NON-LINEAR CONTROL OF WHEELED MOBILE ROBOTS

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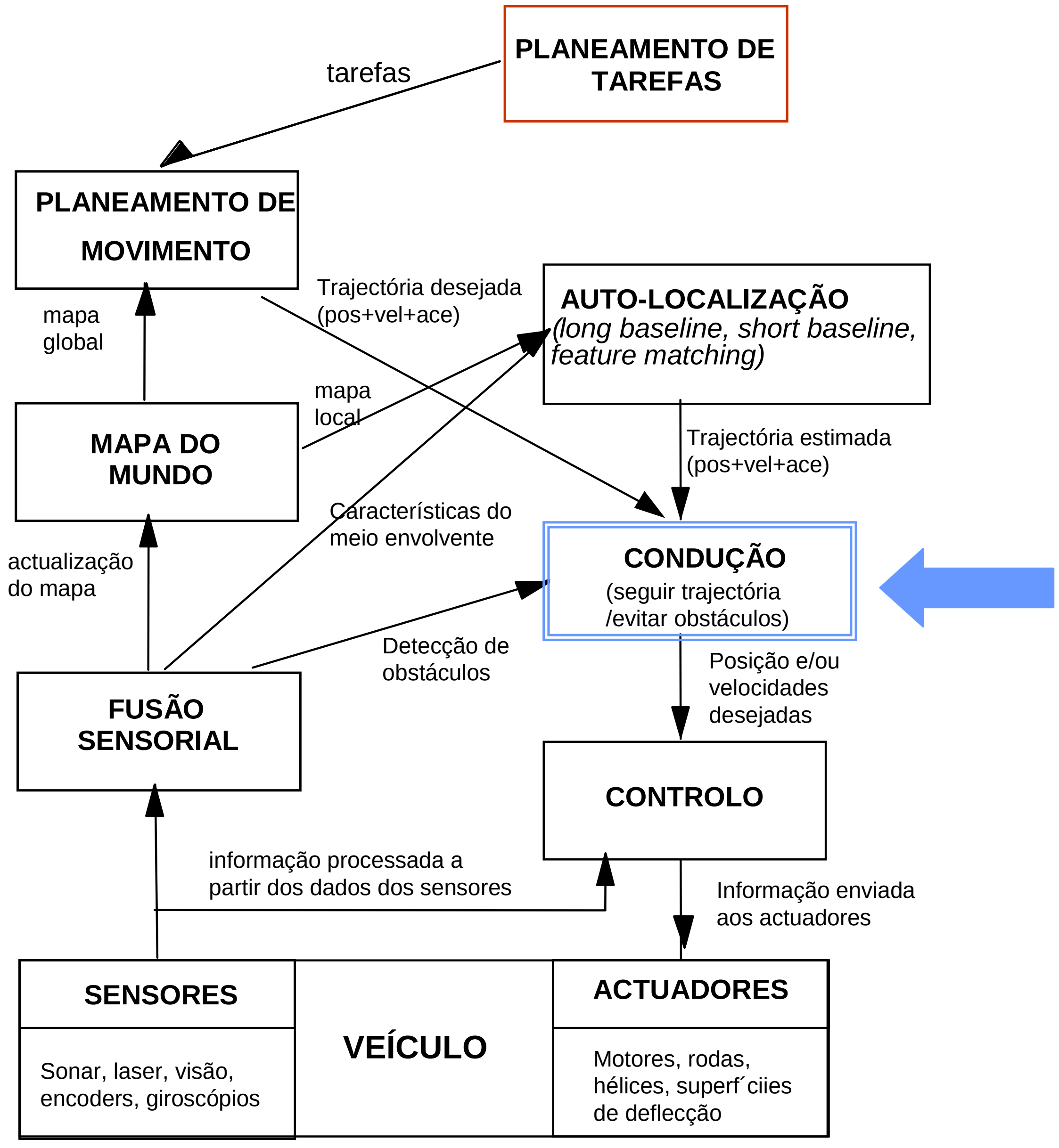
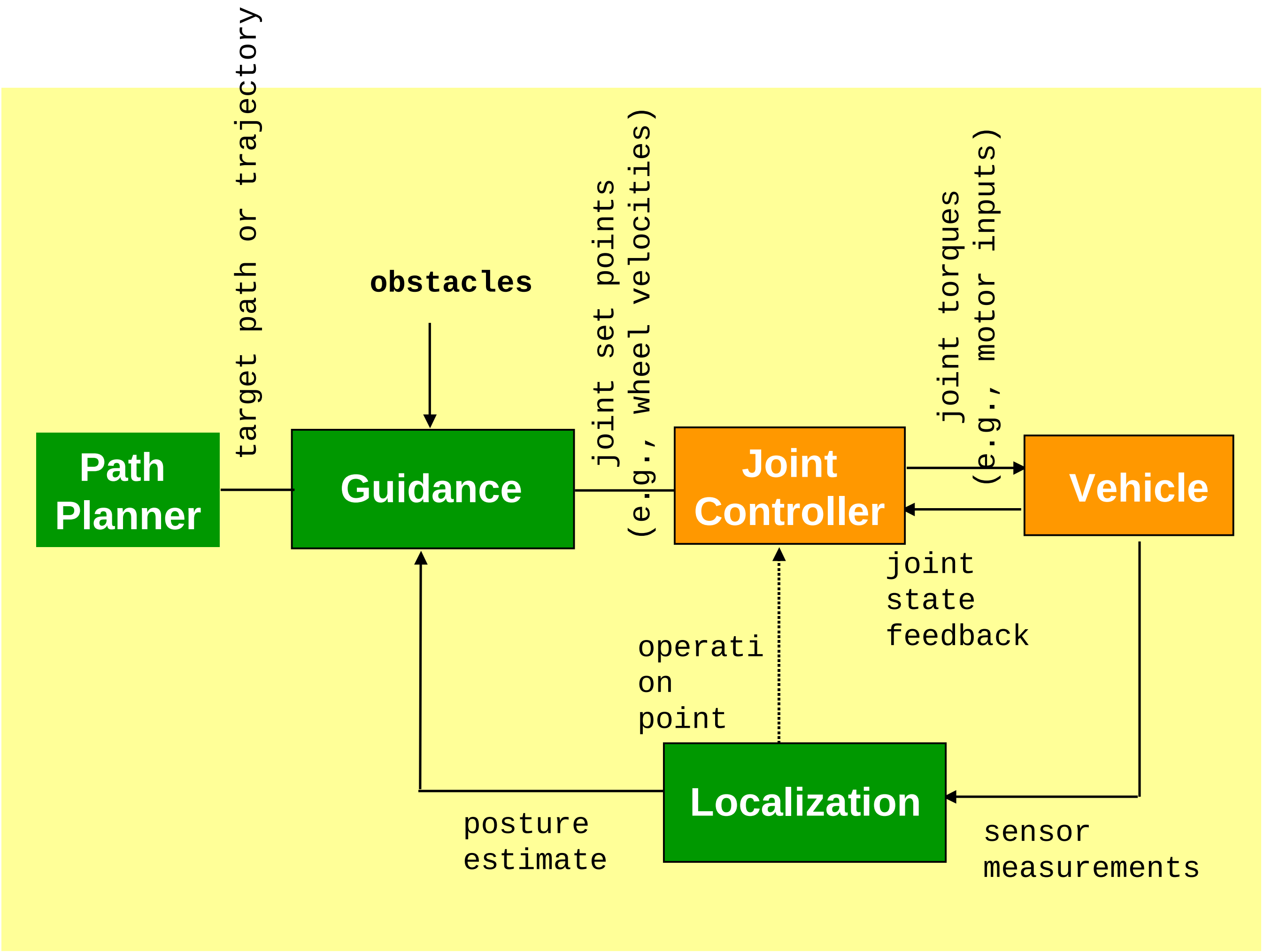
Course Outline



