Algorithms for industrial model predictive control

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This article is concerned with control methods that have been embedded in an industrial model predictive control software package and that have been applied to a wide variety of industrial processes. Three methods are described and the various features are evaluated by considering a constrained multivariable simulation. One method has been in use since 1988 and is widely exploited in industry. The latest methods employ quadratic programming, which has become realistic to employ because of the advances in computing. The relative attributes are contrasted by assessing the ability of the controllers to recover effectively from the impact of a large unmeasured disturbance.

The most well known algorithm for industrial model predictive control (MPC) is that of dynamic matrix control (DMC). This algorithm was developed in the late 1970s in association with the Shell Oil Company. Today DMC still retains its position as market leader in industry. There are various competitors to DMC, such as RMPCT and SMOC. The only UK developed offering that is internationally competitive is the control engineering associated with the software product Connoisseur. Another very strong UK influence in this area of technology has been generalised predictive control (GPC), although it has not been engineered into any industrial product of current day significance.

The initial exploitation of the control engineering of Connoisseur was in 1984, with application to cement kilns, spray drying towers and engine testbeds. More recent applications have related to petrochemical processes, grinding mills, power generation plant and steel annealing furnaces. Over time there has been a slow evolution in the capability of this control engineering, to keep pace with developments in numerical procedures and computing power. This article details three particular methods that are now available as options for use with MPC schemes.

The first, known as the LR method, has been in use since 1988 and employs pragmatic mechanisms to handle constraint violations. The second, known as the LRQP method utilises QP to manage MV constraints and has been in use since 1997. Finally, the full-QP method extends the QP to incorporate CV constraints and has only just been made available in the product. It is primarily a progression in computing speed that has enabled the effective exploitation of QP for control problems of significant dimension.

Table of abbreviations

ARX	auto-regressive-exogenous
CV	controlled variable
DMC	dynamic matrix control
FIR	finite impulse response
FV	feedforward variable
GPC	generalised predictive control
LQ	linear quadratic
LR	long range
LRQP	long range QP
MPC	model predictive control
MV	manipulated variable
QDMC	quadratic DMC
QP	quadratic programming
RBF	radial basis function
SVD	singular value decomposition

Control system design

The basis for the MPC described here is straightforward state-space optimal control theory. A cost function of the general form:

$$J = \sum_{i=1}^{N}(E_{i+1}^{T}.Q.E_{i+1} + U_i^{T}.P.U_i + e_i^{T}.Q1.e_i) \qquad (1)$$

is minimised, where $\boldsymbol{E}$ is a vector of CV errors, $\boldsymbol{e}$ is a vector of MV errors and U is a vector of incremental MV moves. N is chosen large enough to ensure convergence so that the solution corresponds to that for an infinite horizon. Very efficient procedures can be employed to realise the minimisation and thereby solve the companion Riccati equation. The approach generates an optimal controller that has the general form:

$$U_k = K(A1.E1k + B1.U1_k + C.V_k + B2.ek) \qquad (2)$$

where $\boldsymbol{K}$ is a matrix of controller gains that gives rise to the optimal control response and $\boldsymbol{u}$ is a vector of incremental moves. The matrices $\boldsymbol{A1}$, $\boldsymbol{B1}$, $\boldsymbol{C}$ and $\boldsymbol{B1}$ are components of the dynamic ARX or FIR description of the process cause to effect dynamics. The vectors $\boldsymbol{E1}$, $\boldsymbol{U}$ and $\boldsymbol{V}$ are sampled histories of CV errors, MV incremental moves and FV incremental moves, respectively.

The weighting matrices $\boldsymbol{Q}$, $\boldsymbol{P}$ and $\boldsymbol{Q1}$ provide the basis for tuning the character of the controller responses.

Industrial model predictive control

MPC is concerned with the operation of multivariable controllers in the face of process constraints. Such constraints come in two categories: hard constraints (MV minimum and maximum limits, MV incremental move limits) and soft constraints (CV minimum and maximum limits).

It is unusual for a multivariable controller to be viable for industrial process applications without having to cater for constraint issues. With reference to eqn. 2, each MV move is derived in the consideration that all the other MV moves can be implemented. If any MV is constrained so that the computed move is not applied, then the calculated moves for the other MVs are not appropriate and control behaviour will deteriorate (even if only one MV saturates, there will be consequent offset of all CVs from their setpoints). If a MV saturates, a degree of freedom is lost and the number of CVs that can be driven to setpoint is reduced by one.

The next sections review three methods that may be employed to deal with such constraint management issues.

LR method

The LR method employs the LQ design procedure reviewed above in association with prioritisation of CVs to provide a basis for pragmatic management of degrees of freedom in consequence of current and anticipated constraints. This method has been the backbone of most industrial applications associated with Connoisseur to date.

In catering for hard constraints associated with MVs, the procedure is to maintain the number of MVs in play to be greater than or equal to the number of CVs. If an MV saturates, then that MV is locked at the saturation limit and a reduced controller is computed to appropriately take up the remaining reduced degrees of freedom. To maintain balance, this may require a CV to be dropped from the multivariable controller.

