# ROBUST PREDICTIVE ROBOT CONTROL

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*Abstract* - In the last years several control concepts have been applied to robot control. Based on the computed torque method a predictive control law is considered in this paper. It is shown that the predictive control concept is a powerful method to decrease the tracking error in robot manipulator control. Being aware of appearing model-plant mismatches the controller design-procedure must be modified to guarantee disturbance attenuation. Applying an input-output stability analysis it can be shown that the resulting control law is robust against these model-plant mismatches.

## 1 Introduction

For many years the predictive control concept is well known in the literature. The basic idea is to formulate a control signal that minimizes the weighted squared control errors in the future. Based on a known plant model future values of the output signal can be predicted. The plant model can have a parametric or nonparametric structure. Most important disadvantages using predictive controllers are the unknown stability behaviour and the fact that disturbance attenuation is neglected by this controller design method.

The first applications of predicitve control concepts in the former sense can be found in the field of chemical engineering. In early works [3, 9, 10] the basic methods are presented. The well known methods DMC (Dynamic Matrix Control) and MAC (Model Algorithmic Control) seem to be the starting point for all later investigations. Based on nonparametric plant models these concept are especially useful for application in chemical engineering. An overview of predictive control concepts with nonparametric plant models can be found in [11].

One important disadvantage using predictive controllers is that disturbance attenuation is not included in the controller design method. To overcome this problem, underlying control loops must accomplish this design goal as mentioned in [7]. This approach can also be used to guarantee closed-loop stability if the open loop is unstable. Especially the question of closed-loop stability for unstable plants is an unsolved problem using the original predictive control concepts as DMC or MAC. Applying predictive control concepts the utilization of underlying control loops seems to be heuristic and a straight forward controller design is not possible. In this paper the disturbance attenuation problem is solved using a disturbance model. Any degree of disturbance attenuation is possible extending the predictive controller design method by a adequate disturbance model. The utilization of a reference model provides some possibilities to improve the tracking error if future values of the reference trajectory are not available. Considering the rigid robot control problem, the closed loop stability of the double integrator in each axis can be guaranteed with slight modification of the original DMC method. Applying input-output stability conditions the closed-loop robustness against disturbances due to the inertia matrix is discussed. Especially the time varying inertia matrix has a crucial influence on the closed-loop robustness.

## 2 The DMC-Method with Parametric Plant Models

The DMC-method is one of basic preditive control concepts. As almost every predictive control concept this method starts minimizing a quadratic cost function

$$J_k = \sum_{i=1}^{M} q_i\, e_{k+i}^2 + \sum_{i=0}^{N} r_i\, u_{k+i}^2, \tag{1}$$

where

| | |
|---|---|
| $e_{k+i} = w_{k+i} - y_{k+i}$ | the future tracking error |
| $w_{k+i}$ | the future reference trajectory |
| $y_{k+i}$ | the future plant output |
| $u_{k+i}$ | the future control signal |
| $q_i, r_i$ | the weighting factors. |

The parameter $M$ is called the optimizing horizon and $N$ the control horizon. Considering only future values of the tracking error and the control signal is the main idea of predictive controllers. The future values of the control signal will be chosen such that the cost function $J_k$ is minimized at each sampling point. This is only possible if future values of the reference signal are available. Neglecting this effect an increased tracking error can be observed. To minimize the cost function in (1) it is necessary to calculate the future values of the plant output signal. Considering a discrete time state-space descriction

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k, \end{aligned} \tag{2}$$

assuming $M = N$ and a constant control signal over the optimizing horizont ($u_{k+i} = u_k$) future values of $y_k$ can be easily calculated

$$\underbrace{\begin{pmatrix} y_{k+1} \\ y_{k+2} \\ \vdots \\ y_{k+M} \end{pmatrix}}_{Y} = \underbrace{\begin{pmatrix} CA \\ CA^2 \\ \vdots \\ CA^M \end{pmatrix}}_{T} x_k +$$

