Conversion of the Kinematics of a Car with n Trailers into a Chained Form

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Abstract

A new set of coordinates is proposed for the kinematic model of a car with n trailers with only two degrees of freedom. In this paper, the absolute position of the system is given by the location of the rear trailer. By using these coordinates, the kinematic model is locally converted into a nilpotent, chained form. Control strategies for chained systems can, therefore, be applied to locally control a car with n trailers.

1 Introduction

A car with n trailers is a nonholonomic system due to the rolling constraints of the wheels. The configuration of the system is given by two position coordinates and $n+1$ angles, whereas there are only two inputs, namely one tangential velocity and one angular velocity. Thus, the system has two degrees of freedom.

A kinematic model for such a system was presented by [3]. Controllability for this model was proven, but no control law was found. Control of simpler kinematic models of cars and mobile robots have been studied from open-loop and closed-loop points of view. Closed-loop or exact open-loop strategies have, however, not been presented in previous work for a car with n trailers with two degrees of freedom where n is an arbitrary number. An interesting approach to this problem is to seek for a conversion of the kinematic model into a chained form. Then, control schemes for systems on chained form can be applied. A system on chained form is called a chained system. Such systems are nilpotent and have the following form:

$$
\begin{aligned}
\dot{\xi}_1 &= u_1 \\
\dot{\xi}_2 &= u_2 \\
\dot{\xi}_3 &= \xi_2 u_1 \\
\dot{\xi}_4 &= \xi_3 u_1 \\
&\vdots \\
\dot{\xi}_k &= \xi_{k-1} u_1
\end{aligned}
$$

An open-loop strategy to steer nonholonomic systems on a special canonical form which includes chained systems was proposed by [4]. The control strategy used sinusoid-type inputs. A general strategy for steering systems without drift was proposed by [2] and [1]. This approach provides an exact solution for nilpotent or feedback nilpotentizable systems. This strategy can, therefore, also be used to find an exact open-loop solution of the Motion Planning Problem of a car with n trailers if the kinematics can be converted into a chained form.

Also closed-loop strategies have been developed for chained systems. The work on open-loop control using sinusoids was extended to asymptotic stabilization of chained systems by using time-varying feedback [8]. The time-varying feedback law proposed by [6] can also be used to stabilize chained systems. The problem of these smooth time-varying feedback laws is a rather slow convergence. In [7] a new stabilizing control law has been proposed for chained systems having *exponential* convergence. By converting the model of a car with n trailers into a chained form, the system can, therefore, be controlled by using existing control strategies or the control law in [7].

A constructive procedure to transform a nonholonomic system with two inputs into a chained form suitable for control was given by [5] under some assumptions on the input vectors. This was used to locally convert the kinematic model of a car pulling a single trailer into a chained form. However, the algorithm failed when additional trailers were added for the model considered.

In the work of [5] the absolute position of the car with n trailers was given by the position of the pulling car. In this paper, a new model structure is proposed where the absolute position of the car with n trailers is given by the position of the *rear* trailer. This formulation of the model allows for a local conversion of the kinematics of a car with n trailers into a chained form where n is an arbitrary positive number. A change of coordinates and an invertible feedback transformation of the inputs are found which convert this kinematic model into a chained form.

2 Kinematic Model

A car in this context will be represented by two driving wheels connected by an axle. A kinematic model of a car with two degrees of freedom pulling n trailers can be given by:

$$\begin{aligned}
\dot{x} &= \cos\theta_n v_n \\
\dot{y} &= \sin\theta_n v_n \\
\dot{\theta}_n &= \frac{1}{d_n}\sin(\theta_{n-1}-\theta_n)v_{n-1} \\
&\vdots \\
\dot{\theta}_i &= \frac{1}{d_i}\sin(\theta_{i-1}-\theta_i)v_{i-1} \quad i=1,\ldots,n \qquad (1)\\
&\vdots \\
\dot{\theta}_1 &= \frac{1}{d_1}\sin(\theta_0-\theta_1)v_0 \\
\dot{\theta}_0 &= \omega
\end{aligned}$$

where (x, y) is the absolute position of the center of the axle between the two wheels of the *rear* trailer. The use of the position of the rear trailer in the model as proposed here is an original contribution which is a significant improvement with respect to previous models where the position of the *first* car was determining the location of the system, [3] and [5].

