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A receding-horizon framework for optimal control of activated sludge plants

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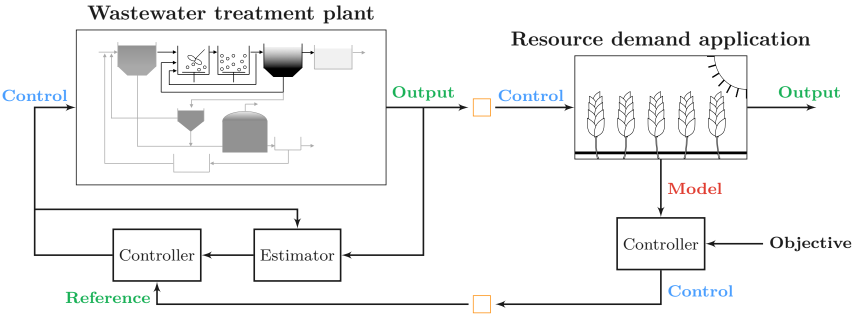
Introduction

A receding-horizon framework for optimal control of ASPs
March 18, 2022

Intro, wastewater treatment plants (WWTP)

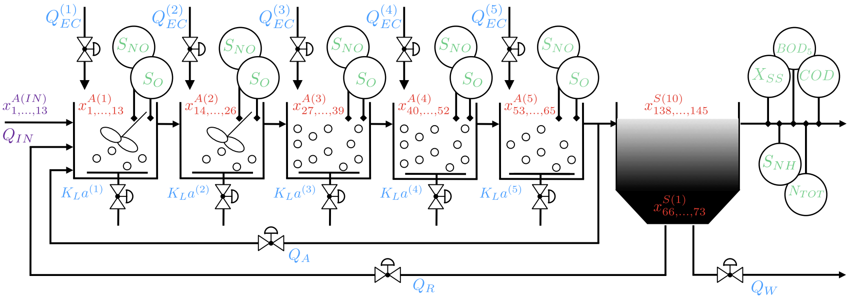
We investigate a general framework for operating biological wastewater treatment plants as water resource recovery facilities (WRRFs)



We consider a conventional **Activated Sludge Process** ⇝ The Benchmark Simulation Model no. 1 (BSM1)[1]

[1] K. Gernaey, U. Jeppsson, P. Vanrolleghem, J. Copp. Benchmarking of Control Strategies for Wastewater Treatment Plants. IWA, 2014.

BSM1, process layout and state-space representation



$$\dot{x}(t) = f(x(t), u(t), w(t)|\theta_x)$$
$$y(t) = g(x(t)|\theta_y)$$

- $\rightsquigarrow$ $x(t) = [x^{A(1)} \ \cdots \ x^{A(5)} \ x^{S(1)} \ \cdots \ x^{S(10)}]^\mathsf{T}$
- $\rightsquigarrow$ $u(t) = [Q_A \ Q_R \ Q_W \ u^{A(1)} \ \cdots \ u^{A(5)}]^\mathsf{T}$
- $\rightsquigarrow$ $y(t) = [y^{A(1)} \ \cdots \ y^{A(5)} \ y^{S(10)}]^\mathsf{T}$
- $\rightsquigarrow$ $w(t) = [Q_{IN} \ x^{A(IN)}]^\mathsf{T}$

- An "expanded model" when compared to common representations

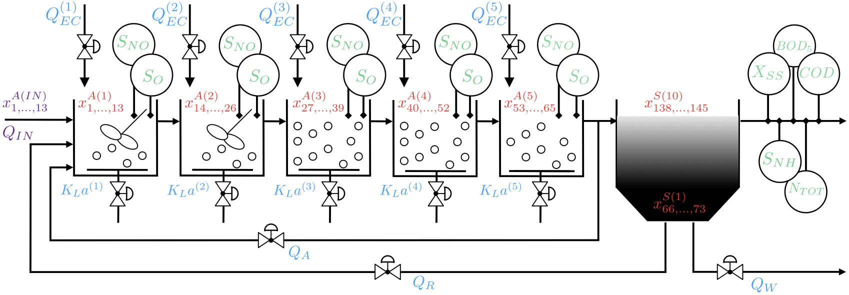
$\rightsquigarrow$ $N_x = 5 \times 13 + 10 \times 8 = 145$ state variables

