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Hardware Experiment of Nonlinear Receding Horizon Adaptive Control

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Abstract

Nonlinear adaptive control algorithm is designed for a two-wheeled car by combining algorithms of nonlinear receding horizon control and nonlinear moving horizon state estimation. The experimental result shows that the proposed nonlinear receding horizon adaptive control algorithm can be implemented with the sampling frequency of 30 Hz and works satisfactorily.

Introduction

The receding horizon control is a kind of feedback control to minimize a performance index with a moving horizon and is a promising approach to control such nonlinear systems of slow dynamics as chemical processes [1]. The receding horizon control is also attractive from the theoretical point of view, because it guarantees closed-loop stability under a certain condition [2,3]. However, it is often prohibitive to implement the receding horizon control for such nonlinear systems of fast dynamics as mechanical systems.

Recently, an efficient algorithm has been proposed for nonlinear receding horizon control, and it has been applied successfully to systems of fast dynamics [4,5]. The stabilized continuation method is used to realize real-time optimization with a moderate amount of calculation and data storage. A nonlinear moving horizon state estimation algorithm [6] has also been proposed as the dual of the nonlinear receding horizon control algorithm.

The main objective of this study is to confirm that nonlinear receding horizon adaptive control is possible by combining the algorithms of nonlinear receding horizon control [5] and nonlinear moving horizon state estimation [6]. A two-wheeled car is employed as the controlled system in the hardware experiment. The state variables and an unknown parameter of the car are estimated, and state feedback control is executed by utilizing the estimated quantities in place of the unknown actual quantities.

Hardware Experiment

The nonlinear receding horizon adaptive control algorithm is designed for a two-wheeled car shown in Fig. 1. The state variables of the car are its position (x_1, x_2), attitude angle x_3, and a half of the distance between the wheels x_4. The velocities at the two wheels are given as the control inputs, and only the position measurement of the car is used in the adaptive control algorithm. The system is expressed by:

$$\begin{cases} \dfrac{dx(t)}{dt} = G[x(t)]u(t) + w(t) \\ y(t) = Cx(t) + v(t) \end{cases} \tag{1}$$

where x denotes the state vector, u the control input vector, w the unknown disturbance vector, y the measured output vector, and v the vector of unknown measurement noises, and $G(x)$ and C are defined by:

$$G(x) = \frac{1}{2}\begin{bmatrix} -\sin x_3 & -\sin x_3 \\ \cos x_3 & \cos x_3 \\ 1/x_4 & 1/x_4 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2}$$

The control objective in this experiment is to track a given reference trajectory. The performance indexes are given by:

- State estimation:

$$J_1 = \frac{1}{2}S_f[\hat{x}_4(t) - X_4(t)]^2 + \frac{1}{2}\int_{t-T_1}^{t} \{[y(t') - C_1\hat{x}(t')]^T Q_1 [y(t') - C_1\hat{x}(t')] + \hat{w}(t')^T R\hat{w}(t')\} dt' \tag{3}$$

where S_f is a scalar weight, Q_1 and R are weighting matrices, $X_4(t)$ denotes the average of $\hat{x}_4$ at previous two sampling times:

$$X_4(t) = \frac{\hat{x}_4(t-\Delta t) + \hat{x}_4(t-2\Delta t)}{2} \tag{4}$$

and the horizon T_1 is given by:

$$T_1(t) = \begin{cases} t\ (1 \le t \le 2) \\ 2\ (t > 2) \end{cases} \tag{5}$$

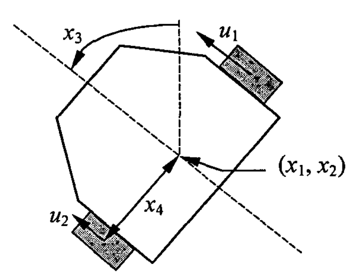

Fig. 1 Two-wheeled car.

• State feedback control:

$$J_2 = \frac{1}{2}\int_t^{t+T_2} \left\{ [p(t') - C_{ref}x(t')]^T Q_2 [p(t') - C_{ref}x(t')] + u(t')^T u(t') \right\} dt' \qquad (6)$$

where the matrix C_{ref} and the reference trajectory $p(t)$ is given by:

$$C_{ref} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (7)$$

$$p(t) = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0.3\sin(0.25t) \\ 0.25\sin(0.5t) \\ -[\pi + \mathrm{atan}(-\dot{p}_1, -\dot{p}_2)] \end{bmatrix} \qquad (8)$$

and the horizon T_2 is given by:

$$T_2(t) = 1 - e^{-0.5t} \qquad (9)$$

The function atan(a,b) $(a, b \in \mathbf{R})$ gives the arctangent of a/b and takes a value in $(-\pi, \pi)$ according to the quadrant in which a point (a, b) belongs to.