θ_i is the orientation angle of trailer i with respect to the x-axis, with $i \in \{1,\ldots,n\}$. θ_0 is the orientation angle of the pulling car with respect to the x-axis. Incidentally, this model is identical to a model of a four-wheeled car pulling $n-1$ trailers where $\theta_0 - \theta_1$ is the angle of the front wheels relative to the orientation θ_1 of the four-wheeled car.

d_i is the distance from the wheels of trailer i to the wheels of trailer $i-1$, where $i \in \{2,\ldots,n\}$. d_1 is then the distance from the wheels of trailer 1 to the wheels of the car.

v_0 is the tangential velocity of the car and is an input to the system. The other input is the angular velocity of the car, ω. We denote

$$\nu = [v_0, \omega]^T$$

The tangential velocity of trailer i, v_i, is given by

$$v_i = \cos(\theta_{i-1}-\theta_i)v_{i-1} = \prod_{j=1}^{i}\cos(\theta_{j-1}-\theta_j)\, v_0 \qquad (2)$$

where $i \in \{1,\ldots,n\}$. An illustration of these definitions is presented in *Fig. 1*.

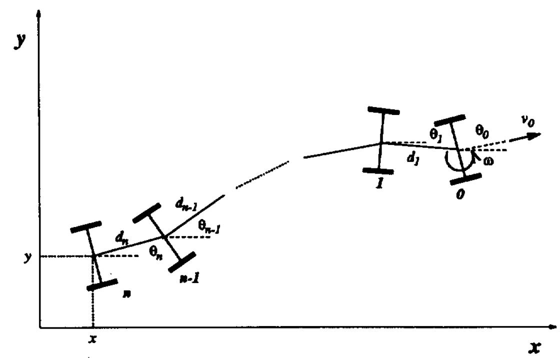

Figure 1: Model of a car with n trailers.

A car with n trailers (1) is a nonholonomic system with rolling constraints for each trailer and the car. Denote by (x_i, y_i) the absolute position of trailer i where $(x_n, y_n) = (x, y)$. The absolute position of the car is denoted by (x_0, y_0). The nonholonomic constraints can then be expressed as, [3],

$$\sin\theta_i \dot{x}_i - \cos\theta_i \dot{y}_i = 0, \quad i \in \{0,1,\ldots,n\} \qquad (3)$$

From a geometrical consideration, *Fig. 1*, we easily see that

$$x_i = x + \sum_{j=i+1}^{n} d_j\cos\theta_j, \quad y_i = y + \sum_{j=i+1}^{n} d_j \sin\theta_j$$

where $(x, y) = (x_n, y_n)$. Eq. (3) then implies

$$\begin{aligned}
0 &= \sin\theta_i\dot{x} - \sin\theta_i\sum_{j=i+1}^{n} d_j\sin\theta_j\dot{\theta}_j \\
&\quad - \cos\theta_i\dot{y} - \cos\theta_i\sum_{j=i+1}^{n} d_j\cos\theta_j\dot{\theta}_j
\end{aligned}$$

which gives the following nonholonomic constraints for $i \in \{0,1,\ldots,n\}$:

$$\sin\theta_i\dot{x} - \cos\theta_i\dot{y} - \sum_{j=i+1}^{n} d_j\cos(\theta_i-\theta_j)\dot{\theta}_j = 0$$

The system has, thus, $(n+3)-(n+1)=2$ degrees of freedom corresponding to the two independent velocity inputs.

