W1—W2

PART 1 Numerical Optimization

- Root finding / Newton's type methods
  - Convergence rates
  - Contraction
  - Invariance
  - Conditions for convergence
- Nonlinear Optimisation (1st + 2nd ORDER OPTIMALITY CONDITIONS)
- Newton's type methods
  - Equality constrained
  - Inequality constrained

- (Automatic Differentiation)
- (Parameter estimation

0

W3 - W4

PART 2 Discrete time Optimal Control

- Formulation, Analysis and Structure
- Dynamic Programming (Discrete state space, LQ, H_∞, LQR)
  - → Gradient of the value function
  - → Minimum principle in DT
  - → Iterative DP
  - → Differential Dynamic Programming.

W5-W6

PART 3 Optimal Control in Continuous-time

- Formulation, overview
- Numerical Integration (Explicit One Step, Orthogonal Collocation)
- Hamilton-Jacobi-Bellman (LQR and Ricatti Eqn's)
- Pontryagin and Indirect Approach (Solution, Regular + Singular Arcs)
- Direct Approaches (Single/Multiple Shooting, Direct Collocation)

W6

(PART 4 MPC and MOVING PREDICTION HORIZON)

- NMPC
- Parametric NLP
- MHE (State and Parameter estimation
- Feedback Linearization

APC 01

CHEM- E7165

- 24 h LECTURES
- 24 h EXERCISES

} HYBRID, AS ALWAYS

THE OBJECTIVE OF THE COURSE IS TO PROVIDE AN INTRODUCTION TO OPTIMAL CONTROL (MATHEMATICS + NUMERICS)

THERE WILL BE SOME CODING

- PYTHON
- MATLAB

- INTUITIVE ROUTINES FOR OPTIMAL CONTROL

TUESDAYS — 2/1 SESSIONS (LECTURE + EXERCISES)

WEDNESDAYS — 1 SESSION (LECTURE)

FRIDAYS — 1/2 SESSION (EXERCISES + CLASSES)

EXAM ON FEB 14 — WE WILL NOT HAVE AN EXAM

→ EXERCISES → PROJECT WORK

APC 01a

OUTCOMES

- UNDERSTAND THE BASIC FORMULATION OF NONLINEAR PROGRAMMING (GENERAL CLASS OF NUMERICAL OPTIMIZATION PROBLEMS)

- UNDERSTAND THE BASIC METHODS OF NUMERICAL INTEGRATION OF DYNAMICAL MODELS (ODE SIMULATION)

- UNDERSTAND HOW TO COMBINE THE ABOVE TO FORMULATE AND SOLVE OPTIMAL CONTROL PROBLEMS
  - DISCRETE TIME
  - CONTINUOUS TIME

OPTIMAL CONTROL CONSIDERS THE OPTIMIZATION OF DYNAMICAL SYSTEMS

IT BRIDGES TWO LARGE FIELDS OF APPLIED MATHEMATICS

* NUMERICAL OPTIMIZATION
* SYSTEM THEORY (NUMERICAL SIMULATION + ANALYSIS)

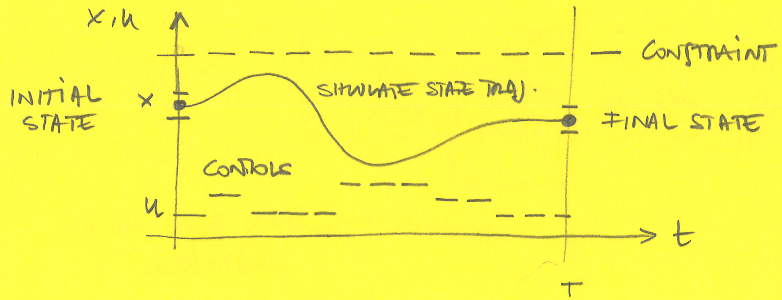
IN THIS COURSE WE REFRESH OR INTRODUCE BASIC CONCEPTS FROM BOTH FIELDS WITH SOME MORE WEIGHT TO NUMERICAL OPTIMIZATION

