Decentralized Continuous Robust Controller for Mobile Robots

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Abstract

A composite system model for the wheeled mobile robot is proposed. The overall system can be thought of as a composite of two subsystems; one is the vehicle with m degrees of freedom, and the other is the robot arm with n degrees of freedom. The interconnections between them are unknown interactive forces. To overcome the effects of the unknown interactive forces and some mismatches between the estimated parameter values and the actual ones, a decentralized continuous robust controller is proposed. Conditions are derived for stability when the interconnections are not known. The theoretical analysis and computer simulation results show that the performance of the new continuous robust controller is much better than that of the non-robust controller. An interesting conclusion is that we can choose arbitrarily large weighting gains to decrease the trajectory errors without increasing the magnitude of the physical control torques.

I. Introduction

A wheeled mobile robot (WMR) is modeled as a rigid robot arm with n joints that moves over a horizontal reference plane on wheels connected to the robot arm by a frame with m degrees of freedom.

Intrinsically, WMR is a multivariable dynamic system. We can model it as an overall dynamic system with (n+m) degrees of freedom by using a conventional approach like Lagrange's equations of motion. If (n+m) is small, the overall model may after all be acceptable as a good choice. However, the complexity of computations increases geometrically with (n+m).

On the other hand, WMR can be thought of as a composite of two subsystems; one is the vehicle with m degrees of freedom, and the other is the robot arm with n degrees of freedom. They form a composite system, in which the interconnections are interactive forces.

For this composite model, there are two major issues that we wish to address in this paper:

1. With a simplified nonlinear state feedback (inner-loop control) acting on this composite model, we would like to show that under some reasonable conditions, two kinds of decentralized continuous robust controller (outer-loop control) work and the trajectory errors can be bounded within a ball whose radius can be made arbitrarily small.
2. We would like to show that the magnitude of the physical control torques is almost independent of the selection of the weighting gain, which means that we might use a very large weighting gain in the controller to decrease the trajectory errors without increasing the magnitude of the control torques.

Many literatures [4 - 8] have addressed about the robust control. In [4 - 6], the asymptotic stability of the dynamical systems with bounded uncertainties can be guaranteed by using the so-called minmax discontinuous controller. That discontinuity is due to the introduction of sign function into the control law. To remove the discontinuity of the controller, a supposed continuous controller was proposed in which a saturation function was used to replace the sign function [7, 8]. However, due to the discontinuity of the derivative of the saturation function, the "continuous" control signals chatter during the operation. In this paper, we proposed a decentralized continuous controller which used a *quasi*-saturation function with a continuous derivative to replace the sign function. The boundedness of errors can be guaranteed while the discontinuity of the control signals can be removed.

II. Dynamic Equations of Vehicle and Robot Arm

In this paper, the motions of the vehicle and robot arm are referred to two coordinate systems. One is the fixed workspace frame (x,y,z), the other is the moving vehicle frame (or the robot arm base frame) (ζ, ξ, ρ) whose origin is at the position of (x,y,h) and the rotation angle from x-axis to ζ-axis is θ.

It is not hard to derive the dynamic equations of the vehicle and robot arm, taking into account the interactive forces, as

$$\begin{aligned} \mathbf{M}_V(\mathbf{q}_V)\ddot{\mathbf{q}}_V + \mathbf{N}_V(\mathbf{q}_V, \dot{\mathbf{q}}_V) &= \mathbf{T}_V + \mathbf{J}_V^{\top}(\mathbf{q}_V)\mathbf{f}_R, \quad (2.1) \\ \mathbf{M}_R(\mathbf{q}_R)\ddot{\mathbf{q}}_R + \mathbf{N}_R(\mathbf{q}_R, \dot{\mathbf{q}}_R) &= \mathbf{T}_R + \mathbf{J}_R^{\top}(\mathbf{q}_R)\mathbf{f}_V, \quad (2.2) \end{aligned}$$

where

$\mathbf{q}_R \in R^n$ is the generalized variables of the robot arm referring to the vehicle frame (ζ, ξ, ρ);

$\mathbf{q}_V \in R^m$ is the generalized variables of the vehicle . For 2-D movement $m = 3$ and we chose $\mathbf{q}_V = [x, y, \theta]^{\top}$;

$\mathbf{f}_R \in R^m$ is the reacting forces acted on the vehicle by the robot arm. For 2-D movement, $\mathbf{f}_R = [f_\zeta, f_\xi, \tau_\rho]^{\top}$.

$\mathbf{f}_V = \left[\bar{f}_x^{\top}, \bar{f}_y^{\top}, \bar{\tau}_z^{\top}\right]^{\top} \in R^{3K\times 1}$ is the inertia forces applied by the vehicle which is moving with linear accelerations $a_x = \ddot{x}, a_y = \ddot{y}$, and angular acceleration $\alpha_z = \ddot{\theta}$. Assuming that the mass of the robot is concentrated on finite discrete points, then by *D'Alembert's principle* $\mathbf{f}_V$ can be expressed as

$$\mathbf{f}_V = \begin{bmatrix} \bar{\mathbf{f}}_{\mathbf{x}} \\ \bar{\mathbf{f}}_{\mathbf{y}} \\ \bar{\tau}_{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{m}} & \bar{\mathbf{0}} & \bar{\mathbf{0}} \\ \bar{\mathbf{0}} & \bar{\mathbf{m}} & \bar{\mathbf{0}} \\ \bar{\mathbf{0}} & \bar{\mathbf{0}} & \bar{\mathbf{ml}}^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{bmatrix} \tag{2.3}$$

