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Online optimal estimation and control for a common class of activated sludge plants

Otacílio B. L. Neto[1], **Michela Mulas**[2], **Francesco Corona**[1]

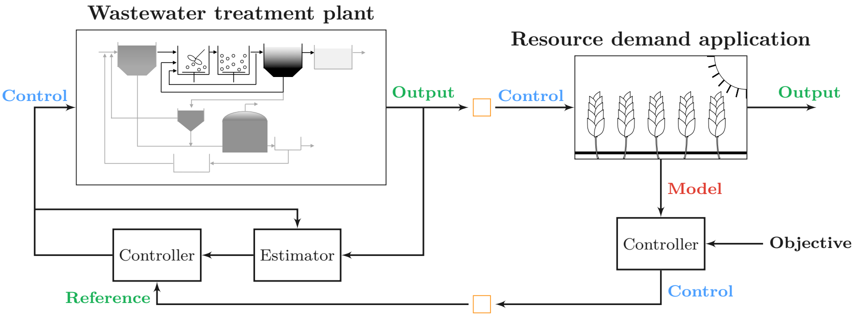
[1] School of Chemical Engineering, Department of Chemical and Metallurgical Engineering, Aalto University, Finland

[2] Department of Teleinformatics Engineering, Federal University of Ceará, Fortaleza-CE, Brazil

Introduction

Intro, wastewater treatment plants (WWTP)

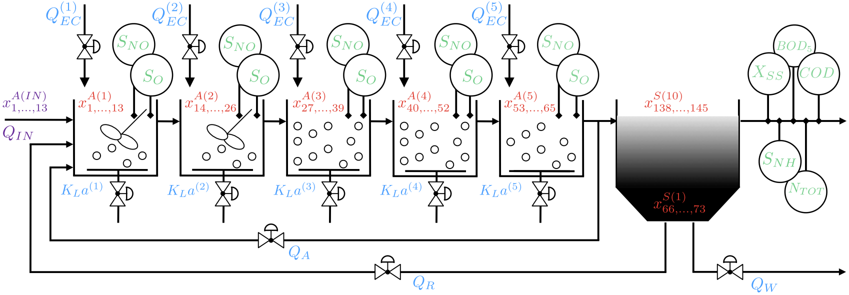
We investigate a general framework for operating biological wastewater treatment plants as water resource recovery facilities (WRRFs)



We consider a conventional **Activated Sludge Process** ⇝ The Benchmark Simulation Model no. 1 (BSM1)[1]

[1] K. Gernaey, U. Jeppsson, P. Vanrolleghem, J. Copp. Benchmarking of Control Strategies for Wastewater Treatment Plants. IWA, 2014.

BSM1, process layout and state-space representation



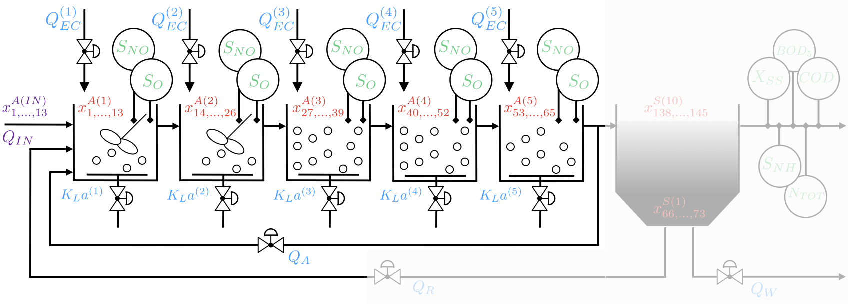
$$\dot{x}(t) = f(x(t), u(t), w(t)|\theta_x)$$
$$y(t) = g(x(t)|\theta_y)$$

- $\rightsquigarrow$ $x(t) = [x^{A(1)} \cdots x^{A(5)}\ x^{S(1)} \cdots x^{S(10)}]^{\mathsf{T}}$
- $\rightsquigarrow$ $u(t) = [Q_A\ Q_R\ Q_W\ u^{A(1)} \cdots u^{A(5)}]^{\mathsf{T}}$
- $\rightsquigarrow$ $y(t) = [y^{A(1)} \cdots y^{A(5)}\ y^{S(10)}]^{\mathsf{T}}$
- $\rightsquigarrow$ $w(t) = [Q_{IN}\ x^{A(IN)}]^{\mathsf{T}}$

