From Linear to Nonlinear Model Predictive Control: Comparison of Different Algorithms

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Performances of predictive control strategies which make use of different forms of linearized models are compared. The linear models range from a simple fixed model, linearized around a reference steady state, to a locally linearized model, which is updated at every sampling time by making use of a full first-principle nonlinear model. An intermediate choice is represented by the use of a weighted sum of different local models. Not only the process nonlinearity but also its directionality and the process-model mismatch affect the differences in performance of the controllers. All these aspects therefore need to be taken into account in order to determine which is the control algorithm that is more advantageous to implement. A terpolymerization reactor is used as a test process.

1. Introduction

Inside the broad denomination of model predictive control (MPC), all the most advanced control strategies are included that have seen favorable acceptance in industry. Their basic advantages are their multivariable nature that permits to address the control problem globally and the possibility of explicitly taking account of the plant constraints.

For some applications, one of the major limitations of MPC techniques in their original formulation comes from the use of a linear model of the process inside the algorithm (Richalet *et al.*, 1978; Cutler and Ramaker, 1979). Such a linear model is used at every sampling time in order to find out which inputs to the process lead its outputs from the current values to future ones, which are as close as possible to desired trajectories to the set points, without violating constraints. The use of this control strategies with nonlinear processes is therefore subject to performance limitations due to process-model mismatch. In particular, it is known (Skogestad and Morari, 1987) that inverse-based controllers are highly sensitive to uncertainties and mismatch when the controlled process is ill conditioned, i.e. when the gain of the process is strongly dependent on the process inputs.

The introduction of nonlinear models inside the control algorithm does not cause major problems from a theoretical point of view (see, e.g., Sistu and Bequette, 1991). However, from a computational point of view, the optimization problem that needs to be solved at every sampling time can become so intensive, particularly for high-dimensional systems, to make on-line applications almost impossible (Sistu *et al.*, 1993).

Quadratic dynamic matrix control (QDMC) and its extension to include nonlinear processes (nonlinear QDMC−NLQDMC) offer the major advantages of having clear objective functions simply connected with closed-loop performance and of not requiring an excessive increase in computational complexity in the nonlinear case (Garcia, 1984; Gattu and Zafiriou, 1992; Lee and Ricker, 1994).

In any case, it is of paramount importance to understand the motivations behind the choice of one or the other control algorithm and to identify the possibility of intermediate solutions that maintain the simplicity of the linear input−output model but include a partial knowledge of the nonlinearity. These are the purposes of this paper. In particular, linear QDMC (LQDMC), NLQDMC with parameter estimation, and a minor modification of LQDMC, which makes use of a local model that is a weighted sum of locally linearized models, are compared. As a test process, a nonlinear ill-conditioned polymerization reactor is used. The comparison of performances in many different cases (set-point variations in the favorable and unfavorable directions with and without parametric uncertainty at different steady-states) makes possible the assessment of conditions for the choice of the algorithm to use.

Previous works concerning comparison of different control strategies for nonlinear systems include the ones of Wright and Edgar (1994) and Ricker and Lee (1995). In their paper Wright and Edgar compare a sequential nonlinear model predictive controller (NMPC) applied to a fixed-bed water-gas shift reactor to an adaptive linear controller and show that NMPC is superior particularly when broad nonlinear operating regions are traversed (e.g. during startup). Ricker and Lee (1995) apply NMPC to the Tennessee Eastman challenge process; in their work they point out that, for such a large scale nonlinear problem, the appropriate choice of controlled and manipulated variables and the use of application-specific procedures are major concerns also when applying NMPC. In any case, the nonlinear controller is able to enhance performance when compared to its linear counterpart. It is the interest of this paper to compare different control strategies applied to a nonlinear test process with particular attention to the effects of directionality and parametric uncertainty in order to give a further contribution toward the development of criteria for the choice of the appropriate control algorithm.

2. The Three Model Predictive Controllers

From a formal point of view, most model predictive control strategies solve at every sampling time t_k an optimization problem of the following kind: compute the set of L future control moves that minimize an objective function and satisfy input and state constraints

$$\min_{\mathbf{u}(k)\ldots\mathbf{u}(k+L-1)} \sum_{i=k+1}^{k+R} \sum_{j=1}^{N} [y_j^{\mathrm{d}}(t_i) - y_j^{\mathrm{pred}}(t_i)]^2 \qquad (1)$$

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$$u_{j,\min} \le u_j(t_i) \le u_{j,\max} \tag{2}$$

$$\Delta u_{j,\min} \le |\Delta u_j(t_i)| \le \Delta u_{j,\max} \tag{3}$$

$$x_{l,\min} \le x_l(t_i) \le x_{l,\max} \tag{4}$$

where the dynamic and measurement equations are as follows:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p}) \tag{5}$$

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) \tag{6}$$

x, **u**, **y**, **p** are the states, inputs, outputs, and parameters, respectively, L is the control horizon, R is the prediction horizon, N is the number of controlled variables, and the objective function includes the square differences between the predicted outputs and their future desired values (usually along an exponential trajectory to the set point). Alternative formulations include different objective functions: linear forms have been used in some cases, and also a penalty on changes in the control has been commonly added.

The main differencies among the existing strategies consist of the choice of the model used to represent the process and of the algorithm chosen for the optimization.

2.1. LQDMC. Linear QDMC (Cutler and Ramaker, 1979) has been widely known for many years. Only its basic aspects are briefly summarized here in order to make it clear which are the similarities and the differences with the approaches that follow.

A linear input−output nonparametric model either in the impulse response form or in the step response form is used to represent the process. The prediction of the future response is built by summing up three contributions:

(1) the effect of past inputs (linear, known),

(2) the effect of future inputs (linear, to be determined), and

(3) an error term due to the difference between current measurements and previous predictions (linear, known).

In this way the outputs are linear in the optimization parameters which appear in the objective function so that an analytical solution is possible if constraints are not violated (a pseudoinversion of an appropriate matrix is needed) or a quadratic programming problem is to be solved at every sampling time.

