ROBUST TRAJECTORY TRACKING ALGORITHMS FOR A WHEELED MOBILE ROBOT

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ABSTRACT

Control algorithms for a Wheeled Mobile Robot to achieve trajectory tracking are presented in this paper. A complete electro-mechanical model is obtained by incorporating the dynamic models of the actuators in the dynamic equations of a tricycle vehicle. The control scheme has been divided into two levels, namely path error control and motor control. A method based on the acceleration feedback is applied at a lower level of control. Simulation results are given to illustrate the controller design procedure and the performance of the controller.

I. INTRODUCTION

The development of control algorithms for Wheeled Mobile Robots (WMRs) to achieve trajectory tracking has recently become the subject of considerable interest [1,2,3,4,5].

This paper proposes dynamically based tracking control algorithms for a tricycle WMR with a front steering wheel. We decompose the control problem into two subproblems, namely path control and motor control. The determination of the structure and parameters of the algorithms is based on the use of the concept of inverse problem of controlled system dynamics [6]. This makes possible the synthesis of such algorithms which can realize a motion in accordance with the prescribed trajectories. A specific feature of these algorithms is the use of control-by-acceleration principle. The control algorithm with acceleration feedback makes it possible to get a control system with a high value gain. Acceleration feedback action reduces the effects of coupling dynamics, parameter uncertainties and external resistive forces and moments.

The paper is organized into five sections. Section II presents a complete electrodynamic model of a tricycle vehicle. The motion control algorithms are given in Section III. Some simulation results are presented in Section IV. Conclusions are given in Section V.

II. WMR DYNAMIC MODEL

A plan view of the model tricycle vehicle with a front steering wheel is shown in Fig.1.

The vehicle moves over horizontal reference plane. The location of the vehicle is measured with respect of the mid-point R between the rear wheels.

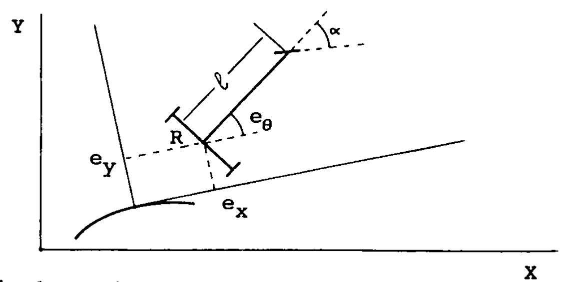

Fig.1. Tricycle Vehicle Configuration and Workspace Coordinates.

In this section a rigid multibody system approach is used to develop the vehicle's equations of motion. The position of the vehicle with respect to an inertial frame is described by a 7x1 vector of generalized coordinates $q = [q_1, q_2, \ldots q_7]^T$ as follows

$$q_1 = x,\quad q_2 = y,\quad q_3 = \theta,\quad q_4 = \alpha,\qquad q_5 = \beta_1,\quad q_6 = \beta_2,\quad q_7 = \beta_3 \tag{1}$$

where

β_1, β_2, β_3 are angles of rotation of the wheels about their axes,

α is front wheel steering angle,

x, y, θ determine the location of the robot body in the reference inertial frame.

The generalized velocities are given by a 7x1 vector $\dot{q} = [\dot{q}_1, \dot{q}_2, \ldots \dot{q}_7]^T$. If no slip occurs at the contact point between the wheels and the plane, the generalized velocities are related to each other by a set of five first order differential equations of nonholonomic constraints

$$\sum_{k=1}^{7} a_{ik}\dot{q}_k = 0,\quad i = 1, \ldots 5, \tag{2}$$

where $a_{ik} = f_{ik}(q_1, q_2, \ldots q_7)$.

We introduce seven nonholonomic velocity parameters (quasivelocities) where the first five quasivelocities correspond to the equations of the nonholonomic constraint conditions (2), and the last two quasivelocities are the vehicle's velocity v

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(the velocity of point R) and the front wheel steering angular velocity α.