where $\bar{\mathbf{m}} = [m_1, \ldots, m_K]^\top$, $\bar{\mathbf{ml}}^2 = [m_1 l_1^2, \ldots, m_K l_K^2]^\top$, $\bar{\mathbf{0}} = [\overbrace{0, \ldots, 0}^{K}]^\top$, K is the number of the finite discrete points; m_i is the mass which is concentrated on the i^{th} point; l_i is the distance from the i^{th} point to the ρ-axis of the vehicle frame. l_i is a function of $\mathbf{q}_R$ so $\mathbf{f}_V$ is a function of $\mathbf{q}_R$ and $\ddot{\mathbf{q}}_V$.

$\mathbf{J}_V(\mathbf{q}_V) \in R^{m\times m}$ maps the m dimensional contact forces into m dimensional vehicle moving space. For 2-D movement,

$$\mathbf{J}_V(\mathbf{q}_V) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.4}$$

$\mathbf{J}_R(\mathbf{q}_R) \in R^{3K\times n}$ is a transformation which maps the $3k$ dimensional inertia forces into n dimensional joint space; it depends on the configuration of the robot arm and may be considered as a Jacobian matrix.

The contact forces $\mathbf{f}_R$ consist of two parts; one owing to the reaction of inertia forces applied onto the robot arm; the other owing to the accelerative action of the robot arm. We can show that

$$\mathbf{f}_R = \begin{bmatrix} f_\zeta \\ f_\xi \\ \tau_\rho \end{bmatrix} = -\begin{bmatrix} \sum_{i=1}^K m_i a_{\zeta i} \\ \sum_{i=1}^K m_i a_{\xi i} \\ \sum_{i=1}^K m_i l_i \dot{\omega}_{\rho i} \end{bmatrix} +$$
$$\begin{bmatrix} \overbrace{-\cos\theta, \ldots, -\cos\theta}^{K} & \overbrace{-\sin\theta, \ldots, -\sin\theta}^{K} & \overbrace{0, \ldots, 0}^{K} \\ \sin\theta, \ldots, \sin\theta & -\cos\theta, \ldots, -\cos\theta & 0, \ldots, 0 \\ 0, \ldots, 0 & 0, \ldots, 0 & -1, \ldots, -1 \end{bmatrix} \mathbf{f}_V,$$

where $a_{\zeta i}, a_{\xi i}$ are the accelerations of the i^{th} point in directions of ζ and ξ, respectively, $\dot{\omega}_{\rho i}$ is the angular acceleration of the i^{th} point with respect to ρ-axis. $\mathbf{f}_R$ is a function of $(\mathbf{q}_R, \ddot{\mathbf{q}}_R, \ddot{\mathbf{q}}_V)$.

Expressing Eqs. (2.1) and (2.2) in state space form, we get the composite system model for WMR as

$$\begin{aligned} \dot{\mathbf{z}}_1 &= \mathbf{f}_1(\mathbf{z}_1) + \check{\mathbf{b}}_1(\mathbf{z}_1)\mathbf{u}_1 + \check{\mathbf{b}}_1(\mathbf{z}_1)\omega_1(\dot{\mathbf{x}}) \\ \dot{\mathbf{z}}_2 &= \mathbf{f}_2(\mathbf{z}_2) + \check{\mathbf{b}}_2(\mathbf{z}_2)\mathbf{u}_2 + \check{\mathbf{b}}_2(\mathbf{z}_2)\omega_2(\dot{\mathbf{x}}) \end{aligned} \tag{2.5}$$

where

$$\mathbf{z}_1 = \begin{bmatrix} \mathbf{q}_V \\ \dot{\mathbf{q}}_V \end{bmatrix}, \quad \mathbf{f}_1(\mathbf{z}_1) = \begin{bmatrix} \dot{\mathbf{q}}_V \\ -\mathbf{M}_V^{-1}(\mathbf{q}_V)\mathbf{N}_V(\mathbf{q}_V, \dot{\mathbf{q}}_V) \end{bmatrix},$$

$$\check{\mathbf{b}}_1(\mathbf{z}_1) = \begin{bmatrix} 0 \\ \mathbf{M}_V^{-1}(\mathbf{q}_V) \end{bmatrix}, \quad \mathbf{u}_1 = \mathbf{T}_V, \ \omega_1(\dot{\mathbf{z}}) = \mathbf{J}_V^\top \mathbf{f}_R;$$

$$\mathbf{z}_2 = \begin{bmatrix} \mathbf{q}_R \\ \dot{\mathbf{q}}_R \end{bmatrix}, \quad \mathbf{f}_2(\mathbf{z}_2) = \begin{bmatrix} \dot{\mathbf{q}}_R \\ -\mathbf{M}_R^{-1}(\mathbf{q}_R)\mathbf{N}_R(\mathbf{q}_R, \dot{\mathbf{q}}_R) \end{bmatrix},$$

$$\check{\mathbf{b}}_2(\mathbf{z}_2) = \begin{bmatrix} 0 \\ \mathbf{M}_R^{-1}(\mathbf{q}_R) \end{bmatrix}, \quad \mathbf{u}_2 = \mathbf{T}_R, \ \omega_2(\dot{\mathbf{z}}) = \mathbf{J}_R^\top \mathbf{f}_V,$$

$$\mathbf{x}^\top = \begin{bmatrix} \mathbf{z}_1 & \mathbf{z}_2 \end{bmatrix}.$$

Note that the interconnections in (2.5) are functions of the derivatives of state variables.

