Robust Control of Full State Tracking of a Wheeled Mobile Robot

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Abstract

This paper proposes a robust trajectory tracking control scheme for a nonholonomic wheeled mobile robot. This control scheme achieves full state tracking in the sense of uniform ultimate boundedness. It is robust against the uncertain inertia parameters, unknown disturbances and initial errors.

1 Introduction

In the past decades, the tracking control problem of nonholonomic wheeled mobile robots was extensively studied and different approaches were proposed. Especially, the static and dynamic input-output feedback linearization techniques were well studied in [1, 2]. Because some useful nonlinearities are cancelled by linearization feedback, it is believed that the system's stability and robustness under the input-output feedback linearization control law are decreased. So, control schemes in terms of the full state tracking were also studied based on the general input-output error dynamics [3] or based on the full order error dynamics [4, 5]. Using Lyapunov method and backstepping method to the full order error dynamics, global tracking control laws were proposed for a two-wheel driven nonholonomic cart [4] and for chained form [5].

The above works do not take into account of the modeling uncertainties in terms of dynamic parameters such as the uncertain inertia parameters and the unknown disturbances. The control schemes that are robust against the modeling uncertainties require other techniques. Su and Stepanenko proposed a robust control strategy based on the reduced-order dynamics in [6] and then [7]. However, it should be pointed out that these control schemes are valid for the tracking problem of partial states or outputs only when the solutions of the original full order system are admissible. On the other hand, sliding mode control is also studied [8, 9]. By appropriate design of the sliding surfaces, some error terms can converge to zero and the others remain within the boundaries in [8], all posture variables are stabilized in [9]. Nonetheless, these methods may be limited to some strict conditions, such as "the reference trajectory and the real trajectory do not cross the origin of the world coordinates" as well as "the heading angle of the robot and the angle coordinate cannot be perpendicular with each other".

In this paper, we propose a novel robust control scheme for nonholonomic wheeled mobile robots. With suitable design of a set of output function, we establish the reduced-order dynamics in terms of the input-output relationship. Then, the robust control scheme is developed based on the reduced-order dynamics. The analysis to the internal stability reveals that the full state tracking can be achieved under an applicable sufficient condition. Simulation results show that the uniformly ultimately bounded full state tracking is achieved with the robustness against uncertain inertia parameters, unknown disturbances and initial errors.

2 Dynamics of a Wheeled Mobile Robot

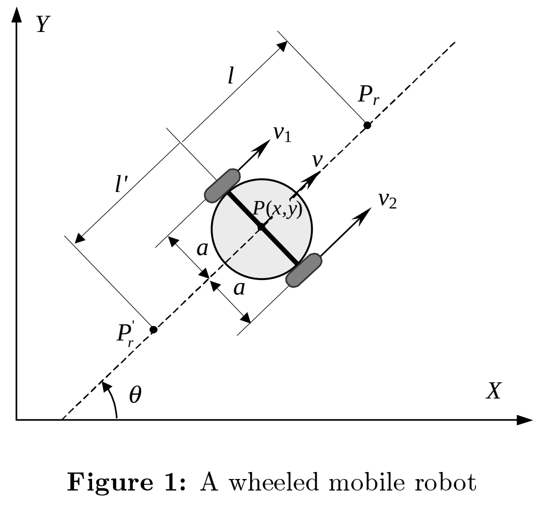

Figure 1: A wheeled mobile robot

We consider wheeled mobile robots moving on the horizontal plane as shown in Figure 1. Suppose that there is no skidding between the wheels and the ground. The robot dynamics can be described as,

$$\dot{q} = G(\theta)\,\mu \tag{1}$$

$$M\dot{\mu} \quad = \quad \tau_r + B\tau \tag{2}$$

where, states $q = [x\ y\ \theta]^T$ are the posture of the robot. States $\mu = [v\ \omega]^T$ consists of the linear velocity v and angular velocity ω. Vector $\tau = [\tau_1\ \tau_2]^T$ consists of control input torques on left driving wheel τ_1 and right driving wheel τ_2 respectively. Vector τ_r is the resistant forces/torques (air resistance and friction) given by

$$\tau_r = \begin{bmatrix} f_a + f_v \\ \tau_\omega \end{bmatrix} \tag{3}$$

where, f_a is the air resistance; f_v is the friction against the linear motion; τ_ω is the friction against the angular motion. Matrices $G(\theta)$, M and B are

$$G(\theta) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \quad M = \begin{bmatrix} m & 0 \\ 0 & I_o \end{bmatrix} \quad B = \frac{r_w}{2}\begin{bmatrix} 1 & 1 \\ -\frac{1}{a} & \frac{1}{a} \end{bmatrix}$$

where, m is the robot mass, I_o is its inertial moment around the vertical axis, r_w is the radius of the wheel, and a is the displacement from each wheel to the center of the robot.

