

14. The Oriented Plane

Mechanics of Manipulation

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Exhortations

“Hands are levers of influence on the world that made intelligence worth having. Precision hands and precision intelligence coevolved in the human lineage, and the fossil record shows that hands led the way.”—Steven Pinker, in *How the Mind Works*

Chapter 1 Manipulation 1

- 1.1 Case 1: Manipulation by a human 1
- 1.2 Case 2: An automated assembly system 3
- 1.3 Issues in manipulation 5
- 1.4 A taxonomy of manipulation techniques 7
- 1.5 Bibliographic notes 8
- Exercises 8

Chapter 2 Kinematics 11

- 2.1 Preliminaries 11
- 2.2 Planar kinematics 15
- 2.3 Spherical kinematics 20
- 2.4 Spatial kinematics 22
- 2.5 Kinematic constraint 25
- 2.6 Kinematic mechanisms 34
- 2.7 Bibliographic notes 36
- Exercises 37

Chapter 3 Kinematic Representation 41

- 3.1 Representation of spatial rotations 41
- 3.2 Representation of spatial displacements 58
- 3.3 Kinematic constraints 68
- 3.4 Bibliographic notes 72
- Exercises 72

Chapter 4 Kinematic Manipulation 77

- 4.1 Path planning 77
- 4.2 Path planning for nonholonomic systems 84
- 4.3 Kinematic models of contact 86
- 4.4 Bibliographic notes 88
- Exercises 88

Chapter 5 Rigid Body Statics 93

- 5.1 Forces acting on rigid bodies 93
- 5.2 Polyhedral convex cones 99
- 5.3 Contact wrenches and wrench cones 102
- 5.4 Cones in velocity twist space 104
- 5.5 The oriented plane 105
- 5.6 Instantaneous centers and Reuleaux's method 109
- 5.7 Line of force; moment labeling 110
- 5.8 Force dual 112
- 5.9 Summary 117
- 5.10 Bibliographic notes 117
- Exercises 118

Chapter 6 Friction 121

- 6.1 Coulomb's Law 121
- 6.2 Single degree-of-freedom problems 123
- 6.3 Planar single contact problems 126
- 6.4 Graphical representation of friction cones 127
- 6.5 Static equilibrium problems 128
- 6.6 Planar sliding 130
- 6.7 Bibliographic notes 139
- Exercises 139

Chapter 7 Quasistatic Manipulation 143

- 7.1 Grasping and fixturing 143
- 7.2 Pushing 147
- 7.3 Stable pushing 153
- 7.4 Parts orienting 162
- 7.5 Assembly 168
- 7.6 Bibliographic notes 173
- Exercises 175

Chapter 8 Dynamics 181

- 8.1 Newton's laws 181
- 8.2 A particle in three dimensions 181
- 8.3 Moment of force; moment of momentum 183
- 8.4 Dynamics of a system of particles 184
- 8.5 Rigid body dynamics 186
- 8.6 The angular inertia matrix 189
- 8.7 Motion of a freely rotating body 195
- 8.8 Planar single contact problems 197
- 8.9 Graphical methods for the plane 203
- 8.10 Planar multiple-contact problems 205
- 8.11 Bibliographic notes 207
- Exercises 208

Chapter 9 Impact 211

- 9.1 A particle 211
- 9.2 Rigid body impact 217
- 9.3 Bibliographic notes 223
- Exercises 223

Chapter 10 Dynamic Manipulation 225

- 10.1 Quasidynamic manipulation 225
- 10.2 Briefly dynamic manipulation 229
- 10.3 Continuously dynamic manipulation 230
- 10.4 Bibliographic notes 232
- Exercises 235

Appendix A Infinity 237

Outline.

Reuleaux's space...what is it?

Formal definitions.

Central projection.

Convexity in the oriented plane.

Relation to polyhedral convex cones.

Rotation centers and the oriented plane.

Reuleaux's space

Reuleaux represents PCC's in planar diff'l twist space.

3D twist space is 6D.

planar twist space is 3D.

