Exercise 01. The following Python code wrongly implements the following functions

$$\begin{cases}
\mathcal{L}(x_{1}, x_{2}, x_{3}, x_{4}) = \prod_{i=1}^{|\mathcal{X}|} \frac{1}{\underbrace{x_{i}\sigma\sqrt{2\pi}}} \exp\left\{\frac{-\left[\ln\left(x_{i}\right) - \mu\right]^{2}}{2\sigma^{2}}\right\} \\
\underbrace{\mathcal{L}(x_{1}, x_{2}, x_{3}, x_{4}|K_{e}, K_{l})}_{l_{i}} = \prod_{i=1}^{|\mathcal{X}|} \frac{1}{\underbrace{x_{i}\sigma\sqrt{2\pi}}} \sum_{k_{e}=1}^{K_{e}} \left\{\frac{-\left[\sum_{k_{l}=1}^{K_{l}} \frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{\underbrace{k_{e}!}}\right\}^{k_{e}} \\
\underbrace{\mathcal{L}(x_{1}, x_{2}, x_{3}, x_{4}|K_{e}, K_{l})}_{l_{i}} = \prod_{i=1}^{|\mathcal{X}|} \frac{1}{\underbrace{x_{i}\sigma\sqrt{2\pi}}} \sum_{k_{e}=1}^{K_{e}} \left\{\frac{-\left[\sum_{k_{l}=1}^{K_{l}} \frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{k_{e}!}\right\}^{k_{e}} \\
\underbrace{\mathcal{L}(x_{1}, x_{2}, x_{3}, x_{4}|K_{e}, K_{l})}_{l_{i}} = \prod_{i=1}^{|\mathcal{X}|} \frac{1}{\underbrace{x_{i}\sigma\sqrt{2\pi}}} \sum_{k_{e}=1}^{K_{e}} \left\{\frac{-\left[\sum_{k_{l}=1}^{K_{l}} \frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{k_{e}!}\right\}^{k_{e}}}_{l_{i}}$$

for $\mathcal{X} = \{x_i\}_{i=1}^{|\mathcal{X}|=4} = \{0.8840, 1.2055, 0.9321, 1.0255\}, K_e = 32, K_l = 32, \mu = 0 \text{ and } \sigma = 0.25.$

Find the errors in the code and suggest possible corrections.

```
def a_logn(x,Kl):
2
       s = 0; k = 1
3
       while k <= Kl:
4
           t = (-1) **k * (x-1) **k / k
5
           s += t; k += 1; a_logn = s
6
       return a_logn
7
8
  def a_exp(x,Ke):
9
       s = 0; k = 0
10
       while k <= Ke:
11
           t = x**k / factorial(k)
12
           s += t; k += 1; a_exp = s
13
       return a_exp
14
15
  def a_lognorm(x,mu,sigma,Ke,Kl):
16
       a_p = 1/x_i/sigma/sqrt(2*pi)*a_exp(-(a_logn(x_i,K1)-mu)**2/2/sigma**2,Ke)
17
       a_lognorm = a_p
18
       return a_log_norm
19
20
  def lognorm(x,mu,sigma):
21
       p = 1/x_i/sigma/sqrt(2*pi)*exp(-(log(x_i)-mu)**2/2/sigma**2)
22
       lognorm = p
23
       return lognorm
24
25 from math import pi, exp, log, sqrt, factorial
26
27 X = [0.8840, 1.2055, 0.9321, 1.0255]
28 \text{ mu} = 0; sigma = 0.25
29
30 \text{ Ke} = 32; \text{ Kl} = 32; \text{ L} = 0; \text{ P} = 1
31 for x in X:
       1_i = a_lognorm(x,mu,sigma,Ke,Kl)
32
33
       p_i = lognorm(x,mu,sigma)
34
       L *= L_i
35
       P *= P_i
```