The usual desired trajectory is an exponential to the set point:

$$y_j^d(t_{k+i}) = \alpha_j^i y_j(t_k) + (1 - \alpha_j^i) y_j^{s-p} \qquad 1 < i < R \tag{7}$$

The basic parameters for the tuning of the controller are the exponential time constants (or the parameters α_j) and the control horizon L; by means of these parameters the appropriate detuning can be accomplished.

2.2. Linear Weighted QDMC (LWQDMC). The closest variation to LQDMC that allows to take into account changes in the process model due to variations in the steady-state conditions is surely the possibility of introducing in the algorithm a new local model that is linearized around the changed steady state. A new process identification any time there are changes in the operating conditions is surely impractical so that a shortcut way of buiding locally linearized models is proposed.

Let us suppose to identify preliminarly n local linear models $\mathbf{M}_i$ linearized around n steady-state controlled variable vectors $\mathbf{y}^i$. For a steady state characterized by the controlled variable vector $\mathbf{y}$, a local model can be defined as a weighted sum of the available local models:

$$\mathbf{M} = \sum_{i=1}^{n} w_i \mathbf{M}_i \tag{8}$$

In eq 8 the linear combination refers to the linear combination of all the coefficients of the impulse or step response models.

The weights need to be defined in such a way that the models nearby in the controlled variable space give a strong contribution and the ones far away a very small one. The definition of a distance is therefore appropriate. First a scaling is accomplished as follows:

$$\bar{\mathbf{y}} = \left[\frac{y_1}{y_{1\max} - y_{1\min}}, \frac{y_2}{y_{2\max} - y_{2\min}}, \ldots, \frac{y_l}{y_{l\max} - y_{l\min}}\right]^{\mathrm{T}} \tag{9}$$

where $y_{j\max} = \max_i(y_j^i)$, $y_{j\min} = \min_i(y_j^i)$. Then the two-norm distance between the operating conditions 1 and 2 is defined as:

$$|\bar{\mathbf{y}}^2 - \bar{\mathbf{y}}^1| = [\sum_{i=1}^{l} (\bar{y}_i^{\,2} - \bar{y}_i^{\,1})^2]^{1/2} \tag{10}$$

If the distance between a generic point $\mathbf{y}$ and the point $\mathbf{y}^i$ (after rescaling) is indicated as d_i, the weights to be used in eq 8 are computed as follows:

$$w_i = \frac{d_i^{-1}}{\sum_{j=1}^{n} d_j^{-1}} \tag{11}$$

In this way the previous specification for the weights is satisfied; moreover, it is easily shown that $\sum_{i=1}^{n} \mathrm{w_i} = 1$ and that if $\mathbf{y} \to \mathbf{y}^i$, $w_i \to 1$ and $w_j(j{\ne}i) \to 0$.

The use of the model **M** in the LQDMC strategy does not cause any change from the previous case in the optimization algorithm once the model **M** has been computed. The computation of a new model can be accomplished once every sampling time or any time the distance from the conditions of the last computed model is larger than a minimum value $d_{\min}$. This control algorithm will be referred to as linear weighted QDMC (LWQDMC) in what follows.

A few final words about the number and the position of the local models $\mathbf{M}_i$ are necessary. It is clear that a large number of models corresponds to a more precise description but also to a greater identification effort. Opposite considerations hold for a small number of models. Even if precise conditions depend strongly on the kind of nonlinearity, we feel that it is important to place the models at the extremes of the region that it is desired to describe. This will be confirmed in the analysis of the test process.

2.3. NLQDMC with Parameter Estimation. A more comprehensive approach to nonlinearity is surely through the use of a detailed first-principle nonlinear model. NLQDMC uses a nonlinear model to obtain at every sampling time a precise local linear model. In order for the local linearization to be possible, also an estimation of all the system states is necessary at every sampling time so that an estimation algorithm needs to be included in the control procedure.

In the NLQDMC framework, the predicted outputs are computed by summing up three contributions, which are completely equivalent to the ones in the linear case:

(1) the effect of past manipulated variables is computed by integrating the nonlinear model starting from the estimated states $\hat{\mathbf{x}}(t_k)$ (DASSL by Petzold (1982) is used as integrator in this work);

(2) the effect of future manipulated variables is computed using a single-step response model which is obtained through local linearization of the nonlinear model (again the estimated states $\hat{\mathbf{x}}(t_k)$ are needed);

(3) the unmodeled effects are taken into account by using the current difference between the actual measurements and their previous estimates as estimates of the error at future times (this contribution adds feedback characteristic to the algorithm).

In this way the problem which needs to be solved at each sampling time is the same quadratic programming problem of linear QDMC (Garcia, 1984) given that the unknown values of the future manipulated variables appear only in the local linear model. As far as the local linearization is concerned, it is suggested in this work to compute the coefficients of the step response model by integrating the nonlinear model with small variations of the manipulated variables starting from the current estimated state $\hat{\mathbf{x}}(t_k)$. This is advantageous over analytical computation of the local continuous linear model and its subsequent discretization (Gattu and Zafiriou, 1992) especially for high-dimensional complex models.

In order to obtain $\hat{\mathbf{x}}(t_k)$, an extended Kalman filter (EKF) is used as state estimator (see, e.g., Ray, 1981). The filter computes the current estimate $\hat{\mathbf{x}}(t_k)$ by solving the model equations starting from the previous estimate $\hat{\mathbf{x}}(t_{k-1})$

$$\dot{\hat{\mathbf{x}}}(t/t_{k-1}) = \mathbf{f}(\hat{\mathbf{x}}(t/t_{k-1}),\mathbf{u},\mathbf{p}) \qquad (12)$$

and updating the estimates at the sampling time when measurements are available

$$\hat{\mathbf{x}}(t_k) = \hat{\mathbf{x}}(t_k/t_{k-1}) + \mathbf{K}(t_k)\{\mathbf{y}(t_k) - \mathbf{g}(\hat{\mathbf{x}}(t_k/t_{k-1}))\} \qquad (13)$$

($\hat{\mathbf{x}}(t/t_{k-1})$ is the estimated value at time t with measurements available until time t_{k-1}).

