# 23. Assembly

## *Mechanics of Manipulation*

Matt Mason

matt.mason@cs.cmu.edu

http://www.cs.cmu.edu/~mason

Carnegie Mellon

**Chapter 1 Manipulation 1**
1.1 Case 1: Manipulation by a human 1
1.2 Case 2: An automated assembly system 3
1.3 Issues in manipulation 5
1.4 A taxonomy of manipulation techniques 7
1.5 Bibliographic notes 8
Exercises 8

**Chapter 2 Kinematics 11**
2.1 Preliminaries 11
2.2 Planar kinematics 15
2.3 Spherical kinematics 20
2.4 Spatial kinematics 22
2.5 Kinematic constraint 25
2.6 Kinematic mechanisms 34
2.7 Bibliographic notes 36
Exercises 37

**Chapter 3 Kinematic Representation 41**
3.1 Representation of spatial rotations 41
3.2 Representation of spatial displacements 58
3.3 Kinematic constraints 68
3.4 Bibliographic notes 72
Exercises 72

**Chapter 4 Kinematic Manipulation 77**
4.1 Path planning 77
4.2 Path planning for nonholonomic systems 84
4.3 Kinematic models of contact 86
4.4 Bibliographic notes 88
Exercises 88

**Chapter 5 Rigid Body Statics 93**
5.1 Forces acting on rigid bodies 93
5.2 Polyhedral convex cones 99
5.3 Contact wrenches and wrench cones 102
5.4 Cones in velocity twist space 104
5.5 The oriented plane 105
5.6 Instantaneous centers and Reuleaux's method 109
5.7 Line of force; moment labeling 110
5.8 Force dual 112
5.9 Summary 117
5.10 Bibliographic notes 117
Exercises 118

**Chapter 6 Friction 121**
6.1 Coulomb's Law 121
6.2 Single degree-of-freedom problems 123
6.3 Planar single contact problems 126
6.4 Graphical representation of friction cones 127
6.5 Static equilibrium problems 128
6.6 Planar sliding 130
6.7 Bibliographic notes 139
Exercises 139

**Chapter 7 Quasistatic Manipulation 143**
7.1 Grasping and fixturing 143
7.2 Pushing 147
7.3 Stable pushing 153
7.4 Parts orienting 162
7.5 Assembly 168
7.6 Bibliographic notes 173
Exercises 175

**Chapter 8 Dynamics 181**
8.1 Newton's laws 181
8.2 A particle in three dimensions 181
8.3 Moment of force; moment of momentum 183
8.4 Dynamics of a system of particles 184
8.5 Rigid body dynamics 186
8.6 The angular inertia matrix 189
8.7 Motion of a freely rotating body 195
8.8 Planar single contact problems 197
8.9 Graphical methods for the plane 203
8.10 Planar multiple-contact problems 205
8.11 Bibliographic notes 207
Exercises 208

**Chapter 9 Impact 211**
9.1 A particle 211
9.2 Rigid body impact 217
9.3 Bibliographic notes 223
Exercises 223

**Chapter 10 Dynamic Manipulation 225**
10.1 Quasidynamic manipulation 225
10.2 Briefly dynamic manipulation 229
10.3 Continuously dynamic manipulation 230
10.4 Bibliographic notes 232
Exercises 235

**Appendix A Infinity 237**

# Outline.

- Role of assembly
- Assembly sequence planning
- Jamming and wedging
- Compliance

# Role of assembly in manufacturing

- Most assembly still manual?
- Assembly automation has a big impact.
- Example: SONY Smart cell
- High volume production uses more specialized machinery.
- Low volume production is usually manual.

# Role of assembly, more generally

Assembly is an application of manipulation. It is also a fundamental manipulation technique.

Consider Smart cell again:

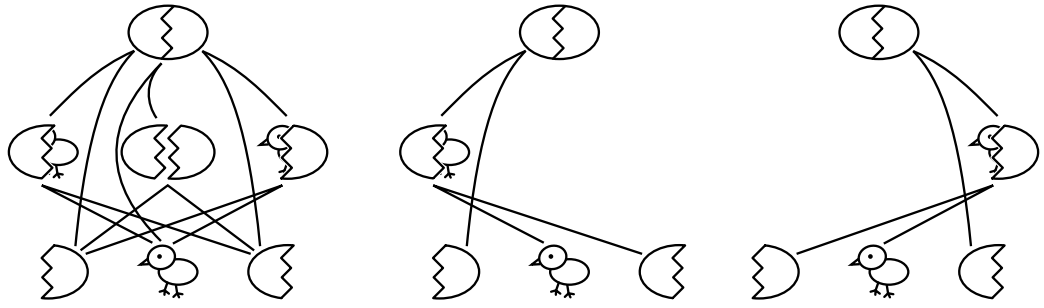
1. Part is oriented in APOS machine. (By assembling it with pallet nests!)
2. Part is grasped by SCARA gripper. (Another assembly.)
3. Part is placed in product. (Assembly again.)
4. Part is released. (*Not* assembly! Disassembly.)

Even placing an object on a table is an assembly. If assembly is defined to be bringing one shape into a desired configuration relative to another shape, then it is obvious that assembly pervades manipulation.

# Issues in assembly

- *Assembly sequence.* What order to put things together?
- *Local constraint analysis.* What motions to join two objects?
- *Path planning and grasp planning.* Finding a free path.
- *Gripper and fixture design.* Being able to apply the desired forces without interfering with the assembly.
- *Stable subassembly.* Partial assemblies must be stable.
- *Assemblability.* Even if path exists, parts might be virtually impossible to assemble, due to jamming.
- *Tolerances.* All issues must also addres variations in shape.
- *Design for assembly.* Product design must be integrated with automation design.

# Assembly sequence



Assume *two-handed* assembly.

Look at all possible partitions of the parts.

Result is an *and/or* tree.

Every subtree with one and-arc per node is a possible assembly.

# Local constraint analysis

To decide if an assembly motion exists,

1. Reverse time, consider disassembly problem.
2. Look at contact constraints, identify all motion freedoms.
3. First order form closure often is sufficient.

Remember the refrigerator problem?

# Jamming and wedging

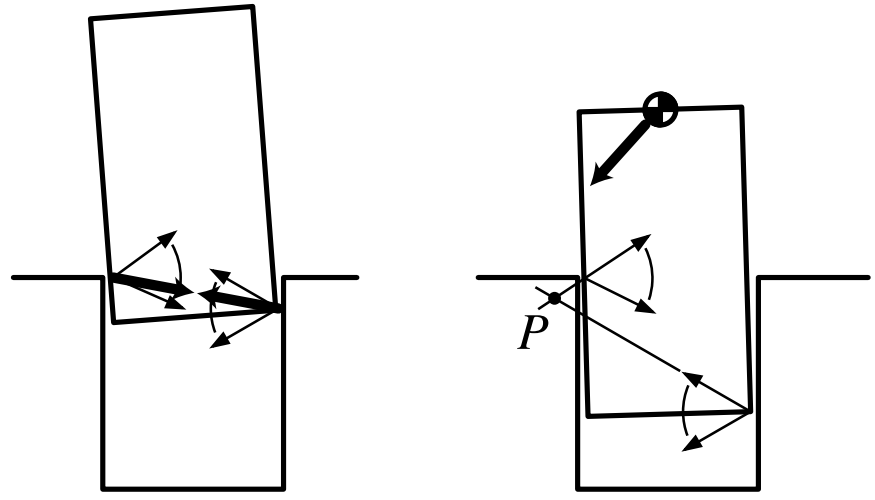
One of the earliest papers on the mechanics of manipulation was by Simunovic, in 1974, studying assembly of peg in hole. This was part of a larger project studying manufacturing issues at the Draper Labs, supervised by Nevins and Whitney.

- Study of several assemblies showed that just a few operations dominate manufacturing. Assembly, joining, etc.
- Also, just a few directions of motion dominate all assemblies. Mostly vertical.
- Simunovic defined jamming and wedging:

  *Jamming.* An assembly force in equilibrium with the contact forces.

  *Wedging.* A configuration such that *every* possibly force is in equilibrium with the contact forces. I.e. force closure.

