An Introduction to Model Predictive Control

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1 Introduction

It is well known that the economic operating point of a typical process unit often lies at the intersection of constraints. A successful industrial controller must therefore maintain the system as close as possible to constraints without violating them. In addition, process units are typically complex, nonlinear, constrained multivariable systems whose dynamic behavior changes with time due to such effects as changes in operating conditions and catalyst aging [26]. This environment led to the development, in industry, of a general model based control methodology for constrained multivariable process systems, in which the control action is determined through the online solution of a dynamic optimization problem at each control execution.

2 The Principle

In the following the principle underlying all Model Predictive Control concepts – linear or nonlinear – is illustrated based on the recent paper by Lee and Cooley [15]. The basic concept of MPC is displayed in Fig. 1. At the k_{th} sample time, information is received about the current state of the system. Based on the information and the system model, the dependence of the future states on the future manipulated inputs can be predicted. The future input trajectory is then determined according to some optimality criterion and implemented until the next sample time, when the entire process is repeated based on new information.

More specifically, let $\mathcal{I}_k$ represent the information vector summarizing the information available at the k_{th} sample time step. $\mathcal{I}_k$ may be as simple as the open-loop state estimate plus the current measurement (as in Dynamic Matrix Control, see below) or substantially more complex like a vector parameterizing the probability distribution of the state vector (in the case of a nonlinear stochastic system). In addition, let $\mathcal{U}_k$ be a finite-dimensional vector parameterizing the future manipulated input trajectory. Then, the relationship between the future state trajectory $x(t)$ and $\mathcal{U}_k$ and $\mathcal{I}_k$ within a chosen prediction horizon ($k \cdot t_s < t < (k+p) \cdot t_s$ where t_s is the sample time) can be established. The following optimization is performed in order to determine $\mathcal{U}_k$:

$$\begin{aligned} &\min_{\mathcal{U}_k} \phi(\mathcal{I}_k, \mathcal{U}_k) \\ &\psi(\mathcal{I}_k, \mathcal{U}_k) \leq 0 \end{aligned} \tag{1}$$

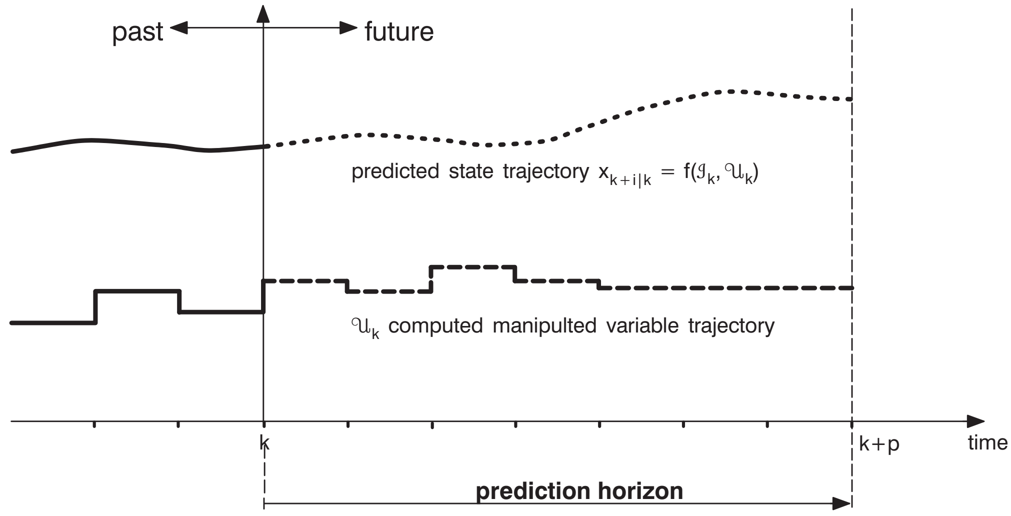

Figure 1: Basic idea of Model Predictive Control.

where the functional ϕ (cost function) is chosen so that the predicted future state trajectory is shaped in a desired manner by minimizing it and vector ψ is chosen so that the inequality represents the constraints of the system. Constraints may arise due to the limits of physical devices as well as safety and economic considerations.

Even though the input trajectory for the entire prediction horizon is computed, it is implemented for only one sample interval and the procedure is repeated at the next sample time. This is referred to as *receding horizon control*. What results is a discrete feedback control law $u_k = f(\mathcal{I}_k)$, but the feedback law is not given explicitly, but implicitly through the optimization problem, which must be solved at each time step.

MPC is rather a methodology than a single technique. The difference between various methods is mainly in the mathematical formulation. All methods, however, consist of six constituting items:

- Process model,
- performance index,
- constraints,
- disturbance rejection,
- optimization and
- receding horizon principle.

