Adaptive Motion Control of a Nonholonomic Vehicle

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Abstract

The stabilization of the motion of a nonholonomic vehicle is considered. The control system developed has a two-level architecture. The lower control level operates within the kinematic model of the vehicle to stabilize its motion to a desired trajectory. The upper control level uses the dynamic model of the vehicle and stabilizes the feedback obtained on the lower control level. The operation of the control system is studied when unknown bounded disturbances affect the motion. The adaptive motion control is proposed to deal with uncertain dynamic parameters of the vehicle.

1 Introduction

The automatic steering of a nonholonomic vehicle attracts a great deal of attention from the research community [1]-[10]. The nonholonomic vehicle is a system with a non-integrable velocity constraint. By Brockett's necessary stability conditions [3], such a system is open-loop controllable, but it can not be stabilized to a point by means of smooth time-invariant state feedback. To stabilize such a system, time-varying feedback laws are developed by Samson [4], piece-wise continuous laws are considered by Canudas de Wit and Sørdalen [5], and discontinuous feedback laws by Guldner and Utkin [6]. Murray and Sastry [7] worked on steering a nonholonomic system between arbitrary points by means of sinusoids. Path planning approaches are developed by Latombe [8] and Laumond *et al.* [9] to generate feasible paths for nonholonomic vehicles. The recent results on the kinematic and dynamic control of the vehicles are presented in [10]. The methods of adaptive control of the autonomous vehicles are considered in [11]-[14].

Our paper deals with the adaptive stabilization of the motion of a nonholonomic vehicle to a desired trajectory when the dynamic parameters of the vehicle are not known precisely and the vehicle's dynamics is subject to unmeasured bounded disturbances. The adaptive control has been much studied for the manipulators while the number of the control inputs equals to the number of the outputs. These results cannot be directly applied to control a nonholonomic vehicle because the number of the control inputs is less than the number of the outputs. To overcome this problem, we propose a two-level hierarchical control architecture. On the lower control level, the kinematic model of the vehicle is used, and the problem is to stabilize the position and orientation of the vehicle to the desired ones by means of a unique control input which is the steering angle while the locomotion velocity is a given function. On the upper control level, the dynamic model of the vehicle is used, and the problem is to stabilize the locomotion velocity and the previously obtained steering angle.

The paper is organized as follows. The kinematic and dynamic models of a nonholonomic vehicle are described in Section 2 where the problem of motion stabilization is formulated. In Section 3, we describe a method of motion control based on the kinematic model. The motion control method based on the dynamic model of the vehicle is considered in Section 4 where the dynamic parameters of the vehicle are supposed to be known. The proposed method ensures the stabilization of the motion to a desired trajectory and is robust with respect to unmeasured bounded disturbances. The adaptive control scheme is considered in Section 5 where the dynamic parameters of the vehicle are supposed to be uncertain. Using a recursive aim inequalities method [15], an adaptation algorithm is designed to estimate the dynamic parameters of the vehicle. Although the asymptotically exact estimation is not possible because of the unknown bounded disturbances, the proposed adaptive control scheme guarantees the motion stabilization with a given accuracy. The experimental results obtained are presented in Section 6.

2 Formulation of the problem

The kinematics of a nonholonomic vehicle with the front steering wheels is described by the equations

$$\begin{cases} \dot{x} = v \cos\psi, \\ \dot{y} = v \sin\psi, \\ \dot{\psi} = \frac{v}{l} \tan\phi, \end{cases} \tag{1}$$

where, as shown in Fig. 1, x and y are the Cartesian coordinates of the midpoint of the rear wheel axle (point **B**), ψ is the orientation angle of the vehicle, v is the velocity of the point **B**, ϕ is the steering angle, and l is the wheel base (a distance between the points **A** and **B**) [16]. When the inertia of the wheels is neglected, the vehicle dynamics can be described by the equation

