Proceedings of the 38th
Conference on Decision & Control
Phoenix, Arizona USA • December 1999

Adaptive Exponential Stabilization of Mobile Robots with Uncertainties

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Abstract

This paper concentrates on the discussions on stabilization of mobile robots with unknown constant-input disturbances. Continuous time-varying adaptive controllers are designed for mobile robots in a chain-form by using Lyapunov approach. With the property of homogeneous systems, uncertain mobile robots governed by the proposed control algorithms become homogeneous of order zero to achieve exponential stability. Simulation results validate the theoretical analysis.

1 Introduction

Feedback stabilization of nonholonomic mobile robots becomes challenging, because the linearized system is not controllable. It is the fact that there does not exist a smooth (or even continuous) time-invariant feedback control algorithm. Therefore, alternative discontinuous control algorithms, time-varying control algorithms or hybrid control algorithms, which combine the former two, are pursued [1]. Bloch [2] designs piecewise analytic stabilizing algorithms for nonholonomic systems, Canudas de Wit [3] proposes a piecewise smooth exponential stabilizing controller by using the circle transformation. In [4], non-smooth state transformations based on Brockett's theory are used to overcome the difficulty in stabilization [1]. Smooth time-varying control algorithms are proposed by Samson [5] to stabilize mobile robots. Thereafter, Coron [6] points out that there could exist smooth time-varying periodic stabilizing control algorithms for controllable driftless systems. In [7], Pomet explicitly constructs the control laws for a class of controllable driftless systems. However, compared with discontinuous control algorithms, which can often give exponential convergence, smooth time-varying control algorithms can only decay at an algebraic rate [8]. It is further verified that smooth time-varying control algorithms can provide time rates of convergence of at most $1/\sqrt{t}$ [5]. Hence, M'Closkey designs ρ-exponentially stable control laws by modifying asymptotic stable control algorithms [9]. With this idea, a lot of continuous time-varying control algorithms, smooth everywhere except at the equilibrium point, are proposed for chain-form systems [11-14]. In [15], these control algorithms are extended to stabilize controllable driftless systems.

But unfortunately, there are few papers in the literature for stabilizing uncertain nonholonomic mobile robots, and ever fewer for stabilizing continuously and exponentially uncertain mobile robots. In [16], Jiang designs an adaptive time varying controller for parameterized chain-form systems. Discontinuous time-invariant adaptive control algorithms are put forward to stabilize chain-form systems with matched input disturbances [17]. But in practice, discontinuous control inputs are inevitably smoothed by dynamics of the systems or the actual control circuits, which may result in the loss of the theoretical exponential stability. In this paper, with the similar thought developed in [11], we concentrate on designing adaptive continuous control algorithms for mobile robots subjected to unknown input disturbances by using Lyapunov approach, in which the equilibrium is ρ-exponentially stable.

This paper is organized as follows. In section 2, related terminology and properties of homogeneous systems are briefly revisited. Mobile robots with unknown input disturbances are modeled in section 3. In section 4, adaptive continuous time-varying control algorithms, which guarantee the system's exponential stability, are proposed. Some simulation results are given in section 5. Finally, this paper ends with some conclusions in section 6.

2 Homogeneous Systems

In this section, we briefly review related terminology and properties of homogeneous systems.

Definition 1. Let $\mathbf{x} \in R^n$, $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T$. With n positive integers $r_1, r_2, \cdots, r_n \geq 1$, we call the map

δ_λ^r for $R^n \mapsto R^n$:

$\delta_\lambda^r \mathbf{x} = (\lambda^{r_1} x_1,\ \lambda^{r_2} x_2,\ \cdots,\ \lambda^{r_n} x_n)$, where $\lambda > 0$, a dilation on R^n.

Definition 2. If a function Φ makes $\Phi \circ \delta_\lambda^r \mathbf{x} = \lambda^m \Phi\ (R^n \mapsto R)$, then Φ is called homogenous of order *m* with respect to δ_λ^r written as $\Phi \in H_m$.