$\rightsquigarrow$ $N_u = 3 + 5 \times 2 = 13$ controls

$\rightsquigarrow$ $N_w = 1 + 13 = 14$ disturbances

$\rightsquigarrow$ $N_y = 5 \times 2 + 5 = 15$ sensors

BSM1, process layout and state-space representation



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- $u(t) = [Q_A \; Q_R \; Q_W \; u^{A(1)} \;\cdots\; u^{A(5)}]^\mathsf{T}$
- $y(t) = [y^{A(1)} \;\cdots\; y^{A(5)} \; y^{S(10)}]^\mathsf{T}$
- $w(t) = [Q_{IN} \; x^{A(IN)}]^\mathsf{T}$

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- $N_x = 5 \times 13 + 10 \times 8$ $= 145$ state variables
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- $N_w = 1 + 13$ $= 14$ disturbances
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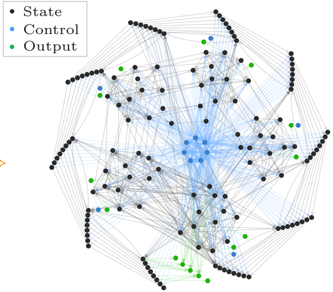
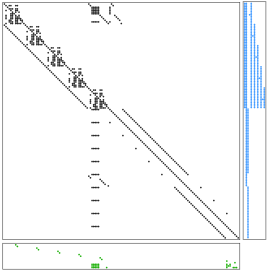
BSM1, structural controllability and observability[1]

Network
$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ $\Rightarrow$





$\Leftarrow$ **System**
(A, B, C)

Pair (A, B) is **structurally controllable**

$\Updownarrow$

The plant described by $\dot{x}(t) = f(\cdot|\theta_x)$ is **controllable** for almost all possible realisations of matrices A and B

Pair (A, C) is **structurally unobservable**

$\Updownarrow$

The plant $\dot{x}(t) = f(\cdot|\theta_x)$ with $y(t) = g(\cdot|\theta_y)$ is **unobservable** for every possible realisations of matrices A and C

[1] O. Neto, M. Mulas, F. Corona. About the classical and structural controllability and observability of a common class of activated sludge plants. Journal of Process Control, 111:8-26, 2022.

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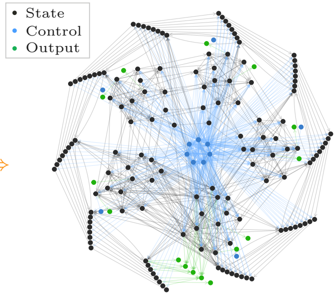

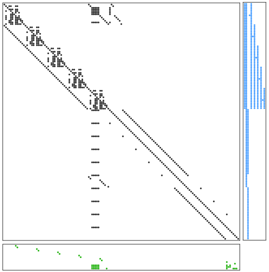

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Output model predictive control

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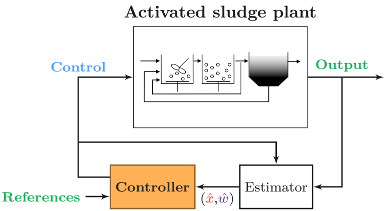
Model predictive control, formulation



Online optimal control problem

$$\min_{\substack{x_k,\ldots,x_{k+N}\\ u_k,\ldots,u_{k+N-1}}} \sum_{n=k}^{k+N-1} L(x_n,u_n) + L_f(x_N)$$

$$\begin{aligned}\text{s.t.}\ (\forall n\in[k,k+N-1]) \quad & x_{n+1} = f_{\Delta t}(x_n,u_n,\hat{w}_k|\theta_x)\\ & x_n \in \mathcal{X}, \quad u_n \in \mathcal{U},\\ & \Phi(x_k,x_{k+N}) = 0\end{aligned}$$

