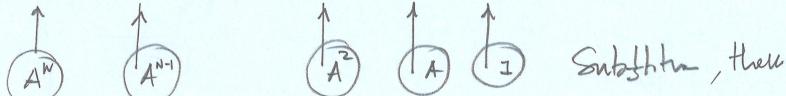


CAYLEY-HAMILTON THEOREM

"Every square matrix satisfies its own characteristic (eigenvalue) equation"

$$\Rightarrow \det(A - \lambda I) = 0 \quad \text{EIGENVALUE EQUATION}$$

$$\lambda^n + a_{n-1}\lambda^{n-1} + \dots + a_2\lambda^2 + a_1\lambda + a_0 = 0$$



Substitution, then

$$A^n + a_{n-1}A^{n-1} + \dots + a_2A^2 + a_1A + a_0I = 0 \quad \text{It is satisfied to } \text{TOO}$$

$$\text{AND THEN } A^n = -a_0I - a_1A - a_2A^2 - a_3A^3 - \dots - a_{n-1}A^{n-1}$$

AND WE ALSO HAVE THAT

$$A^{n+1} = -a_0I + a_1I + \dots + a_{n-1}A^{n-1} + a_nA^n$$

$$A^{n+1} = \sum_{j=0}^{n-1} \alpha_j A^j$$

$$\text{If we consider } x(t) = e^{At}x_0 \quad \text{with } e^{At} = I + At + A^2t^2/2! + \dots$$

so that we can write

$$\alpha_0(t)I + \alpha_1(t)A + \alpha_2(t)A^2 + \dots + \alpha_{n-1}(t)A^{n-1}$$

THE SOLUTION TO minimize $J(u)$ s.t. $\dot{x} = Ax + Bu$
 $u \in \mathbb{R}^{N_u}$

$$\Rightarrow u(t) = -Kx(t) \quad \text{WITH} \quad K = R^{-1}B^T(S)$$

$N_x \times N_x \text{ (symmetric)}$

KNOWN THIS IS THE SOLUTION TO
 THE ALGEBRAIC RICCATI
 EQUATION

How to solve the Algebraic Riccati Eq.
for S (there may exist multiple solutions)

$$A^T S + SA - SBR^{-1}B^T S + Q = 0$$

$$\dot{x} = Ax + Bu \quad ; \quad y = \underbrace{x}_{\text{ }}_x$$

$$u = -Kx$$

If the system is controllable, we can find a $u(t)$ that steers it from any initial to any final state value, i.e. the

WE ARE ALWAYS ASSUMING THAT WE CAN ALWAYS MEASURE THE FULL STATE VECTOR

$K \rightarrow$ MANUALLY, BY EIGEN VALUE PLACEMENT

OPTIMALLY, BY OPTIMAL CONTROL FOR LINEAR SYSTEMS
(LQR)

THE FULL STATE FEEDBACK CONTROLLED SYSTEM

$$\dot{x} = (A - BK)x$$