Fast-model predictive control of multivariable systems

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Abstract: The paper extends a simplified fast-model predictive control technique to a multivariable system. The strategy is based on identifying drive combinations which will maximise the time at which predicted variables are 'offside', having a sign which opposes that of the drive signal. It is relevant to systems such as multi-axis robot configurations, in which each variable can be associated with an input which dominates it, so that, if driven by an input of limiting value, the variable will ultimately take the same sign as the input. Billingsley's SISO strategy is restated and the multivariable strategy, as extended by Mo, is illustrated by a simulation for which results are presented.

1 Introduction

Fast-model predictive control is based on the use of a fast iterative model of the process to be controlled. A variety of trial inputs are applied to the model, and, from the resulting responses, a decision is made on the input to be applied to the plant. The method is primarily applicable to bang-bang control.

Since the first application of fast-model predictive control to closed-loop control systems was described by Coales and Noton [1], many advances have been made. Chestnut *et al.* [2, 3] presented a simplified strategy for second-order systems. Gul'ko and Kogan [8] suggested a strategy for a third-order system, and, later [9], Gul'ko *et al.* proposed several solutions for the *n*th-order problem. The optimality of Gul'ko's strategy has been proved by Fuller [4, 5].

Billingsley and Coales proposed a family of simple time-suboptimal predictive control algorithms for high-order systems [6]. For the third-order case, Fuller has shown that these algorithms give a performance very close to the optimal. Megson and Stiles have applied predictive control to multivariable systems [10, 12]. Nehrir considered the use of an adaptive fast model [11]. Ryan [13] studied time-optimal control of fourth-order systems and, later, in 1981, published a book which includes reports of the works of Fuller and of Billingsley and Coales [14].

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Recently, Billingsley proposed a new time-suboptimal algorithm for high-order plants which centred on the concept of the model state being 'offside' [7]. The performance is closely similar to that achieved by the algorithm published in 1968 [6], but is greatly simplified in concept; it is based on selecting the sign of drive for which the fast-model state is offside at the greatest value of model future time [7]. It yields time-suboptimal control for a SISO plant and is conceptually easy to extend to plants with arbitrarily high-order models.

The present paper considers the extension of similar strategies to multivariable systems. The methods are particularly appropriate for systems such as multiple robot axes with dynamic interaction. As an example, a control algorithm is devised for a double pendulum with linearised model. Simulation results are presented.

2 Predictive control based on the value of model offside time

Assume that the plant to be controlled is linear, stationary and consists of a cascade of integrators with a single input. The state of the plant is then described by the output of the final integrator and its derivatives. The input F is now bounded, such that

$$F_{min} \leqslant F \leqslant F_{max}$$

The model state is defined to be offside when the sign of the output is opposite to that of the applied input. For this simple plant, where the response is a polynomial in time, it can readily be shown that the 'eventual' sign of the output must match that of the input; for each value of model state, there will be a time beyond which the state will always be 'onside'. Similarly, there will be a time when each derivative will be locked 'onside', and it can be shown that the model run can be terminated when all the states are first onside together.

The basic strategy of predictive control, based on the concept of the value of model offside time, may be summarised as follows:

(*a*) Set the model state to correspond to the plant state

(*b*) Run the model with maximum positive drive. Note the greatest value of model time at which the model is offside

(*c*) When all states are onside, reset the model state to the plant state

(*d*) Run the model with extreme negative drive. Again note the greatest value of time for which the model is offside

(*e*) Whichever value of offside time is greater, apply that drive to the plant. Repeat the cycle.

For less simple systems which do not consist of cascaded integrators, the trajectory of the plant must, by definition, approach the origin within quadrant 2 or quadrant 4 of the phase plane. The strategy described above can be used for noncascade integrator systems; simulations have shown that this control strategy can result in good performance and details will be submitted in a later paper.

