IEEE/RSJ International Workshop on Intelligent
Robots and Systems IROS '91, Nov. 3-5, 1991,
Osaka, Japan. IEEE Cat. No. 91TH0375-6

Feedback Stabilization of a Nonholonomic Wheeled Mobile Robot *

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Abstract

When a nonlinear system is not stabilizable by smooth state feedbacks, it is common to think of discontinuous feedbacks as being the alternative. However, another interesting possibility consists of considering smooth time-varying *feedbacks in which the independent time index plays the role of an extra variable. This possibility is here illustrated in the case of two degrees-of-freedom nonholonomic wheeled mobile robot for which globally stabilizing smooth feedbacks are derived.*

1 Introduction

Feedback control of nonholonomic mobile robots is motivating an increasing number of communications and publications in Robotics and Automatic Control conferences and journals [1]-[12]. While the topic may be seen as a logical extension of the much studied case of holonomic robot manipulators, nonholonomy induces new specific control problems. For instance, like robot manipulators, mobile robots subject to classical nonholonomic constraints are usually (when the number of actuators is equal to the number of degrees of freedom) completely controllable in their configuration space [1] [3] [4] [13]-[18]. However, unlike robot manipulators, they cannot be stabilized to a desired configuration by using smooth state-feedback control [1] [3] [4] [10]. This difficulty is often occulted by the fact that, for many practical purposes, the dimension of the output controlled space does not have to be as large as the dimension of the configuration space: state-feedback stabilization of output vectors the dimension of which does not exceed the number of degrees of freedom may usually be achieved in a way much similar to what is done in the case of holonomic robot manipulators, and classical design control techniques may then be applied with success [6], [7], [9]. Another peculiarity, pointed out in [10] [11] for the specific case of a cart with two independent motorized wheels on the same axis, is that the system's stabilizability properties are modified when considering dynamic tracking objectives instead of regulation to a static desired configuration. For instance, feedback control laws that ensure global asymptotic convergence to zero of the configuration error between the cart and a *moving* reference model subject to the same nonholonomic constraints exist and can be derived. This type of result already points out the interest of considering feedback control laws which depend not only on the configuration state variables but also on the independent time variable, used in this case to parametrize the reference cart's motion. Nevertheless, the possibility of having the cart converge to a desired configuration by only using feedback control still remained an open question.

Non-existence of stabilizing smooth state-feedbacks is a well known problem in nonlinear control theory [19] [20], and it usually steers the conclusion that discontinuous feedback is the alternative. This idea has been exploited in [2] where a discontinuous feedback strategy is proposed to stabilize a knife edge moving in point contact on a plane surface. However, there is another alternative which consists of considering a possible dependency of the feedback control law on the exogenous time index. This possibility, briefly studied in [12] to stabilize the cart evoked above, is here revisited to derive other sets of globally stabilizing smooth *time-varying* feedbacks. We believe that this type of result, and the specifically tailored technique used to establish it, are of interest not only for wheeled mobile robot (WMR) control but also in the more general framework of feedback stabilization of nonlinear systems.

The paper is organized as follows. In Section 2, the main points of a stability analysis proposed in [12] for a set of nonlinear systems, the equations of which encompass stable linear invariant systems, are recalled. Application to the considered mobile robot is done in Section 3 where it is shown how the analysis can be utilized to design smooth *time-varying* feedback stabilizers. Velocity control, where the control inputs are assumed to be the motorized wheels' angular velocities, is first considered. Feedback stabilization of the cart to the desired configuration is then illustrated by simulation results. Finally, extension of the method to the case of torque control, where the control variables are the torques applied to the cart's motorized wheels, is briefly explained.

To avoid any misenterpretation of the results proposed in this paper, it may be useful to forewarn the reader that we are here more concerned with conceptual new possibilities than with realistic implementation aspects. These will be the subject of other studies.