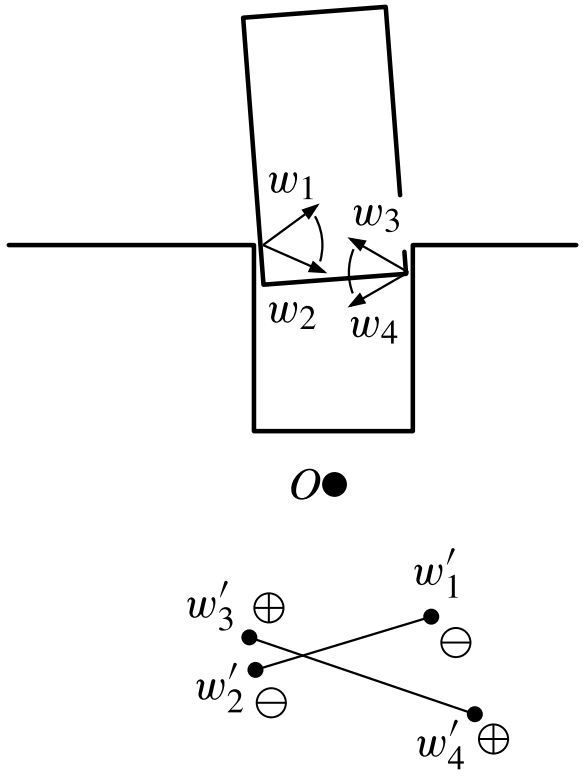
# Jamming and wedging

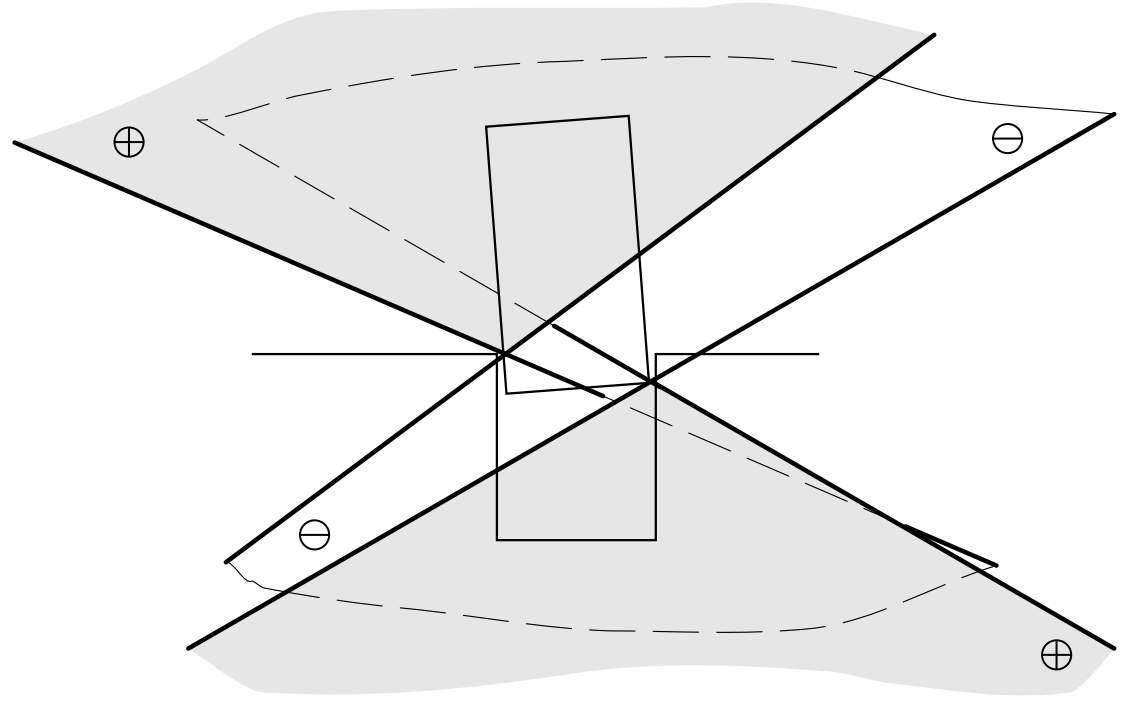



Simunovic's analysis of wedging anticipated Nguyen's approach to planar force closure.