The filter gain $\mathbf{K}(t_k)$ is computed at each sampling time as a function of the Jacobians of the system and measurement equations ($\mathbf{F} = \partial\mathbf{f}/\partial\mathbf{x}$, $\mathbf{G} = \partial\mathbf{g}/\partial\mathbf{x}$) and of the covariance matrices of the process noise, the measurement noise, and the error on the initial conditions. The addition of a closed-loop estimator makes this procedure applicable also to open-loop unstable processes, which is not the case for the previous approaches. The algorithm which is adopted is therefore similar to the one in Gattu and Zafiriou (1992) with the introduction of an EKF which substitutes the steady-state Kalman filter and the numeric computation of the local linear model instead of the analytical computation of the local continuous linear model and its subsequent discretization as specified above. As explicitly stated by Gattu and Zafiriou (1992) and also discussed by Ricker (1990), it is necessary in the prediction of the future outputs to add the estimate of the error at future times to the effects of past and future manipulated variables in order to eliminate offset, even though the model states are compensated for unmodeled effects.

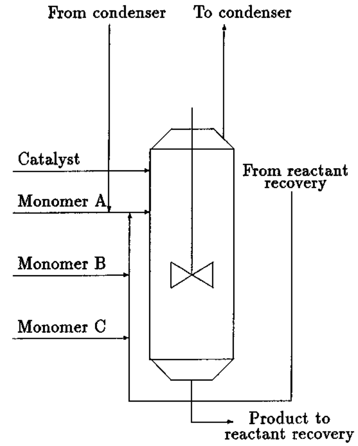

Figure 1. Schematic representation of the reactor.

A further characteristic of the NLQDMC used in this paper is the addition of parameter estimation. In fact the Kalman filter theory is based on the assumption that noises and errors are random, Gaussian, and noncorrelated and is therefore inappropriate in the case of model parametric uncertainty. When systematic errors on the model parameters are present, the EKF as above is affected by offsets in the values of the state estimates (Semino *et al.*, 1996), which degrades the nonlinear model prediction ability and can involve offsets in the controlled variables as well. In any case, if parametric errors are present, the filter structure can still be used by adding a number of model parameters as fictitious states to be updated in order to take the systematic error into account.

This is easily accomplished by including in the dynamic model the equations

$$\dot{\mathbf{p}} = 0 \qquad \mathbf{p}(0) = \mathbf{p}_0 \qquad (14)$$

where the off-line parameter estimates (taken from the literature or from off-line experiments) are used as initial values. In this way the advantageous possibility of changing the model on-line is added to the algorithm.

An alternative approach to state and parameter estimation, which has not been considered in this paper, is the use of horizon-based optimization methods (Ramamurthi *et al.*, 1993; Robertson *et al.*, 1996).

The *three predictive controllers* which have been presented can be seen as three different attempts of using a linear model in a control algorithm for a nonlinear process. The first one uses a fixed local model obtained at the beginning and never changed. The second uses a weighted sum of local models that tries to follow on-line the changes due to nonlinearities. The third and more comprehensive approach accomplishes a local linearization at every sampling time taking advantage of a full nonlinear model of the process. It is interesting to understand under what circumstances an improvement in performance can be obtained with the more complicated controllers. This will be the effort of the analysis of the test process.

3. Test Process

The test process which is used to compare the control strategies is a continuous stirred-tank reactor in which a terpolymerization from three monomers A, B, and C in the presence of a catalyst is accomplished. A simple and schematic representation of the reactor is shown in Figure 1. A large condenser, which feeds back

Table 1. Product Specifications and Steady-State Characteristics

	% B	% C	C_A (mol/L)	C_B (mol/L)	C_C (mol/L)
product range	24−30	3−6			
steady state 1	27	4.4	1.69	2.81	0.136
steady state 2	24	6	2.27	3.41	0.245
steady state 3	30	3	1.26	2.65	0.0753
steady state 4	30	6	2.97	6.73	0.393
steady state 5	24	3	0.983	1.39	0.0486

evaporated reactants to the reactor, is used to eliminate the heat of reaction. The kinetic scheme of the polymerization, a detailed model of the process, and the values of the kinetic and physical constants are provided in the Supporting Information.

The purposes of the complete control system of the reactor are to maintain at specified values both the production rate and product quality indicators, namely, the melt viscosity index (which is closely related to the molecular weight distribution) and polymer composition (weight percent A and percent B in the polymer). Since the product quality measurements usually have both a large delay (30−40 min) and a large sampling time (up to a few hours), a reasonable approach for the control system is to control at a first level the reactant compositions, which are measured frequently (every 5 min) and with a short delay (10−15 min) and correct their setpoints with an outer level controller when the product quality measurements are available (Ogunnaike, 1994).

In this work attention is focused on the first-level controller in order to interpret correctly the improvements in performance due to the first level control algorithm without mixing them with the improvements associated with the correct operation of the two-level controller. The compositions of monomers A, B, and C in the reacting mixture are therefore the controlled variables, and their feed flow rates the manipulated ones.

The same reactor is used to manifacture several different grades of the same polymer. In this work a single product is considered whose specifications in terms of polymer composition are listed in Table 1. It is clear that there are wide ranges of compositions of the reacting mixture which correspond to products within the specifications. In particular, five versions of the same product are considered (Table 1). All of them satisfy the specifications on the polymer compositions even if the process behaves very differently in proximity of each of the different steady states.

