Stability Analysis of Mobile Robot Path Tracking†

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Abstract

This paper presents a new approach that analyzes the stability of a general class of path tracking algorithms taking into account the pure delay in the control loop. The analysis has been done for straight paths and paths of constant curvature. This has sufficient generality since most usual paths can be decomposed in pieces of constant curvature. The method has been applied to the pure pursuit path tracking algorithm, one of the most widely used. The experiments performed with a computer controlled HMMWV show good agreement with the theoretical predictions of the proposed method.

1 Introduction

Path tracking is a basic function of most mobile robot or autonomous vehicle control systems. The objective of path tracking is to generate control commands for the vehicle to follow a previously defined path by taking into account the actual position and the constraints imposed by the vehicle and its lower level motion controllers.

The estimation of the mobile robot position can be performed using several sources of data including the perception system (image, range data, sonars) as well as internal sensors (inclinometers, gyroscopes, accelerometers) and dead reckoning techniques. Navigation in structured environments is usually based on the tracking of visual features such as lines. In this case the position estimation with respect to the path to follow is given by the perception system. Thus, there are path tracking formulations [1] which respond to a controller incorporating real time image processing to estimate the vehicle position.

In more complex situation the path to follow is previously computed by a path planner. The objective of a robot path planner is to find a path from a start position to a goal position with no collisions while minimizing a cost measure. Furthermore, there are also path generation methods which interpolate between way points computed by a global path planner [2]. In these situations the path tracker input is the path defined by the path planning/generation system.

Finally, there are also architectures in which the perception and path planning are integrated with path tracking and low level control in order to provide fast response for real time obstacle avoidance [3].

Path tracking is directly related with the lateral vehicle motion and steering control. Vehicle´s control also involves speed control. Obviously, both are coupled problems. However, path tracking has been usually studied for constant velocity. Thus, the path tracking algorithm implements a steering control law by using the error between the current estimated vehicle position/orientation and the path to follow. The inputs of the path tracker are variables defining the state of the vehicle with respect to the path, and the output is the steering command to be executed by the low-level motion controllers.

From the point of view of the control law, linear proportional feedback of errors [4][5], and PI control algorithms [6], have been implemented by using error coordinate systems. These control laws are derived using kinematic relations.

The most significant dynamic effects in the vehicle´s control come from the vehicle-terrain interaction and from the dynamic of the actuation systems. The former is hard to consider in real time due to the complex relations involved and the difficulties in sensing. However, it can be shown from experimentation that the steering actuation dynamics is relatively easy to model and has a significant impact in the path tracking performance of several vehicles [7].

On the other hand, for short intervals, the kinematic equations can be linearized for constant velocity using vehicle´s fixed coordinates or path dependent coordinates. Thus, the path tracking problem can be formulated for every value of the vehicle speed as a linear control problem and the dynamic of the steering actuation system can be considered as additional linear equations. Thus, linear control methods can be applied for the vehicle automated steering [8][9]. Recently, Generalized Predictive Control (GPC) theory has

†This research has been partially supported by the CICYT projects TAP93-0581 and TAP94-1159-E, and the grant program for stays in foreign countries from the University of Seville.

been applied obtaining good results [10] when the vehicle is close the path in position and heading, and there are not significant perturbations. However if, due to measuring errors or perturbations, the vehicles separates from the path, the linearization conditions are violated and the tracking deteriorates.

Furthermore, fuzzy control methods have been also applied with good results both for automatic tuning of other methods [11] or to generate directly the steering command by using heuristic rules [12]. This strategy leads to a nonlinear control law on the lateral error, heading and distance from the vehicle to a goal point in the path. The only problem pointed out is the difficulty for designing and tuning the fuzzy controller.

Most existing path tracking implementations are pure pursuit strategies. Due to this reason, this paper considers pure pursuit as the basic technique. However, the conclusions can be extended to other algorithms.

The results of all these control laws greatly depend on parameter tuning. Thus, oscillations or even instability can arise for some values of the controller parameters. These instability conditions are related with both the vehicle´s characteristics and navigation conditions including speed, path to follow and terrain. This paper investigates the dynamic behavior of path tracking. Particularly stability conditions of the pure pursuit algorithm are studied.

