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Optimal Feedback Position Control for an Autonomous Mobile Robot

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1 Introduction

High precision motion control of autonomous mobile robots is of crucial importance for the successful performance of their tasks. Such robots have to move in time varying environments and require extremely complex information processing algorithms on different levels. Current research interest is focussed on optimal feedback position control which consists of controlling the posture of the robot in order to reach a specified goal posture using the velocities of translation and rotation as inputs. Wheeled autonomous mobile robots underlie nonholonomic constraints and therefore they belong to a special class of nonholonomic systems. For this class of systems feedback stabilization plays a central role because they cannot be stabilized by continuously differentiable time invariant control laws. Nevertheless, alternative control techniques such as time varying feedback laws can be used [1]. In this contribution a time varying feedback control law that minimizes the input energy [2] is designed for solving the optimal feedback control problem for high precision positioning.

2 Optimal Feedback Position Control

Consider the following kinematical model for a synchronous driven autonomous mobile robot

$$\dot{\tilde{x}} = v\cos(\tilde{\theta}), \qquad \dot{\tilde{y}} = v\sin(\tilde{\theta}), \qquad \dot{\tilde{\theta}} = \omega \tag{1}$$

where the state vector $z = (\tilde{x}, \tilde{y}, \tilde{\theta})^T$ describes position, $\tilde{x}$ and $\tilde{y}$, and orientation $\tilde{\theta}$ of the robot. The inputs are the translational velocity v and the angular velocity ω. An important characteristic of this type of systems (1) is their complete controllability, i.e., they can be steered from any initial state to any final state in finite time by using finite inputs. Our purpose is to develop a feedback control law that transfers the autonomous mobile robot from any given initial state at time $t = 0$ to any prescribed final state at given time $t = T$ minimizing a certain performance index J. We apply the transformation of coordinates from z to x and the transformation of inputs from $(v, \omega)^T$ to $(u_1, u_2)^T$ suggested in [3] to system (1) obtaining a system in chained form. Together with a transformation of the initial state that allows to take as final condition $x(T) = 0$ described in [3] the optimal control problem is stated in the following terms:

From all the vectors $x(t) = (x_1(t), x_2(t), x_3(t))^T, u(t) = (u_1(t), u_2(t))^T$ with $0 \le t \le T$ that fulfill the differential equations

$$\dot{x}_1 = u_1, \qquad \dot{x}_2 = u_2, \qquad \dot{x}_3 = u_1 x_2 \tag{2}$$

and the boundary conditions

$$x(0) = x^0, \qquad x(T) = 0 \tag{3}$$

those that minimize the performance index

$$J = \int_0^T \frac{1}{2}\|u(t)\|^2 dt = \frac{1}{2}\int_0^T (u_1^2(t) + u_2^2(t))dt \tag{4}$$

are sought. The solution of the optimal control problem is denoted by

$$x^*(t) = (x_1^*(t), x_2^*(t), x_3^*(t))^T, u^*(t) = (u_1^*(t), u_2^*(t))^T. \tag{5}$$

The quadratic performance index J is to be minimized during the control process. The physical interpretation of the chosen performance index is the minimization of the input energy. As referred to in [3] from the *Maximum Principle* [2] the optimal control law can be found by minimizing the Hamiltonian with respect to the transformed inputs together with the equations for the co-states. In this particular case the system of coupled differential equations can be solved analytically and the optimal input vector $(u_1^*, u_2^*)^T$ can be also calculated analytically [3] containing free constants which