The linearization of the system around steady state 1 gives the following transfer function matrix relating controlled to manipulated variables (a normalization of both manipulated and controlled variables with respect to their steady-state value is accomplished; time is in hours):

$$\mathbf{G}_1(s) = \begin{pmatrix} -\dfrac{0.838(-4s+1)}{(0.4s+1)(6s+1)} & +\dfrac{0.771}{7.5s+1} & +\dfrac{1.16}{5s+1} \\ -\dfrac{2.08}{7.5s+1} & +\dfrac{2.08}{5s+1} & +\dfrac{1.18}{7.5s+1} \\ -\dfrac{1.71}{5s+1} & +\dfrac{1.09}{7.5s+1} & +\dfrac{2.24}{4.5s+1} \end{pmatrix} \quad (15)$$

The process is highly interactive and strongly ill conditioned with the following relative gain array (RGA; Bristol (1966)) and singular value decomposition (SVD; Klema and Laub (1980)):

$$\mathbf{\Lambda} = \begin{pmatrix} -4.004 & +2.885 & +2.119 \\ +1.363 & +0.3138 & -0.6770 \\ +3.641 & -2.199 & -0.442 \end{pmatrix} \quad (16)$$

$$\mathbf{G}_1(0) = \mathbf{V\Sigma U}^{\mathrm{T}} \qquad \gamma = 31 \quad (17)$$

$$\mathbf{V} = \begin{pmatrix} -0.3529 & -0.2251 & 0.9082 \\ -0.676 & 0.7324 & -0.0812 \\ -0.6469 & -0.6426 & -0.4107 \end{pmatrix} \quad (18)$$

$$\mathbf{\Sigma} = \begin{pmatrix} 4.5375 & 0 & 0 \\ 0 & 1.0887 & 0 \\ 0 & 0 & 0.1458 \end{pmatrix} \quad (19)$$

$$\mathbf{U} = \begin{pmatrix} 0.6188 & -0.2167 & 0.7550 \\ -0.5238 & 0.6024 & 0.6022 \\ -0.5854 & -0.7682 & 0.2593 \end{pmatrix} \quad (20)$$

The decomposition shows in particular that an input in the unfavorable direction (i.e. the one associated with the minimum singular value), corresponding to the vector [0.755, 0.6022, 0.2593], moves the output in the direction [0.9082, −0.0812, −0.4107] with a gain of 0.1458.

Let us understand the role played by the nonlinearities. If steady state 4 is considered, the gain matrix changes as follows (the same normalization as in eq 15 is used):

$$\mathbf{G}_4(0) = \begin{pmatrix} -1.08 & 0.956 & 1.18 \\ -2.56 & 2.29 & 1.20 \\ -1.96 & 1.39 & 2.24 \end{pmatrix} \quad (21)$$

With the exception of column 3, whose gains are almost unchanged, all gains vary by 10−20%. However, these changes affect much more relevantly the directional behavior of the plant. In this case, the same input vector as before [0.755, 0.6022, 0.2593] moves the output in the direction [0.256, −0.937, −0.239] with a gain of 0.259. The effect of the input is completely different, and the gain is almost twice as before. If one considers that partial changes take place in the time constants as well, it is easy to imagine how difficult it becomes for the linear controller to predict and follow the process behavior. Such variations are not as strong for the favorable direction (i.e. the one associated with the maximum singular value) as will be confirmed by the simulations.

4. Results

The three kind of controllers have been applied to the test process for set-point tracking in the favorable and in the unfavorable direction starting from different steady states. Moreover the effect of parametric uncertainty has been addressed.

The parameters of all predictive controllers have been chosen as $\alpha_j = 0.9$ ($j = 1, 2, 3$), $L = 3$, $R = 6$, $N = 300$ with a sampling time $T_s = 5$ min. A delay $\theta_m = 2T_s$

due to the analyzer has been considered for all composition measurements. The model used in the LWQDMC is updated at every sampling time.

4.1. Comparison of LQDMC and LWQDMC. As a first test, the behavior of the process for a step set-point change in the favorable direction around steady state 1 has been analyzed.

Performances of three controllers have been compared:

(1) a LQDMC which uses a model linearized around steady state 1 (L1),

(2) a LQDMC which uses a model linearized around steady state 5 (L5), and

(3) a LWQDMC which weights four models linearized around steady states 2−5 (LW2345).

Figure 2 shows the dynamic behavior of the three controlled variables when each of the three controllers is used; percentual variations from the starting steady state are used for the concentrations both here and in the examples that follow. Both L1 and LW2345 follow desired exponential trajectories to the steady state; L5 contains wrong steady state and dynamic information and takes some more time to recognize the correct control actions to lead the process to the correct steady state. Model weighting is therefore successful in this case to describe correctly the nonlinear process with precision similar to the one of the correct locally linearized model. Similar results are obtained if simple SISO control problems are considered (not shown); in these cases the weighted model is a good substitute of the nonlinear model.

4.2. Effect of Directionality. The behavior of the same three controllers as above in the case of a step set-point change in the unfavorable direction around steady state 1 has been analyzed (Figure 3). Still the controller LW2345 behaves very similarly to the controller L1, but their performance degrades relevantly when compared to the one in the favorable direction. This is related to the effect of nonlinearities: small uncertainties due to the linear approximation are largely amplified in the unfavorable direction for this ill-conditioned plant. This results in a slow approach to the steady state.

The controller built around a different steady state (L5), which has wrong information about the gains, addresses the change in set point with stronger actions. This results in a more oscillatory behavior but also in a somewhat faster approach to the steady state.

4.3. Effect of the Position of the Steady States. In the example above, model weighting has been used to obtain the approximation of a local linear model whose characteristic values are included in the ranges delimited by the weighted models. It is important to understand that if such is not the case, model weighting can cause misleading results.

The behavior of the process for a step set-point change in the favorable and in the unfavorable direction around steady state 5 has been analyzed.

Performances of three controllers have been compared:

(1) a LQDMC which uses a model linearized around steady state 5 (L5),

(2) a LQDMC which uses a model linearized around steady state 1 (L1), and

(3) a LWQDMC which weights four models linearized around steady states 1−4 (LW1234).

Figures 4 and 5 show the dynamic behavior for the favorable and the unfavorable direction, respectively.

