

## A New Control Strategy for Tracking in Mobile Robots and AGV's

A. Hemami

M.G. Mehrabi

R.M.H. Cheng

Department of Mechanical Engineering  
Concordia University  
1455 de Maisonneuve Blvd., West  
Montréal, (Québec)  
Canada, H3G 1M8

### Abstract

In this paper the tracking problem of a wheeled mobile robot or an automated guided vehicle is considered. For the vehicles with a steering wheel, a new control strategy is proposed to determine the steering angle at each instant based on measured errors, the offset from the path and the deviation in orientation. The results of computer simulation of the dynamics of the system and comparisons with other proposed control policies are presented. It is illustrated that by implementing the new policy significant improvement in tracking capability can be achieved.

### 1. Introduction

An Automated Guided Vehicle (AGV) is a vehicle which is guided by underground wires carrying an electrical current at some frequency. Wires with different frequencies are used for defining different paths. A Wheeled Mobile Robot (WMR) is a similar vehicle with the exception of the guidance tool being a memory addressed map or a colour line on the floor, similar to an AGV, with different colours for different paths.

In both systems, the vehicle is supposed to accurately follow a desired path. In this sense, any deviation from the path, either in the form of an offset (being away from the path) or orientation (making an angle with the line tangent to the path) is undesirable, since both lead to the path being lost. A good controller, thus, becomes necessary for a stable and efficient performance and tracking ability.

Steering can be achieved either by steering wheel(s), usually in the front, or can be combined with the traction system by running two driving wheels at different speeds. This causes the vehicle to turn to right or left, as necessary, in addition to maintaining the desired forward speed.

In this paper, the first steering system is considered. For such a system it is necessary at each instant to control the angle of the steering wheel so that any deviation from the path is corrected in a stable manner and as fast as possible, and without oscillations about the path (hunting). The control strategy must define the steering angle (the angle of the steering wheel with the vehicle longitudinal axis) as a function of the two measurable errors hereafter referred to by offset (the positional error, the distance from the vehicle mass centre to the line tangent to the path at the point nearest to the mass centre) and orientation error (the angle between the vehicle longitudinal axis and the line tangent to the path).

The tracking stability and motion control of AGV's (or WMR's) have been the subject of numerous research works [2,7,9,11,15] based on the steering system, various control strategies have been used [1,12,13]. In cases where the steering wheel is in the front, it is well established that a control law based on a proportional feedback of the offset only is not sufficient and can lead in hunting rather than following the path. Inclusion of a derivative term in the control law provides a predictive element and results in improved tracking [13,16].

In order to study the effect of a control strategy by simulation it becomes necessary to formulate the dynamic equations of the vehicle. Numerous studies, conducted in vehicle system dynamics and models with various complexity and using sophisticated representation of the vehicle's tires, suspension, etc., have been published [4,5,10]. Also the concepts of friction circle and ellipse are introduced to represent the tire forces that are used in determining the necessary constraints to be imposed in order to follow a given path without slippage [5,8]. However, simplified models can be sufficient in describing the dynamic behaviour of the vehicle in plane motion [10,14,17], based on the assumption that for small side slip angles (less than 5 degrees) the side forces are proportional to the sideslip angles.

## 2. Vehicle Dynamics

Formulation of the dynamic equations of wheeled vehicles is complicated. Simple models have been proposed [6] while very complicated models are also available [4,5]. In this work, a simplified model that can adequately describe the dynamic behaviour of a vehicle is employed. It is assumed that the side slip angles of tires (angle between the path and the plane of tire) are small and, therefore, the side forces on each tire is proportional to its side slip angle.