We will make a change of inputs under the assumption that the state $q=[x,y,\theta_n,\dots,\theta_0]$ is in a neighborhood D of the origin where D is given by

$$\begin{aligned}(x,y) &\in \mathbb{R}^2\\ \theta_i &\in (-\frac{\pi}{4}+\varepsilon,\frac{\pi}{4}-\varepsilon)\ , \quad i\in\{0,\dots,n\}\end{aligned}$$

where ε is a small constant. We introduce the transformed input v as

$$v=\cos\theta_n\ v_n=\cos\theta_n\prod_{j=1}^{n}\cos(\theta_{j-1}-\theta_j)\ v_0 \qquad (4)$$

The transformed input v is the velocity of trailer n in x-direction. This transformation from v_0 to v is nonsingular and smooth in D. The velocity of v_i from (2) can then be rewritten

$$v_i=\frac{1}{\cos\theta_n\prod_{j=i+1}^{n}\cos(\theta_{j-1}-\theta_j)}v=\frac{1}{p_i(\underline{\theta}_i)}v \qquad (5)$$

for $i\in\{0,\dots,n\}$ where

$$\begin{aligned}\underline{\theta}_i &\triangleq [\theta_i,\dots,\theta_n]^T &(6)\\ p_i(\underline{\theta}_i) &\triangleq \cos\theta_n\prod_{j=i+1}^{n}\cos(\theta_{j-1}-\theta_j)\\ &= \prod_{j=i}^{n}\cos(\theta_j-\theta_{j+1}) &(7)\end{aligned}$$

for $i\in\{0,\dots,n\}$ where $\theta_{n+1}\triangleq 0$. Eq. (5) then gives $v=p_0(\underline{\theta}_0)v_0$.

System (1) can now be represented (locally) at the following form:

$$\begin{aligned}\dot{x} &= v\\ \dot{\theta}_0 &= \omega\\ \dot{\theta}_1 &= \frac{1}{d_1}\frac{\tan(\theta_0-\theta_1)}{p_1(\underline{\theta}_1)}v\\ &\vdots\\ \dot{\theta}_i &= \frac{1}{d_i}\frac{\tan(\theta_{i-1}-\theta_i)}{p_i(\underline{\theta}_i)}v\ , \quad i\in\{1,\dots,n\} \qquad (8)\\ \dot{y} &= \tan\theta_n v\end{aligned}$$

where v is given by (4). We denote for $i\in\{1,\dots,n\}$

$$\begin{aligned}f_i(\underline{\theta}_{i-1}) &= \frac{1}{d_i}\frac{\tan(\theta_{i-1}-\theta_i)}{p_i(\underline{\theta}_i)} &(9)\\ \underline{f}_i(\underline{\theta}_{i-1}) &= [f_i(\underline{\theta}_{i-1}),\dots,f_n(\underline{\theta}_{n-1})]^T &(10)\end{aligned}$$

This means that we can write

$$\begin{aligned}\dot{\theta}_i &= f_i(\underline{\theta}_{i-1})v &(11)\\ \dot{\underline{\theta}}_i &= \underline{f}_i(\underline{\theta}_{i-1})v &(12)\end{aligned}$$

After a reordering of the state variables, we denote the state by the vector

$$z=[z_1,\dots,z_{n+3}]^T=[x,\theta_0,\dots,\theta_n,y]^T$$

which has dimension $n+3$. We note from (8) that this kinematic model has a special triangular structure where $\dot{z}_i$ is not a function of $z_1,\dots,z_{i-2}$, where $i\in\{3,\dots,n+3\}$.

3 Conversion into a Chained Form

In this section we will exploit the special structure of (8) to convert system (1) into a chained form. The chained form will be obtained by a constructive procedure. This is formulated in the following theorem where $\xi=[\xi_1,\dots,\xi_{n+3}]^T$.

