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On the controllability of activated sludge plants

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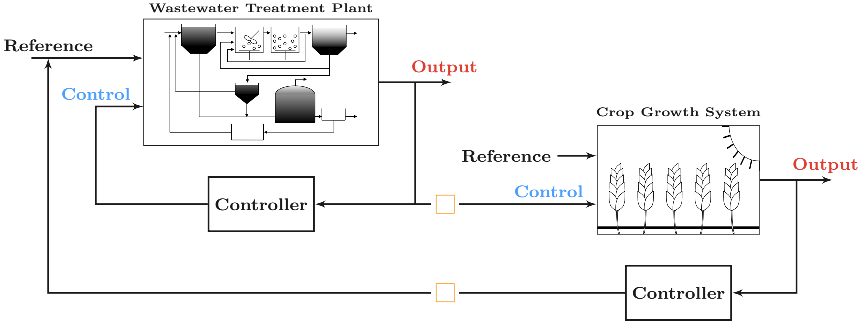
Introduction and motivation

Control and Estimation for Wastewater Treatment Plants (WWTP)

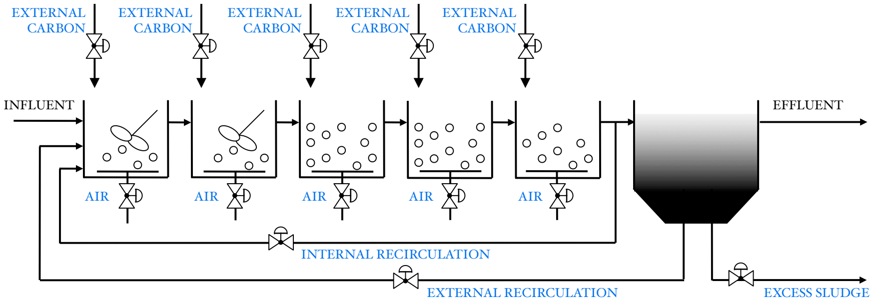
- **Objective:** Sustainable reuse of wastewater
  - ⇝ A treatment-reclamation system for agricultural purposes



Motivation

Is it possible to operate the treatment plant to produce reusable wastewater of a specified quality, on demand?

Activated Sludge Process, description



For the task, we considered a conventional Activated Sludge Process (ASP)

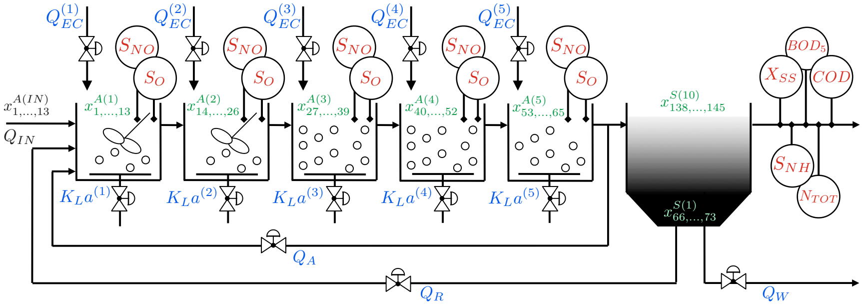
- The **Benchmark Simulation Model no. 1 (BSM1)**[1]

Plant layout

- 5 sequential bio-reactors (Activated Sludge Model no. 1)
- A non-reactive settler (10-layers double-exponential settling model)

[1] Gernaey, K., Jeppsson, U., Vanrolleghem, P., Copp, J., 2014. Benchmarking of Control Strategies for Wastewater Treatment Plants. IWA.

System-oriented description of the process



Reactor $(k = 1, \cdots, 5)$

State variables

⇝ $x^{A(k)} \in \mathbb{R}^{13}_{\geq 0}$

Input variables

⇝ $u^{A(k)} = [K_La^{(k)} \; Q_{EC}^{(k)}]$ ⇝ $[Q_A]$

⇝ $d^{A(1)} = [Q_{IN} \quad x^{A(IN)}]$

Measurement variables

⇝ $y^{A(k)} = [S_O^{A(k)} \quad S_{NO}^{A(k)}]$

Settler $(m = 1, \cdots, 10)$

State variables

⇝ $x^{S(m)} \in \mathbb{R}^{8}_{\geq 0}$

Input variables

⇝ $[Q_R \; Q_W]$

Measurement variables

⇝ $y^{S(10)} = [X_{SS}^{S(10)} \quad S_{NH}^{S(10)} \quad BOD_5^{S(10)} \quad COD^{S(10)} \quad N_{tot}^{S(10)}]$