To facilitate the robust controller design, we consider the reduced-order dynamics of the mobile robots based on the input-output relationships. To this end, a set of output variable $z \in R^2$ is defined as

$$z = h\,(q) = \begin{bmatrix} x \\ y \end{bmatrix} + R^T\,(\theta)\begin{bmatrix} l \\ 0 \end{bmatrix} \tag{4}$$

where, $R(\theta)$ is the rotation matrix. Output $h(q)$ represents the coordinates of a virtual reference point P in front of the robot when l is positive, and a virtual reference point P_r behind the robot when l is negative.

Taking time derivative to (4) yields

$$\dot{z} = E\,(\theta)\,\mu \tag{5}$$

where,

$$E\,(\theta) = \frac{\partial h\,(q)}{\partial q} G\,(\theta) = R^T(\theta)\begin{bmatrix} 1 & 0 \\ 0 & l \end{bmatrix} \tag{6}$$

Clearly, if $l \neq 0$, the decoupling matrix $E\,(\theta)$ is nonsingular and the output (4) is input-output decoupled.

We define the velocity vector of output as

$$\mu_z = \dot{z} \tag{7}$$

Using the invertible property of the decoupling matrix $E(\theta)$ and equation (5), the generalized velocity μ is expressed in terms of the output velocity as

$$\mu = E^{-1}\,(\theta)\,\mu_z \tag{8}$$

Substituting (8) and its time derivative into dynamics model (2), a modified dynamics is obtained as

$$ME^{-1}\,(\theta)\,\dot{\mu}_z = -M\dot{E}^{-1}\,(\theta)\,\mu_z + \tau_r + B\tau \tag{9}$$

Suppose that the desired trajectory is given by $z_d(t)$, which is twice differentiable. We define the output tracking error as

$$\tilde{z} = z - z_d \tag{10}$$

Furthermore, define the combined output tracking error as

$$r = \dot{\tilde{z}} + \Lambda\tilde{z} \tag{11}$$

and the modified desired velocity as

$$\mu_s = \dot{z}_d - \Lambda\tilde{z} \tag{12}$$

where Λ is a positive definite matrix whose all eigenvalues have strictly positive real parts. Then, the output velocity (7) is represented as

$$\mu_z = \dot{z} = r + (\dot{z}_d - \Lambda\tilde{z}) = r + \mu_s \tag{13}$$

Using (13) into (9) leads to the dynamics in terms of the combined output tracking error r as

$$\begin{aligned} ME^{-1}\,(\theta)\,\dot{r} \quad = \quad & -M\dot{E}^{-1}\,(\theta)\,r + B\tau + \tau_r \\ & -ME^{-1}\,(\theta)\,\dot{\mu}_s - M\dot{E}^{-1}\,(\theta)\,\mu_s \end{aligned} \tag{14}$$

Note that, there exist disturbances in the external resistant force vector τ_r and unknown inertia parameters in the matrix M. For the controller design that is robust against disturbances and unknown parameters, we would like to have the disturbances and unknown parameters linearized.

First, we show that $ME^{-1}\,(\theta)\,\dot{\mu}_s + M\dot{E}^{-1}\,(\theta)\,\mu_s$ is linearly parameterizable. We know that $M\mu$ is linearly parameterizable, i.e.,

$$M\dot{\mu} = Y_1(\dot{\mu})\phi_m$$

where,

$$Y_1(\dot{\mu}) = \begin{bmatrix} \dot{v} & 0 \\ 0 & \dot{\omega} \end{bmatrix} \qquad \phi_m = [\,m \quad I_o\,]^T$$

