sum in a Python function

$\rightsquigarrow$ x and n are the input arguments

$\rightsquigarrow$ The sum s is the return output

```
#######################################################################
def L(x, n):                                                          #
 s = 0                                                                #
 for i in range(1, n+1):                                              #
  s += (1.0/i)*(x/(1.0+x))**i                                         #
 return s                                                             #
#######################################################################
```

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Summation (cont.)

It can be shown that $L(x;n)$ is an approximation to $\ln(1+x)$

- For a finite n and for $x \geq 1$

The approximation becomes exact in the limit

$$\rightsquigarrow \lim_{n\to\infty} L(x;n) = \ln(1+x)$$

Instead of having L return only the value of the sum s, it would be also interesting to return additional information on the approximation error

Summation (cont.)

$$L(x;n) = \sum_{i=1}^{n} \frac{1}{i}\Big(\frac{x}{1+x}\Big)^{i}$$

The size of the terms decreases with `n`

- ⤳ The first neglected term (`n+1`) is bigger than all remaining terms
- ⤳ (those calculated for `n+2,n+3,...`)

Yet, it is not necessarily bigger than their sum

The first neglected term is hence an indication of the size of the total error

- ⤳ We may use this term as a crude estimate of the error

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Summation (cont.)

We return the exact error (we calculate the `log` function by `math.log`)

Example

```
########################################################################
def L2(x, n):                                                          #
 s = 0                                                                 #
 for i in range(1, n+1):                                               #
  s += (1.0/i)*(x/(1.0+x))**i                                          #
 value_of_sum = s                                                      #
                                                                       #
 first_neglected_term = (1.0/(n+1))*(x/(1.0+x))**(n+1)                 #
                                                                       #
 from math import log                                                  #
 exact_error = log(1+x) - value_of_sum                                 #
                                                                       #
 return value_of_sum, first_neglected_term, eactual_error              #
########################################################################

value, approximate_error, exact_error = L2(x, 100)
```

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No returns

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No returns

Sometimes a function can be defined to performs a set of statements

- Without necessarily computing objects returned to calling code

In such situations, the return statement is not needed

- The function without return values

No returns (cont.)

Example

Consider the construction of a table of the accuracy of function $L(x; n)$

$\rightsquigarrow$ It is an approximation to $\ln(1 + x)$

```
################################################################################
def L2(x, n):                                                                  #
 s = 0                                                                         #
 for i in range(1, n+1):                                                       #
  s += (1.0/i)*(x/(1.0+x))**i                                                  #
 value_of_sum = s                                                              #
                                                                               #
 first_neglected_term = (1.0/(n+1))*(x/(1.0+x))**(n+1)                         #
                                                                               #
 from math import log                                                          #
 exact_error = log(1+x) - value_of_sum                                         #
                                                                               #
 return value_of_sum, first_neglected_term, eactual_error                      #
################################################################################

################################################################################
def table(x):                                                                  #
 print '\nx=%g, ln(1+x)=%g' % (x, log(1+x))                                    #
                                                                               #
 for n in [1, 2, 10, 100, 500]:                                                #
  value, next, error = L2(x, n)                                                #
  print 'n=%-4d %-10g (next term: %8.2e '\                                     #
        'error: %8.2e)' % (n, value, next, error)                              #
################################################################################
```

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No returns (cont.)

```
################################################################################
def L2(x, n):                                                                  #
 s = 0                                                                         #
 for i in range(1, n+1):                                                       #
  s += (1.0/i)*(x/(1.0+x))**i                                                  #
 value_of_sum = s                                                              #
                                                                               #
 first_neglected_term = (1.0/(n+1))*(x/(1.0+x))**(n+1)                         #
                                                                               #
 from math import log                                                          #
 exact_error = log(1+x) - value_of_sum                                         #
                                                                               #
 return value_of_sum, first_neglected_term, eactual_error                      #
################################################################################

################################################################################
def table(x):                                                                  #
 print '\nx=%g, ln(1+x)=%g' % (x, log(1+x))                                    #
                                                                               #
 for n in [1, 2, 10, 100, 500]:                                                #
  value, next, error = L2(x, n)                                                #
  print 'n=%-4d %-10g (next term: %8.2e '\                                     #
        'error: %8.2e)' % (n, value, next, error)                              #
################################################################################
```

```
>>> table(10)
 x=10, ln(1+x)=2.3979
 n=1    0.909091  (next term: 4.13e-01 error: 1.49e+00)
 n=2    1.32231   (next term: 2.50e-01 error: 1.08e+00)
 n=10   2.17907   (next term: 3.19e-02 error: 2.19e-01)
 n=100  2.39789   (next term: 6.53e-07 error: 6.59e-06)
 n=500  2.3979    (next term: 3.65e-24 error: 6.22e-15)
```

