

Tiered Live Variable Analysis for Heavy-Tailed CFG Distributions

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Abstract

Live range analysis is typically taught as a monotone fixed-point computation with pessimistic worst-case iteration bounds. This paper shows that, on real compiler workloads, both control-flow structure and convergence are far more regular than that view suggests.

Across 290,000 functions from the Go toolchain, CFGs fall into three regimes: 68% are acyclic, 24% contain exactly one loop kernel (median 6 blocks), and only 8% exhibit multiple cyclic regions; overall sizes are heavy-tailed ($\text{dispersion} \approx 175$). More strikingly, *every* strongly connected component in this dataset reaches a liveness fixed point in exactly three traversals when we alternate postorder and reverse postorder (PO–RPO–PO).

These observations motivate a tiered solver: give acyclic code a one-pass fast path, isolate cyclic regions via SCC decomposition, and apply the three-pass scheme only inside non-trivial SCCs. Our contribution is empirical—a large-scale CFG characterization and a convergence regularity that enables a simple, uniform implementation with predictable behavior.

1 Introduction

Live-variable (live-range) analysis is the ubiquitous backward data-flow pass that drives interference construction, spilling, and many SSA-based optimizations. In principle it is a monotone fixed-point problem; in practice, its cost is dominated by how often information must circulate around cycles in the control-flow graph (CFG).

The literature and textbooks largely reason about arbitrary CFG instances and worst-case iteration [8, 1, 4]. Modern compilers, however, run the same solver millions of times on a *workload distribution*. When that distribution has distinct structural regimes, a single “one-size-fits-all” solver leaves performance on the table.

This paper makes two empirical claims from a 290,000-function study of the Go toolchain: (i) CFGs are heavy-tailed and stratify into three regimes (68% acyclic, 24% single-loop, 8% multi-cycle), and (ii) within each strongly

connected component (SCC), liveness converges in exactly three alternating traversals (postorder, reverse postorder, postorder). Together, these facts justify a *tiered* architecture that optimizes the common case while remaining robust on the tail.

From examples to distributions. Figure 1 shows a textbook-sized CFG fragment. Such examples are invaluable for correctness arguments, but they hide the question that matters for compiler engineering: *what kinds of CFGs occur in bulk, and what solver structure matches that distribution?*

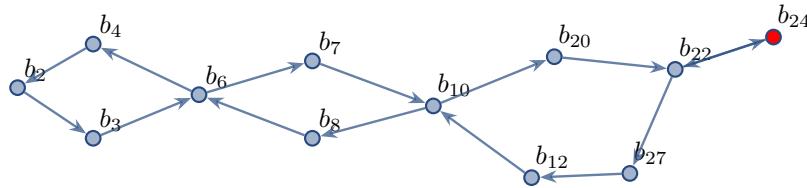


Figure 1: A typical CFG fragment used in algorithm exposition. Single-instance analysis reveals local structure but says nothing about the *distribution* of such structures a compiler must handle.

But this focus on individual instances is overrated for engineering purposes. A production compiler does not process one CFG—it processes hundreds of thousands, drawn from a distribution with complex structure. The question is not “how does the algorithm behave on *this* graph?” but “how does it behave across the full workload?” Recent data-driven work has begun to address this gap, using profiling and traces to guide allocation decisions in ways that outperform purely static analyses [16, 7, 12]. We contribute to this empirical turn by characterizing the *structural distribution* of CFGs in a production workload.

The core observation. We analyzed 290,334 CFG instances from a full build-and-test of the Go toolchain. The size distribution is striking:

- **Median:** 9 blocks, 7 SCCs
- **90th percentile:** 44 blocks, 31 SCCs
- **Maximum:** 12,676 blocks
- **Dispersion index:** ≈ 175 (vs. 1 for Poisson)

But the *structural* distribution is even more revealing. CFGs fall into three regimes based on their cyclic complexity:

- **68% acyclic:** All SCCs are singletons—no back edges, no iteration needed.
- **24% single-loop:** Exactly one non-trivial SCC (median size 6 blocks)—one localized region requiring iteration.

- **8% complex:** Multiple non-trivial SCCs—genuinely complex control flow.

The variance is $175 \times$ the mean—there is no “characteristic scale” for CFG complexity. Simple parametric models (Poisson, negative binomial) fail to capture this structure.

Design implications. This distribution fundamentally shapes how we should build live variable solvers:

1. **Average-case metrics mislead.** A mean of 20 blocks hides that end-to-end time is dominated by tail functions, not the median.
2. **Simple cases deserve simple treatment.** The 50% of CFGs with ≤ 9 blocks need minimal overhead—elaborate data structures hurt more than they help.
3. **Tail cases are more tractable than theory suggests.** While textbook worst-case analysis implies $O(|B|)$ iterations, our empirical study reveals that *all* SCCs in our dataset converge in exactly three passes with an alternating traversal order.
4. **Stratification reveals structure.** The poor fit of global models is *good news*: it indicates exploitable heterogeneity across packages, compilation stages, and code patterns.