Then,

$$\begin{aligned} ME^{-1}\,(\theta)\,\dot{\mu}_z + M\dot{E}^{-1}\,(\theta)\,\mu_z \quad &= \quad M\dot{\mu}|_{\dot{\mu}=\dot{E}^{-1}\mu_z + E^{-1}\dot{\mu}_z} \\ &= \quad Y_2(q, \mu, \mu_z, \dot{\mu}_z)\phi_m \end{aligned}$$

where,

$$Y_2(q, \mu, \mu_z, \dot{\mu}_z) = Y_1(\dot{\mu})|_{\dot{\mu}=\dot{E}^{-1}\mu_z + E^{-1}\dot{\mu}_z}$$

Using the modified desired velocity μ_s instead of the output velocity μ_z, then, $ME^{-1}\,(\theta)\,\dot{\mu}_s + M\dot{E}^{-1}\,(\theta)\,\mu_s$ is linearly parameterized as

$$ME^{-1}\,(\theta)\,\dot{\mu}_s + M\dot{E}^{-1}\,(\theta)\,\mu_s = Y_2(q, \mu, \mu_s, \dot{\mu}_s)\phi_m \tag{15}$$

Note that, regression matrix $Y_2(q, \mu, \mu_s, \dot{\mu}_s)$ in (15) is measurable.

On the other hand, we model the resistant forces/torques in (3) as

$$\begin{aligned} f_a &= k_{a1}v + k_{a2}v^2 \\ f_v &= k_{fv}\mathrm{sat}(v/0.01)mg \\ f_\omega &= k_{f\omega}\mathrm{sat}\left(\omega/0.01\right)mg \end{aligned}$$

where, g is the gravitational acceleration, k_{a1}, k_{a2} are the air resistance coefficients, $k_{fv}, k_{f\omega}$ are the friction coefficients. Uncertain coefficients k_{a1}, k_{a2}, k_{fv} and $k_{f\omega}$ characterize uncertain resistances and unknown disturbances. Then, the resistant vector τ_r can be linearly parameterized as follows

$$\tau_r = D(\mu)\phi_\tau \tag{16}$$

where

$$\phi_r = [\, k_{a1} \quad k_{a2} \quad k_{fv}m \quad k_{f\omega}m\,]^T$$

$$D(\mu) = \begin{bmatrix} v & v^2 & \mathrm{sat}(v/0.01)g & 0 \\ 0 & 0 & 0 & \mathrm{sat}(\omega/0.01)g \end{bmatrix}$$

Combining (15) and (16) into (14) leads to

$$ME^{-1}\dot{r} = -M\dot{E}^{-1}r + B\tau - Y\phi \tag{17}$$

where, $\phi \in R^6$ is the uncertain parameters given by

$$\phi = \begin{bmatrix} \phi_m \\ \phi_r \end{bmatrix} = [\, m \quad I_o \quad k_{a1} \quad k_{a2} \quad k_{fv}m \quad k_{f\omega}m\,]^T$$

Y is the augmented regression matrix given by

$$Y\left(q, \mu, \mu_s, \dot{\mu}_s\right) = [\, Y_2(q, \mu, \mu_s, \dot{\mu}_s) \quad -D(\mu)\,]$$

Equation (17) is the reduced-order dynamics of the full-order dynamics (1)(2) with (4) as the output. Because all the uncertain parameters are linearized in ϕ, reduced-order dynamics (17), which is based on the relationships between input τ and combined output tracking error r, is now suitable to develop controllers that are robust to the uncertain parameter ϕ.

3 Robust Controller Design

Taking into account of the uncertain parameter ϕ in (17), a simple robust control law is provided as follows.

Suppose that, there exist $\hat{\phi} \in R^6$ and $w \in R_+$, both known, such that

$$|\tilde{\phi}| = |\phi - \hat{\phi}| \leq w \tag{18}$$

Then, consider the following control law

$$B\tau = Y\left(\hat{\phi} + u_\phi\right) - E^T\left(\theta\right)\Pi r \tag{19}$$

where, Π is a positive definite 2×2 matrix; u_ϕ is an additional control input given by

$$u_\phi = \begin{cases} -w\frac{\bar{r}}{\|\bar{r}\|} & \text{if } \|\bar{r}\| > \sigma \\ -w\frac{\bar{r}}{\sigma} & \text{if } \|\bar{r}\| \leq \sigma \end{cases} \tag{20}$$

with $\bar{r} = Y^T E^{-1} r$ a vector and σ a positive constant.