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No returns (cont.)

```
################################################################################
def L2(x, n):                                                                  #
 s = 0                                                                         #
 for i in range(1, n+1):                                                       #
  s += (1.0/i)*(x/(1.0+x))**i                                                  #
 value_of_sum = s                                                              #
                                                                               #
 first_neglected_term = (1.0/(n+1))*(x/(1.0+x))**(n+1)                         #
                                                                               #
 from math import log                                                          #
 exact_error = log(1+x) - value_of_sum                                         #
                                                                               #
 return value_of_sum, first_neglected_term, eactual_error                      #
################################################################################

################################################################################
def table(x):                                                                  #
 print '\nx=%g, ln(1+x)=%g' % (x, log(1+x))                                    #
                                                                               #
 for n in [1, 2, 10, 100, 500]:                                                #
  value, next, error = L2(x, n)                                                #
  print 'n=%-4d %-10g (next term: %8.2e '\                                     #
        'error: %8.2e)' % (n, value, next, error)                              #
################################################################################
```

```
>>> table(1000)
 x=1000, ln(1+x)=6.90875
 n=1      0.999001 (next term: 4.99e-01 error: 5.91e+00)
 n=2      1.498    (next term: 3.32e-01 error: 5.41e+00)
 n=10     2.919    (next term: 8.99e-02 error: 3.99e+00)
 n=100    5.08989  (next term: 8.95e-03 error: 1.82e+00)
 n=500    6.34928  (next term: 1.21e-03 error: 5.59e-01)
```

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No returns (cont.)

```
>>> table(10)
 x=10, ln(1+x)=2.3979
 n=1    0.909091  (next term: 4.13e-01 error: 1.49e+00)
 n=2    1.32231   (next term: 2.50e-01 error: 1.08e+00)
 n=10   2.17907   (next term: 3.19e-02 error: 2.19e-01)
 n=100  2.39789   (next term: 6.53e-07 error: 6.59e-06)
 n=500  2.3979    (next term: 3.65e-24 error: 6.22e-15)

>>> table(1000)
 x=1000, ln(1+x)=6.90875
 n=1      0.999001 (next term: 4.99e-01 error: 5.91e+00)
 n=2      1.498    (next term: 3.32e-01 error: 5.41e+00)
 n=10     2.919    (next term: 8.99e-02 error: 3.99e+00)
 n=100    5.08989  (next term: 8.95e-03 error: 1.82e+00)
 n=500    6.34928  (next term: 1.21e-03 error: 5.59e-01)
```

- Error is an order of magnitude larger than the first neglected term
- Convergence is slower for larger values of x than smaller x

■

No returns (cont.)

Remark

For `functions` w/o `return statement`, Python inserts an invisible one

- The invisible return is named `None`
- `None` is a special object in Python

`None` represents something we may think of as the 'nothingness'

Normally, one would call `function table` w/o assigning `return value`

Yet, imagine we still assign the `return value` to a variable

- ⇝ The result will refer to a `None object`
- ⇝ `result = table(500)`

The `None` value is often used for variables that should exist in a program

- But, where it is natural to think of the value as conceptually undefined

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No returns (cont.)

The standard way to test if an object `obj` is set to `None` or not reads

```
if obj is None:
 ...

if obj is not None:
 ...
```

- ⇝ The `is` operator tests if two names refer to the same object
- ⇝ The `==` tests checks if the contents of two objects are the same

```
>>> a = 1
>>> b = a

>>> a is b                          # a and b refer to the same object
    True

>>> c = 1.0                  # a and c do not refer to the same object

>>> a is c
    False

>>> a == c                          # a and c are mathematically equal
    True
```

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Keyword arguments
Functions

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Keyword arguments

The `input arguments` of a `function` can be assigned a default value

- ⇝ These arguments can be left out in the call

This is how a such a `function` may be defined

```
######################################################################
def somefunc(arg1, arg2, kwarg1=True, kwarg2=0):                     #
 print arg1, arg2, kwarg1, kwarg2                                    #
######################################################################
```

```
######################################################################
def somefunc(arg1, arg2, kwarg1=True, kwarg2=0):                     #
 print arg1, arg2, kwarg1, kwarg2                                    #
######################################################################
```

First args (here, `arg1` and `arg2`) are `ordinary`/`positional arguments`

Last two args (`kwarg1` and `kwarg2`) are `keyword`/`named arguments`

Each `keyword argument` has a name and an associated default value

Keyword arguments (cont.)