Contributions. This paper makes two contributions:

1. **Structural characterization of real-world CFGs.** We provide the first large-scale empirical analysis of CFG cyclic structure, showing that production workloads fall into distinct regimes (68% acyclic, 24% single-loop, 8% complex) that demand tiered treatment.
2. **Three-pass convergence for SCCs.** We report the empirical finding that data-flow equations over strongly connected components converge in exactly three passes when using alternating traversal order (postorder \rightarrow reverse postorder \rightarrow postorder). This is significantly better than the theoretical $O(|B|)$ bound and, to our knowledge, has not been previously documented.

1.1 Positioning Our Claims in the Literature

Our results are not a new formulation of data-flow analysis; they are measurements that sharpen what parts of the standard story matter in production, and they suggest a simpler solver organization than worst-case reasoning alone would imply.

What is standard. Live-variable analysis sits in the classic monotone-framework tradition [4, 3, 9]. Worklist iteration, and SCC decomposition to localize cyclic dependence, are textbook tools for obtaining linear-time behavior on acyclic regions and convergence on cyclic ones [1, 13, 14]. Traversal order (postorder, reverse postorder) is also a familiar engineering knob, typically discussed as a heuristic for faster convergence rather than as a property that can be stated at the workload level.

What we add empirically. First, we characterize CFG cyclic structure at scale and show that real workloads are not well summarized by a single “typical” CFG: they mix regimes with very different solver needs. Second, and more surprisingly, we observe a workload-wide convergence regularity: every SCC in our dataset reaches a fixed point in exactly three alternating traversals. To our knowledge, this “PO–RPO–PO in three” phenomenon has not been previously documented for liveness in production compiler workloads.

Why this matters for design. Because most CFGs are acyclic and most cyclic regions are small, a solver can be structured as a dispatcher rather than a monolith: a one-pass acyclic fast path, an SCC-local solver for reducible cycles, and a rare fallback for irreducible control flow. This paper develops that tiered architecture and evaluates how well the three-pass SCC solver matches the observed distribution.

Our approach. We advocate a *tiered architecture* that naturally adapts to the distribution: SCC decomposition [13, 14] partitions the CFG into acyclic regions that need only one pass each, plus cyclic kernels. Within each kernel, we apply the three-pass algorithm with alternating traversal order. This combination is fast on typical small CFGs, efficient on cyclic regions, and avoids the complexity of general worklist solvers—achieving both the speed linear-scan methods promise [10] and predictable behavior on complex cases.

The remainder of this paper develops this approach. Section 2 summarizes the empirical CFG distribution and structural regimes. Section 3 translates these statistics into design principles. Section 4 presents the tiered solver, including the three-pass algorithm. Appendix A details the data-flow equations and their iterative solution, Appendix B provides full statistical analysis, and Appendix C gives algorithm pseudocode.

2 The Statistical Reality of CFG Structure

Table 1 summarizes the distribution. The dispersion index $D = \text{Var}(X)/\mathbb{E}[X]$ quantifies over-dispersion: $D \approx 1$ for Poisson processes, but we observe $D \approx 175\text{--}180$. This indicates the workload contains many distinct “regimes” of CFG complexity, not a single characteristic scale.

Attempting to fit a negative binomial model (the standard extension for over-dispersed counts) yields shape parameters near $r \approx 0.1$, indicating an

Metric	Median	p_{90}	Max	Mean	Var	Var/Mean
Blocks	9	44	12,676	20.10	3521	175
SCCs	7	31	12,676	15.31	2749	180

Table 1: CFG statistics from 290,656 functions in Go build-and-test workload.

extremely heavy tail. Yet even this model fits poorly—the empirical distribution shows multi-modal structure from distinct code populations (small leaf functions, medium inlined code, large state machines). Full details appear in Appendix B.

Key takeaway. The distribution is *heterogeneous* and *heavy-tailed*. Solver design must account for both the common case (small, simple CFGs) and the tail (large, complex CFGs that dominate worst-case time and memory).

2.1 Structural Regimes: Where Cycles Live

Size statistics alone do not capture what matters for iterative solvers: *cyclic structure*. A CFG with 100 blocks but no back edges needs only one pass; a 20-block CFG with nested loops may need many. We classified each CFG by its non-trivial SCC count (SCCs with >1 block).

Regime	Count	Fraction
Acyclic (0 non-trivial SCCs)	197,091	67.9%
Single-loop (1 non-trivial SCC)	68,357	23.5%
Multi-loop (2+ non-trivial SCCs)	24,886	8.6%

Table 2: CFG structural regimes. Over two-thirds of CFGs have no cycles at all.

