PATH-FOLLOWING AND POINT-STABILIZATION CONTROL LAWS FOR A WHEELED MOBILE ROBOT

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ABSTRACT

Two main problems, concerning wheeled mobile robots, have attracted extensively the attention of many authors in the literature: the path following and the stabilization problems. In this paper, we present a nonlinear pure-state feedback based on partial state feedback linearization and Lyapunov method for the path following problem, and a discontinuous time-varying state feedback for the stabilization problem. The desired path is described by the motion of a fictitious reference robot with the same kinematics constraints as the real mobile robot.

1. INTRODUCTION

It is well known that wheeled mobile robots are nonholonomic systems, characterized by nonintegrable constraint equations involving the time derivatives of the configuration variables. This fact can be assigned to the restricted mobility in the direction of wheels axis which prevents the robot to move sideways. Therefore, the number of degrees of freedom is less than the number of the configuration variables that are necessary to completely describe the kinematics behavior of such systems. In this paper a kinematics model based on reference mobile robot tracking is used to derive a nonlinear stationary control law (i.e. pure-state feedback) for the path following problem. In the same time the kinematics model for the stabilization problem is derived from the first model by vanishing the linear and rotational velocities (v_r and $\dot{\theta}_r$) of the reference mobile robot. At the first view, it seems obvious that the stabilization problem is a particular case of the path following problem when the reference robot is stopped. Unfortunately, it is not possible to control the position and orientation errors when the linear and rotational velocities of the reference robot are equal to zero. For this reason, the stabilization of nonholonomic systems to an arbitrary configuration in the state space has received much attention in the literature; especially, since Brockett [1] has proved that this class of systems, in Cartesian space, can not be stabilized via smooth time-invariant state feedback. However, other solutions based on continuous time-varying or (and) discontinuous time-invariant state feedback have been proposed in the literature (see for example [6-8], [14] and [9]).

In this work we propose an hybrid state feedback (i.e. variable structure and time-varying controller) to solve the point-stabilization problem.

This paper is organized as follows: In section II, we present a nonlinear pure state feedback for the path following problem. In section III we propose a variable structure time-varying state feedback for the parking problem. In section IV, simulation results are given to highlight the effectiveness of the proposed controllers. Section V concludes the paper.

2. PATH-FOLLOWING PROBLEM

2.1. Problem Statement

The mobile robot under consideration is a unicycle-like vehicle. The motion control of this vehicle can be achieved by dealing with the linear and rotational velocities (v, $\dot{\theta}$). It is assumed that the vehicle moves on a horizontal ground. Its kinematics model is derived under rolling-without-slippage assumption. The desired path is represented by a fictitious reference robot with the same nonholonomic constraints. The configuration of the real robot is described by its orientation θ and the position of the point M located at mid-distance of the rear-wheels. (M_r, θ_r) is the equivalent configuration of the reference robot (Fig. 1).

The main objective is to superpose the real robot and the reference one by vanishing the error configuration $X_e = (x_e, y_e, \theta_e)$. The following notations are used :

$R_0 = (O, \vec{i}_0, \vec{j}_0)$: fixed global frame.

$R_1 = (M, \vec{i}_1, \vec{j}_1)$: mobile frame linked to the real robot.

(x_e, y_e): coordinates of the position-error-vector $\overrightarrow{MM_r}$ in the basis of the frame R_1 .

θ, θ_r: respectively, the orientation of the real robot and the reference one with respect to $\vec{i}_0$.

v, v_r: respectively, the linear velocity of M and M_r.

$\dot{\theta}, \dot{\theta}_r$: respectively, the rotational velocity of the real robot and the reference one.

$\theta_e = \theta - \theta_r$: the orientation error.

