Robust Tracking Control for a Wheeled Mobile Robot

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Abstract – **This paper deals with the tracking problem for a wheeled mobile robot considering the measurement uncertainties. A new robust control law is performed via Lyapunov method to ensure a good path following under bounded measurement perturbations.**

I. INTRODUCTION

It is well known that wheeled mobile robots are subjected to nonholonomic constraints due to the restricted mobility, in the direction of the wheels axis, which prevents the robot to move sideways Therefore, the degrees of freedom are less than the dimension of the configuration space. As a consequence, their control is more difficult than the control of robot manipulators which are holonomic systems.

The mobile robot under consideration is a two degrees of freedom (2 DOF) unicycle-like vehicle, its motion control can be achieved by dealing with the linear and rotational velocities. The kinematics model is derived under rolling-without-slippage assumption, and the desired path is represented by a fictitious reference robot with the same nonholonomic constraints (see Fig.1).

In order to behave properly, the controller must know the localization of the robot at any time, by fusing the exteroceptive measurements with the dead reckoning.
Unfortunately the measurements are not error-free and the controller must be robust with respect to the measurement uncertainties.

The robust control proposed in this paper is an extension of the control law proposed in [2], to ensure a good path following in forward-motion ($v_r > 0$) and backward-motion ($v_r < 0$), under bounded measurement perturbations.

The paper is organized as follows. In section II, a kinematics error model based on fictitious reference mobile robot tracking is derived. In section III, we present a nonlinear control law, which ensures local asymptotic stability of the equilibrium point when the measurements are assumed to be noise-free. This control law is extended to ensure a robust tracking in the case of bounded measurement uncertainties. Section IV is devoted to simulation results to highlight the effectiveness of the proposed control law. Section V concludes the paper.

II. KINEMATICS ERROR MODEL

The configuration of the real robot is described by the vector $(x,y,\theta)^T$, where (x,y) are the coordinates of the point M, located at mid-distance of the rear-wheels, in the global frame $(O,\vec{i}_0,\vec{j}_0)$, and θ is the orientation of the vehicle, taken counterclockwise from the global X-axis.
$(x_r,y_r,\theta_r)^T$ is the equivalent configuration for the fictitious reference robot.
$(x_e,y_e,\theta_e)^T$ is the error configuration vector, where (x_e,y_e) are the coordinates of the error position vector $\vec{MM_r}$ in the mobile frame linked to the real mobile robot $(M,\vec{i}_1,\vec{j}_1)$, and θ_e is the orientation error with respect to $\vec{i}_0$.
x, y and θ are updated using exteroceptive and dead-reckoning measurements, and x_e, y_e and θ_e are computed as follows:

$$\begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{pmatrix}$$

v and v_r are respectively, the linear velocities of M and M_r, and $\dot\theta, \dot\theta_r$ are respectively, the rotational velocities of the real robot and the reference one.

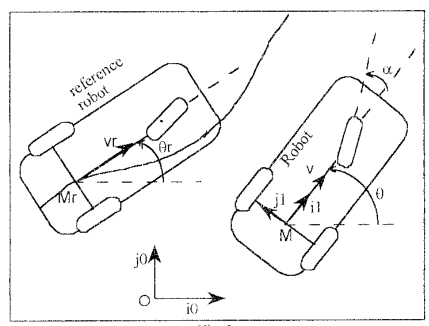

Fig.1.

Our aim is to superpose the real robot with the reference one by vanishing the error-configuration $(x_e, y_e, \theta_e)^T$.
In the mobile frame $(M,\vec{i}_1,\vec{j}_1)$ one has

$$\vec{MM_r} = x_e\,\vec{i}_1 + y_e\,\vec{j}_1 \tag{1}$$

$$\frac{d\,\vec{MM_r}}{dt} = \dot{x}_e\vec{i}_1 + \dot{y}_e\vec{j}_1 - y_e\dot\theta\,\vec{i}_1 + x_e\dot\theta\,\vec{j}_1 \tag{2}$$

The latter can also be written as follows:

$$\frac{d\,\vec{MM_r}}{dt} = \frac{d\,\vec{OM_r}}{dt} - \frac{d\,\vec{OM}}{dt} \tag{3}$$

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where,

$$\frac{d\,\overrightarrow{OM_r}}{dt} = v_r \cos\theta_e \vec{i}_1 - v_r \sin\theta_e \vec{j}_1 \quad (4)$$

$$\frac{d\,\overrightarrow{OM}}{dt} = v\vec{i}_1 \quad (5)$$

Substituting (4) and (5) in equation (3) and identifying with equation (2) yields:

$$\begin{aligned} \dot{x}_e &= v_r \cos\theta_e + y_e\dot{\theta} - v \\ \dot{y}_e &= -v_r \sin\theta_e - x_e\dot{\theta} \end{aligned} \quad (6)$$

Let u_1 and u_2 be the control variables defined as:

$$\begin{aligned} u_1 &= v - v_r \\ u_2 &= \dot{\theta} - \dot{\theta}_r \end{aligned} \quad (7)$$