O seguinte código Python implementa erroneamente as seguintes funções

$$\begin{cases}
\mathcal{L}(x_{1}, x_{2}, x_{3}, x_{4}) = \prod_{i=1}^{|\mathcal{X}|} \frac{1}{x_{i}\sigma\sqrt{2\pi}} \exp\left\{\frac{-\left[\ln(x_{i}) - \mu\right]^{2}}{2\sigma^{2}}\right\} \\
\frac{1}{2\sigma^{2}} \left\{\frac{-\left[\sum_{k_{l}=1}^{K_{l}} \frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{k_{l}}\right\} \\
\mathcal{P}(x_{1}, x_{2}, x_{3}, x_{4} | K_{e}, K_{l}) = \prod_{i=1}^{|\mathcal{X}|} \frac{1}{x_{i}\sigma\sqrt{2\pi}} \sum_{k_{e}=1}^{K_{e}} \left\{\frac{\left\{\frac{-\left[\sum_{k_{l}=1}^{K_{l}} \frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{k_{e}!}\right\} \\
\frac{2\sigma^{2}}{k_{e}!} \right\} \\
\frac{p_{i}}{\sqrt{2\pi}} \left\{\frac{\left\{\frac{-\left[\sum_{k_{l}=1}^{K_{l}} \frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{k_{e}!}\right\} \\
\frac{p_{i}}{\sqrt{2\pi}} \left\{\frac{-\left[\sum_{k_{l}=1}^{K_{l}} \frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{k_{e}!}\right\} \\
\frac{p_{i}}{\sqrt{2\pi}} \left\{\frac{p_{i}}{\sqrt{2\pi}} \left(\frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{k_{e}!}\right\} \\
\frac{p_{i}}{\sqrt{2\pi}} \left\{\frac{p_{i}}{\sqrt{2\pi}} \left(\frac{(-1)^{k_{l}-1}(x_{i}-1)^{k_{l}}}{k_{l}} - \mu\right]^{2}}{k_{e}!}\right\} \\
\frac{p_{i}}{\sqrt{2\pi}} \left(\frac{p_{i}}{\sqrt{2\pi}} - \mu\right)^{2}}{k_{i}!}\right\} \\
\frac{p_{i}}{\sqrt{2\pi}} \left(\frac{p_{i}$$

com $\mathcal{X} = \{x_i\}_{i=1}^{|\mathcal{X}|=4} = \{0.8840, 1.2055, 0.9321, 1.0255\}, K_e = 32, K_l = 32, \mu = 0 \text{ e } \sigma = 0.25.$

Encontre os erros e sugere possíveis correções.

```
1 def a_logn(x,Kl):
2
       s = 0; k = 1
3
       while k <= Kl:
4
           t = (-1)**k * (x-1)**k / k
5
           s += t; k += 1; a_logn = s
6
       return a_logn
7
8
  def a_exp(x,Ke):
9
       s = 0; k = 0
10
       while k <= Ke:</pre>
11
           t = x**k / factorial(k)
           s += t; k += 1; a_exp = s
12
13
       return a_exp
14
15
  def a_lognorm(x,mu,sigma,Ke,Kl):
16
       a_p = 1/x_i/sigma/sqrt(2*pi)*a_exp(-(a_logn(x_i,Kl)-mu)**2/2/sigma**2,Ke)
17
       a_lognorm = a_p
18
       return a_log_norm
19
20 def lognorm(x,mu,sigma):
21
       p = 1/x_i/sigma/sqrt(2*pi)*exp(-(log(x_i)-mu)**2/2/sigma**2)
22
       lognorm = p
23
       return lognorm
24
25 from math import pi, exp, log, sqrt, factorial
26
27 X = [0.8840, 1.2055, 0.9321, 1.0255]
28 \text{ mu} = 0; sigma = 0.25
29
30 \text{ Ke} = 32; \text{ Kl} = 32; \text{ L} = 0; \text{ P} = 1
31 for x in X:
32
       l_i = a_lognorm(x,mu,sigma,Ke,Kl)
33
       p_i = lognorm(x,mu,sigma)
34
       L *= L_i
35
       P *= P i
```