- An "expanded model" when compared to common representations

$\rightsquigarrow$ $N_x = 5 \times 13 + 10 \times 8 = 145$ state variables
$\rightsquigarrow$ $N_u = 3 + 5 \times 2 = 13$ controls
$\rightsquigarrow$ $N_w = 1 + 13 = 14$ disturbances
$\rightsquigarrow$ $N_y = 5 \times 2 + 5 = 15$ sensors

BSM1, process layout and state-space representation



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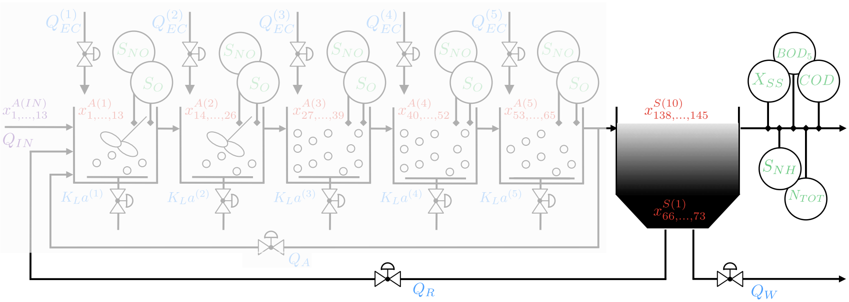

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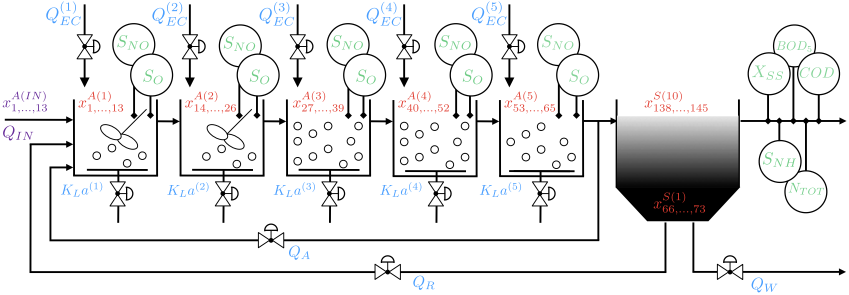

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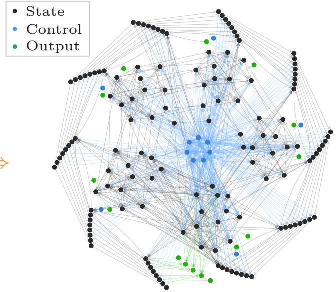
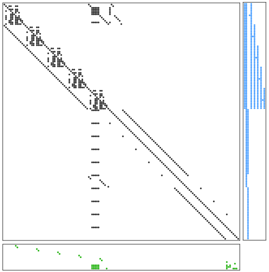
BSM1, structural controllability and observability[1]

Network
$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ $\Rightarrow$





$\Leftarrow$ **System**
(A, B, C)

Pair (A, B) is **structurally controllable**

The plant described by $\dot{x}(t) = f(\cdot|\theta_x)$ is **controllable** for almost all possible realisations of matrices A and B

Pair (A, C) is **structurally unobservable**

The plant $\dot{x}(t) = f(\cdot|\theta_x)$ with $y(t) = g(\cdot|\theta_y)$ is **unobservable** for every possible realisations of matrices A and C

[1] O. Neto, M. Mulas, F. Corona. About the classical and structural controllability and observability of a common class of activated sludge plants. Journal of Process Control, 111:8-26, 2022.