Substituting (7) in (6) we obtain the following state representation

$$\begin{cases} \dot{x}_e = -u_1 + y_e u_2 + y_e\dot{\theta}_r + v_r(\cos\theta_e - 1) \\ \dot{y}_e = -x_e u_2 - x_e\dot{\theta}_r - v_r \sin\theta_e \\ \dot{\theta}_e = u_2 \end{cases} \quad (8)$$

III. CONTROL SYNTHESIS

A. Control Law Without Robustness: Noiseless Case

Proposition 1:

If v_r and $\dot{\theta}_r$ are not both equal to zero, the equilibrium point $(0,0,0)$ of the system (8) is locally asymptotically stable under the following control law:

$$\begin{cases} u_1 = v_r(\cos\theta_e - 1) + K_1 x_e \\ u_2 = v_r(K_2 y_e - K_3 sign(v_r)\sin\theta_e) \end{cases} \quad (9)$$

where K_1, K_2 and K_3 are positive constant parameters.

Proof:

Let us use the Lyapunov function proposed by Kanayama et al. in [2]:

$$V(X_e) = \frac{1}{2}(x_e^2 + y_e^2) + \frac{1-\cos\theta_e}{K_2} \quad (10)$$

which is positive semi-definite and whose time derivative is:

$$\dot{V}(X_e) = -K_1 x_e^2 - \frac{K_3}{K_2}\left| v_r \right| (\sin\theta_e)^2 \quad (11)$$

By using La Salle's theorem when v_r and $\dot{\theta}_r$ are constants and not both equal to zero, or Barbalat's Lemma when v_r and $\dot{\theta}_r$ are bounded time varying functions and do not both converge to zero, we can prove easily that $(x_e = 0, y_e = 0, \theta_e = 0)$ is a locally asymptotically stable equilibrium point for the system (8).

□

B. Robust Control Law

In practice, the state variables x_e, y_e and θ_e are only available through their éstimations $\hat{x}_e, \hat{y}_e$ and $\hat{\theta}_e$. Hence, the control law should be a function of these estimated values.

The estimated states are defined as follows:

$$\hat{x}_e = x_e + \delta x_e\,;\ \hat{y}_e = y_e + \delta y_e\,;\ \hat{\theta}_e = \theta_e + \delta\theta_e \quad (12)$$

where $\delta x_e, \delta y_e$ and $\delta\theta_e$ are the measurement uncertainties assumed to be bounded as follows:

$$\begin{gathered} \left|\delta x_e\right| \le \lambda_1,\ \left|\delta y_e\right| \le \lambda_2 \text{ and } \left|\delta\theta_e\right| \le \lambda_3 \\ \lambda = (\lambda_1, \lambda_2, \lambda_3) \in IR_+^3 \end{gathered} \quad (13)$$

where λ is known.

Proposition 2:

Consider system (8) with the following state feedback:

$$\begin{cases} \hat{u}_1 = v_r(\cos\hat{\theta}_e - 1) + K_1\hat{x}_e + \eta_1 \\ \hat{u}_2 = v_r(K_2\hat{y}_e - K_3 sign(v_r)\sin\hat{\theta}_e) + \eta_2 \end{cases} \quad (14)$$

with

$$\eta_1 = \begin{cases} -(K_1\lambda_1 + \left|v_r\right|\lambda_3)\, sign(\hat{x}_e) & if \quad \left|\hat{x}_e\right| \ge 2\lambda_1 \\ 0 & if \quad \left|\hat{x}_e\right| < 2\lambda_1 \end{cases} \quad (15)$$

$$\eta_2 = \begin{cases} -\left|v_r\right|(K_2\lambda_2 + K_3\lambda_3)\, sign(\sin\hat{\theta}_e) & if\ \left|\hat{\theta}_e\right| \ge 2\lambda_3 \\ 0 & if\ \left|\hat{\theta}_e\right| < 2\lambda_3 \end{cases} \quad (16)$$

where, K_1, K_2 and K_3 are positive parameters, v_r and $\dot{\theta}_r$ are not both equal to zero.

Then the states are uniformly ultimate bounded (UUB) and converge towards the following attractive domain:

$$\Phi = \left\{ (x_e, y_e, \theta_e) \in \Re^3 \mid \left|x_e\right| \le \lambda_1, \left|y_e\right| \le \lambda_2 + \frac{K_3}{K_2}(1 + \lambda_3), \left|\theta_e\right| \le \lambda_3 \right\}.$$

Proof:

The control variables $\hat{u}_1$ and $\hat{u}_2$ can be written as:

$$\begin{cases} \hat{u}_1 = u_1 + \delta u_1 + \eta_1 \\ \hat{u}_2 = u_2 + \delta u_2 + \eta_2 \end{cases} \quad (17)$$

where u_1, u_2 are defined in (9) and δu_1, δu_2 are unknown control uncertainties due to the measurement uncertainties. They can be approximated using Taylor's expansion of $\hat{u}_1$ and $\hat{u}_2$ in the neighborhood of (x_e, y_e, θ_e):

$$\begin{aligned} \delta u_1 &\approx -v_r \delta\theta_e \sin\theta_e + K_1 \delta x_e \\ \delta u_2 &\approx v_r (K_2 \delta y_e - K_3 sign(v_r) \delta\theta_e \sin\theta_e) \end{aligned} \tag{18}$$

which can be bounded as follows:

$$\begin{aligned} |\delta u_1| &\le K_1 \lambda_1 + |v_r| \lambda_3 \\ |\delta u_2| &\le |v_r| K_2 \lambda_2 + |v_r| K_3 \lambda_3 \end{aligned} \tag{19}$$