The problem is not easy due to the non linear nature of the feedback tracking system. Thus, only few results have been presented in this significant problem. In [14] a non-linear control law is presented and the stability is proved. However, the method requires the assumption of perfect velocity tracking which is unrealistic in many vehicles, particularly in outdoor navigation. In [15], a nonlinear feedback is obtained, and asymptotic input-output stability and Lagrange stability of the overall system for straight line and circular paths are proven.

Furthermore, notice that, in addition of kinematic equations and the dynamic of the steering system, path tracking involves a pure delay in the control loop. This delay is due mainly to the mobile robot position estimation, particularly when involving environment perception, as well as to other computing and communication delays. The consideration of the pure delay in the path tracking loop introduces an additional complexity in the analysis.

Most path tracking methods have a parameter related with the selection of the goal point in the path to follow. This parameter has a significant effect on the tracking performance. In the pure pursuit method this critical parameter is the lookahead distance. In this paper we study the stability of the tracking system in terms of the lookahead.

In the existing stability criteria for time-delay systems, mainly two ways of approach have been adopted. Namely, one direction is to obtain stability conditions which do not include information on the delay. This approach generally provides nice algebraic conditions [16][17]. However, not using information on the delay necessarily causes conservativeness of the criteria. Reported delay-dependent criteria consider explicitly the delay [18][19], and correspond to the trade-off between simplicity and sharpness. In this paper, stability criteria are derived directly from the trascendental equations, giving the exact stability regions.

The remaining of the paper is organized as follows. Section 2 presents the formulation of the path tracking problem. The basic stability analysis is presented in section 3. Section 4 and 5 present the stability analysis with time delays of straight and circular paths. The experiments validating the proposed method are included in Section 6. Sections 7, 8 and 9 are for the Conclusions, Acknowledgments and References.

2 Path tracking

Let (x,y,z) be the vehicle´s coordinates, (ψ, ϕ, θ) the orientation angles, and $\gamma = d\theta/ds$ the vehicle's radius of curvature. For 2D navigation the posture of the vehicle is given by (x,y,θ). Then, the control problem can be formulated in terms of driving the vehicle from its actual configurations (x,y,θ) to the desired or target configurations (x^d, y^d,θ^d).

Let the vehicle´s motion equation be given by:

$$dx = -\sin\theta ds \quad (1)$$

$$dy = \cos\theta ds \quad (2)$$

$$d\theta = \gamma ds \quad (3)$$

where γ is the vehicle´s curvature, $d\theta$ is the increment in vehicle's heading and ds is the distance travelled.The motion equations in world coordinates can also be expressed as:

$$\dot{x}_w = -V\sin\theta \quad (4)$$

$$\dot{y}_w = V\cos\theta \quad (5)$$

$$\dot{\theta} = V\gamma \quad (6)$$

where V is the longitudinal velocity, or vehicle speed, and γ is the angular velocity. These velocities can be considered as the control variables in (4)-(6). However, in some vehicles, it has been shown that the dynamic behavior of the mechanisms to apply these velocities from the control signal is very significant for the vehicle´s driving and then, should be considered for the vehicle´s control.

Let the dynamic of the steering actuation system be represented by means of the following first order model:

$$\frac{d\gamma}{dt} = -\frac{1}{T}(\gamma - \gamma_R) \quad (7)$$

where T is the time constant.

Thus, the steering control system can be represented by means of equations (4),(6) and (7) where x,θ and γ are the state variables and γ_R is the control variable computed by the steering control algorithm.

In order to simplify the equations and decrease the number of parameters in the model, consider the following nondimensional variables:

$$t' = \frac{t}{T} \qquad x_w' = \frac{x_w}{VT} \qquad \theta' = \theta \qquad \gamma' = VT\gamma \quad (8)$$

and the nondimensional lookahead is:

$$L' = \frac{L}{VT} \quad (9)$$

In this paper, the nondimensional form of the equations is used, and the $'$ in variables and parameters is skipped for clarity. The equations are:

$$\begin{aligned} \dot{x} &= -\sin\theta \\ \dot{\theta} &= \gamma \\ \dot{\gamma} &= -\gamma + \gamma_R \end{aligned} \quad (10)$$

where γ_R is the non-dimensional form of the control law.