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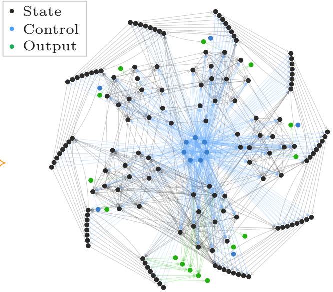

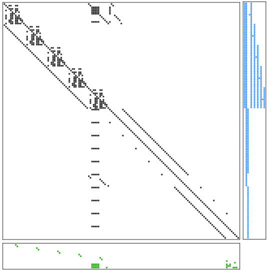

$\Leftarrow$ **System** (A, B, C)

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$\Downarrow$

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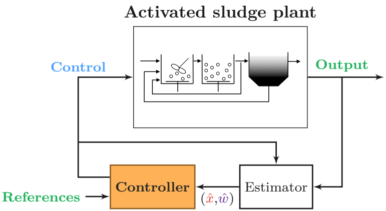
Output model predictive control

Online optimal estimation and control for a common class of ASPs

March 18, 2022

Model predictive control, formulation



Online optimal control problem

$$\min_{\substack{x_k,\dots,x_{k+N}\\u_k,\dots,u_{k+N-1}}} \sum_{n=k}^{k+N-1} L(x_n,u_n) + L_f(x_N)$$

$$\text{s.t.}\ (\forall n\in[k,k+N-1])\quad x_{n+1} = f_{\Delta t}(x_n,u_n,\hat{w}_k|\theta_x)$$

$$x_n \in \mathcal{X}, \quad u_n \in \mathcal{U},$$

$$\Phi(x_k,x_{k+N}) = 0$$

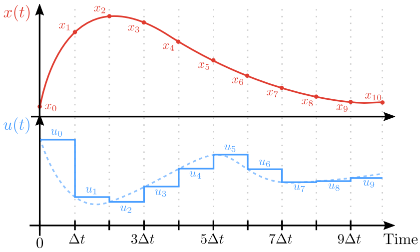
Discretise-then-optimise: The problem is transcribed into a NLP then solved numerically

- Zero-order hold of inputs (given $\Delta t > 0$):

$$u(t) = u(t_n)\ \ (t\in[t_n,t_{n+1}))\,, \quad t_n = n\Delta t$$

- Evolution is given by transition function

$$\underbrace{x_{n+1} = x_n + \int_{t_n}^{t_{n+1}} f(x(t),u_n,w_n)dt}_{f_{\Delta t}(x_n,u_n,w_n)}$$



Model predictive control, affine quadratic regulators

Constrained affine quadratic regulator

$$\min_{\substack{x_k,\dots,x_{k+N}\\ u_k,\dots,u_{k+N-1}}} \sum_{n=k}^{k+N-1} L(x_n, u_n) + L_f(x_{k+N})$$

$$\text{s.t.}\ (\forall n \in [k, k+N-1]) \quad \begin{aligned} x_{n+1} &= f_{\Delta t}(x_n, u_n, \hat{w}_k | \theta_x) \\ x_n &\in \mathcal{X}, \quad u_n \in \mathcal{U}, \\ \Phi(x_k, x_{k+N}) &= 0 \end{aligned}$$

- Quadratic[1] cost functions (given (x^{sp}, u^{sp}))

$$L(\cdot) = \|x_n - x_n^{sp}\|_Q^2 + \|u_n - u_n^{sp}\|_R^2$$
$$L_f(\cdot) = \|x_{k+N} - x_{k+N}^{sp}\|_{Q_f}^2$$

- Linearisations of $f(\cdot)$ around $P_n \coloneqq (x_n^{sp}, u_n^{sp}, w_n^{sp})$

$$x_{n+1} = z_{\Delta t}^{(n)} + A_{\Delta t}^{(n)} x_n + B_{\Delta t}^{(n)} u_n + G_{\Delta t}^{(n)} \hat{w}_k$$

- Constraint sets

$$\mathcal{X} = \{x \in \mathbb{R}^{N_x} | H_x x \le h_x,\ H(x, u) \le 0\}$$
$$\mathcal{U} = \{u \in \mathbb{R}^{N_u} | H_u u \le h_u,\ H(x, u) \le 0\}$$

- Fixed initial state (given $\hat{x}_k$)

$$\Phi(x_k, x_{k+N}) = x_k - \hat{x}_k$$

[1] Quadratic form $\|z\|_W^2 = z^\mathsf{T} W z$

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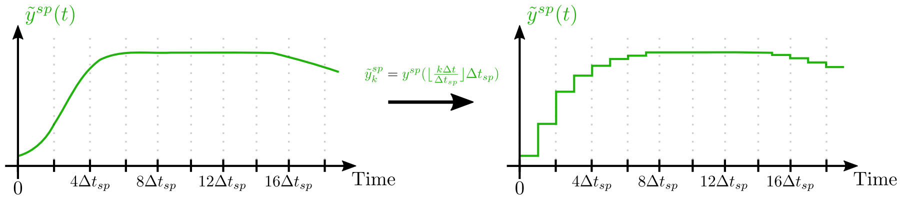
Model predictive control, reference optimisation