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$$+\underbrace{\begin{pmatrix} CB \\ CAB \\ \vdots \\ CA^M B + CA^{M-1}B + \ldots + CB \end{pmatrix}}_{S} u_k \tag{3}$$

by recursively substituting of (2). On the right-hand side of (3) $Tx_k$ can be interpreted as the free motion depending on the initial state vector $x_k$. The influence of future control signals can be found in $Su_k$. The coefficients in $S$ are the values of the discrete-time step response. Using the vector matrix description the cost function in (1) can be reformulated to

$$J_k = (W-Y)^T Q (W-Y) + u_k R u_k, \tag{4}$$

where $Q$ and $R$ represent the weighting faktors $q_i$ and $r_i$. The vector $W$ represents the future values of the reference trajectory. Minimizing the cost function $J_k$ the optimal future control signal

$$u_k = \left(R + S^T Q S\right)^{-1} S^T Q \left(W - T x_k\right) \tag{5}$$

can be found. With the abbrevations

$$g^T = \left(R + S^T Q S\right)^{-1} S^T Q \tag{6}$$

$$k^T = \left(R + S^T Q S\right)^{-1} S^T Q T \tag{7}$$

the predictive control law

$$u_k = g^T W - k^T x_k \tag{8}$$

can be interpreted as a linear state feedback. The only difference appears in the scalar product $g^T W$. This can be interpreted as a FIR- filter. Condering SISO systems the predictive controller yields a combination of a state feedback and a suitably chosen FIR-filter. It can be easily seen in (8) that the M-step future value of the reference trajectory must be known. If this value is not available and only the actual value $w_k$ is used a delay time of $M$ sampling periods appears. Realising the control law in (8) a state observer or Kalman-filter is necessary, which was also mentioned in [11, 14].

Free parameters are the weighting factors $q_i, r_i$ and the optimizing horizont $M$. For the first approach the parameters can be chosen to $q_i = 1, r_i = 0$ i.e. $Q = I$ and $R = 0$. The parameter $M$ acts directly on the closed-loop dynamic. Increasing the value of $M$ a slower dynamic with a settling time of almost $3M$ sampling periods can be observed. Decreasing $M$ a faster dynamic appears causing higher control signals. With these modifications of the orginal DMC method, the predictive controller is capable of stabilizing the double integrator for $M > 1$ [12].

# 3 Predictive Control with Disturbance und Reference Model

As mentioned in the introdution, disturbance attenuation is neglected in the original predictive control concept. Disturbance attenuation, due to faster closed-loop dynamic is always possible but a powerfull reduction of the disturbance transfer function cannot be achived. Avoiding underlying control loops, a new predictive contol law securing disturbance attenuation will be presented in the sequel.

To consider the effects of external disturbances $z_k$, the state-space description in (2) must be extended to

$$\begin{aligned} x_{k+1} &= A x_k + B u_k + E z_k \\ y_k &= C x_k + F z_k. \end{aligned} \tag{9}$$

With this extension the future values of the plant output

$$Y = T x_k + S u_k + \underbrace{\begin{pmatrix} F & 0 & \ldots & 0 \\ 0 & CE+F & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ 0 & CA^M E & \ldots & CE+F \end{pmatrix}}_{S_z} \underbrace{\begin{pmatrix} z_{k+1} \\ z_{k+2} \\ \vdots \\ z_{k+M} \end{pmatrix}}_{Z} \tag{10}$$

can be obtained. Assuming that the disturbance signal is the free motion of a dynamic system with suitable initial values and no input

$$\begin{aligned} v_{k+1} &= A_z v_k \\ z_k &= C_z v_k, \end{aligned} \tag{11}$$

called disturbance model the future values of disturbance signal

$$\begin{pmatrix} z_{k+1} \\ z_{k+2} \\ \vdots \\ z_{k+M} \end{pmatrix} = \underbrace{\begin{pmatrix} C_z A_z \\ C_z A_z^2 \\ \vdots \\ C_z A_z^M \end{pmatrix}}_{T_z} v_k \tag{12}$$

can be easily calculated. The state vector of the disturbance model can be estimated using a disturbance observer. Modified in this way the predictive controller is capable of rejecting the effects of external disturbances if the disturbance dynamic is known. If no disturbance model is available a first order model can be used similar to the integral component of conventionel controllers. Considering the evaluation of the predictive control law in section 2 a good tracking performance can only be guaranteed if future values of the reference trajectory are available. If only the actual reference value $w_k$ is known a delay time of $M$ sampling periods can be observed. Considering long sampling periods this effect increases the tracking error. To overcome this problem a reference model

$$\begin{aligned} r_{k+1} &= A_r r_k \\ w_k &= C_r r_k \end{aligned} \tag{13}$$

with suitable initial values can be assumed for many applications. Based on this model the future values of the reference trajectorie

$$\begin{pmatrix} w_{k+1} \\ w_{k+2} \\ \vdots \\ w_{k+M} \end{pmatrix} = \underbrace{\begin{pmatrix} C_r A_r \\ C_r A_r^2 \\ \vdots \\ C_r A_r^M \end{pmatrix}}_{T_r} r_k \tag{14}$$

can be easily calculated and used in the predictive control law. The reference model state vector can be estimated using a reference observer.