2.1 Pure pursuit path tracking

The pure pursuit strategy [4] is based on very simple geometric considerations. The path is tracked by repeatedly fitting circular arcs to different goal points on the path as the vehicle moves forward. The steering command is the requested new curvature (γ_R=1/r) which can be computed by:

$$\gamma_R = \frac{2x}{L^2} \quad (11)$$

where x is the x displacement of the goal point in vehicle coordinates (lateral displacement) and L is the lookahead distance. Thus, the pure pursuit method is a proportional controller of the steering angle using the lookahead for the gain and the local coordinate x as the error. It is necessary to choose a good goal point in the map according with the current navigation conditions. In fact, if the goal point is too far, the vehicle may cut corners and if too near, oscillations may result. As we will see in the next section there is a minimum value of L to maintain the stability of the system. Observe how for $L \to \infty$ in (11) the controller gain tends to zero and no steering corrections at all are introduced. However, for short lookahead the gains is high.

To conclude this section note that the conventional pure pursuit strategy does not consider the vehicle's dynamics; however from experimentation it can be shown that the steering dynamics (7) has a significant influence on the behavior of the vehicle. Thus, this dynamics will be considered in the following sections.

3 Closed Loop Basic Stability Analysis

Local stability of an equilibrium state of a nonlinear system may be examined by stability of the linearized system around the equilibrium state. This analysis apply only in the neighborhood of the equilibrium state. In the path tracking problem, the vehicle is trying to follow the path, so the vehicle's state will be in the vicinity of the equilibrium state, and therefore the above assumption is acceptable.

Consider the motion equations in non-dimensional form (10). The linearized system around the origin is:

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \dot{\gamma} \end{bmatrix} = J \begin{bmatrix} x \\ \theta \\ \gamma \end{bmatrix} \quad (12)$$

J is the Jacobian of the nonlinear system:

$$J = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ \varphi_x & \varphi_\theta & (\varphi_\gamma - 1) \end{bmatrix} \quad (13)$$

where $\varphi_x = \left.\frac{\partial \gamma_R}{\partial x}\right|_0$, $\varphi_\theta = \left.\frac{\partial \gamma_R}{\partial \theta}\right|_0$ and $\varphi_\gamma = \left.\frac{\partial \gamma_R}{\partial \gamma}\right|_0$ are the partial derivatives of γ_R at the origin.

Stability problems for linear systems can be reduced to the analysis of roots of the characteristic equations. A given system is said to be stable if all the roots of the characteristic equation have negative real part, and unstable if there exist at least one root with positive real part.

Let $P(s)$ be the characteristic polynomial of the linearized system:

$$P(s) = |sI - J| = s^3 + s^2 - s^2\varphi_\gamma - s\varphi_\theta + \varphi_x \quad (14)$$

The loss of stability will occur when one root crosses the imaginary axis. This condition can be obtained from the equation $P(j\omega) = 0$, where $j = \sqrt{-1}$. Applying this condition for the real and imaginary parts:

$$\begin{aligned} -\omega^2 + \omega^2\varphi_\gamma + \varphi_x &= 0 \\ -\omega^3 - \omega\varphi_\theta &= 0 \end{aligned} \quad (15)$$

and eliminating ω, the stability is given by the equation:

$$\varphi_\theta(1 - \varphi_\gamma) + \varphi_x \geq 0 \quad (16)$$

3.1 Pure Pursuit Basic Stability

Let us consider the tracking of a straight line as shown in Figure 1. In this case, the steering command γ_R is:

$$\gamma_R = \frac{2}{L^2}\left[x\cos\theta - \sqrt{L^2 - x^2}\sin\theta\right] \quad (17)$$

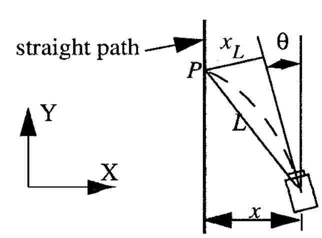

Figure 1 Pure pursuit straight path tracking

and

$$\varphi_\theta = \frac{-2}{L} \qquad \varphi_x = \frac{2}{L^2} \qquad \varphi_\gamma = 0 \tag{18}$$

So, the stability condition is:

$$L \geq 1 \tag{19}$$

4 Tracking constant-curvature paths

The expressions that have been obtained in Section 3 refers to the tracking of a straight line. Consider now the tracking of a constant curvature path. Tracking these paths, is more convenient to express the equations in polar coordinates (see Figure 2).