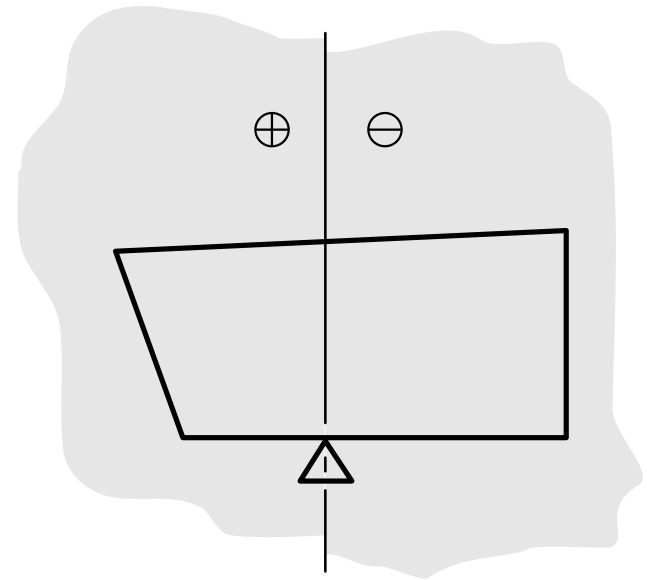
Reuleaux does it in 2D!

What space do signed rotation centers live in?

Plane with $+$ and/or $-$ label,

and points at infinity?

Stolfi and Guibas: the *oriented plane*.



Formal definition

Consider homogeneous coordinates $(x, y, w) \neq (0, 0, 0)$.

A **point in the oriented plane** is a ray of triples:

$$\{(kx, ky, kw) \mid k > 0\} \quad (1)$$

Three cases:

$w > 0$: Signed Euclidean point $(x/w, y/w, \oplus)$.

$w < 0$: Signed Euclidean point $(x/w, y/w, \ominus)$.

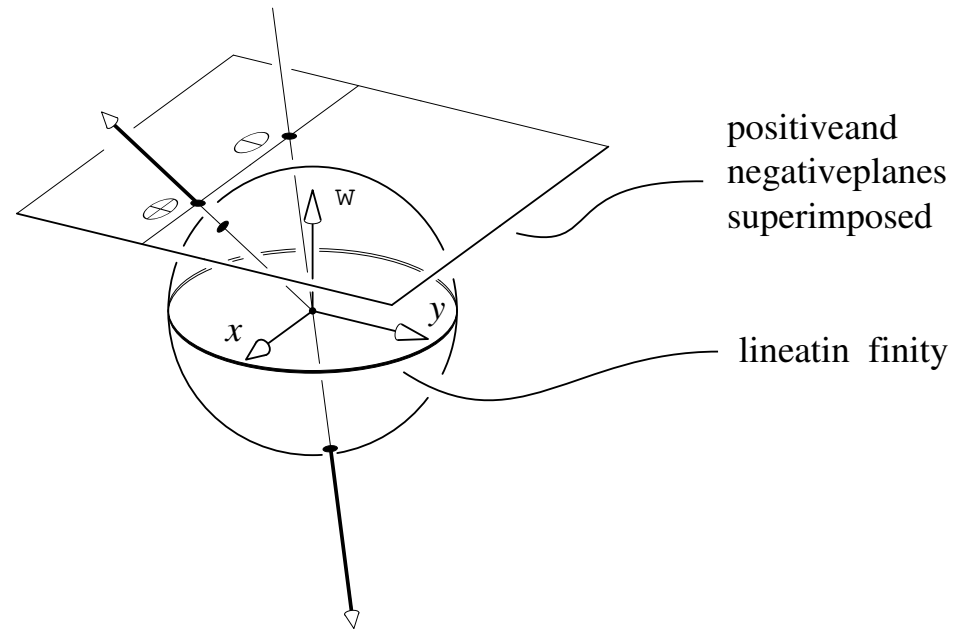
$w = 0$: Ideal point: point at infinity.

Central projection

Project each point (x, y, w) to $w = 1$ plane.

Attach appropriate sign.

Ideal points miss the plane but hit the equator.



Relation to projective plane

Maybe we should call it the *Oriented Projective Plane*.

Projective plane is set of lines through the origin of \mathbb{E}^3 .

Oriented plane is set of *directed* lines through the origin of \mathbb{E}^3 .

