Robust Adaptive Control of Nonholonomic Wheeled Mobile Robot

Hyun-Sik Shim   Jong-Hwan Kim*

Abstract— **This paper proposes a robust adaptive control algorithm for non-holonomic mobile robot locomotion based upon Lyapunov direct method. In this scheme, if the velocity error is greater than the prespecified error bound, the feedback gains are adjusted adaptively such that the predetermined tracking error precision is achieved under disturbances. If there exist no disturbances, proposed algorithm guarantees global convergence of overall system and sero tracking velocity error. And also, to improve the position tracking performance, position error feedback term is added to the control law. Computer simulations demonstrate the effectiveness of the scheme.**

I. Introduction

In this paper, robust adaptive control algorithm for stable locomotion tracking control of the mobile robot is proposed based upon Lyapunov direct method. The dynamic model using this paper is derived under some assumptions. Provided that the assumptions are not satisfied(e.g. flexible tire, existing slippage, traveling mound or rough ground, friction on motor, etc.), the model may be unreliable. In this case, we consider those as disturbances. The mass or center of gravity of the mobile robot like loader can be changed while doing a task. Considering such points as the existing disturbances and the model parameter variations, two robust adaptive controller are designed to accomplish stable tracking control of the robot. Firstly, robust adaptive control algorithm for velocity tracking is derived. If the velocity error is greater than the prespecified error bound, the feedback gains are adjusted such that the predetermined velocity tracking error precision is achieved under disturbances. And also, if there exist no disturbances, the algorithm guarantees zero tracking velocity error and global convergence of the overall adaptive system. However, it may not guarantee zero tracking posture error or its boundedness. So, secondly, posture error feedback term is added to the control law with only velocity feedback term to improve the posture tracking performance. Computer simulation illustrates the effectiveness of the proposed scheme.

II. Kinematics and Dynamics

A. Kinematics

There is a mobile robot which is located on a 2D plane in which a global cartesian coordinate system is defined. There are global coordinate X-Y and local coordinate x-y attached on the center of robot body as in Figure 1. The robot in the world possesses three degrees of freedom in its relative positioning which are represented by a *posture* $\mathbf{p}$,

$$\mathbf{p} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \tag{1}$$

where the heading direction θ is taken counterclockwise from the X-axis to x-axis. The angle θ denotes the orientation of the vehicle or the wheels. Since the robot has a locomotion capability in the plane, the posture $\mathbf{p}$ is in fact a function of t. The entire locus of the point$(x(t), y(t))$ is called a *path trajectory*. In this control system, two postures are used: a *reference posture* $\mathbf{p}_r = (x_r\ y_r\ \theta_r)^T$ and a *current posture* $\mathbf{p}_c = (x_c\ y_c\ \theta_c)^T$. The reference posture is a goal posture of the robot and the current posture is its real posture at the moment.

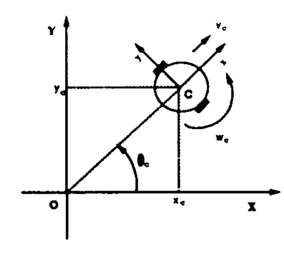

Fig. 1. Coordinate of mobile robot($\mathbf{p}_c$ is current posture.)

The robot's motion is controlled by its *linear velocity* v and *angular velocity* ω at the center of the robot, which are also functions of time. The robot's kinematics is defined by a Jacobian matrix $J(\theta)$:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \dot{\mathbf{p}} = \mathrm{J}(\theta)\mathbf{q} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} \tag{2}$$

where $\mathbf{q} = (v\ \ \omega)^T$. This equation can be applied to all kinds of vehicles except for omni-directional vehicles.

We define a *robot based error posture* $\mathbf{p}_{ec}$, which is a transformation of the posture $(\mathbf{p}_r - \mathbf{p}_c)$ with an origin in

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a local coordinate and a direction of θ_c:

$$\mathbf{p}_{ec} = \begin{pmatrix} x_{ec} \\ y_{ec} \\ \theta_{ec} \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} (\mathbf{p}_r - \mathbf{p}_c) \quad (3)$$

where x_{ec} represents the projection of the vector from current x_c to reference x_r into x_c axis and y_{ec} represents the projection of the vector from current y_c to reference y_r into y_c axis, respectively.

The derivative of error posture, i.e., velocity of error posture can be described in the following lemma similarly as in[1].