Definition 3. A vector field $\mathbf{v}(\mathbf{x}) = \sum v_j(\mathbf{x})(\partial/\partial x_j)$ is homogenous of order *m* with respect to the dilation δ_λ^r, if $v_j \in H_{m+r_j}$, $j = 1,\ 2,\ \cdots,\ n$.

Definition 4. A homogeneous p-norm with respect to the dilation δ_λ^r is defined as follows.

$$\rho(\mathbf{x}) = \|\mathbf{x}\|_{\delta,p} = (\sum |x_i|^{p/r_i})^{1/p}$$

From above, homogeneous p-norm is homogeneous of order one with respect to δ_λ^r, i.e., $\rho(\mathbf{x}) \in H_1$.

Definition 5. Let $\mathbf{x} = 0$ be the equilibrium of the system $\dot{\mathbf{x}} = f(t, \mathbf{x})$. If there is a neighborhood U of the origin and two constants $c_1 > 0$ and $c_2 > 0$ such that

$$\rho(\mathbf{x}(t)) \le c_1 \rho(\mathbf{x}(0)) e^{-c_2 t}, \quad \forall t \ge 0,\ \forall \mathbf{x}(0) \in U,$$

the equilibrium is locally exponentially stable with respect to the homogeneous norm $\rho(\mathbf{x})$, denoted as ρ-exponentially stable.

Lemma 1 [10] If the vector field **F** is continuous and homogeneous of order zero, then (uniformly) asymptotically stable is equivalent to (globally) ρ-exponentially stable for the equilibrium $\mathbf{x} = 0$ of the system $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$.

3 Problem Formulation

In an ideal situation, the kinematics of a mobile robot is described by:

$$\begin{cases} \dot{x} = v \cos\theta \\ \dot{y} = v \sin\theta \\ \dot{\theta} = \omega \end{cases} \tag{1}$$

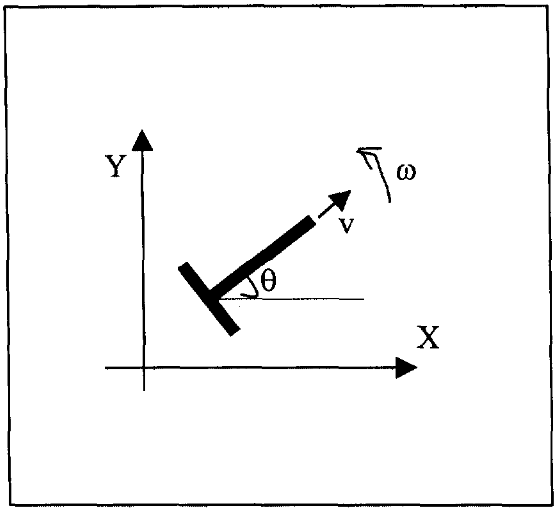

Fig. 1 The kinematics of a mobile robot

It can be transformed into a chain-form system

$$\begin{cases} x_1 = \theta \\ x_2 = x\cos\theta + y\sin\theta \\ x_3 = x\sin\theta - y\cos\theta \\ u_1 = \omega \\ u_2 = v - x_3\omega \end{cases} \tag{2}$$

with the following state transformation

$$\begin{cases} \dot{x}_1 = u_1 \\ \dot{x}_2 = u_2 \\ \dot{x}_3 = x_2 u_1 \end{cases} \tag{3}$$

Let us now consider the situation when the mobile robot is subject to unknown constant-input disturbances.

$$v_a = v + \xi_1 \quad \omega_a = \omega + \xi_2 \tag{4}$$

where v_a and ω_a are actual control inputs. With the same state transformation (3), the kinematics of the mobile robot with unknown constant-input disturbances is obtained.

$$\begin{cases} \dot{x}_1 = u_1 + \xi_2 \\ \dot{x}_2 = u_2 + \xi_1 - x_3\xi_2 \\ \dot{x}_3 = x_2 u_1 + x_2\xi_2 \end{cases} \tag{5}$$

Thus, the problem of our concern is stated as below.