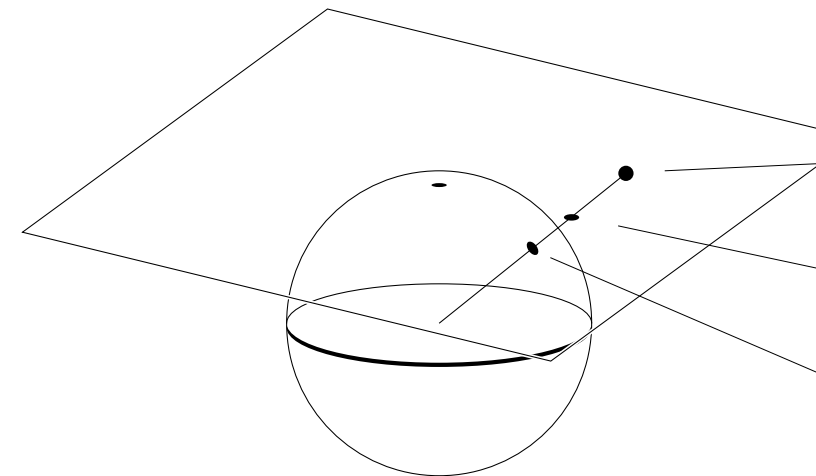
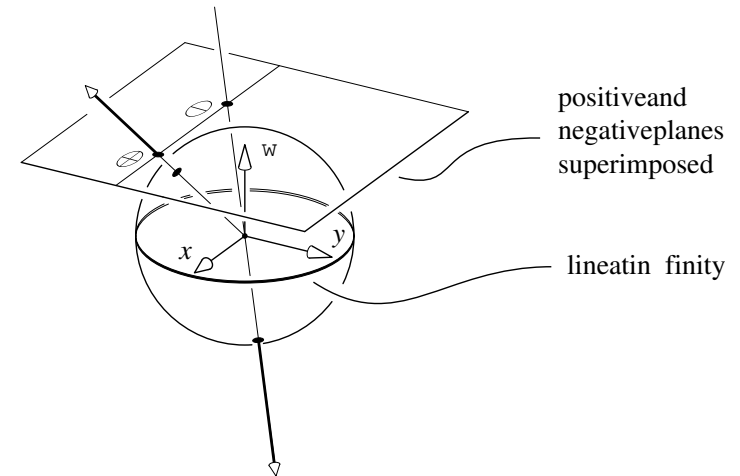
The projective plane is the sphere $\mathbb{S}(2)$ with antipodes identified.

The oriented plane is the sphere $\mathbb{S}(2)$.

Northern hemisphere is the positive plane,

Southern hemisphere is the negative plane,

Equator is the ideal line.



Why do we need it?

Because, e.g. Reuleaux's method is easier than analyzing polyhedral convex cones in planar differential twist space.

(or rather, Reuleaux's method is an easy way to analyze PCCs in planar differential twist space.)

In practice, we work directly in the oriented plane, as Reuleaux did, not worrying too much about the projection.

But to answer deep questions, refer back to the projection.

Geometry

An example question. We know what points are in the oriented plane. Are there lines?

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So ...

- Two non-antipodal points determine a line.
(The ideal line, or a Euclidean line labeled “ \pm ”.)
- Every pair of lines intersects in *two* antipodal points.

Convexity

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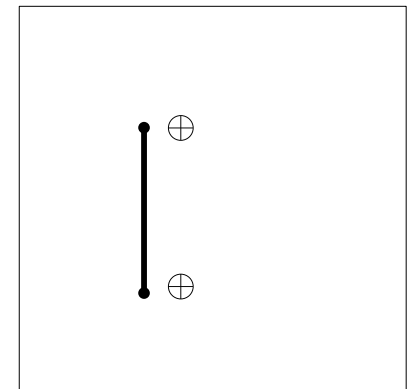
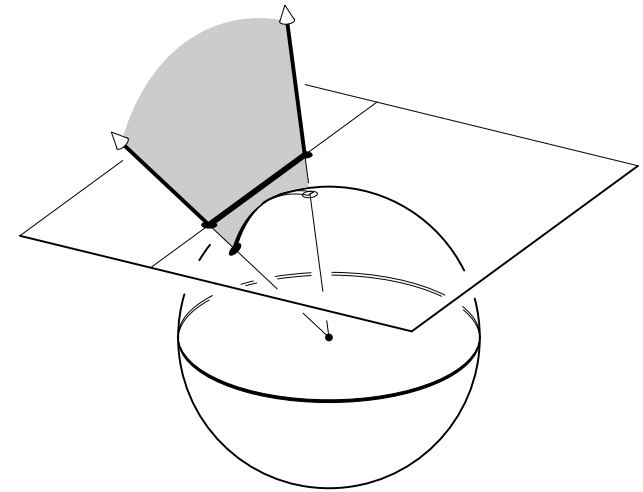
What is \overline{ab} in the oriented plane? What is $\text{conv}(\{a, b\})$?