Figure 1 shows the terms for various dimensions and angles between the coordinate system ucw attached to the vehicle at its mass centre and the global or world coordinate system xoy.  $v_u$  and  $v_w$  are the longitudinal and lateral components of the velocity  $v$  of the vehicle mass centre in the ucw frame, and  $v_x$  and  $v_y$  represent the components of  $v$  in the x- and y- direction, respectively. Similarly,  $a_u$ ,  $a_w$ ,  $a_x$  and  $a_y$  are used for components of the acceleration in the two coordinate frames.  $\psi$  is the orientation of the vehicle at each instant of time and  $\omega = \dot{\psi}$  is the angular velocity in both ucw and xoy. The positional offset and the orientational error are denoted by  $\epsilon_d$  and  $\epsilon_\theta$ , respectively, and finally  $\delta$  is the steering angle, i.e., the angle between the steering wheel and the vehicle longitudinal axis cu.

In this sense, the relationship for transformation of components of any vector like  $v$ , the velocity, in the two coordinate frame (when necessary) are in the form of

$$v_x = v_u \sin \psi + v_w \cos \psi \quad (1)$$

$$v_y = v_u \cos \psi - v_w \sin \psi \quad (2)$$

The schematic diagram of a three-wheeled vehicle considered in this study is depicted in Figure 2. As shown, the resultant of the forces exerted from tires to the vehicle have components in the longitudinal (u) and lateral (w) directions. In what follows, the subscripts R and L are used to refer to Right and Left (for rear tires), and the subscripts r and f represent rear and front, respectively. Thus  $(F_{wr})_L$  denotes, for instance, the lateral (w) components of the force (F) from rear (r) left (L) tire. However, the longitudinal components of the forces from the two rear tires are the same and represented by  $F_{ur}$ .

Considering only the plane motion of the system and neglecting rotations about cu and cw axes, and displacement normal to them, the equation of motion of the vehicle at any time can be written as follows:

$$2 F_{ur} + F_{uf} \cos \delta - F_{wf} \sin \delta = m a_u \quad (3)$$

$$(F_{wr})_L + (F_{wr})_R + F_{wf} \cos \delta + F_{uf} \sin \delta = m a_w \quad (4)$$

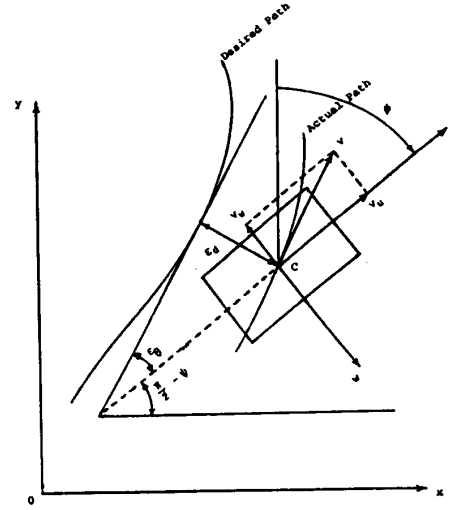


Fig. 1

$$-\left[(F_{wr})_R + (F_{wr})_L\right] b + (F_{wf} \cos \delta + F_{uf} \sin \delta) a = I \dot{\omega} \quad (5)$$

where a and b are the distances as shown in Fig. 2, I is the moment of inertia about a vertical axis passing through the centre of mass,  $a_u$  and  $a_w$  are the u- and w- components of the acceleration of point c, the mass centre, and  $\dot{\omega}$  is the angular acceleration.

It can be shown that  $a_u$  and  $a_w$  can be written in the form of

$$a_u = \dot{v}_u - v_w \omega \quad (6)$$

$$a_w = \dot{v}_w + v_u \omega \quad (7)$$

where  $v_u$  and  $v_w$  are the components of the velocity in the vehicles coordinate system [3]. Then the dynamic equations of the plane motion for the vehicle are,

$$m \dot{v}_u = m v_w \omega + 2 F_{ur} + F_{uf} \cos \delta - F_{wf} \sin \delta \quad (8)$$

$$m \dot{v}_w = -m v_u \omega + (F_{wr})_L + (F_{wr})_R + F_{wf} \cos \delta + F_{uf} \sin \delta \quad (9)$$

$$I \dot{\omega} = -\left[(F_{wr})_R + (F_{wr})_L\right] b + (F_{wf} \cos \delta + F_{uf} \sin \delta) a \quad (10)$$