$$A(q, \phi, \theta)\dot{q} + b(q, \phi, \theta) = u(t) + w(t), \tag{2}$$

where $q = \mathrm{col}\,(v, \dot{\phi})$, $u = \mathrm{col}\,(u_1, u_2)$, u_1 is the engine torque, u_2 is the steering torque, θ is the vector of dynamic parameters of the vehicle; $A(q, \phi, \theta)$ is the symmetrical and positively definite inertia matrix; $b(q, \phi, \theta)$ represents the vector of centripetal, Coriolis, and friction forces; $w(t)$ is the vector of external disturbances. The disturbance is unknown, but it is bounded by a known constant:

$$\sup_t \|w(t)\| \leq C_w, \quad C_w > 0. \tag{3}$$

The equation (2) is so called equation of dynamics in quasicoordinates [16].

Let **P** be a smooth curve of a desired trajectory: $\mathbf{P} = \{\ z_d(s) \mid s \in \mathbf{R}_+\}$ where $z_d(s) = \mathrm{col}(x_d(s), y_d(s))$

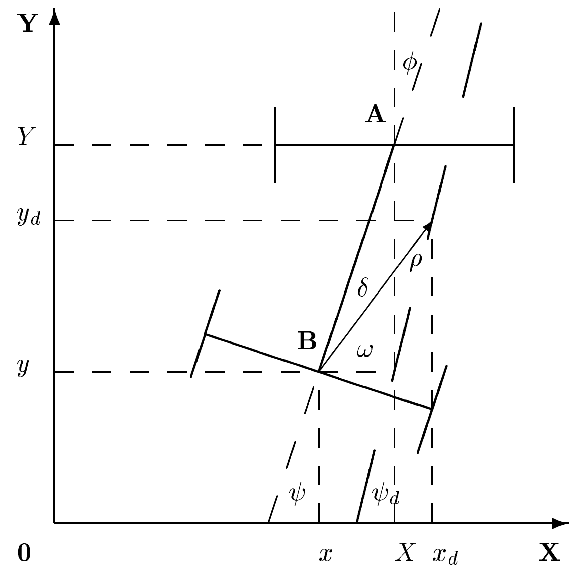

Figure 1: Kinematics of a nonholonomic vehicle

is a smooth function and $\mathbf{R}_+$ denotes a set of the non-negative numbers. It is supposed that this parametrization is nonsingular, $|\frac{d}{ds}z_d(s)| \neq 0$ for all $s \in \mathbf{R}_+$ and the first and second derivatives of $z_d(s)$ are bounded: $|\frac{d}{ds}z_d(s)| \leq C_z^{(1)}$ and $|\frac{d^2}{ds^2}z_d(s)| \leq C_z^{(2)}$.

Let $z = z(t) = \mathrm{col}\,(x, y)$ denote the vector of the Cartesian coordinates of the point **B** of the vehicle, and $Z = Z(t) = \mathrm{col}\,(X, Y)$ - the vector of coordinates of the point **A**, as depicted in Fig. 1. The coordinates X and Y can be expressed in terms of the coordinates x, y and ψ:

$$X = x + l\cos\psi, \quad Y = y + l\sin\psi. \tag{4}$$

Let $\psi_d(s)$ denote the angle between the abscissae axis and the tangent vector to the curve **P** at a point $z_d(s)$. The curvature of **P** is assumed to be bounded

$$|\frac{d}{ds}\psi_d(s)| \leq C_\psi. \tag{5}$$

This constraint arises naturally, because the steering angle of a nonholonomic vehicle is physically limited: $|\phi| \leq \phi_{max}$. From the kinematic equations (1) it follows that the curvature of **P** must not exceed $C_\psi = \sin\phi_{max}/\,l$. The curve **P** describes the geometry of a desired motion, and the motion dynamics is related to a desired locomotion velocity $v_d(s)$.