Example

```
#######################################################################
def somefunc(arg1, arg2, kwarg1=True, kwarg2=0):                      #
 print arg1, arg2, kwarg1, kwarg2                                     #
#######################################################################
```

```
>>> somefunc('Hello', [1,2])
    Hello [1, 2] True 0

>>> somefunc('Hello', [1,2], kwarg1='Hi')
    Hello [1, 2] Hi 0

>>> somefunc('Hello', [1,2], kwarg2='Hi')
    Hello [1, 2] True Hi

>>> somefunc('Hello', [1,2], kwarg2='Hi', kwarg1=6)
    Hello [1, 2] 6 Hi
```

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Keyword arguments (cont.)

Remark

`Keyword arguments` must be listed AFTER `positional arguments`

Suppose that ALL `input arguments` are explicitly referred to (`name=value`)

The sequence is not relevant, `positional` and `keyword` can be mixed up

```
>>> somefunc(kwarg2='Hello', arg1='Hi', kwarg1=6, arg2=[1,2])
    Hi [1, 2] 6 Hello
```

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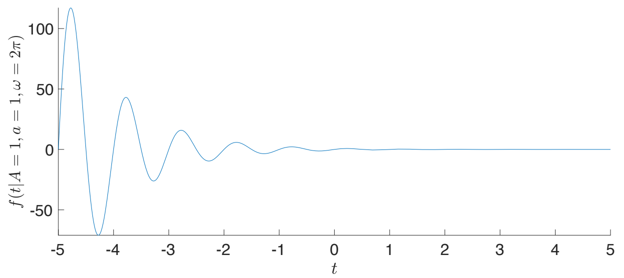
Keyword arguments (cont.)

Example

Consider some function of t also containing some parameters A, a and ω

$$f(t; A, a, \omega) = Ae^{-at}\sin(\omega t)$$

We have,



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Keyword arguments (cont.)

$$f(t; A, a, \omega) = Ae^{-at}\sin(\omega t)$$

We implement f as function of independent variable t, `ordinary argument`

We set parameters A, a, and ω as `keyword arguments` with default values

```
from math import pi, exp, sin

#######################################################################
def f(t, A=1, a=1, omega=2*pi):                                       #
 return A*exp(-a*t)*sin(omega*t)                                      #
#######################################################################
```

Keyword arguments (cont.)

```
##########################################################################
def f(t, A=1, a=1, omega=2*pi):                                          #
 return A*exp(-a*t)*sin(omega*t)                                         #
##########################################################################
```

We can call function f with only argument t specified

```
>>> v1 = f(0.2)
```

Some of the other possible function calls

```
>>> v2 = f(0.2, omega=1)
>>> v3 = f(1, A=5, omega=pi, a=pi**2)
>>> v4 = f(A=5, a=2, t=0.01, omega=0.1)
>>> v5 = f(0.2, 0.5, 1, 1)
```

■

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Keyword arguments (cont.)

Example

Consider $L(x;n)$ and functional implementations `L(x,n)` and `L2(x,n)`

$$L(x;n) = \sum_{i=1}^{n} \frac{1}{i}\Big(\frac{x}{1+x}\Big)^i, \text{ with } \lim_{n\rightarrow\infty} L(x;n) = \ln(1+x), \text{for } x \geq 1$$

We can now specify a minimum tolerance value ε for the accuracy

$\leadsto$ (Instead of specifying the number n of terms in the sum)

We can use the first neglected term as an estimate of the accuracy

- Add terms as long as the absolute value of next term is greater than ε

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Keyword arguments (cont.)

$$L(x;n) = \sum_{i=1}^{n} \frac{1}{i}\Big(\frac{x}{1+x}\Big)^i$$

It is natural to provide a default value for ε

```
##########################################################################
def L3(x, epsilon=1.0E-6):                                               #
 x = float(x)                                                            #
 i = 1                                                                   #
 term = (1.0/i)*(x/(1+x))**i                                             #
 s = term                                                                #
                                                                         #
 while abs(term) > epsilon:                                              #
  i += 1                                                                 #
  term = (1.0/i)*(x/(1+x))**i                                            #
  s += term                                                              #
                                                                         #
 return s, i                                                             #
##########################################################################
```

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Keyword arguments (cont.)