Guidance



GUIDANCE

–take the robot from the current posture to the desired posture, possibly following a pre-determined path or trajectory, while avoiding obstacles

Guidance Methodologies

Some Guidance methodologies

- **State(posture)-feedback methods**:
  - **posture stabilization** (initial and final postures given; no path or trajectory pre-determined; obstacles not considered; may lead to large unexpected paths)
  - **trajectory tracking** (requires pre-planned path)
  - **virtual vehicle tracking** (requires pre-planned trajectory)
- **Potential-Field like methods**
  - **potential fields** (holonomic vehicles)
  - **generalized potential fields** (non-holonomic vehicles)
  - **modified potential fields** (non-holonomic vehicles)
- **Vector Field Histogram (VHF) like methods**
  - **nearness diagram navigation** (holonomic vehicles)
  - **freezone** (non-holonomic vehicles)

NON-LINEAR CONTROL DESIGN
FOR MOBILE ROBOTS

Control of Mobile Robots

- Three distintic problems:
  - Trajectory Tracking or Posture Tracking
  - Path Following
  - Point Stabilization

Trajectory Tracking

- **Vehicle of unicycle type**

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \omega$$

kinematic model

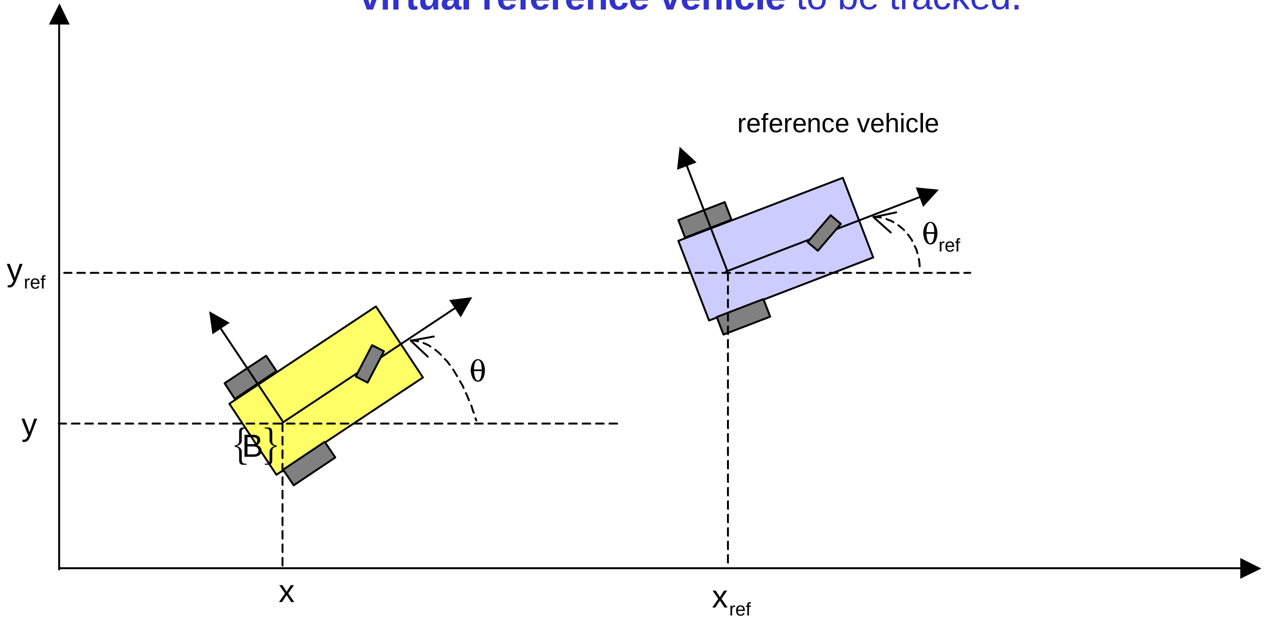
- Is a simplified model, but
- Captures the nonholonomy property which characterizes most WMR and is the core of the difficulties involved in the control of these vehicles

$z = (x, y, \theta)$ position and orientation with respect to a fixed frame

v linear velocity

ω angular velocity

The trajectory tracking problem for a WMR of the unicycle type is usually formulated with the introduction of a **virtual reference vehicle** to be tracked.