III. Decentralized Continuous Robust Controller

Now we will introduce local nonlinear feedback control laws into the composite system (2.5) and investigate the stability of the closed-loop error system. If we know the parameters of the robot arm and the vehicle very well and if we can measure the state variables without measurement errors, then we could get the "exact" nonlinear state feedback. For that case, we have shown that [10] the closed-loop error systems are stable and the tracking errors can be bounded within a small ball. However, in practice it is very hard, if not impossible, to implement the "exact" nonlinear feedback control scheme because of the incomplete knowledge of the robot parameters and noisy measurements of the state variables. It is much more reasonable to suppose that the inner loop control laws are chosen as

$$\begin{aligned} \mathbf{u}_1 &= \hat{\mathbf{M}}_V \mathbf{v}_1 + \hat{\mathbf{N}}_V, \\ \mathbf{u}_2 &= \hat{\mathbf{M}}_R \mathbf{v}_2 + \hat{\mathbf{N}}_R. \end{aligned} \tag{3.1}$$

where $(\hat{*})$ represents estimated or computed version of $(*)$. With this reasonable inner-loop control laws, (2.5) becomes

$$\dot{\mathbf{z}}_i = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \mathbf{z}_i + \begin{bmatrix} 0 \\ I \end{bmatrix} \bar{\omega}_i \tag{3.2}$$

where $\bar{\omega}_i = \mathbf{M}_i^{-1}\hat{\mathbf{M}}_i\mathbf{v}_i - \mathbf{M}_i^{-1}\tilde{\mathbf{N}}_i + \mathbf{M}_i^{-1}\mathbf{J}_i^\top\mathbf{f}_i$. In the sequel the symbol I will denote the *identity matrix* with proper dimension and $(\tilde{*})$ will denote the mismatch between the actual value $(*)$ and the estimate $(\hat{*})$, that is, $(\tilde{*}) = (*) - (\hat{*})$.

Given the desired trajectories $(\mathbf{z}_{id}(t), \dot{\mathbf{z}}_{id}(t)), i = 1, 2$, define the tracking error as

$$\mathbf{e}_i(t) = \mathbf{z}_i(t) - \mathbf{z}_{id}(t) \qquad i = 1, 2,$$

then by choosing the new input $\mathbf{v}_i$ as

$$\mathbf{v}_i = \begin{bmatrix} -K_{ip} & -K_{iv} \end{bmatrix} \mathbf{e}_i + \begin{bmatrix} 0 & I \end{bmatrix} \dot{\mathbf{z}}_{id} + \Delta\mathbf{v}_i, \tag{3.3}$$

we get the closed-loop error system

$$\begin{aligned} \dot{\mathbf{e}}_i &= \begin{bmatrix} 0 & I \\ -K_{ip} & -K_{iv} \end{bmatrix} \mathbf{e}_i + \begin{bmatrix} 0 \\ I \end{bmatrix} \delta_i + \begin{bmatrix} 0 \\ I \end{bmatrix} \mathbf{w}_i \\ &= \mathbf{A}_i\mathbf{e}_i + \mathbf{b}_i\delta_i + \mathbf{b}_i\mathbf{w}_i, \end{aligned} \tag{3.4}$$

where δ_i, hereafter called the *mismatch term*, is given by

$$\delta_i = \mathbf{M}_i^{-1}\tilde{\mathbf{M}}_i\{\begin{bmatrix} K_{ip} & K_{iv} \end{bmatrix}\mathbf{e}_i - \ddot{\mathbf{q}}_{id}\} - \mathbf{M}_i^{-1}\tilde{\mathbf{N}}_i + \mathbf{M}_i^{-1}\mathbf{J}_i^\top\mathbf{f}_i,$$

and $\mathbf{w}_i$, the *compensating term*, is

$$\mathbf{w}_i = \mathbf{M}_i^{-1}\hat{\mathbf{M}}_i\Delta\mathbf{v}_i.$$

The control objective is to select a proper $\Delta\mathbf{v}_i$ such that $\mathbf{w}_i$ can reduce the effect of δ_i on (2.5) to the minimum.

In order to estimate the bound on η_i we assume that

$$\begin{aligned} &\|\tilde{\mathbf{M}}_i\| \le \delta M_i, && \|\tilde{\mathbf{N}}_i\| \le n_i(\mathbf{e}_i, t), \\ &\|\mathbf{f}_i\| \le f_{i\max}, && \|\ddot{\mathbf{q}}_{id}\| \le Q_i, \end{aligned} \tag{3.5}$$

where $n_i(\mathbf{e}_i, t)$ is a known function of $\mathbf{e}_i$ bounded in t; $\delta M_i, f_{i\max}, Q_i$ are positive constants which are chosen based on knowledge of the range of the system parameters and the range of the measurement noise of state variables as well as the expected loads. The structure of $n_i(\mathbf{e}_i, t)$ and the selection of δM_i depend on how we get the estimates for $\mathbf{N}_i$ and $\mathbf{M}_i$. For example, from [3] we know that $\mathbf{M}_i$ has upper bound and lower bound $m_{i\max}, m_{i\min}$. If we select