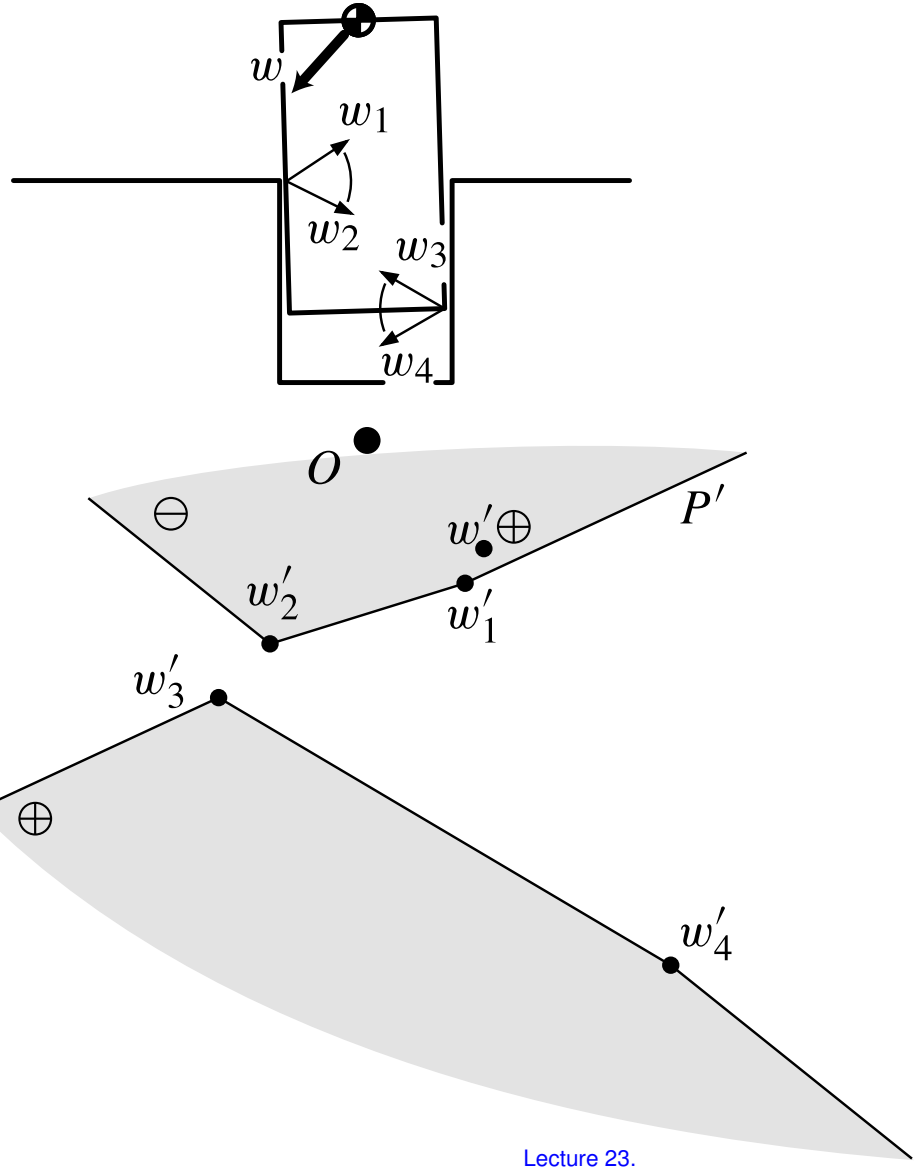
Simunovic's analysis of jamming anticipated the moment labeling method. Jamming may occur if the applied force makes positive moment at $P$.

