

11. Kinematic models of contact

Mechanics of Manipulation

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Outline.

Grübler

- Review of kinematic mechanisms
- Mobility and connectivity
- Grübler's formula

Salisbury

- Taxonomy of contacts
- Mobility and connectivity of grasp

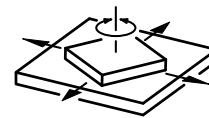
Kinematic mechanisms

Link: a rigid body;

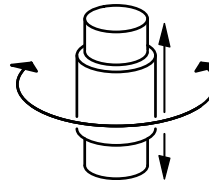
Joint: imposes one or more constraints on the relative motion of two links;

Kinematic mechanism: a bunch of links joined by joints;

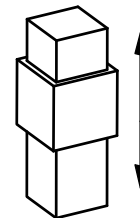
lower pairs joints involving positive contact area.



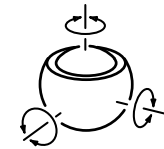
Planar
3 freedoms



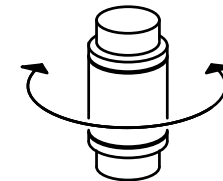
Cylindrical
2 freedoms



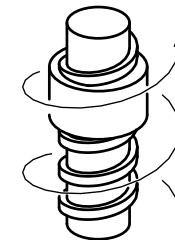
Prismatic
1 freedom



Spherical
3 freedoms



Revolute
1 freedom



Helical
1 freedom

Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

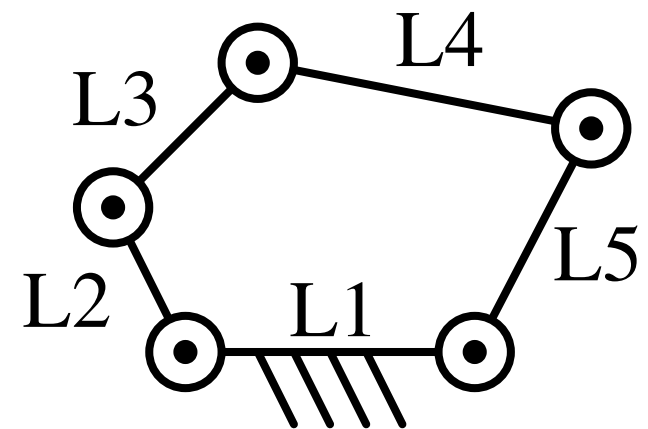
What is the mobility of the five bar linkage at right?

What is the connectivity of

Link 1 relative to link two?

Link 3 relative to link 1?

Link 3 relative to link 4?



Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

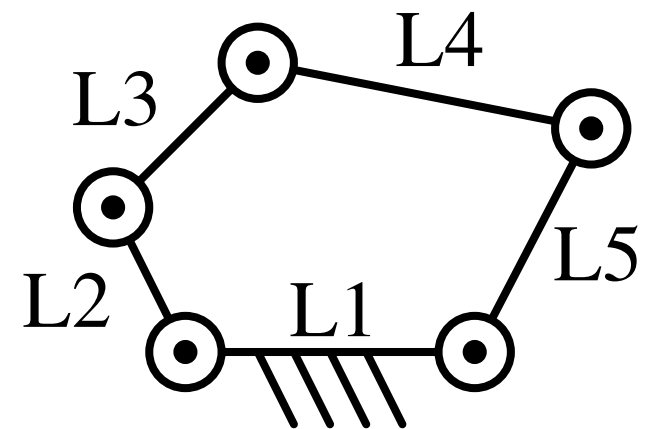
What is the mobility of the five bar linkage at right? Two.

What is the connectivity of

Link 1 relative to link two?

Link 3 relative to link 1?

Link 3 relative to link 4?



Mobility and connectivity

mobility of a mechanism: DOFs with one link fixed.

connectivity DOFs of one link relative to another.