Theorem 1 *The following change of coordinates, $\xi=F(z)$, and feedback transformation, $u=G(z)\nu$, convert locally the model (1) of a car with n trailers into a chained form:*

$$\begin{aligned}\xi_1 &= x &(13)\\ \xi_2 &= \frac{\tan(\theta_0-\theta_1)}{c_2(\underline{\theta}_1)}+\tau_2(\underline{\theta}_1) &(14)\\ &\vdots\\ \xi_i &= \frac{\tan(\theta_{i-2}-\theta_{i-1})}{c_i(\underline{\theta}_{i-1})}+\tau_i(\underline{\theta}_{i-1}) &(15)\\ &\vdots\\ \xi_n &= \frac{\tan(\theta_{n-2}-\theta_{n-1})}{c_n(\underline{\theta}_{n-1})}+\tau_n(\underline{\theta}_{n-1}) &(16)\\ \xi_{n+1} &= \frac{\tan(\theta_{n-1}-\theta_n)}{d_n\cos^3\theta_n} &(17)\\ \xi_{n+2} &= \tan\theta_n &(18)\\ \xi_{n+3} &= y &(19)\end{aligned}$$

where

$$\begin{aligned}c_i(\underline{\theta}_{i-1}) &= \prod_{j=i}^{n+1}\cos^{j-i+3}(\theta_{j-1}-\theta_j)d_{n+i-j}\\ &= p_{i-1}^2(\underline{\theta}_{i-1})\prod_{j=i-1}^{n}d_jp_j(\underline{\theta}_j) &(20)\\ \tau_i(\underline{\theta}_{i-1}) &= \frac{\partial\xi_{i+1}}{\partial\underline{\theta}_i}\underline{f}_i(\underline{\theta}_{i-1}) &(21)\end{aligned}$$

where $p_i(\underline{\theta}_i)$ is given by (7).

The feedback transformation of the inputs, $u = G(z)\nu$, is given by

$$u_1 = p_0(\underline{\theta}_0)v_0 \quad (22)$$

$$u_2 = \frac{1}{\cos^2(\theta_0-\theta_1)c_2(\underline{\theta}_1)}\omega + \tau_1(\underline{\theta}_0)p_0(\underline{\theta}_0)v_0 \quad (23)$$

where $p_0(\underline{\theta}_0)$ is given by (7) and $\tau_1(\underline{\theta}_0)$ is given by (21).

Proof: We are free to choose

$$\xi_{n+3} = y \quad (24)$$

From (8), differentiation with respect to time gives

$$\dot{\xi}_{n+3} = \dot{y} = \tan\theta_n v \quad (25)$$

We get $\dot{\xi}_{n+3} = \xi_{n+2}v$ by choosing

$$\xi_{n+2} = \tan\theta_n \quad (26)$$

From (8), differentiation gives

$$\dot{\xi}_{n+2} = \frac{1}{\cos^2\theta_n}\dot{\theta}_n = \frac{\tan(\theta_{n-1}-\theta_n)}{d_n\cos^3\theta_n}v \quad (27)$$

We get $\dot{\xi}_{n+2} = \xi_{n+1}v$ by choosing

$$\xi_{n+1} = \frac{\tan(\theta_{n-1}-\theta_n)}{d_n\cos^3\theta_n} \quad (28)$$

We will show by induction that $\dot{\xi}_{i+1} = \xi_i v$ by choosing

$$\xi_i = \frac{\tan(\theta_{i-2}-\theta_{i-1})}{c_i(\underline{\theta}_{i-1})} + \tau_i(\underline{\theta}_i) \quad (29)$$

for $i \in \{2,\ldots,n\}$ where

$$c_i(\underline{\theta}_{i-1}) = p_{i-1}^2(\underline{\theta}_{i-1})\prod_{j=i-1}^{n} d_j p_j(\underline{\theta}_j) \quad (30)$$

