

Exercise 1. Write Python functions that receive x as input argument and computes and returns the following function values:

1.

$$f(x) = \frac{\sin \left[e^{x^2} - \cos(x) + 2 \sin(x^2) \sqrt{1 + 2 \sin^2(x)} \right]}{2 \sin^2(x)};$$

2.

$$g(x) = \frac{\sqrt{e^{x^2} - \cos(x) + 2 \sin(x^3) \sqrt{1 + 2 \sin(x^3)}}}{2 \sin^3 x};$$

3.

$$h(x) = \frac{x^{\arctan(x)} - x^{(\pi/2)}}{(1+x)^{1+\pi/2} \sqrt{\log(x)}}.$$

Test your functions when $x \in \mathcal{X}$, for some \mathcal{X} of your choice such that $|\mathcal{X}| > 1$.

Exercise 2. Given the functions in Exercise 1, $f(x)$, $g(x)$ and $h(x)$, write Python functions that receive x and N , (x, N) , as input arguments and computes and returns the corresponding function values, when the following approximations are used:

•

$$e^x \approx \sum_{n=0}^N \frac{x^n}{n!};$$

•

$$\sin(x) \approx \sum_{n=0}^N \left[\frac{(-1)^n x^{(2n+1)}}{(2n+1)!} \right];$$

•

$$\cos(x) \approx \prod_{n=1}^N \left[1 - \frac{x^2}{\pi^2(n-1/2)^2} \right];$$

•

$$\arctan(x) \approx \sum_{n=0}^N \left[\frac{(-1)^n}{2n+1} x^{(2n+1)} \right].$$

Test your functions when $x \in \mathcal{X}$, for the same \mathcal{X} of your choice such that $|\mathcal{X}| > 1$ in Exercise 1, and for $N \in \{2, 4, 8, 16\}$.

Compare the function values with what obtained in Exercise 1.