3 Predictive control of multivariable systems

Suppose that the system is composed of N interacting subsystems, each subsystem being affected by all N input values. Suppose also that the crosscoupling is subject to domination by the input designated to each subsystem, so that each output will eventually assume the sign of the corresponding input. Let the controller drive every input to either the maximum or minimum limit of magnitude. There are two possible bang-bang input values for the input of a SISO system, the maximum and minimum drive. For a multivariable system with N inputs, there are 2^N possible values of the input set for the fast model. The control strategy for such a system is as follows:

Step 1: For i = each of the 2^N possible sets of input values:

(*a*) set the model state to the plant state

(*b*) run the model with the set of N inputs:

inp $(i, 1)$, inp $(i, 2) \cdots$ inp (i, N)

(*c*) note the set of the N greatest offside times of each output:

ofs $(i, 1)$, ofs $(i, 2) \cdots$ ofs (i, N)

(*d*) When all the outputs have become onside, each run can be terminated and the model reset for the next value of i.

Step 2:

(*a*) After all possible sets of inputs have been tried, for each output ($j = 1$ to N), compare values of the maximum offside times recorded for each input set:

ofs $(1, j)$, ofs $(2, j) \cdots$ ofs $(2^N, j)$

Note the value of i ($i = 1$ to 2^N) which gives the greatest offside time of the output j and choose, for the value of the jth input, the corresponding sign of drive inp (i, j)

(*b*) When all such drives have been deduced, apply these drives to the plant. Go to step 1.

4 Example

The applicability of the proposed predictive control strategy is illustrated here by a simple example, the control of a double pendulum with linearised model. This is shown in Fig. 1.

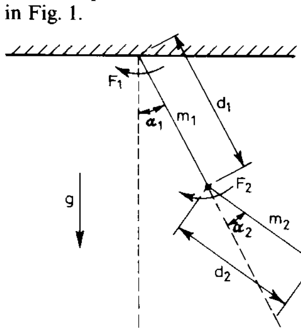

Fig. 1 *Double pendulum example*

The linearised dynamic equations of the motion are as follows:

$$D\ddot{e}(t) + 2S_v\dot{e}(t) + \{S_d + S_c + S_g\}e(t) = F(t)$$

where

$$D = \begin{pmatrix} d_1^2(m_1 + m_2) + d_2^2 m_2 + 2d_1 d_2 \cos(\alpha_2) & d_2^2 m_2 + d_1 d_2 m_2 \cos(\alpha_2) \\ d_2^2 m_2 + d_1 d_2 \cos(\alpha_2) & d_2^2 m_2 \end{pmatrix}$$

$$S_v = \begin{pmatrix} -d_1 d_2 m_2 \sin(\alpha_2)\dot{\alpha}_2 & -d_1 d_2 m_2 \sin(\alpha_2)(\dot{\alpha}_1 + \dot{\alpha}_2) \\ d_1 d_2 m_2 \sin(\alpha_2)\dot{\alpha}_1 & 0 \end{pmatrix}$$

$$S_d = \begin{pmatrix} 0 & -d_1 d_2 m_2 \sin(\alpha_2)(2\ddot{\alpha}_1 + \ddot{\alpha}_2) \\ 0 & -d_1 d_2 m_2 \sin(\alpha_2)\ddot{\alpha}_1 \end{pmatrix}$$

$$S_c = \begin{pmatrix} 0 & -d_1 d_2 m_2 \cos(\alpha_2)\dot{\alpha}_2(2\dot{\alpha}_1 + \dot{\alpha}_2) \\ 0 & d_1 d_2 m_2 \sin(\alpha_2)\ddot{\alpha}_1 \end{pmatrix}$$

$$S_g = \begin{pmatrix} d_1 g(m_1 + m_2)\cos(\alpha_1) + d_2 g \cos(\alpha_1 + \alpha_2) & d_2 g m_2 \cos(\alpha_1 + \alpha_2) \\ d_2 g \cos(\alpha_1 + \alpha_2) & d_2 g m_2 \cos(\alpha_1 + \alpha_2) \end{pmatrix}$$

