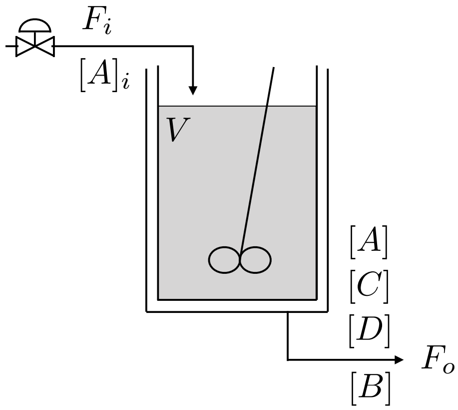
[Student number]

CHEM-E7140/2019: Assignment II

Exercise 1. Consider a continuous stirred tank reactor in which the reaction scheme occurs



$$A \xrightarrow{k_1} B \xrightarrow{k_2} C \tag{1a}$$

$$2A \xrightarrow{k_3} D \tag{1b}$$

Component B is the desired product and we assume that we can measure its composition in the reactor, $[B](t)$. We also assume that the feed only contains component A, whose composition $[A]_i(t)$ can be set, and that density, temperature and volume in the reactor are constant.

Let $F(t)$ [lt min^{-1}] be the volumetric flow-rate of the inlet and outlet stream and let $F^{SS}/V = 4/7$ [min^{-1}] be the dilution-rate/space-velocity at some steady-state operation point, V [lt] indicates the volume. Let $[A]_i^{SS} = 10$ [mol lt^{-1}] be the concentration of component A in the feed at that steady-state. Corresponding steady-state concentrations for A, B, C and D are $[A]^{SS} = 3$, $[B]^{SS} = 1.117$, $[C]^{SS} = 3.258$ and $[D]^{SS} = 1.3125$, all expressed in [mol lt^{-1}].

The state-space model of the reactor as linearised at the given steady-state condition is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -2.4048 & 0 & 0 & 0 \\ 0.8333 & -2.2381 & 0 & 0 \\ 0 & 1.6667 & -0.5714 & 0 \\ 0.5 & 0 & 0 & -0.5714 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 7 & 10 \\ -1.117 & 0 \\ -3.258 & 0 \\ -1.3125 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix},$$

with state variables $x_1 = [A] - [A]^{SS}$, $x_2 = [B] - [B]^{SS}$, $x_3 = [C] - [C]^{SS}$ and $x_4 = [D] - [D]^{SS}$, and control variables $u_1 = F/V - F^{SS}/V$ and $u_2 = [A]_i - [A]_i^{SS}$

- Study the stability of the process A. Comment and plot the results of you analysis;
- Study the controllability of the pair (A, B). Discuss the results of your analysis;
- Design a full-state feedback controller using i) the eigenvalue procedure and ii) the linear quadratic regulator. For both solutions, motivate your procedure, discuss the results and comment on the performance of the closed-loop systems that you developed.
- Study the observability of the pair (A, C) for i) $C = [1\ 1\ 1\ 1]$ and ii) $C = [0\ 1\ 0\ 0]$. Comment on the meaning of the considered matrices C and the results of your analysis.