*This work was supported by the Ministère de la Recherche et de la Technologie and ALCATEL-ALSTHOM under contract 1 91 E008

2 A stability analysis

Let $\mathcal{S}$ denote the set of matrix functions $f(.,t)$ defined on $R^k \times R^+$ ($k \in N$), of class C^∞, uniformly bounded with respect to the independent time variable t, and with successive partial derivatives also uniformly bounded with respect to t.

By choosing functions in this set, technicalities which are not essential to the analysis are avoided.

We consider a class of systems defined by an equation in the following form:

$$\dot{X} = -P^{-1}(Q(X,t) + \tilde{C}(X,t))X \tag{1}$$

where:

- $X \in R^n$
- $\dot{X}(t) = \frac{d}{dt}X(t)$
- P is a positive-definite symmetric (*p.d.s.*) matrix (meaning that $P = P^T$ and $X^TPX > 0$ for all $X \in R^n$ and $X \neq 0$)
- $Q(X,t) \in \mathcal{S}$ is a positive symmetric (*p.s.*) matrix function (meaning that $Q(X,t) = Q(X,t)^T$ and $X^TQ(X,t)X \geq 0$ for all $(X,t) \in R^n \times R^+$)
- $\tilde{C}(X,t) \in \mathcal{S}$ is a skew-symmetric (*s.s.*) matrix function (meaning that $\tilde{C}(X,t) + \tilde{C}(X,t)^T = 0$).

As explained in [12] the set of systems defined by 1 encompasses all *stable* linear systems in the form:

$$\dot{X} = AX \tag{2}$$

where A is a *stability* matrix (i.e. a matrix the eigenvalues $\lambda_i(A)$ of which have a strictly negative real part).

Moreover, when the functions Q and $\tilde{C}$ are constant, we have the following result:

Lemma 2.1 ([12])
If:

- P *is a* $(n \times n)$ p.d.s. *matrix*
- Q *is a* $(n \times n)$ *(semi)* p.s. *matrix*
- $\tilde{C}$ *is a* $(n \times n)$ s.s. *matrix*

then the matrix:
$A = -P^{-1}(Q + \tilde{C})$
is a stability matrix if and only if:

$$rank \begin{bmatrix} Q \\ QP^{-1}\tilde{C} \\ \vdots \\ Q(P^{-1}\tilde{C})^{n-1} \end{bmatrix} = n$$

Lemma 2.1 provides us with a matrix rank test which is usually simpler to perform than the calculation of the eigenvalues of the matrix $A = -P^{-1}(Q + \tilde{C})$. It is thus of interest when the system under study is directly given in the form $\dot{X} = -P^{-1}(Q + \tilde{C})X$ and the matrices P and $(Q + \tilde{C})$ are known to be positive.

An extension of this lemma, that may be used to study the stability properties of nonlinear systems in the form 1, is given in the next lemma which may also be seen as an adaptation of Lasalle's theorem [21]. The following notations are used: $||X||_P = (X^TPX)^{1/2}$ and $||X|| = (X^TX)^{1/2}$.

Lemma 2.2 ([12]) *Consider the subset:*
$\mathcal{D} = \{Y(t) \in \mathcal{S} \mid \forall t \in R^+ \ : \ ||Y(t)||_P = constant\}$
of vector functions defined from R^+ *to* R^n*, and a sequence of matrices* $\{Q_i\}_{i\in N}$ *such that:*

- *for* $i = 0$*:*
$$\left.\begin{array}{l} Y \in \mathcal{D} \\ \lim_{t\to\infty} Q(Y(t),t)Y(t) = 0 \end{array}\right\} \Rightarrow \lim_{t\to\infty} Q_0Y(t) = 0$$
- *for* $i \geq 1$*:*
$$\left.\begin{array}{l} Y \in \mathcal{D} \\ \lim_{t\to\infty} Q_{i-1}Y(t) = 0 \\ \lim_{t\to\infty} Q_{i-1}P^{-1}\tilde{C}(Y(t),t)Y(t) = 0 \end{array}\right\} \Rightarrow Q_iY(t) \to 0$$

then the solutions $X(t)$ *to the system 1 are such that:*
$\lim_{t\to\infty} Q_iX(t) = 0 \quad ; \forall i \in N$

Note that, although the conditions of Lemma 2.2 are always satisfied by the null matrix, the practical purpose of the determination of the sequence $\{Q_i\}_{i\in N}$ is clearly to reach a matrix Q_i of maximum rank.