State-space representation of the process

$$\dot{x}(t) = f(x(t), u(t), d(t)|\theta_x)$$
$$y(t) = g(x(t)|\theta_y)$$

- $\rightsquigarrow$ $x(t) = [x^{A(1)} \, \cdots \, x^{A(5)} \; x^{S(1)} \, \cdots \, x^{S(10)}]^T$
- $\rightsquigarrow$ $u(t) = [Q_A \; Q_R \; Q_W \; u^{A(1)} \, \cdots \, u^{A(5)}]^T$
- $\rightsquigarrow$ $y(t) = [y^{A(1)} \, \cdots \, y^{A(5)} \; y^{S(10)}]^T$
- $\rightsquigarrow$ $d(t) = [d^{A(1)}]^T$
- $\rightsquigarrow$ $\{\theta_x, \theta_y\}$: Model parameters

- An "expansion" of the state-vector compared to common representations
- All possible control and sensors that do not require changes in the plant layout

$\rightsquigarrow$ $N_x = 5 \times 13 + 10 \times 8$ $= 145$ state variables

$\rightsquigarrow$ $N_u = 3 + 5 \times 2$ $= 13$ controls

$\rightsquigarrow$ $N_y = 5 \times 2 + 5$ $= 15$ sensors

We try to address our initial question by studying two properties of this model

Full-state Controllability
Can we manipulate $u(t)$ to steer the state-vector $x(t)$ to a desired value?

Full-state Observability
Can we reconstruct the state-vector $x(t)$ from measurements $y(t)$?

Controllability analysis

Controllability

A system is *controllable* if there exists a control $u(t)$ transferring initial state $x(0) = x_0$ to any final state $x(t_f) = x_{t_f}$, for $0 < t_f < \infty$.

Kalman's Controllability Test

Given a linear(-ised) system (A, B) and matrix $\mathcal{C} = [B \quad AB \quad A^2B \; \cdots \; A^{N_x - 1}B]$

$\rightsquigarrow$ (A, B) is controllable $\Leftrightarrow$ $\texttt{rank}(\mathcal{C}) = N_x$

Advantages

- A direct test when state-space is low-dimensional
- Allows for a direct definition of
  - $\rightsquigarrow$ Controllable subspace
  - $\rightsquigarrow$ Uncontrollable subspace

Disadvantages

- Becomes ill-posed when state-space is high-dimensional
- Requires a specific linearisation

(We should consider an alternative approach...)

Structural controllability, definition

The system $\dot{x}(t) = f(\cdot|\theta_x)$, with $y(t) = g(\cdot|\theta_y)$, from a structural perspective

$$A_{i,j} = \frac{\partial f_i}{\partial x_j} \begin{cases} \neq 0 & (x_j \text{ affects } x_i) \\ = 0 & \text{o/w} \end{cases} \quad B_{i,k} = \frac{\partial f_i}{\partial u_k} \begin{cases} \neq 0 & (u_k \text{ affects } x_i) \\ = 0 & \text{o/w} \end{cases} \quad C_{k,j} = \frac{\partial g_k}{\partial x_j} \begin{cases} \neq 0 & (x_j \text{ affects } y_k) \\ = 0 & \text{o/w} \end{cases}$$

This structural system describes a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

$$\begin{aligned} \mathcal{V} &= \mathcal{V}_A \cup \mathcal{V}_B \cup \mathcal{V}_C \\ &= \{x_1, \cdots, x_{N_x}\} \cup \{u_1, \cdots, u_{N_u}\} \\ &\quad \cup \{y_1, \cdots, y_{N_y}\} \end{aligned} \qquad \begin{aligned} \mathcal{E} &= \mathcal{E}_A \cup \mathcal{E}_B \cup \mathcal{E}_C \\ &= \{(x_j, x_i) | A_{i,j} \neq 0\} \cup \{(u_k, x_i) | B_{i,k} \neq 0\} \\ &\quad \cup \{(x_j, y_k) | C_{k,j} \neq 0\} \end{aligned}$$