For arbitrary unknown constant-input disturbances ξ_1 and ξ_2, adaptive continuous control algorithms u_1 and u_2, which guarantee the ρ-exponential *stability of the equilibrium of the system, are to be found for the system described by Eq.(5),*

4 Design of Adaptive ρ-Exponentially Stable Controllers

In the system described by Eq.(5), we let $\hat{\xi}_1$ and $\hat{\xi}_2$ be the estimated values of unknown constant disturbances ξ_1 and ξ_2 respectively, and denote $\tilde{\xi}_1 = \xi_1 - \hat{\xi}_1$, $\tilde{\xi}_2 = \xi_2 - \hat{\xi}_2$. Then Eq.(5) can be re-written as

$$\dot{\mathbf{x}} = \mathbf{X}(t, \mathbf{x}) + \mathbf{R}(t, \mathbf{x}) \tag{6}$$

where

$$\mathbf{X}(t, \mathbf{x}) = \begin{bmatrix} u_1 + \xi_2 \\ u_2 + \xi_1 - x_3 \xi_2 \\ x_2 u_1 + x_2 \xi_2 \end{bmatrix} \quad \text{and}$$

$$\mathbf{R}(t, \mathbf{x}) = \begin{bmatrix} 0 \\ -x_3 \tilde{\xi}_2 \\ 0 \end{bmatrix}$$

In the following, we firstly design ρ-exponentially stable controllers for the system $\dot{\mathbf{x}} = \mathbf{X}(t, \mathbf{x})$. For 3-dimensional chain-form systems, we introduce the following dilation.

$$\delta_\lambda \mathbf{x} = (\lambda x_1, \ \lambda x_2, \ \lambda^2 x_3) \tag{7}$$

Its corresponding homogeneous norm is given by

$$\rho(\mathbf{x}) = (x_1^4 + x_2^4 + x_3^2)^{1/4} \tag{8}$$

To design continuous control algorithms, time-varying function $\beta(t) = \sin t$ is used. Then a new variable is defined by

$$\tilde{x}_2 = x_2 - \frac{x_3}{\rho(\mathbf{x})} \beta \tag{9}$$

Corresponding to the new states $\tilde{\mathbf{x}} = (x_1, \ \tilde{x}_2, \ x_3)^T$ and the dilation Eq.(7), we give a new homogeneous norm

$$\rho(\tilde{\mathbf{x}}) = (x_1^4 + \tilde{x}_2^4 + x_3^2)^{1/4} \tag{10}$$

With Eqs. (8) and (9), we have

$$\frac{d}{dt}\rho(\mathbf{x}) = \frac{1}{\rho^3(\mathbf{x})}(x_1^3 \dot{x}_1 + x_2^3 \dot{x}_2 + \frac{1}{2} x_3 \dot{x}_3) \tag{11}$$

$$\begin{aligned} \dot{\tilde{x}}_2 &= \dot{x}_2 - \frac{\beta}{\rho(\mathbf{x})}\dot{x}_3 + \frac{x_3 \beta}{\rho^2(\mathbf{x})}\dot{\rho}(\mathbf{x}) - \frac{x_3}{\rho(\mathbf{x})}\dot{\beta} \\ &= P x_1^3 \dot{x}_1 + (1 + P x_2^3)\dot{x}_2 + (\frac{1}{2} P x_3 - \frac{\beta}{\rho(\mathbf{x})})\dot{x}_3 \\ &\quad - \frac{x_3}{\rho(\mathbf{x})}\dot{\beta} \end{aligned} \tag{12}$$

where $P = \dfrac{x_3 \beta}{\rho^5(\mathbf{x})}$. Then a Lyapunov candidate is given by