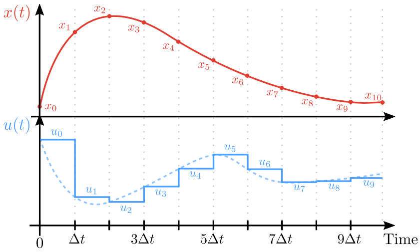
Discretise-then-optimise: The problem is transcribed into a NLP then solved numerically

- Zero-order hold of inputs (given $\Delta t > 0$):

$$u(t) = u(t_n) \quad (t \in [t_n, t_{n+1}))\,, \qquad t_n = n\Delta t$$

- Evolution is given by transition function

$$\underbrace{x_{n+1} = x_n + \int_{t_n}^{t_{n+1}} f(x(t),u_n,w_n)dt}_{f_{\Delta t}(x_n,u_n,w_n)}$$



Model predictive control, affine quadratic regulators

Constrained affine quadratic regulator

$$\min_{\substack{x_k,\dots,x_{k+N}\\ u_k,\dots,u_{k+N-1}}} \sum_{n=k}^{k+N-1} L(x_n, u_n) + L_f(x_{k+N})$$

$$\text{s.t.}\ (\forall n \in [k, k+N-1]) \quad \begin{aligned} x_{n+1} &= f_{\Delta t}(x_n, u_n, \hat{w}_k | \theta_x) \\ x_n &\in \mathcal{X}, \quad u_n \in \mathcal{U}, \\ \Phi(x_k, x_{k+N}) &= 0 \end{aligned}$$

- Quadratic[1] cost functions (given (x^{sp}, u^{sp}))

$$L(\cdot) = \|x_n - x_n^{sp}\|_Q^2 + \|u_n - u_n^{sp}\|_R^2$$

$$L_f(\cdot) = \|x_{k+N} - x_{k+N}^{sp}\|_{Q_f}^2$$

- Convex constraint sets

$$\mathcal{X} = \{x \in \mathbb{R}^{N_x} | H_x x \le h_x\}$$

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- Linearisations of $f(\cdot)$ around $P_n := (x_n^{sp}, u_n^{sp}, w_n^{sp})$

$$x_{n+1} = z_{\Delta t}^{(n)} + A_{\Delta t}^{(n)} x_n + B_{\Delta t}^{(n)} u_n + G_{\Delta t}^{(n)} \hat{w}_k$$

- Fixed initial state (given $\hat{x}_k$)

$$\Phi(x_k, x_{k+N}) = x_k - \hat{x}_k$$

[1] Quadratic form $\|z\|_W^2 = z^\mathsf{T} W z$

Model predictive control, affine quadratic regulators

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$$\min_{\substack{x_k,\dots,x_{k+N}\\u_k,\dots,u_{k+N-1}}} \sum_{n=k}^{k+N-1} \left(\|x_n - x_n^{sp}\|_Q^2 + \|u_n - u_n^{sp}\|_R^2\right) + \|x_{k+N} - x_{k+N}^{sp}\|_{Q_f}^2$$

s.t. $(\forall n \in [k, k+N-1])$

$$x_{n+1} = f_{\Delta t}(x_n, u_n, \hat{w}_k | \theta_x)$$

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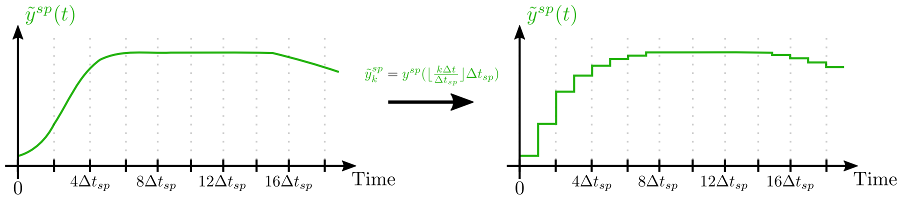
Model predictive control, reference optimisation

Usually, reference trajectories are available only for output variables

(For continuous-time trajectories, we consider a discretisation with $\Delta t_{sp} > 0$)