Theorem 1 *The closed-loop system of (19)(20) and (17) is uniformly ultimately bounded if the solutions $q(t)$ and $\mu(t)$ are admissible for all t.*

Proof: Using control (19) into dynamics (17), the closed-loop system is obtained as

$$ME^{-1}\dot{r} = -M\dot{E}^{-1}r - E^T\Pi r + Y\left(u_\phi - \tilde{\phi}\right) \tag{21}$$

Consider the following positive definite function

$$V\left(r,t\right) = \frac{1}{2}\left(E^{-1}r\right)^T M\left(E^{-1}r\right) \tag{22}$$

Its time derivative is

$$\dot{V}\left(r,t\right) = \left(E^{-1}r\right)^T M\dot{E}^{-1}r + \left(E^{-1}r\right)^T ME^{-1}\dot{r} \tag{23}$$

Substituting (21) into (23) yields

$$\dot{V}\left(r,t\right) = -r^T\Pi r + \bar{r}^T\left(u_\phi - \tilde{\phi}\right) \tag{24}$$

Clearly, the first term is negative definite. The second term, if $\|\bar{r}\| > \sigma$, becomes

$$\begin{aligned} \bar{r}^T\left(u_\phi - \tilde{\phi}\right) &= \bar{r}^T\left(-w\frac{\bar{r}}{\|\bar{r}\|} - \tilde{\phi}\right) \\ &= -w\|\bar{r}\| - \bar{r}^T\tilde{\phi} \\ &\leq -w\|\bar{r}\| + \|\bar{r}\|\cdot\left\|\tilde{\phi}\right\| \\ &= \|\bar{r}\|\left(-w + \left\|\tilde{\phi}\right\|\right) \leq 0 \end{aligned} \tag{25}$$

If $\|\bar{r}\| \leq \sigma$, we have

$$\begin{aligned} \bar{r}^T\left(u_\phi - \tilde{\phi}\right) &\leq \bar{r}^T u_\phi + \|\bar{r}\|\cdot\left\|\tilde{\phi}\right\| \\ &\leq -\bar{r}^T w\frac{\bar{r}}{\sigma} + w\|\bar{r}\| \\ &= -\frac{w}{\sigma}\|\bar{r}\|^2 + w\|\bar{r}\| \end{aligned} \tag{26}$$

To complete the proof, it suffices to notice the following. The decoupling matrix $E(\theta)$ is uniformly bounded and regular if $l \neq 0$. The matrix M is positive definite and bounded. So, there exist a P_m and a P_M such that

$$P_m I \leq \frac{1}{2}\left(E^{-1}\right)^T ME^{-1} \leq P_M I$$

and hence,

$$P_m\|r\|^2 \leq V\left(r,t\right) \leq P_M\|r\|^2$$

Inequality (26) shows that

$$\dot{V}\left(r,t\right) \leq -P_\Pi\|r\|^2 - \frac{w}{\sigma}\|r\|^2 + w\|r\|$$

where constant $P_\Pi = \lambda_{\min}\left(\Pi\right) > 0$. Uniform ultimate boundedness thus follows using Lemmas in [10]. ■

Remark 1 *The ultimate boundary depends on the value of σ. The smaller the value of σ is, the smaller the ultimate boundary is. As $\sigma \to 0$, the ultimate boundary approaches zero.*

4 Full State Tracking

Suppose that a feasible desired trajectory $q_d(t), \mu_d(t)$ for the mobile robot in Figure 1 is pre-specified such that the dynamics (1)(2) are satisfied for a uniformly bounded input $\tau_d(t)$, i.e.,

$$\begin{aligned} \dot{q}_d &= G(\theta_d)\,\mu_d & (27)\\ M\dot{\mu}_d &= \tau_{dr} + B\tau_d & (28) \end{aligned}$$

Clearly, the desired trajectory can also be expressed in the form of output (4) as

$$z_d = h(q_d) \tag{29}$$

We say that the system (1)(2) with output (4) achieves *uniformly ultimately bounded output tracking* to the desired trajectory (29) if a control law τ makes the output tracking error $\tilde{z} = z - z_d$ uniformly ultimately bounded. Similarly, we say that the system (1)(2) achieves *uniformly ultimately bounded full state tracking* to the desired trajectory (27)(28) if a control law τ makes the tracking errors $\tilde{q} = q - q_d$ and $\tilde{\mu} = \mu - \mu_d$ uniformly ultimately bounded.