The above equations have been used for the simulations carried out in this study. As it was mentioned earlier, it is assumed that the side slip angles are small and thus the corresponding side forces are determined from

$$F_{wf} = C_f \beta_f \quad (11)$$

and

$$(F_{wr})_{L,R} = C_r (\beta_r)_{L,R} \quad (12)$$

where  $\beta_f$ ,  $\beta_{rL}$  and  $\beta_{rR}$  are the slip angles for the front, rear left and rear right tires, respectively, and  $C_f$  and  $C_r$  are the cornering stiffness of the front and rear wheels, respectively. The slip angles for the front and rear tires can be determined as follows (Fig. 2):

$$\beta_f = \delta - \tan^{-1} \frac{v_w + a \omega}{v_u} \quad (13)$$

$$(\beta_r)_R = \tan^{-1} \frac{b \omega - v_w}{v_u - d \omega} \quad (14)$$

$$(\beta_r)_L = \tan^{-1} \frac{b \omega - v_w}{v_u + d \omega} \quad (15)$$

where  $2d$  is the distance between the rear tires.

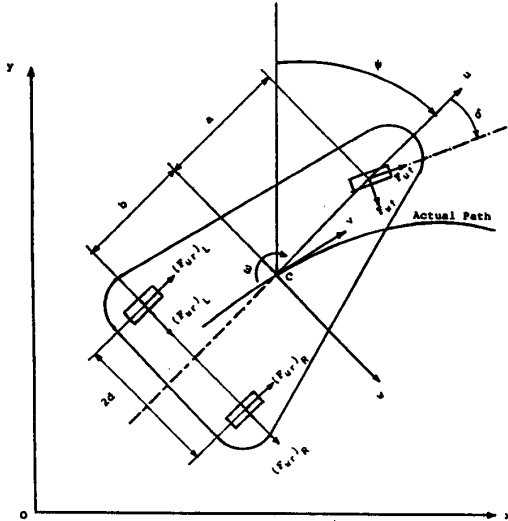


Fig. 2

### 3. Motion Control Policy

The two errors that arise in the process of tracking a trajectory are the offset and the orientation errors, denoted by  $\epsilon_d$  and  $\epsilon_\theta$ , respectively (See Figure 1). These two errors are measurable by sensors, either directly or indirectly depending on the system, the type and location of sensor(s). The measurement devices and the corresponding relationships, to find offset for the vehicle's mass centre, are not discussed in this paper. In the previous studies a linear proportional feedback of the errors in the form of

$$\delta = K_1 \epsilon_\theta + K_2 \epsilon_d \quad (16)$$

has been used for the steering angle.  $K_1$  and  $K_2$  are two constants the best values of which could be decided by trial and error. It has been shown that eliminating the first term can lead to undesirable oscillatory results and even instability. Also, omitting the second term leads to a steady state offset error.

In this work, a nonlinear feedback law in the form of

$$\delta = K_1 \tan^{-1} \frac{v \sin \epsilon_\theta + a \omega}{v \cos \epsilon_\theta} + K_2 \epsilon_d \quad (17)$$

is proposed. The expression is equation (17), in fact, makes use of the angular velocity of the vehicle  $\omega$ ; in other words, if this term is omitted, equation (16) will result. The inclusion of  $\omega$  in the feedback greatly improves the behaviour of a vehicle in tracking a trajectory, as it is shown in the simulation results in the next section.

### 4. Simulation Results

In this section, a control law in the form of equation (17) has been implemented to a three-wheeled vehicle. A tuning factor  $g$  has been also included such that the steering angle is determined according to

$$\delta = g \left[ K_1 \tan^{-1} \frac{v \sin \epsilon_\theta + a \omega}{v \cos \epsilon_\theta} + K_2 \epsilon_d \right] \quad (18)$$

The numerical values of the physical properties and other constants are given in appendix A. The forces on the tires to be included in the dynamic equations (8), (9) and (10) are determined from equations (11)-(15). Furthermore, in the simulation studies, it is assumed that the velocity of the vehicle is maintained, that is, it is assumed for simplicity of simulation that the traction power input takes care of keeping the velocity constant. Otherwise, we had to treat the problem, having an extra output, the velocity of the vehicle, to be controlled as well.