In the general case any desired performance index (objective function) can be used. Plant dynamics are described by an explicit process model which can take, in principle, any required mathematical form. Process input and output constraints are included directly in the problem formulation so that future constraint violations are anticipated and prevented. Although a constrained controller in fact always is a nonlinear controller, the terminology in Model Predictive Control separates *linear* and *nonlinear* schemes solely on the basis of the model type (linear/nonlinear) used for predicting the future process behavior.

2.1 Degrees of Freedom

In order to be able to fulfill the control goals (e.g. driving outputs to setpoints) with limitations on both manipulated inputs and controlled outputs, significant degrees of freedom have to exist. They exits when there are more manipulated inputs than controlled outputs (a so called"fat" process). For a "fat" process no unique steady state solution exists. This leaves room for optimization motivated by economics or by operational objectives. In case the process has an equal amount of manipulated inputs and controlled outputs (a "square" process), a unique constrained steady state solution exists. The dynamic solution depends on control horizon, controller parameters, etc. In case the process has fewer manipulated inputs than controlled outputs (a "thin" process), the steady state solution is determined by the optimization formulation. MPC is not intended for this to be a normal operating mode. However, the way a controller responds when degrees of freedom are lost (due to instrument failure or other activation constraints) is an important issue.

2.2 Relationship to Optimal Control

It is worth noting that the future input trajectory computed as above is a deterministic trajectory computed solely on the basis of the current information. Hence, a discrepancy exists in that open-loop control is assumed in the feedback control computation. This is a key feature distinguishing MPC from optimal feedback control. The optimal feedback control approach assumes an optimal relation between future input actions and future information but leads to an intractable dynamic programming problem (with a few notable exceptions like the LQG problem). The open-loop control assumption allows for a significant reduction in the complexity, but it can also lead to significantly worse performance than what is achievable with feedback control [26]. In the purely deterministic cases, however, MPC provides a computationally attractive way to solve an optimal control problem.

3 History and State of the Art

Even though the current industrial and academic interest in MPC was primarily generated by the 1978 paper of Richalet *et al.* [29], and by the 1979 Shell publication [5], what have since become recognized as the central MPC concepts actually predate these first reports of application by some twenty years [21]. Apparently, the first publication to introduce, explicitly, the now standard MPC concept of the finite moving prediction horizon seems to be Propoi [25]. Nevertheless, Model Algorithmic Control (MAC), as the forerunner of the scheme commercially marketed under the name of IDCOM, was in fact originally conceived in the late 1960s by Adersa/Gerbios, subsequently developed, and successfully applied to several multivariable industrial processes [29, 30].

The Dynamic Matrix Control (DMC) technique first appeared in the the open literature in 1979 (after having been applied with notable success on several Shell processes for a number of years), when Cutler and Ramaker, and Prett and Gillette reported, its application to a fluid catalytic cracking unit [5]. A reformulation of the original DMC problem as a quadratic program by Garcia and Morshedi [9] subsequently led to the development of its most popular variation to date: Quadratic Dynamic Matrix Control (QDMC).

The first attempt to put MPC (primarily as embodied by DMC and MAC) within the same framework as the so-called "classical control schemes" was by Garcia and Morari [8], in which the Internal Model Control (IMC) paradigm was proposed. They showed that the IMC structure –in which an internal model of the plant operates in parallel with the plant, and in which the controller is some appropriate inverse of this plant model – is inherent in all MPC schemes. This and subsequent publications by Morari and coworkers provided insight into the stability, robustness and performance of MPC schemes [18]. Several additional linear MPC techniques like Generic Model Control or Adaptive Generalized Predictive Control [4] have since been developed, and some have become commercial products.

3.1 Linear Model Predictive Control - Theory

Several authors have published excellent reviews of MPC theoretical issues, including the papers of Garcia *et al.* [10], Ricker [31], and Muske and Rawlings [20]. The main focus of MPC research during the past five years has been on devising algorithms for which stability can be proven (see e.g. [28, 36, 35]). The main concepts like

- infinite horizon MPC with finite number of input moves,
- infinite horizon MPC with fixed linear feedback relation,
- infinite horizon MPC with end constraint and
- MPC with contraction constraints

are reviewed in a recent paper by Lee and Cooley [15].

A second key issue has been the detection of limitations of existing industrial MPC technology. These include:

- over-parameterization of impulse and step response models and limitation of application of the algorithms to strictly stable processes,
- sub-optimal solution of the dynamic optimization,
- sub-optimal feedback (constant output disturbance assumption),
- tuning requirements to achieve nominal stability and
- inadequate handling of model uncertainty.

Muske and Rawlings have demonstrated how better performance can be achieved by using a state space model and an optimal state observer [19, 20].