- Each pair (x_n^{sp}, u_n^{sp}) satisfying $\tilde{y}_n^{sp} \in \mathbb{R}^{N_{\tilde{y}}}$ is the solution of an optimisation[1]:

Steady-state optimisation

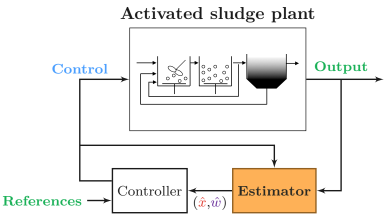
$$\begin{aligned} \min_{x_n^{sp}, u_n^{sp}} \quad & \| H g(x_n^{sp}) - \tilde{y}_n^{sp} \|^2_{W_{\tilde{y}}} + \| u_n^{sp} - \tilde{u}_n^{sp} \|^2_{W_u} \\ \text{s.t.} \quad & f(x_n^{sp}, u_n^{sp}, w_n^{sp} | \theta_x) = 0, \\ & x_n^{sp} \in \mathcal{X}^{sp}, \quad u_n^{sp} \in \mathcal{U}^{sp} \end{aligned}$$

- Search for stationary point $(x_n^{sp}, u_n^{sp}, w_n^{sp})$
- The $N_{\tilde{y}} \leq N_y$ outputs of interest are selected by matrix $H \in \{0,1\}^{N_{\tilde{y}} \times N_y}$
- $W_{\tilde{y}}, W_u \succeq 0$ are tuning parameters

(We consider fixed $w_n^{sp} = w^{SS}$ and $\tilde{u}_n^{sp} = 0$)

[1] Rawlings, J., Mayne, D., and Diehl, M., 2020. Model Predictive Control: Theory, Computation and Design, 2^{nd} edition. Nob Hill Publishing, LLC.

Moving-horizon estimation, formulation



Moving-horizon estimation problem

$$\min_{\substack{\hat{x}_{k-N_e+1}, \dots, \hat{x}_k \\ \hat{w}_{k-N_e+1}, \dots, \hat{w}_k}} \quad L_0(\hat{x}_{k-N_e+1}) + \sum_{n=k-N_e+1}^{k} L(\hat{x}_n, \hat{w}_n | y_n)$$

$$\underset{(\forall n \in [k-N_e, k])}{\text{s.t.}} \quad \hat{x}_{n+1} = f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n | \theta_x)$$

$$\hat{x}_n \in \mathcal{X}, \quad \hat{w}_n \in \mathcal{W}$$

⇝ The optimal estimation problem derives from a *maximum a posteriori* estimate solution

- **Stochastic state-space**

$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t), \hat{w}(t) | \theta_x)$$
$$y(t) = g(\hat{x}(t) | \theta_y) + v(t)$$

with
$$\hat{x}(0) \sim e^{-L_0(\hat{x}(0) | Q_{x_0})}$$
$$\hat{w}(t) \sim e^{-L_w(\hat{w}(t) | R_w)}$$
$$v(t) \sim e^{-L_v(v(t) | Q_v)}$$

- Zero-order hold of disturbances:
(Given the rate of measurement Δt_e)

$$\hat{w}(t) = \hat{w}(t_n) \quad (t \in [t_n, t_{n+1})), \quad t_n = n \Delta t_e$$

- Evolution given by *transition function*

$$\hat{x}_{n+1} = \underbrace{\hat{x}_n + \int_{t_n}^{t_{n+1}} f(\hat{x}(t), u_n, \hat{w}_n) dt}_{f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n)}$$

Moving-horizon estimation, Gauss-Markov estimators

Constrained Gauss-Markov estimator

$$\begin{aligned}\min_{\substack{\hat{x}_{k-N_e+1},\ldots,\hat{x}_k\\ \hat{w}_{k-N_e+1},\ldots,\hat{w}_k}} \quad & L_0(\hat{x}_{k-N_e+1}) + \sum_{n=k-N_e+1}^{k} L(\hat{x}_n, \hat{w}_n | y_n)\\ \underset{(\forall n\in[k-N_e,k])}{\text{s.t.}} \quad & \hat{x}_{n+1} = f_{\Delta t_e}(\hat{x}_n, u_n, \hat{w}_n | \theta_x)\\ & \hat{x}_n \in \mathcal{X}, \quad \hat{w}_n \in \mathcal{W}\end{aligned}$$