We should note that the output function $h(q)$ defined in (4) is an epimorphism. So, the uniformly ultimately bounded full state tracking implies the uniformly ultimately bounded output tracking. However, the reverse might not be true. Next, we shall show that, under certain condition, robust controller (19)(20) may achieve both the uniformly ultimately bounded output tracking and the uniformly ultimately bounded full state tracking.

Instead of the original system of (q, μ), we study its diffeomorphic system. Because the decoupling matrix $E(\theta)$ is nonsingular, one may check that the following map

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \eta \end{bmatrix} = \bar{\Phi}(q,\mu) = \begin{bmatrix} h(q) \\ E(\theta)\,\mu \\ \theta \end{bmatrix} \tag{30}$$

ia a diffeomorphism. We define an auxiliary control

$$u = R^T(\theta)\begin{bmatrix} 0 & -l \\ 1 & 0 \end{bmatrix}\mu + M^{-1}(\tau_r + B\tau) \tag{31}$$

In the new coordinates (ξ_1, ξ_2, η), robot dynamics (1)(2) are described as,

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 & (32)\\ \dot{\xi}_2 &= u & (33)\\ \dot{\eta} &= \frac{1}{l}[\,-\sin\eta \quad \cos\eta\,]\xi_2 & (34) \end{aligned}$$

Similar operations on (27)(28) lead to the desired state in new coordinates $(\xi_{1d}, \xi_{2d}, \eta_d)$. Taking difference between the dynamics of (ξ_1, ξ_2, η) and $(\xi_{1d}, \xi_{2d}, \eta_d)$, we obtain the dynamics of tracking errors in terms of $\tilde{\xi} = \xi_i - \xi_{id}$ $(i = 1, 2)$ and $\tilde{\eta} = \eta - \eta_d$ as follows

$$\begin{aligned} \dot{\tilde{\xi}}_1 &= \tilde{\xi}_2 & (35)\\ \dot{\tilde{\xi}}_2 &= u - \dot{\xi}_{2d} & (36)\\ \dot{\tilde{\eta}} &= \varphi(\tilde{\xi}_2, \tilde{\eta}, t) & (37) \end{aligned}$$

where,

$$\begin{aligned} \varphi(\tilde{\xi}_2, \tilde{\eta}, t) &= \frac{1}{l}[\,-\sin(\tilde{\eta}+\eta_d) \quad \cos(\tilde{\eta}+\eta_d)\,](\tilde{\xi}_2 + \xi_{2d}) \\ &\quad -\frac{1}{l}[\,-\sin\eta_d \quad \cos\eta_d\,]\xi_{2d} & (38) \end{aligned}$$

This set of tracking error dynamic equations (35)-(37) consists of two parts. The first part is the ξ-subsystem (35)(36), which characterizes the input-output dynamics. The second part is the η-subsystem (37), which is not controllable and characterizes the internal dynamics. In the case that the ξ-subsystem stabilized by an input-output control law, the stability property of η-subsystem determines whether the full state tracking and even the output tracking can be achieved. Especially, the zero dynamics, equation (37)(38) when the system output $(\tilde{z} = \tilde{\xi}_1)$ is set to zero, is given as

$$\begin{aligned} \dot{\tilde{\eta}} &= \varphi_o(\tilde{\eta}, t) & (39)\\ &= \frac{\xi_{2d}}{l}[\,\sin\eta_d - \sin(\tilde{\eta}+\eta_d) \quad \cos(\tilde{\eta}+\eta_d) - \cos\eta_d\,] \end{aligned}$$

and its stability plays a critical role. To gain more insights to the tracking error zero dynamics, (39) may be expressed in the original mobile robot coordinates as follows

$$\begin{aligned} \dot{\tilde{\theta}} &= f_o(\tilde{\theta}, v_d(t), \omega_d(t)) & (40)\\ &= -\frac{v_d(t)}{l}\sin\tilde{\theta} - \omega_d(t)\left(1 - \cos\tilde{\theta}\right) \end{aligned}$$

where, $\tilde{\theta} = \theta - \theta_d$; $v_d(t)$ and $\omega_d(t)$ are the linear and angular velocities of the desired trajectory respectively.

Now, we may propose a sufficient condition for the uniformly ultimately bounded full state tracking by the following theorem.