Figure 2 shows the distribution of largest SCC size across all CFGs. The dominant spike at size 1 represents the acyclic majority—nearly 200,000 CFGs where every block is its own SCC.

For the 24% of CFGs with exactly one non-trivial SCC, Figure 3 shows the size distribution of that single loop kernel. The median is just 6 blocks, with 90% under 27 blocks—but the tail extends to 376 blocks.

Implications for iteration. These regimes have direct algorithmic consequences:

- **Acyclic (68%)**: One pass suffices. No worklist, no convergence check needed.
- **Single-loop (24%)**: Iteration confined to one small region (median 6 blocks). The rest of the CFG is processed in a single pass.

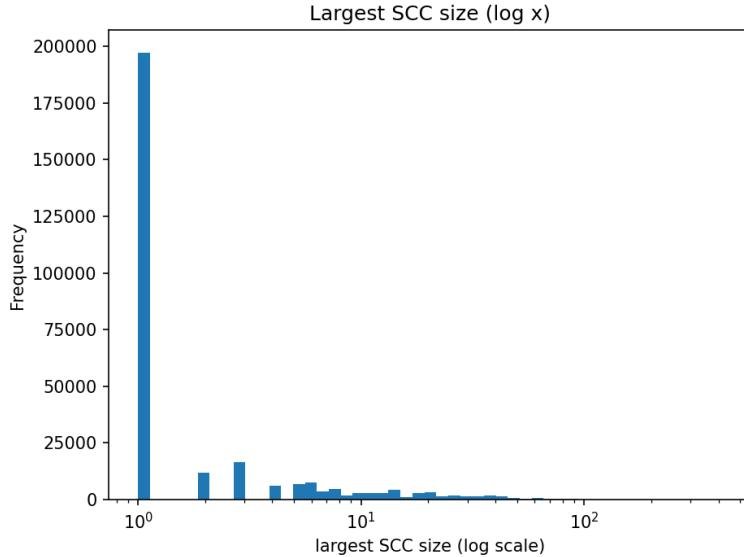


Figure 2: Distribution of largest SCC size (log scale). The spike at 1 represents the 68% of CFGs with no cycles. The tail extends to SCCs with hundreds of blocks.

- **Multi-loop (8%):** Multiple iteration regions, but still isolated from each other by SCC decomposition.

A tiered solver that exploits this structure can avoid iteration overhead for 68% of CFGs, minimize it for another 24%, and reserve full generality for the remaining 8%.

3 Design Implications

The structural regime data directly informs solver architecture. Rather than designing for a generic “CFG,” we design for three distinct populations:

Regime 1: Acyclic CFGs (68%). These need exactly one backward pass—no iteration, no worklist, no convergence checking. The solver should recognize this case (all SCCs are singletons) and take the fast path. Setup overhead matters here: avoid allocating iteration-related data structures when they won’t be used.

Regime 2: Single-loop CFGs (24%). One non-trivial SCC, typically small (median 6 blocks). The solver iterates within this single kernel while processing everything else in one pass. This is the “common loop” case—worth optimizing specifically because it covers a quarter of all functions.

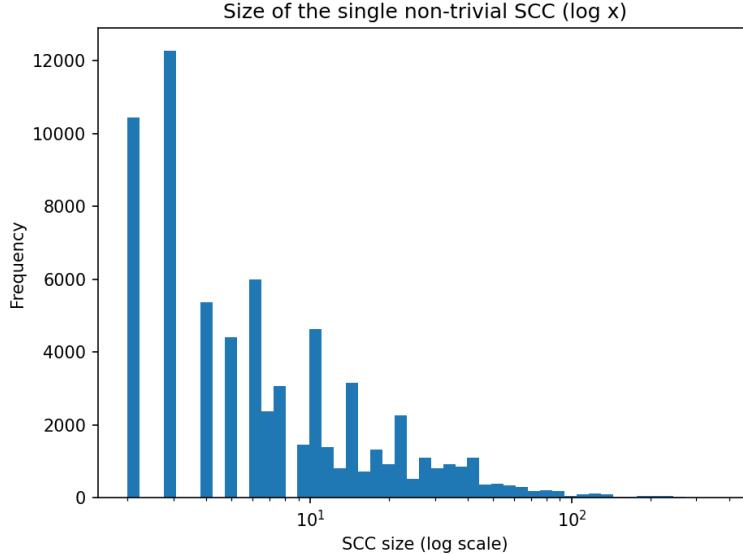


Figure 3: Size of the single non-trivial SCC (when exactly one exists). Median 6 blocks, $p_{90} = 27$, max = 376.

Regime 3: Multi-loop CFGs (8%). Multiple cyclic regions, potentially including the largest and most complex functions. Each SCC is solved independently, and our empirical finding (Section 4.2) shows that even these complex cases converge in exactly three passes with alternating order.