- Gaussian distributions for the initial state, disturbances, and measurement noise (Given $\{\bar{x}_n, \bar{w}_n\}_{k-N_e+1}^{k}$ the solutions from previous horizon)

$$\hat{x}_{k-N_e+1} \sim \mathcal{N}(\bar{x}_{k-N_e+1}, Q_{x_0}), \qquad \hat{w}_n \sim \mathcal{N}(\bar{w}_n, R_w), \qquad v_n \sim \mathcal{N}(0, Q_v)$$

- Linearisations around each $P_n := (\bar{x}_n, u_n, \bar{w}_n)$

$$\hat{x}_{n+1} = z_{f_{\Delta t_e}}^{(n)} + A_{\Delta t_e}^{(n)} \hat{x}_n + B_{\Delta t_e}^{(n)} u_n + G_{\Delta t_e}^{(n)} \hat{w}_n$$

$$\hat{y}_n = z_g^{(n)} + C^{(n)} \hat{x}_n$$

- Convex support sets

$$\mathcal{X} = \{x \in \mathbb{R}^{N_x} \mid H_x x \le h_x\}$$

$$\mathcal{W} = \{w \in \mathbb{R}^{N_w} \mid H_w w \le h_w\}$$

Moving-horizon estimation, Gauss-Markov estimators

Constrained Gauss-Markov estimator

$$\min_{\substack{\hat{x}_{k-N_e+1},\cdots,\hat{x}_k\\ \hat{w}_{k-N_e+1},\cdots,\hat{w}_k}} \|\hat{x}_{k-N_e+1}-\bar{x}_{k-N_e+1}\|^2_{Q_{x0}} + \sum_{n=k-N_e+1}^{k}\left(\|\hat{y}_n-y_n\|^2_{Q_v}+\|\hat{w}_n-\bar{w}_n\|^2_{R_w}\right)$$

$$\underset{(\forall n\in[k-N_e,k])}{\text{s.t.}} \quad \hat{x}_{n+1}=f_{\Delta t_e}(\hat{x}_n,u_n,\hat{w}_n|\theta_x)$$

$$\hat{x}_n\in\mathcal{X},\quad \hat{w}_n\in\mathcal{W}$$

- Gaussian distributions for the initial state, disturbances, and measurement noise
  (Given $\{\bar{x}_n,\bar{w}_n\}_{k-N_e+1}^{k}$ the solutions from previous horizon)

$$\hat{x}_{k-N_e+1}\sim\mathcal{N}(\bar{x}_{k-N_e+1},Q_{x0}),\qquad \hat{w}_n\sim\mathcal{N}(\bar{w}_n,R_w),\qquad v_n\sim\mathcal{N}(0,Q_v)$$

- Linearisations around each $P_n := (\bar{x}_n,u_n,\bar{w}_n)$

$$\hat{x}_{n+1}=z^{(n)}_{f_{\Delta t_e}}+A^{(n)}_{\Delta t_e}\hat{x}_n+B^{(n)}_{\Delta t_e}u_n+G^{(n)}_{\Delta t_e}\hat{w}_n$$

$$\hat{y}_n=z^{(n)}_g+C^{(n)}\hat{x}_n$$

- Convex support sets

$$\mathcal{X}=\{x\in\mathbb{R}^{N_x}\mid H_x x\le h_x\}$$

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- Gaussian distributions for the initial state, disturbances, and measurement noise

(Given $\{\bar{x}_n, \bar{w}_n\}^k_{k-N_e+1}$ the solutions from previous horizon)

$$\hat{x}_{k-N_e+1} \sim \mathcal{N}(\bar{x}_{k-N_e+1}, Q_{x0}), \qquad \hat{w}_n \sim \mathcal{N}(\bar{w}_n, R_w), \qquad v_n \sim \mathcal{N}(0, Q_v)$$