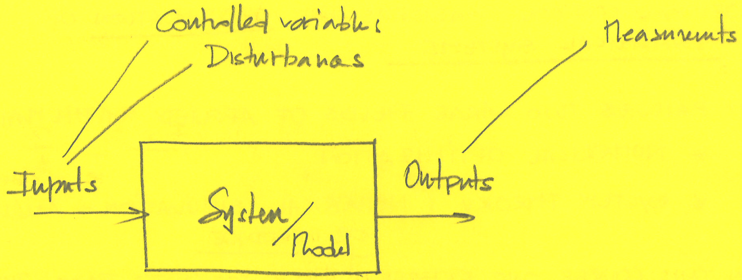
WHAT IS REQUIRED?
- LINEAR ALGEBRA
- NUMERICAL ANALYSIS
- CODING

THERE ARE ~~(FOUR)~~ (THREE) MAJOR PARTS

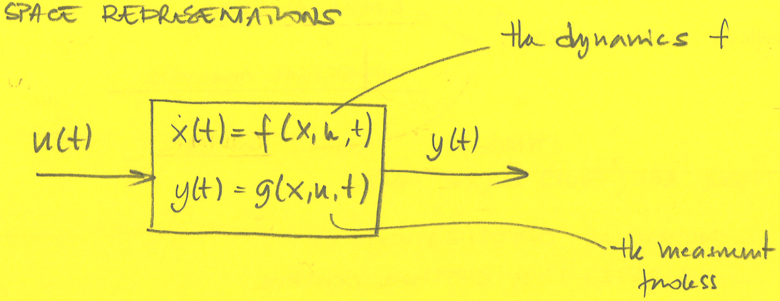
- NUMERICAL OPTIMIZATION
- DISCRETE-TIME OPTIMAL CONTROL
- CONTINUOUS-TIME OPTIMAL CONTROL
- (ONLINE OPTIMAL CONTROL)

FOR A GIVEN SYSTEM STATE X, WHICH CONTROLS U LEAD TO THE BEST (IN SOME SENSE) OBJECTIVE, WITHOUT VIOLATING THE CONSTRAINTS?

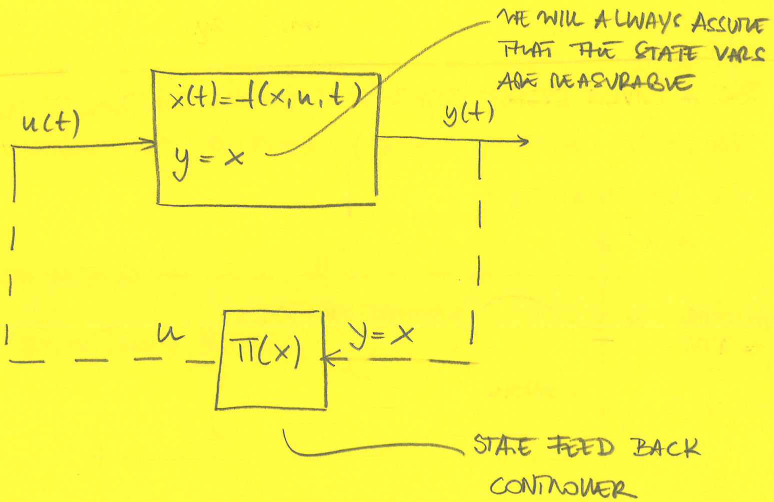




STATE SPACE REPRESENTATIONS



x are the state variables $\longrightarrow$ they evolve in time.



- WE IDENTIFY DYNAMICAL SYSTEMS AS PROCESSES THAT EVOLVE IN TIME

$\rightsquigarrow$ They can be characterized by STATES x that allow us to predict the future behaviour of the system

$\rightsquigarrow$ Often the dynamics can be controlled by a suitable choice of inputs, we denote them as CONTROLS u

TYPICALLY THE CONTROLS SHOULD BE OPTIMALLY CHOSEN

$\rightsquigarrow$ In such a way that an OBJECTIVE FUNCTION is optimized subjected to some CONSTRAINTS

TECHNIQUES FOR NUMERICALLY CHOSING OPTIMAL CONTROLS IS THE MAIN TOPIC OF THIS COURSE

An important property of a dynamical system is the INITIAL VALUE OF THE STATE VARIABLES, x_0 and the CONTROL INPUT TRAJECTORY $u(t)$ for all $t \in [0, T]$