Structural Controllability

The pair (A, B) with network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is *structurally controllable* IFF

- Any realisation of A and B results in a controllable system (classical sense)

Conditions for Structural Controllability

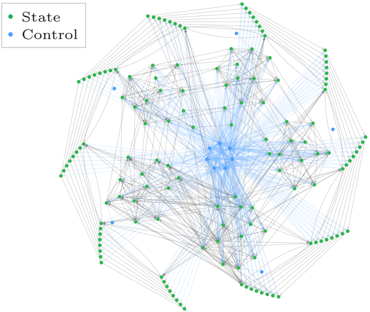
- **Accessibility**
  There exists at least one path starting from any $u_k \in \mathcal{V}_B$ to each $x_i \in \mathcal{V}_A$
- **Dilation-free**
  For every $\mathcal{S} \subseteq \mathcal{V}_A$, $|T(\mathcal{S})| \geq |\mathcal{S}|$, where $T(\mathcal{S})$ is the neighborhood set of $\mathcal{S}$

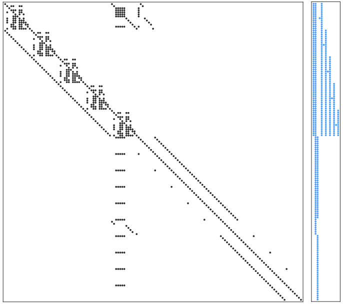
BSM1 - Structural representation

We analyse the structural controllability of the BSM1 given the network $\mathcal{G}_\mathcal{C} = (\mathcal{V}_\mathcal{C}, \mathcal{E}_\mathcal{C})$

$$\mathcal{V}_\mathcal{C} = \mathcal{V}_A \cup \mathcal{V}_B = \{x_1, \cdots, x_{145}\} \cup \{u_1, \cdots, u_{13}\} \qquad A = \partial f / \partial x \quad (A \in \mathbb{R}^{145 \times 145})$$

$$\mathcal{E}_\mathcal{C} = \mathcal{E}_A \cup \mathcal{E}_B = \{(x_j, x_i) | A_{i,j} \neq 0\} \cup \{(u_k, x_i) | B_{i,k} \neq 0\} \qquad B = \partial f / \partial u \quad (B \in \mathbb{R}^{145 \times 13})$$



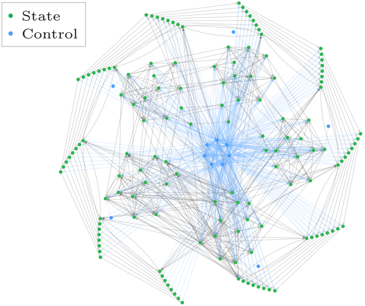


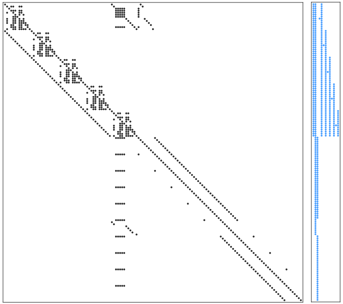
- Network $\mathcal{G}_\mathcal{C} = (\mathcal{V}_\mathcal{C}, \mathcal{E}_\mathcal{C})$
  - ⇝ Self-loops are not shown
- Structural pair (A, B)
  - ⇝ Colored according to $\mathcal{G}_\mathcal{C} = (\mathcal{V}_\mathcal{C}, \mathcal{E}_\mathcal{C})$

BSM1 - Structural controllability results

- Accessibility:
  All state vertices are reachable from the given set of control vertices
- Dilation-free:
  As all state vertices have self-loops, it always holds that $|T(\mathcal{S})| \geq |\mathcal{S}|$, for every $\mathcal{S} \subseteq \mathcal{V}_A$

- The topology of $\mathcal{G}_\mathcal{C} = (\mathcal{V}_\mathcal{C}, \mathcal{E}_\mathcal{C})$ indicates that (A, B) is **structurally controllable**





The plant described by $\dot{x}(t) = f(\cdot|\theta_x)$ is controllable for almost all possible realisations of matrices A and B