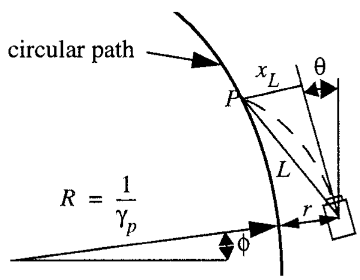

Figure 2 Pure pursuit circular path tracking

Variables are defined as follows: r is the radial distance from the path, θ is the angle between the vehicle heading and the normal to the $\vec{r}$ vector and the curvature γ is $\gamma = \gamma_1 - \gamma_p$, where γ_1 is the absolute curvature, and $\gamma_p = 1/R$ is the constant path curvature.

In this coordinate system, the nondimensional equations of motion are:

$$\begin{aligned} \dot{r} &= -\sin\theta \\ \dot{\theta} &= \gamma + \gamma_p - \frac{\gamma_p \cos\theta}{1+\gamma_p r} \\ \dot{\gamma} &= -(\gamma + \gamma_p) + \gamma_R \end{aligned} \tag{20}$$

Tracking of straight paths is a particular case with $\gamma_p = 0$.

5 Stability analysis with time delays

Let consider now there is a pure delay (τ) in the feedback control loop, due to computing and communication delays. Now the steering command γ_R is delayed τ with respect to the other variables. To obtain the stability conditions, the system is locally linearized around the origin:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\theta}(t) \\ \dot{\gamma}(t) \end{bmatrix} = J \begin{bmatrix} x(t) \\ \theta(t) \\ \gamma(t) \end{bmatrix} + J_\tau \begin{bmatrix} x(t-\tau) \\ \theta(t-\tau) \\ \gamma(t-\tau) \end{bmatrix} \tag{21}$$

The Jacobian matrices are:

$$J = \begin{bmatrix} 0 & -1 & 0 \\ \gamma_p^2 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \qquad J_\tau = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \varphi_r & \varphi_\theta & \varphi_\gamma \end{bmatrix} \tag{22}$$

where J_τ is the Jacobian with respect to the delayed variables.

Let $Q(s)$ *be* the characteristic quasi-polynomial of the linearized system. $Q(s)$ is defined as:

$$Q(s) = det[sI - J - J_\tau e^{-s\tau}] \tag{23}$$

It can be shown that the condition for the asymptotic stability of solutions of linear equations with delayed arguments is that the real parts of all roots of the characteristic quasi-polynomial be negative. So, the stability condition is $Q(j\omega) = 0$. For system (20), $Q(s)$ is:

$$\begin{aligned} Q(s) &= s^3 + s^2 + \gamma_p^2 s + \gamma_p^2 + \\ &\quad \left[-s^2\varphi_\gamma - s\varphi_\theta + \left(\varphi_r - \gamma_p^2 \varphi_\gamma \right) \right] e^{-s\tau} \end{aligned} \tag{24}$$

Provided that $e^{-j\omega\tau} = \cos(\tau\omega) - j\sin(\tau\omega)$, we obtain two conditions for the real and imaginary parts:

$$\begin{aligned} &-\omega^2 + \left(\omega\varphi_\gamma + \varphi_r - \gamma_p^2 \varphi_\gamma \right) \cos(\tau\omega) - \\ &\qquad \omega\varphi_\theta \sin(\tau\omega) + \gamma_p^2 = 0 \\ &-\omega^3 + \gamma_p^2 \omega - \omega\varphi_\theta \cos(\tau\omega) + \\ &\qquad \left(-\omega^2\varphi_\gamma - \varphi_r + \gamma_p^2 \varphi_\gamma \right) \sin(\tau\omega) = 0 \end{aligned} \tag{25}$$

These equations define the stability conditions of (20).