Corollary 1 (of Lemma 2.2 [12]) *If one of the matrices* Q_i *is nonsingular then:*

- $\lim_{t\to\infty} X(t) = 0$
- *the rate of convergence to zero is exponential in the neighbourhood of zero if, in addition, one of the following conditions is satisfied:*
  - *i)* $Q(0,t) \geq \alpha I_n$ *for some strictly positive number* α *(meaning that* $X^TQ(0,t)X \geq \alpha||X||^2$, $\forall X \in R^n$*)*
  - *ii)* $Q(0,t)$ *and* $\tilde{C}(0,t)$ *are constant matrices and the matrix* $-P^{-1}(Q(0,t) + \tilde{C}(0,t))$ *is a stability matrix*

3 Feedback stabilization of a cart

3.1 Velocity control

The cart that is considered is schematized in **Fig.1** and **Fig.2**. It is equipped with two motorized wheels that share the same rotation axis. It is assumed that it moves on a horizontal ground. The model equations of the cart's motion are derived under the usual rolling-without-slippage assumption and are thus simply obtained by expressing the fact that the point of each motorized wheel in contact with the ground has zero velocity (see [5], [10] for example).

The following notations are used:

- M: the cart's point located at mid-distance of the motorized wheels
- $\dot{q}_1$, $\dot{q}_2$: the motorized wheels' angular velocities
- $\mathcal{F}_0 = (O; \vec{\imath}_0, \vec{\jmath}_0)$: a fixed reference frame such that the point M belongs to the plane $(O; \vec{\imath}_0, \vec{\jmath}_0)$
- $\mathcal{F}_1 = (M; \vec{\imath}_1, \vec{\jmath}_1)$: a frame rigidly linked to the cart

- x, y: the coordinates of the vector $\vec{OM}$ in the basis of the frame $\mathcal{F}_1$

and also:

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad (3)$$

$$D = \begin{bmatrix} r/2 & r/2 \\ r/2R & -r/2R \end{bmatrix} \quad (4)$$

with r: the radius of the motorized wheels, and $2R$: the distance between the two wheels.

$$U = \begin{bmatrix} v \\ \dot{\theta} \end{bmatrix} \quad (5)$$

with v: the cart's advancement velocity ($\frac{d}{dt}\vec{OM} = v\vec{i_1}$). We then obtain the following relation:

$$U = D\dot{q} \quad (6)$$

Since the matrix D is nonsingular and known, it is equivalent to choose $\dot{q}$ or U as the control vector.

With these notations, the system's equations are:

$$\begin{aligned} \dot{x} &= u_2 y + u_1 \\ \dot{y} &= -u_2 x \\ \dot{\theta} &= u_2 \end{aligned} \quad (7)$$

From these equations, it is quite simple to verify that the system is completely controllable and also, by application of a Brockett's theorem [19], that no differentiable (or even continuous [20]) state feedback $U(x, y, \theta)$ is able to stabilize it to zero.