$$V(\tilde{\mathbf{x}}) = \frac{1}{2}\rho^2(\tilde{\mathbf{x}}) + \frac{1}{2\Gamma_1}\tilde{\xi}_1^2 + \frac{1}{2\Gamma_2}\tilde{\xi}_2^2 \tag{13}$$

where $\Gamma_1 > 0, \Gamma_2 > 0$. With $\dot{\tilde{\xi}}_1 = -\dot{\hat{\xi}}_1$, $\dot{\tilde{\xi}}_2 = -\dot{\hat{\xi}}_2$. Computing the time derivative of $V(\tilde{\mathbf{x}})$ along with the solutions of $\dot{\mathbf{x}} = \mathbf{X}(t, \mathbf{x})$ yields:

$$\begin{aligned} \dot{V}(\tilde{\mathbf{x}}) &= \frac{1}{\rho^2(\tilde{\mathbf{x}})}(x_1^3 \dot{x}_1 + \tilde{x}_2^3 \dot{\tilde{x}}_2 + \frac{1}{2} x_3 \dot{x}_3) - \frac{1}{\Gamma_1}\tilde{\xi}_1 \dot{\hat{\xi}}_1 - \frac{1}{\Gamma_2}\tilde{\xi}_2 \dot{\hat{\xi}}_2 \\ &= \frac{1}{\rho^2(\tilde{\mathbf{x}})}(x_1^3 + \frac{1}{2} x_2 x_3)(u_1 + \hat{\xi}_2) + \frac{\tilde{x}_2^3}{\rho^2(\tilde{\mathbf{x}})}[(1 + P x_2^3) u_2 + (1 + P x_2^3)\hat{\xi}_1 \\ &\quad + (P x_1^3 + \frac{1}{2} P x_2 x_3 - \frac{x_2 \beta}{\rho(\mathbf{x})}) u_1 + (P x_1^3 + \frac{1}{2} P x_2 x_3 - \frac{x_2 \beta}{\rho(\mathbf{x})} - x_3 - P x_3 x_2^3)\hat{\xi}_2 \\ &\quad - \frac{x_3}{\rho(\mathbf{x})}\dot{\beta}] + \tilde{\xi}_1[\frac{\tilde{x}_2^3}{\rho^2(\tilde{\mathbf{x}})}(1 + P x_2^3) - \frac{1}{\Gamma_1}\dot{\hat{\xi}}_1] + \tilde{\xi}_2[\frac{1}{\rho^2(\tilde{\mathbf{x}})}(x_1^3 + \frac{1}{2} x_2 x_3) \\ &\quad + \frac{\tilde{x}_2^3}{\rho^2(\tilde{\mathbf{x}})}(P x_1^3 + \frac{1}{2} P x_2 x_3 - \frac{x_2 \beta}{\rho(\mathbf{x})}) - \frac{1}{\Gamma_2}\dot{\hat{\xi}}_2] \end{aligned} \tag{14}$$

Now, we choose the stabilizing control algorithm:

$$\begin{cases} u_1 = -\hat{\xi}_2 - c_1(x_1^3 + \frac{1}{2} x_2 x_3)/\rho^2(\tilde{\mathbf{x}}) \\ u_2 = \dfrac{1}{(1 + P x_2^3)}[\dfrac{x_3}{\rho(\mathbf{x})}\dot{\beta} - (1 + P x_2^3)\hat{\xi}_1 - \\ \qquad (P x_1^3 + \frac{1}{2} P x_2 x_3 - \dfrac{x_2 \beta}{\rho(\mathbf{x})})(u_1 + \hat{\xi}_2) \\ \qquad - c_2 \tilde{x}_2] + x_3 \hat{\xi}_2 \end{cases} \tag{14a}$$

To cancel the influence of disturbances, the terms including $\tilde{\xi}_1$ and $\tilde{\xi}_2$ must be zero. Then the adaptive updating algorithm is obtained.