5.1 Pure pursuit

Consider the tracking of the curvature constant path shown in Figure 2. For pure pursuit path tracking, γ_R is:

$$\gamma_R = \left[\frac{2}{L} \frac{\gamma_p (r^2 + L^2) + 2r}{2L(1+\gamma_p r)} \cos\theta - \sqrt{1 - \left(\frac{\gamma_p (r^2 + L^2) + 2r}{2L(1+\gamma_p r)} \right)^2} \sin\theta \right] \tag{26}$$

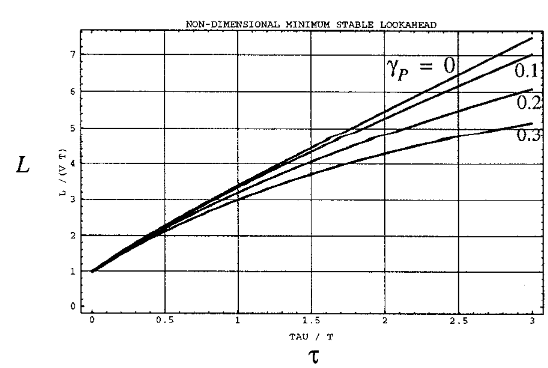

Figure 3 Circular path stable lookahead limits

and

$$\varphi_r = \frac{2}{L^2}\left(1 - \frac{\gamma_p^2 L^2}{2}\right) \quad \varphi_\theta = -\frac{2}{L^2}\sqrt{1 - \frac{\gamma_p^2 L^2}{4}} \quad \varphi_\gamma = 0 \quad (27)$$

Substituting (27) in (25), the stability conditions are:

$$\begin{aligned} & -\omega^2 + \frac{2}{L^2}\left(1 - \frac{\gamma_p^2 L^2}{2}\right)\cos(\tau\omega) + \\ & \qquad \frac{2}{L}\sqrt{1 - \frac{\gamma_p^2 L^2}{4}}\omega\sin(\tau\omega) + \gamma_p^2 = 0 \\ & -\omega^3 + \gamma_p^2\omega + \frac{2}{L}\sqrt{1 - \frac{\gamma_p^2 L^2}{4}}\omega\cos(\tau\omega) - \\ & \qquad \frac{2}{L^2}\left(1 - \frac{\gamma_p^2 L^2}{2}\right)\sin(\tau\omega) = 0 \end{aligned} \quad (28)$$

The stable limit values of the nondimensional lookahead L are shown in Figure 3 for several path curvatures.

Figure 4 RedZone's HMMWV

6 Experimental Results

A series of test runs were performed in order to determine the validity of the analysis. The testbed used was a computer controlled HMMWV (High Mobility Multi-purpose Wheeled Vehicle) from RedZone Robotics, Inc.

The tests were carried out in a 500 m. long and 10 m. width road. This road is almost straight, with a curved section of small curvature. Therefore, the theoretical limits for $\gamma_P = 0$ have been used. The estimated time delay for the test vehicle is $\tau = 0.55s$. This includes computing, communication and actuator delays. The time constant of the steering system is estimated to be $T = 1.3s$.

The objective of the tests was to determine the value of the lookahead that was the limit between stable and unstable motion. In each test, the vehicle was commanded to follow the center path of the road, starting at the same point at the beginning of the path. When the motion was unstable (see Figure 5), small deviations from the path were amplified quickly, and the supervisor had to take control of the vehicle to avoid hitting obstacles on the sides of the road. When the motion was stable (see Figure 6), the vehicle tracked the road path with considerable oscillation around it. This is due to the fact that we used lookaheads only slightly higher than the ones that made the trajectories unstable.