$$\tau_i(\underline{\theta}_{i-1}) = \frac{\partial\xi_{i+1}}{\partial\underline{\theta}_i}\underline{f}_i(\theta_{i-1}) \quad (31)$$

where $i \in \{2,\ldots,n\}$. This means that $\xi_i = \xi_i(\underline{\theta}_{i-2})$. Assume that (29) is satisfied for $i = m$. Eqs. (7), (8), (12), (30) and (31) imply

$$\begin{aligned}
\dot{\xi}_m &= \frac{\partial\xi_m}{\partial\theta_{m-2}}\dot{\theta}_{m-2} + \frac{\partial\xi_m}{\partial\underline{\theta}_{m-1}}\dot{\underline{\theta}}_{m-1} \\
&= \frac{1}{\cos^2(\theta_{m-2}-\theta_{m-1})c_m(\underline{\theta}_{m-1})}\cdot \\
&\quad \frac{1}{d_{m-2}}\frac{\tan(\theta_{m-3}-\theta_{m-2})}{p_{m-2}(\underline{\theta}_{m-2})}v \\
&\quad +\frac{\partial\xi_m}{\partial\underline{\theta}_{m-1}}\underline{f}_{m-1}(\underline{\theta}_{m-2})v \\
&= (\frac{\tan(\theta_{m-3}-\theta_{m-2})}{c_{m-1}(\underline{\theta}_{m-2})} + \tau_{m-1}(\underline{\theta}_{m-2}))v
\end{aligned}$$

Note from (7) and (20) that

$$\begin{aligned}
c_{m-1} &= \cos^2\alpha_{m-2}c_m d_{m-2}p_{m-2} \\
&= \cos^2\alpha_{m-2}p_{m-1}^2(\prod_{j=m-1}^{n} d_j p_j)d_{m-2}p_{m-2} \\
&= p_{m-2}^2(\underline{\theta}_{m-2})\prod_{j=m-2}^{n} d_j p_j(\underline{\theta}_j)
\end{aligned}$$

where $\alpha_{m-2} = \theta_{m-2}-\theta_{m-1}$, $c_m = c_m(\underline{\theta}_{m-1})$, and $p_j = p_j(\underline{\theta}_j)$.

We have thus shown that if ξ_i is given by (29) for $i = m$ then

$$\dot{\xi}_m = \xi_{m-1}v$$

by choosing ξ_{m-1} as in (29) with $i = m-1$. It remains to be shown that if ξ_n is given by (29) with $i = n$ and ξ_{n+1} is given by (28) then

$$\dot{\xi}_{n+1} = \xi_n v$$

We find from (28), (8), (7), (12), (30) and (31) that

$$\begin{aligned}
\dot{\xi}_{n+1} &= \frac{\partial\xi_{n+1}}{\partial\theta_{n-1}}\dot{\theta}_{n-1} + \frac{\partial\xi_{n+1}}{\partial\theta_n}\dot{\theta}_n \\
&= \frac{1}{\cos^2(\theta_{n-1}-\theta_n)d_n\cos^3\theta_n}\cdot \\
&\quad \frac{1}{d_{n-1}}\frac{\tan(\theta_{n-2}-\theta_{n-1})}{\cos\theta_n\cos(\theta_{n-1}-\theta_n)}v \\
&\quad +\frac{\partial\xi_{n+1}}{\partial\theta_n}f_n(\underline{\theta}_{n-1})v \\
&= (\frac{\tan(\theta_{n-2}-\theta_{n-1})}{c_n(\underline{\theta}_{n-1})} + \tau_n(\underline{\theta}_{n-1}))v
\end{aligned}$$

This means that

$$\dot{\xi}_{n+1} = \xi_n v$$

by choosing

$$\xi_n = \frac{\tan(\theta_{n-2}-\theta_{n-1})}{c_n(\underline{\theta}_{n-1})} + \tau_n(\underline{\theta}_{n-1}) \quad (32)$$