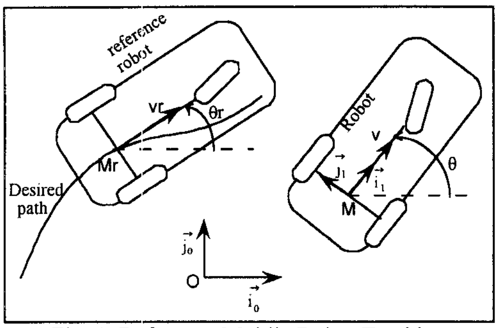

Fig. 1. Reference Mobile Robot Tracking

In the mobile frame R_1 , one has

$$\overrightarrow{MM_r} = x_e \vec{i_1} + y_e \vec{j_1} \tag{1}$$

$$\frac{d\,\overrightarrow{MM_r}}{dt} = \dot{x}_e \vec{i_1} + \dot{y}_e \vec{j_1} - y_e \dot{\theta}\, \vec{i_1} + x_e \dot{\theta}\, \vec{j_1} \tag{2}$$

The latter can also be written as follows

$$\frac{d\,\overrightarrow{MM_r}}{dt} = \frac{d\,\overrightarrow{OM_r}}{dt} - \frac{d\,\overrightarrow{OM}}{dt} \tag{3}$$

where

$$\frac{d\,\overrightarrow{OM_r}}{dt} = v_r \cos\theta_e\, \vec{i_1} - v_r \sin\theta_e\, \vec{j_1} \tag{4}$$

$$\frac{d\,\overrightarrow{OM}}{dt} = v\vec{i_1} \tag{5}$$

Substituting (4) and (5) in equation (3) and identifying with equation (2) yield

$$\begin{aligned} \dot{x}_e &= v_r \cos\theta_e + y_e \dot{\theta} - v \\ \dot{y}_e &= -v_r \sin\theta_e - x_e \dot{\theta} \end{aligned} \tag{6}$$

Let u_1 and u_2 be the control variables defined as follows

$$\begin{aligned} u_1 &= v - v_r \\ u_2 &= \dot{\theta} - \dot{\theta}_r \end{aligned} \tag{7}$$

Finally, we obtain the overall state representation for the tracking problem as follows

$$\begin{cases} \dot{x}_e = -u_1 + y_e u_2 + y_e \dot{\theta}_r + v_r(\cos\theta_e - 1) \\ \dot{y}_e = -x_e u_2 - x_e \dot{\theta}_r - v_r \sin\theta_e \\ \dot{\theta}_e = u_2 \end{cases} \tag{8}$$

2.2. Tracking Control Law Synthesis

Using the same kinematics model and a particular Lyapunov function depending on $(1-\cos\theta_e)$, the authors in [11] have proposed a nonlinear control law which locally stabilizes the equilibrium point $(0,0,0)$ for $v_r > 0$. Otherwise, this control law globally stabilizes the equilibrium points $(0,0,0\,mod(2\pi))$, it means that θ_e can-converges to a multiple of 2π as it is shown in Fig.2.

Figure 3 shows the vehicle motion under the control law proposed in [11] for the following initial conditions: $(x_{e0} = 2m, y_{e0} = 2m,\ \theta_{e0} = 3\frac{\pi}{4} rd)$. We can see the looping made by the vehicle before reaching the reference path, this is due to the attractiveness of the equilibrium point $(0,0,2\pi)$.

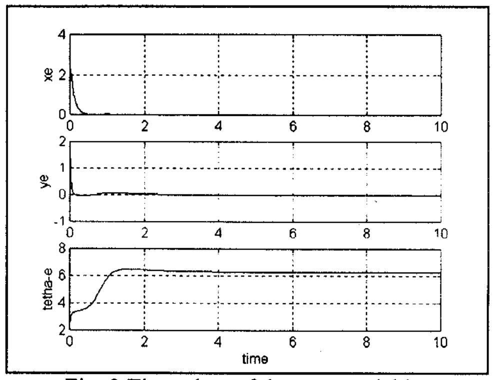

Fig. 2 Time plots of the state variables

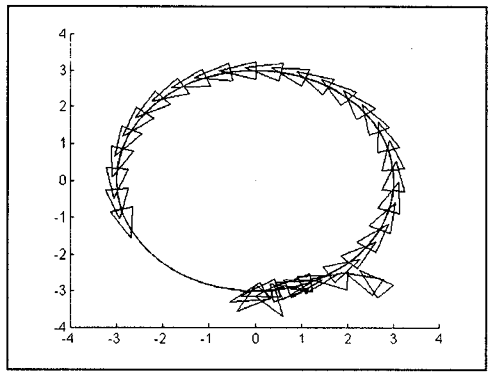

Fig. 3 Vehicle motion

In the sequel, we propose a nonlinear control law which globally stabilizes the equilibrium point $(0,0,0)$ provided that $v_r(t)$ does not converge to zero when t tends to infinity.