To deal with this in a sensibly managed way, the MVs and the CVs are prioritised. The CVs of least importance are eliminated first. The MVs that are retained in play are consistent with an acceptable condition number for the steady-state gain matrix that relates to the controlled structure. The design engineer defines such priorities in consideration of the needs of the process and in consideration of the relative steady-state sensitivities of the cause to effect paths. For the latter considerations, simulation of the various options quickly provides an appropriate perspective for design, particularly when used in association with SVD tools for condition number evaluation. Given definition of the priorities, mechanisms for selection of the signals that are to be involved in the controller can then be automatic, altering as required in the face of variations in the encountered hard constraints.

The priority-based approach has the advantage that it is simple to set up on the basis of sensible judgment by the process engineer. It does not individually address, however, all the permutations of cause and effect that might arise. It is possible that certain selections might be inappropriate, for example it may be necessary to drop the highest priority CV rather than the lowest, in order to maintain an effective condition number. Fortunately, operating experience does suggest that exceptions of this nature are not common. It is important, however, that the engineer be given the facility to trap such exceptions and to override the standard procedure with more appropriate structure selections. This mechanism has been provided for the algorithms described here by an interpretative command language known as Director. Director subroutines, which are supplied with condition number and saturation status, may be called at critical stages in order to analyse and perhaps override the standard decision making procedures. Alternative cause and effect signals may also be selected for the controller. The introduction of such Director subroutines usually evolves with the experience of controller operations.

For control purposes, CVs divide into two categories, those for control to setpoints (termed setpoint CVs) and those that are to be maintained within soft constraint boundaries (termed constrained CVs). The normal situation is that a constrained CV is not addressed directly by the controller (i.e. it does not have a setpoint). At each instant for execution of the controller, a closed-loop simulation is carried out across a defined horizon into the future (the long-range or LR horizon). It is closed loop so that the simulation reflects as closely as possible circumstances that are really going to happen. Should a constrained CV be predicted to exceed any constraint boundary at any stage over the horizon, then that CV is introduced into the controller and given a setpoint. This simulation procedure is multi-pass, with soft constraints being brought in successively in consideration of priorities and available degrees of freedom as appropriate. It would be usual to allocate constrained CVs a higher priority than setpoint CVs so that in a crisis the controller diverts its attention to maintaining the process within bounds rather than maintaining required quality targets.

When a constrained CV comes under control, there is the issue of selection of an appropriate setpoint to control to. In principle this should be the constraint boundary itself (or inside it by a small comfort zone), on the presumption that an optimiser is pushing the controller to the boundaries. However, it is not wise to drive the process forcefully towards a boundary if a violation is predicted (it is going to get there anyway and extra impetus might lead to unwelcome overshoot). Therefore the setpoint is manipulated so that the CV will move gently towards the boundary (for example the setpoint might be continuously adjusted to be 30% of the way towards the boundary).

In a crisis, the controller maintains the process within bounds rather than maintaining required quality targets

The closed-loop simulation for the detection of soft constraint violations can employ either a linear or a non-linear model (e.g. a neural network RBF model), whereas the control design mechanism requires a linear model. A non-linear model provides the opportunity to predict potential constraint violations with greater accuracy with the consequent possibility of holding the process closer to constraint boundaries for improved economic benefit. This ability to employ nonlinear prediction within the MPC procedure is an important strength of this LR approach.

Note that every MV and CV combination that is encountered requires an individual solution of the Riccati equation that gives rise to a specific set of gains K for that situation. Certain combinations are encountered very frequently and it is more efficient, computationally, to save these gains in a table as they are computed. Then, when they are required again, the controller simply needs to point to the appropriate reference within the table. However, if any weighting is changed or if any signal is switched on or off, it is necessary to throw away all stored gains and recompute from scratch. For large systems, computation can be heavy in the initial stages as the tables are being created, but demand decreases significantly as the look-up mechanism phases in.

The capability to set up a comprehensive real-time simulation of the complete mechanism for this form of MPC is essential. Such simulation would normally be able to run faster than real time and provide the engineer with the capability to feel for and to tune and refine the modes of operation of the control system, prior to any encounter with the real process.

Quadratic programming

The numerical procedure of quadratic programming provides an alternative for solution of the control issue as expressed above. QP has also been employed to produce an enhanced version of DMC known as QDMC. QDMC has been exploited widely by the Shell Oil Company but has not been generally marketed. It is not clear to what extent and in what manner other market vendors exploit QP in their offerings.

The QP algorithm requires the presentation of information that encapsulates predictive behaviour across the complete design horizon N, together with the cost weightings and constraints that are to apply (i.e. a Hessian matrix, a gradient vector and constraint inequality matrices). At each control instant k, the complete problem may be resolved by just a single call to the QP procedure. In contrast to the Riccati approach, which produces a set of controller gains, the QP procedure gives rise to a profile of current and future MV moves and only the current move is employed (this mechanism is known as 'moving horizon'). With reference to eqn. 1, it is only when N is large enough to ensure convergence that a succession of first moves (as k iterates) produces the same responses as would arise by the implementation of the complete profile. In terms of time for solution, the QP methods are more demanding, particularly if N is large and the number of active constraints is significant. The choice of N becomes an issue for design. The high computational demand has been the main reason that the QP approaches have been slow to emerge in application to industrial control problems of significant scale.