Let us then introduce the following *time-varying* state vector:

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (8)$$

with:

$$z = \theta + k(y, t) \quad (9)$$

where $k(y, t)$ is any scalar function in $\mathcal{S}$ such that:

- $\forall t \in R^+,\ k(0, t) = 0$
- $\forall y \neq 0,\ \frac{\partial k}{\partial t}(y, t)$ does not converge to zero when t tends to infinity

From 7, we have:

$$\begin{aligned} \dot{x} &= u_2 y + u_1 \\ \dot{y} &= -u_2 x \\ \dot{z} &= u_2 - \frac{\partial k}{\partial y} u_2 x + \frac{\partial k}{\partial t} \end{aligned} \quad (10)$$

or, in a matrix form chosen so as to make a skew-symmetric matrix function $\tilde{C}_1$ appear:

$$\dot{X} = -\tilde{C}_1(u_2, y, t)X + BV \quad (11)$$

with:

$$V = \begin{bmatrix} u_1 - \frac{\partial k}{\partial y} u_2 z \\ u_2 + \frac{\partial k}{\partial t} \end{bmatrix} \quad (12)$$

$$\tilde{C}_1 = \begin{bmatrix} 0 & -u_2 & -\frac{\partial k}{\partial y} u_2 \\ u_2 & 0 & 0 \\ \frac{\partial k}{\partial y} u_2 & 0 & 0 \end{bmatrix} \quad (13)$$

and:

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (14)$$

The analysis of Section 2 then suggests choosing the auxiliary control vector V as follows:

$$V = -(\tilde{C}_2(X, t) + R(X, t))B^T X \quad (15)$$

with:

- $\tilde{C}_2(X, t)$: a (2×2) *s.s.* matrix function in $\mathcal{S}$
- $R(X, t)$: a (2×2) *p.d.s* matrix function in $\mathcal{S}$

so as to obtain a system in the form 1 with $P = I_3$, $\tilde{C} = \tilde{C}_1 + B\tilde{C}_2 B^T$ and $Q = BRB^T$.

Expressions of the control variables u_1 and u_2 are then obtained from 12 and 15 by elimination of V.

To simplify, and since our objective here is only to show the existence of stabilizing smooth feedbacks, we will choose $\tilde{C}_2$ equal to zero and R equal to a constant positive diagonal matrix: $R = diag\{r_1, r_2\}$. With this choice, the control expressions are:

$$\begin{aligned} u_2 &= -r_2 z - \frac{\partial k}{\partial t} \\ u_1 &= -r_1 x + \frac{\partial k}{\partial y} u_2 z \end{aligned} \quad (16)$$

We first notice that u_1 and u_2 are indeed smooth functions of x, y, θ and t. The controlled system is also in the form 1 with $\tilde{C} = \tilde{C}_1$ and:

$$Q = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_2 \end{bmatrix} \quad (17)$$

Thus, a matrix Q_0 which satisfies the condition of Lemma 2.2 is:

$$Q_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (18)$$

and x and z converge to zero.

We also have:

$$Q_0\tilde{C}(X, t) = \begin{bmatrix} 0 & -u_2 & -\frac{\partial k}{\partial y} u_2 \\ 0 & 0 & 0 \\ \frac{\partial k}{\partial y} u_2 & 0 & 0 \end{bmatrix} \quad (19)$$

Therefore, if $Y(t)$ is a (3×1) vector function in $\mathcal{S}$, one easily verifies that $||Y(t)|| = l$, $\lim_{t\to\infty} Q_0 Y(t) = 0$ and $\lim_{t\to\infty} Q_0\tilde{C}(Y(t), t)Y(t) = 0$ together imply:

$$\lim_{t\to\infty} Y(t) = \begin{bmatrix} 0 \\ \pm l \\ 0 \end{bmatrix} \quad (20)$$
$$\lim_{t\to\infty} \frac{\partial k}{\partial t}(\pm l, t) l = 0$$

and, in view of the assumption made on the choice of $k(y,t)$, l must be equal to zero. A matrix Q_1 which satisfies the conditions of Lemma 2 thus is the identity matrix. According to the lemma, this in turn implies that all solutions $X(t)$ converge to zero. Returning to the definition 8 and 9 of X, and the fact that $k(0,t) = 0$, we finally obtain that the cart's configuration variables x, y and θ converge to zero.

We thus have shown that the smooth *time-varying* feedback control 16 is globally asymptotically stabilizing.