After applying the Boltzmann-Hamel formulation [7] in quasicoordinates for nonholonomic systems, and incorporating the dynamic models of the actuators in the vehicle's equations of motion, the complete dynamic WMR model is derived in the form

$$M(q)\ddot{p} + V\dot{p} + C(q,\dot{q}) + F(\dot{q}) = U, \quad (3)$$

where

$M(q)$ is 2x2 matrix comprising the inertia terms,

V is 2x2 diagonal matrix comprising the viscous terms,

$C(q,\dot{q})$, $F(\dot{q})$ are 2x1 vectors comprising respectively the Coriolis and frictional forces,

U is 2x1 vector of the actuator inputs (armature voltages),

p is 2x1 vector of the quasicoordinates s an α (where $\dot{s}=v$).

III. CONTROLLER DESIGN PROCEDURES

The tracking control problem of a WMR in the presence of a parametric uncertainties is considered in this section. The deviation from the desired trajectory, while the vehicle is moving along the route is a matter of interest. The vehicle motion can be described with respect to an inertial frame, or related to a moving reference frame. For our tricycle type WMR with a front steering and driving wheel, the guidepoint (the point of the vehicle, that is determined to follow the reference trajectory) is chosen at the mid-point between the rear wheels.

Let $(x_r,y_r,\theta_r)^T$ and $(x,y,\theta)^T$ are respectively the desired and current position-heading WMR vectors. The position-heading error vector $(e_x,e_y,e_\theta)^T$ is defined as the difference between the actual and the desired position-heading vectors

$$\begin{pmatrix} e_x \\ e_y \\ e_\theta \end{pmatrix} = \begin{pmatrix} \cos\theta_r & \sin\theta_r & 0 \\ -\sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x - x_r \\ y - y_r \\ \theta - \theta_r \end{pmatrix}. \quad (4)$$

Since in practice the position error vector is very small, it can be shown that the velocity error vector $(\dot{e}_x,\dot{e}_y,\dot{e}_\theta)^T$ is described in component form as

$$\begin{aligned} \dot{e}_x &= v - v_r \\ \dot{e}_y &= v e_\theta \qquad (5) \\ \dot{e}_\theta &= \dot{\theta} - \dot{\theta}_r. \end{aligned}$$

The curvature function of the desired trajectory is $k_r(s)$ where s is the path length. As the lateral position error $e_y \ll 1/k_r(s)$, the angular velocity error e_θ may be written as

$$\dot{e}_\theta = v[\frac{tg\alpha}{l} - k_r(s)], \quad (6)$$

where

α is the front wheel steering angle.

l is the wheelbase length.

Hierarchical layers of control are a common approach to the WMRs motion control problem. The trajectory tracking controller is usually presented as a two layer structure, namely a path error control and a motor control. With regard to this, the control algorithms design procedure described in this section is based on the principle of decomposition of the control problem. During the synthesis of each outer loop, the structure and parameters of the inner loops are supposed to have already been designed.

III.1.Designing the Motor Control Algorithms

In order to simplify the design of the control system it is suggested for the control scheme to be divided into two stages. The angular velocity control algorithm has to be designed at the first stage (inner control loop) and at the second stage the steering control algorithm has to be design (outer control loop). A method based on acceleration feedback is applied at the lowest level of control. One usually strives at the controller with acceleration feedback rejecting the parameter variations, coupling effects and external resistive forces/torques. Block diagram of the corresponding angular velocity control loop with acceleration feedback is shown in Fig.2.

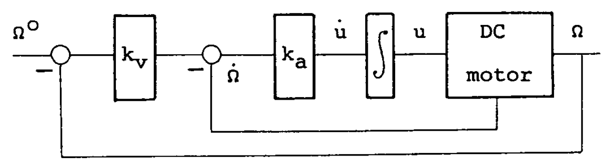

Fig.2. Angular Velocity Control Structure with Acceleration Feedback.