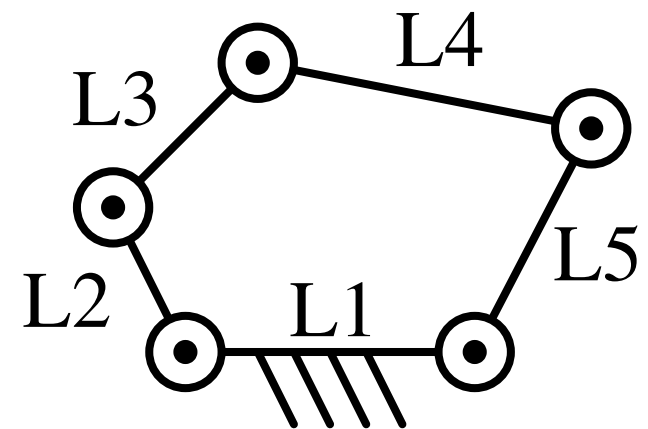
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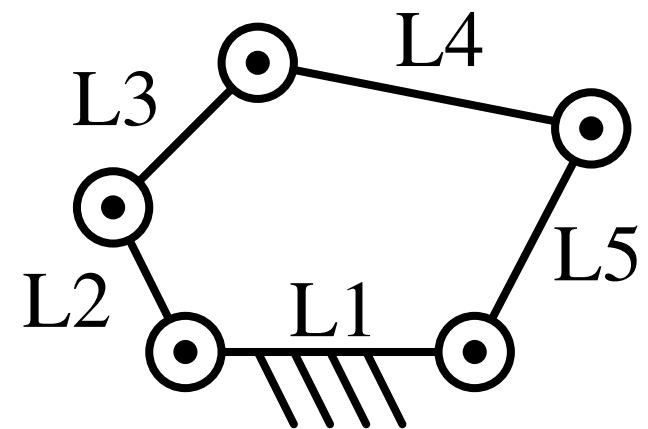
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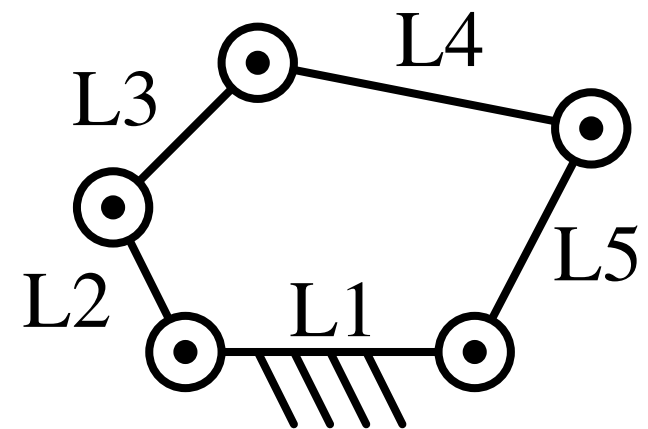
What is the mobility of the five bar linkage at right? Two.

What is the connectivity of

Link 1 relative to link two? One.

Link 3 relative to link 1? Two.

Link 3 relative to link 4? One.



Grübler's formula

Given n links joined by g joints,

with u_i constraints and f_i freedoms at joint i . (Note that $u_i + f_i = 6$.)

Assume one link is fixed and constraints are all independent.

The mobility M is

$$\begin{aligned} M &= 6(n - 1) - \sum u_i \\ &= 6(n - 1) - \sum (6 - f_i) \\ &= 6(n - g - 1) + \sum f_i \end{aligned}$$

Or, for a planar mechanism:

$$\begin{aligned} M &= 3(n - 1) - \sum u_i \\ &= 3(n - g - 1) + \sum f_i \end{aligned}$$

Grübler: special case for loops

The previous formula works (sort of) for all mechanisms.

For loops there is a variant.

One loop: $n = g$, so

$$M = \sum f_i + 6(-1)$$

Two loops: make a second loop by adding k links and $k + 1$ joints:

$$M = \sum f_i + 6(-2)$$

Every loop increases excess of joints over links by 1. For l loops:

$$M = \sum f_i - 6l$$

for a spatial linkage, and

$$M = \sum f_i - 3l$$

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gröbler's formula:

$$M = 3(n - 1) - \sum u_i =$$

$$M = 3(n - g - 1) + \sum f_i =$$

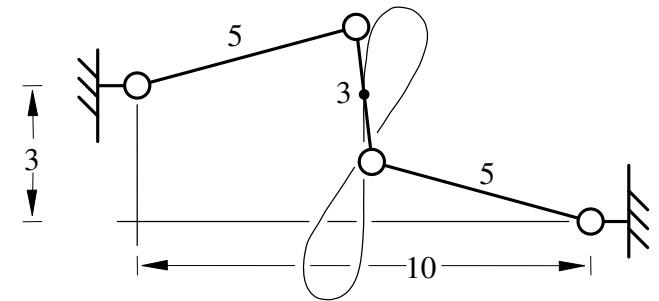
$$M = \sum f_i - 3l =$$

Spatial Gröbler's formula:

$$M = 6(n - 1) - \sum u_i =$$

$$M = 6(n - g - 1) + \sum f_i =$$

$$M = \sum f_i - 6l =$$



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gröbler's formula:

$$M = 3(n - 1) - \sum u_i = 1$$

$$M = 3(n - g - 1) + \sum f_i =$$

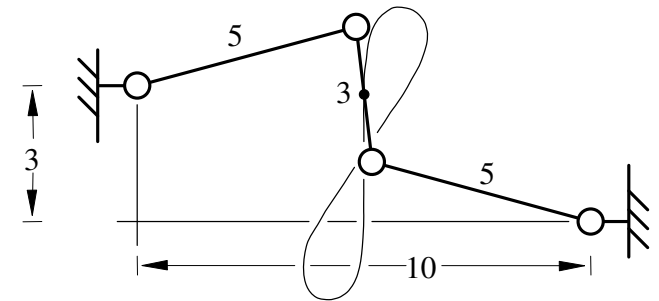
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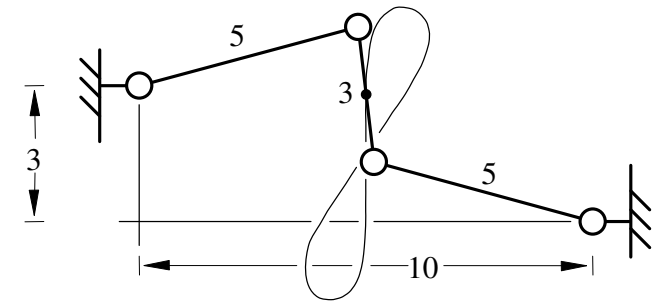
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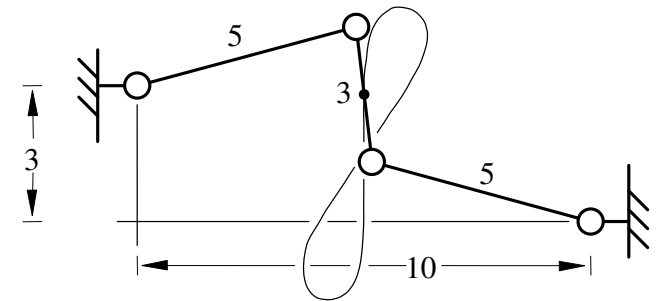
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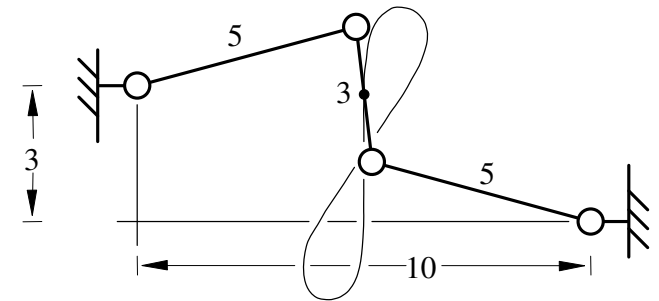
$$M = \sum f_i - 3l = 1$$

Spatial Gröbler's formula:

$$M = 6(n - 1) - \sum u_i = -2$$

$$M = 6(n - g - 1) + \sum f_i =$$

$$M = \sum f_i - 6l =$$



Independent constraints is a very strong assumption.

Common sense

Example: what is the mobility of Watt's linkage?

Planar Gröbler's formula:

$$M = 3(n - 1) - \sum u_i = 1$$

$$M = 3(n - g - 1) + \sum f_i = 1$$

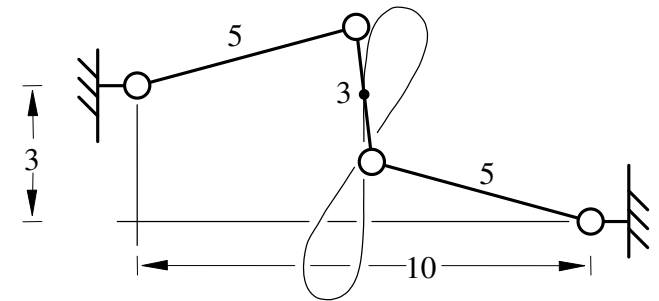
$$M = \sum f_i - 3l = 1$$

Spatial Gröbler's formula:

$$M = 6(n - 1) - \sum u_i = -2$$

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Independent constraints is a very strong assumption.

Common sense

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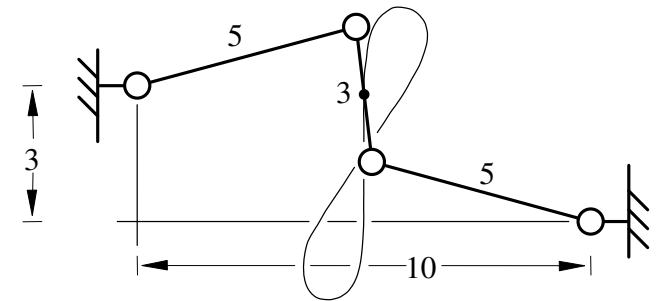
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$$M = 6(n - g - 1) + \sum f_i = -2$$

$$M = \sum f_i - 6l = -2$$



Independent constraints is a very strong assumption.

