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# Stability Constrained Model Predictive Control for Nonlinear Systems

Xu Cheng
Process Control Division
Westinghouse Electric Corporation
Pittsburgh, PA 15238
chengx@westinghouse.com

Bruce H. Krogh
Dept. of Electrical and Computer Engineering
Carnegie Mellon University
Pittsburgh, PA 15213-3890
krogh@ece.cmu.edu

**Abstract**

A new method for nonlinear model predictive control with guaranteed stability is proposed as an extension to the stability constrained model predictive control (SCMPC) for linear time-invariant systems. The method applies to a class of nonlinear systems that can be transformed to a controllable companion form and depends on the existence of a nonlinear deadbeat control (although this is not necessarily the control that is used). Asymptotic stability is proved for the case when all state variables are measurable.

## 1. Introduction

Model predictive control (MPC), also known as receding horizon control or moving horizon control, is a control technique where a control input sequence is solved from an optimization problem over a finite prediction horizon. The first calculated control input is actually implemented and the whole optimization process is repeated at the next sampling time using the new information from the process measurements. MPC is attractive since it allows the designer to incorporate process constraints and intuitive control objectives directly into the on-line control calculations, and it handles multi-input and multi-output (MIMO) systems easily. MPC was initially developed for linear time invariant (LTI) systems [15],[32], and has enjoyed many successful industrial applications, especially in the chemical and petrochemical process control industry [5], [27].

In recent years, MPC research has focused on handling nonlinear processes. One approach is to linearize nonlinear dynamics along the operating trajectory, applying linear MPC using the appropriate linearized model at each sampling instant. Extensive nonlinear MPC research along this line can be found in the literature [4], [6], [17], [18], [19], [22], [23], [25], [26]. Another approach is to use the nonlinear plant model directly [1],[14]. In this work closed-loop stability is guaranteed by either imposing a final state equality constraint or extending the prediction horizon to infinity. Meadows et al. [21] showed that a class of feedback linearizable systems can be stabilized by an end-state-constrained MPC. Nonlinear MPC based on Volterra models have also been studied [8], [20]. Application examples are given in [10], [29], [28].

In this paper we extend stability constrained model predictive control (SCMPC) proposed in [2] to nonlinear systems. In this approach a *stability constraint* is imposed on the squared-magnitude of the state vector over the prediction horizon. The stability constraint for the next time step is set equal to the maximum of the squared-magnitude of the predicted state vectors. It is shown that for a class of multi-input nonlinear systems that can be transformed into a controllable form, the stability constraint is a contraction mapping leading to asymptotic stability. One important feature of the SCMPC is that controller parameters (weighting parameters in the objective function, prediction horizon, etc.) can be freely assigned as needed to tune for desired performance since asymptotic stability is guaranteed by the imposed stability constraint. In order to simplify the problem statement in the following, only regulation control with full state measurement is considered. It is also assumed that the system has no constraints on control inputs or state variables.

## 2. Preliminaries

Consider an autonomous discrete-time nonlinear system, with $n$ state variables and $m$ input variables, that can be expressed by the following state equation:

$$x(k+1) = Ax(k) + B_f(x(k), u(k)) \tag{1}$$

where

$$A = \begin{bmatrix} O_{(n-m)\times m} & I_{(n-m)} \\ A_1 & A_2 \end{bmatrix},$$

$$B_f(x(k), u(k)) = \begin{bmatrix} O_{(n-m)\times 1} \\ B_2(x(k), u(k)) \end{bmatrix} \tag{2}$$

and $I_{(n-m)}$ is the identity matrix with dimension of $n-m$; $O_{(n-m)\times m}$ is the zero matrix with dimension of $(n-m)\times m$; and $A_1 \in \Re^{m\times m}$, $A_2 \in \Re^{m\times(n-m)}$, $B_2 \in \Re^{m\times 1}$ are matrices of possible nonzero elements. The form in (2) is called *controllable companion form* in some nonlinear control literature (e.g. [30]). It is a counterpart to the controllable canonical form in the linear system case. For simplicity we use the term *controllable form.*

In the following development, components of vectors are also indicated by subscript indices. For a given state vector $x \in \Re^n$, the notation $x^1$ is used to denote the first $m$ components of $x$, and $x^2$ is used to denote the remaining $n-m$ components. It is assumed that for arbitrary $b \in \Re^m$ and $x \in \Re^n$ there exists $u \in \Re^m$ such that $B_2(x, u) = b$.

