May 10, 2018

CK0030: Partial evaluation II

Exercise 01 (50%).

Consider the function $y(x) = x + \sin(x) + 2\cos(2x) - 0.00004e^{(x)}$, $x \in \mathbb{R}$.

Write a program based on `FOR-loops` that implements the approximation

$$y(x) \approx x + \sum_{k_1=0}^{N_1} \frac{(-1)^{k_1} x^{(2k_1+1)}}{(2k_1+1)!} + 2\sum_{k_2=0}^{N_2} \frac{(-1)^{k_2}(2x)^{2k_2}}{(2k_2)!} - 0.00004\sum_{k_3=0}^{N_3} \frac{x_3^k}{k_3!}$$

Let $x = 6$ and use $N_1 = 16$, $N_2 = 16$ and $N_3 = 32$.

Solution:

```
from math import factorial as fact

x = [6.]; nx = len(x)                              # x = [1.,2.,3.,4.,5.,6.,7.]

k2_max = 16
k3_max = 16
k4_max = 32

s1 = nx*[0]
s2 = nx*[0]
s3 = nx*[0]
s4 = nx*[0]

y  = nx*[0]

for ix in range(0,nx):

 s1[ix] = x[ix]

 s2[ix] = 0.
 for k2 in range(0,k2_max+1):
  t2_num = (-1)**k2 * x[ix]**(2*k2+1)
  t2_den = fact(2*k2+1)
  t2 = t2_num/t2_den
  s2[ix] = s2[ix] + t2

 s3[ix] = 0.
 for k3 in range(0,k3_max+1):
  t3_num = (-1)**k3 * (2*x[ix])**(2*k3)
  t3_den = fact(2*k3)
  t3 = t3_num/t3_den
  s3[ix] = s3[ix] + t3

 s4[ix] = 0.
 for k4 in range(0,k4_max+1):
  t4 = (x[ix]**k4)/fact(k4)
  s4[ix] = s4[ix] + t4

 y[ix] = s1[ix] + s2[ix] + 2*s3[ix] - 0.00004*s4[ix]
```

Exercise 02 (50%).

Consider the following Python program

```
1 z = 3**(-1)*n*(n+1)*(n+2)
2
3 n = 32
4 s = 0
5 for i in range(0,n+1):
6  for j in range(0,i):
7   for k in range(j,i+(j+1)):
8    s += 1
```

The code is a bad implementation of the following identity

$$\sum_{i=1}^{n}\Big(\sum_{j=1}^{i}\Big(\sum_{k=j}^{i+j}1\Big)\Big) = (n+2)(n+1)(n+0)/3$$

Find and correct the potential mistakes that are in the code.

Solution:

```
1 n=32                                          # 1 (before z)
2 z = 3**(-1)*n*(n+1)*(n+2)
3
4 s=0
5 for i in range(1,n+1):                        # 2 (start at 1)
6  for j in range(1,i+1):                       # 3, 4 (start at 1, end at i)
7   for k in range(j,i+(j+1)):
8    s += 1
```