Firstly, consider the MV hard constraints. MV moves computed by QP analysis are derived with consideration of the hard constraints. The approach deals with transient constrained situations more effectively than the pragmatic address utilised by the LR method. Thus the QP approach will better manage situations that involve MVs that are subject to incremental or rate of change constraint. However, if an MV locks at a constraint boundary there is still the loss of a degree of freedom and continued effective management of setpoints may still require re-computation with a reduced structure subject to CV priorities. The interpretation of 'lock' is that the QP

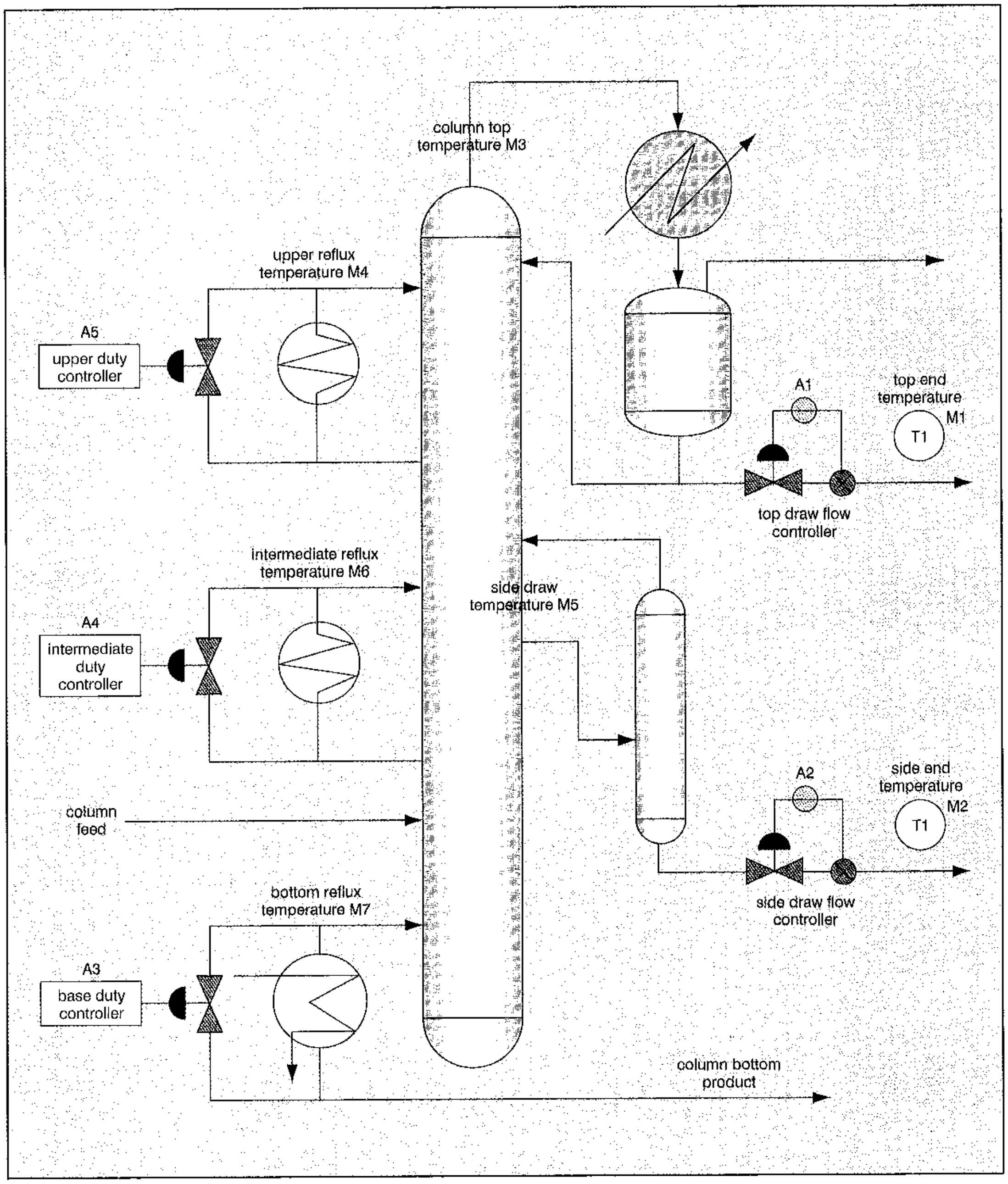


Fig. 1 Schematic of the Shell heavy oil fractionator problem

generates an MV move that is bounded at a constraint for the complete design horizon of N stages.

A QP solver may also be presented with a matrix of constraint inequalities that relate to the CVs in addition to the aspects discussed above. This provides an elegant mechanism for solving within soft constraint bounds. However, from the control engineering perspective, there are both strengths and weaknesses to the approach.