Proposition 1. Assume that $v_r(t)$ and $\dot{\theta}_r(t)$ are bounded and consider system (8) with the following control law

$$\begin{cases} u_1 = v_r(\cos\theta_e - 1) + y_e(\dot{\theta}_r + u_2) + k_1 x_e \\ u_2 = -k_2\theta_e + k_3 v_r y_e \dfrac{\sin\theta_e}{\theta_e} \end{cases} \tag{9}$$

where k_1, k_2 and k_3 strictly positive parameters.

Then, the equilibrium point $(0,0,0)$ is globally asymptotically stable provided that $v_r(t)$ does not converge to zero when t tends to infinity.

□

Proof. A global exponential stabilization of x_e to zero is obtained using the linearizing feedback u_1 in the first equation of (8), which becomes:

$$\dot{x}_e = -k_1 x_e \tag{10}$$

Let k_1 be chosen sufficiently large such that x_e converges to zero as fast as possible, and consider the following subsystem obtained from (8), when x_e is sufficiently small

$$\begin{cases} \dot{y}_e = -v_r \sin\theta_e \\ \dot{\theta}_e = u_2 \end{cases} \tag{11}$$

Let us take the following Lyapunov function

$$V(X_e) = \frac{1}{2}(\theta_e^2 + k_3 y_e^2) \tag{12}$$

which is positive semi-definite and whose time derivative is given by

$$\dot{V}(X_e) = -k_3 v_r y_e \sin\theta_e + \theta_e u_2 \tag{13}$$

Substituting u_2 in (13) leads to

$$\dot{V}(X_e) = -k_2 \theta_e^2 \tag{14}$$

It is clear that $\dot{V}(X_e)$ is always negative semi-definite, therefore $V(X_e)$ decreases with respect to time and tends to a finite positive value when t tends to infinity. Thus, $\dot{V}(X_e)$ tends to zero, and so do θ_e and $\dot{\theta}_e$, when t tends to infinity. Then, from (11) u_2 tends to zero. Moreover, from (9) u_2 tends to $k_3 v_r y_e$. Hence, the convergence of y_e to zero immediately follows since $v_r(t)$ is bounded and does not converge to zero.

□

Remark 1. Form the previous proof, it appears clearly that the proposed control law is valid only if $v_r \neq 0$. For this reason, it can not be applied for the stabilization problem because the convergence of y_e can not be ensured. (i.e. if $v_r = 0$ we can ensure only the convergence to zero of the two variables x_e and θ_e).

Remark 2. It is worth noticing that, the stability analysis of the closed loop system (8)-(9) can be studied using singular perturbation method. In fact, system (8) under the control law (9) can be viewed as a connection of two subsystems with fast (x_e) and slow (y_e, θ_e) dynamics when k_1 is sufficiently large.

2.3. Choice of the Control Parameters

In the previous section we have demonstrated that the equilibrium point $(0,0,0)$ is globally asymptotically stable under the control law (9) for all strictly positive values of k_1, k_2 and k_3. However, since we want an optimal response we have to find an optimal parameter set. In order to simplify the analysis we consider the case in which the reference robot is moving with a constant linear velocity. Then, in a neighborhood of the equilibrium point, the closed loop system (8)-(9) becomes

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{pmatrix} = \begin{pmatrix} -k_1 & 0 & 0 \\ -\dot{\theta}_r & 0 & -v_r \\ 0 & k_3 v_r & -k_2 \end{pmatrix} \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} \tag{15}$$

This system is characterized by its eigenvalues which are solutions of the following equation

$$(\lambda + k_1)(\lambda^2 + k_2 \lambda + k_3 v_r^2) = 0 \tag{16}$$

As shown in the previous section $1/k_1$ corresponds to the time constant for the first order system which describe the global behavior of the coordinate x_e.
The local behavior of y_e and θ_e is described by the solutions of

$$(\lambda^2 + k_2 \lambda + k_3 v_r^2) = 0 \tag{17}$$

Hence, we can choose k_2 and k_3 in accordance to the desired dynamics.