The SCC decomposition advantage. Remarkably, SCC-based processing handles all three regimes *without explicit case analysis*. Acyclic CFGs yield all-singleton SCCs processed in one pass. Single-loop CFGs yield one non-trivial SCC that iterates while singletons don't. Multi-loop CFGs yield multiple non-trivial SCCs, each solved independently. The algorithm naturally adapts to the regime.

Tail-aware engineering. The 8% complex regime includes functions with 12,000+ blocks and SCCs with hundreds of nodes. Our three-pass algorithm handles these efficiently:

- Fixed iteration count (three passes) gives predictable performance
- No dynamic worklist data structures needed
- Memory access patterns are sequential and cache-friendly

Metric selection. Use quantiles and regime fractions rather than means. The fraction of CFGs in each regime, the 99th percentile of non-trivial SCC size, and

tail exceedance rates (e.g., fraction with SCC >100 blocks) are better stability indicators than averages.

4 A Tiered Solver Architecture

Our solver combines SCC decomposition with a novel three-pass iteration scheme to handle all three structural regimes efficiently and uniformly.

4.1 SCC Decomposition

We partition the CFG into strongly connected components using Kosaraju–Sharir (see Appendix C for pseudocode). Contracting each SCC yields a DAG—the *condensation graph*. For backward analysis, we process SCCs in reverse topological order: when solving any SCC, all successor SCCs are already complete.

Automatic regime adaptation. The key insight is that SCC decomposition *implicitly* handles all three regimes:

- **Acyclic CFGs:** All SCCs are singletons. Each is processed once, in topological order. No iteration occurs—the algorithm degenerates to a single backward pass.
- **Single-loop CFGs:** One SCC has multiple blocks; the rest are singletons. Singletons are processed in one pass; the non-trivial SCC iterates internally.
- **Multi-loop CFGs:** Multiple non-trivial SCCs, but each is solved independently. Inter-SCC dependencies are resolved by topological ordering, not iteration.

No explicit regime detection is needed—the structure emerges from the SCC partition.

4.2 Three-Pass Algorithm for SCCs

The standard approach for cyclic SCCs is either a worklist solver or repeated full passes until convergence. Textbook analysis suggests worst-case $O(|B|)$ iterations for a cycle of $|B|$ blocks.

However, our empirical study reveals a striking result: **all SCCs in our 290,000-function dataset converge in exactly three passes** when using an alternating traversal order—postorder, then reverse postorder, then postorder again.

Postorder and reverse postorder. A *postorder* traversal visits each node *after* visiting all its successors in the DFS tree. For a CFG, this means nodes with no outgoing edges (exits) appear early in the sequence, while the entry node appears last. Concretely, if DFS from entry e visits $e \rightarrow b_1 \rightarrow b_2 \rightarrow \dots$, the postorder records nodes as their subtrees complete: exits first, entry last.

Reverse postorder (RPO) is simply this sequence reversed: entry first, exits last. For a DAG, RPO is a topological order. For backward data-flow analysis, RPO ensures that when we process a node, we have already processed its successors—ideal for propagating liveness information “upward” from exits toward the entry.

Why alternation helps on cycles. On acyclic graphs, a single RPO pass suffices. But cycles break the topological property: some successor will inevitably be processed after its predecessor, requiring iteration. The key insight is that *alternating directions* accelerates convergence:

Consider a cycle $b_1 \rightarrow b_2 \rightarrow b_3 \rightarrow b_1$. Suppose postorder is $[b_3, b_2, b_1]$ (exits-first) and reverse postorder is $[b_1, b_2, b_3]$ (entry-first).

- **Pass 1 (postorder $[b_3, b_2, b_1]$):** Information flows $b_3 \rightarrow b_2 \rightarrow b_1$. Facts from b_3 reach b_1 , but b_1 ’s facts don’t yet reach b_3 .
- **Pass 2 (reverse postorder $[b_1, b_2, b_3]$):** Information flows $b_1 \rightarrow b_2 \rightarrow b_3$. Now b_1 ’s updated facts propagate forward to b_3 .
- **Pass 3 (postorder):** Reconciles any remaining dependencies—facts that needed to “go around” the cycle twice.

With single-direction iteration, a fact at b_1 takes 3 passes to return to b_1 . With alternating order, information flows both ways simultaneously, converging faster.

Algorithm inputs. The algorithm takes three inputs:

- **Initial facts** (called `start` in implementations): The data-flow values at SCC boundaries, inherited from successor SCCs that have already been solved. For liveness, this is the LiveIn of successor blocks outside the current SCC.
- **Entry node:** The block where DFS begins. For the full CFG, this is the function entry. Within an SCC, any block can serve as entry; the choice affects traversal order but not correctness.
- **The SCC subgraph:** The blocks and edges forming the strongly connected component.