Using the Lyapunov function (10) with the control law (14), yields:

$$\begin{aligned} \dot{V}(X_e) = &-K_1 x_e^2 - \frac{K_3}{K_2} |v_r| (\sin\theta_e)^2 \\ &+ (\delta u_1 + \eta_1) x_e + \frac{(\delta u_2 + \eta_2)}{K_2} \sin\theta_e \end{aligned} \tag{20}$$

Substituting (15) and (16) in (20), leads to the following facts:

If $|\delta\theta_e| < |\theta_e|$ and $|\delta x_e| < |x_e|$, the orientation error θ_e and its estimate value $\hat{\theta}_e$ have the same sign and similarly for x_e and $\hat{x}_e$. Therefore, $\dot{V}$ remains nonpositive, since δu_1 and δu_2 are bounded as in (19).

$\dot{V}$ remains also nonpositive if $x_e = \sin\theta_e = 0$.

Thus, x_e and θ_e converge towards a bounded domain defined by: $\Phi_1 = \{ (x_e, \theta_e) \in \Re^2 \; ; \; |x_e| \le \lambda_1 , |\theta_e| \le \lambda_3 \}$.

Assuming that θ_e varies slowly within Φ_1, then from (8), $\hat{u}_2$ tends to zero. This implies that u_2 tends to $(-\delta u_2)$, since η_1 and η_2 are both equal to zero when x_e and θ_e are within Φ_1.

In fact, y_e tends to $(\frac{K_3}{K_2} sign(v_r)(1+\delta\theta_e)\sin\theta_e - \delta y_e)$.

Finally, we can conclude that y_e converges to the bounded domain defined by:

$$\Phi_2 = \left\{ y_e \in \Re \;\; | \;\; |y_e| \le \lambda_2 + \frac{K_3}{K_2}(1+\lambda_3) \right\}.$$

□

IV. SIMULATION RESULTS

The simulation results presented have been obtained with and without robustness when the real robot tracks a circular path defined by the kinematics characteristics of a fictitious reference robot ($v_r = 0.3 m/s$ and $\dot{\theta}_r = 0.1 rd/s$), under the following measurement perturbations:

$$\delta x_e = \delta y_e = 0.2(1 - \exp(-0.1\,t))$$

$$\delta\theta_e = \frac{\pi}{20}(1 - \exp(-0.1\,t))$$

Figure 2 shows the harmful effects when neglecting the measurement uncertainties in the control design. Indeed, the error-configuration (x_e, y_e, θ_e) converge towards another equilibrium point different from zero.

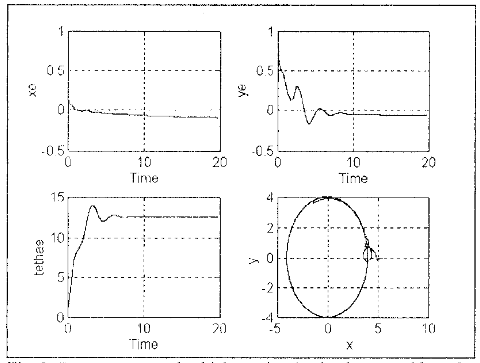

Fig. 2. System states and vehicle motion (under the control law (9), considering measurement uncertainties).

Figure 3 shows the convergence of the error-configuration towards a bounded domain around zero which is defined in accordance with the bounds of the measurement uncertainties.

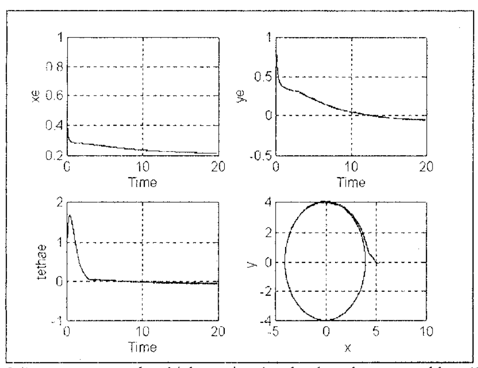

Fig 3 System states and vehicle motion (under the robust control law (14), considering measurement uncertainties).

V. CONCLUSION

In this paper, a robust control scheme, for the tracking problem of a wheeled mobile robot, has been performed via Lyapunov method. This control law ensures a global boundedness of the system states, under measurement uncertainties.

If we assume that the measurements are noise-free, the control law in equations (9) is sufficient to guarantee local asymptotic stability of the equilibrium point $(0,0,0)$. But in practice, the measurements are not noise-free and can instigate harmful effects in the path following. Hence, it is necessary to extend this control law by adding supplement control variables η_1 and η_2 according with the bounds of the measurement uncertainties, for bringing the system states into an attractive domain around zero.

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