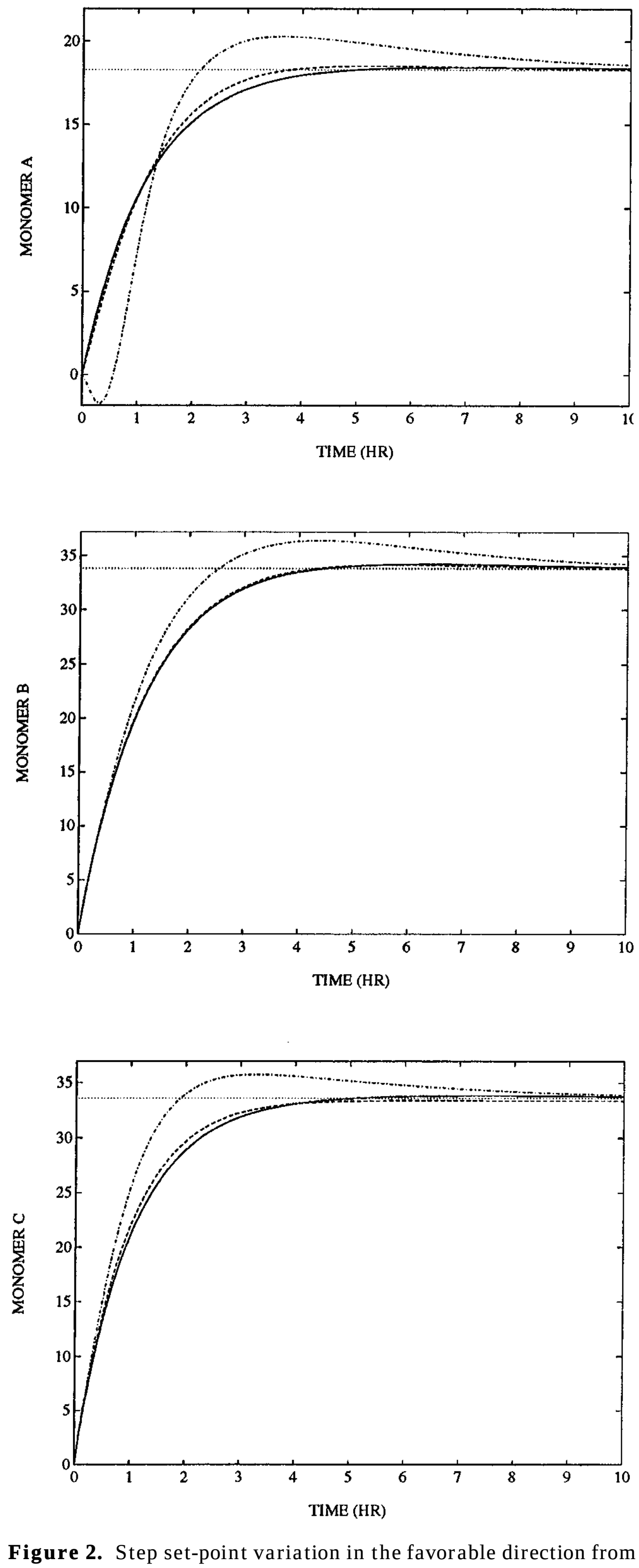

Figure 2. Step set-point variation in the favorable direction from steady state 1: —, L1; —•—, L5; - - -, LW2345.

In this case the weighted controller LW1234 does not manage to describe the process similarly to the local model (L5); this unsuccessfulness is due to the fact that steady state 5 is outside the region delimited by the four weighted models. Performance of LW1234 is in this case similar to the one of the controller which uses a wrong steady state (L1). LW1234 has a slow approach to the steady state in the favorable direction and a much worse degradation in performance in the unfavorable direction.

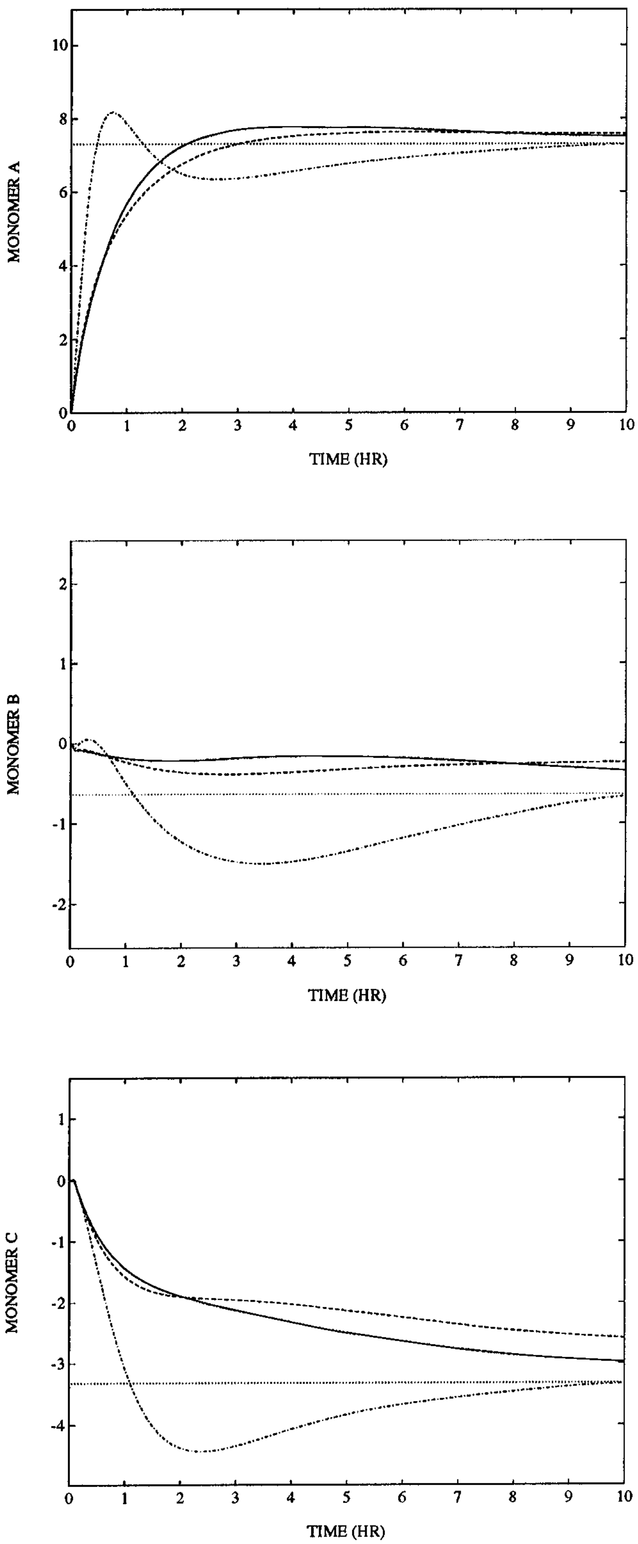

Figure 3. Step set-point variation in the unfavorable direction from steady state 1: —, L1; —•—, L5; - - -, LW2345.