Notice that the choice $k = 0$, for which the control is stationary, would only ensure convergence of x and θ to zero while y would tend to some limit value not necessarily equal to zero.

3.2 Simulation results

The stabilizing feedback control 16, with $k(y,t) = y\ sint$, $r_1 = .5$ and $r_2 = 1$ has been simulated. Results are presented in **Fig.3-Fig.6**.

- **Fig.3** shows the motion of the cart starting from the point ($x = 0,\ y = 1,\ \theta = 0$). This particular configuration was chosen because it is an equilibrium point when $k = 0$.
- **Fig.4** shows the convergence of the Lyapunov function $W(X) = x^2 + y^2 + z^2$ to zero. As expected from the controlled system's equation, this function decreases monotically.
- **Fig.5** shows the convergence of the squared configuration distance $W' = x^2 + y^2 + \theta^2$ to zero. The decreasing of this function is not monotonic. Monotonicity would be obtained by choosing $k = 0$ at the expense of non-convergence to zero.
- **Fig.6** shows that $t\ W(X(t))$ asymptotically converges. This tells us that the configuration error tends to zero like $t^{-1/2}$. Possibilities of improving this convergence rate remain to be explored.

3.3 Torque control

Let us now assume that the control variables are no longer the motorized wheels velocities $\dot{q}_1$ and $\dot{q}_2$, but the torques γ_1 and γ_2 applied to these wheels.

Neglecting wheel-ground contact forces other than those involved in rolling-without-slippage, and assuming that the ground is flat (so that gravity terms have no influence on the cart's motion), the dynamics equation which relates the applied torques to the cart's accelerations is of the form [12]:

$$\Gamma = M\ddot{q} \quad (21)$$

with:

- $\Gamma = (\gamma_1, \gamma_2)^T$
- M: constant *p.d.s inertia* matrix

The dynamics of the system are thus described by the three equations 6, 7 and 21.

Introducing the following state vector:

$$\mathcal{X} = \begin{bmatrix} X \\ D^{-1}(U+W) \end{bmatrix} \quad (22)$$

where D, X and U have been previously defined and:

$$W(X,t) = \begin{bmatrix} \frac{\partial k}{\partial y}\frac{\partial k}{\partial t} z \\ \frac{\partial k}{\partial t} \end{bmatrix} \quad (23)$$

it is simple to verify that the system's equation may be written as follows:

$$\dot{\mathcal{X}} = \begin{bmatrix} -\tilde{C}_1(u_2,y,t) & BA(y,z,t)D \\ 0 & 0 \end{bmatrix} \mathcal{X} + \begin{bmatrix} 0 \\ M^{-1}\Gamma + D^{-1}h(X,\dot{q},t) \end{bmatrix} \quad (24)$$

with:

$$A(y,z,t) = \begin{bmatrix} 1 & -\frac{\partial k}{\partial y} z \\ 0 & 1 \end{bmatrix} \quad (25)$$

and:

$$h(X,\dot{q},t) = \frac{d}{dt}W(X,t) \quad (26)$$

Lemma 2.2 of Section 2 then suggests choosing, for example, a control such as:

$$\Gamma = -MD^{-1}h - RD^{-1}(U+W) - D^T A^T B^T X \quad (27)$$

with R being a (2×2) *p.d.s.* matrix, so as to obtain a controlled system in the form 1 with:

$$\begin{aligned} P &= \begin{bmatrix} I_2 & 0 \\ 0 & M \end{bmatrix} \\ \tilde{C} &= \begin{bmatrix} \tilde{C}_1 & -BAD \\ D^T A^T B^T & 0 \end{bmatrix} \\ Q &= \begin{bmatrix} 0 & 0 \\ 0 & R \end{bmatrix} \end{aligned} \quad (28)$$

We leave to the interested reader the task of verifying that, with the same choice of the function $k(y,t)$ as before, Lemma 2.2 applies to this system with:

$$\begin{aligned} Q_0 &= \begin{bmatrix} 0 & 0 \\ 0 & I_2 \end{bmatrix} \\ Q_1 &= \begin{bmatrix} \begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} & 0 \\ 0 & I_2 \end{bmatrix} \\ Q_2 &= I_5 \end{aligned} \quad (29)$$

and thus that the smooth time-varying feedback torque control 27 yields the convergence of x, y and θ to zero, whatever the initial conditions.