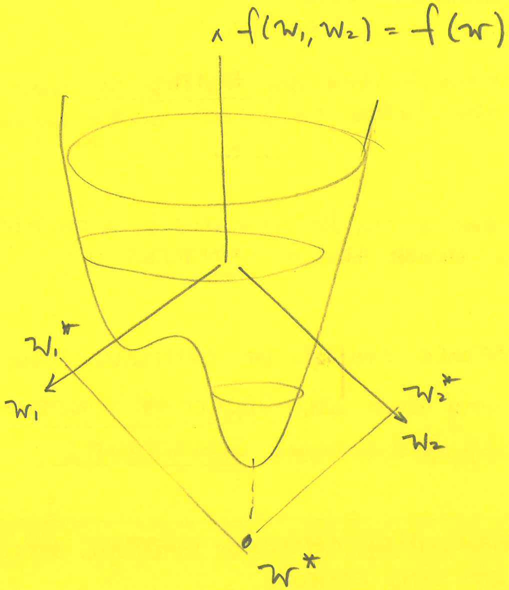
Their knowledge allow us to determine the state trajectory $x(t)$, for all $t \in [0, T]$

WE DISCUSS IMPORTANT CLASSES OF DYNAMICAL MODELS

2 DISCRETE - TIME

1 CONTINUOUS - TIME

THEN WE OVERVIEW IMPORTANT CLASSES OF OPTIMIZATION PROBLEMS



WHAT IS OPTIMISATION?

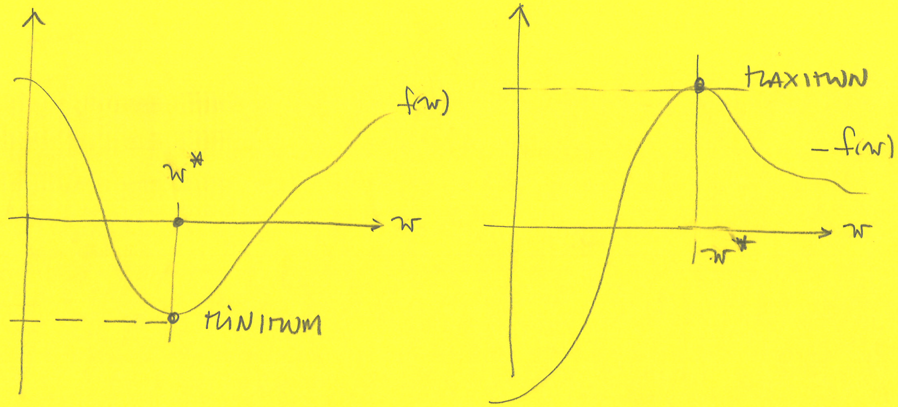
OPTIMISATION MEANS 'SEARCH FOR THE BEST SOLUTION'

OPTIMAL

IN SOME SENSE

MATHEMATICALLY, THIS MEANS 'MINIMISATION' OR 'MAXIMISATION' OF SOME OBJECTIVE FUNCTION

- $f(w)$ IS USED TO DENOTE THE OBJECTIVE FUNCTION
- w ARE THE 'DECISION VARIABLES'

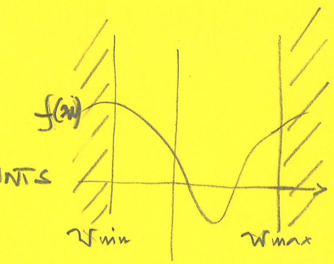


SOLVING MAXIMISATION PROBLEMS IS EQUIVALENT TO SOLVING MINIMISATION PROBLEM

→ SAME w^*

FROM NOW ON, MINIMISATION

VARIABLES X ARE SUBJECTED TO CONSTRAINTS



APC 04

PYTHON (or MATLAB) + CASADI

WE WILL HAVE A QUICK INTRO

WE ALREADY HAD A QUICK DEMO (GIVE SLIDES)

WE WILL HAVE A QUICK INTRO

APC 05