We make a table of the approximation error as ε decreases

```
##########################################################################
def L3(x, epsilon=1.0E-6):                                               #
 x = float(x)                                                            #
 i = 1                                                                   #
 term = (1.0/i)*(x/(1+x))**i                                             #
 s = term                                                                #
                                                                         #
 while abs(term) > epsilon:                                              #
  i += 1                                                                 #
  term = (1.0/i)*(x/(1+x))**i                                            #
  s += term                                                              #
                                                                         #
 return s, i                                                             #
##########################################################################

##########################################################################
def table2(x):                                                           #
 from math import log                                                    #
                                                                         #
 for k in range(4, 14, 2):                                               #
  epsilon = 10**(-k)                                                     #
  approx, n = L3(x, epsilon=epsilon)                                     #
  exact = log(1+x)                                                       #
  exact_error = exact - approx                                           #
##########################################################################
```

Keyword arguments (cont.)

The output from calling `table2(10)`

```
>>> table2(10)
epsilon: 1e-04, exact error: 8.18e-04, n=55
epsilon: 1e-06, exact error: 9.02e-06, n=97
epsilon: 1e-08, exact error: 8.70e-08, n=142
epsilon: 1e-10, exact error: 9.20e-10, n=187
epsilon: 1e-12, exact error: 9.31e-12, n=233
```

The `epsilon` estimate is about ten times smaller than the exact error

- regardless of the size of `epsilon`

`epsilon` follows the exact error over many orders of magnitude

We may view `epsilon` as a valid indication of error size

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Doc strings

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Doc strings

There is a convention to augment functions with some documentation

- The documentation string, known as a doc string
- A short description of the purpose of the function
- It explains what arguments and return values are
- Placed after the `def funcname:` line of definition

`Doc strings` are usually enclosed in triple double quotes `"""`

- This allows the string to span several lines

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Doc strings (cont.)

Example

Consider the following Python function with plain documentation

```
#######################################################################
def C2F(C):                                                           #
 """                                                                  #
 Convert Celsius degrees (C) to Fahrenheit.                           #
                                                                      #
 C: Input argument, temperature in Celsius                            #
 return: Temperature in Fahrenheit                                    #
 """                                                                  #
                                                                      #
 return (9.0/5)*C + 32                                                #
#######################################################################
```

Doc strings (cont.)

Example

Consider the following Python function with documentation and arguments

```
############################################################################
def line(x0, y0, x1, y1):                                                  #
 """                                                                       #
 Compute the coefficients a and b in the mathematical expression for      #
 a straight line y = a*x + b that goes through two points (x0, y0)        #
 and (x1, y1).                                                             #
                                                                           #
 x0, y0: a point on the line (floats).                                     #
 x1, y1: another point on the line (floats).                               #
 return: coefficients a, b (floats) for the line (y=a*x+b).                #
 """                                                                       #
                                                                           #
 a = (y1 - y0)/float(x1 - x0)                                              #
 b = y0 - a*x0                                                             #
 return a, b                                                               #
############################################################################
```

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Doc strings (cont.)

To extract doc strings from source code use `funcname.__doc__`

```
>>> print line.__doc__
```

```
Compute the coefficients a and b in the mathematical expression for
a straight line y = a*x + b that goes through two points (x0, y0)
and (x1, y1).

x0, y0: a point on the line (float objects).
x1, y1: another point on the line (float objects).
return: coefficients a, b (floats) for the line (y=a*x+b).
```

If function `line` is in a file `funcs.py`, we can run `pydoc funcs.line`

- Shows the documentation of function `line`
- Function signature and doc string

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Doc strings (cont.)

Doc strings often contain interactive sessions, from the Python shell

- They are used to illustrate how the function can be used

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Doc strings (cont.)