This form of QP implementation works by satisfying two objectives: firstly, to manoeuvre the CVs to be within soft constraints and, secondly, to minimise the cost function that specifies control engineering performance. Thus if the QP can satisfy a soft constraint issue by employing violent MV moves, it will do so, irrespective of cost function weightings. These exaggerated moves are simple to reduce by the use of the hard incremental move constraints. However, such constraints can then reduce unfeasibility (in the sense that no allowable set of MV

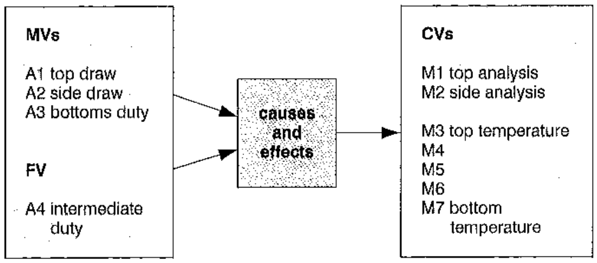


Fig. 2 Cause and effect diagram for the heavy oil fractionator

moves can bring the CVs to within their associated constraint boundaries). These perspectives make the proper selection of the MV incremental constraints a different and a far more important design issue with the QP method than with the LR method.

If the QP is not able to establish a solution that satisfies the soft constraint requirements, the issue is then unfeasible for solution and the algorithm cannot give an answer, which is of course unacceptable in an online control situation. In such a case it is necessary to relax the soft constraint boundaries to a degree that allows a solution to prevail. Such relaxation is achieved by introducing an auxiliary cost function that involves dummy MVs (or slack variables). A dummy MV adds to the soft constraint values that are within the inequality matrices that are presented to the QP, in order to broaden the range of validity. A very high cost penalty is imposed on the dummy MVs so that they are only exploited if there is no alternative, i.e. if the QP could not otherwise obtain a solution. The dummy MVs may also be assigned relative cost weightings so that certain constraints are relaxed in preference to others. However, if such relaxation arises, it is at the expense of the control engineering that is expressed in the primary cost function, and unwelcome overall effects can arise. These effects can take the form of offset from setpoints and response dynamics that are dictated by open-loop process characteristics.

LRQP method

The LRQP method restricts the constraints handled by the QP to the MV hard constraints (both incremental and absolute) whereas the soft constraints are managed in similar fashion to those of the LR method (i.e. by simulation and prioritisation). This approach was first employed in 1997 and the lack of inclusion of soft constraint considerations was then justified on the following basis:

- It avoids the need to have to address the issue of unfeasibility that can arise if CV constraints are processed. The basis for the selection of incremental MV constraints is compatible with that of the LR method.
- The dimension of the QP problem is kept to a minimum, with no CV constraint inequalities and no dummy variables to constraint relaxation. Thus, time to compute a solution is also kept to a minimum.
- The CV constraints are managed by error feedback, rather than by direct manipulation of the model in a fashion that is not cost function related.

The downside of this method of utilisation of QP, relative to the LR method, is the computational effort to achieve solution. Each control step will involve multiple passes of the QP so that the optimisation calculations have to be repeatedly implemented (the luxury of tables of gains for fast reference is not available). The approach is therefore more expensive computationally during normal operation. However, it is not subject to extensive computation during initialisation, which is a weakness of the LR method.

Each execution of the QP implicitly involves a simulation across the design horizon, since the QP delivers a complete set of MV moves for that horizon. Therefore, embedded within the QP is a basis for directly calculating future CV behaviour, without need for any further simulation and control calculations. This approach may be employed within the LRQP method as a compromise for the sake of computational efficiency. However, there is a price to pay in terms of control effectiveness because variations in behaviour in