Lemma 1

$$\dot{\mathbf{p}}_{ec} = f(\mathbf{p}_{ec}, \mathbf{q}_r, \mathbf{q}_c) = \begin{pmatrix} y_{ec}\omega_c + v_c - v_r\cos\theta_{ec} \\ -x_{ec}\omega_c + v_r\sin\theta_{ec} \\ \omega_{ec} \end{pmatrix} \quad (4)$$

Proof : Using (3) and an equality $\dot{x}\sin\theta = \dot{y}\cos\theta$ from (2), this lemma can be derived by similiar method to that in Kanayama[1]. ∎

B. Dynamics

Let us consider a wheeled mobile robot having two drive wheels and an auxiliary wheel shown in Figure 2. The auxiliary wheel is a kind of caster and its radius is smaller than the drive wheel's. The two drive wheels are cylindrical shape having same radius and mass. For practical application, it is considered that all the mass of the robot except wheel's mass is concentrated on G, not on C.

We introduce the following four practical assumptions to make the modeling problem tractable:

1. The robot does not contain flexible parts.
2. All steering axes are perpendicular to the ground.
3. The robot moves on a planar ground.
4. The contact between wheels and the ground satisfies the condition of pure rolling and non-slipping.

Provided that assumptions are not satisfied(e.g., flexible tire, existing slippage, traveling mound or rough ground, etc.), those phenomena are considered as disturbances or noises.

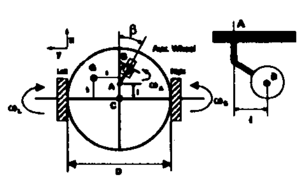

Fig. 2. Wheeled mobile robot model

Using the Lagrangian fomalism, dynamic equation can be derived easily(See [2][5].) Derived dynamic equation of the mobile robot is represented as follow:

$$\begin{cases} \tau = \mathbf{H}(\beta)\dot{\boldsymbol{\Omega}} + \mathbf{G}(\beta, \boldsymbol{\Omega}) \\ v = \frac{r}{2}(\omega_R + \omega_L) \\ \omega = \frac{r}{D}(\omega_R - \omega_L) \\ \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \\ \dot{\beta} = \frac{1}{d}(\sin\beta v - (d + l\cos\beta)\omega) \end{cases} \quad (5)$$

where

$\boldsymbol{\Omega}$:	drive wheel angular velocity,	$\boldsymbol{\Omega} \in R^{2\times1}$
τ :	applied torque,	$\tau \in R^{2\times1}$
$\mathbf{H}$:	inertia momentum matrix,	$\mathbf{H} \in R^{2\times2}$
$\mathbf{G}$:	coriolis and centripetal force term,	$\mathbf{G} \in R^{2\times1}$,

$\boldsymbol{\Omega}$ is $(\omega_L\ \omega_R)^T$ and τ is $(\tau_L\ \tau_R)^T$. ω_L, ω_R, τ_L and τ_R are left ,right angular velocity, applied left and right torque, respectively.

Above dynamic equation is similar to the dynamic equation of robot manipulator and each term in the equation has similar physical meaning. It can be considered that H represents inertia moment matrix and G represents the coriolis and centripetal terms. The difference is that it has no gravity term due to Assumption 2.

III. Robust adaptive control of mobile robot

The inputs to the robust adaptive controller are the desired trajectories of posture and velocity and its output is input torque τ to the mobile robot as Figure 3 shows. In the figure, MR represents the mobile robot model derived in the previous section as a dynamic equation. Although it is difficult to produce desired trajectories because of non-holonomic problem [4], we deal with controller design under situation that predesigned path generator produces continuous desired trajectories $\mathbf{p}_r$, $\boldsymbol{\Omega}_r$ and $\dot{\boldsymbol{\Omega}}_r$(*or* $\mathbf{q}_r$ and $\dot{\mathbf{q}}_r$).

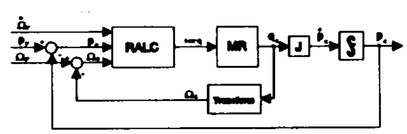

Fig. 3. Control structure using robust adaptive control(subscript 'c' is a "current" and 'r' is a "reference".)

A. Robust adaptive velocity tracking control

The derived dynamic equation has some unconsidered factors, e.g., changing mass of robot and inertia moment due to moving object in robot, friction, parameter uncertainties, etc., in real world. Under these circumstances, robust adaptive controller is designed to achieve stable tracking control of mobile robot.