Example

Consider the following Python function with complete documentation

```
############################################################################
def line(x0, y0, x1, y1):                                                  #
 """                                                                       #
 Compute the coefficients a and b in the mathematical expression for      #
 a straight line y = a*x + b that goes through two points (x0, y0)        #
 and (x1, y1).                                                             #
                                                                           #
 x0, y0: a point on the line (floats).                                     #
 x1, y1: another point on the line (floats).                               #
 return: coefficients a, b (floats) for the line (y=a*x+b).                #
                                                                           #
 Example:                                                                  #
 >>> a, b = line(1, -1, 4, 3)                                              #
 >>> a                                                                     #
     1.3333333333333333                                                    #
 >>> b                                                                     #
     -2.333333333333333                                                    #
 """                                                                       #
                                                                           #
 a = (y1 - y0)/float(x1 - x0)                                              #
 b = y0 - a*x0                                                             #
 return a, b                                                               #
############################################################################
```

Functions (cont.)

The usual convention in Python

- Function arguments represent input data to the function
- Returned objects represent output data from function

Definition

The general structure of a Python function

```
def somefunc(i1, i2, i3, io4, io5, i6=value1, io7=value2):

    # modify io4, io5, io6
    # compute o1, o2, o3

    return o1, o2, o3, io4, io5, io7
```

⇝ *i1, i2, i3 are positional arguments, input data*

⇝ *io4 and io5 are positional arguments, input and output data*

⇝ *i6 and io7 are keyword arguments, input and input/output data*

⇝ *o1, o2, and o3 are computed in the function, output data*

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Functions as arguments to functions

Functions

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Functions as arguments to functions

We can have functions to be used as arguments to other functions

A math function $f(x)$ may be needed for specific Python functions

Numerical root finding

- Solve $f(x) = 0$, approximately

Numerical differentiation

- Compute $f'(x)$, approximately

Numerical integration

- Compute $\int_a^b f(x)\mathrm{d}x$, approximately

Numerical solution of differential equations

- Compute $x(t)$ from $\frac{\mathrm{d}x}{\mathrm{d}t} = f(x)$, approximately

In such functions, function $f(x)$ can be used as input argument (`f`)

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Functions as arguments to functions (cont.)

This is straightforward in Python and hardly needs any explanation

- In most other languages, special constructions must be used
- Transfer a function to another function as argument

Functions as arguments to functions (cont.)

Example

Compute the 2nd-order derivative of some function $f(x)$, numerically

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

h is a small number

A Python `function` that implements this expression

```
##################################################################
def diff2nd(f, x, h=1E-6):                                       #
 r = (f(x-h) - 2*f(x) + f(x+h))/float(h*h)                       #
 return r                                                        #
##################################################################
```

`f` is a name for a `function object`

- It is treated as the other `input arguments`

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Functions as arguments to functions (cont.)

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

```
##################################################################
def g(t):                                           # g(t) = t^(-6) #
 return t**(-6)                                                  #
##################################################################

##################################################################
def diff2nd(f, x, h=1E-6):                                       #
 r = (f(x-h) - 2*f(x) + f(x+h))/float(h*h)                       #
 return r                                                        #
##################################################################

t = 1.2
d2g = diff2nd(g, t)

print "g''(%f)=%f" % (t, d2g)
```

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Functions as arguments to functions (cont.)

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

Asymptotically, the approximation of the derivative get more accurate

- For $h \rightarrow 0$

We show this property by making a table of the second-order derivatives

- $g(t) = t^{-6}$, for $t = 1$ and $h \rightarrow 0$

```
for k in range(1,15):
 h = 10**(-k)
 d2g = diff2nd(g, 1, h)
 print 'h=%.0e: %.5f' % (h, d2g)
```

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Functions as arguments to functions (cont.)