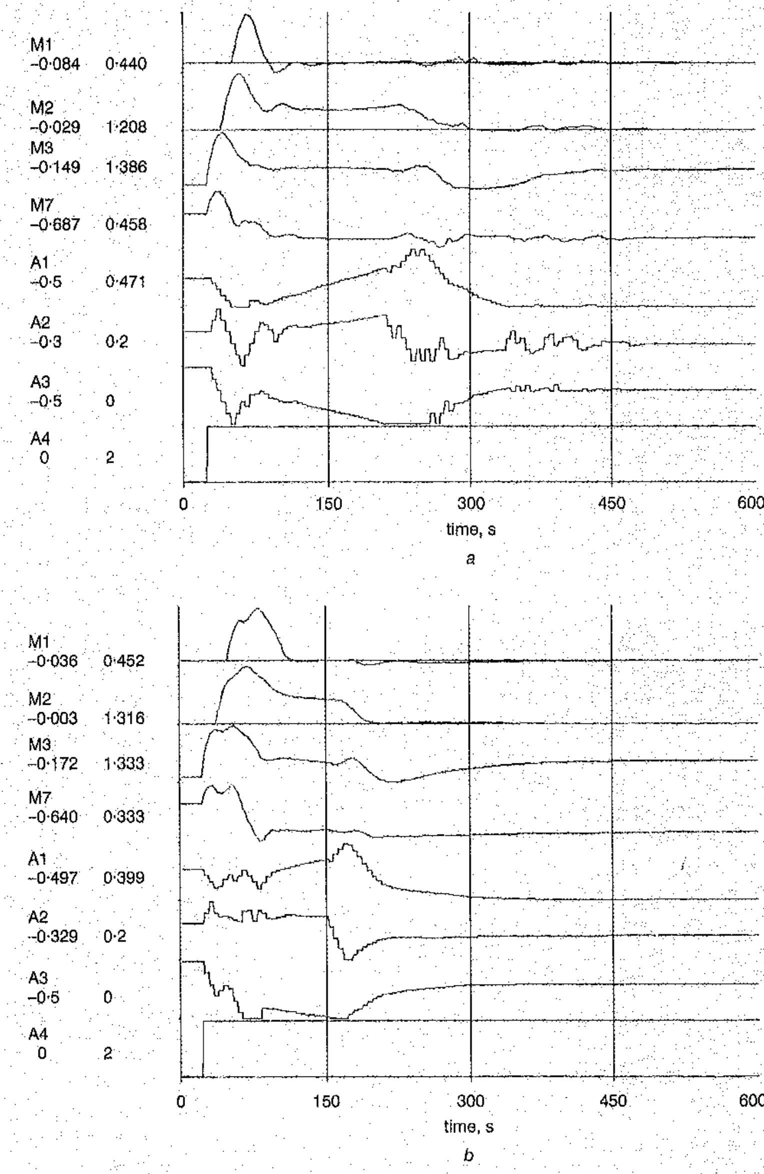
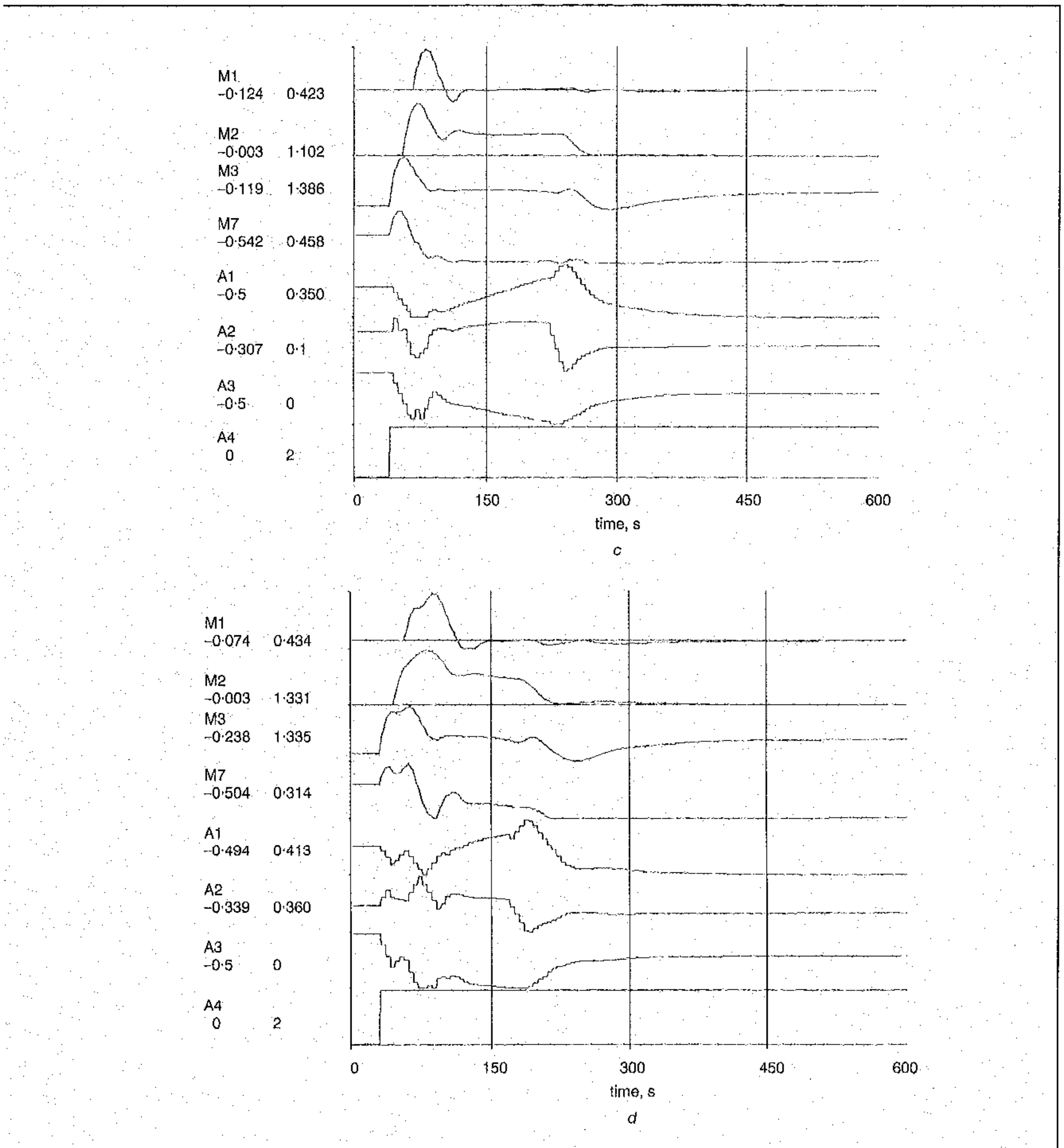


Fig. 3 Succession of responses: (*a*) the LR method; (*b*) the LRQP method (single pass); (*c*) the LRQP method (multi-pass); and (*d*) the full-QP method

managing soft constraints are not accommodated in the horizon simulation (and the ability to employ a non-linear simulation is lost).