3. POINT-STABILIZATION PROBLEM

3.1. Problem Statement

Now, our objective is to stabilize the vehicle to a desired final configuration starting from any initial one. To this end, we make use of a discontinuous time-varying state feedback. The kinematics model is derived from (8) by setting $v_r = 0$ and $\dot{\theta}_r = 0$.

$$\begin{cases} \dot{x}_e = -u_1 + y_e u_2 \\ \dot{y}_e = -x_e u_2 \\ \dot{\theta}_e = u_2 \end{cases} \tag{18}$$

where x_e and y_e are the coordinates of the error vector $\overrightarrow{MO}$ in the basis of the mobile frame linked to the mobile robot, θ_e is the orientation error and u_1, u_2 are respectively the linear and rotational velocities of the mobile robot.
To study the controllability of system (18), let us rewrite it in a matrix form

$$\dot{X}_e = A(X_e)U \tag{19}$$

where

$$A(X_e) = \begin{pmatrix} A_1(X_e) & A_2(X_e) \end{pmatrix} = \begin{pmatrix} -1 & y_e \\ 0 & -x_e \\ 0 & 1 \end{pmatrix},$$

$X_e = [x_e \quad y_e \quad \theta_e]^T$ and $U = [u_1 \quad u_2]^T$.

In order to determine the dimension of the Control Lie Algebra, we calculate the following Lie bracket:

$$A_3 = [A_1, A_2] = \frac{\partial A_2}{\partial X_e} A_1 - \frac{\partial A_1}{\partial X_e} A_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Finally we obtain a full rank Control Lie Algebra generated by A_1, A_2 and A_3 as follows

$$L(X_e) = [A_1 \quad A_2 \quad A_3] = \begin{pmatrix} -1 & y_e & 0 \\ 0 & -x_e & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$det[\ L(X_e)] = 1 \quad \Rightarrow \quad rank[\ L(X_e)] = 3$$

This result implies the existence of a time-varying feedback $[u_1(X_e,t)\ u_2(X_e,t)]^T$ which drives the vehicle from any initial configuration X_0 to any final one X_f [2]. Nevertheless, system (18) is not stabilizable via a smooth pure-state feedback. In fact, the last of the three following Brockett necessary conditions [1], for the stabilizability via smooth time-invariant state feedback, is not satisfied. It is easy to verify this fact by just choosing $\xi_3 = 0$ and $\xi_2 \neq 0$ in *(iii)*.

(i) The linearized system of (18) has no uncontrollable modes associated with eigenvalues with positive real part.

*(ii)*The system (18) is completely controllable.

(iii) The mapping Γ: $IR^2 \times [0\ \ 2\pi] \times IR^2 \to IR^3$ defined by Γ: $(X_e, u) \mapsto f(X_e,\ u)$ is onto an open set containing zero. This means that we can solve $f(X_e, u) = \xi$ for all ξ sufficiently small,

where, $f(X_e, u) = \begin{pmatrix} -u_1 + y_e u_2 \\ -x_e u_2 \\ u_2 \end{pmatrix}$ and $\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix}$.

3.2. Parking Control Law Synthesis

In this section we present a variable structure time-varying control law which allows to drive the vehicle to any final configuration starting from any initial one. Without loss of generality, we assume that the final configuration is the origin.

Proposition 2. Consider system (18) with the following variable structure time-varying control law:

$$\begin{cases} u_1 = \begin{cases} \dfrac{df(t)}{dt} + k_1\left(x_e + f(t)\right) & \text{if } |y_e| \geq \varepsilon \\ k_1 x_e & \text{if } |y_e| < \varepsilon \end{cases} \\ u_2 = \begin{cases} -k_2\left(y_e f(t) + \theta_e\right) & \text{if } |y_e| \geq \varepsilon \\ -k_2 \theta_e & \text{if } |y_e| < \varepsilon \end{cases} \end{cases} \tag{20}$$

where ε, k_1 and $k_2$are positive parameters, $f(t)$ is any C^1 bounded function of time which has its derivative $\frac{df}{dt}$ also bounded and not vanishing when t tends to infinity, then:

(i) x_e and θ_e converge asymptotically to zero.