The exact answer is $g''(t = 1) = 42$

```
h=1e-01: 44.61504
h=1e-02: 42.02521
h=1e-03: 42.00025
h=1e-04: 42.00000
h=1e-05: 41.99999
h=1e-06: 42.00074
h=1e-07: 41.94423
h=1e-08: 47.73959
h=1e-09: -666.13381
h=1e-10: 0.00000
h=1e-11: 0.00000
h=1e-12: -666133814.77509
h=1e-13: 66613381477.50939
h=1e-14: 0.00000
```

Computations start returning very inaccurate results for $h < 10^{-8}$

- For small h rounding errors blow up and destroy accuracy
- It is necessary to switch from standard floating-point numbers (`float`) to numbers with arbitrary high precision (`module decimal`)

■

The main program
Functions

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The main program

In programs with functions, a part of the program is the main program

- It is the collection of all statements outside the functions
- Plus, the definition of all functions

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The main program

Example

```
from math import *                                                  # In main

#########################################################################
def f(x):                                                 # A function, in main #
 e = exp(-0.1*x)                                                         #
 s = sin(6*pi*x)                                                         #
 return e*s                                                              #
#########################################################################

x = 2                                                               # In main
y = f(x)                                                            # In main
print 'f(%g)=%g' % (x, y)                                           # In main
```

Execution always starts with the first line in the main

When a function is encountered, its statements are used to define it

- Nothing is computed inside a function before it is called

Variables initialised in the main program become global variables

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The main program (cont.)

```
from math import *                                                  # In main

#########################################################################
def f(x):                                                 # A function, in main #
 e = exp(-0.1*x)                                                         #
 s = sin(6*pi*x)                                                         #
 return e*s                                                              #
#########################################################################

x = 2                                                               # In main
y = f(x)                                                            # In main
print 'f(%g)=%g' % (x, y)                                           # In main
```

1. Import functions from the math module
2. Define function f(x)
3. Define x
4. Call f and execute the function body
5. Define y as the value returned from f
6. Print a string

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Lambda functions

A convenient one-line construction of `functions` used to make code compact

```
f = lambda x: x**2 + 4
```

This so-called lambda function is equivalent to the usual form

```
##########################################################################
def f(x):                                                                #
 return x**2 + 4                                                         #
##########################################################################
```

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Lambda functions

Definition

In general, we have the following

```
##########################################################################
def g(arg1, arg2, arg3, ...):                                            #
 return expression                                                       #
##########################################################################
```

This can be re-written

```
g = lambda arg1, arg2, arg3, ...: expression
```

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Lambda functions (cont.)

`Lambda functions` are used as `function argument` to `functions`

Example

```
##########################################################################
def g(t):                                                # g(t) = t^(-6) #
 return t**(-6)                                                          #
##########################################################################

##########################################################################
def diff2nd(f, x, h=1E-6):                                               #
 r = (f(x-h) - 2*f(x) + f(x+h))/float(h*h)                               #
 return r                                                                #
##########################################################################

t = 1.2
d2g = diff2nd(g, t)

print "g''(%f)=%f" % (t, d2g)
```

Consider the `diff2nd` `function` used to differentiate $g(t) = t^{-6}$ twice

- We first make a `g(t)` then pass `g` as `input argument` to `diff2nd`

Lambda functions (cont.)

```
#########################################################################
def diff2nd(f, x, h=1E-6):                                              #
 r = (f(x-h) - 2*f(x) + f(x+h))/float(h*h)                              #
 return r                                                               #
#########################################################################
```

We skip the step of defining `g(t)` and use a `lambda function` instead

```
>>> d2 = diff2nd(lambda t: t**(-6), 1, h=1E-4)
```

- A `lambda function f` as `input argument` into `diff2nd`

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Lambda functions (cont.)

Remark

`Lambda functions` can also take `keyword arguments`

```
d2 = diff2nd(lambda t, A=1, a=0.5: -a*2*t*A*exp(-a*t**2), 1.2)
```

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Branching

Functions and branching

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Branching

The flow of computer programs often needs to branch

- If a condition is met, we do one thing
- If it is not met, we do some other thing

Branching

Example

Consider the multi-case function

$$f(x) = \begin{cases} \sin(x), & 0 \leq x \leq \pi \\ 0, & \text{elsewhere} \end{cases}$$

Implementing this function requires a test on the value of x

Consider the following implementation

```
##########################################################################
def f(x):                                                                #
                                                                         #
 if 0 <= x <= pi:                                                        #
  value = sin(x)                                                         #
 else:                                                                   #
  value = 0                                                              #
                                                                         #
 return value                                                            #
##########################################################################
```

■

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IF-ELSE blocks

Definition

The general structure of the ***IF-ELSE test***

```
if condition:
 <block of statements,
  executed if condition is True>

else:
 <block of statements,
  executed if condition is False>
```

- *If `condition` is `True`, the program flow goes into the first block of statements, indented after the `if:` line*
- *If `condition` is `False`, program flow goes into the second block of statements, indented after the `else:` line*

The blocks of statements are indented, and note the two two-points

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IF-ELSE blocks (cont.)