$$\hat{\mathbf{M}}_i = \frac{m_{i\max} + m_{i\min}}{2} I,$$

then

$$\delta M_i = \frac{m_{i\max} - m_{i\min}}{2}.$$

The point is that we can always choose at least one set of the above constants to satisfy assumptions (3.5). It is reasonable to confine the estimate for $\mathbf{M}_i$ within necessary limit, that is,

$$m_{i\min} \le \|\hat{\mathbf{M}}_i\| \le m_{i\max}. \tag{3.6}$$

Now let us define two bounded, positive functions

$$\begin{aligned}\phi_i(\mathbf{e}_i) &= \frac{m_{i\max}}{m_{i\min}^2}\{(\delta M_i Q_i + f_{i\max}) + \delta M_i \kappa_i \|\mathbf{e}_i\| + n_i(\mathbf{e}_i, t)\} \\ &\ge 0, \qquad i = 1, 2;\end{aligned} \tag{3.7}$$

where κ_i is the norm of $[K_{ip} \;\; K_{iv}]$, and two filtered error vectors

$$\mathbf{E}_i = \mathbf{b}_i^\top \mathbf{P}_i \mathbf{e}_i = \begin{bmatrix} P_{i2} & P_{i3} \end{bmatrix} \mathbf{e}_i, \qquad i = 1, 2;$$

where P_{i2} and P_{i3} are submatrices of $\mathbf{P}_i$ which is the solution of Lyapunov equation

$$\mathbf{A}^\top \mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}.$$

Using (3.7) we have

$$\|\delta_i\| \le \alpha_i \phi_i(\mathbf{e}_i) \tag{3.8}$$

where $\alpha_i = m_{i\min}/m_{i\max} \le 1$.

With these definitions we have the following theorem.

Theorem 3.1 The composite system (2.5) is totally ultimately bounded by using control laws (3.1) and (3.3) with

$$\Delta \mathbf{v}_i = -\phi_i(\mathbf{e}_i)\frac{\mathbf{E}_i}{\epsilon_i + \|\mathbf{E}_i\|}, \qquad i = 1, 2; \tag{3.9}$$

where ϵ_i is a small positive number.

Moreover, the tracking errors can be limited within a ball about zero with the radius $r(\epsilon_i)$ which can be reduced by choosing a small ϵ_i.

Proof : Let a positive definite continuous function be

$$v(\mathbf{e}_1, \mathbf{e}_2) = \sum_{i=1}^{2} \mathbf{e}_i^\top \mathbf{P}_i \mathbf{e}_i. \tag{3.10}$$

Along the solutions of (3.4), $\dot{v}(\mathbf{e}_1, \mathbf{e}_2)$ is

$$\begin{aligned}\dot{v}(\mathbf{e}_1, \mathbf{e}_2) &= \sum_{i=1}^{2}\{\dot{\mathbf{e}}_i^\top \mathbf{P}_i \mathbf{e}_i + \mathbf{e}_i^\top \mathbf{P}_i \dot{\mathbf{e}}_i\} \\ &= \sum_{i=1}^{2}\{-\mathbf{e}_i^\top \mathbf{Q}_i \mathbf{e}_i + 2\mathbf{e}_i^\top \mathbf{P}_i \mathbf{b}_i \delta_i + 2\mathbf{e}_i^\top \mathbf{P}_i \mathbf{b}_i \mathbf{w}_i\} \\ &= \sum_{i=1}^{2}\{-\mathbf{e}_i^\top \mathbf{Q}_i \mathbf{e}_i + 2\mathbf{E}_i^\top \mathbf{M}_i^{-1}\hat{\mathbf{M}}_i \Delta v i + 2\mathbf{E}_i^\top \delta_i\}.\end{aligned} \tag{3.11}$$

By control law (3.9), the second term on the right-hand side of (3.11) has a quadratic form

$$2\mathbf{E}_i^\top \mathbf{M}_i^{-1}\hat{\mathbf{M}}_i \Delta \mathbf{v}_i = -2\phi_i(\mathbf{e}_i)\frac{\mathbf{E}_i^\top \mathbf{M}_i^{-1}\hat{\mathbf{M}}_i \mathbf{E}_i}{\epsilon_i + \|\mathbf{E}_i\|}, \tag{3.12}$$

which is a negative number for any non-zero $\mathbf{E}_i$.

Since

$$\frac{1}{m_{i\max}} I \le \mathbf{M}_i^{-1} \le \frac{1}{m_{i\min}} I, \quad m_{i\min} I \le \hat{\mathbf{M}}_i \le m_{i\max} I,$$

we have the following inequality

$$\begin{aligned}2\mathbf{E}_i^\top \mathbf{M}_i^{-1}\hat{\mathbf{M}}_i \Delta \mathbf{v}_i &= -2\phi_i(\mathbf{e}_i)\frac{\mathbf{E}_i^\top \mathbf{M}_i^{-1}\hat{\mathbf{M}}_i \mathbf{E}_i}{\epsilon_i + \|\mathbf{E}_i\|} \\ &\le -2\alpha_i \phi_i(\mathbf{e}_i)\frac{\|\mathbf{E}_i\|}{\epsilon_i + \|\mathbf{E}_i\|}.\end{aligned} \tag{3.13}$$

From (3.8), the norm of the third term on the right-hand side of (3.11) has a upper bound as

$$\begin{aligned}\|2\mathbf{E}_i^\top \delta_i\| &\le 2\|\mathbf{E}_i\|\|\delta_i\| \\ &\le 2\alpha_i \phi_i(\mathbf{e}_i)\|\mathbf{E}_i\|,\end{aligned} \tag{3.14}$$

or equivalent to

$$\|2\mathbf{E}_i^\top \delta_i\| = 2\alpha_i(\phi_i(\mathbf{e}_i) - \Delta\phi_i)\|\mathbf{E}_i\|, \tag{3.15}$$

where $\Delta\phi_i$ is a positive number representing the difference between the two sides of (3.14).