Algorithm 1 gives the pseudocode. Each pass applies the transfer function f_b to every block, updating LiveIn from LiveOut.

Algorithm 1 Three-Pass SCC Solver

Require: SCC K (blocks and edges), entry node e , initial boundary facts from successor SCCs

Ensure: Converged LiveIn and LiveOut for all blocks in K

```
1: function SOLVETHREEPASS( $K, e, boundaryFacts$ )
2:    $po \leftarrow \text{POSTORDER}(K, e)$                                  $\triangleright$  DFS postorder starting from  $e$ 
3:    $rpo \leftarrow \text{REVERSE}(po)$                                   $\triangleright$  Reverse postorder

4:   // Initialize LiveOut from boundary (successor SCCs already
   solved)
5:   for  $b \in K$  do
6:      $\text{LiveOut}(b) \leftarrow \bigcup_{s \in \text{Succ}(b)} \text{LiveIn}(s)$      $\triangleright$  Includes external successors
7:   end for

8:   APPLYPASS( $po$ )                                               $\triangleright$  Pass 1: postorder (exits  $\rightarrow$  entry)
9:   APPLYPASS( $rpo$ )                                               $\triangleright$  Pass 2: reverse postorder (entry  $\rightarrow$  exits)
10:  APPLYPASS( $po$ )                                               $\triangleright$  Pass 3: postorder (exits  $\rightarrow$  entry)
11: end function

12: function APPLYPASS( $order$ )
13:   for  $b \in order$  do
14:      $\text{LiveOut}(b) \leftarrow \bigcup_{s \in \text{Succ}(b)} \text{LiveIn}(s)$ 
15:      $\text{LiveIn}(b) \leftarrow \text{Use}(b) \cup (\text{LiveOut}(b) \setminus \text{Def}(b))$ 
16:   end for
17: end function
```

Complexity. Each pass visits every block once: $O(|K| + |E_K|)$ per pass. With exactly three passes, the total cost is $O(|K| + |E_K|)$ per SCC. Summing over all SCCs (which partition the CFG), the total cost is $O(|B| + |E|)$ —linear in CFG size.

Empirical validation. We validated convergence by comparing three-pass results against a reference worklist solver on all 290,000 functions. In every case, the three-pass algorithm produced identical results, confirming that three passes suffice for this workload. While we lack a formal proof of universal convergence, the empirical evidence—spanning reducible and irreducible control flow, simple loops and complex nests—suggests this is a robust practical bound.

Comparison to worklist solvers. A worklist solver guarantees convergence but incurs overhead: maintaining the queue, checking for changes, and reprocessing blocks in unpredictable order. The three-pass algorithm trades theoretical generality for simplicity: fixed iteration count, predictable memory access patterns, and no dynamic data structures. For the rare case where three passes

might not suffice, a convergence check after pass 3 could trigger worklist fallback.

4.3 Complete Tiered Algorithm

Combining SCC decomposition with the three-pass solver yields the complete algorithm:

1. Compute SCCs of the CFG using Kosaraju–Sharir (Appendix C).
2. Process SCCs in reverse topological order of the condensation DAG.
3. For each SCC:
 - If singleton (acyclic): apply transfer function once.
 - If non-trivial (cyclic): apply three-pass algorithm.

This achieves $O(|B| + |E|)$ complexity for the entire CFG, regardless of cyclic structure.

A Solving the Data-Flow Equations

This appendix reviews the standard formulation of live-range analysis as a data-flow problem and the iterative algorithm used to solve it [8, 4].

A.1 The Data-Flow Equations

Let the CFG have basic blocks $b \in B$ with edges $b \rightarrow s$ to successors. For each block, define:

- $\text{Def}(b)$: variables defined (written) in b
- $\text{Use}(b)$: variables used (read) in b before any local definition

Live variable analysis computes, for each block b :

$$\text{LiveOut}(b) = \bigcup_{s \in \text{Succ}(b)} \text{LiveIn}(s) \quad (1)$$

$$\text{LiveIn}(b) = \text{Use}(b) \cup (\text{LiveOut}(b) \setminus \text{Def}(b)) \quad (2)$$

This is a *backward* data-flow problem: information flows from successors to predecessors, opposite to control flow [1].

A.2 The Monotone Framework

The equations above instantiate a *monotone framework* [3, 4]. The key properties ensuring a well-defined solution are:

1. **Lattice structure.** The domain is the powerset 2^V of program variables, ordered by subset inclusion. The bottom element is \emptyset ; the top is V .
2. **Monotone transfer functions.** The transfer function $f_b(X) = \text{Use}(b) \cup (X \setminus \text{Def}(b))$ is monotone: $X \subseteq Y \Rightarrow f_b(X) \subseteq f_b(Y)$.
3. **Finite height.** The lattice has height $|V|$, bounding the number of times any fact can change.