- Linearisations around each $P_n := (\bar{x}_n, u_n, \bar{w}_n)$

$$\hat{x}_{n+1} = z^{(n)}_{f_{\Delta t_e}} + A^{(n)}_{\Delta t_e}\hat{x}_n + B^{(n)}_{\Delta t_e}u_n + G^{(n)}_{\Delta t_e}\hat{w}_n$$

$$\hat{y}_n = z^{(n)}_g + C^{(n)}\hat{x}_n$$

- Convex support sets

$$\mathcal{X} = \{x \in \mathbb{R}^{N_x} \mid H_x x \le h_x\}$$

$$\mathcal{W} = \{w \in \mathbb{R}^{N_w} \mid H_w w \le h_w\}$$

Moving-horizon estimation, Gauss-Markov estimators

Constrained Gauss-Markov estimator

$$\min_{\substack{\hat{x}_{k-N_e+1},\dots,\hat{x}_k\\ \hat{w}_{k-N_e+1},\dots,\hat{w}_k}} \|\hat{x}_{k-N_e+1}-\bar{x}_{k-N_e+1}\|^2_{Q_{x0}} + \sum_{n=k-N_e+1}^{k}\left(\|\hat{y}_n - y_n\|^2_{Q_v} + \|\hat{w}_n-\bar{w}_n\|^2_{R_w}\right)$$

$$\underset{(\forall n\in[k-N_e,k])}{\text{s.t.}} \quad \hat{x}_{n+1} = z^{(n)}_{\Delta t_e} + A^{(n)}_{\Delta t_e}\hat{x}_n + B^{(n)}_{\Delta t_e} u_n + G^{(n)}_{\Delta t_e}\hat{w}_n$$

$$H_x\hat{x}_n \leq h_x, \quad H_w\hat{w}_n \leq h_w$$

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Experimental results

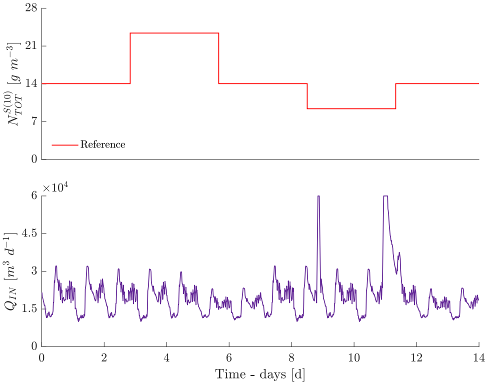
Experiment, objective and simulation results

- **Goal:** Tracking references for $N_{TOT}^{S(10)}$

$$N_{TOT}^{S(10)}(t) = \begin{cases} \frac{5}{3} N_{TOT}^{SS}, & t \in [2.8, 5.6) \text{ d} \\ \frac{2}{3} N_{TOT}^{SS}, & t \in [8.4, 11.2) \text{ d} \\ N_{TOT}^{SS}, & \text{otherwise} \end{cases}$$

Summary of results:

- Tracking accuracy (RMSE): 2.05 g m^{-3}
- The references are satisfied by manipulating $NO_2^- + NO_3^-$ nitrogen inside the reactors



Output MPC Parameters (c-AQR and c-AGM, Reference tracking)

General parameters:

Simulation time $[T]$ = 14 days,
MPC horizon $[N]$ = 12 (12h),
MHE horizon $[N_e]$ = 12 (3h)

Sampling periods:

Control interval $[\Delta t] = (1/24)d$ (1h),
Measurement rate $[\Delta t_e] = (1/96)d$ (15m)

Influent conditions:

$w(\cdot) =$ STORMY WEATHER
$(Q_{IN}^{avg} = 19744\ m^3/d)$
$(S_{NH}^{avg} = 29.48\ d/m^3)$