Full-QP method

The continuing progression in the capability of low-cost computing has encouraged the recent introduction of QP managed soft constraints into the algorithms employed by Connoisseur. These have been added to reinforce rather than replace the approach for the LRQP method. Thus, the use of simulation and CV prioritisation to determine when CV soft constraints are to be incorporated is retained, as is the redirection of available degrees of freedom to manage potential CV excursions beyond constraint boundaries. The QP based CV constraint inequalities are therefore introduced to reinforce the LRQP approach, rather than to replace it. The QP constraints amount to clamps placed around the CVs to prevent excursions beyond boundaries unless an unfeasible situation exists. The benefit of this full-QP method is clearly evident when making large changes in setpoints. The LR and LRQP methods do not prevent CV



constraint boundary violations during responses to changes in setpoint (they address such violations as they are predicted to arise in order to minimise such excursions). The full-QP method prevents any such violations from arising at all. When a process is under full-QP control, it is only the influence of process disturbances that can drive CVs beyond constraints.

Simulated case studies

The 'Shell heavy oil fractionator problem' is used as the basis for simulated comparisons and examples. This problem was posed by Shell in 1987 to provide a benchmark for the assessment of multivariable control procedures. Although the model does not relate directly to any real existing process, the simulation was devised to contain all the significant elements of real fractionator control problems. A schematic of the Shell heavy oil fractionator is given in Fig. 1. In the petrochemical industry, fractionators are used to break down heavy oil feeds into lighter, more useful, components. This process occurs in distillation columns in which heated heavy oils vaporise and are separated. In the model, the heavy oil is

Table 1 Minimum/maximum values and recovery times of CVs during control response

	LR	LRQP1	LRQP2	Full-QP
M1	−0·840/0·440; 90 s	−0·036/0·452; 90 s	−0·124/0·423; 90 s	−0·074/0·434; 90 s
M2	−0·029/1·208; 250 s	−0·003/1·316; 180 s	−0·003/1·102; 250 s	−0·003/1·331; 195 s
M3	−0·149/1·386	−0·172/1·333	−0·119/1·386	−0·238/1·335
M7	−0·687/0·458	−0·640/0·333	−0·542/0·458	−0·504/0·314

fractionated into three components, drawn off from the top, side and bottom draws for products at decreasing temperatures, respectively. The model also incorporates three circulating loops (upper, intermediate and bottom reflux), which sequentially remove heat from the distillation column to achieve the desired product separation. For the standard control problem, the seven CVs are top end point analysis (signal M1), side end point analysis (both temperatures), and five column temperatures (signals M3 to M7 from column top to bottom). There are three MVs, flow rates from the top draw (signal A1) and side draw (signal A2), plus the power input measurement bottom duty (signal A3). There is a single FV, signal A4, the intermediate duty, which is also a measure of input power. A simplified structure of cause and effect is chosen, as shown in Fig. 2.

In mathematical terms, the model consists of 35 'first order plus time delay' transfer functions. These transfer functions are normalised so that all process parameters have a nominal standard range of between −0·5 and +0·5. These transfer functions appear to have been chosen deliberately so that the column is ill conditioned, i.e. column interactions make it difficult to control multiple issues simultaneously because of heavy interaction and a mix of fast and slow time constants and short and long time delays.

For the purpose of comparing the various methods of MPC that are reviewed above, an example is contrived which drives the process to an extreme constraint situation, but for which there is a final acceptable solution that can satisfy setpoint demands within both hard and soft constraint boundaries. This involves the process being initially at a steady state at the centre of the bounded region. A large and unmeasured disturbance is then applied to the intermediate duty (signal A4) and the comparison discussions are concerned with the manner in which the control engineering manages recovery.

Fig. 3 presents four sets of responses: Fig. 3*a* for the situation with the LR method; Figs. 3*b* and 3*c* with the LRQP; and Fig. 3*d* with the QP. In each situation, the configuration and tuning selections are identical and the controller updates every 5 s. The controller is required to hold the two analysers (M1 and M2) at setpoint and to maintain the other five CVs within minimum and maximum temperature bounds. In fact, only two of these temperatures, M3 (top) and M7 (bottom) prove to have relevance. The CVs are prioritised in order of importance with M7 highest, then the other temperatures including M3, then M2 and then M1. Thus the management of CV constraints takes priority over setpoints, which would be the normal application situation.