In this simulation, first the effect of changing the ratio between  $K_1$  and  $K_2$  is studied. Then, the effect of the tuning parameter  $g$  is investigated. For a number of initial conditions, that is, initial errors in the offset and orientation of the vehicle, the simulation studies are carried out. Figures 3 to 9 illustrate samples of the simulation results. The results of this study can be outlined as follows:

- 1 - The best ratio for values  $K_1$  and  $K_2$  coefficients is one ( $K_1 = K_2$ ). The most appropriate value for parameter  $g$  can then be determined based on any particular system's requirement. Taking the overall results, in the example used in this study a value of around 1 has shown to be more appropriate for a fixed gain. For larger values of  $g$  ( $g \geq 1.5$ ) the system's behaviour becomes oscillatory in both offset and orientation settling. In all the graphs shown,  $g = 1$  and  $K_1 = K_2 = 1$ .
- 2 - Comparison with the results of simulation when the steering angle is adjusted according to equation (16) reveals that much better performance is obtained when the new control law is implemented. The results of this comparison are shown in Figures 7, 8 and 9 for the offset, orientation error and the steering angle, respectively. In the simulation results shown, the initial offset ( $\epsilon_d$ ) is 0.2 m, the initial orientation error ( $\epsilon_\theta$ ) is 22.5 degrees ( $\pi/8$  Rad.) and the value of  $\delta$  is in radians. In Figures 7, 8, and 9, the continuous line corresponds to the control law in equation (17) and the curve shown by "\*" corresponds to the proportional control law in equation (16). For both  $K_1 = K_2 = 1$ .

### 5. Conclusion

For a three-wheeled vehicle with a front steering wheel, a control strategy to adjust the steering angle is proposed. A simplified dynamic model of the vehicle is used to study the effect of this control law by simulation on a digital computer. The simulation results indicate the excellent tracking capability of the controlled system and the superiority over the previous techniques.

### References

- [1] J. Borenstein and Y. Koren, "Motion Control Analysis of a Mobile Robot," ASME Journal of Dynamic Systems Measurement and Control, Vol. 109, June 1987, pp. 73-79.
- [2] J. Borenstein and Y. Koren, "A Mobile Platform for Nursing Robots," IEEE Transactions on Industrial Electronics, Vol. IE-32, No. 2, June 1985, pp. 158-165.
- [3] A.F. D'Souza and V.K. Garg, "Advanced Dynamics: Modeling and Analysis," Prentice Hall, 1984.
- [4] H. Dugoff and B.J. Brown, B.J., "Measurement of Tire Shear Forces," Paper No. 700092, SAE Transactions, 1970, pp 316-324.
- [5] H. Dugoff et al., "An Analysis of Tire Traction Properties and Their Influence on Vehicle Dynamic Performance," Paper No. 700377, SAE Transactions, 1970, pp 1219-1243.
- [6] J.R. Ellis, "Vehicle Dynamics," Business Books, London, 1969.
- [7] K. Fujiwara et al., "Development of Guideless Robot Vehicle," Proceedings of the 11th International Symposium on Industrial Robots, Tokyo, 1981, pp 195-202.
- [8] T.J. Grattinger and B.H. Krogh, "Evaluation and Time-Scaling of Trajectories for Wheeled Mobile Robots," ASME Journal of Dynamic Systems Measurement and Control, Vol. 111, June 1989, pp 222-231.
- [9] G. Hammond, "Automated Guided Vehicle Systems," IFS publications, Bedford, 1986.
- [10] H. Hatwal and E.C. Mikulcik, "Some Inverse Solutions to an Automobile Path-Tracking Problem with Input Control of Steering and Brakes," Vehicle System Dynamics, Vol. 15, 1986, pp 61-71.
- [11] R.H. Hollier, "Automated Guided Vehicle Systems," IFS Publications, Bedford, 1987.
- [12] M. Julliere et al., "A Guidance System for a Mobile Robot," Proceedings of the 13th ISIR/Robots 7 Conference, Chicago, April 1983, pp 13.58-13.68.
- [13] M.H.E. Larcombe, "Tracking Stability of Wire Guided Vehicles," Proceedings of the 1st International Conference on Automated Guided Vehicle Systems, Stratford-upon-Avon, U.K., June 1981, pp 137-144.
- [14] T. Legouis et al., "Characterization of Dynamic Vehicle Stability Using Two Models of the Human Pilot Behaviour," Vehicle System Dynamics, Vol. 15, 1986, pp 1-18.
- [15] H.P. Moravec, "The Stanford Cart and the CMU Rover," Proceedings of the IEEE, Vol. 71, No. 7, July 1983, pp 872-884.
- [16] T. Tsumura et al., "An Experimental System for Automatic Guidance of Robot Vehicle Following the Route Stored in Memory," Proceedings of the 11th International Symposium on Industrial Robots, Tokyo, 1981, pp 187-193.
- [17] J.Y. Wong, "Theory of Ground Vehicles," John Wiley & Sons, N.Y., 1978.