Substituting disturbance and reference model in the cost function in (1) the minimization of $J_k$ with respect to $u_k$ yields the control law

$$u_k = \left(R + S^T Q S\right)^{-1} S^T Q \left(T_r r_k - T x_k - S_z T_z v_k\right). \quad (15)$$

# 4 Predictive Robot Control

Under rigid body assumptions, the robot dynamics can be described by a set of coupled, nonlinear second-order ordinary differential equations of the form

$$M(q, m_L)\ddot{q} + h(q, \dot{q}) = u(t), \quad (16)$$

where

| | |
|---|---|
| $M(q, m_L)$ | is the positive definite inertia matrix, |
| $h(q, \dot{q})$ | is the vector of Coriolis and centripetal forces, forces due to gravity as well as viscous friction, |
| $u(t)$ | is the torque vector, |
| $q$ | is the vector of joint coordinate positions, |
| $m_L$ | is the load mass. |

Therefore the parameter in (16) depend on the actual manipulator configuration (joint positions and velocities) as well as on the normally usually unknown load mass.

The applied control law consists of a model-based feedforward compensation of the nonlinear robot dynamic

$$u(t) = \hat{M} v + \hat{h}, \quad (17)$$

where

| | |
|---|---|
| $\hat{M}, \hat{h}$ | are linear or nonlinear models of $M, h$ |
| $v$ | is the output of a linear controller. |

This control law is a special case of nonlinear decoupling [5, 6], that is well known as the *computed torque method* [8].

Substituting this control law in (16) a linear system description with additional disturbances

$$\ddot{q} = v + \eta \quad (18)$$

is obtained, where

$$\eta = (M^{-1}\hat{M} - I)v + M^{-1}(h - \hat{h}) \quad (19)$$

represents the model-plant mismatches between $M, h$ and $\hat{M}, \hat{h}$. The linear part of the system in (18) can now be stabilized with a linear predictive controller, which has also the task of reducing the effects of the nonlinearities $\eta$.

# 5 A Robustness Approach

Considering the system in (18) the predictive controller must guarantee the closed-loop robustness against the model-plant mismatches $\eta$. Appearing in the feedforward path the time varying inertia matrix has the most important influence on the closed-loop stability. Other nonlinearities such as Coriolis, centripetal and forces due to gravity are acting as additional disturbances on the linear part of the system. Therefore their effects can be easily compensated with a linear controller and suitable chosen feedback gains.

To discuss the question of robustness against the model-plant mismatches due to the time-varying inertia matrix, input-output stability conditions are considered in the sequel. Based on the ideas in [13] and [1] an extended small gain theorem yields sufficient conditions for robust stability. As one disadvantage this conditions can only be used for continuous-time controllers. If digital controllers are used, this approach has to be modified, using discrete time operator norms instead of the well known induced operator norms for continuous time systems [4]. Using the discrete time norms the problem of estimating the nonlinearities in the continuious-time part of the system must be solved.

To overcome this problem, consider the continuous-time solution

$$x(t) = e^{A(t-kT)}x_k + \int_{kT}^{t} e^{A(t-\tau)}B(v + \eta)d\tau \quad (20)$$

of a state-space description of (18), where $T$ is the sampling time. Considering digital controllers, $v(\tau) = v_k$ is constant over the sampling intervall and so (20) can be reformulated to

$$\begin{aligned} x(t) = e^{A(t-kT)}x_k \; &+ \int_{kT}^{t} e^{A(t-\tau)}Bd\tau \, v_k \\ &+ \int_{kT}^{t} e^{A(t-\tau)}B\eta d\tau, \end{aligned} \quad (21)$$

where the first row is a standard expression on the way to a discrete time description of the linear part of the system. In the second row of (21) the model-plant mismatches appear. Neglecting friction of rest the nonlinearities $\eta$ are continuous and therefore the second row can be written as