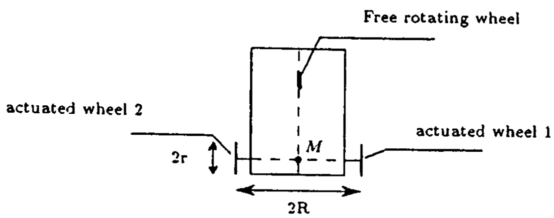

Fig.1 : Above view of the cart

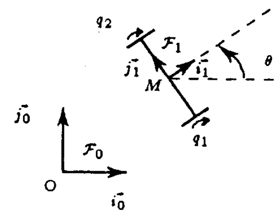

Fig.2 : Cart's parameters

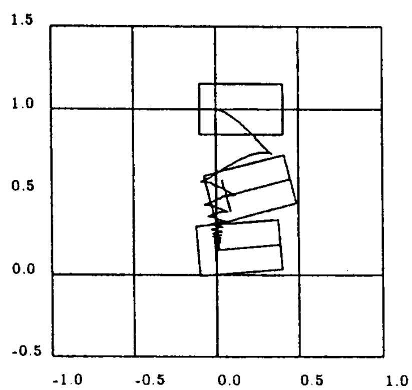

Fig.3 : Cart's motion

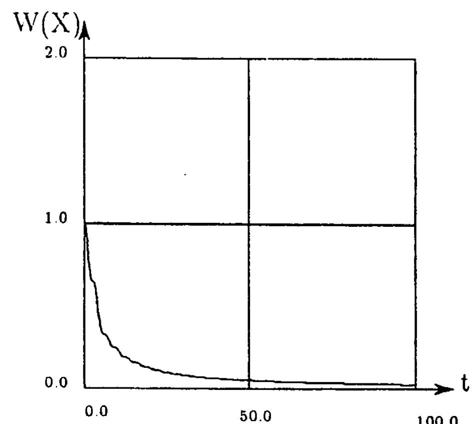

Fig.4 : convergence of $x^2 + y^2 + z^2$ to zero

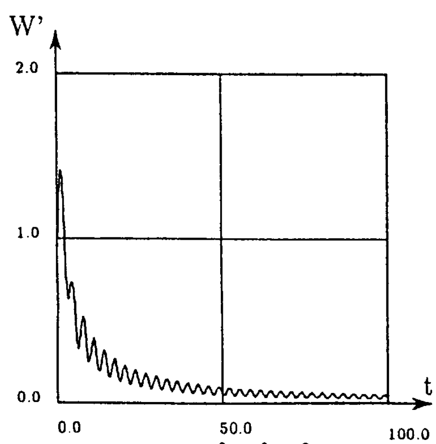

Fig.5 : convergence of $x^2 + y^2 + \theta^2$ to zero

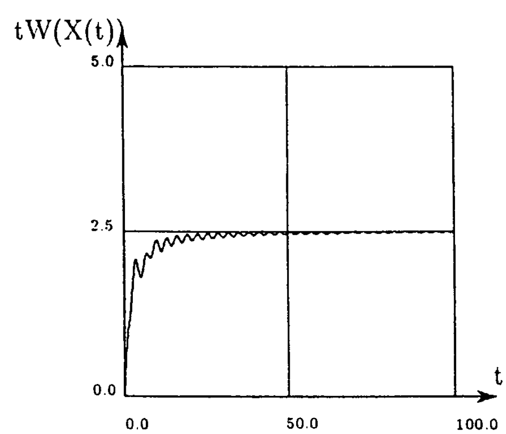

Fig.6 : Linear convergence rate of W(X(t))

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