Convexity: plus and plus

Consider the polyhedral convex cone determined by the two rays.

Project it to the plane.

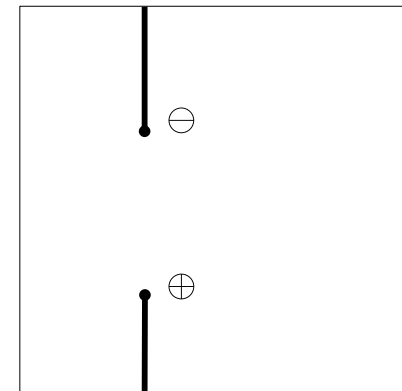
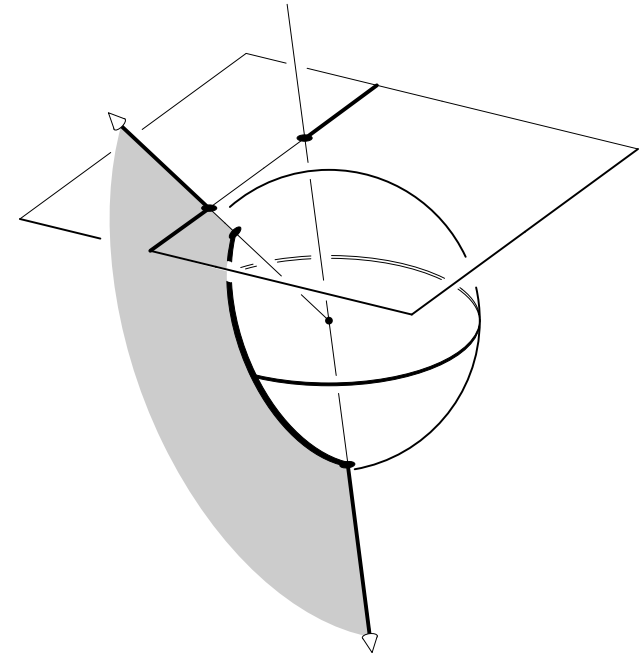
So, if a and b have the same sign then $\text{conv}(\{a, b\})$ is the line segment between them with the same sign.



Convexity: plus and minus

Consider the PCC ...

Define the *external* line segment.



Problem Set 4

Let's do PS4!

How do you construct the convex hull of some set of points?

1. Pick two points, and construct their convex hull.
2. Repeat step 1, ad nauseum.

Reuleaux's method

Given a differential velocity twist $(v_{0x}, v_{0y}, \omega_z)$, what is the rotation center?

Velocity at a point $\mathbf{p} = (x, y)$ is

$$\begin{aligned} & \mathbf{v}_0 + \omega \times \mathbf{p} \\ &= \begin{pmatrix} v_{0x} - \omega y \\ v_{0y} + \omega x \end{pmatrix} \end{aligned}$$

Set equal to $(0, 0)$ and solve for $(x, y) \dots$

Rotation center is at

$$\begin{pmatrix} -v_{0y}/\omega \\ v_{0x}/\omega \end{pmatrix}$$

Central projection, and rotation of coordinates.

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- 3.4 Bibliographic notes 72
- Exercises 72

Chapter 4 Kinematic Manipulation 77

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- 4.4 Bibliographic notes 88
- Exercises 88

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- 5.10 Bibliographic notes 117
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- 9.3 Bibliographic notes 223
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- 10.3 Continuously dynamic manipulation 230
- 10.4 Bibliographic notes 232
- Exercises 235

Appendix A Infinity 237