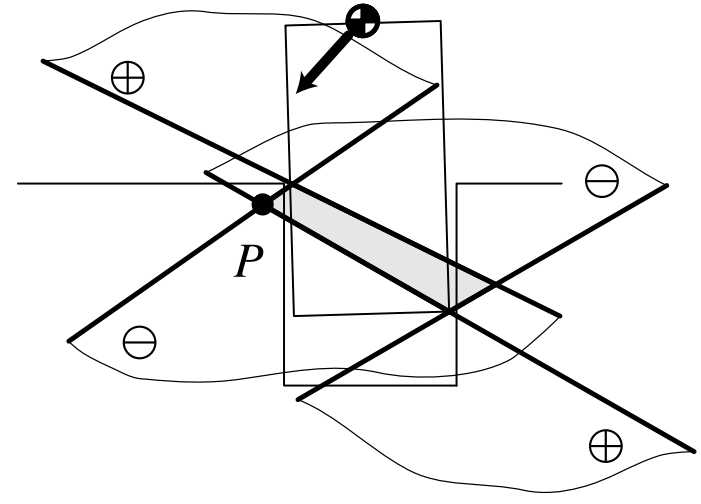
# Analysis of wedging



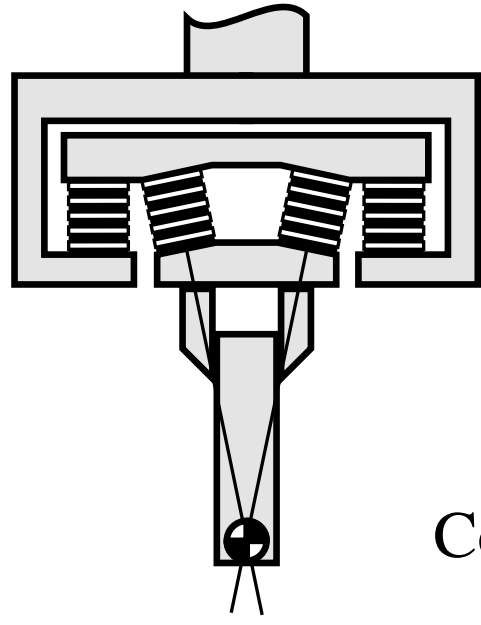



# Analysis of jamming



# Compliance



Compliance elements

rubber/steel sandwiches

stiff in compression

soft in sheer

Compliance center

- Active force control is an option.
- Remote Center Compliance is a passive mechanism.
- Compliance Center: behaves like decoupled translational and rotational springs.
- Put compliance center at tip of peg or at lip of hole.

**Chapter 1 Manipulation 1**
1.1 Case 1: Manipulation by a human 1
1.2 Case 2: An automated assembly system 3
1.3 Issues in manipulation 5
1.4 A taxonomy of manipulation techniques 7
1.5 Bibliographic notes 8
Exercises 8

**Chapter 2 Kinematics 11**
2.1 Preliminaries 11
2.2 Planar kinematics 15
2.3 Spherical kinematics 20
2.4 Spatial kinematics 22
2.5 Kinematic constraint 25
2.6 Kinematic mechanisms 34
2.7 Bibliographic notes 36
Exercises 37

**Chapter 3 Kinematic Representation 41**
3.1 Representation of spatial rotations 41
3.2 Representation of spatial displacements 58
3.3 Kinematic constraints 68
3.4 Bibliographic notes 72
Exercises 72

**Chapter 4 Kinematic Manipulation 77**
4.1 Path planning 77
4.2 Path planning for nonholonomic systems 84
4.3 Kinematic models of contact 86
4.4 Bibliographic notes 88
Exercises 88

**Chapter 5 Rigid Body Statics 93**
5.1 Forces acting on rigid bodies 93
5.2 Polyhedral convex cones 99
5.3 Contact wrenches and wrench cones 102
5.4 Cones in velocity twist space 104
5.5 The oriented plane 105
5.6 Instantaneous centers and Reuleaux's method 109
5.7 Line of force; moment labeling 110
5.8 Force dual 112
5.9 Summary 117
5.10 Bibliographic notes 117
Exercises 118

**Chapter 6 Friction 121**
6.1 Coulomb's Law 121
6.2 Single degree-of-freedom problems 123
6.3 Planar single contact problems 126
6.4 Graphical representation of friction cones 127
6.5 Static equilibrium problems 128
6.6 Planar sliding 130
6.7 Bibliographic notes 139
Exercises 139

**Chapter 7 Quasistatic Manipulation 143**
7.1 Grasping and fixturing 143
7.2 Pushing 147
7.3 Stable pushing 153
7.4 Parts orienting 162
7.5 Assembly 168
7.6 Bibliographic notes 173
Exercises 175

**Chapter 8 Dynamics 181**
8.1 Newton's laws 181
8.2 A particle in three dimensions 181
8.3 Moment of force; moment of momentum 183
8.4 Dynamics of a system of particles 184
8.5 Rigid body dynamics 186
8.6 The angular inertia matrix 189
8.7 Motion of a freely rotating body 195
8.8 Planar single contact problems 197
8.9 Graphical methods for the plane 203
8.10 Planar multiple-contact problems 205
8.11 Bibliographic notes 207
Exercises 208

**Chapter 9 Impact 211**
9.1 A particle 211
9.2 Rigid body impact 217
9.3 Bibliographic notes 223
Exercises 223

**Chapter 10 Dynamic Manipulation 225**
10.1 Quasidynamic manipulation 225
10.2 Briefly dynamic manipulation 229
10.3 Continuously dynamic manipulation 230
10.4 Bibliographic notes 232
Exercises 235

**Appendix A Infinity 237**