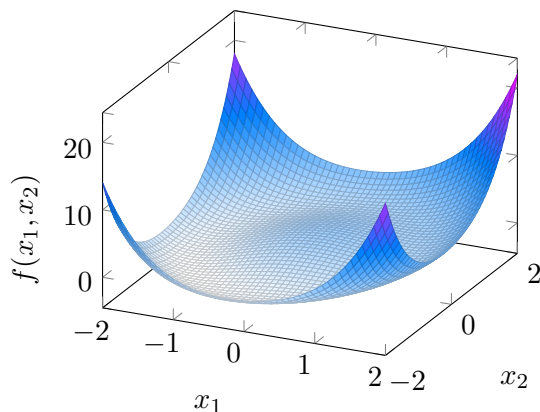
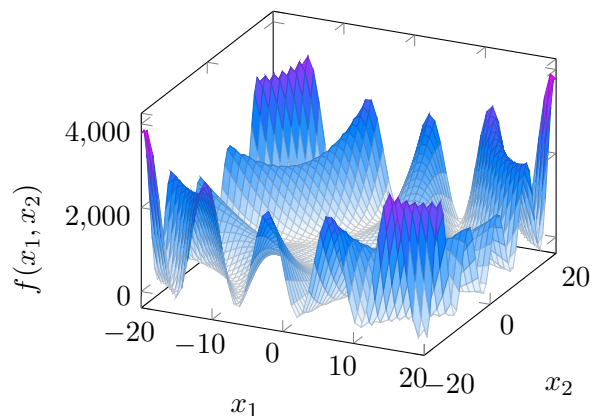


**Exercise 1.** Consider the objective functions

$$f(\mathbf{x}) = x_1 + x_2 + 0.5(x_1^2 + x_2^2 - 2)^2$$



$$f(\mathbf{x}) = [\sin(\frac{2}{3}\pi x_1 x_2 + 1)](2x_1^2 + 3x_2^2 + 4)$$



1. Compute expressions for the gradient vector  $\nabla f(\mathbf{x})$  and the Hessian matrix  $\nabla^2 f(\mathbf{x})$
2. Write code to implement two line-search methods to minimise  $f(\mathbf{x})$

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \alpha_k \mathbf{d}^{(k)}, \quad \text{with } \mathbf{d}^{(k)} = \begin{cases} -[\nabla f(\mathbf{x}^{(k)})] \\ -[\nabla^2(\mathbf{x}^k)^{-1}] \end{cases}$$

Test with fixed values of  $\alpha_k$  and different initial solutions  $\mathbf{x}^{(0)}$