Combining (3.13) and (3.15), the last two terms on the right-hand side of (3.11) has a upper bound as

$$\begin{aligned}& 2\mathbf{E}_i^\top \mathbf{M}_i^{-1}\hat{\mathbf{M}}_i \Delta \mathbf{v}_i + 2\mathbf{E}_i^\top \delta_i \\ \le\; & -2\alpha_i \phi_i(\mathbf{e}_i)\frac{\|\mathbf{E}_i\|}{\epsilon_i + \|\mathbf{E}_i\|} + 2\alpha_i \phi_i(\mathbf{e}_i)\|\mathbf{E}_i\| - 2\alpha_i \Delta\phi_i \|\mathbf{E}_i\| \\ =\; & \frac{2\alpha_i \epsilon_i \phi_i(\mathbf{e}_i)\|\mathbf{E}_i\|}{\epsilon_i + \|\mathbf{E}_i\|} - 2\alpha_i \Delta\phi_i \|\mathbf{E}_i\|\end{aligned} \tag{3.16}$$

which attains a maximum value of $2\alpha_i\epsilon_i(\sqrt{\phi_i} - \sqrt{\Delta\phi_i})^2$ when $\|\mathbf{E}_i\| = \epsilon_i(\sqrt{\phi_i/\Delta\phi_i} - 1)$.

Substituting (3.16) into (3.11) yields

$$\begin{aligned}\dot{v}(\mathbf{e}_1, \mathbf{e}_2) &\le \sum_{i=1}^{2}\{-\mathbf{e}_i^\top \mathbf{Q}_i \mathbf{e}_i + 2\alpha_i\epsilon_i(\sqrt{\phi_i} - \sqrt{\Delta\phi_i})^2\} \\ &\le -\bar{\mathbf{e}}^\top \bar{\mathbf{Q}} \bar{\mathbf{e}} + 2\epsilon_{\max}(\phi_1 + \phi_2) \\ &\le -\lambda_{\min}(\bar{\mathbf{Q}})\|\bar{\mathbf{e}}\|^2 + 2\epsilon_{\max}(\phi_1 + \phi_2),\end{aligned} \tag{3.17}$$

where $\bar{\mathbf{Q}} \triangleq diagonal\{\mathbf{Q}_1, \mathbf{Q}_2\}$, $\bar{\mathbf{e}} \triangleq [\mathbf{e}_1^\top, \;\; \mathbf{e}_2^\top]^\top$, $\epsilon_{\max} \triangleq \max\{\epsilon_1, \epsilon_2\}$. Thus, the sufficient condition for $\dot{v} < 0$ is

$$\|\bar{\mathbf{e}}\| > \sqrt{\frac{2\epsilon_{\max}(\phi_1 + \phi_2)}{\lambda_{\min}(\bar{\mathbf{Q}})}}. \tag{3.18}$$

By Lemma A.1 and A.2, the tracking errors for system (2.5) is bounded by

$$\|\bar{\mathbf{e}}\| \le \sqrt{\frac{2\epsilon_{\max}(\phi_1 + \phi_2)}{k^{\gamma-1}}} \tag{3.19}$$

if we select $k_{ip} = k_{iv} = k$ and $\mathbf{Q}_i = diagonal\{2k^\gamma\}$. In this case,

$$\mathbf{E}_i = k^{\gamma-1}[\mathbf{e}_i + (1 + \frac{1}{k})\dot{\mathbf{e}}_i]. \tag{3.20}$$

(3.19) said that the error bounds can be reduced if we select a small ϵ and large k and γ.

□

IV. Discussion

(1) In last section we defined two bounded, positive functions $\phi_1(\mathbf{e}_1)$ and $\phi_2(\mathbf{e}_2)$. Actualy there are many procedures to define ϕ_i. The simplest method may be to select a "very large" constant as ϕ_i. However, as we can see from the proof of Theorem 3.1, the larger ϕ_i, the larger tracking errors.

(2) The new robust controller is continuous in $\mathbf{e}_i$ and t, which leads to the conclusion of stability for the composite system (2.5). The continuity of the controller is attributed to the introduction of the small positive number ϵ_i into control laws (3.9). Without this small ϵ_i, the controller is discontinuous at $\|\mathbf{e}_i\| = 0$. In that case, we cannot say anything about the stability behavior of (2.5). Further more, we can see that the norm of the tracking errors is proportional to the square root of ϵ_i, which means that the tracking errors can be reduced to a small limit by choosing a very small ϵ_i as long as $\epsilon_i \neq 0$.