Trajectory Tracking

Kinematic model of the Reference Vehicle

$$\dot{x}_{ref} = v_{ref} \cos\theta_{ref}$$
$$\dot{y}_{ref} = v_{ref} \sin\theta_{ref}$$
$$\dot{\theta}_{ref} = \omega_{ref}$$

$$z_{ref} = (x_{ref}, y_{ref}, \theta_{ref})$$

$v_{ref}(t) \quad \omega_{ref}(t)$

- Bounded
- Bounded derivatives
- Do not tend to zero as t tends to infinity

Control Objective

Drive the errors $x - x_{ref},\ y - y_{ref},\ \theta - \theta_{ref}$ to zero

Express the errors in the {B} frame

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_{ref} \\ y - y_{ref} \\ \theta - \theta_{ref} \end{bmatrix}$$

Differentiating
Introducing the change of inputs

$$u_1 = -v + v_{ref} \cos e_3$$
$$u_2 = \omega_{ref} - \omega$$

Trajectory Tracking

$$\dot{e} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} e + \begin{bmatrix} 0 \\ \sin e_3 \\ 0 \end{bmatrix} v_{ref} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

non-linear dynamic system

control variables

Question?

- Is it possible to design a feedback law u=f(e) such that the error converges to zero?
- Is this law linear or non-linear ?

Two different solutions:

- Linear feedback control
- Nonlinear feedback control

LINEAR FEEDBACK CONTROL

$$\dot{e} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} e + \begin{bmatrix} 0 \\ \sin e_3 \\ 0 \end{bmatrix} v_{ref} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

linearize about the equilibrium point $e = 0$, $u = 0$

$$\dot{e} = \begin{bmatrix} 0 & \omega_{ref}(t) & 0 \\ -\omega_{ref}(t) & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} e + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

linear time-varying dynamic system

Assuming

$\omega_{ref}(t) = \omega_{ref}$

$v_{ref}(t) = v_{ref}$

linear time invariant dynamic system

Is it possible to design a linear feedback law u=f(e) such that the error converges to zero?

$\Updownarrow$

Is the dynamic system controllable ?

Trajectory Tracking

$$\Gamma_c = \begin{bmatrix} 1 & 0 & 0 & 0 & -\omega_{ref}^2 & v_{ref}\omega_{ref} \\ 0 & 0 & -\omega_{ref} & v_{ref} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

If $v_{ref} = \omega_{ref} = 0$ the SLIT is non-controllable

the reference robot at rest

$u_1 = -k$ the error cannot be taken to zero in finite time

otherwise

$$u = Ke$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

K_{ij} Chosen by pole placement

undetermined system

closed loop poles $\longrightarrow$ $(s + 2\xi a)(s^2 + 2\xi as + a^2) = 0$

$$u_1 = -k_1 e_1$$
$$u_2 = -k_2 \operatorname{sgn}(v_{ref}) e_2 - k_3 e_3$$

$$k_1 = 2\xi a$$
$$k_2 = \frac{a^2 - \omega_r^2}{|v_{ref}|}$$
$$k_3 = 2\xi a$$

If v_r tends to zero, k_2 increases withouth bound

Trajectory Tracking

To avoid the previous problem:

The closed-loop poles depend on the values of v_r and w_r

closed loop poles $\longrightarrow$ $(s + 2\xi a)(s^2 + 2\xi as + a^2) = 0$

$$a = \sqrt{w_{ref}^2 + bv_{ref}^2}$$

$$u_1 = -k_1 e_1$$
$$u_2 = -k_2\, \mathrm{sgn}(v_r) e_2 - k_3 e_3$$

$$k_1 = 2\xi\sqrt{w_{ref}^2 + bv_{ref}^2}$$
$$k_2 = b\,|\,v_{ref}\,|$$
$$k_3 = 2\xi\sqrt{w_{ref}^2 + bv_{ref}^2}$$

Trajectory Tracking

NONLINEAR FEEDBACK CONTROL

$$\dot{e} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} e + \begin{bmatrix} 0 \\ \sin e_3 \\ 0 \end{bmatrix} v_{ref} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$u_1 = -k_1(v_{ref}, w_{ref}) e_1$$

$$u_2 = -k_4 v_{ref} \frac{\sin e_3}{e_3} e_2 - k_3(v_{ref}, w_{ref}) e_3$$

k_4 positive constant

k_1 continuos function strictly positive in RxR-(0,0)

k_3 continuos function strictly positive in RxR-(0,0)

See the analogy with the linear contol

Property: This control globally asymptotically stabilizies the origin e=0

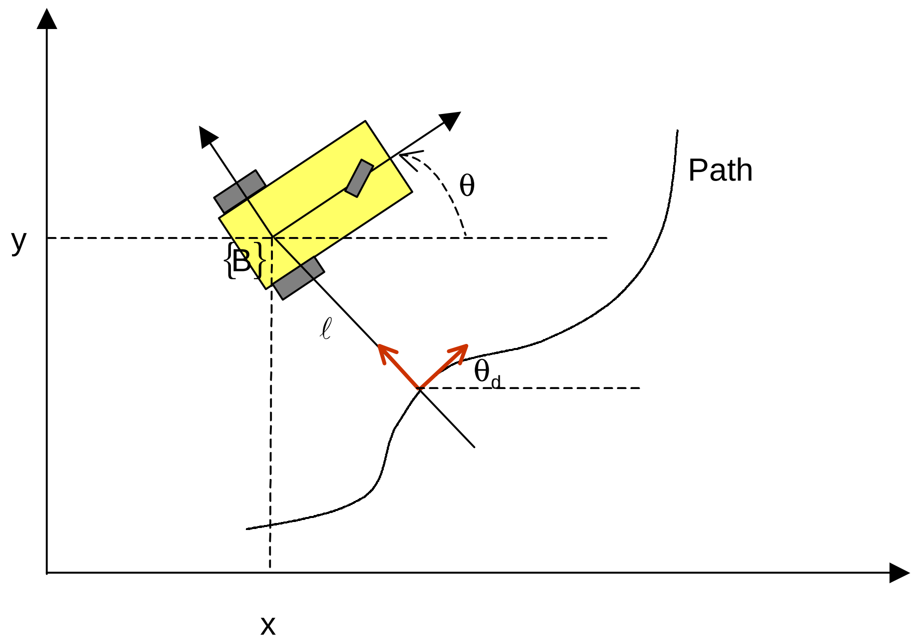
demonstration using Lyapunov stability theory

$$k_1(v_{ref}, w_{ref}) = k_3(v_{ref}, w_{ref}) = 2\xi\sqrt{w_{ref}^2 + b v_{ref}^2}$$