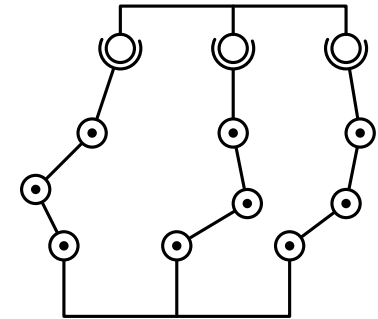
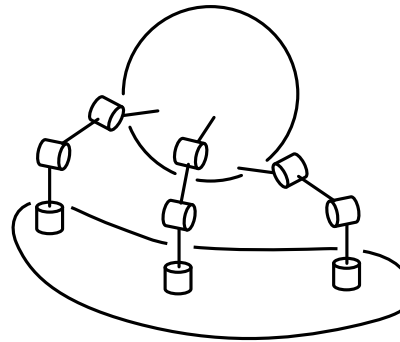
Kinematic models of contact

A grasp is like a kinematic mechanism.

Assume fingers do not lift or slip.

Model each contact as a joint

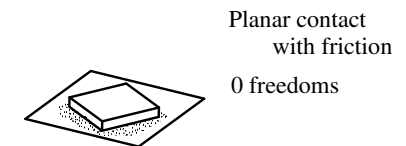
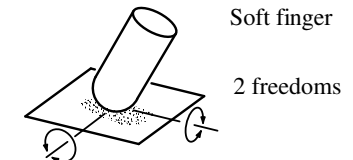
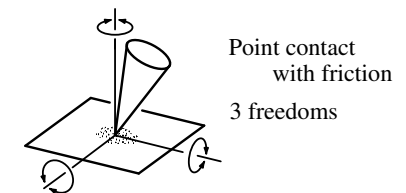
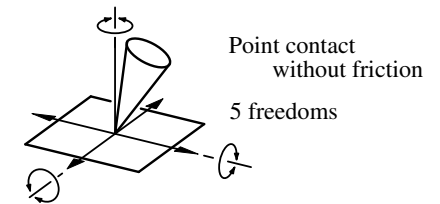
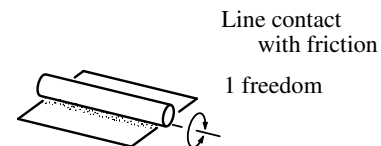
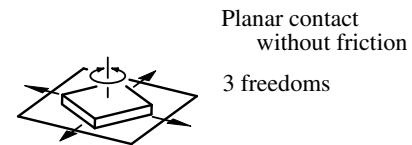
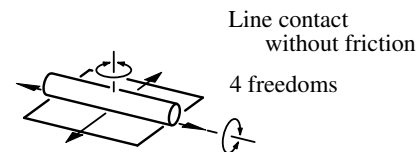
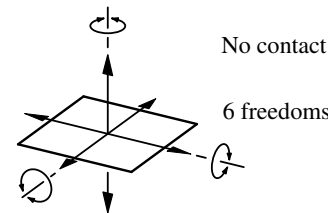
Apply Grübler's formula!



Taxonomy of contact types

In previous slide, contact was modeled as spherical joint. Are there other possibilities?

Salisbury's PhD thesis, 1982, included a taxonomy.



Mobility and connectivity of grasp

Salisbury suggests four measures:

M Mobility of the entire system with the finger joints free.

M' Mobility of the entire system, with the finger joints locked.

C Connectivity of the object relative to a fixed palm, with the finger joints free.

C' Connectivity of the object relative to a fixed palm, with the finger joints locked.

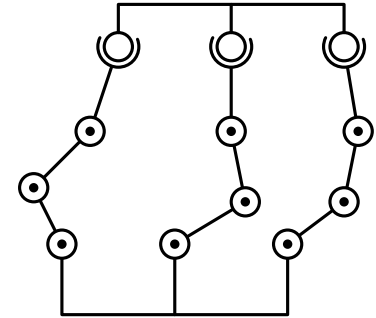
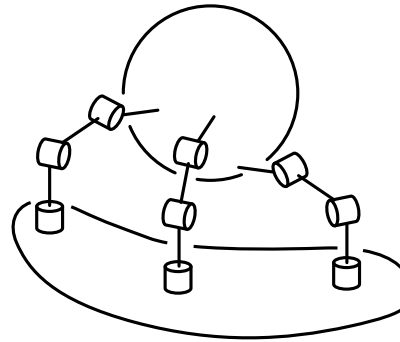
If $C = 6$ then object can make general motions.

If $C' \leq 0$ then hand can immobilize object.

Example: the Salisbury hand

What is C ?

What is C' ?



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