Usually, reference trajectories are available only for output variables

(For continuous-time trajectories, we consider a discretisation with $\Delta t_{sp} > 0$)



⇝ Each pair (x_n^{sp}, u_n^{sp}) satisfying $\tilde{y}_n^{sp} \in \mathbb{R}^{N_{\tilde{y}}}$ is the solution of an optimisation[1]:

Steady-state optimisation

$$\min_{x_n^{sp}, u_n^{sp}} \quad \|Hg(x_n^{sp}) - \tilde{y}_n^{sp}\|^2_{W_{\tilde{y}}} + \|u_n^{sp} - \tilde{u}_n^{sp}\|^2_{W_u}$$

$$\text{s.t.} \quad f(x_n^{sp}, u_n^{sp}, w_n^{sp}|\theta_x) = 0,$$

$$x_n^{sp} \in \mathcal{X}^{sp}, \quad u_n^{sp} \in \mathcal{U}^{sp}$$

(We consider fixed $w_n^{sp} = w^{SS}$ and $\tilde{u}_n^{sp} = 0$)

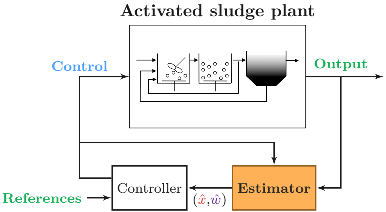
⇝ Search for stationary point $(x_n^{sp}, u_n^{sp}, w_n^{sp})$

⇝ The $N_{\tilde{y}} \leq N_y$ outputs of interest are selected by matrix $H \in \{0,1\}^{N_{\tilde{y}} \times N_y}$

⇝ $W_{\tilde{y}}, W_u \succeq 0$ are tuning parameters

[1] Rawlings, J., Mayne, D., and Diehl, M., 2020. Model Predictive Control: Theory, Computation and Design, 2^{nd} edition. Nob Hill Publishing, LLC.

Moving-horizon estimation, formulation



Moving-horizon estimation problem

$$\min_{\substack{\hat{x}_{k-N_e+1},\dots,\hat{x}_k\\ \hat{w}_{k-N_e+1},\dots,\hat{w}_k}} L_0(\hat{x}_{k-N_e+1}) + \sum_{n=k-N_e+1}^{k} L(\hat{x}_n, \hat{w}_n | y_n)$$

$$\underset{(\forall n \in [k-N_e,k])}{\text{s.t.}} \quad \hat{x}_{n+1} = f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n | \theta_x)$$

$$\hat{x}_n \in \mathcal{X}, \quad \hat{w}_n \in \mathcal{W}$$

⇝ The optimal estimation problem derives from a *maximum a posteriori* estimate solution

- **Stochastic state-space**

$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t), \hat{w}(t) | \theta_x)$$
$$y(t) = g(\hat{x}(t)|\theta_y) + v(t)$$

with $\hat{x}(0) \sim e^{-L_0(\hat{x}(0)|Q_{x_0})}$

$\hat{w}(t) \sim e^{-L_w(\hat{w}(t)|R_w)}$

$v(t) \sim e^{-L_v(v(t)|Q_v)}$

- Zero-order hold of disturbances:
  (Given the rate of measurement Δt_e)

$$\hat{w}(t) = \hat{w}(t_n) \ \ (t \in [t_n, t_{n+1})), \quad t_n = n\Delta t_e$$

- Evolution given by *transition function*

$$\underbrace{\hat{x}_{n+1} = \hat{x}_n + \int_{t_n}^{t_{n+1}} f(\hat{x}(t), u_n, \hat{w}_n)dt}_{f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n)}$$