It is assumed that the velocity $\boldsymbol{\Omega}$ is measurable by encoder or tacometer, the matrices $\mathbf{H}$ and vector $\mathbf{G}$ are known structure and their physical parameters are bounded. The desired trajectory is represented by bounded time functions in terms of posture trajectory $\mathbf{p}_r$, velocity trajectory $\boldsymbol{\Omega}_r$ and acceleration trajectory $\dot{\boldsymbol{\Omega}}_r$.

Let us define drive wheel angular velocity error, drive angular acceleration error and robot's velocity error as

$$\boldsymbol{\Omega}_e = \boldsymbol{\Omega}_c - \boldsymbol{\Omega}_r$$
$$\dot{\boldsymbol{\Omega}}_e = \dot{\boldsymbol{\Omega}}_c - \dot{\boldsymbol{\Omega}}_r$$
$$\mathbf{q}_e = \mathbf{q}_c - \mathbf{q}_r$$

and robot based error posture $\mathbf{p}_{ec}$ as a posture error.

The control objective is to make $\mathbf{p}_{ec}$ and $\boldsymbol{\Omega}_e(\mathbf{q}_e)$ zero in disturbance free case or to maintain $\mathbf{p}_{ec}$ and $\boldsymbol{\Omega}_e$ within boundaries in finite time under bounded disturbances.

A control law is given as the following equation:

$$\begin{aligned} \tau &= \mathbf{H}(\beta)\dot{\boldsymbol{\Omega}}_r + \mathbf{G}(\beta,\boldsymbol{\Omega}) - \frac{1}{2}\dot{\mathbf{H}}(\beta)\boldsymbol{\Omega}_e - K(t)\boldsymbol{\Omega}_e \\ &= \mathbf{W}(\beta,\boldsymbol{\Omega},\boldsymbol{\Omega}_e,\dot{\boldsymbol{\Omega}}_r)\boldsymbol{\Theta} - K(t)\boldsymbol{\Omega}_e \end{aligned} \quad (6)$$

where,

$$K(t) = \begin{pmatrix} k(t) & 0 \\ 0 & k(t) \end{pmatrix}$$

$\mathbf{W}$ and Θ are known function and unknown paramater, respectively. we can get a linear regressor form as follows:

$$\mathbf{H}(\beta)\dot{\boldsymbol{\Omega}}_r + \mathbf{G}(\beta,\boldsymbol{\Omega}) - \frac{1}{2}\dot{\mathbf{H}}(\beta)\boldsymbol{\Omega}_e = \mathbf{W}(\beta,\boldsymbol{\Omega},\boldsymbol{\Omega}_e,\dot{\boldsymbol{\Omega}}_r)\boldsymbol{\Theta}$$

where

$\boldsymbol{\Theta}$:	unknown parameter vector,	$\boldsymbol{\Theta} \in R^{p\times 1}$
$\mathbf{W}$:	matrix constructed by known function,	$\mathbf{W} \in R^{2\times p}$

Since we cannot know parameter true values of the system, $\hat{\boldsymbol{\Theta}}$ is used in the above control law instead of $\boldsymbol{\Theta}$ as follows.

$$\begin{aligned} \tau &= \hat{\mathbf{H}}\dot{\boldsymbol{\Omega}}_r + \hat{\mathbf{G}} - \frac{1}{2}\hat{\dot{\mathbf{H}}}\boldsymbol{\Omega}_e - K(t)\boldsymbol{\Omega}_e \\ &= \mathbf{W}(\beta,\boldsymbol{\Omega}_e,\dot{\boldsymbol{\Omega}}_r)\hat{\boldsymbol{\Theta}} - K(t)\boldsymbol{\Omega}_e \end{aligned} \quad (7)$$

where $\hat{\mathbf{H}}$, $\hat{\mathbf{G}}$ and $\hat{\dot{\mathbf{H}}}$ are estimation matrix of $\mathbf{H}$, $\mathbf{G}$ and $\dot{\mathbf{H}}$, respectively. Substituting equation (7) into equation (5), the following error equation is obtained:

$$\mathbf{H}\dot{\boldsymbol{\Omega}}_e = \tilde{\mathbf{H}}\dot{\boldsymbol{\Omega}}_r + \tilde{\mathbf{G}} - \frac{1}{2}\tilde{\dot{\mathbf{H}}}\boldsymbol{\Omega}_e - K(t)\boldsymbol{\Omega}_e \quad (8)$$

where $\tilde{\mathbf{H}} = \hat{\mathbf{H}} - \mathbf{H}$, $\tilde{\mathbf{G}} = \hat{\mathbf{G}} - \mathbf{G}$ and $\tilde{\dot{\mathbf{H}}} = \hat{\dot{\mathbf{H}}} - \dot{\mathbf{H}}$.