$$\begin{cases} \dot{\hat{\xi}}_1 = \dfrac{\Gamma_1}{\rho^2(\tilde{\mathbf{x}})}\tilde{x}_2^3(1 + P x_2^3) \\ \dot{\hat{\xi}}_2 = \dfrac{\Gamma_2}{\rho^2(\tilde{\mathbf{x}})}[(x_1^3 + \frac{1}{2} x_2 x_3) + \tilde{x}_2^3(P x_1^3 + \frac{1}{2} P x_2 x_3 - \dfrac{x_2 \beta}{\rho(\mathbf{x})})] \end{cases} \tag{14b}$$

With Eqs.(14a) and (14b), there is

$$\dot V(\tilde{\mathbf x}) = -c_1\frac{(x_1^3+\frac{1}{2}x_2x_3)^2}{\rho^2(\tilde{\mathbf x})} - c_2\frac{\tilde x_2^4}{\rho^2(\tilde{\mathbf x})} \le 0 \qquad (15)$$

From Eq. (15), we know $\dot V(\tilde{\mathbf x}) = 0$ iff $x_1^3 + \frac{1}{2}x_2x_3 = 0$ and $\tilde x_2 = 0$. Furthermore, with

$\tilde x_2 = x_2 - \frac{x_3}{\rho(\mathbf x)}\beta = 0$

we obtain

$$x_1^3 + \frac{1}{2}\frac{x_3^2}{\rho(\mathbf x)}\sin t = 0 \qquad (16)$$

Assume $\dot V(\tilde{\mathbf x}) = 0$ when $t = T$. Hence, we know that $\hat\xi_2$, $\dot x_1$ are constant from Eqs (14a), (14b) and (6). Without loss of generality, let $\dot x_1 = c = \text{const}$. Then $\forall t \ge T$, there is

$$x_1(t) = c(t-T) + x_1(T) \qquad (17)$$

Substituting Eq.(17) into Eq.(16) and using the fact t, $\sin t$ are linearly independent yield

$$c = 0,\ \ x_1(T) = 0,\ \ x_3 = 0 \qquad (18)$$

$x_1(T) = 0$ implies that $\dot V(\tilde{\mathbf x}) = 0$ is sufficient for $x_1 = 0$. Moreover, $\tilde x_2 = 0$ implies $x_2 = 0$. Therefore, we conclude as follows.

Proposition 1 The equilibrium $\mathbf x = 0$ of the system $\dot{\mathbf x} = \mathbf X(t,\mathbf x)$ with adaptive control algorithm (14a) and (14b) is uniformly asymptotically stable.

In order to illustrate ρ-exponential stability of the equilibrium $\mathbf x = 0$ of the system $\dot{\mathbf x} = \mathbf X(t,\mathbf x)$, we investigate the closed-loop system with the control algorithm (14a). It results in

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\end{bmatrix} = \begin{bmatrix} -c_1(x_1^3+\frac{1}{2}x_2x_3)/\rho^2(\tilde{\mathbf x})+\tilde\xi_2 \\ \frac{1}{(1+Px_2^2)}\frac{1}{\rho(\mathbf x)}[\frac{x_3}{\rho(\mathbf x)}\dot\beta + c_1(x_1^3+\frac{1}{2}x_2x_3)(Px_1^3+\frac{1}{2}Px_2x_3 - \frac{x_2\beta}{\rho(\mathbf x)}) - c_2\tilde x_2] + \tilde\xi_1 \\ x_2\tilde\xi_2 - c_1(x_1^3+\frac{1}{2}x_2x_3)x_2/\rho^2(\tilde{\mathbf x}) \end{bmatrix} \qquad (19)$$