Consider Fig. 3*a* for the LR method. This shows, for a time span of 10 minutes:

- Analyser signals M1 (top) and M2 (side) and their associated setpoints which are constant at 0.
- Temperatures M3 (top) and M7 (bottom) and their associated setpoints, which are manoeuvred to accommodate anticipated soft constraint violations. The bounds are −0·5 to 0·5 for both M3 and M7.
- The three MV signals (A1, A2 and A3), which are bounded to the range −0·5 to 0·5 and to a move constraint of no more than 0·1 per update.
- The 'unmeasured' disturbance A4, which is seen to undergo a large change from 0 to 2 to the left side of the trends.

Fig. 3*a* is reviewed in detail to highlight the characteristics of the contrived problem that the various approaches are required to resolve. Progressing from left to right along the trends, detail may be observed as follows, following the disturbance impact:

- M1 peaks at 0·44 after 40 s and recovers to setpoint after 90 s, following some undershoot.
- M2 peaks at 1·208 after 35 s, but takes more than 250 s to recover to setpoint.
- M3 peaks at 1·386 after 25 s. It is quickly brought back within bounds but stays active as a soft constraint until about 190 s after the impact. It is at this point that the degree of freedom that is directed to hold M3 is released and redirected to bring M2 back to setpoint.
- M7 peaks at 0·458 after just 10 s. It then moves down to the lower constraint boundary and it thereafter remains at that boundary.
- A1 takes five steps to reach its minimum of −0·5 s and then stays locked at this minimum for a further four steps. It then freely manoeuvres for about 260 s before finally locking at the minimum of −0·5.
- A2 manipulates up and down with maximum moves of ±0·1 for a period of 60 s and thereafter moves around with less aggression at no time moving outside the range −0·3 to 0·2.
- A3 drives in sympathy with A1 to the minimum of −0·5 but then steps off immediately. It returns to this minimum after 150 s, stays there for a further 60 s and then floats away freely.

It takes about 480 s for the process to recover to a steady state. There are five main phases to this recovery:

- The first, for a period of about 60 s, is crisis management to attempt to hold the process within soft constraint bounds and the MVs move as aggressively as they can for this purpose. Setpoints for M1 and M2 are completely abandoned.
- There is then a period of some 120 s during which M2 is not controlled because both M3 and M7 are under control to prevent soft constraint violation. The MVs can be seen to be slowly moving during this phase as the longer time constants of the process prevail.
- Eventually, the constraint on M3 ceases to be predicted to be violated and the third phase is evident. The MVs then manoeuvre to bring M2 down to setpoint, which takes about 70 s.
- Thereafter, for the fourth phase, the MVs move in a gentle fashion compensating for the long time constants, maintaining both M1 and M2 at setpoint at the same time as holding M7 within soft constraint bounds.
- Eventually A1 reaches the lower limit of −0·5 and the controller is just able to hold the two setpoints and the soft constraint associated with M7, despite having only two degrees of freedom left. This is because, in the final steady state, M7 settles just inside the lower soft constraint boundary.

Now consider Fig. 3*b* for the LRQP fast method that bases soft constraint evaluation from a single pass of the QP procedure. The responses follow the same general pattern as for the LR method but with the following major differences:

- The reaction to the initial crisis is less fierce. The two temperatures are brought back within bounds more slowly and the MVs do not drive to their limits. Initial excursions of these two CVs are in general slightly larger and are outside bounds for about twice the period.
- The time for recovery to setpoint is faster at 180 s rather than 250 s.

Fig. 3*c* illustrates the LRQP method but this time with horizon behaviour being computed by multiple use of the QP to more accurately reflect the reality of the moving horizon mechanism. In this case the response timing