Practical systems have measurement errors, parameter uncertainties, model uncertainties, etc. These can be considered as input disturbances. Denoting the disturbances as a vector function $\nu(t) \in R^{2\times 1}$, real applied input is represented as follows:

$$\tau^* = \tau + \nu(t)$$

Substituting above equation into equation (5), the following error equation is obtained as follows.

$$\mathbf{H}\dot{\boldsymbol{\Omega}}_e = \tilde{\mathbf{H}}\dot{\boldsymbol{\Omega}}_r + \tilde{\mathbf{G}} - \frac{1}{2}\tilde{\dot{\mathbf{H}}}\boldsymbol{\Omega}_e - K(t)\boldsymbol{\Omega}_e + \nu(t). \quad (9)$$

We assume that there exists a bounded positive constant α which satisfies the following relation for any nonzero 2×1 vector $\boldsymbol{\Omega}_e$:

$$\text{A:} \qquad \|\nu(t)\boldsymbol{\Omega}_e\| \le \alpha\|\boldsymbol{\Omega}_e\|$$

The unknown parameter α is estimated by a parameter adaptation algorithm and the estimates of α is denoted as $\hat{\alpha}$.

Let us consider the following parameter adaptation algorithm estimating unknown parameters:

$$\dot{\hat{\boldsymbol{\Theta}}} = -\Gamma\mathbf{W}(\beta,\boldsymbol{\Omega},\boldsymbol{\Omega}_e,\dot{\boldsymbol{\Omega}}_r)^T\boldsymbol{\Omega}_e \quad (10)$$

$$\dot{\hat{\alpha}} = \begin{cases} w\|\boldsymbol{\Omega}_e\|, & \text{if } \|\boldsymbol{\Omega}_e\| > \epsilon \\ 0, & \text{if } \|\boldsymbol{\Omega}_e\| \le \epsilon \end{cases} \quad (11)$$

$$k(t) = \frac{\hat{\alpha}}{\epsilon} \quad (12)$$

where ϵ is a positive constant, $\Gamma = \mathrm{diag}(\gamma_1\ \gamma_2\ \cdots\ \gamma_p)$, $\gamma_i > 0$, w is a positive constant. The following theorem is derived from the above parameter adaptation algorithm and control law.

Theorem 1 *Consider the mobile robot given by (5) under the assumption A with the control law (7) and the adaptation algorithms (10) (11)(12) : If $\nu(t) = 0$, velocity error go to zero as $t \to \infty$. And if $\nu(t) \neq 0$, there exists a finite time T such that*

$$\|\boldsymbol{\Omega}_e(t)\| \le \epsilon^* \quad \textit{for all } t > T$$

where ϵ^* is a constant satisfying $\epsilon^* > \epsilon$.

Proof: Let us choose the Lyapunov function candidate as follows:

$$V(t) = \frac{1}{2}\boldsymbol{\Omega}_e^T\mathbf{H}\boldsymbol{\Omega}_e + \frac{1}{2}\tilde{\boldsymbol{\Theta}}^T\Gamma^{-1}\tilde{\boldsymbol{\Theta}} + \frac{1}{2w}(\alpha - \hat{\alpha})^2 \quad (13)$$

The time derivative of $V(t)$ is as follows:

$$\dot{V}(t) = \boldsymbol{\Omega}_e^T\mathbf{H}\dot{\boldsymbol{\Omega}}_e + \frac{1}{2}\boldsymbol{\Omega}_e^T\dot{\mathbf{H}}\boldsymbol{\Omega}_e + \tilde{\boldsymbol{\Theta}}^T\Gamma^{-1}\dot{\tilde{\boldsymbol{\Theta}}} - \frac{1}{w}(\alpha - \hat{\alpha})\dot{\hat{\alpha}}$$