It is of great practical importance that the proposed control law u(t) can be used without any acceleration measurements necessary. This makes it possible to simplify the sensor system of a WMR. In that case the control law can be written in the form

$$u(t) = k_v k_a \int(\Omega^o - \Omega)dt + k_a[\Omega(0) - \Omega], (7)$$

where

Ω is the angular velocity of the actuator,

$\Omega(t=0) = \Omega(0)$ and $\Omega(\infty) = \Omega^o$,

k_a and k_v are positive gains.

The gain k_a in Eq.(7) is determined by the requirement that the acceleration loop has much quicker response as compared to the angular velocity loop. In that case the acceleration loop does not affect seriously the closed loop dynamics. Such an influence can be witnessed only at the beginning of the transient response. Thus the process for $\Omega(t)$ in the closed loop system can be approximated with a great degree of precision by the process in the reference system

$$\dot{\Omega}^*(t) + k_v\Omega^*(t) = k_v\Omega^o, \qquad k_v>0, \qquad (8)$$

where

$\Omega^*(t)$ represents the process in the reference angular velocity system.

In addition to everything said so far, all the on-line computational problems, associated with computing a WMR inverse dynamics can be avoided and the WMR control problem can be essentially reduced to that of computing independent PI controllers for the different motor control units.

During the synthesis of the steering angle control algorithm (outer control loop) it is supposed that the structure and parameters of the angular velocity control algorithm are known and they fulfil the role of a controlled plant, Eq.(8). Thus the control law is reduced to the determination of $\Omega^o=\Omega^o(t)$ for the angular velocity control algorithm. For simplicity the desired steering angle position control loop can be considered as a first order differential equation(we note that a more detailed study would require a second order differential equation)

$$\dot{\alpha}^*(t) + k_p\alpha^*(t) = k_p\alpha^o, \qquad k_p>0, \qquad (9)$$

where

$\alpha(t) = 1/n\Omega$ and n is the gear ratio, $\alpha^o = \alpha(\infty)$ is the desired steering angle.

The control signal for $\Omega^o(t)$ now looks as follows

$$\Omega^o(t) = nk_p(\alpha^o - \alpha). \qquad (10)$$

The differential equation for the closed loop system with the control law (10) is

$$\ddot{\alpha}(t) + k_v\dot{\alpha}(t) + k_vk_p\alpha(t) = k_vk_p\alpha^o. \qquad (11)$$

Parameters k_v and k_p are determined by the requirement that the inner control loop for Ω must be much quicker that the outer loop for α. It is only in such cases that the real motion trajectory (11) practically coincides with the reference trajectory (9).

III.2. Designing the Path Error Control Algorithm

The purpose of the path error controller is to reduce the position and heading errors of the WMR to zero. Further on it is assumed for simplicity that velocity control loop takes care of keeping the vehicle velocity constant. We will also discuss the problem of the WMR movement stabilization along a straight-line trajectory. According to the procedure of designing, it is supposed that the closed loop dynamics of the motor control subsystem is known in the form of Eq.(9).

The path error control problem can be formulated as follows. The initial position of the WMR in respect to the desired trajectory in t=0 is $e_y(0)=e_{yo}$ and $e_\theta(0)=e_{\theta o}$.

It is necessary to design the control function $\alpha^o=\alpha^o(t)$, with the assistance of which the WMR will move back to the desired path given, in accordance with the solution of the reference differential equation

$$\dddot{e}_y(t) + k_p\ddot{e}_y(t) + k_1\dot{e}_y(t) + k_oe_y(t) = 0, \qquad (12)$$

where the gain $k_p>0$ is already chosen (Eq.(9)); $k_1,k_o>0$ are determined with the aid of a simulation process; $k_pk_1>k_o$.