$$k_4 = b$$

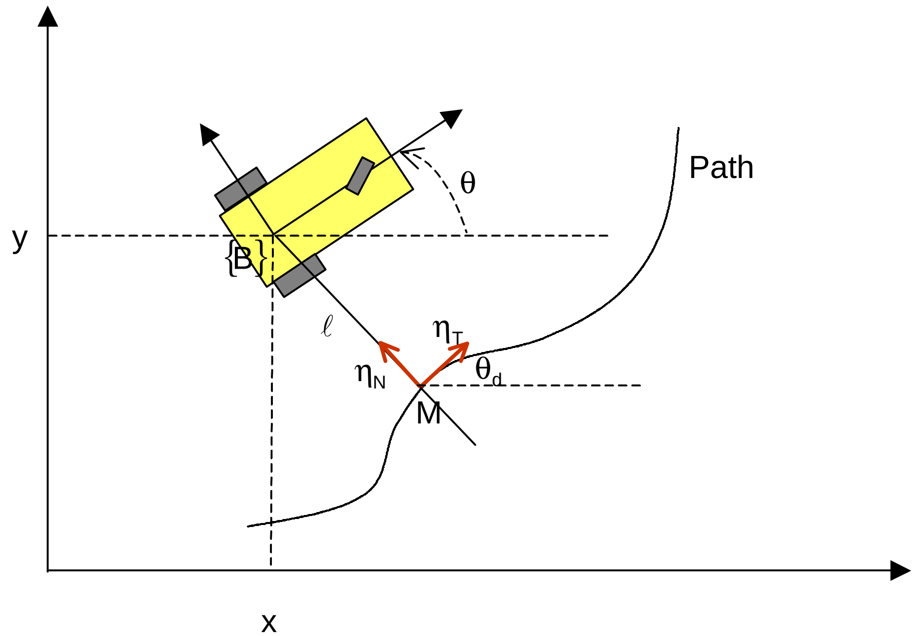
Path Following

- **Objective:**
  – Steer the vehicle at a constant forward speed along a predefined geometric path that is given in a time-free parametrization.



- **Approach:**
  – The controller should compute
    – The distance of the vehicle to the path
    – The orientation error between the vehicle's main axis and the tangent to the path
  – The controller should act on the angular velocity to drive both to zero

Path Following



- M is the orthogonal projection of the robot's position P on the path
  - M exists and is uniquely defined if the path satisfies some conditions

(P, η_T, η_N)

- Serret-Frenet frame moving along the path.
  - The vector η_T is the tangent vector to the path in the closest point to the vehicle
  - The vector η_N is the normal

- ℓ is the distance between P and M
- **s** is the signed curvilinear distance along the path, from some initial path to the point M
- **θ_d(s)** is the angle betwen the vehicle's x-axis and the tange to the path at the point M.
- **c(s)** is the path's curvature at the point M, assumed uniformly bounded and differentiable
- $\tilde{\theta} = \theta - \theta_d$ is the orientation error

Path Following

- A new set of state coordinates for the mobile robot

$$(s, \ell, \tilde{\theta})$$

They coincide with x,y,θ in the particular case where the path coincides with the x-axis

- The error dynamics can be derived writing the vehicle kinematic model in the Serret-Frenet frame:

$$\dot{s} = \frac{v\cos\tilde{\theta}}{1 - c(s)\ell}$$

$$\dot{\ell} = v\sin\tilde{\theta}$$

$$\dot{\tilde{\theta}} = w - \frac{v\cos\tilde{\theta}\, c(s)}{1 - c(s)\ell}$$

PROBLEM FORMULATION

Given a path in the x-y plane and the mobile robot translational velocity, v(t), (assumed to be bounded) together with its time-derivative dv(t)/dt, the path following problem consists of finding a (smooth) feedback control law

$$\omega = k(s, \ell, \tilde{\theta}, v(t))$$

such that

$$\lim_{t\to\infty} \ell(t) = 0$$

$$\lim_{t\to\infty} \tilde{\theta}(t) = 0$$

Path Following

$$\dot{s} = \frac{v\cos\tilde{\theta}}{1-c(s)\ell}$$
$$\dot{\ell} = v\sin\tilde{\theta}$$
$$\dot{\tilde{\theta}} = w - \frac{v\cos\tilde{\theta}c(s)}{1-c(s)\ell}$$

$$u = w - \frac{v\cos\tilde{\theta}c(s)}{1-c(s)\ell}$$

$$\dot{s} = \frac{v\cos\tilde{\theta}}{1-c(s)\ell}$$
$$\dot{\ell} = v\sin\tilde{\theta}$$
$$\dot{\tilde{\theta}} = u$$