(3) Note that $\Delta\mathbf{v}_i$ is a vector of *quasi*-saturation function. Its partial derivatives with respect to $\mathbf{E}_i$

$$\left\{ \frac{1}{\epsilon_i + \|\mathbf{E}_i\|} - \frac{\mathbf{e}_i^2}{(\epsilon_i + \|\mathbf{E}_i\|)\|\mathbf{E}_i\|} \right\}$$

is a vector of continuous function in $\mathbf{e}_i$ and t, which makes the control signals continuous and smooth. This property is very important and useful in practice. It guarantees that the control signals are realizable.

(4) The conditions for the validity of Theorem 3.1 are not rigorous. The assumptions (3.5) are very reasonable and general. The new robust controller works even if we select the "worst case" in (3.5), for example, select $\delta M_i = m_{i\,\max} - m_{i\,\min}$ or large δN_{ij}. However, selecting "bad" constants in (3.5) should lead to a large $\phi_i(\mathbf{e}_i)$ and, consequently, a large error bound since the magnitude of $\phi_i(\mathbf{e}_i)$ and the error bound are functions of these constants. The "best case" is that we can use the exact $\mathbf{M}_i, \mathbf{N}_i$ in controller and, therefore, select $\delta M_i = n_i(\mathbf{e}_i, t) = 0$. In this case, the tracking error is limited by

$$\|\bar{\mathbf{e}}\| \leq \sqrt{\frac{2\epsilon_{\max}(\mathbf{f}_{1\max} + \mathbf{f}_{2\max})}{k^{\gamma-1}}}, \tag{4.1}$$

which is the minimum error bound for composite system (2.5) with control laws (3.1), (3.3), and (3.9).

(5) From the proof of Theorem 3.1 we see that the error bound can be reduced by choosing a big positive integer number γ and feedback gain k. However, big γ and k may result in large control energy due to (3.9) and (3.20). Therefore, it is necessary to investigate the dependence of $\Delta\mathbf{v}_i$ on k and γ. Plugging (3.20) into (3.9) gives

$$\begin{aligned} \Delta\mathbf{v}_i &= -\phi_i \frac{k^{\gamma-1}[\mathbf{e}_i + (1+1/k)\dot{\mathbf{e}}_i]}{\epsilon_i + \|k^{\gamma-1}[\mathbf{e}_i + (1+1/k)\dot{\mathbf{e}}_i]\|} \\ &\approx -\phi_i \frac{\mathbf{e}_i + \dot{\mathbf{e}}_i}{\|\mathbf{e}_i + \dot{\mathbf{e}}_i\|} \end{aligned} \tag{4.2}$$

if k is big enough and ϵ_i is small enough. (4.2) shows that the magnitude of $\Delta\mathbf{v}_i$ is independent of the selection of k and γ.

(6) Now consider outer-loop control laws (3.3). If we select a very big feedback gain k in order to reduce the error bound, could the control torques $\mathbf{v}_i$ increase infinitely ? Let us check the control torques $\mathbf{v}_i$:

$$\mathbf{v}_i = \begin{bmatrix} -kI & -kI \end{bmatrix} \mathbf{e}_i + \begin{bmatrix} 0 & I \end{bmatrix} \dot{\mathbf{z}}_{id} + \Delta\mathbf{v}_i.$$

The only two terms in $\mathbf{v}_i$ and $\Delta\mathbf{v}_i$ which depend on the selection of feedback gain k are $\begin{bmatrix} kI & kI \end{bmatrix} \mathbf{e}_i$ and $\kappa_i\|\mathbf{e}_i\|$ where κ_i is the norm of $[K_{ip} \;\; K_{iv}] = [kI \;\; kI]$.

These terms will be bounded by

$$\begin{aligned} \left\| \begin{bmatrix} kI & kI \end{bmatrix} \mathbf{e}_i \right\| &\leq \kappa_i\|\mathbf{e}_i\| \\ &= \sqrt{2}k \; \|\mathbf{e}_i\|. \end{aligned} \tag{4.3}$$

But from (3.19) we know that $\|\mathbf{e}_i\|$ is bounded by $\sqrt{2\epsilon_i\phi_i}/k$ if we set $\gamma = 3$. Thus (4.3) becomes

$$\| \begin{bmatrix} kI & kI \end{bmatrix} \mathbf{e}_i\| \leq \kappa_i\|\mathbf{e}_i\| \leq \sqrt{4\epsilon_i\phi_i}$$

which is independent of the selection of the feedback gain k.

Therefore, we can select a big enough feedback gain k to decrease the trajectory errors without increasing too much the magnitude of control torques .

V. Computer Simulations

To show the effectiveness of the decentralized controller for the mobile robot, we study a simple two-link planar rotary manipulator (n=2) that mounts on a wheeled frame which can move only along the direction of x (m=1, $\theta \equiv 0$) (Fig.1) . We assume that the parameters of the system are unknown but their bounds are given as follows: link length ($0.8m < l_1, l_2 < 1.4m$); link mass ($4.5kg < m_1, m_2 < 5.5kg$); vehicle height ($0.8m < h < 1.3m$); vehicle mass ($40kg < M < 60kg$) . The mass of the i^{th} link is assumed to concentrated on the middle of the i^{th} link.