Substituting the error equation (9) in the above equation instead of $\mathbf{H}\dot{\boldsymbol{\Omega}}_e$, $\dot{V}$ is represented as follows.

$$\begin{aligned} \dot{V}(t) &= \boldsymbol{\Omega}_e^T(\tilde{\mathbf{H}}\dot{\boldsymbol{\Omega}}_r + \tilde{\mathbf{G}} - \frac{1}{2}\tilde{\dot{\mathbf{H}}}\boldsymbol{\Omega}_e + \frac{1}{2}\dot{\mathbf{H}}\boldsymbol{\Omega}_e - K(t)\boldsymbol{\Omega}_e \\ &\quad + \nu(t)) + \tilde{\boldsymbol{\Theta}}^T\Gamma^{-1}\dot{\tilde{\boldsymbol{\Theta}}} - \frac{1}{w}(\alpha - \hat{\alpha})\dot{\hat{\alpha}} \\ &= -\boldsymbol{\Omega}_e^TK(t)\boldsymbol{\Omega}_e + \tilde{\boldsymbol{\Theta}}^T(\mathbf{W}(\beta,\boldsymbol{\Omega},\dot{\boldsymbol{\Omega}}_r,\boldsymbol{\Omega}_e)^T\boldsymbol{\Omega}_e + \Gamma^{-1}\dot{\tilde{\boldsymbol{\Theta}}}) \\ &\quad - \frac{1}{w}\alpha\dot{\hat{\alpha}} + \frac{1}{w}\hat{\alpha}\dot{\hat{\alpha}} + \boldsymbol{\Omega}_e^T\nu(t) \\ &= -\boldsymbol{\Omega}_e^TK(t)\boldsymbol{\Omega}_e - \frac{1}{w}\alpha\dot{\hat{\alpha}} + \frac{1}{w}\hat{\alpha}\dot{\hat{\alpha}} + \boldsymbol{\Omega}_e^T\nu(t) \end{aligned}$$

Let us consider the above equation in case of three case.
Case 1) if $\nu(t) = 0$ and $\|\boldsymbol{\Omega}_e\| > \epsilon$

In this case, $\dot{V}(t)$ represented as follows has a negative value.

$$\begin{aligned} \dot{V}(t) &= -\mathbf{\Omega}_e^T K(t)\mathbf{\Omega}_e - \frac{1}{w}\alpha\dot{\hat{\alpha}} + \frac{1}{w}\hat{\alpha}\dot{\hat{\alpha}} \\ &= -\|\mathbf{\Omega}_e\|^2\frac{\hat{\alpha}}{\epsilon} - \alpha\|\mathbf{\Omega}_e\| + \hat{\alpha}\|\mathbf{\Omega}_e\| < 0\,. \end{aligned}$$

Case 2) if $\nu(t) = 0$ and $\|\mathbf{\Omega}_e\| \leq \epsilon$
In this case, $\dot{V}(t)$ is a function of only $\mathbf{\Omega}_e$ term. So velocity error converges to zero.

$$\dot{V}(t) = -\mathbf{\Omega}_e^T K(t)\mathbf{\Omega}_e < 0$$

That is, in case of $\nu(t) == 0$, it is shown that the tracking error converges to zero.
Case 3) if $\nu(t) \neq 0$, from assumption A, we get

$$\begin{aligned} \dot{V}(t) &= -\mathbf{\Omega}_e^T K(t)\mathbf{\Omega}_e - \frac{1}{w}\alpha\dot{\hat{\alpha}} + \frac{1}{w}\hat{\alpha}\dot{\hat{\alpha}} + \mathbf{\Omega}_e\nu(t) \\ &\leq -\mathbf{\Omega}_e^T K(t)\mathbf{\Omega}_e - \frac{1}{w}\alpha\dot{\hat{\alpha}} + \frac{1}{w}\hat{\alpha}\dot{\hat{\alpha}} + \alpha\|\mathbf{\Omega}_e\| \\ &= \begin{cases} -\mathbf{\Omega}_e^T K(t)\mathbf{\Omega}_e + \frac{1}{w}\hat{\alpha}\dot{\hat{\alpha}} < 0, & \text{if } \|\mathbf{\Omega}_e\| > \epsilon \\ -\mathbf{\Omega}_e^T K(t)\mathbf{\Omega}_e + \alpha\|\mathbf{\Omega}_e\|, & \text{otherwise} \end{cases} \end{aligned}$$

Therefore, $\dot{V}(t)$ has a bounded value for any t and $\hat{\alpha}$ and $\mathbf{\Omega}_e$ is also bounded.