Two different solutions:

- Linear feedback control
- Nonlinear feedback control

LINEAR FEEDBACK CONTROL

Linearize the dynamics around the equilibrium point $(\ell = 0, \tilde{\theta} = 0)$

$$\dot{\ell}(t) = v(t)\tilde{\theta}(t)$$
$$\dot{\tilde{\theta}}(t) = u(t)$$

Linearization of the last two equations

Path Following

If

$v(t)=v=cte \neq 0$ → The linear system is

CONTROLLABLE

↓

ASYMPTOTICALY STABILIZABLE BY LINEAR STATE FEEDBACK

Linear stabilizing feedback

$$u = -k_2 v \ell - k_3 |v| \tilde{\theta}$$

$k_2 > 0, k_3 > 0$

↓

$$\ddot{\ell} + k_3 |v| \dot{\ell} + k_2 v^2 \ell = 0$$

Closed-loop differential equation

transformation

$$\ell' = \frac{\partial \ell}{\partial \gamma}$$

$$\gamma = \int_0^t |v| d\tau$$

Distance gone by point M along the path

$$\ell'' + k_3 \ell' + k_2 \ell = 0$$

$$s^2 + 2\xi a s + a^2 = 0$$

Desired closed-loop characteristic equation

Path Following

NONLINEAR FEEDBACK CONTROL

$$\dot{s} = \frac{v\cos\tilde{\theta}}{1 - c(s)\ell}$$

$$\dot{\ell} = v\sin\tilde{\theta}$$

$$\dot{\tilde{\theta}} = u$$

Nonlinear control law

$$u = -k_2 v \ell \frac{\sin\tilde{\theta}}{\tilde{\theta}} - k(v)\tilde{\theta}$$

with

$k_2 > 0$

$k(.)$ continuous function strictly positive

In order to have the two (linear and nonlinear) controllers behave similarly near $\ell = 0,\ \tilde{\theta} = 0$ chose

$$k(v) = k_3\,|\,v\,|$$

with $k_3 = 2\xi a$

$k_2 = a^2$

Path Following

- **Property**

Under the assumption

$$\lim_{t\to\infty} v(t) \neq 0$$

the non-linear control

$$u = -k_2 v \ell \frac{\sin\tilde{\theta}}{\tilde{\theta}} - k(v)\tilde{\theta}$$

asymptotically stabilizes $(\ell = 0, \tilde{\theta} = 0)$

provided that the vehicle's initial configuration is such that

$$\ell(0)^2 + \frac{1}{k_2}\tilde{\theta}(0)^2 < \frac{1}{\lim\sup(c(s))^2}$$

Condition to guarantee that
1-c(s)l remains positive

The vehicle's location along the path is characterized by the value of **s** (the distance gone along the path)

Depends on v(t)

This degree of freedom can be used to stabilize **s** about a prescribed value $\mathbf{s_d}$

Point Stabilization

- Given
  - An arbitrary posture

$$z_d = (x, y, \theta)$$

- Find
  - A control law

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = k(z - z_d, t)$$

which stabilizes asymptotically $z\text{-}z_d$ about zero, whatever the initial robot's posture z(0)

COROLLARY

There is **no** smooth control law k(z) that can solve the point stabilization problem for the considered class of systems.

3 ALTERNATIVES

- Smooth (differentiable) time-varying nonlinear feedback k(z,t)
- Piecewise continuous control laws k(z)
- Time-varying piecewise continuous control laws k(z,t)

References

- C. Canudas de Wit, H. Khennouf, C. Samson, O. Sordalen, “Nonlinear Control Design for Mobile Robots” in Recent Developments in Mobile Robots, World Scientific, 1993.
  - **Reading assignment**

- Carlos Canudas de Wit, Bruno Siciliano and Georges Bastin (Eds), "Theory of Robot Control", 1996.