$$\begin{aligned} \int_{kT}^{t} e^{A(t-\tau)}B\eta d\tau \;&=\; \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \int_{kT}^{t} e^{A(t-\tau)}Bd\tau \, \eta(\xi_1) + \\ &+ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \int_{kT}^{t} e^{A(t-\tau)}Bd\tau \, \eta(\xi_2), \end{aligned} \quad (22)$$

where $\xi_i$ lies in the interval $[kT, (k+1)T]$. With the abbreviations

$$\Phi = e^{AT}, \qquad \Gamma_v = \int_0^T e^{A\tau}Bd\tau$$

and

$$\begin{aligned} \Gamma_{z_1} &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \int_0^T e^{A\tau}Bd\tau, \qquad z_{1_k} = \eta(\xi_1) \\ \Gamma_{z_2} &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \int_0^T e^{A\tau}Bd\tau, \qquad z_{2_k} = \eta(\xi_2) \end{aligned} \quad (23)$$

(21) can be reformulated to

$$x_{k+1} = \Phi x_k + \Gamma_v v_k + \Gamma_{z_1} z_{1k} + \Gamma_{z_2} z_{2k}. \quad (24)$$

Considering only the time varying inertia matrix, the nonlinearities $z_i$ can be estimated by

$$|z_i| \le \alpha |v_k|. \quad (25)$$

The value of $\alpha$ can be found by searching the maximum

$$\alpha = \max_{q, m_L} ||M^{-1}\hat{M} - I||, \quad (26)$$

where $\|.\|$ denotes the spectral norm of a matrix. Using the idea of the small gain theorem the influence of the nonlinearities on the linear part of the system togther with a predictive controller can be described with the input-output relation

$$v_k = g_{s_1} * z_{1k} + g_{s_2} * z_{2k} + g_w * w_k, \tag{27}$$

where $g_{s1}, g_{s2}, g_w$ are the impulse responses of the disturbance transfer functions and the transfer function from $w_k$ to $v_k$. Using the extended discrete $L_\infty$-norm

$$\|v\|_\infty^K = \max_{0<k<K} |v_k| \tag{28}$$

and the induced operator norm

$$\beta = \sum_{k=0}^{K} |g_k| \tag{29}$$

the disturbance influence on the linear system part can be estimates by

$$\|v\|_\infty^K \leq \beta_{s_1}\|z_1\|_\infty^K + \beta_{s_2}\|z_2\|_\infty^K + \beta_w\|w\|_\infty^K. \tag{30}$$

Substituting (25) in (30) the inequality

$$\|v\|_\infty^K \leq \frac{\beta_w}{1-\alpha(\beta_{s_1}+\beta_{s_2})}\|w\|_\infty^K \tag{31}$$

can be obtained. Applying (31) the closed-loop dynamic is stable if the sufficient stability conditon

$$\lim_{K\to\infty} \alpha(\beta_{s_1}+\beta_{s_2}) < 1 \tag{32}$$

is satisfied. As shown in [2] the model-plant mismatches due to the time varying inertia matrix yields the estimate $\alpha = 0.73$ for the MANUTEC r3 robot using a constant model matrix. Therefore the discrete induced operator norm $\beta_s$ must be less than 1.37 to guarantee robust stability. It can be shown that robust stability can only be guaranteed for the dead-beat observer with a optimizing horizont $M > 15$. For a shorter horizont the induced operator norm $\beta_s$ tends to high values. This is due to the fact that the closed-loop poles of the system are conjugate complex. Therefore the impulse response tends to oscillations.

# 6 Conclusion

In this paper a predictive control concept based on a parametric plant model is applied to the robot control problem. With slight modifications this control concept is capable of stabilizing the unstable plant. Tuning guidelines for the free parameters are given. Being aware of the appearing model-plant mismatches the disturbance attenuation problem must be solved. This was possible using a disturbance model. To guarantee a good tracking behaviour a reference model is necessary if future values of the reference trajectory are not available. As shown in this paper the extension of the original predictive control concept by disturbance and reference model is very easy and seems to be the right way toward practical applications of predictive controllers.

Using input-output stability conditions, it can be shown that the predictive controller can guarantee robustness against model-plant mismatches due to the time varying inertia matrix. Located in the feedforward path the inertia matrix has a crucial influence on the closed-loop stability. As one result it can be stated that a better compensation of this matrix is necessary for a higher degree of robustness considering the MANUTEC r3 robot.

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