Therefore, ξ_i is given by (29) for all $i \in \{2,\ldots,n\}$ and the transformations (14)-(19) imply that

$$\dot{\xi}_i = \xi_{i-1}v\ , \quad \forall i \in \{3,\ldots,n+3\}$$

To complete the proof we have to show that

$$\dot{\xi}_2 = u_2\ , \qquad \dot{\xi}_1 = u_1$$

We have shown that ξ_2 is given by (29) with $i = 2$. Differentiation gives

$$\begin{aligned}
\dot{\xi}_2 &= \frac{\partial\xi_2}{\partial\theta_0}\dot{\theta}_0 + \frac{\partial\xi_2}{\partial\underline{\theta}_1}\dot{\underline{\theta}}_1 \\
&= \frac{1}{\cos^2(\theta_0-\theta_1)c_2(\underline{\theta}_1)}\omega + \frac{\partial\xi_2}{\partial\underline{\theta}_1}\underline{f}_1(\underline{\theta}_0)v \\
&= \frac{1}{\cos^2(\theta_0-\theta_1)c_2(\underline{\theta}_1)}\omega + \tau_1(\underline{\theta}_0)p_0(\underline{\theta}_0)v_0
\end{aligned}$$

since $v = p_0(\underline{\theta}_0)v_0$. This implies that the transformation (23) makes

$$\dot{\xi}_2 = u_2 \tag{33}$$

From (8) it follows directly that

$$\dot{\xi}_1 = u_1 \tag{34}$$

by choosing $\xi_1 = x$ and $u_1 = v = p_0(\underline{\theta}_0)v_0$. We can thus conclude that the transformation (13)-(19) implies that

$$\begin{aligned} \dot{\xi}_1 &= u_1 \\ \dot{\xi}_2 &= u_2 \\ \dot{\xi}_3 &= \xi_2 u_1 \\ &\vdots \\ \dot{\xi}_{n+3} &= \xi_{n+2} u_1 \end{aligned}$$

where u_1 and u_2 are given by (22)-(23).

□

Several control strategies have been presented where $\xi = [\xi_1, \ldots, \xi_{n+3}]^T$ is controlled to zero using $u = [u_1, u_2]^T$, [5], [6], [8] and [7]. In order to show that the convergence of ξ to zero implies that z converges to zero, we have to show that the transformation given in Theorem 1 has an inverse

$$z = F^{-1}(\xi)$$

which is continuously differentiable in a neighborhood of zero and $F^{-1}(0) = 0$. Moreover, the feedback input transformation, $u = G(z)\nu$, has to have an inverse

$$\nu = G^{-1}(z)u$$

First we note from (7) and (9) that $f_i(\underline{\theta}_{i-1})$ is smooth, i.e. C^∞, in D. This implies that $\underline{f}_i(\underline{\theta}_{i-1})$ in (10) is smooth as well. Then it is straightforward to show by induction from the transformations in Theorem 1 that $\xi_i(\underline{\theta}_{i-2})$ is smooth in D for $i \in \{2, \ldots, n+2\}$ since $\xi_{n+1}(\theta_{n-1}, \theta_n)$ is smooth in D. The other transformations $\xi_1(x)$, $\xi_{n+2}(\theta_{n+2})$ and $\xi_{n+3}(y)$ are obviously smooth in D. Therefore the transformation $F(z)$ is smooth. We will then show that the Jacobian matrix

$$J = \frac{\partial F(z)}{\partial z}$$

is nonsingular for all z in D. The components of z are $[z_1, \ldots, z_{n+3}]^T = [x, \theta_0, \ldots, \theta_n, y]^T$. We see from the transformation $\xi = F(z)$ in Theorem 1 that

$$J_{ij} = \frac{\partial F_i(z)}{\partial z_j} = 0\,, \quad \text{if} \quad i > j$$