where $F(t) = [F_1(t), F_2(t)]^T$ is the control torque vector, m_1 and m_2 are masses of links, g is the gravitational constant, d_1 and d_2 are lengths of links, α_1 and α_2 are values of position, about which the dynamic equations are linearised, and $\dot{\alpha}_1$, $\dot{\alpha}_2$, $\ddot{\alpha}_1$ and $\ddot{\alpha}_2$ are first and second derivatives of α_1 and α_2, respectively. $e(t) = [e_1(t), e_2(t)]^T$, $\dot{e}(t) = [\dot{e}_1(t), \dot{e}_2(t)]$ and $\ddot{e}(t) = [\ddot{e}_1(t), \ddot{e}_2(t)]$ are the position, velocity and acceleration of error vector, respectively.

Assuming

$$m_1 = m_2 = 1.0 \text{ kg}, d_1 = d_2 = 1.0 \text{ m}, \quad + |F_1(t)| < 10 \text{ Nm}, |F_2(t)| < 5 \text{ Nm}$$

and

$$\alpha_1 = \alpha_2 = 0, \dot{\alpha}_1 = \dot{\alpha}_2 = 0, \ddot{\alpha}_1 = \ddot{\alpha}_2 = 0$$

so that the equations are linearised about the rest state,

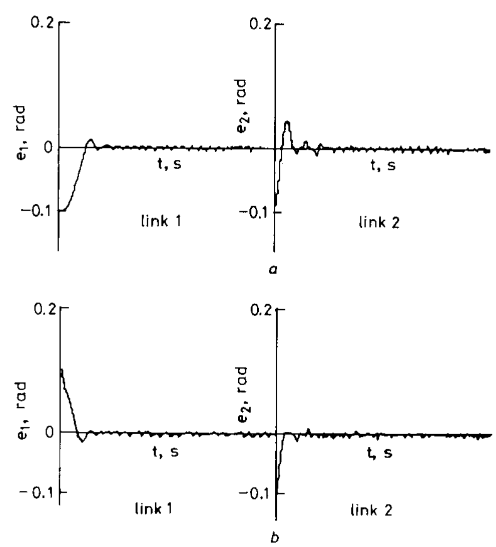

Fig. 2 *Simulation results of the predictive algorithm*

the linearised dynamic equations are

$$5\ddot{e}_1(t) + 2\ddot{e}_2(t) + 39.8e_1(t) + 9.8e_2(t) = F_1(t)$$

$$2\ddot{e}_1(t) + \ddot{e}_2(t) + 9.8e_1(t) + 9.8e_2(t) = F_2(t)$$

Defining the state vector as

$$E(t) = [e_1(t), \dot{e}_1(t), e_2(t), \dot{e}_2(t)]^T = [E_1, E_2, E_3, E_4]^T$$

we have state-space equations:

$$\dot{E}_1(t) = E_2(t)$$

$$\dot{E}_2(t) = -9.8E_1(t) + 9.8E_3(t) + F_1(t) - 2F_2(t)$$

$$\dot{E}_3(t) = E_4(t)$$

$$\dot{E}_4(t) = 9.8E_1(t) - 3 \times 9.8E_3(t) - 2F_1(t) + 5F_2(t)$$

Fig. 2 shows the results of simulation.

5 Conclusion

A fast-model predictive control algorithm for multivariable systems has been developed. This control strategy is based on the concept of an offside value of the model state. Simulation, performed for a double pendulum using linearised equations, has demonstrated the feasibility of applying the algorithm proposed in this paper. Although the performance is seen to be 'good', a comparison with that of more conventional controllers has not been included here.

Since the 1970s, research into fast-model predictive control has been somewhat subdued. With the explosion of the availability of digital computer systems applicable to real-time control and with the growing interest in strategies based on logical inference, fast-model control shows renewed promise.

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