The first term in Eq. (3) is added to suppress variation of x_4, since x_4 should be constant. Furthermore, x_4 is unobservable when the car moves straight. In such a case, the first term is necessary to determine x_4 avoiding failure of the estimation algorithm. It should also be noted that the performance index Eq. (6) does not include the reference control input that realizes the reference trajectory and simply penalizes the magnitude of the control input. Although the reference control input should be included to achieve perfect tracking, the receding horizon control achieves satisfactory tracking performance with such a simple performance index without the reference control input.

The adaptive control algorithm is implemented in the C language on a computer (CPU: MMX Pentium 200 MHz) with the sampling frequency of 30 Hz. The sampling frequency is determined by the frame frequency of the CCD camera used as the sensor.

A typical experimental result is shown in Fig. 2 for time histories and in Fig. 3 for the trajectory, respectively. The initial condition is $[x_1, x_2, x_3, x_4] = [0.001, -0.064, 0.79, 0.124]$. The initial estimation error in the attitude angle x_3 is set at 0.1 [rad], and the initial estimate of the unknown parameter x_4 is set at 140% of the actual value. In spite of the initial estimation errors, the two-wheeled car follows the reference trajectory after the estimation algorithm starts at t = 1 [s]. The proposed adaptive control algorithm generates the accurate optimal solutions in the real time and achieves satisfactory performance.

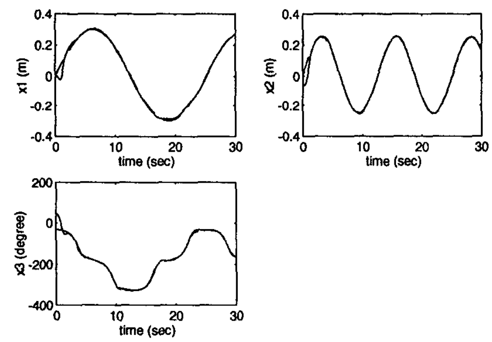

Fig. 2 Time histories of the adaptive control experiment: actual state variables (solid), and reference state variables (broken).

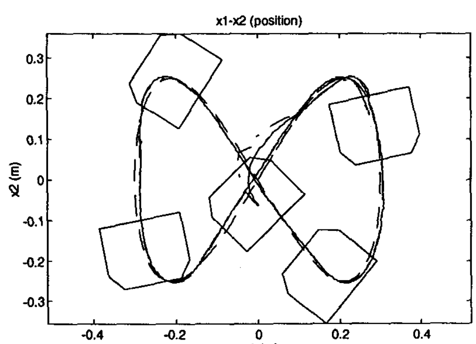

Fig. 3 Trajectory of the adaptive control experiment: estimated position (dash-dotted), actual position (solid), and reference position (broken).

Conclusions

A nonlinear adaptive control algorithm is designed by combining algorithms of nonlinear receding horizon control and nonlinear moving horizon state estimation. Numerical difficulty is avoided by adding a penalty on variation of the estimate, even if a state variable to be estimated is unobservable. The experimental result shows that the proposed nonlinear receding horizon adaptive control algorithm can be implemented for the real time control and works satisfactorily in spite of initial estimation errors.

References

[1] M. A. Henson, and D. E. Seborg (Eds.), *Nonlinear Process Control*, Prentice Hall, NJ, 1997.

[2] D. Q. Mayne, and H. Michalska, "Receding Horizon Control of Nonlinear Systems," *IEEE Trans. Automat. Contr.*, Vol. AC-35, No. 7, pp. 814-824, 1990.

[3] P. O. M. Scokaert, D. Q. Mayne, and J. B. Rawlings, "Suboptimal Model Predictive Control (Feasibility Implies Stability)," *IEEE Trans. Automat. Contr.*, Vol. 44, No. 3, pp. 648-654, 1999.

[4] T. Ohtsuka, and H. A. Fujii, "Real-Time Optimization Algorithm for Nonlinear Receding-Horizon Control," *Automatica*, Vol. 33, No. 6, pp. 1147-1154, 1997.

[5] T. Ohtsuka, "Time-Variant Receding-Horizon Control of Nonlinear Systems," *J. Guidance, Contr., and Dynamics*, Vol. 21, No. 1, pp. 174 - 176, 1998.

[6] T. Ohtsuka, "Nonlinear Receding-Horizon State Estimation with Unknown Disturbances," to appear in *Trans. Soc. Instrument and Contr. Engineers*, 1999.