ASPECTS ON KINEMATIC MODELLING AND SIMULATION OF WHEELED ROBOTS

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ABSTRACT

For a three wheeled robot planned to be constructed in the near future, the direct and inverse kinematic models are built. These models are then tested for various inputs and considering several perturbations in the geometric and dynamic parameters of the robot.

KEYWORDS: Mobile Robots, Wheeled Robots, Kinematic Modelling, Dead Reckoning.

1. INTRODUCTION

A robot can only manipulate objects that it can reach. To overcome the problems caused by the limited reach of fixed robot arms, two approaches are under investigation. One is the flexible manufacturing cell, where the robot is fixed in place and the machines that it services are placed around it. A second approach is to mobilize the robot. In the car manufacturing industry for example, the car chassis is moved from cell to cell using AGV's (Automated Guided Vehicles) which follow guide wires buried in the floor of the factory. The motion of these vehicles is restricted to certain predefined paths, and once the guidance mechanism is installed it is expensive to alter. In the industries where customers expect a variety of options and fast delivery, smaller, more flexible mobile robots are needed. A way to reduce the constraints of following a physical path, although at the cost of increased computing power on the robot is the following of a preprogrammed path by dead reckoning.

Dead reckoning is the calculation of the robot's position and orientation from measurements of wheel motion, also called odometry. Dead reckoning control suffers from several sources of inaccuracy: poor mechanical alignment of wheels, slop in gears, noise in sensor signal, wheel slippage and trajectory variations due to surface unevenness. Despite of these it is still used (in conjunction with beacons to correct errors in position) because of its low cost.

2. CHAPTER 1: Kinematic models for a three wheeled robot

The structure of the robot considered in the paper is presented in figure 1. It is a three wheeled robot with two driven wheels and a castor.

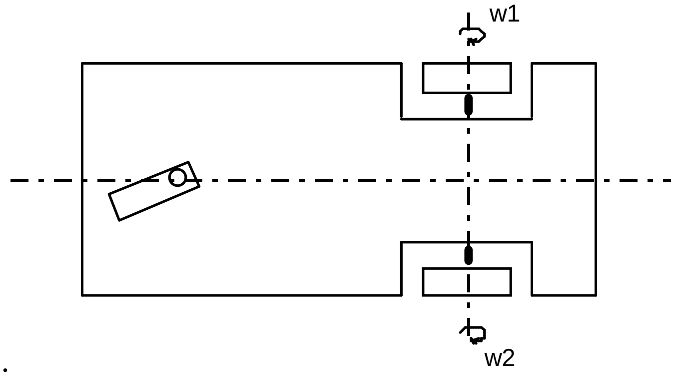

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Figure 1.

The direction of motion is determined by the speeds ω_1 and ω_2 of the two driven wheels.

The development of a kinematic model for such a structure can be done in several ways.

For example in [2] is presented the method used by Muir and Newman (1986) which use the assignment of coordinate frames (Sheth - Uicker convention) and the homogeneous transforms. While this is a rigorous approach suitable for any structure, for the simple structure considered we used a more direct method:

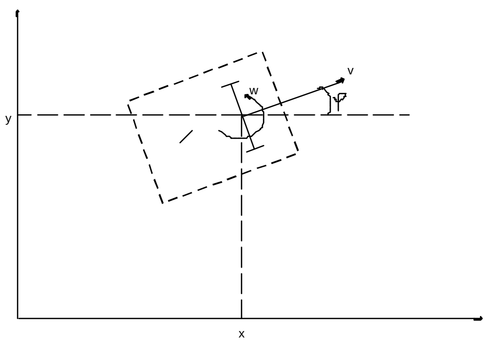


Figure 2.

With notations in figure 2 we can write:

$$\begin{cases} \dot{x} = v \cdot \cos\varphi \\ \dot{y} = v \cdot \sin\varphi \\ \dot{\varphi} = \omega \end{cases} \qquad (1)$$

Here x, y and φ are the position and orientation of the vehicle and v and ω are the linear and angular velocity of the robot.

If we consider the plan-parallel motion of a segment of length l and center **O** (see figure 3) we can write:

$$\begin{cases} v_1 = -(\frac{l}{2} - d) \cdot \omega \\ v_2 = (\frac{l}{2} + d) \cdot \omega \end{cases} \qquad (2)$$

Here point **I** is considered to be the center of rotation, the distance between **I** and **O** being d.

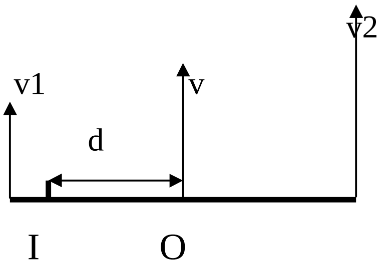


Figure 3.