These properties guarantee that iterative application of the equations converges to the *least fixed point*—the smallest solution satisfying all constraints [9].

A.3 The Standard Iterative Algorithm

The textbook algorithm processes blocks in reverse postorder (RPO) of a depth-first traversal [8, 1, 11]:

1. Compute a DFS postorder of the CFG.
2. Initialize $\text{LiveIn}(b) = \text{LiveOut}(b) = \emptyset$ for all b .
3. Repeat in reverse postorder until no changes:
 - (a) Compute $\text{LiveOut}(b)$ from successors via Equation 1.
 - (b) Compute $\text{LiveIn}(b)$ via Equation 2.

For a backward analysis, reverse postorder processes blocks “close to exits first,” propagating information in the predominant direction of data flow.

A.4 Complexity and Convergence

Per-iteration cost. Each pass visits every block and, for each block, examines its successors and performs set operations. With $|B|$ blocks and $|E|$ edges, one iteration costs $O(|B| + |E|)$ assuming set operations are $O(|V|)$ or use efficient representations (bit vectors, sparse sets).

Number of iterations. In the worst case, a single new fact may propagate one block per iteration. For a CFG with a long chain or deep loop nest, this yields $O(|B|)$ iterations, giving overall worst-case complexity:

$$O(|B| \cdot (|B| + |E|)) = O(|B|^2 + |B| \cdot |E|)$$

On reducible CFGs (most structured programs), convergence is typically much faster—often 2–3 passes suffice in practice [1]. However, irreducible control flow or adversarial loop structures can approach the worst case.

The ordering assumption. The standard algorithm assumes a *fixed* DFS-based ordering computed once before iteration. This ordering is optimal for acyclic graphs (one pass suffices) and near-optimal for reducible graphs with natural loops.

Notably, **there has been little research on dynamic or adaptive ordering schemes** that might accelerate convergence on complex CFGs. The literature universally adopts DFS postorder [8, 1, 11, 9].

Alternating order: an empirical improvement. Our empirical study (Section 4.2) reveals that *alternating* between postorder and reverse postorder dramatically improves convergence. Specifically, processing blocks in the sequence postorder \rightarrow reverse postorder \rightarrow postorder achieves convergence in exactly three passes for *all* SCCs in our 290,000-function dataset—including irreducible control flow and complex loop structures.

This is significantly better than the theoretical $O(|B|)$ worst case. While we lack a formal proof that three passes suffice universally, the empirical evidence suggests this alternating strategy exploits structure in real-world CFGs that single-direction iteration misses.

A.5 Distance-to-Next-Use Extension

For register allocation, we often want not just *whether* a variable is live, but *how soon* it will be used [6]. We generalize to a map $L_b : V \rightarrow (\mathbb{N} \cup \{\infty\})$ where $L_b(v)$ estimates instructions until next use. At joins:

$$L_b(v) = \min_{s \in \text{Succ}(b)} (\delta(b, s) + L_s^{\text{in}}(v))$$

where $\delta(b, s) \geq 1$ is an edge cost (modeling branch likelihood, transfer overhead, etc.).

This remains a monotone framework: the lattice is $(\mathbb{N} \cup \{\infty\})^V$ ordered pointwise by \geq , and distances decrease monotonically from ∞ toward smaller values as uses become reachable. The same iterative algorithm applies, with the same complexity bounds.

A.6 Motivation for SCC-Based Solving

The theoretical $O(|B|^2)$ worst case motivates the SCC-based approach in Section 4. By decomposing the CFG into strongly connected components:

- Acyclic portions (68% of CFGs in our study) require exactly one pass.
- Cyclic SCCs are solved independently, confining iteration to the cyclic subgraph.
- The condensation DAG ensures no redundant reprocessing across SCCs.

Combined with the three-pass alternating-order algorithm (Section 4.2), this approach achieves $O(|B| + |E|)$ complexity on *all* CFGs in our study—not just typical ones, but including the complex tail cases that would otherwise dominate compile time.

B Statistical Analysis Details

This appendix provides the full statistical analysis of CFG structure from our Go toolchain study.

B.1 Dataset

Each line of the collected `*_scc.csv` files corresponds to one analyzed CFG instance, recording the number of basic blocks and the number of SCC “kernels” (SCCs in the condensation). Across 240 input files, the dataset contains $n = 290,656$ CFG instances.

B.2 Heavy Tails and Over-Dispersion

Metric	n	Min	Median	p_{90}	Max	Mean	Var	Var/Mean
Blocks	290,656	3	9	44	12,676	20.10	3521.00	175.14
SCCs	290,656	2	7	31	12,676	15.31	2749.30	179.59

Table 3: Complete CFG statistics. The dispersion index is $\approx 175\text{--}180$, far exceeding Poisson ($D = 1$).