The matrix J is thus upper triangular. The diagonal elements are found to be

$$\begin{aligned} J_{11} &= 1 \\ J_{ii} &= \frac{\partial \xi_i}{\partial \theta_{i-2}} = \frac{1}{\cos^2(\theta_{i-2} - \theta_{i-1}) c_i(\underline{\theta}_{i-1})} \\ J_{n+2,n+2} &= \frac{1}{\cos^2 \theta_n} \\ J_{n+3,n+3} &= 1 \end{aligned}$$

where $i \in \{2, \ldots, n+1\}$. From the definition of $c_i(\underline{\theta}_{i-1})$, (20), we have that $J(z)$ is nonsingular for all $z \in D$. From the Inverse Function Theorem we can conclude that in a neighborhood of any $z \in D$, the transformation $\xi = F(z)$ has an inverse

$$z = F^{-1}(\xi)$$

which is smooth.

We see from $F(z)$ given in Theorem 1 that $F(0) = 0$ which implies that $F^{-1}(0) = 0$. Since $F^{-1}(\xi)$ is smooth we can conclude that convergence of ξ to zero implies convergence of z to zero.

From (7), (9), (22) and (23) we have that the feedback input transformation, $u = G(z)\nu$ is well defined for all $z \in D$ since

$$\frac{\partial \xi_2}{\partial \underline{\theta}_1} \underline{f}_1(\underline{\theta}_0) = \tau_1(\underline{\theta}_0)$$

is smooth. We find the inverted transformation, $\nu = G^{-1}(z)u$, from (21) and (22)-(23):

$$\begin{bmatrix} v_0 \\ \omega \end{bmatrix} = G^{-1}(z) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where

$$G^{-1}(z) = \begin{bmatrix} \frac{1}{p_0(\underline{\theta}_0)} & 0 \\ -\cos^2(\theta_0 - \theta_1)c_2\tau_1, & \cos^2(\theta_0 - \theta_1)c_2 \end{bmatrix}$$

where $c_2 = c_2(\underline{\theta}_1)$, (20), and $\tau_1 = \tau_1(\underline{\theta}_0)$, (21). This transformation is nonsingular and smooth for all $z \in D$ which implies that a control law for $u = [u_1, u_2]^T$ can be transformed to a control law for $\nu = [v_0, \omega]^T$. Therefore, locally in D a control problem for a car with n trailers can be solved by using a control law for a chained system via the transformations $\xi = F(z)$ and $\nu = G^{-1}(z)u$.

Remark: The proof of Theorem 1 is in fact similar to the constructive proof of Proposition 4 in [5] which was given in terms of Lie brackets. The proof given here, therefore, shows that the new kinematic model (8) satisfies the necessary conditions in [5] for converting systems into a chained form.

4 Conclusions

It has been shown how a kinematic model of a car with n trailers can be converted locally into a nilpotent chained form by a change of coordinates and an invertible feedback transformation of the inputs. In order to achieve this conversion, the location of the car-trailer system was modeled with the position of the *rear* trailer.

The conversion holds when the orientation angles of the trailers are less than $\pi/4 - \varepsilon$ in magnitude where ε is an arbitrary small constant. In fact, the conversion holds if the less conservative condition $\cos(\theta_{i-1} - \theta_i) > 0$ holds for all $i \in \{1, \ldots, n\}$ and $\cos\theta_n > 0$. There is no condition on the position (x, y) of the system. Since the angels θ_i will tend towards zero as the pulling car advances along the x-axis in positive x-direction. Therefore, the local assumption on the orientations will be satisfied in finite time by driving the pulling car along the x-axis.

Under the assumption of exact modeling of the car/trailer system, the proposed conversion of the kinematic model into a chained form makes it possible to use control strategies developed for chained systems to control a car with n trailers. In particular, a stabilizing feedback law for a chained system can be used to locally stabilize a car with n trailers.

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