# Appendix A

## Values of Vehicle Characteristics Employed in Simulation Study

Symbol	Name	Value
a	Distance of c.g. to the front wheel axis	4.5 (ft)
b	Distance of c.g. to the rear wheel axis	5.5 (ft)
I	Yaw moment of inertia	3000 slug-ft <sup>2</sup>
C <sub>f</sub> , C <sub>r</sub>	Cornering stiffness	6000 lb/rad
m	Mass	124 (slug)
d	Distance between two rear wheels	2.5 (ft)

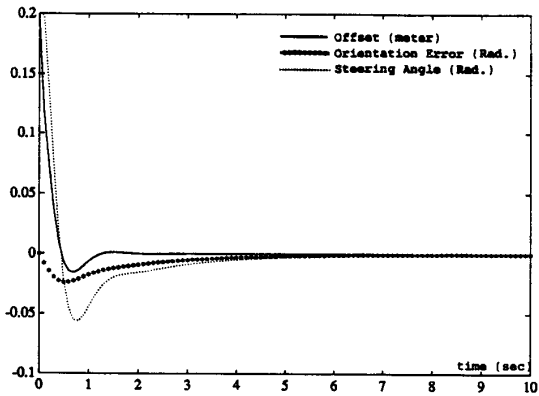


Fig. 3. Initial Offset = 0.2 m, Initial Orientation Error = 0

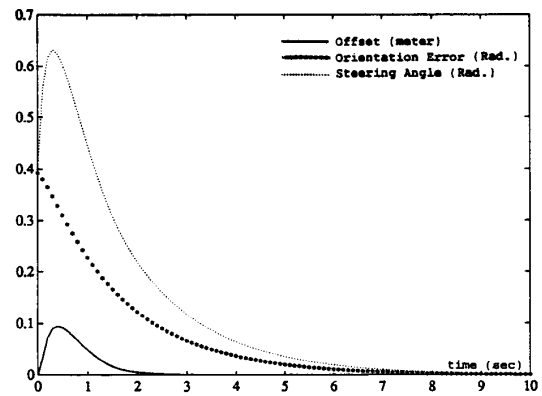


Fig. 4. Initial Offset = 0, Initial Orientation Error =  $\pi/8$

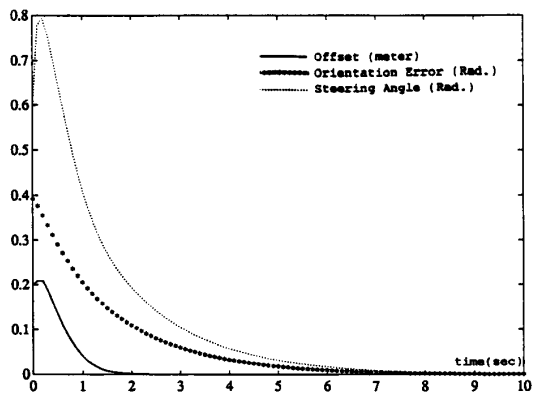


Fig. 5. Initial Offset = 0.2 m, Initial Orientation Error =  $\pi/8$