3.2 Linear Model Predictive Control - Commercial Tools and Applications

In an excellent review paper Qin and Badgwell [26] have recently reviewed the current status of commercial MPC technology surveying five MPC vendors (DMC Corp., Setpoint Inc., Honeywell IAC, Adersa and Treiber Controls)[1]. Most commercial MPC technologies are still based on linear input-output models. These models are usually identified from plant test data. The identification procedure and particularly the design of plant tests is crucial for the performance of MPC algorithms.

Now the third generation of MPC technology (IDCOM-M, HIECON, PCT) is on the marked. Main characteristics are possibilities to include hard and soft constraints with hard constraints ranked in order of priority, a controllability supervisor to screen out ill-conditioned plant subsets and a multi-objective function formulation with a quadratic output objective followed by a quadratic input objective.

The total number of reported MPC applications is currently over 2200. All of the vendors report a considerable number of applications in progress so it is likely that this number will continue to increase rapidly. The majority of applications (67 %) are in refining, one of the original application areas where MPC technology has a solid track record of success. A significant number of applications can also be found in petrochemicals and chemicals, although it has taken longer for MPC technology to break into these areas. Significant growth areas include the chemicals, pulp and paper, food processing, aerospace and automotive industries.

3.3 Nonlinear Model Predictive Control - Theory

The use of nonlinear models in predictive control is an area that has attracted a large number of researchers in the last years. The main obstacle in nonlinear model predictive control is the necessity to solve a nonlinear dynamic optimization problem online. Rawlings *et al.* [27] and Mayne [17] summarize the very latest technical developments in nonlinear MPC control theory. The number of report applications is still very low with the notable exceptions of [33, 34].

Algorithms for nonlinear model predictive control are often extensions of linear model predictive control algorithms. For those approaches based on state space model representations mainly three different classes of nonlinear MPCs can be distinguished.

The first class covers nonlinear extensions of Dynamic Matrix Control [5] and Quadratic Dynamic Matrix Control (DMC) [9]. Several researchers have contributed to this area. In nonlinear quadratic dynamic matrix control (NLQDMC), proposed by Garcia [9], a nonlinear model is used to compute the effect of past manipulated variables on the predicted output. A linear model, obtained by linearizing the nonlinear model at each sampling time, is used to compute the manipulated variable values. An advantage with this approach is that only one quadratic program is solved at each sampling time. Gattu and Zafiriou [11, 12] extended the NLQDMC algorithm by incorporating state estimation using a steady-state Kalman filter gain computed at each sampling time. This algorithm can be used both for nonlinear input-output models and state-space models. The algorithm proposed by Peterson

[1]DMC Corp. and Setpoint Inc. are now part of Aspen Tech Inc.

et al. [24] extends the linear DMC algorithm by extending the disturbance vector to account also for the effect of nonlinearities in the prediction horizon, which are determined from a nonlinear state-space model. No observer is included. The future control moves are determined from an iteration, where the control moves are calculated from the ordinary DMC formula, and a new "disturbance vector" is calculated from the nonlinear model. The step response matrix used in DMC calculations are calculated by linearizing the nonlinear model at a given steady state or by perturbing the nonlinear model by a unit step. Conditions for convergence of the iterations are stated and proved. The algorithm can trivially be extended to a nonlinear QDMC algorithm by determining the control moves from the solution of a quadratic program.

The second class covers Newton-type nonlinear MPC algorithms which have mainly been developed by Biegler and co-workers [16, 22]. Their approach is to linearize a nonlinear state-space model around a nominal trajectory determined by the input sequence computed at the previous sampling time. Impulse or step response models are determined from the linear and time-varying state-space models obtained from the linearization. A new input sequence is computed by solving a quadratic program once over the time horizon, followed by a line search where the quadratic optimization criterion is computed based on the nonlinear model. Through the action of the line search, global convergence of the method can be enforced as long as the method exhibits descent directions. The optimization method used by Biegler and co-workers can be considered a special form of a sequential quadratic programming (SQP) strategy.

Finally, several researchers have proposed nonlinear MPC algorithms which utilize general nonlinear programming (NLP) techniques for solving the optimization [14, 6, 23, 2, 1]. These techniques are often used in conjunction with nonlinear, continuous state-space models. Their main differences are in the way that the model equations are solved and in the specific optimization method that is utilized. SQP algorithms are often used for solving these NLP problems. These algorithms are computationally more demanding than the previously referred nonlinear MPC algorithms.

The positive experience with input/output models in linear MPC suggests that empirical input/output models are strong candidates also for nonlinear MPC. The step from linear to nonlinear will be smaller when the model representations are similar. Input/output models are typically more computationally efficient than first principles models, and empirical models can often be found at a significantly lower cost than first principles models. Various kinds of input/output model have been studied in conjunction with MPC:

- Neural networks [3, 33],
- Fuzzy and multiple linear models [7],
- Volterra Models [13],
- Polynomial NARX models [32].

Although differing in terms of the nonlinear model representation, all nonlinear MPCs from this class are very similar with respect to formulation and solution of the optimization problem.

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