# CAYLEY–HAMILTON THEOREM

"Every square matrix satisfies its own characterist (eigenvalue) eq"

$\rightsquigarrow$ $\det(A - \lambda I) = 0$ EIGENVALUE EQUATION

$$\lambda^n + a_{n-1}\lambda^{n-1} + \text{——} + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

$$A^N \qquad A^{N-1} \qquad A^2 \qquad A \qquad I$$

Substitute, then

$$A^N + a_{n-1}A^{n-1} + \text{——} + a_2 A^2 + a_1 A + a_0 I = 0$$

It is satisfied too

$\rightsquigarrow$ THEN $A^N = -a_0 I - a_1 A - a_2 A^2 - a_3 A^3 - \text{——} - a_{n-1} A^{n-1}$

AND WE ALSO HAVE THAT

$$A^{N+1} = -a_0 I + a_1 I + \text{——} + a_{n-1} A^{N-1} + a_n A^N$$

$$A^{\geq n} = \sum_{j=0}^{N-1} \alpha_j A^j$$

If we consider $x(t) = e^{At} x_0$ with $e^{At} = I + At + A^2 t^2/2! + \text{——}$

so that we can write

$$\alpha_0(t) I + \alpha_1(t) A + \alpha_2(t) A^2 + \text{——} + \alpha_{n-1}(t) A^{n-1}$$

THE SOLUTION TO $\underset{u \in \mathbb{R}^{N_u}}{\text{minimize}}\ J(u)$ s.t. $\dot{x} = Ax + Bu$

$\rightsquigarrow u(t) = -Kx(t)$ WITH $K = R^{-1}B^T S$

$N_x \times N_x$ (symmetric)

KNOWN

THIS IS THE SOLUTION TO THE ALGEBRAIC RICATTI EQUATION

$$A^T S + SA - SBR^{-1}B^T S + Q = 0$$

How to solve the Algebraic Ricatti Eq. for $S$ (there may exist multiple solutions)

$\dot{x} = Ax + Bu$ ; $\underbrace{y = x}$

$u = -Kx$

If the system is controllable, we can find a u(t) that steers it from any initial to any final state value, finite time

WE ARE ALWAYS ASSUMING THAT WE CAN ALWAYS MEASURE THE FULL STATE VECTOR

K → { MANUALLY, BY EIGEN VALUE PLACEMENT
OPTIMALLY, BY OPTIMAL CONTROL FOR LINEAR SYSTEMS (LQR) }

THE FULL STATE FEEDBACK CONTROLLED SYSTEM

$\dot{x} = (A - BK)x$