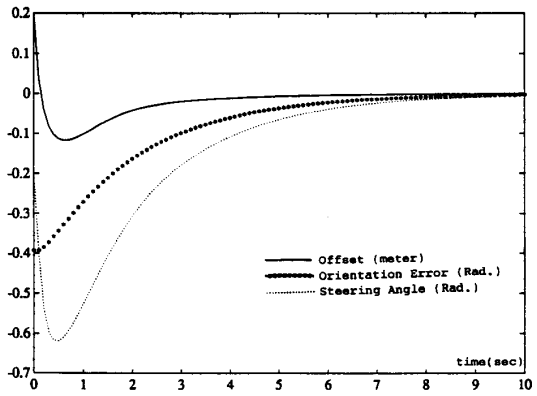


Fig. 6. Initial Offset = 0.2 m, Initial Orientation Error =  $-\pi/8$

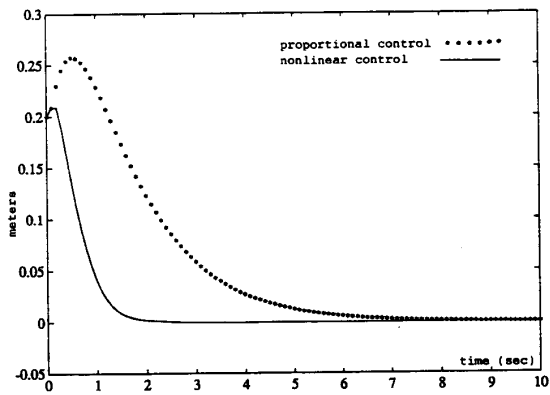


Fig. 7. Comparison of Methods: Variation of Offset for the initial conditions  $\epsilon_d = .2$ ,  $\epsilon_\theta = \pi/8$

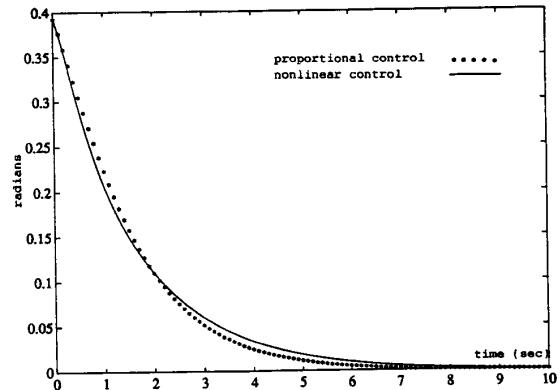


Fig. 8. Comparison of Methods: Variation of Orientation Error for the initial condition  $\epsilon_d = .2$ ,  $\epsilon_\theta = \pi/8$

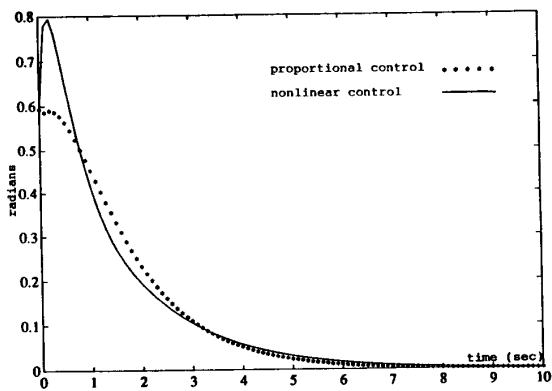


Fig. 9. Comparison of Methods: variation of steering angle for the initial condition  $\epsilon_d = .2$ ,  $\epsilon_\theta = \pi/8$