Without loss of generality, it is assumed that the prediction horizon equals the control horizon denoted by $N$. The predicted input and state sequences at time $k$ are denoted by $u^p(k+i|k)$ and $x^p(k+i+1|k)$, respectively, where $i = 0$,

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1,..., $N$-1, and the superscript $p$ stands for prediction. Note that $x(k+1) = x^p(k+1|k)$ in the full state feedback case. $\| \bullet \|$ will be used to denote the Euclidean norm.

Following from the above assumptions, it is natural to ask whether for a general nonlinear system given by

$$z(k+1) = f(z(k), u(k)) \tag{3}$$

it is possible to find a state transformation such that (3) can be transformed into the form of (1). This is the case when the system is *feedback linearizable* [11],[30]. To describe the desired state transformation, we introduce the following definition.

***Definition 1***: A function $\phi: \Re^n \to \Re^n$, defined in a region $\Omega$, is called a *diffeomorphism* if it is smooth, and if its inverse $\phi^{-1}$ exists and is smooth.

If the region $\Omega$ is the whole space $\Re^n$, then $\phi(z)$ is called a *global* diffeomorphism. Global diffeomorphisms are rare, and therefore one often looks for local diffeomorphisms, i.e., for transformations defined only in a finite neighborhood of a given point. A diffeomorphism can be used to transform a nonlinear system into another nonlinear system in terms of a new set of state variables, similar to what is commonly done in the analysis of linear systems. For general nonlinear systems given in an arbitrary form, the exact formula for calculation of a diffeomorphism is not available. However, for nonlinear systems which are linear in control, i.e., the system which can be represented as:

$$z(k+1) = f(z(k)) + g(z)u(k),$$

a standard method of computing diffeomorphism can be found in [11].

If the nonlinear system dynamics is originally expressed by an input-output difference equation, the controllable form state equation may be obtained by defining appropriate state variables directly. For example, if a SISO nonlinear system is defined by the following equation

$$y(k+1) = F(y(k), \ldots, y(k-n+1), u(k)) \tag{4}$$

then one can define internal state variables as

$$\begin{aligned} x_1(k) &= y(k-n+1) \\ x_2(k) &= y(k-n+2) \\ &\vdots \\ x_{n-1}(k) &= y(k-1) \\ x_n(k) &= y(k). \end{aligned}$$

As a result, the corresponding state space equation can be written as:

$$x(k+1) = \begin{bmatrix} O_{(n-1)\times 1} & I_{(n-1)\times(n-1)} \\ O_{1\times(n-1)} & 0 \end{bmatrix} x(k) + \begin{bmatrix} O_{(n-1)\times 1} \\ F(x((k), u(k))) \end{bmatrix} \tag{5}$$

where $x = [x_1, x_2, \ldots, x_n]^T$.

Equation (5) is a special form of (1). A class of feedback linearizable discrete-time systems falls into this nonlinear system category [9], [13], [16].

The development of the SCMPC for nonlinear systems uses the notion of a deadbeat control, which is defined as follows.

***Definition 2:*** A control sequence $u(k), u(k+1), \ldots$ is a *deadbeat control* for system (3) if it drives the state variables to zero within $n/m$ steps.

The above definition for deadbeat control is fairly standard, and it can be shown that feedback linearizable systems admit a deadbeat control. Some earlier work on nonlinear deadbeat control design can also be found in [7]. The existence of a nonlinear deadbeat control for systems given by (5) is equivalent to the existence of a control sequence beginning at $k-n$ such that $F(x_n(k), \ldots, x_1(k), u(k)) = 0$ regardless of the current state. If a nonlinear deadbeat control always exists for the given system, all previous results for SCMPC of LTI systems can be developed similarly.

### 3. Main Result

Before we proceed to the computation algorithm and stability analysis, two lemmas need to be introduced as preliminaries.