The dispersion index $D = \text{Var}(X)/\mathbb{E}[X]$ diagnoses over-dispersion. For a Poisson model, $D \approx 1$. Observing $D \approx 175$ implies variance two orders of magnitude larger than a homogeneous process predicts.

One interpretation: if $X \mid \Lambda \sim \text{Poisson}(\Lambda)$ with latent rate Λ varying across CFG instances, then

$$D = 1 + \frac{\text{Var}(\Lambda)}{\mathbb{E}[\Lambda]}.$$

A dispersion index near 175 implies $\text{Var}(\Lambda) \approx 174\mathbb{E}[\Lambda]$: the workload spans many “regimes” of CFG complexity.

B.3 Why Negative Binomial Fits Poorly

The negative binomial (NB) extends Poisson for over-dispersed counts via a Gamma–Poisson mixture. Moment-matching yields extremely small shape parameters ($r \approx 0.115$ for blocks, $r \approx 0.086$ for SCCs), indicating a very heavy tail.

Yet this simple NB model remains a poor global fit for several reasons:

- 1. Structural constraints.** CFGs have minimum sizes and discrete construction artifacts (entry/exit blocks, synthetic lowering blocks) that create mass at specific small counts not captured by continuous mixtures.
- 2. Multi-modal mixtures.** Small leaf functions, medium inlined functions, generated code, and large dispatcher functions form distinct populations. A single Gamma mixing distribution cannot capture multiple modes.
- 3. Different tail mechanisms.** The far tail often arises from specific sources (parser tables, regexp engines, large switch lowering) that may follow lognormal or power-law-like behavior rather than NB.
- 4. SCC structure is not independent.** SCC count correlates with blocks but depends on loop structure, irreducibility, and canonicalization—additional heterogeneity beyond simple count models.

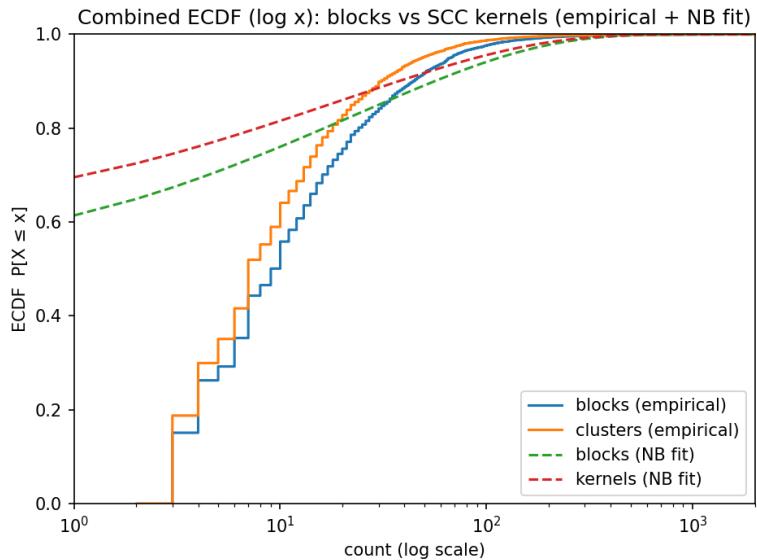


Figure 4: Empirical ECDFs (log x -axis) with moment-matched NB overlay. The poor fit confirms multi-regime workload structure.

B.4 Implications

The poor global NB fit is *good news*: it indicates exploitable structure. Stratifying by package, compilation stage, or structural features (presence of large switch lowering, irreducible loops, inlining depth) often yields per-stratum distributions far less pathological. Those strata-level models are the right abstraction for predicting and improving register-allocation performance.

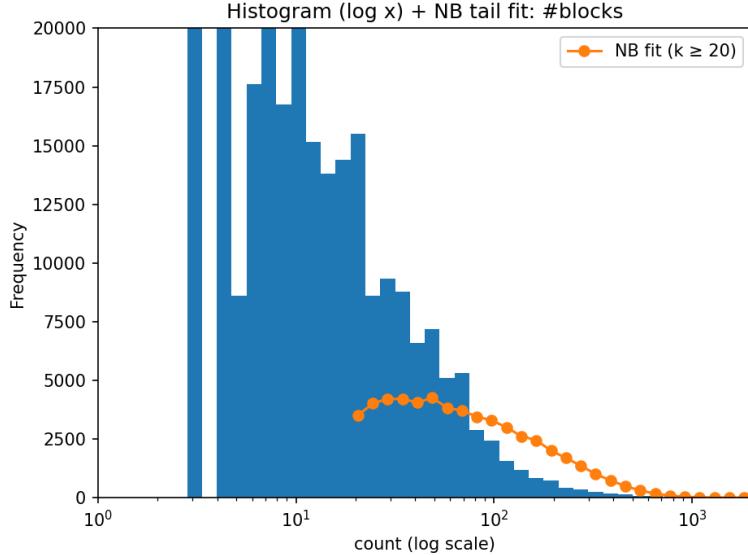


Figure 5: Block count histogram (log x -axis) with NB tail overlay. Even in the tail region, the fitted curve diverges from empirical counts.