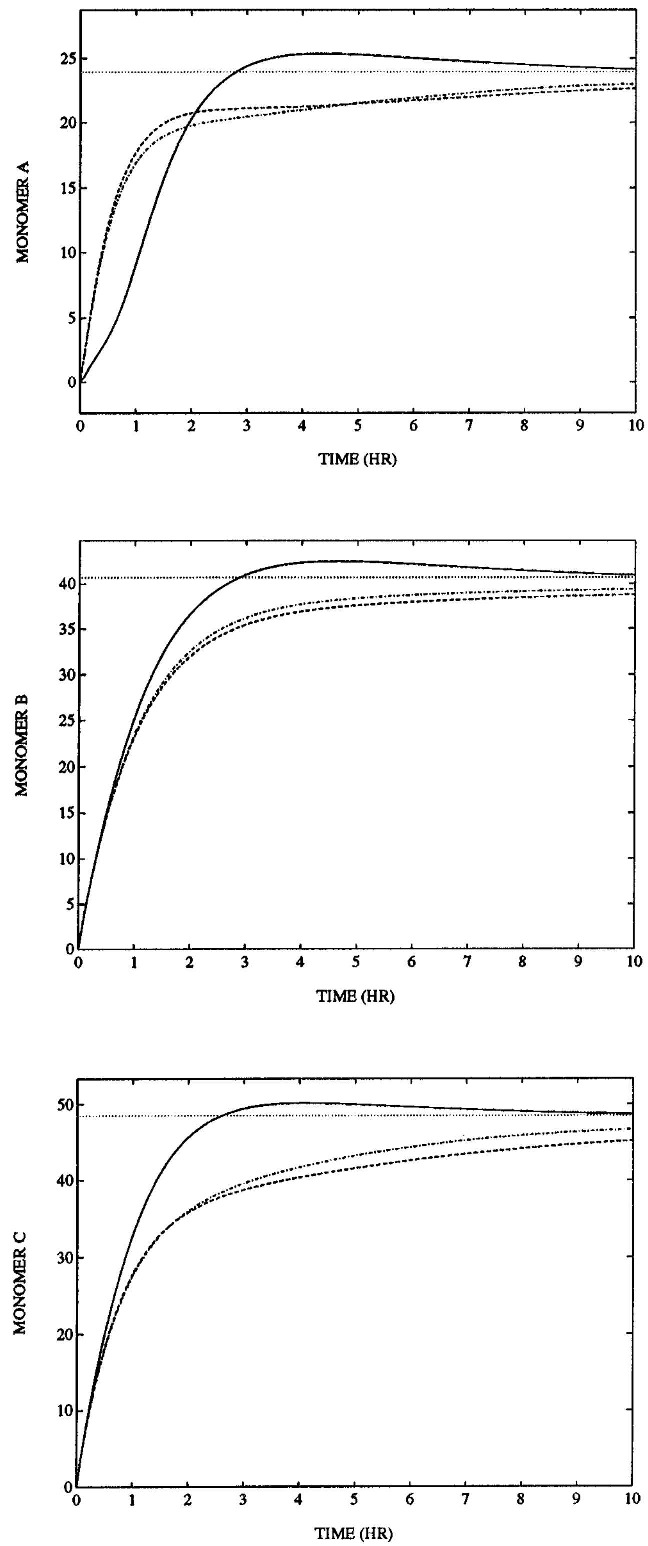

Figure 4. Step set-point variation in the favorable direction from steady state 5: —, L5; —•—, L1; - - -, LW1234.

4.4. Comparison with NLQDMC. Given the unsuccessful performance of all linear controllers when a set-point variation in the unfavorable direction takes place, the nonlinear controller is tested in this case. Steady state 1 is used as a starting point.

Three controllers are compared:

(1) a LWQDMC which weights four models linearized around steady states 2−5 (LW2345),

(2) a LQDMC which uses a model linearized around steady state 5 (L5), and

(3) the NLQDMC controller with parameter estimation.

It is clear that only the nonlinear controller is able to give good performance for all controlled variables (Figure 6). The behavior of the other two controllers had already been reported in Figure 3. It suffices to note that the linear weighted controller (LW2345) gives performance similar to the one of the correct local linear controller (not reported here) but despite of this is characterized by a slow approach to the steady state.