For the composite system model, we have two subsystems :

1. for the vehicle, the dynamic equation is

$$M\ddot{x} = F + f_\zeta,$$

2. for the two-link robot arm, the dynamic equation is

$$\begin{aligned} &\begin{bmatrix} (0.25m_1 + m_2)l_1^2 + 0.25m_2l_2^2 + m_2l_1l_2\cos\theta_2 & 0.25m_2l_2^2 + 0.5m_2l_1l_2\cos\theta_2 \\ 0.25m_2l_2^2 + 0.5m_2l_1l_2\cos\theta_2 & 0.25m_2l_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} \\ + &\begin{bmatrix} -0.5m_2l_1l_2\sin\theta_2\dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}_2) - 0.5m_1l_1g\sin(\theta_1+\theta_2) - (m_2 + 0.5m_1)l_1g\sin\theta_1 \\ 0.5m_2l_1l_2\sin\theta_2\dot{\theta}_1^2 - 0.5m_2l_2g\sin(\theta_1+\theta_2) \end{bmatrix} \\ = &\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} -0.5l_1\cos\theta_1 & -l_1\cos\theta_1 - 0.5l_2\cos(\theta_1+\theta_2) \\ 0 & -0.5l_2\cos(\theta_1+\theta_2) \end{bmatrix} \cdot \mathbf{f}_x. \end{aligned}$$

In this example, we assume that the interactive forces f_ζ and $\tilde{f}_x$ are unknown.

The desired positions of x, θ_1, θ_2 are

$$\begin{aligned} x_d(t) &= \begin{cases} 0.666\ t^3; & t \leq 5\mathbf{sec.} \\ 2.5\ t^2 - 12.5\ t + 20.833; & t \leq 10\mathbf{sec.} \\ 37.5\ t - 229.166; & t \leq 30\mathbf{sec.} \\ 0.125t^3 - 15t^2 + 600t - 6979.16. & t \leq 40\mathbf{sec.} \end{cases} \\ \theta_{1d}(t) &= -1.318\sin(0.5\pi t), \\ \theta_{2d}(t) &= 2.636\sin(0.5\pi t). \end{aligned}$$

The desired path for the end-effector of the robot arm in Cartesian space is shown in Fig. 2. The end-effector moves from (x=0 m, z=3 m)($x = 0, \theta_1 = \theta_2 = 0$) to (x=1,020 m, z= 3m) ($x = 1,020, \theta_1 = \theta_2 = 0$) in 40 seconds. At the end-points of the path, the velocities and accelerations in both of x- and z-direction are equal to zero. Two kinds of controller, non-robust and robust, are used in this simulation to compare the performance behavior of the control schemes. For simplicity we select a large positive number (100) as ϕ_i, $\epsilon_1 = \epsilon_2 = 0.0001$. The feedback gain k is selected as 30 for both of non-robust controller and robust controller.

The simulation results are shown in Figs. 3 - 6. Figs. 3 and 4 show the position errors in joint space for non-robust control and robust control, respectively. Figs.5 and 6 show the Cartesian position errors in both of x- and z-direction for non-robust control and robust control, respectively. From the simulations results we can see that, with the same inner-loop nonlinear feedback (using estimated or computed versions in control law) and the same feedback gain ($k = 30$), the tracking errors by using robust controller is only the tenth of those by using non-robust controller in joint space. In Cartesian space, when the end-effector moving 1,020 m in 40 seconds along x-direction, the tracking error is limited within 0.01m in robust case. In non-robust case, on the other hand, the tracking error is about 0.2 m.

VI. Conclusion

We have derived a decentralized continuous robust controller for a wheeled mobile robot by considering the mobile base and attached robot arm as two subsystems. Theoretical analysis and computer simulation results show that the closed-loop error systems of the wheeled mobile robot considered as two interconnected subsystems are stable if the unknown interconnections are bounded, and the trajectory errors can be decreased to certain degree by choosing small ϵ.

An interesting conclusion from Section 4 is that the magnitudes of the control torques are independent of the choice of the feedback gains beyond a certain point. This conclusion is important from the engineering point of view since we can choose a big enough feedback gain to decrease the trajectory errors without worrying about the control torques. This conclusion can be extended to the other robotic systems such as two-arm robotic systems. In fact, the two-arm robotic systems can be thought of as two isolated robot arm systems. They can form a composite system with the contact forces as the interconnections between them.

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Appendix

Lemma A.1 For Lyapunov equation

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}, \tag{A.1}$$

if we chose

$$\mathbf{A} = \begin{bmatrix} 0 & I \\ -kI & -\alpha kI \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 2k^\gamma I & 0 \\ 0 & 2k^\gamma I \end{bmatrix}, \tag{A.2}$$

then

$$\mathbf{P} = \begin{bmatrix} \frac{1}{\alpha}[(\alpha^2+1)k^\gamma + k^\gamma - 1]I & k^{\gamma-1}I \\ k^{\gamma-1}I & \frac{1}{\alpha}k^{\gamma-1}(1+\frac{1}{k})I \end{bmatrix}$$

and

$$\sqrt{\frac{\lambda_{\max}(\mathbf{P})}{\lambda_{\min}(\mathbf{P})\lambda_{\min}(\mathbf{Q})}} < \sqrt{\frac{1+\alpha^2}{2k^{\gamma-1}}} \tag{A.3}$$

as long as k be selected large enough. In (A.2) and (A.3), k, γ, and α are prescribed positive constants, $\lambda_{max}(*)(\lambda_{min}(*))$ represents the maximum (minimum) eigenvalue of $(*)$.