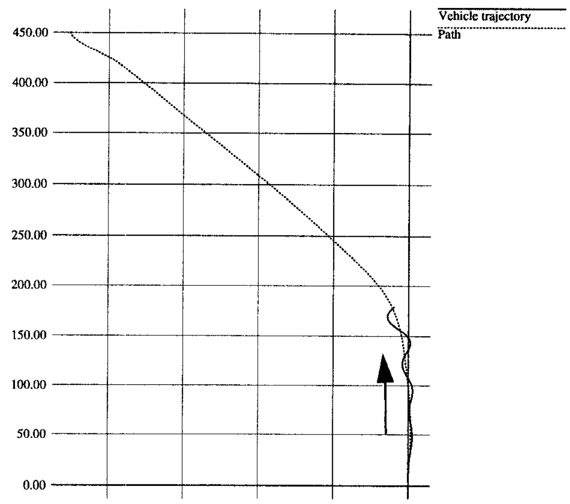

Figure 5 Unstable trajectory

Several runs were performed for three vehicle velocities (3, 6 and 9 m/s). For each velocity, the minimum lookahead that made the motion stable and the maximum lookahead that made it unstable were determined. The stability limit will be between these two values.

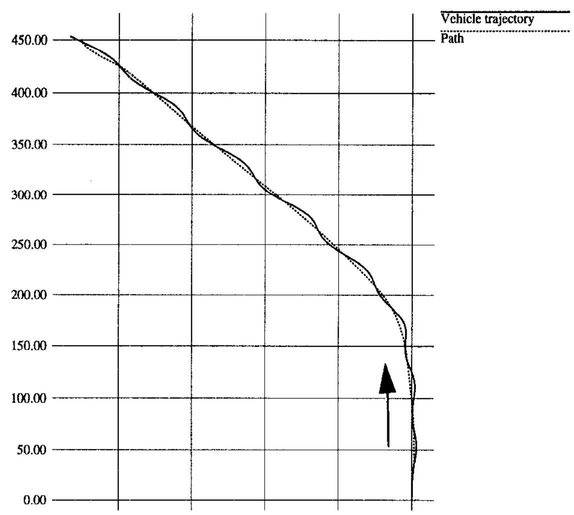

Figure 6 Stable trajectory

The results are presented in Figure 7. The two lines show the stability limits, considering the time delay and without doing it. The bars show the experimental stability limits: the bottom point is the maximum unstable lookahead and the top point is the minimum stable lookahead. The obtained values are close to the limit lookaheads predicted with the stability analysis considering the delay, and are much higher than the theoretical limits without considering the time delay. The discrepancies can be due to the nonlinear phenomena not modelled in the equations (like tire slippage) and the estimation of the parameters.

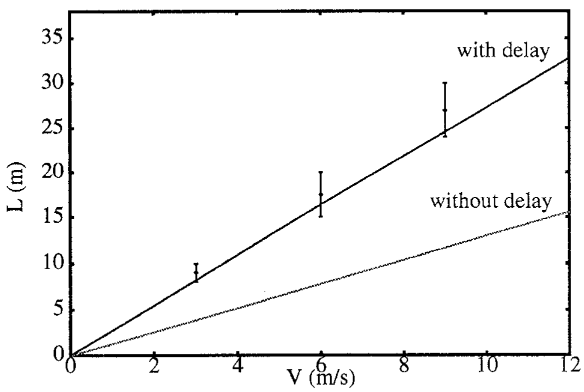

Figure 7 Experimental and theoretical stability limits

7 Conclusions

Path tracking implementations tipically involves a pure delay in the control loop that can affect significantly its stability.

In this paper, we have presented a new approach that analyzes the stability of a general class of path tracking algorithms taking into account the time delay. The analysis has been done for straight paths and paths of constant curvature. This has sufficient generality since most usual paths can be decomposed in pieces of constant curvature.

The method has been applied to the pure pursuit path tracking algorithm, one of the most widely used. The experiments performed with a computer controlled HMMWV show good agreement with the theoretical predictions of the proposed method.

This technique can be used in developing path tracking algorithms for real time control and supervision, and in adaptive lookahead strategies. Future work include a more accurate determination of parameters, more testing with different time delays, the analysis of another path tracking algorithms and the inclusion of more terms of the dynamics in the equations.

8 Acknowledgments

We gratefully acknowledge assistance from Omead Amidi at the VASC (Robotics Institute, CMU), and Ben Motazed, Jeff Callen and Demetri Patukas from RedZone Robotics, Inc. (Pittsburgh, PA, USA)

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