Solving the system (2) for ω we obtain: $\omega = \frac{v_2 - v_1}{l}$ (3), while solving for d gives:

$$d = \frac{v_1 + v_2}{v_2 - v_1} \cdot \frac{l}{2} \qquad (4)$$

The linear speed of the center **O** is: $v = \omega \cdot d$. From (3) and (4) this yields to:

$$v = \frac{v_1 + v_2}{2} \qquad (5)$$

The linear speeds v_1 and v_2 are given by the angular speeds of the wheels ω_1 and ω_2:

$$\begin{cases} v_1 = \omega_1 \cdot R_1 \\ v_2 = \omega_2 \cdot R_2 \end{cases} \qquad (6)$$

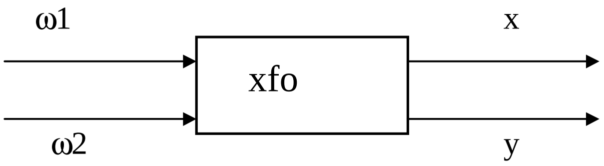
where R_1 is the radius of the left wheel and R_2 is the radius of the right wheel.

From (1), (3), (5), and (6) results the kinematic model:

$$\begin{cases} \dot{x} = \frac{\omega_1 \cdot R_1 + \omega_2 \cdot R_2}{2} \cdot \cos\varphi \\ \dot{y} = \frac{\omega_1 \cdot R_1 + \omega_2 \cdot R_2}{2} \cdot \sin\varphi \\ \dot{\varphi} = \frac{\omega_2 \cdot R_2 - \omega_1 \cdot R_1}{l} \end{cases} \qquad (7)$$

By integrating this system we can found the position of the robot based on the angular velocities of its driven wheels ω_1 and ω_2 (physically obtained with tachogenerators). The solution can be obtained assuming an initial start point: (x_0, y_0, φ_0).

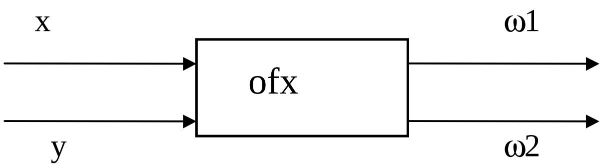
We shall symbolize this model as **xfo** (xy function of omega):



The inverse of **xfo** can be obtained solving the system (7) for ω_1 and ω_2. This yields to:

$$\begin{cases} \omega_1 = \frac{1}{R_1} \cdot \left(\frac{\dot{x}}{\cos\varphi} - \frac{l \cdot \dot{\varphi}}{2} \right) \\ \omega_2 = \frac{1}{R_2} \cdot \left(\frac{\dot{x}}{\cos\varphi} + \frac{l \cdot \dot{\varphi}}{2} \right) \\ \varphi = arctg\left(\frac{\dot{y}}{\dot{x}} \right) \end{cases} \tag{8}$$

We shall symbolize this model as **ofx** (omega function of xy):



3. CHAPTER 2: Simulation of a three wheeled robot.

With the symbols defined in chapter 1 we have considered the following scheme for the robot simulation:

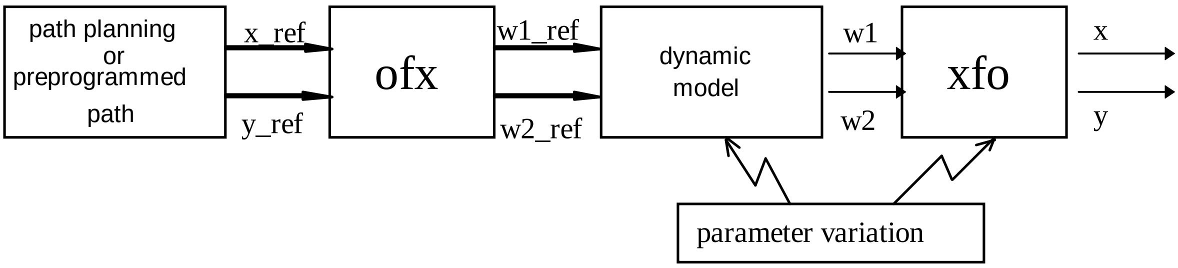


Figure 4.

ofx was solved using finite differences and **xfo** was solved using the Runge Kutta method.

We have approximated the dynamic model with a first order transfer function: $H(s) = \frac{1}{T \cdot s + 1}$.

The dimensions of the robot are: $2R_1 = 2R_2 = 275$ mm;

l = 650 mm;

The parameters of the robot considered to vary were:

l - distance between wheels

R_1, R_2 - wheels radii

T_1, T_2 - dynamic model time constants

Δl models the wheels backlash on X axis and ΔR models the tyre pressure variations.