Particularly, if we select $\alpha = 1$, then

$$\sqrt{\frac{\lambda_{max}(\mathbf{P})}{\lambda_{min}(\mathbf{P})\lambda_{min}(\mathbf{Q})}} < \sqrt{\frac{1}{k^{\gamma-1}}}. \tag{A.4}$$

Proof : Straightforward substituting (A.2) into (A.1) and solving for $\mathbf{P}$ yields (A.3).

□

Lemma A.2 If there exist three positive constants λ_1, λ_2, and λ_3 and a positive function $f(x(t))$ such that for a continuous function $v(x(t))$,

$$\lambda_1 \|x(t)\|^2 \leq v(x(t)) \leq \lambda_2 \|x(t)\|^2,$$
$$\dot{v}(x(t)) \leq -f(x(t))(\|x(t)\| - \lambda_3), \quad \forall t \in [t_0, \infty),$$

then

$$\|x(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}}\lambda_3. \tag{A.5}$$

Proof : See [6].

□

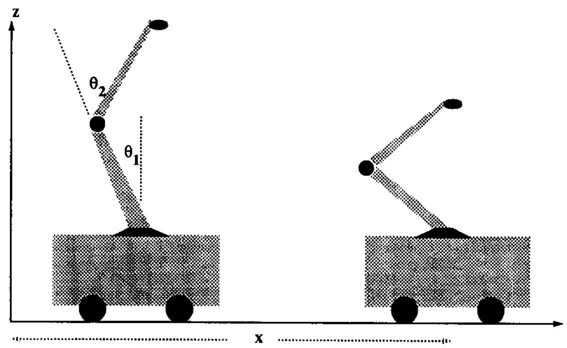

Fig.1 Two Link Mobile Robot

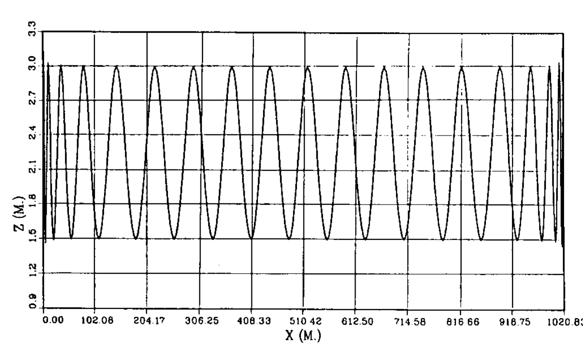

Fig.2 Desired Path in Cartesian Space

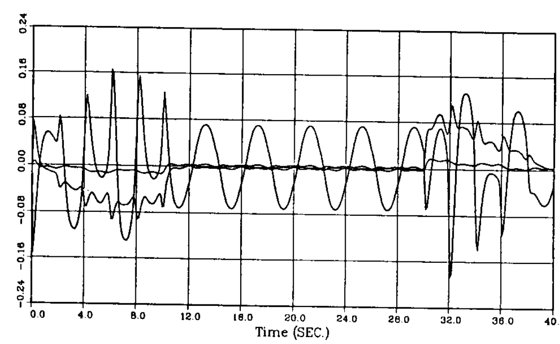

Fig.3 Non-Robust: Errors in Joint Space

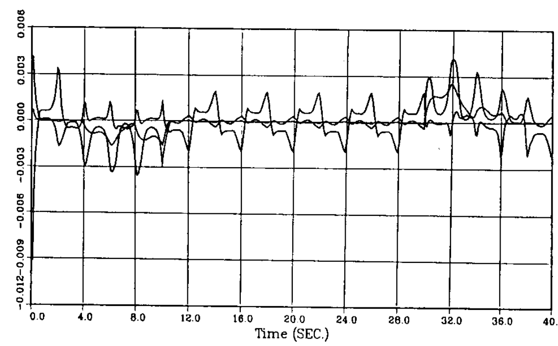

Fig.4 Robust: Errors in Joint Space

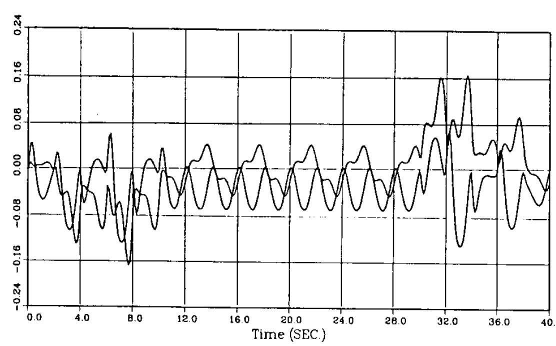

Fig.5 Non-Robust: Errors in Cartesian Space

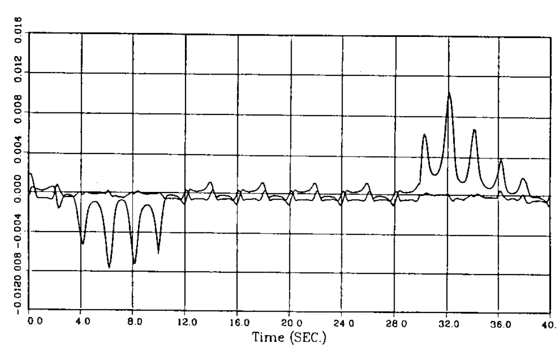

Fig.6 Robust: Errors in Cartesian Space