Classical analysis

The benchmark suggests a linearisation using steady-state $SS \equiv (x^{SS}, u^{SS}, d^{SS}, y^{SS})$

$$\dot{x}(t) = A^{SS}x(t) + B^{SS}u(t) + G^{SS}d(t)$$
$$y(t) = C^{SS}x(t)$$

- A realisation of the structural model

$$A^{SS} = \frac{\partial f}{\partial x}\bigg|_{SS} \qquad B^{SS} = \frac{\partial f}{\partial u}\bigg|_{SS} \qquad C^{SS} = \frac{\partial g}{\partial x}\bigg|_{SS}$$

Verify if the structural controllability result holds for (A^{SS}, B^{SS})

The computation of controllability matrix $[B^{SS} \quad A^{SS}B^{SS} \quad \cdots \quad (A^{SS})^{N_x - 1}B^{SS}]$ suffers from round-off errors $\rightsquigarrow$ Need for an alternative scalable controllability test

Classical analysis, PBH controllability test

Popov-Belevitch-Hautus (PBH) Controllability Test

The pair (A, B) is controllable IFF

- $\leadsto$ $\texttt{rank}(\begin{bmatrix}\lambda I - A & B\end{bmatrix}) = N_x, \forall \lambda \in \mathbb{C}$
- $\leadsto$ $\texttt{rank}(\begin{bmatrix}\lambda_i I - A & B\end{bmatrix}) = N_x, \forall \lambda_i \in \sigma(A) \subset \mathbb{C}$ $\quad (\sigma(A) = \{\lambda_i\}_{i=1}^{N_x}$, spectrum of $A)$

Requires a total of N_x rank evaluations for a $N_x \times (N_x + N_u)$ matrix

- BSM1: The spectrum $\sigma(A^{SS})$ consists of 69 distinct eigenvalues $\{\lambda_i(A^{SS})\}$
  - $\leadsto$ $\{\lambda_1, \cdots, \lambda_{31}\} \subset \mathbb{R}$
  - $\leadsto$ $\{\lambda_{32}, \lambda_{32}^*, \cdots, \lambda_{69}, \lambda_{69}^*\} \subset \mathbb{C}$

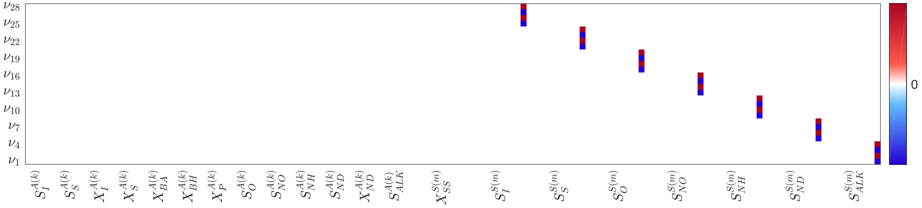
Provides a relationship between eigenvectors $\nu_i(\lambda_i)$ and controllability subspaces

- If $\texttt{rank}([\lambda_i I - A \;\; B]) < N_x$ then ν_i lies in the uncontrollable subspace

BSM1 - PBH controllability test results

The PBH test indicates that (A^{SS}, B^{SS}) is **uncontrollable in the classical sense**

- A real eigenvalue failing the test
  - Algebraic multiplicity: 28
  - Geometric multiplicity: 7
- The non-zero entries of associated eigenvectors $\nu_1, \cdots, \nu_{28}$ correspond to soluble matter in the effluent



For linearisation (A^{SS}, B^{SS}), we cannot control the effluent concentrations of soluble matter

Contradiction between classical and structural results

We found a contradiction between the controllability results

- (A, B) is **controllable in a structural sense**
- (A^{SS}, B^{SS}) is **uncontrollable in a classical sense**

Dilation-free condition: A known issue whenever some self-loops weights are identical

- **Non-reacting matter in reactors:** $S_a^{A(k)}$ $(a \in \{I, ALK\})$ and $X_b^{A(k)}$ $(b \in \{I, P\})$
  - $$\frac{\partial \dot{S}_a^{A(k)}}{\partial S_a^{A(k)}} = \frac{\partial \dot{X}_b^{A(k)}}{\partial X_b^{A(k)}} = -\frac{Q_A + Q_R + Q_{IN} + \sum_{j=1}^{k} Q_{EC}^{(j)}}{V_A^{(k)}}$$