$\mathbf{\Omega}_e$ is continuous for time t and its time derivative is bounded. Then, since α is also bounded, $\mathbf{\Omega}_e$ satisfies Lipschitz condition for time t. So the following relation satisfies for bounded time T.

$$\|\mathbf{\Omega}_e\| < \epsilon, \quad \text{for all } t \geq T \quad \blacksquare$$

From the above theorem, although we can show $\mathbf{\Omega}_e \to 0$ as $t \to \infty$, we cannot guarantee to make $\mathbf{p}_{ec}$ zero and that $\mathbf{p}_{ec}$ is bounded. So, in next subsection, we add compensative posture error term to this control law.

B. Robust Adaptive Control Algorithm with posture feedback

We propose a robust adaptive control law with posture feedback to track both the desired posture and velocity.

Adding a new term to the previous control law, the following equation is given:

$$\begin{aligned} \tau &= \mathbf{H}\dot{\mathbf{\Omega}}_r + \mathbf{G} - \frac{1}{2}\dot{\mathbf{H}}\mathbf{\Omega}_e - K(t)\mathbf{\Omega}_e + X \\ &= \mathbf{W}(\beta, \mathbf{\Omega}, \mathbf{\Omega}_e, \dot{\mathbf{\Omega}}_r)\Theta - K(t)\mathbf{\Omega}_e + X \end{aligned} \tag{14}$$

where,

$$X = \begin{pmatrix} k_x & -k_y & -k_\theta \\ k_x & k_y & k_\theta \end{pmatrix} \mathbf{p}_{ec}$$

$$\begin{aligned} k_x &= \frac{-v_r\cos\theta_{ec} + v_c - k(t)x_{ec}}{\omega_e + v_e} \\ k_y &= \frac{v_r\sin\theta_{ec} + k(t)y_{ec}}{v_e - \omega_e} \\ k_\theta &= \frac{\omega_e + k(t)\theta_{ec}}{v_e - \omega_e} \end{aligned}$$

Let us consider the parameter adaptation algorithm estimating unknown parameters as follows.

$$\dot{\hat{\Theta}} = -\Gamma\mathbf{W}(\beta, \mathbf{\Omega}, \mathbf{\Omega}_e, \dot{\mathbf{\Omega}}_r)^T\mathbf{\Omega}_e \tag{15}$$

$$\dot{\hat{\alpha}} = \begin{cases} w\|\mathbf{\Omega}_e\| + w\|\mathbf{p}_{ec}\|, & \text{if } \|\mathbf{\Omega}_e\| > \epsilon,\ \|\mathbf{p}_{ec}\| > \epsilon \\ w\|\mathbf{\Omega}_e\|, & \text{if } \|\mathbf{\Omega}_e\| > \epsilon,\ \|\mathbf{p}_{ec}\| \leq \epsilon \\ w\|\mathbf{p}_{ec}\|, & \text{if } \|\mathbf{\Omega}_e\| \leq \epsilon,\ \|\mathbf{p}_{ec}\| > \epsilon \\ 0, & \text{if } \|\mathbf{\Omega}_e\| \leq \epsilon,\ \|\mathbf{p}_{ec}\| \leq \epsilon \end{cases} \tag{16}$$

$$k(t) = \frac{\hat{\alpha}}{\epsilon} \tag{17}$$

where ϵ is positive constant.

Then, if $\nu(t) = 0$, we can show $\dot{V}(t) < 0$ for all $\mathbf{\Omega}_e \neq 0, \mathbf{p}_{ec} \neq 0$.

We can get the following theorem using above equations.

Theorem 2 *Consider the mobile robot given by (5) subject to assumption A with the control law (14) and the adaptation algorithms (15) (16)(17): If $\nu(t) = 0$, posture error and velocity error go to zero as $t \to \infty$. And if $\nu(t) \neq 0$, there exists a finite time t_f such that*

$$\|\mathbf{p}_{ec}(t)\| \leq \epsilon^*, \ \|\mathbf{\Omega}_e(t)\| \leq \epsilon^* \quad \textit{for all} \quad t \geq t_f$$

where ϵ^* is a constant satisfying $\epsilon^* > \epsilon$.

proof: It can be easily proved from the above result. $\blacksquare$

The problem, that is, the control law with only velocity feedback does not satisfy to track the desired velocity and posture, is solved by Theorem 2. Although disturbance exists in system, we can limit both the magnitude of the robot's velocity and posture error within a certain allowable value.