***Lemma 1***: Given $(A, B_f)$ in controllable form (2), at any time $k$ and for any input sequence $u(k), u(k+1), \ldots, u\left(k + \left\lceil \frac{n}{m} \right\rceil - 1\right)$, where

$$\left\lceil \frac{n}{m} \right\rceil = \begin{cases} n/m & \text{if } n/m \text{ is an integer} \\ int(n/m)+1 & \text{otherwise} \end{cases},$$

the state vector $x(k)$ can be expressed as:

$$x(k) = \left[ I_n \;\middle|\; O_{n \times \left(m\left\lceil \frac{n}{m} \right\rceil - n\right)} \right] \begin{bmatrix} x^1(k) \\ x^1(k+1) \\ \vdots \\ x^1\left(k + \left\lceil \frac{n}{m} \right\rceil - 1\right) \end{bmatrix} \tag{6}$$

*<Proof>* Equation (6) follows from the definition of controllable form $(A, B_f)$ and repeated application of the state equation (1). □

***Lemma 2:*** Given $(A, B_f)$ in controllable form (2), starting at any time $k$, if there always exists a $u^*(k+i)$ such that

$$A_1 x^1(k+i) + A_2 x^2(k+i) + B_2(x(k), u^*(k)) = 0 \tag{7}$$

$$\left(i = 0 \ldots \left\lceil \frac{n}{m} \right\rceil - 1\right)$$

then $\|x(k+i)\|^2 = \|x(k)\|^2 - \sum_{j=0}^{i-1} \|x^1(k+j)\|^2$, and moreover, $x\left(\left\lceil k + \frac{n}{m} \right\rceil\right) \equiv 0$.

*<Proof>* Without loss of generality, let $k = 0$.

It follows from the controllable form (2) and Lemma 1 that $x(i+1) = \begin{bmatrix} x^2(i) \\ x^1\left(i+\left\lceil \frac{n}{m} \right\rceil\right) \end{bmatrix}$. By applying $u^*(i)$, we have:

$$x^1\left(i+\left\lceil \frac{n}{m} \right\rceil\right) = [A_1 \quad A_2]x(i) + B_2(x(k), u^*(k)) = 0$$

Therefore, $x(i+1) = \begin{bmatrix} x^2(i) \\ 0 \end{bmatrix}$

$$\Rightarrow \quad \|x(i+1)\|^2 = \|x(i)\|^2 - \|x^1(i)\|^2 \qquad (8)$$

Repeated application of (6) beginning from $i = 0$ gives us $\|x(i)\|^2 = \|x(0)\|^2 - \sum_{j=0}^{i-1} \|x^1(j)\|^2$.

Let $i = \left\lceil \frac{n}{m} \right\rceil$, we have

$$\left\|x\left(\left\lceil \frac{n}{m} \right\rceil\right)\right\|^2 = \|x(0)\|^2 - \sum_{j=0}^{\left\lceil \frac{n}{m} \right\rceil - 1} \|x^1(j)\|^2.$$

By using Lemma 1 again, it yields $\left\|x\left(\left\lceil \frac{n}{m} \right\rceil\right)\right\|^2 = 0$, which implies $x\left(\left\lceil \frac{n}{m} \right\rceil\right) = 0$. □

Lemma 1 is essential to the SCMPC development. When control $u^*$ defined in Lemma 2 is applied from $i = 0$ to $i = \lceil n/m \rceil - 1$, it is essentially a deadbeat control, since, by a usual definition, a deadbeat control is a control action that zeros the state vector in $\lceil n/m \rceil$ steps. For convenience, we term the control sequence defined in Lemma 2 as deadbeat control even when $i < \lceil n/m \rceil - 1$. Based on the context, this slight abuse of terminology should not cause confusion.

We now turn our attention to the computation algorithm. Suppose a diffeomorphism $x = \phi(z)$ exists for a nonlinear system (3) such that the transformed system is in the form of equation (1), i.e.,

$$x(k+1) = Ax(k) + B_f(F) = Ax(k) + B_f(F(x, u)),$$

where $A$ and $B_f(F(x,u))$ are in controllable form. Similar to the SCMPC for a LTI system [2], one can set up the optimization objective function in terms of the original state variables $z(k)$, while the stability constraint will be imposed in terms of state variables $x(k)$ in the transformed coordinates. Specifically, given a general optimization objective function:

$$\Phi(Z^P, U^P),$$

where $Z^P = [z^P(k+1|k), \ldots, z^P(k+N|k)]$ and $U^P = [u^P(k|k), \ldots, u^P(k+N|k)]$, we can formulate the nonlinear SCMPC computations by the following steps at each sampling stage $k = 0, 1, \ldots$