Theorem 2 *Suppose that a feedback control law makes the reduced-order dynamics (17) uniformly ultimately bounded. Then, this control law locally achieves uniformly ultimately bounded full state tracking of the robot dynamics (1)(2) to the moving desired trajectory $(q_d(t), \mu_d(t))$, which satisfies (27)(28) with $\mu_d(t)$ uniformly bounded, if parameter l has the same sign with the velocity $v_d(t)$ of the desired trajectory.*

Proof: Due to (30) is a diffeomorphism, the uniform ultimate boundedness of tracking error system (35)-(37) is equivalent to that of the robot dynamics (1)(2) to a moving desired trajectory $(q_d(t), \mu_d(t))$. Next, we prove that the ξ-subsystem (35)(36) and the η-subsystem (37) are uniformly ultimately bounded, respectively.

1. ξ-subsystem (35)(36) is uniformly ultimately bounded.

Since the reduced-order input-output dynamics (17) of r is uniformly ultimately bounded, by the definition of combined tracking error r in (11), there exist certain d and T, such that

$$\dot{\tilde{z}} = -\Lambda \tilde{z} + r \qquad \forall \|r\| \leq d, t \geq T \tag{41}$$

Take r in (41) as a perturbation term, it is easy to check (by Lemma 4.8 in [11]) that the output tracking error $\tilde{z}$ in (41) is uniformly ultimately bounded. This in turn means that $\dot{\tilde{z}}$ in (41) is uniformly ultimately bounded. Therefore, by the definitions of $\tilde{\xi}_1 = \tilde{z}$ and $\xi_2 = \dot{\tilde{z}}$, the ξ-subsystem is uniformly ultimately bounded.

2. η-subsystem (37) is locally uniformly ultimately bounded.

(a) Firstly, we show that the tracking error zero dynamics (40) and hence (39) are locally uniformly exponentially stable.

Clearly, the function

$$\frac{\partial f_o}{\partial \tilde{\theta}}\left(\tilde{\theta}, t\right) = -\frac{v_d(t)}{l}\cos\tilde{\theta} - \omega_d(t)\sin\tilde{\theta}$$

is globally bounded and Lipschitz, uniformly in t, if $\mu_d(t) = [v_d(t)\ \omega_d(t)]^T$ is uniformly bounded. Let

$$A(t) = \left.\frac{\partial f_o}{\partial \tilde{\theta}}\left(\tilde{\theta}, t\right)\right|_{\tilde{\theta}=0} = -\frac{v_d(t)}{l}$$

The linear system

$$\dot{\tilde{\theta}} = A(t)\tilde{\theta} = -\frac{v_d(t)}{l}\tilde{\theta}$$

is clearly uniformly exponentially stable if parameter l and $v_d(t)$ are positive and negative simultaneously. Therefore, (by Theorem 4.4 in [11]) the tracking error zero dynamics (40) and hence (39) are locally uniformly exponentially stable.

(b) Let

$$g(\tilde{\eta}, t) = \varphi(\tilde{\xi_1}(t), \tilde{\xi_2}(t), \tilde{\eta}, t) - \varphi_o(\tilde{\eta}, t)$$

The η-subsystem (37) can be written as

$$\dot{\tilde{\eta}} = \varphi_o(\tilde{\eta}, t) + g(\tilde{\eta}, t) \tag{42}$$

Take $g(\tilde{\eta}, t)$ as a perturbed term. Since the uniform boundedness of μ_d implies the uniform boundedness of ξ_{2d}, by (38) and (**??**), both $\varphi(\tilde{\xi_1}(t), \tilde{\xi_2}(t), \tilde{\eta}, t)$ and $\varphi_o(\tilde{\eta}, t)$ are uniform boundedness. This in turn means that $g(\tilde{\eta}, t)$ is uniform boundedness. We have proven that $\dot{\tilde{\eta}} = \varphi_o(\tilde{\eta}, t)$ is locally uniformly exponentially stable if $v_d(t)/l \geq 0$. Therefore, by Lemma 4.8 in [11], one may check that the η-subsystem (42) is locally uniformly ultimately bounded.

This completes the proof. ■

Theorems 2 give the applicable approach to achieve the uniformly ultimately bounded full state tracking of the robot in Figure 1 by using control laws based on the input-output dynamics such as the robust control law in Theorems 1. It indicates that the uniformly ultimately bounded full state tracking can be locally achieved by adjusting the parameter l in the output function according to the behavior of the desired trajectory, i.e, setting $l > 0$ when the desired trajectory moves forwards ($v_d > 0$) and setting $l < 0$ when the desired trajectory moves backwards ($v_d < 0$).