With the definition of a new state variable $\mathbf x' = (x_1,\ \tilde x_2,\ x_3,\ \tilde\xi_1,\ \tilde\xi_2)^T$ and a dilation $\delta_\lambda \mathbf x' = (\lambda x_1,\ \lambda\tilde x_2,\ \lambda^2 x_3,\ \lambda\tilde\xi_1,\ \lambda\tilde\xi_2)$, $\lambda > 0$, the corresponding homogeneous norm is written as

$$\rho(\mathbf x') = (x_1^4 + \tilde x_2^4 + x_3^2 + \tilde\xi_1^4 + \tilde\xi_2^4)^{1/4} \qquad (20)$$

From definition 3, we know that the system expressed by Eq.(19) is homogeneous of order zero with respect to the dilation $\delta_\lambda \mathbf x'$. Therefore, $\rho(\mathbf x')$ converges to zero exponentially. Then we can write $\dot\rho(\mathbf x') = Q(t,\ \mathbf x')\rho(\mathbf x')$ and denote $m = \sup_{(t,\mathbf x')} Q(t,\ \mathbf x')$, which is apparently dependent on controller parameters c_1 and c_2.

In the sequel, we step back to the $\tilde N$-exponential stability of the original system expressed by Eq. (5). First of all, we have the following lemma.

Lemma 2[15] Let $\mathbf x = 0$ be the asymptotic stable equilibrium of the continuous homogeneous of order zero vector field $\dot{\mathbf x} = \mathbf X(t,\mathbf x)$. If each item of $\mathbf R(t,\mathbf x)$ in the perturbed system $\dot{\mathbf x} = \mathbf X(t,\mathbf x) + \mathbf R(t,\mathbf x)$ satisfies

$$|R_i(t,\mathbf x)| \le m\rho^{r_i+1}(\mathbf x),\qquad i=1,\ 2,\ \cdots,\ n,\qquad \forall \mathbf x\in U \qquad (21)$$

where U is a neighborhood of the origin, $\rho(\mathbf x)$ denotes the corresponding homogeneous norm. Then $\mathbf x = 0$ is locally $\tilde N$-exponentially stable equilibrium of the perturbed system.

Now we investigate $\mathbf R(t,\mathbf x)$ in Eq. (6). Clearly, when $|\rho(\mathbf x'(0))| \le 1$, we have $|\rho(\mathbf x'(t))| \le 1$ from ρ-exponential stability of the system. It follows

$$|x_3(t)| \le \rho^2(\mathbf x'(t)) \le 1 \qquad (22)$$

and

$$\begin{aligned}\rho^2(\mathbf x') &= (x_1^4 + \tilde x_2^4 + x_3^2 + \tilde\xi_1^4 + \tilde\xi_2^4)^{1/2} \\ &\ge (x_3^4 + \tilde\xi_2^4)^{1/2} \ge \sqrt{2}|x_3\tilde\xi_2|\end{aligned} \qquad (23)$$

i.e., $|x_3\tilde\xi_2| \le \frac{\sqrt 2}{2}\rho^2(\mathbf x')$. As long as we choose the controller parameters c_1 and c_2 such that $m \ge \frac{\sqrt 2}{2}$, $\mathbf R(t,\mathbf x)$ in Eq.(6) satisfies the condition (21). With proposition 1, lemma 1, lemma 2 and Eq. (19), we arrive at the conclusion.

Proposition 2 For the mobile robot system with unknown constant-input disturbances described by Eq.(5) with adaptive continuous time-varying control algorithms (14a) and (14b), if $|\rho(\mathbf x'(0))| \le 1$ $\mathbf x = 0$ is the ρ-exponentially stable equilibrium of the system.

Remark 1 Dividing u_1 by $\rho^2(\tilde{\mathbf{x}})$ in the control algorithm (14a) is to make the feedback system homogeneous of order zero.