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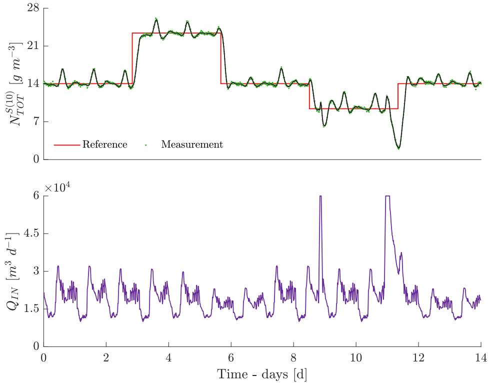

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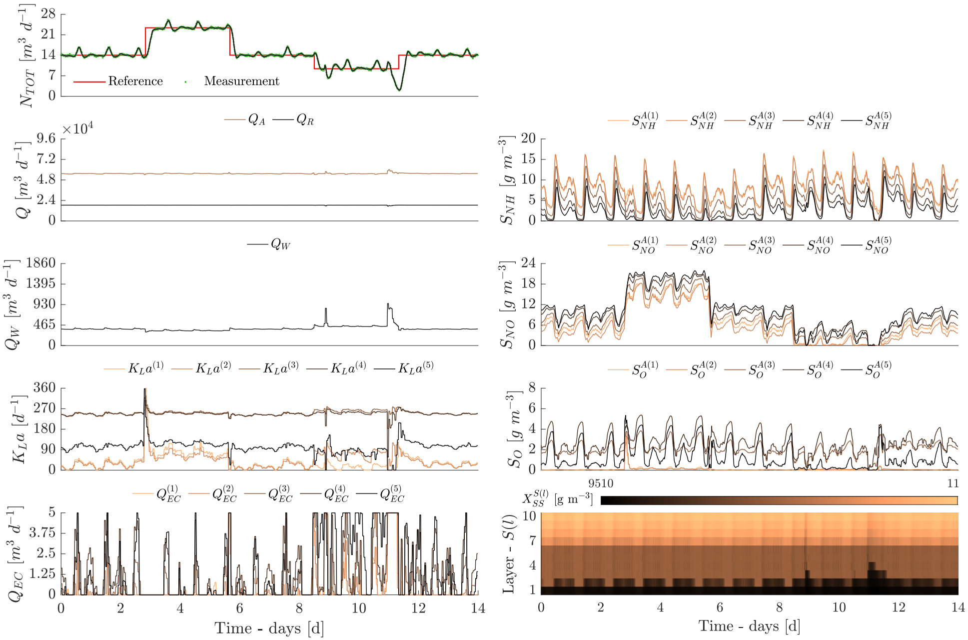
Reference
Measurement
N_{TOT} [m^3 d^{-1}]
Q_A Q_R
$\times 10^4$
Q [m^3 d^{-1}]
Q_W
Q_W [m^3 d^{-1}]
$K_La^{(1)}$ $K_La^{(2)}$ $K_La^{(3)}$ $K_La^{(4)}$ $K_La^{(5)}$
K_La [d^{-1}]
$Q_{EC}^{(1)}$ $Q_{EC}^{(2)}$ $Q_{EC}^{(3)}$ $Q_{EC}^{(4)}$ $Q_{EC}^{(5)}$
Q_{EC} [m^3 d^{-1}]
Time - days [d]
$S_{NH}^{A(1)}$ $S_{NH}^{A(2)}$ $S_{NH}^{A(3)}$ $S_{NH}^{A(4)}$ $S_{NH}^{A(5)}$
S_{NH} [g m^{-3}]
$S_{NO}^{A(1)}$ $S_{NO}^{A(2)}$ $S_{NO}^{A(3)}$ $S_{NO}^{A(4)}$ $S_{NO}^{A(5)}$
S_{NO} [g m^{-3}]
$S_O^{A(1)}$ $S_O^{A(2)}$ $S_O^{A(3)}$ $S_O^{A(4)}$ $S_O^{A(5)}$
S_O [g m^{-3}]
$X_{SS}^{S(l)}$ [g m^{-3}]
9510
11
Layer - $S(l)$
Time - days [d]

Thank you!

Questions?

A receding-horizon framework for optimal control of ASPs
March 18, 2022