B.5 Structural Regime Analysis

Beyond size distributions, we analyzed the cyclic structure of each CFG by counting non-trivial SCCs (those with more than one block).

Non-trivial SCC count	CFGs	Fraction
0	197,091	67.9%
1	68,357	23.5%
2	14,918	5.1%
3	5,273	1.8%
4	1,884	0.6%
5+	2,811	1.0%

Table 4: Distribution of non-trivial SCC counts. The tail drops off rapidly.

For the 68,357 CFGs with exactly one non-trivial SCC, Table 5 characterizes the size of that single loop kernel.

We also measured what fraction of each CFG's nodes participate in non-trivial SCCs:

- **Median:** 0% (the acyclic majority)
- p_{90} : 60%
- p_{99} : 87.5%

Metric	n	Min	Median	p_{90}	p_{99}	Max
Single non-trivial SCC size	68,357	2	6	27	86	376

Table 5: Size statistics for the single non-trivial SCC (conditional on exactly one existing).

- **Max:** 99.1%

Even in CFGs with cycles, the cyclic portion is often a minority of the total blocks. This reinforces the value of SCC decomposition: iteration is confined to a (usually small) subset of the CFG.

C SCC Algorithm Pseudocode

We use Kosaraju–Sharir [13] for SCC decomposition. Compared to Tarjan’s algorithm [14]:

- Straightforward iterative implementation without explicit stack management.
- No auxiliary data (lowlink, index) required on graph nodes.
- The postorder from the first pass is typically already cached by the compiler, making that phase effectively free.

Using BFS instead of DFS for the second pass simplifies implementation while maintaining correctness.

C.1 Algorithm Overview

1. **Forward pass:** Compute postorder traversal via DFS on forward edges.
2. **Reverse pass:** Process blocks in reverse postorder, performing BFS on predecessor edges to discover each SCC.

Processing in reverse postorder on the transposed graph ensures that starting a new component cannot reach any previously discovered component.

C.2 Pseudocode

Algorithm 2 Kosaraju–Sharir SCC Partition

Require: CFG $G = (B, E)$ with blocks B and edges E

Ensure: List of SCCs in topological order of the condensation DAG

```
1: function SCCPARTITION( $G$ )
2:    $po \leftarrow \text{POSTORDER}(G)$             $\triangleright$  DFS postorder on forward edges
3:    $seen \leftarrow \emptyset$ 
4:    $reachable \leftarrow \{b.ID : b \in po\}$ 
5:    $result \leftarrow []$ 

6:   for  $i \leftarrow |po| - 1$  downto 0 do            $\triangleright$  Reverse postorder
7:      $leader \leftarrow po[i]$ 
8:     if  $leader.ID \in seen$  then
9:       continue
10:      end if

11:      $scc \leftarrow \text{BFSREVERSED}(G, leader, seen, reachable)$ 
12:     APPEND( $result, scc$ )
13:   end for

14:   return  $result$ 
15: end function
```

Algorithm 3 BFS on Reversed Edges

Require: Block *leader*, sets *seen* and *reachable*

Ensure: SCC containing *leader*; updates *seen* in place

```
1: function BFSREVERSED( $G, \text{leader}, \text{seen}, \text{reachable}$ )
2:    $\text{queue} \leftarrow [\text{leader}]$ 
3:    $\text{scc} \leftarrow []$ 
4:    $\text{seen} \leftarrow \text{seen} \cup \{\text{leader.ID}\}$ 

5:   while  $\text{queue} \neq \emptyset$  do
6:      $b \leftarrow \text{DEQUEUE}(\text{queue})$ 
7:     APPEND( $\text{scc}, b$ )

8:     for  $e \in \text{PREDS}(b)$  do                                 $\triangleright$  Traverse reversed edges
9:        $\text{pred} \leftarrow e.\text{block}$ 
10:      if  $\text{pred.ID} \in \text{reachable}$  and  $\text{pred.ID} \notin \text{seen}$  then
11:         $\text{seen} \leftarrow \text{seen} \cup \{\text{pred.ID}\}$ 
12:        ENQUEUE( $\text{queue}, \text{pred}$ )
13:      end if
14:    end for
15:  end while

16:  return  $\text{scc}$ 
17: end function
```

C.3 Properties

1. The first SCC contains only the entry block (assuming no predecessors).
2. Unreachable blocks are excluded.
3. SCCs are returned in topological order of the condensation DAG.
4. Block order within each SCC is deterministic for a given input.

Complexity. Time: $O(|B| + |E|)$ —each block and edge visited once per pass.
Space: $O(|B|)$ for visited sets and queue.

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