*Step 1: Initialization:* Given measured $z(k)$ and $l_k$ from the previous iteration (set $l_0$ to be an arbitrary number), and $c \geq 1$, define

$$x(k) = \phi(z(k))$$

$$\hat{l}_k = max\{l_k, \ \|x(k)\|^2\}$$

$$\varepsilon_k = \frac{\|x^1(k)\|^2}{c \cdot \hat{l}_k}$$

$$x^P(k|k) = x(k)$$

*Step 2: Optimization:*

$$\text{Min } \Phi(Z^P, U^P) = \Phi(\phi^{-1}(X^P), U^P)$$

$$\text{s.t. } x^P(k+i+1|k) = Ax^P(k+i|k) + B_f(F(x^P, u)) \quad (i = 0, \ldots, N\text{-}1)$$

$$\|x^P(k+i|k)\|^2 \leq (1-\varepsilon_k)\hat{l}_k \qquad (i = 1, \ldots, N)$$

(stability constraint)

*Step 3: Assignment:*

$$u(k) = u^P(k|k)$$

$$l_{k+1} = max\left\{\|x^P(k+i|k)\|^2, \quad \text{i=1,2,}\ldots\text{,N}\right\}$$

*Step 4: Implementation:* Apply control $u(k)$. Set $k = k+1$ and repeat from step 1. □

The following Lemma concerns feasibility of the online optimization.

***Lemma 3:*** Given $(A, B_f)$ in controllable form (2), the optimization problem outlined in the nonlinear SCMPC computation is always feasible for all $k = 0, 1, \ldots$.

*<Proof >* We show the control action specified in Lemma 2 is always a feasible solution to the optimization at any sample $k$. When the control $u^*$ specified in Lemma 2 is applied, we have:

$$\|x(k+1)\|^2 = \|x(k)\|^2 - \|x^1(k)\|^2$$

$$\begin{aligned}\|x(k+i)\|^2 &= \|x(k)\|^2 - \sum_{j=0}^{i-1} \|x^1(k+j)\|^2 \\ &\leq \|x(k)\|^2 - \|x^1(k)\|^2 \\ &= \|x(k)\|^2 - c\hat{l}_k \cdot \frac{\|x^1(k)\|^2}{c\hat{l}_k} \\ &\leq \hat{l}_k - c\hat{l}_k \cdot \varepsilon_k \\ &= \hat{l}_k(1 - c\varepsilon_k) \quad \leq \hat{l}_k(1-\varepsilon_k)\end{aligned}$$

□

The proof of Lemma 3 shows that the control defined in Lemma 2, which is essentially a deadbeat control, is feasible at each stage. The deadbeat control is not necessarily the control that is selected by the optimization, however. The SCMPC optimization is free to choose any feasible control, and will typically choose a control other than the deadbeat control.

The stability result for the nonlinear SCMPC with full state measurement can now be stated as the following theorem.

***Theorem 1:*** For controllable nonlinear systems (3) that can be transformed by a diffeomorphism into the controllable form (2), if a deadbeat control exists, the nonlinear SCMPC asymptotically stabilizes the closed-loop system.

*<Proof>* By Lemma 3, the SCMPC is well defined for all $k$. In the perfect state information case, $x(k) = x^P(k|k-1)$, which implies from computation *Step 1* that $l_k = \hat{l}_k$. Moreover, the sequence $l_k$ $(k = 0, 1, ...)$ is nonincreasing due to the stability constraint and assignment in *Step 3*. Two cases are considered for the sequence $\varepsilon_k$ $(k = 0, 1, ...)$ and it will be shown that $\|x(k)\| \to 0$ in both cases.

*Case 1*: Suppose the sequence $\varepsilon_k$ does not converge to zero. This implies there exists some $\varepsilon_{min} > 0$ and an infinite sequence of indices $0 \le k_1 < k_2 < \dots$ such that $\varepsilon_{k_j} \ge \varepsilon_{min}$ for all $j = 1, 2, \dots$.

From the stability constraint and definition of $l_k$, it follows that

$$l_{k+1} \le \prod_{j=0}^{k} (1-\varepsilon_j) l_0$$

where $0 \le \varepsilon_j \le 1$. Therefore, for any $J \ge 1$,

$$l_{k_J+1} \le \prod_{j=1}^{J} (1-\varepsilon_{k_j}) l_0$$

which implies:

$$l_{k_J+1} \le (1-\varepsilon_{min})^J l_0$$

Thus, $l_{k_j+1} \to 0$ as $j \to \infty$, which implies $l_k \to 0$ as $k \to \infty$ since the sequence $l_k$ is nonincreasing. This implies $\|x(k)\| \to 0$, since $x(k) = x^P(k|k-1)$ and $\|x^P(k|k-1)\|^2 \le l_{k-1}$.