Remark 2 In the expression of u_2, it can be argued that $(1+Px_2^3)$ equals to zero. In fact,

$$|Px_2^3| = \left|\frac{x_3 x_2^3 \beta}{\rho^5(\mathbf{x})}\right| = \left|\frac{x_3}{\rho^2(\mathbf{x})}\right| \cdot \left|\frac{x_2^3}{\rho^3(\mathbf{x})}\right| \cdot |\beta| < 1$$

and thus $(1 + Px_2^3) > 0$. It means that the control algorithm u_2 is significant everywhere.

Remark 3 From Eqs. (14a) and (14b), we know that the control algorithms are continuous and smooth everywhere except at $\mathbf{x} = 0$.

5 Simulation Results

To validate the proposed control algorithms, simulations are done for mobile robots using MATLAB. The sampling period is 10 ms , controller parameters are $c_1 = c_2 = 1$ and $\Gamma_1 = \Gamma_2 = 5$, and the initial state $\mathbf{x}_0 = (0,\ 0,\ 0.5)^T$. Simulation results are given in Fig.2 and Fig.3. To illustrate ρ-exponential stability of the system with the proposed control algorithms, we purposefully plot the curve of $\ln\rho(\mathbf{x}')$. The fact that the whole curve lies under some straight line implies that $\rho(\mathbf{x}')$ converges to zero exponentially.

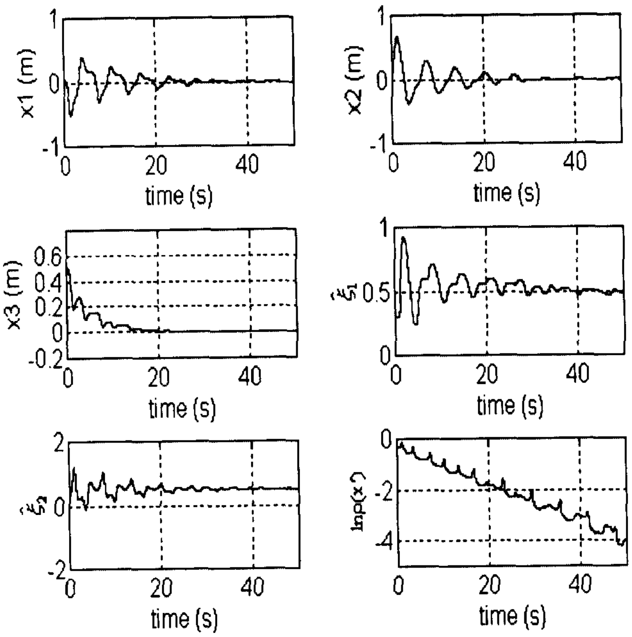

Fig.2 Time history of states and estimates of disturbances

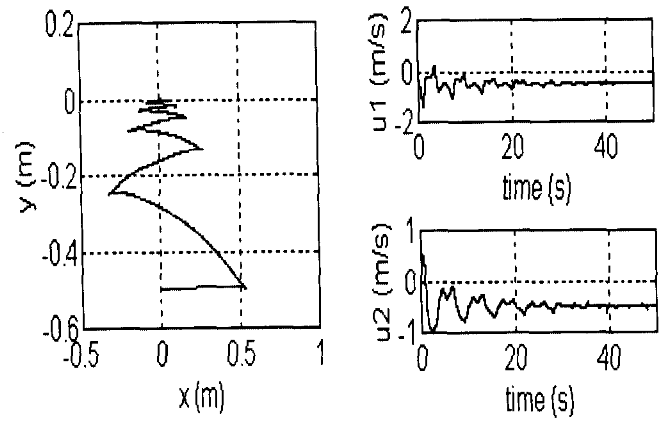

Fig.3 Phase trajectory and control inputs

6 Conclusions

This paper discusses stabilization of mobile robots with unknown constant or slowly variant input disturbances. We have designed continuous time-varying adaptive control algorithms, which are smooth everywhere except at the equilibrium. By using the proposed algorithms, the feedback closed-loop system is homogeneous of order zero. With the property of homogeneous systems of order zero, the equilibrium of mobile robots with input disturbances is ρ-exponentially stable.

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