(ii) y_e is uniformly ultimate bounded and converge to the attractive domain defined by $|y_e| < \varepsilon$ □

Proof. First, assume that $|y_e| \geq \varepsilon$, and let us use the following time-varying Lyapunov function

$$V(X_e,t) = \frac{1}{2}\left[\left(x_e + f(t)\right)^2 + y_e^2 + \theta_e^2\right] \tag{21}$$

its time derivative is

$$\dot{V}(X_e,t) = -x_e y_e u_2 + \theta_e u_2 + (x_e + f(t))(-u_1 + y_e u_2 + \frac{df(t)}{dt}) \tag{22}$$

Using (20) gives

$$\dot{V}(X_e,t) = -k_1\left(x_e + f(t)\right)^2 - k_2\left(y_e f(t) + \theta_e\right)^2 \tag{23}$$

which is negative semi-definite. Therefore, $V(X_e,t)$ decreases with respect to time and tends to a finite positive value when t tends to infinity, and $\dot{V}(X_e,t)$ tends to zero. Thus,

$$y_e f(t) + \theta_e \to 0 \tag{24}$$

and

$$(x_e + f(t)) \to 0. \tag{25}$$

Differentiating (24) with respect to time gives

$$\dot{y}_e f + y_e \frac{df(t)}{dt} + \dot{\theta}_e \to 0 \tag{26}$$

From (20) $u_2 \to 0$, and from (18) $\dot{y} \to 0$ and $\dot{\theta}_e \to 0$, therefore (26) leads to

$$y_e \frac{df(t)}{dt} \to 0. \tag{27}$$

From the latter it appears clearly that $|y_e|$ decreases whenever $|y_e| \geq \varepsilon$, since $\frac{df(t)}{dt}$ is bounded and is not vanishing when t tends to infinity. Hence, from (24) and (25) one can easily conclude that x_e and θ_e are bounded since $f(t)$ is a bounded function.

Now, assume that y_e reaches the attractive domain defined by $|y_e| < \varepsilon$, and consider the following time-invariant Lyapunov function

$$V(X_e) = \frac{1}{2}\left[x_e^2 + y_e^2 + \theta_e^2\right] \qquad (28)$$

its time derivative then becomes:

$$\dot{V}(X_e) = -k_1 x_e^2 - k_2 \theta_e^2 \qquad (29)$$

which is always negative semi-definite. Using La Salle's theorem one can easily conclude that the largest invariant set defined by $\dot{V}(X_e) = 0$ is restricted to $\{(x_e, y_e, \theta_e) \in \Re^3 \,/\, x_e = 0, \theta_e = 0, |y_e| < \varepsilon\}$.
In particular if $\varepsilon \to 0$ then x_e, y_e and θ_e converge asymptotically to zero. □

Remark 3. Notice that the convergence of y_e and θ_e is quite satisfactory, this is not the case for x_e which converge to zero only if y_e is within the attractive domain that depends on ε. The convergence rate of x_e can be improved by increasing the tuning parameter ε to the detriment of y_e accuracy.

4. SIMULATION RESULTS

4.1. Path-following Case

In this section we present simulation results for the path following problem when the robot tracks a circular path ($v_r = 0.3m/s$ and $\dot{\theta}_r = 0.1rd/s$) and a rectilinear path ($v_r = 0.3m/s$ and $\dot{\theta}_r = 0$), with the following control parameters $k_1 = 1, k_3 = 10$ and $k_2 = 2|v_r|\sqrt{k_3}$.
Figure 4 shows the time evolution of the system states (i.e. position and orientation errors), and Fig.5 shows the vehicle motion along the circular reference path, for the following initial conditions: ($x_{e0} = 2m, y_{e0} = 2m$, $\theta_{e0} = 3\frac{\pi}{4}rd$). In rectilinear path tracking case, the time evolution of the system states is depicted in Fig. 6, and the vehicle motion is shown in Fig. 7, for the following initial conditions: ($x_{e0} = 1m, y_{e0} = 1m, \theta_{e0} = \frac{\pi}{4}rd$).