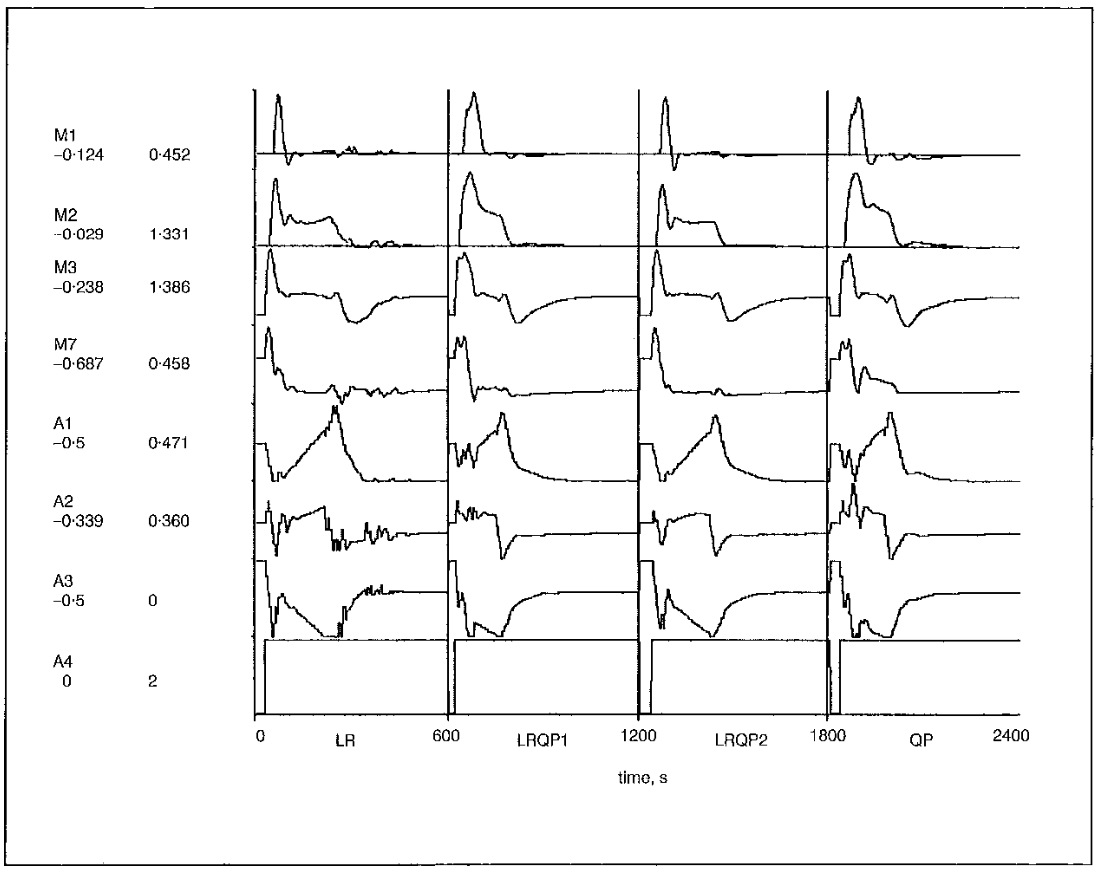


Fig. 4 Compact view of all the responses of Fig. 3

closely follows the pattern of the LR method which also re-computes the MVs for each step of the simulation horizon. The time for recovery to setpoint is back to 250 s, a price to pay for the improved management of the CV constraints during the initial crisis. A major difference from the LR method is much smoother control action during the recovery period following the initial crisis management phase and reduced amplitude in these manipulations and in the CV responses. This smoother action is as a result of better management of the MV constraints that arises from the application of QP.

Now consider Fig. 3*d* for the full-QP method. This is based on a single pass of the QP procedure and the CV constraint inequalities are processed within the QP in addition to the LRQP processing employed for the LRQP method. The only significant difference between the responses of the LRQP method (Fig. 3*b*) and the full-QP method is the behaviour of temperature M7. Under LRQP management this drops to a low of –0·64 (with the boundary at –0·5). Under full-QP management the temperature is held at the boundary with the lowest value being –0·504, this apparently being achieved through more extensive manipulation of A2.

The LR method has, to date, been the most popular because it is robust, reliable and efficient

Table 1 shows the maximum and minimum deviations of the CV responses, together with some recovery times, during the 10 minute sequence. It may be seen that the LRQP method (multi-pass) LRQP2, achieves the control objectives with, on average, the smallest deviations of the CVs. The full-QP method shows improvement on the LRQP1 method with which it is comparable, albeit at the price of slightly extended recovery time for M2. Extension of the full-QP method to include multiple passes of the LR procedure would therefore be expected to yield further improvements, albeit at a high computational cost.

Fig. 4 presents a compact view of all of the responses of Fig. 3. The LRQP method (multi-pass), LRQP2, is seen as the most effective in managing overall recovery. Excursions and manipulations are generally smoother, although the recovery of M2 to setpoint is slightly prolonged, relative to the LRQP (single pass) and the full-QP (single pass) approaches.

Conclusions

This article describes MPC algorithms that are in use to address a broad range of industrial process control situations. The LR method has, to date, been the most popular for exploitation because, in spite of pragmatism in dealing with constraint issues, it has proven to be robust, reliable and computationally efficient. The progress with the efficiency of computation (both hardware and software) now makes it practicable to consider the use of QP, at least for medium-scale industrial problems. QP provides a more elegant address for the management of constraints. The QP methods, which are a combination of the best attributes of the LR and QP approaches, have therefore been introduced. They use prioritised control engineering to manage the soft constraints and QP to deal with the manipulated variable constraints and to reinforce the soft constraints. LRQP is now considered the favoured method for most industrial applications, ideally with multi-pass closed-loop simulation if computing resources will allow. The LRQP method gives rises to a computational demand that is generally heavier than that of the LR method (particularly if the multi-pass approach is adopted), except during initialisation when a complete set of gain matrices K have to be computed for effective LR management of constraints. The strongest advantage of LRQP over LR is that no such initialisation is required if weightings or model coefficients are altered.

It is anticipated that the 'full-QP' approach will eventually supersede the LRQP. At the moment, even with the power of the latest Pentium III processor, solution times for the full-QP method during crisis management on the simulation example described above are more than five times that of the LRQP. Current development focus is being given to the QP solver itself.

Acknowledgments

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