Example

Consider the following code

```
if C < -273.15:
 print '%g degrees Celsius is non-physical!' % C
 print 'The Fahrenheit temperature will not be computed.'

else:
 F = 9.0/5*C + 32
 print F

print 'end of program'
```

We have,

- The two `print statements` in the **IF-block** are executed if and only if condition `C < -273.15` evaluates as `True`
- Otherwise, execution skips the `print statements` and carries out with the computation of the statements in the **ELSE-block** and prints `F`

IF-ELSE blocks (cont.)

```
if C < -273.15:
 print '%g degrees Celsius is non-physical!' % C
 print 'The Fahrenheit temperature will not be computed.'

else:
 F = 9.0/5*C + 32
 print F

print 'end of program'
```

The `end of program` bit is printed regardless of the condition check outcome

⇝ This statement is not indented

It is neither part of the `IF-block` nor of the `ELSE-block`

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IF-ELSE blocks (cont.)

Definition

The `else` part of the `IF-ELSE test` can be skipped

```
if condition:
 <block of statements>
<next statement>
```

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IF-ELSE blocks (cont.)

Example

Consider the following code

```
if C < -273.15:
 print '%s degrees Celsius is non-physical!' % C

F = 9.0/5*C + 32
```

The computation of `F` will always be carried out

- The statement is not indented
- It is not part of the `IF-block`

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IF-ELSE blocks (cont.)

Definition

With `elif` (for else if) several mutually exclusive `IF-test` are performed

```
if condition1:
 <block of statements>

elif condition2:
 <block of statements>

elif condition3:
 <block of statements>

else:
 <block of statements>
<next statement>
```

This construct allows for multiple branching of the program flow

IF-ELSE blocks (cont.)

Example

Let us consider the so-called HAT function

$$N(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x \leq 2 \\ 0, & x \geq 2 \end{cases}$$

Write a Python `function` that implements it

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IF-ELSE blocks (cont.)

$$N(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x \leq 2 \\ 0, & x \geq 2 \end{cases}$$

Consider the following implementation

```
##########################################################################
def N(x):                                                                #
                                                                         #
 if x < 0:                                                               #
  return 0.0                                                             #
                                                                         #
 elif 0 <= x < 1:                                                        #
  return x                                                               #
                                                                         #
 elif 1 <= x < 2:                                                        #
  return 2 - x                                                           #
                                                                         #
 elif x >= 2:                                                            #
  return 0.0                                                             #
##########################################################################
```

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IF-ELSE blocks (cont.)

$$N(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x \leq 2 \\ 0, & x \geq 2 \end{cases}$$

Consider an alternative implementation

```
##########################################################################
def N(x):                                                                #
                                                                         #
 if 0 <= x < 1:                                                          #
  return x                                                               #
                                                                         #
 elif 1 <= x < 2:                                                        #
  return 2 - x                                                           #
                                                                         #
 else:                                                                   #
  return 0.0                                                             #
##########################################################################
```

■

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Inline IF-tests

Branching

Inline IF-test

Variables are often assigned a value based on some boolean expression

Consider the following code using a common `IF-ELSE test`

```
if condition:
 a = value1
else:
 a = value2
```

The equivalent one-line syntax (**inline IF-test**)

```
a = (value1 if condition else value2)
```

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Inline IF-test (cont.)

Example

Consider the following multiple-case mathematical function

$$f(x) = \begin{cases} \sin(x), & 0 \leq x \leq \pi \\ 0, & \text{elsewhere} \end{cases}$$

We are interested in the corresponding Python function

We have,

```
########################################################################
def f(x):                                                              #
 return (sin(x) if 0 <= x <= 2*pi else 0)                              #
########################################################################
```

Alternatively, we have

```
f = lambda x: sin(x) if 0 <= x <= 2*pi else 0
```

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Inline IF-test (cont.)

Remark

The `IF-ELSE test` cannot be used inside a `lambda function`

Notice that the test has more than one single expression

- `Lambda functions` cannot have statements
- Only a single expression is accepted