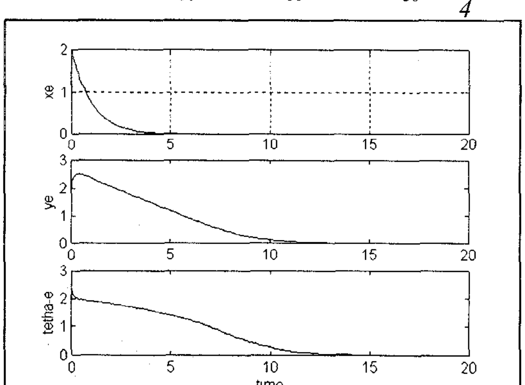

Fig. 4 Time plot of the state variables

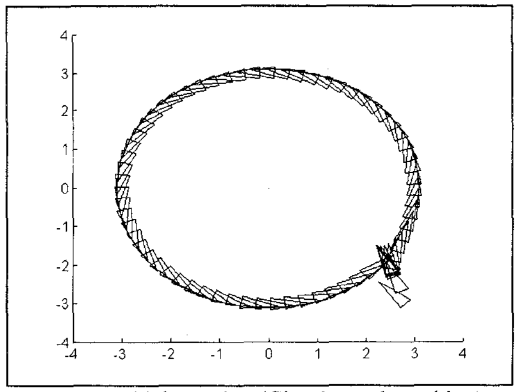
Fig. 5. Vehicle motion (Circular path tracking)

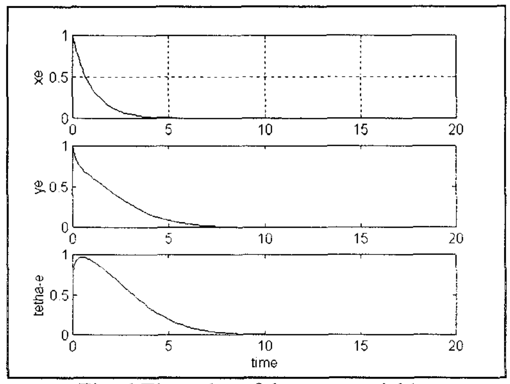

Fig. 6 Time plot of the state variables

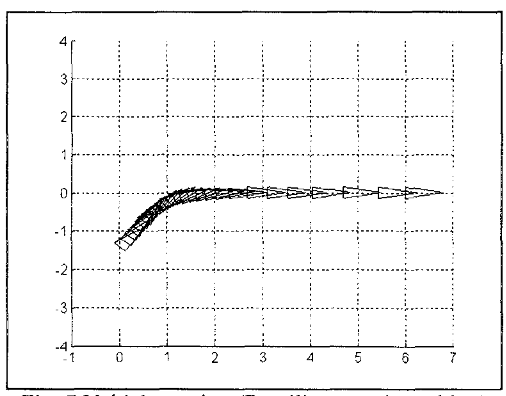
Fig. 7 Vehicle motion (Rectilinear path tracking)

4.2 .Point-Stabilization Case

In this section we present some simulation results for the parking problem. Our objective is to drive the vehicle from the initial configuration ($xe_0 = 0, ye_0 = 2$, $\theta e_0 = 0$) to the final configuration $X_f = (0,0,0)$. The function $f(t)$ and the control parameters have been chosen as follows $f(t) = sin(t)$, $k_1 = 1$, $k_2 = 1$ and $\varepsilon = 0.05$.
Figure 8 shows the convergence of the system states towards zero and the generated trajectory. In Fig. 9 we can see the vehicle motion in the parking maneuver under the proposed control law.

