Linear systems

Advanced topics in machine learning

Francesco Corona

Department of Computer Science
Federal University of Ceará, Fortaleza

Linear systems

UFC/DC
CK0255|TIP8244
2018.2

Content
Deterministic Markov functions
Stochastic Markov functions
Logistic
Evaluation

General content

Linear systems and ATML

AUG 08, 2018 _ FC _

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General content

Consider the general problem of system analysis and controller design

- We have at our disposal a wealth of knowledge
- Deterministic analysis and control theory

We can investigate the system structure and modes of its response

- We can design compensators that alter these characteristics

We can design controllers that provide appropriate inputs

- To generate desired system responses

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General content (cont.)

Why do we need to go beyond these results?

- Why stochastic models?

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General content (cont.)

To observe the actual system behaviour, we construct measurement devices

- They output data signals, proportional to variables of interest

These signals and the known inputs are the only available information

- Information discernible about the system

If we design a controller, these signals are the only available inputs

- Direct inputs to the controller

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General content (cont.)

No mathematical system model is considered perfect

- Only dominant/critical modes are modelled
- Computationally feasible (light)

Systems are not only driven by our control inputs

- Disturbances that we cannot control
- Disturbances that we cannot model

Sensors are not perfect nor they are complete

- Not all interesting information
- No exact readings and noise
- Own system dynamics

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General content (cont.)

We cannot assume perfect knowledge of all quantities or perfect control

We must develop a system model that accounts for uncertainties

- ⇝ How to optimally estimate the quantities of interest?
- ⇝ How to optimally control the system to perform?

How to evaluate the performance of these estimates and control?

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General content (cont.)

The course is an introduction to modelling of dynamic processes

- Deterministic and stochastic state-space models
- Deterministic and random Markov functions

We overview dynamic system analysis and probability theory

Deterministic Markov functions

General content

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Deterministic Markov functions

Deterministic Markov systems/models and their classification

- Input-output and state-space representations
- ⇝ Focus on state-space models (a richer class)

General properties of deterministic systems

- Dynamical/Instantaneous
- Linear/Nonlinear
- Stationary/Nonstationary
- Proper/improper
- With/Without delay

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Deterministic Markov functions (cont.)

The analysis of state-space models only in the time-domain

- ⇝ Linear and stationary models

A general scheme to determine the state transition matrix

- ⇝ **The Sylvester expansion**

A general procedure to solve the analysis problem

- ⇝ **The Lagrange formula**

Similarity transformations and canonical forms

- ⇝ **Diagonalisation**
- ⇝ **Jordan's form**

The system modes and their intepretation

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Stochastic Markov functions

General content

Stochastic Markov functions

Stochastic Markov system/models, their classification and their properties

- Continuous-time continuous-state Markov processes
- Continuous-time discrete-state Markov processes
- Discrete-time discrete-state Markov processes

Main focus on discrete-time discrete-state processes

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Stochastic Markov functions (cont.)

Analysis of discrete-time discrete-state Markov processes (chains)

- General concepts and definitions
- **Classification of states**
- **Irreducibility**

Some important matrices

- **Fundamental matrix**
- **Reachability matrix**
- **Potential matrix**

Some important distributions

- **Steady-state distribution**
- **Stationary distribution**
- **Limiting distribution**

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Stochastic Markov functions (cont.)

Analysis of continuous-time discrete-state Markov processes

- General concepts and definitions
- **Transition probabilities**
- **Transition rates**

Some important distributions

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Stochastic Markov functions (cont.)

Continuous-time continuous-space (general) Markov processes

- **General Markov state density function**
- **Chapman-Kolmogorov equations**
- **Kramers-Moyal equations**
- **The Markov propagator**

Time-integral and **time-evolution of Markov processes**

- General moment evolution equations

Stochastic Markov functions (cont.)

The continuous propagator and its characterising functions

- Time-evolution equations

Three important continuous Markov processes

- The Liouville process
- The Weiner process
- Ornstein-Uhlenbeck process

The Fokker-Planck equation and the Langevin equation

Logistics

Linear systems and ATML

Logistic

SIGAA doesn't know the course location

- Here, mostly

Timetable, as you have known it

- Wednesdays afternoons
- $14:00 \rightarrow 18:00$

I do not care about presence

- If you deliver

$\rightsquigarrow$ fkorona.github.io/ATML

Evaluation

Linear systems and ATML

Evaluation

Two, maybe three, intermediate tests (AP) here in the classroom (70%)

- Train by exercising and participating

One, maybe two, home assignment/project (30%)

To pass the course you need a 5, with 7 you pass earlier (8, I am happy)

- You can have a final test (AF), if you ask