5 Simulation Results

A simulation program is developed to verify the robust control law (19)(20). In the simulation, the true values of the robot are set as

$$a = 0.5\text{m},\ r_w = 0.3\text{m},\ m = 100\text{kg},\ I_o = 25\text{kgm}^2,$$

$$k_{a1} = 10\text{Ns/m},\ k_{a2} = 0\text{Ns/m},\ k_{fv} = 0.05,\ k_{f\omega} = 0.2$$

The estimation $\bar{\phi}$ of the unknown parameter in (19) is chosen as

$$\bar{\phi} = [0.8m\ \ 1.2I_o\ \ 1.2k_{a1}\ \ k_{a2}\ \ 0.64k_{fv}m\ \ 0.72k_{f\omega}m]^T$$

Parameters w and σ in additional control law (20) are chosen as

$$w = 0.2\left\|\bar{\phi}\right\| \qquad \sigma = 0.1$$

The control gain Π and Λ are taken as

$$\Pi = \text{diag}\{1, 1\} \qquad \Lambda = \text{diag}\{1, 1\}$$

The desired trajectory consists of three straight lines and two curves, which starts from the origin along the x axis with the constant linear velocity $v_d = 2.5$m/s. The initial configurations of the robot are set as

$$[x(0), y(0), \theta(0), v(0), \omega(0)] = [0, -1, 0, 0, 0]$$

so that the real robot is initially off the desired trajectory with the following initial errors

$$[\tilde{x}(0), \tilde{y}(0), \tilde{\theta}(0), \tilde{v}(0), \tilde{\omega}(0)] = [0, -1, 0, -2.5, 0]$$

The parameter l in the output function is chosen as $l = \text{sgn}(v_d)0.6$m.

The obtained results are shown in Figure 2 and Figure 3. The time responses in Figure 2 show that the state tracking error $\tilde{q} = [\tilde{x}(t)\ \tilde{y}(t)\ \tilde{\theta}(t)]^T$, $\tilde{\mu} = [\tilde{v}(t)\ \tilde{\omega}_d(t)]^T$ exhibit uniform ultimate boundedness around zero. The Cartesian motion in Figure 3 shows more clearly the convergence of the configurations of the robot to the desired trajectory. This observation indicates that the uniformly ultimately bounded full state tracking is achieved by (a) using control law (19)(20), (b) setting parameter l a value with the same sign of $v_d(t)$.

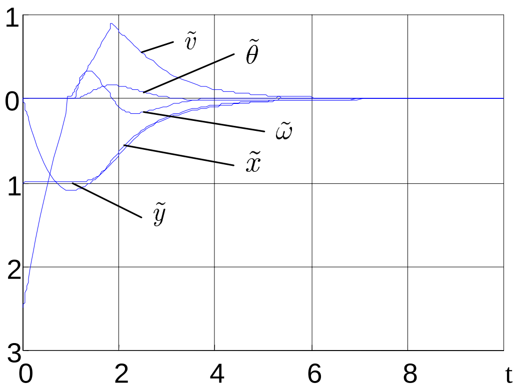

Figure 2: Uniform ultimate boundedness

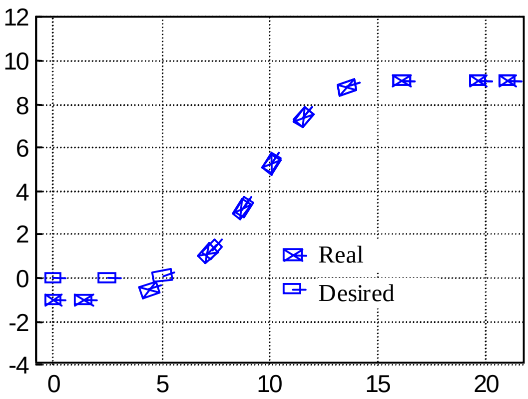

Figure 3: Cartesian motion

6 Conclusions

Through establishing the reduced-order input-output dynamics of a wheeled mobile robot, the robust control scheme is proposed. The analysis to the internal dynamics reveals that the full state tracking can be achieved under the proposed control scheme by adjusting the output function.

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