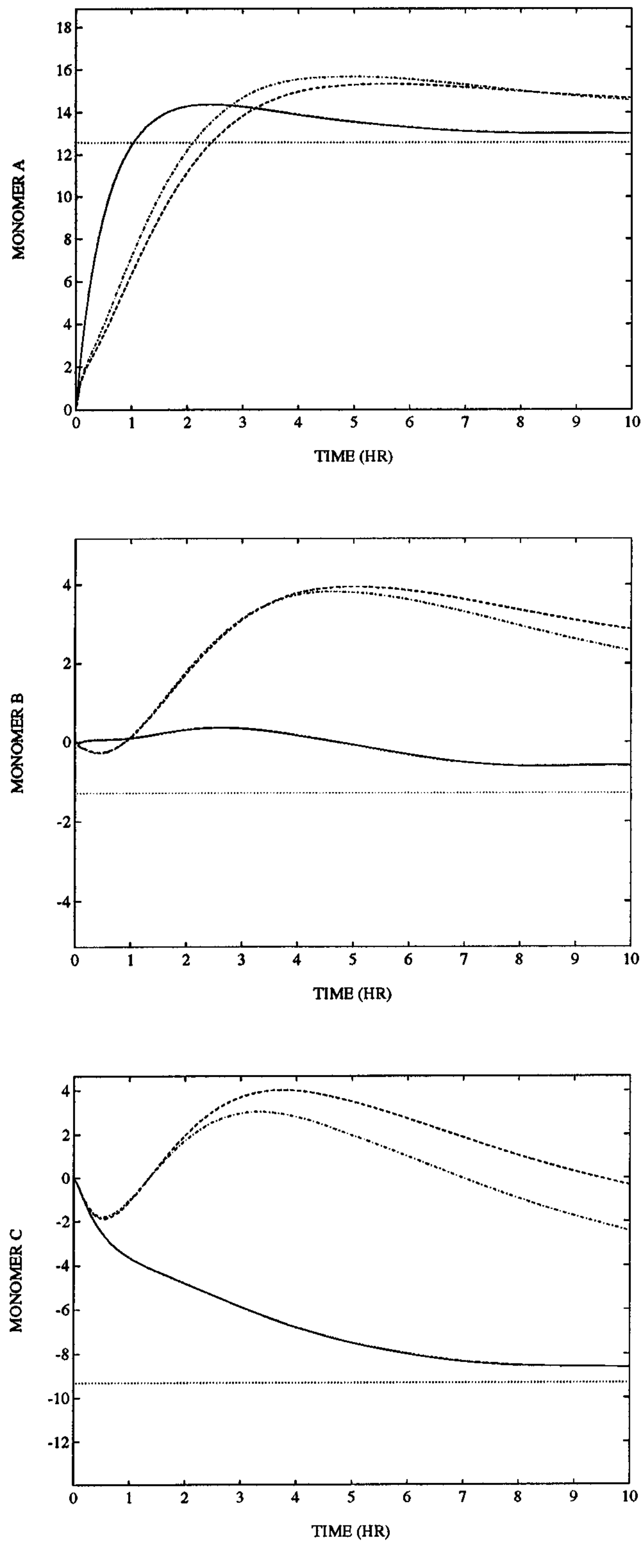

Figure 5. Step set-point variation in the unfavorable direction from steady state 5: —, L5; —•—, L1; - - -, LW1234.

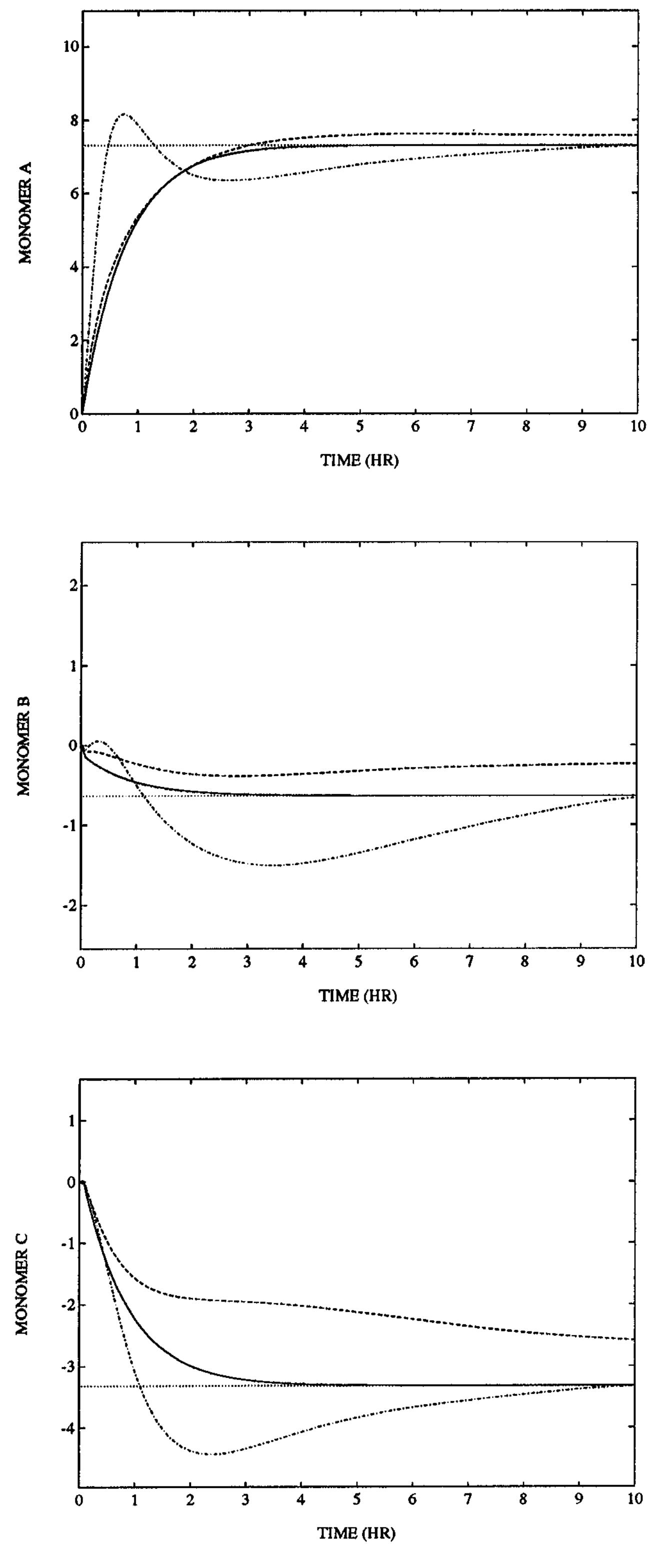

Figure 6. Step set-point variation in the unfavorable direction from steady state 1: —, NLQDMC; —•—, L5; - - -, LW2345.

4.5. Effect of Uncertainty. It is very interesting to note what happens if parametric uncertainty on the model kinetic constants is present. A very demanding case in which there are errors between the originally estimated values and the operating values ranging from 10% to 30% for all 15 kinetic constants has been considered. The nonlinear controller updates on-line 5 of the 15 kinetic constants, which have been chosen analyzing both the observability properties of the system and the quantitative effect of parameter variations on the system states. Details on which model parameters are chosen and of how large uncertainties are are reported in the Supporting Information. Together with parametric uncertainty, noise on the measurements (a random error with 2% of maximum amplitude) is present.