We have studied the responses at these perturbations for linear and circular imposed trajectories, the results being plotted in fig. 5 - fig. 8.

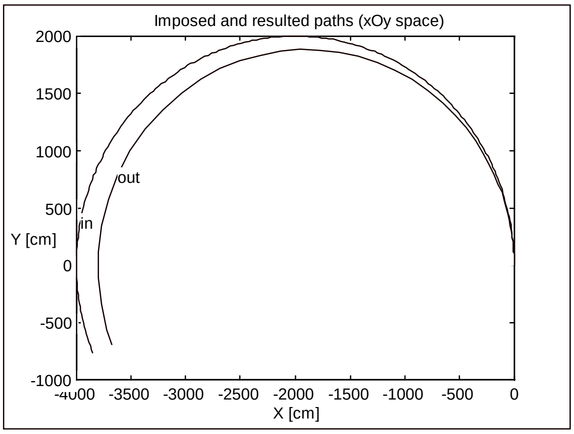


Figure 5.

Circular trajectory;

$\Delta l = 0.05l$

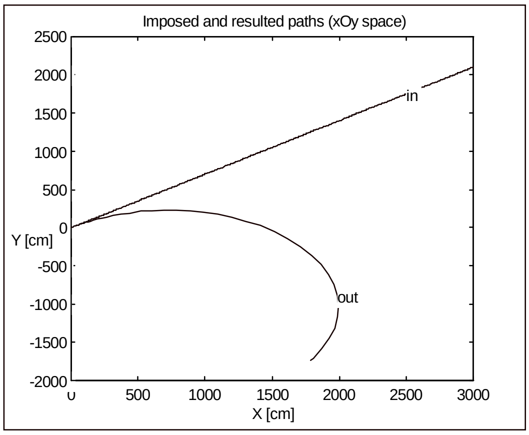


Figure 6.

Linear trajectory;

$R_2 = 0.95 R_1$

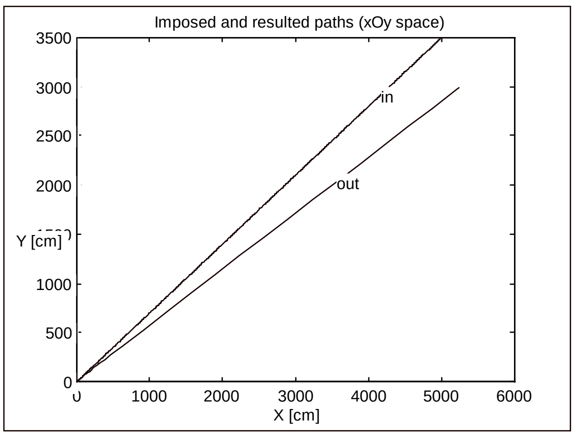


Figure 7.

Linear trajectory;

Constant speed;

$T_1 = 0.5$; $T_2 = 0.55$;

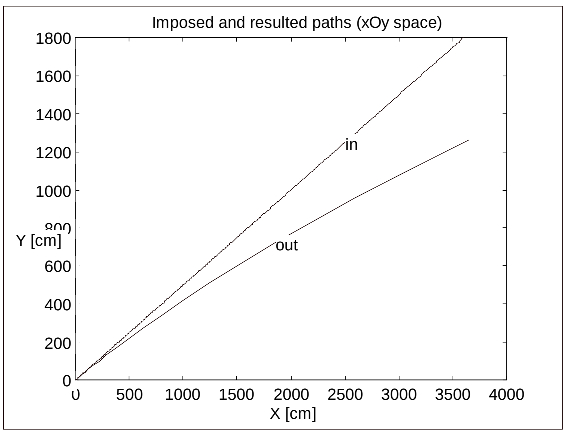


Figure 8.

Linear trajectory;

Linear variable speed;

$T_1 = 0.5$; $T_2 = 0.55$;

4. CONCLUSION

As expected, if R_1 and R_2, respective T_1 and T_2 vary in the same manner, the trajectory is not distorted but is passed across in advance or in delay with respect to the reference. This shows the importance that the parameters of the two speed loops be rigorous equal.

The Δl variations lead to distorted trajectories (except for linear trajectories) but in practice these variations are very small.

The deducted models permit both qualitative and quantitative judgments about the robot structure and they will be utilized on line to control the robot planned to be realized in our laboratory.

The approximation of the dynamic model by a first order transfer function is very rough. Yet, the simulation programs have been conceived to accept any kind of transfer function and so the model that will result from experimenting a real robot can easily be included.

A future task for us will be to design and implement certain trajectory correction algorithms based on dead reckoning and error detection systems.

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