Moving-horizon estimation, Gauss-Markov estimators

Constrained Gauss-Markov estimator

$$\min_{\substack{\hat{x}_{k-N_e+1},\dots,\hat{x}_k \\ \hat{w}_{k-N_e+1},\dots,\hat{w}_k}} L_0(\hat{x}_{k-N_e+1}) + \sum_{n=k-N_e+1}^{k} L(\hat{x}_n, \hat{w}_n | y_n)$$

$$\underset{(\forall n \in [k-N_e,k])}{\text{s.t.}} \quad \hat{x}_{n+1} = f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n | \theta_x)$$

$$\hat{x}_n \in \mathcal{X}, \quad \hat{w}_n \in \mathcal{W}$$

- Gaussian distributions for the initial state, disturbances, and measurement noise

(Given $\{\bar{x}_n, \bar{w}_n\}_{k-N_e+1}^{k}$ the solutions from previous horizon)

$$\hat{x}_{k-N_e+1} \sim \mathcal{N}(\bar{x}_{k-N_e+1}, Q_{x_0}), \qquad \hat{w}_n \sim \mathcal{N}(\bar{w}_n, R_w), \qquad v_n \sim \mathcal{N}(0, Q_v)$$

- Linearisations around each $P_n := (\bar{x}_n, u_n, \bar{w}_n)$

$$\hat{x}_{n+1} = z^{(n)}_{f_{\Delta t_e}} + A^{(n)}_{\Delta t_e}\hat{x}_n + B^{(n)}_{\Delta t_e} u_n + G^{(n)}_{\Delta t_e}\hat{w}_n$$

$$\hat{y}_n = z^{(n)}_g + C^{(n)}\hat{x}_n$$

- Support sets

$$\mathcal{X} = \{x \in \mathbb{R}^{N_x} \mid H_x x \leq h_x\}$$

$$\mathcal{W} = \{w \in \mathbb{R}^{N_w} \mid H_w w \leq h_w\}$$

Moving-horizon estimation, Gauss-Markov estimators

Constrained Gauss-Markov estimator

$$\min_{\substack{\hat{x}_{k-N_e+1},\cdots,\hat{x}_k \\ \hat{w}_{k-N_e+1},\cdots,\hat{w}_k}} \|\hat{x}_{k-N_e+1} - \bar{x}_{k-N_e+1}\|^2_{Q_{x0}} + \sum_{n=k-N_e+1}^{k} \left(\|\hat{y}_n - y_n\|^2_{Q_v} + \|\hat{w}_n - \bar{w}_n\|^2_{R_w} \right)$$

$$\text{s.t.}\ (\forall n \in [k-N_e, k]) \quad \hat{x}_{n+1} = f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n | \theta_x)$$

$$\hat{x}_n \in \mathcal{X}, \quad \hat{w}_n \in \mathcal{W}$$

- Gaussian distributions for the initial state, disturbances, and measurement noise
  (Given $\{\bar{x}_n, \bar{w}_n\}_{k-N_e+1}^{k}$ the solutions from previous horizon)

$$\hat{x}_{k-N_e+1} \sim \mathcal{N}(\bar{x}_{k-N_e+1}, Q_{x0}), \qquad \hat{w}_n \sim \mathcal{N}(\bar{w}_n, R_w), \qquad v_n \sim \mathcal{N}(0, Q_v)$$

- Linearisations around each $P_n := (\bar{x}_n, u_n, \bar{w}_n)$

$$\hat{x}_{n+1} = z^{(n)}_{f_{\Delta t_e}} + A^{(n)}_{\Delta t_e}\hat{x}_n + B^{(n)}_{\Delta t_e} u_n + G^{(n)}_{\Delta t_e}\hat{w}_n$$

$$\hat{y}_n = z^{(n)}_g + C^{(n)}\hat{x}_n$$

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$$H_x\hat{x}_n \le h_x, \quad H_w\hat{w}_n \le h_w$$

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Experimental results

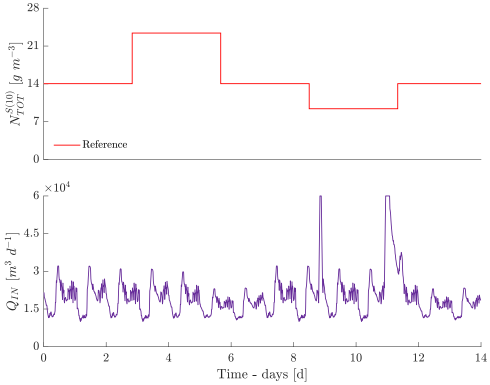
Experiment, objective and simulation results