A step set-point variation in the unfavorable direction starting from set point 1 has been considered. The behavior of three controllers (L1, LW2345, NLQDMC) is reported in Figure 7. At short times all three controllers suffer from the difference between the model and the process. However, in a few hours the nonlinear

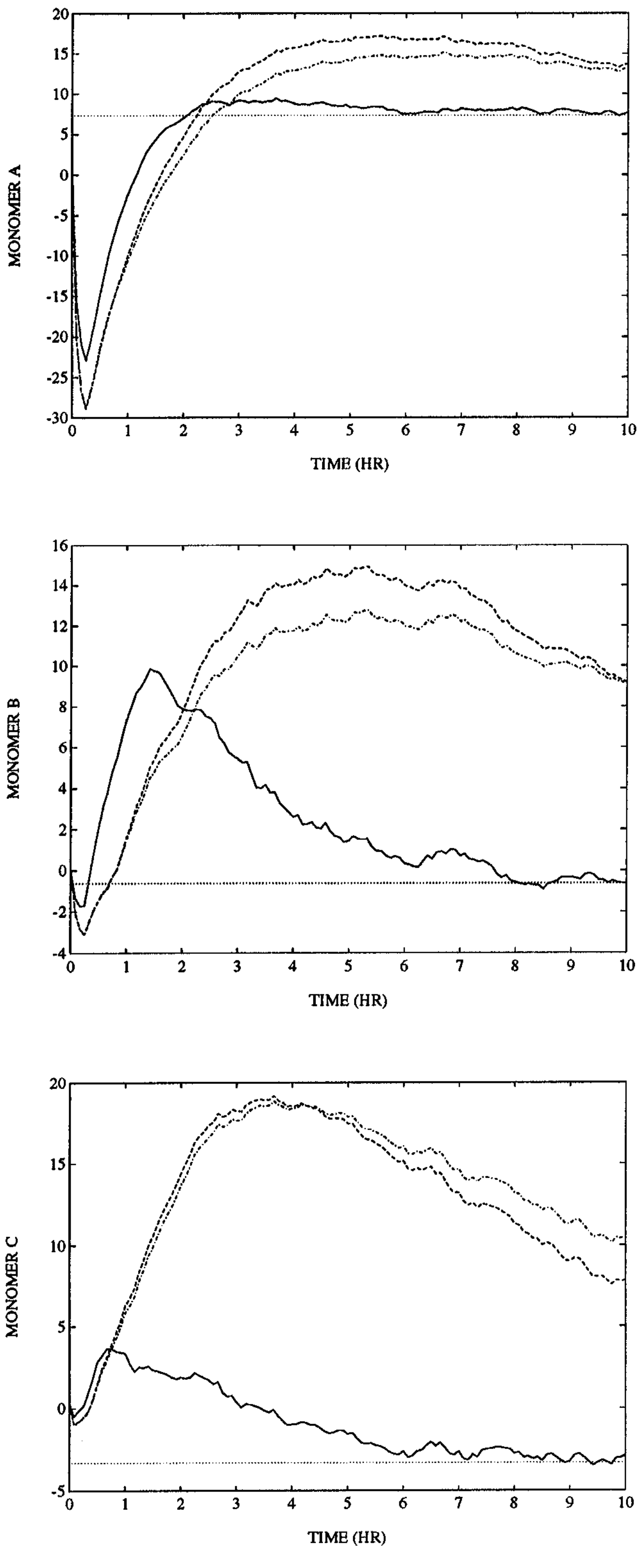

Figure 7. Step set-point variation in the unfavorable direction from steady state 1 in the presence of parametric uncertainty and noise on the measurements: —, NLQDMC; —•—, L5; - - -, LW2345.

controller manages to modify its parameters and correct the initial actions so that the controlled variables are led with desired trajectories to the steady state. This is possible because of the parameter estimation ability of the controller. On the other side, neither of the linear controllers is able to recover in a reasonable amount of time from the initial incorrect behavior so that after 10 h all controlled variables are still far away from their steady-state values.

5. Discussion and Conclusions

Three different predictive control strategies have been compared. The first one is the usual linear quadratic dynamic matrix Control (LQDMC) which uses for the prediction a fixed locally linearized model in the impulse or step response form. The second one makes use of a local model which is a weighted sum of locally linearized models (linear weighted QDMC−LWQDMC). The algorithm is not more involved than the first one from a computational point of view, but a more intensive identification effort is needed. The third strategy uses a first-principle nonlinear model which is locally linearized at every sampling time (nonlinear QDMC−NLQDMC). This algorithm requires a detailed description of the physics and of the kinetics of the process; moreover, a state and parameter estimation algorithm must be included in the controller so that the computational complexity increases.

It is relevant to sum up the results of the application of the different control strategies to a nonlinear ill-conditioned terpolymerization reactor in order to give indications about the appropriateness of use of the various predictive controllers.

It gives good results to use the LQDMC if the process has not strong nonlinearities or if strong variations from the original steady state do not take place during operation (due to wide changes in the operating variables or relevant uncertainties) and the process is not characterized by a strong directionality.

The LWQDMC addresses some of the problem connected with the nonlinearities. However, the availability of steady-state data which cover the full range of variations of the system variables during operation is required. The problems connected with the behavior in the unfavorable direction for a highly directional process and with the behavior in the case of large uncertainties are not solved by the use of the linear weighted controller. Moreover, it is expected that the precision with which the weighted model describes the nonlinear process strongly depends on the kind of nonlinearity. One possible reason for this unsuccess is the lack of information about the transient state of the process. Since no estimation of the states is included, the method is based on the hypothesis that the current outputs are an adequate indication of the states; this may add process-model mismatch, which is detrimental for ill-conditioned processes. In any case, for a nonlinear process with no severe problems connected with directionality or uncertainty, the use of the LWQDMC is encouraged.

For the most severe cases, the nonlinear controller is the best choice as expected. A nonlinear, directional, and uncertain process can be controlled reasonably well by the NLQDMC. However, it is of paramount importance that a sufficient number of parameters are updated on-line in order to take into account all sorts of uncertainties which are present during operation. The absence of the parameter estimation algorithm may result in steady-state offsets in the uncertain case.

Supporting Information Available: Details on the polymerization model with the complete information used in the paper (10 pages). Ordering information is given on any current masthead page.

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