*Case 2:* Suppose $\varepsilon_k \to 0$, i.e., for any $\eta > 0$ there exists some $K_\eta$ such that for all $k > K_\eta$ we have $\varepsilon_k < \eta$.

Now let $\eta = \gamma / \left(c \left\lceil \frac{n}{m} \right\rceil l_0\right)$, where $\gamma > 0$ is an arbitrary small number, $c \ge 1$, and $l_0$ is chosen arbitrarily. From the definition of $\varepsilon_k$, it becomes clear that

$$\|x^1(k)\|^2 = c\varepsilon_k l_k \quad \Rightarrow \quad \|x^1(k)\|^2 < c\eta l_k \le c\eta l_{K_\eta} \le c\eta l_0 \quad (k > K_\eta)$$

Furthermore, since $k + i > K_\eta$ for $i \ge 0$, we have:

$$\|x^1(k+i)\|^2 < c\eta l_0 \qquad \left(i = 1, 2, \dots, \left\lceil \frac{n}{m} \right\rceil - 1\right)$$

By using Lemma 1, it follows:

$$\|x(k)\|^2 \le \sum_{i=0}^{\lceil \frac{n}{m} \rceil - 1} \|x^1(k+i)\|^2 < \left\lceil \frac{n}{m} \right\rceil \cdot c\eta l_0 \qquad (k > K_\eta)$$

Therefore, substituting the definition of $\eta$ yields $\|x(k)\|^2 < c\left\lceil \frac{n}{m} \right\rceil l_0 \cdot \frac{\gamma}{c\left\lceil \frac{n}{m} \right\rceil l_0} = \gamma$, and this is equivalent to the following statement:

*For any small number $\gamma$, there exists a time $K_\eta$ such that for all $k \ge K_\eta$, $\|x(k)\|^2 < \gamma$ holds.* $\Rightarrow$ $\|x(k)\| \to 0$.

Based on the results of case 1 and 2, and the definition of diffeomorphism, it follows that $z(k) \to 0$. □

As in the application of SCMPC to LTI systems, since the stability property of SCMPC only depends on the stability constraint, all other parameters, including the prediction horizon and objective function $\Phi$, can be chosen to tune for performance. Of course, in order to make the optimization numerically stable, a convex objective function $\Phi$ is desirable.

We note that SCMPC parameter tuning can be performed continuously during real-time control operation without destroying stability. This is not possible for MPC formulations with either an end-state constraint or an infinite horizon, since their stability proofs depend on choosing the objective function as a Lyapunov function. Consequently, parameters in the objective function can not be varied arbitrarily during on-line operation, since it may damage some properties of the objective function which are necessary for the stability proof.

Finally, note that although Theorem 1 is stated for controllable discrete-time nonlinear systems, it is not difficult to extend the result to stabilizable systems by performing an appropriate decomposition first.

## 4. Example

Consider a discrete-time nonlinear system described by:

$$z_1(k+1) = \sqrt[3]{z_3(k)}$$

$$z_2(k+1) = \sqrt[3]{z_3(k)} \cdot (z_1^2(k) + z_2(k) + u(k))$$

$$z_3(k+1) = \left(\frac{z_2(k)}{z_1(k)}\right)^3$$

A transformation $z_1 = x_1$, $z_2 = x_1 \cdot x_3$, and $z_3 = x_2^3$ yields a nonlinear system in controllable form:

$$x_1(k+1) = x_2 k$$

$$x_2(k+1) = x_3(k)$$

$$x_3(k+1) = x_1^2(k) + x_1(k)x_3(k) + u(k)$$

If an initial condition $z(0) = [-2 \;\; 1 \;\; -3]^T$ is assumed and the optimization objective function is selected as:

$$\Phi = \sum_{i=0}^{N} z^P(k+i+1|k)^T Q z^P(k+i+1|k) + \\ + \sum_{i=0}^{N} u^P(k+i|k)^T R u^P(k+i|k)$$

where prediction horizon $N = 3$ and $Q$, $R$ are identity matrices, it can be shown by a simulation (Figures 1,2 and 3) that the closed-loop system is asymptotically stable when the nonlinear SCMPC presented in section 3 is applied.

Other simulations show increasing the prediction horizon helps to produce a smoother dynamic response. It can also be shown that for the same on-line optimization problem, the closed-loop system is unstable for a short prediction horizon without any stability constraints.,

## 5. Discussion

A nonlinear stability constrained MPC is presented in this paper. This approach provides an alternative solution to the Jacobian linearization-based nonlinear MPC controller design. In addition to guaranteed closed-loop stability, other features (flexible parameter tuning, short prediction horizon, etc.) of SCMPC previously developed for LTI systems are also preserved in the nonlinear framework. We conclude with some remarks regarding the nonlinear SCMPC.