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are determined by the solution of the boundary value problem for the initial and end condition of the states. The optimal control law is transformed into an optimal feedback control law by first expressing the optimal steering inputs $u_1^*(t)$, $u_2^*(t)$ depending on the initial states x^0 and then computing the dependence on the initial conditions of the states and substituting these in the optimal control law. The optimal feedback control law

$$\begin{aligned} u_1^*(t,x(t)) &= \tfrac{1}{2}\left(x_2(t)\lambda - x_1(t)\lambda\tfrac{\sin(\lambda t)}{1+\cos(\lambda t)}\right), \\ u_2^*(t,x(t)) &= -\tfrac{1}{2}\left(x_1(t)\lambda + x_2(t)\lambda\tfrac{\sin(\lambda t)}{1+\cos(\lambda t)}\right) \end{aligned} \tag{6}$$

is obtained. This control law has singularities at $t = \frac{(2K+1)\pi}{\lambda}$ with K an integer number. As the problem was stated for a time interval $0 \le t \le T$ we chose $T \le \frac{(2K+1)\pi}{\lambda}$ and therefore during the valid time interval the first singularity is not reached.

3 Experimental Results

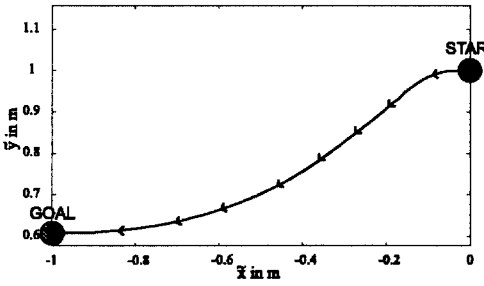

Figure 1: Driven path on the $\tilde{x}$-$\tilde{y}$-plane.

The designed optimal feedback position controller is implemented on the autonomous mobile robot B21 from Real World Interface. Several experiments have been carried out and in Fig. 1 the result for one specific task, i.e. driving backwards and sidewards simultaneously, is shown. The measured smooth trajectory of the robot for initial and final states in the movement plane and initial and final states in the transformed coordinates given by

$$\begin{array}{llll} \tilde{x}(0)=0, & \tilde{x}(T)=-1, & x_1(0)=0, & x_1(T)=0, \\ \tilde{y}(0)=1, & \tilde{y}(T)=1-\frac{\pi}{8}, & x_2(0)=1, & x_2(T)=0, \\ \tilde{\theta}(0)=0, & \tilde{\theta}(T)=0, & x_3(0)=-\frac{\pi}{8}, & x_3(T)=0 \end{array} \tag{7}$$

is depicted. In Fig. 3 the steering inputs and the evolution in time of the states in the transformed coordinates are shown for this particular experiment. Due to the fact that the inputs v and ω are combinations of u_1 and u_2 it can be deduced that these signals are also smooth. As it can be observed in Fig. 3 the states in the transformed coordinates evolve from the initial state to the zero state as required. These experimental results show that using the designed position controller the driven paths are smooth and that for the steering inputs only smooth functions are needed.

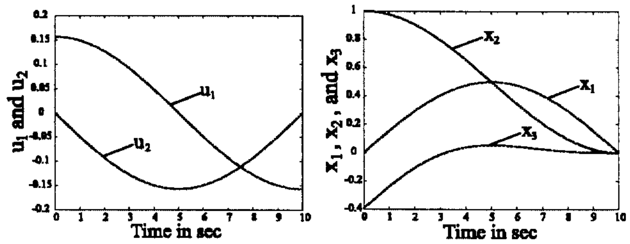

Figure 2: Steering inputs and states.

4 Conclusions

In this contribution we have developed a position controller for an autonomous mobile robot which allows to drive the robot from any initial to any final state configuration while minimizing the input energy. For this the autonomous mobile robot has been modelled as a nonlinear nonholonomic control system. A preliminary change of coordinates together with a change of inputs and a transformation of boundary conditions converts the original system into a system belonging to the class of chained systems. In these new coordinates the input vector is to be chosen to drive the system from any initial to any final state configuration while minimizing the input energy. From the analytical open loop solution the feedback control law is computed. For obtaining experimental results the designed control law is implemented on the robot B21 and several experiments have been carried out. The results are highly satisfactory because the driven paths are smooth and for the steering inputs smooth functions are used. The advantage of this approach in contrast to other time varying control laws, i.e. exponential stabilizing control laws [1], is that the driven path is smooth and that the steering inputs are also smooth functions.

References

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