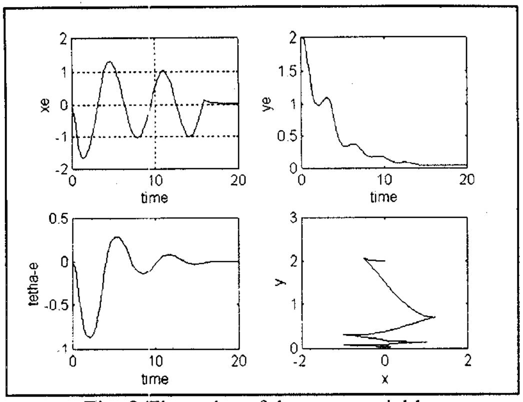

Fig. 8 Time plot of the state variables

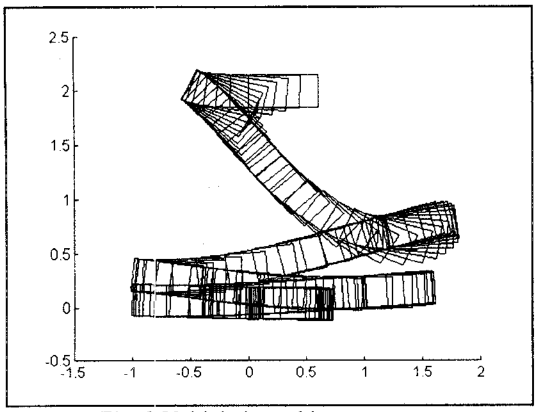
Fig. 9 Vehicle in parking maneuver

5. CONCLUSION

State feedback control schemes for path following and point-stabilization problems have been proposed for a nonholonomic wheeled mobile robot. The difficulty to solve the second problem via smooth time-invariant state feedback is pointed out when the reference robot is stopped. As an alternative solution, a discontinuous time-varying control law is proposed. It is important to notice that the proposed variable structure time-varying control law generates nonsmooth oscillating trajectories which are not very realistic. Other more realistic issues are subject to future studies.

REFERENCES

[1] R. W. Brockett, "Asymptotic stability and feedback stabilization", *In: R. W. Brockett, R.S. Millman and H.J. Sussmann, Eds., Differential Geometric Control Theory (Birkhauser, Basel-Boston, 1983), pp. 181-191.*

[2] J. M. Coron, "Global asymptotic stabilization for controllable systems without drift", *Mathematics of Control, Signal, and Systems. **N0. 5** (Springer-Verlag, New York, 1992), pp. 295-312.*

[3] G. Campion , B. d'Andrea-Novel and G. Bastin , "Controllability and state-feedback Stabilization of Nonholonomic Mechanical Systems", *Inter. Workshop in Adaptive and Nonlinear Control: Issues in Robotics, Grenoble (Springer-Verlag, Berlin-New York, 1990) , pp. 106-124.*

[4] A. M. Bloch and N.H. McClamroch, "Control of mechanical systems with classical nonholonomic constraints", *In: Proc. 28th IEEE Conf On Deci. Control, Tampa, FL, (1989), pp. 201-205.*

[5]A. M. Bloch, M. Reyhanoglu, and N.H. Mc Clamroch, "Control and stabilization of nonholonomic dynamic systems", *IEEE Trans. On Autom. Control 37 (1992), pp. 1746-1757.*

[6] C. Samson, "Control of chained systems application to path following and time-varying point-stabilization of mobile robots", *IEEE Trans. on automatic control, **40** (1995)*, pp.*64-77.*

[7] C. Samson and K. Ait-Abderrahim, "Feedback control of a nonholonomic wheeled cart in Cartesian space", *In: Proc. IEEE Int. Conf. on Robotics and Automation, Sacramento, CA, (1991), pp.1136-1141.*

[8] J. B. Pomet, "Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift", *System and Control Letters. **18** (1992), pp.147-158.*

[9] C. Canudas de Wit and O. J. Sordalen, "Exponential stabilization of mobile robots with nonholonomic constraints", *IEEE Tran. On Autom. Control 37 (1992), pp.1791-1797.*

[10] O.J. Sordalen and C. Canudas de Wit, "Exponential Control Law for a Mobile Robot: Extension to path Following", *in: Proc. Int. Conf. On Robotics and Automation, Nice, France (1992), pp. 2158-2163.*

[11] Y. Kanayama, Y. Kimura, F. Myazaki and T. Noguchi, "A stable tracking control method for a nonholonomic mobile robot", *In: Proc. IEEE Int. Workshop on Int. Robots and Systems, Osaka.(1991), pp. 1236-1241.*

[12] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: Steering using sinusoids", *IEEE Trans. On Autom. and Control, **38**, **No.5**, (1993),pp. 700-716.*

[13] J. J. Slotine and W. Li, *"Applied Nonlinear Control"*, Prentice-Hall International, Ed. 1991.

[14] A. Tayebi and A. Rachid, "A time-varying-based robust control for the parking problem of a wheeled mobile robot," *In Proc. IEEE Int. Conf. On Robotics and Automation, Minneapolis, Minnesota, April 1996, pp.3099-3104.*

[15] A.Tayebi and A. Rachid, "Robust tracking control for a wheeled mobile robot, " *In Proc. of IEEE Mediterranean Electrotechnical Conference, Bari, ITALY, May 1996.*

[16] A. Tayebi, M. Tadjine and A. Rachid, "A simple control law for the path following problem of a wheeled *mobile robot" In Proc. Word Automation Congress, Montpellier, France, May 1996.*