The concept of enforcing stability through a contraction mapping for nonlinear MPC algorithms has been considered by Economo [3] and Oliveira and Morari [26]. They do not, however, show the optimization problem is guaranteed to be feasible at each stage as what has been shown in Lemma 3 with perfect state information. Moreover, we do not require a long prediction horizon or open-loop stable system as is the case in some model predictive control stability proofs [25], [34].

One of the important issues of nonlinear SCMPC concerns the efficiency of on-line optimization. Unlike in the LTI system case, even for a general feedback linearizable system, the stability constraint may not be convex. Therefore, the on-line computation could be quite involved. This is one reason the Jacobian linearization method still remains an attractive alternative to consider for nonlinear systems. Fortunately, the SCMPC allows the use of a short prediction horizons compared to other MPC approaches, so the practicality of applying a nonlinear SCMPC for real-time implementation is enhanced.

The application of nonlinear MPC is not only limited to feedback-linearizable systems. A broader class of nonlinear systems can be expected to be handled, although we have not identified the scope of the problem exactly. Control input and other process constraints can all be incorporated into the optimization directly as long as the optimization is always feasible.

One common drawback for the nonlinear model based control system design is that all state variables have to be measurable. Direct incorporation of a state estimator is usually difficult since nonlinear state estimator design is not a trivial task. This limitation will constrain the applications of SCMPC to certain extent. Encouragingly, conceivable advancement in nonlinear state observer design has been made in recent years [12], [24], [31], [33]. Future work will address the issue of incorporating state estimation when state variables are not fully measurable. Selecting appropriate objective functions for efficient on-line computation is also of interest.


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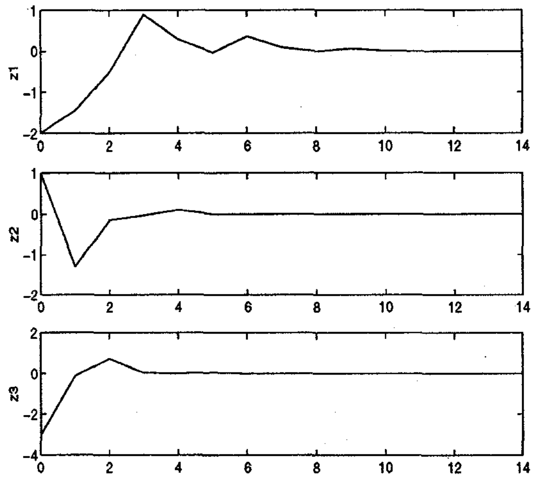


Fig. 1. Original state variables $z(k)$

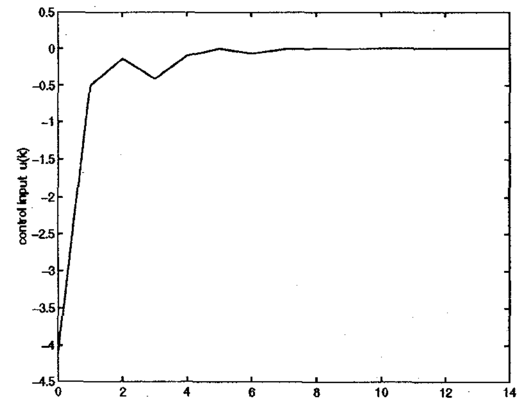


Fig. 2 Control input $u(k)$

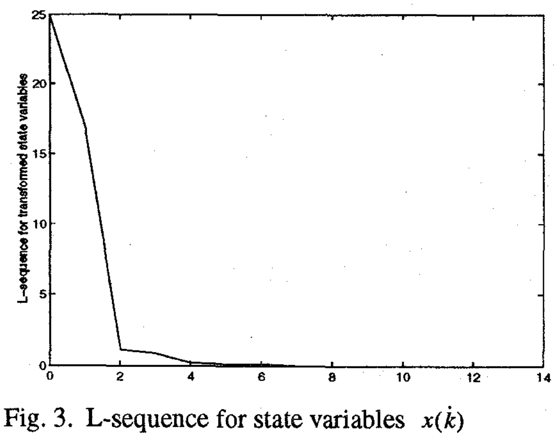


Fig. 3. L-sequence for state variables $x(k)$