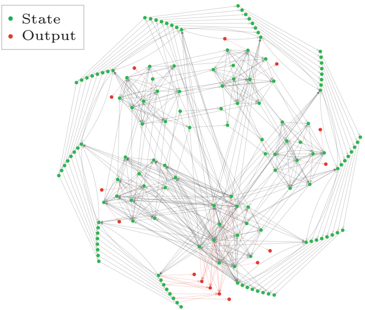
- **Soluble matter in the settler:** $S_c^{S(m)}$ $(c \in \{I, S, O, NO, NH, ND, ALK\})$
  - For $m = 1, \cdots, 5$
    $$\frac{\partial \dot{S}_c^{S(m)}}{\partial S_c^{S(m)}} = \frac{-Q_R - Q_W}{V_S^{(m)}}$$
  - For $m = 6$
    $$\frac{\partial \dot{S}_c^{S(m)}}{\partial S_c^{S(m)}} = \frac{-Q_{IN} + Q_R}{V_S^{(m)}}$$
  - For $m = 7, \cdots, 10$
    $$\frac{\partial \dot{S}_c^{S(m)}}{\partial S_c^{S(m)}} = \frac{Q_W - Q_{IN}}{V_S^{(m)}}$$

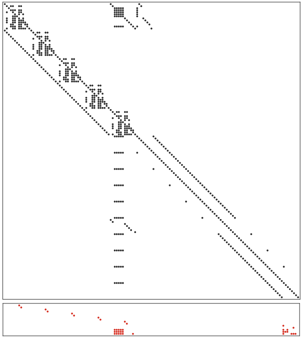
We can never control the full state-space for the model $\dot{x}(t) = f(\cdot|\theta_x)$, regardless of the linearisation being used

BSM1 - Structural observability results

- Accessibility: There are no paths from vertices $S_O^{S(7\rightsquigarrow 10)}$, $S_{ALK}^{A(1\rightsquigarrow 5)}$ and $S_{ALK}^{S(1\rightsquigarrow 10)}$ to any output vertex
- Dilation-free: As all state vertices have self-loops, it always holds that $|T(\mathcal{S})| \geq |\mathcal{S}|$, for every $\mathcal{S} \subseteq \mathcal{V}_A$

- The topology of $\mathcal{G}_\mathcal{O} = (\mathcal{V}_\mathcal{O}, \mathcal{E}_\mathcal{O})$ indicates that (A, C) is **structurally unobservable**



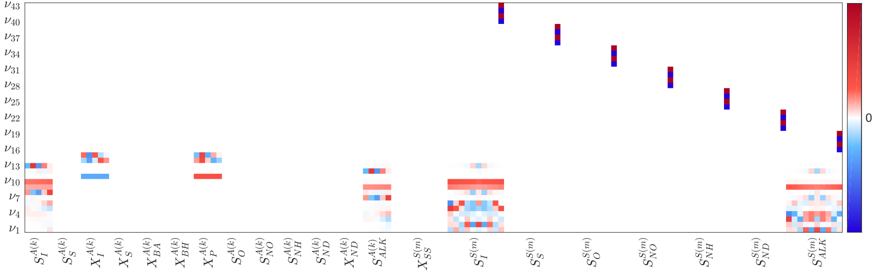


The plant $\dot{x}(t) = f(\cdot|\theta_x)$ with measurements $y(t) = g(\cdot|\theta_y)$ is unobservable for all possible realisations of matrices A and C

BSM1 - PBH observability test results

The PBH test indicates that (A^{SS}, C^{SS}) is **unobservable in the classical sense** (as expected)

- 10 distinct eigenvalues failing the test
  - Including 5 complex pairs and 2 real values with multiplicity larger than 1
- Total of 43 eigenvectors $(\nu_1, \cdots, \nu_{43})$
- Non-zero entries correspond to
  - All non-reacting components
  - Soluble matter in the effluent



For linearisation (A^{SS}, C^{SS}), we cannot unequivocally determine the state-vector from a sequence of outputs over a finite time interval

Final Remarks

The controllability and observability of an Activated Sludge Process were studied

Our results show that

- Pair (A, B): controllable but unobservable in the structural sense
- Pair (A^{SS}, B^{SS}): uncontrollable and unobservable in the classical sense
- A large portion of the state-space is still controllable (and observable)

These results will be the backbone to the design of optimal controllers for the treatment-reclamation application we described

Thank you!