- **Goal:** Tracking references for $N_{TOT}^{S(10)}$

$$N_{TOT}^{S(10)}(t) = \begin{cases} \frac{5}{3} N_{TOT}^{SS}, & t \in [2.8, 5.6) \text{ d} \\ \frac{2}{3} N_{TOT}^{SS}, & t \in [8.4, 11.2) \text{ d} \\ N_{TOT}^{SS}, & \text{otherwise} \end{cases}$$

Summary of results:

- Tracking accuracy (RMSE): 2.05 g m^{-3}
- The references are satisfied by manipulating $NO_2^- + NO_3^-$ nitrogen inside the reactors



Output MPC Parameters (c-AQR and c-AGM, Reference tracking)

General parameters:

Simulation time $[T]$ = 14 days,
MPC horizon $[N]$ = 12 (12h),
MHE horizon $[N_e]$ = 12 (3h)

Sampling periods:

Control interval $[\Delta t] = (1/24)d$ (1h),
Measurement rate $[\Delta t_e] = (1/96)d$ (15m)

Influent conditions:

$w(\cdot) =$ STORMY WEATHER
$(Q_{IN}^{avg} = 19744 \text{ m}^3/\text{d})$
$(S_{NH}^{avg} = 29.48 \text{ d/m}^3)$

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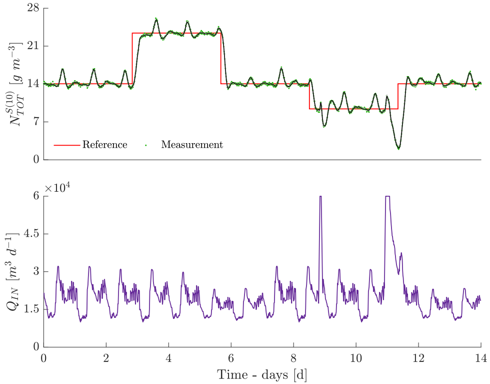

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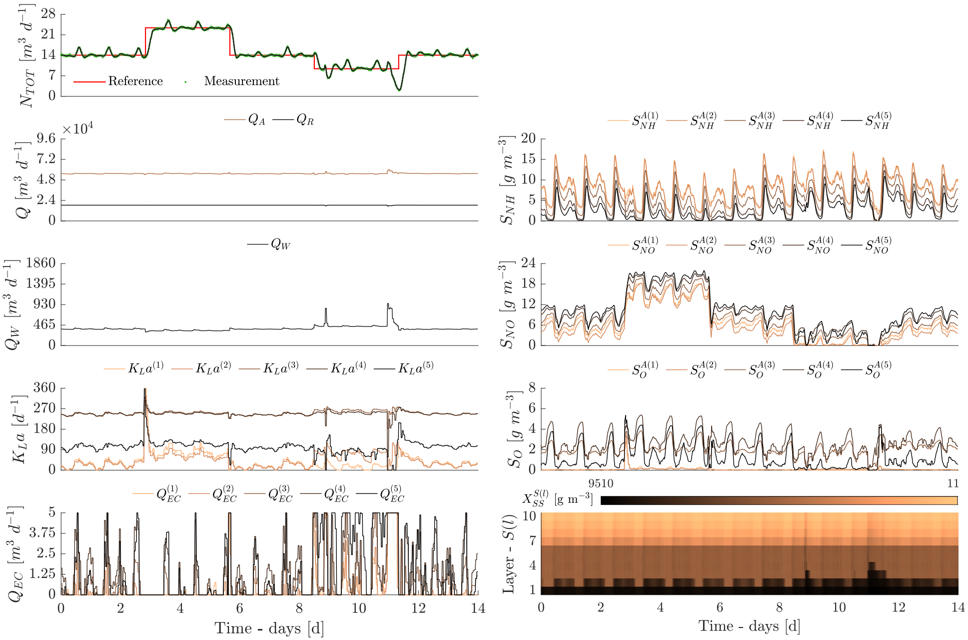
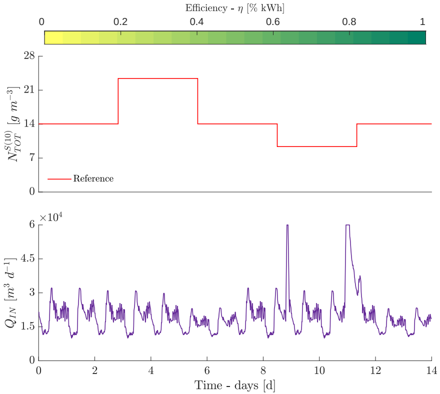