IV. Computer Simulation

We simulate the proposed control law to demonstrate its effectiveness. In simulation program, kinematics and dynamics described in Section 2 are used as a mobile robot model. Specification of the model is shown in Table 1. We assume that unknown parameters are mass of body, position of center of mass, inertia of robot body. The number of estimated parameters related to the above terms is four and those initial values are $M = 2.5$, $M(a^2 + b^2) = 0.0$, $I_G = 0.0$ and $Mb = 0.0$. Figure 4 and 5 show simulation results of posture and velocity error norms using control law (7) in cases of no disturbance and under disturbance which is random noise of mean 0.01 and variance 0.01. Velocity error converges to zero asymptotically in cases of no disturbance or remain within bounded value under noise. But posture errors remains within bounded value or have diverging values. Figure 6 and 7 show simulation results of posture and velocity error norms using control law (14) in cases of no disturbance and under disturbance which is random noise of mean 0.01 and variance 0.01. Using control law (14), posture errors as well as velocity errors converge to zero asymptotically or remain within bounded values in contrast to control law (7)

V. Conclusion

In this paper, we have a proposed robust adaptive control of wheeled mobile robot to track the desired trajectories. Firstly, a robust adaptive control algorithm for velocity tracking is derived. If the velocity error is greater than the prespecified error bound, the feedback gains are adjusted such that the predetermined velocity tracking error precision is achieved under disturbances. And also, if there exist no disturbances, the algorithm guarantees zero tracking velocity error and global convergence of the overall adaptive system. However, it may not guarantee zero tracking posture error or its boundedness. So, a posture error feedback term is added to the control law with only velocity feedback term to improve the posture tracking performance. Then, although disturbance exists in the system, we can limit both the magnitude of the robot's velocity and posture error within the certain allowable value.

Table 1. Parameter values used in simulation

Notation	value	explanation
M	20 Kg	mass of body
D	0.5 m	diameter of body
l	0.25 m	position of auxiliary wheel
d	0.05 m	length of auxiliary wheel rod
r	0.07 m	radius of drive wheel
r_A	0.02 m	radius of auxiliary wheel
m	0.2 Kg	mass of drive whee
m_A	0.1 Kg	mass of auxiliary wheel
a	0.05 m	x position of center of mass
b	0.05 m	y position of center of mass

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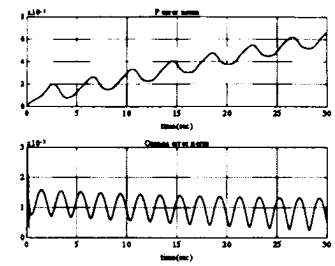

Fig. 4. Norms of velocity error and posture error using control law with only velocity feedback term

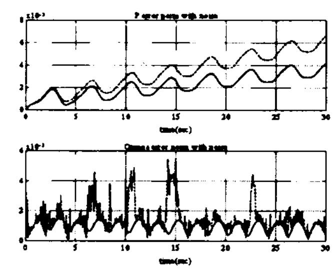

Fig. 5. Norms of velocity error and posture error using control law with only velocity feedback term under random noise(mean 0.01 and variance 0.01:dashed line is a case of disturbance.)

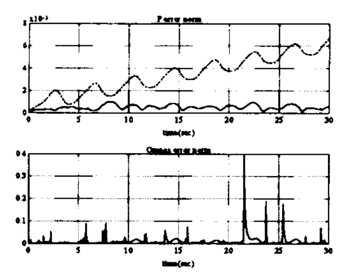

Fig. 6. Norms of velocity error and posture error using control law with posture feedback term (dashed line is a result of control law(7).)

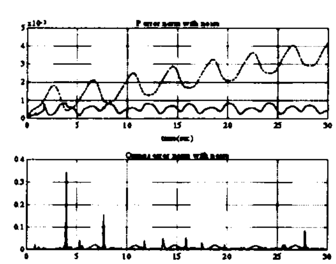

Fig. 7. Norms of velocity error and posture error using control law with posture feedback term under random noise(mean 0.01 and variance 0.01: dashed line is a result of control law(7).)