

Each recursive call (**queen-cols** i) generates the sequence of possible solutions for the first i columns on the $k \times k$ board. Let q_i denote the length of that sequence and T_i the time required to generate it. Then

$$T_i = T_{i-1} + T'_i$$

where T'_i is the time required to generate the sequence of solutions for i columns from the sequence of solutions for $i - 1$ columns.

$$T'_i = T_{\text{filter}} + T_{\text{flatmap}}$$

Filtering the sequence of proposed solutions requires $i - 1$ checks for each of the $q_{i-1}k$ proposed solutions, so

$$T_{\text{filter}} = q_{i-1}k(i - 1)$$

In the build-up step we append each of the k column indices to each of the q_{i-1} solutions from the previous step and use **flatmap** to combine these q_{i-1} sequences of proposed solutions to a single sequence.

$$T_{\text{flatmap}} = q_{i-1}k + T_{\text{append}}$$

flatmap is implemented in terms of **append**. Assuming that **append** requires time proportional to the length of its first argument, we have $q_{i-1} - 1$ calls to **append** where the length of the first sequence is k , $2k$, $3k$ and so on until $(q_{i-1} - 1)k$.

$$\begin{aligned} T_{\text{append}} &= k \cdot \sum_{j=1}^{q_{i-1}-1} j = \dots = k \cdot \frac{q_{i-1}(q_{i-1} - 1)}{2} \\ T_{\text{flatmap}} &= k \cdot \left(q_{i-1} + \frac{q_{i-1}(q_{i-1} - 1)}{2} \right) \\ &= q_{i-1}k \cdot \left(1 + \frac{q_{i-1} - 1}{2} \right) \\ T'_i &= q_{i-1}k(i - 1) + q_{i-1}k \cdot \left(1 + \frac{q_{i-1} - 1}{2} \right) \\ &= q_{i-1}k \cdot \left(i + \frac{q_{i-1} - 1}{2} \right) \end{aligned}$$