Reference
Measurement
N_{TOT} [m^3 d^{-1}]
Q_A Q_R
$\times 10^4$
Q [m^3 d^{-1}]
Q_W
Q_W [m^3 d^{-1}]
$K_La^{(1)}$ $K_La^{(2)}$ $K_La^{(3)}$ $K_La^{(4)}$ $K_La^{(5)}$
K_La [d^{-1}]
$Q_{EC}^{(1)}$ $Q_{EC}^{(2)}$ $Q_{EC}^{(3)}$ $Q_{EC}^{(4)}$ $Q_{EC}^{(5)}$
Q_{EC} [m^3 d^{-1}]
Time - days [d]
$S_{NH}^{A(1)}$ $S_{NH}^{A(2)}$ $S_{NH}^{A(3)}$ $S_{NH}^{A(4)}$ $S_{NH}^{A(5)}$
S_{NH} [g m^{-3}]
$S_{NO}^{A(1)}$ $S_{NO}^{A(2)}$ $S_{NO}^{A(3)}$ $S_{NO}^{A(4)}$ $S_{NO}^{A(5)}$
S_{NO} [g m^{-3}]
$S_{O}^{A(1)}$ $S_{O}^{A(2)}$ $S_{O}^{A(3)}$ $S_{O}^{A(4)}$ $S_{O}^{A(5)}$
S_O [g m^{-3}]
$X_{SS}^{S(l)}$ [g m^{-3}]
9510
11
Layer - $S(l)$
Time - days [d]

Experiment, objective and simulation results

- **Goal:** Tracking same $N_{TOT}^{S(10)}$ references while performing energy-recovery

$$\eta \mathrm{OCI}_{\mathrm{kWh}} \leq \frac{\eta_D}{N} \sum_{n=1}^{N} Q_W \left(\frac{X_{SS}^{S(1)}}{0.75} + S_I^{S(1)} + S_S^{S(1)} \right)$$



Summary of results:

- + Disposed sludge, − Aeration
- Energy autonomy achievable up to 60%

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$(S_{\mathrm{NH}}^{\mathrm{avg}} = 29.48\ \mathrm{d/m}^3)$

[*] $\mathrm{OCI}_{\mathrm{kWh}} = \frac{1}{1000N} \sum_{n=1}^{N} \left(8 \sum_{r=1}^{5} V^{A(r)} K_L a^{(r)} + (4Q_A + 8Q_R + 50Q_W) + 120 \sum_{r=1}^{5} V^{A(r)} \mathbf{1}(20 - K_L a^{(r)}) \right)$

Experiment, objective and simulation results

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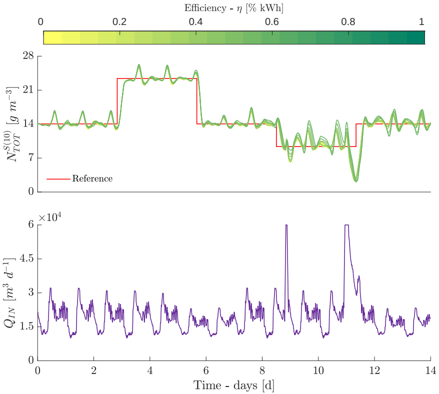

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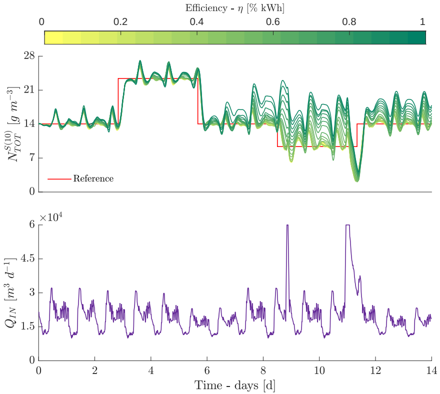

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Thank you!

Questions?