Taking Eqs. (5) and (6) into consideration the control law assumes the following expression

$$\alpha^o = \frac{1}{k_pv^2}(-k_1\dot{e}_y - k_oe_y). \qquad (13)$$

IV. SIMULATION RESULTS

For numerical purposes parameters of the vehicle are considered as:

Wheelbase l=0,16 m;

Wheelradii r=0,06 m;

Mass of the platform - 22 kg;

Mass of the front steering and driving mechanism including the front wheel - 2,5 kg.

In the first example it is assumed that the controller takes care of keeping the motor angular velocity constant. Figure 3 shows the difference between the step responses of the angular velocity control loop and the reference angular velocity control loop.

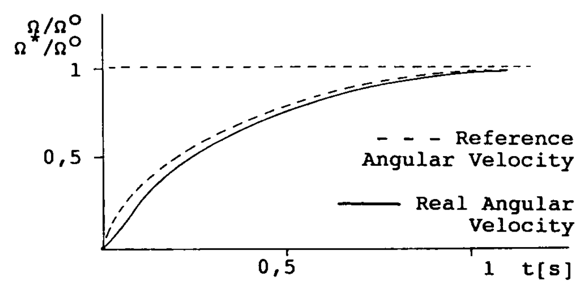

Fig.3: Simulation Results of the Responses for the Real Angular Velocity Control Loop and the Reference Control Loop; k_a=300.

In the example presented above the big value of k_a is determined by the small value of the front steering mechanism inertia, while the reference system has considerably bigger inertia than the real system.

In Fig.4. and Fig.5. are shown trajectories followed by the vehicle with different initial positions. It is assumed that the WMR desired motion is along the X axis to the positive direction with a constant velocity. Let us propose a differential equation (12) in the form

$$\dddot{e}_y(t) + 3k\ddot{e}_y(t) + 3k^2\dot{e}_y(t) + k^3 e_y(t) = 0.$$

The control steering angle α^o is determined from Eq.(13) as

$$\alpha^o = \frac{1}{3kv^2}(-3k^2\dot{e}_y - k^3 e_y).$$

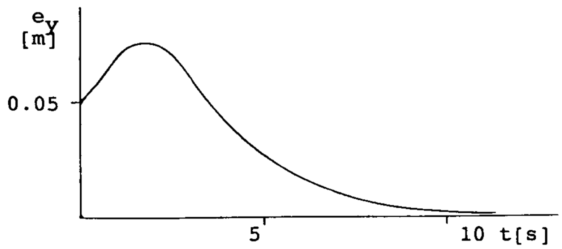

Fig.4. Example of a Simulation Trajectory. Initial Position e_{yo}=0,05m, $\dot{e}_{yo}$=0,043m/s; v=0,167m/s.

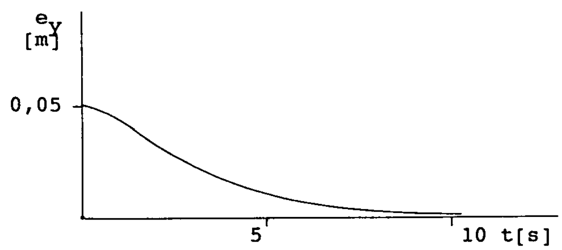

Fig.5. Example of a Simulation Trajectory. Initial Position e_{yo}=0,05m, $\dot{e}_{yo}$=0; v=0,167m/s.

V. CONCLUSIONS

In this paper are proposed control algorithms for a WMR to achieve trajectory tracking through using moving reference frame. The controller design procedure has been divided into two levels. This approach makes it easier to determine the structure and parameters of the control scheme and, at the same time, better take into account, the features of the drive systems. Acceleration feedback action can improve the dynamic response characteristics of the control system. Further more, the controller design requires a minimal knowledge of the mobile robot dynamics. Since actual measurement or calculation of the acceleration signal is not required, practical implementation problems associated with acquisition of the acceleration are avoided. Although the proposed control technique is applied to the WMR with a front steering and driving wheel the same method can be generalized for WMRs with a different structure from the above discussed.

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