We shall say that the control aim is fulfilled if for given constants $\varepsilon_z > 0$ and $\varepsilon_v > 0$ there exists a smooth time parametrization $s(t)$ of **P** such that the inequalities

$$||Z(t) - z_d(s(t))|| < \varepsilon_z, \tag{6}$$

$$|v(t) - v_d(s(t))| < \varepsilon_v \tag{7}$$

hold for all $t \in \mathbf{R}_+$. In other words, the control law has to provide that the trajectory of the vehicle is enclosed in ε_z-neighborhood of $\mathbf{P}$ and the velocity of the vehicle belongs to ε_v-neighborhood of v_d.

3 Motion control based on a kinematic model

For the low-speed motion, the dynamics of the vehicle can be neglected and the motion is described by the equations (1) where the control inputs are the locomotion velocity v and steering angle ϕ. In order to focus on providing the inequality (6), we assume that the inequality (7) is satisfied and

$$v(t) = v_d(s(t)). \tag{8}$$

The problem is to find a time parametrization $s(t)$ of $\mathbf{P}$ and a steering angle $\phi(t)$ which ensure (6).

Let the distance between the current position $z(t)$ of the vehicle and the desired position $z_d(s(t))$ be given by the function $\rho\,(z(t), z_d(s(t))) = \|\, z_d(s(t)) - z(t)\,\|$. Let a time parametrization $s(t)$ of $\mathbf{P}$ be chosen in such a way that

$$\rho\,(z(t), z_d(s(t)) = l. \tag{9}$$

Instead of solving this nonlinear algebraic equation online, the following differential equation is used:

$$\frac{d}{dt}\rho + \gamma_\rho(\rho - l) = 0, \quad \gamma_\rho > 0 \tag{10}$$

which ensures that the function $\rho\,(z(t), z_d(s(t)))$ exponentially converges to l.

Let functions

$$\omega\,(z(t), z_d(s(t))) = \arg\,(z_d(s(t)) - z(t)) \tag{11}$$

and

$$\delta\,(z(t), \psi(t), z_d(s(t))) = \omega\,(z(t), z_d(s(t))) - \psi(t) \tag{12}$$

represent *direction* and *deviation* angles respectively, as shown in Fig. 1. By substituting the full derivative of $\rho\,(z(t), z_d(s(t)))$ along the trajectory of the system (1) into the reference equation (10), the following differential equation is obtained:

$$\dot{s} = \frac{v\cos\delta - \gamma_\rho(\rho - l)}{|\frac{d}{ds}z_d(s)|\cos(\omega - \psi_d)}, \quad s(0) = 0. \tag{13}$$

The equation (13) determines the desired time parametrization $s(t)$ of $\mathbf{P}$. The key idea of the control law is to provide such a steering angle ϕ that the deviation angle δ tends to zero. This condition together with the equation (9) will ensure that the position $Z(t)$ of the point $\mathbf{A}$ of the vehicle will tend to the point $z_d(s(t))$ of the desired trajectory.

In order the deviation angle δ exponentially converges to zero, the following reference equation is considered:

$$\frac{d}{dt}\delta + \gamma_\delta\ \delta = 0, \quad \gamma_\delta > 0. \tag{14}$$

By substituting the full derivative of the deviation angle δ given by (12) into the equation (14), the control law is derived as

$$\phi = \Phi(z, \psi, s, v) \overset{def}{=} \arctan\frac{l\,(\dot\omega - \gamma_\delta(\psi - \omega))}{v}, \tag{15}$$

where

$$\begin{aligned}\dot\omega = \tfrac{1}{\rho}((\tfrac{d}{ds}y_d(s)\dot{s} - v\,\sin\psi)\,\cos\omega -\\ (\tfrac{d}{ds}x_d(s)\dot{s} - v\,\cos\psi)\,\sin\omega).\end{aligned} \tag{16}$$

Theorem 1 *Suppose that the initial conditions satisfy the inequalities*

$$\begin{cases} 0 < \rho\,(z(0), z_d(0)) \le l, \\ |\psi_d(0) - \omega(z(0), z_d(0)| < \varepsilon. \end{cases} \tag{17}$$

Then, for small enough ε and C_ψ the solution of the closed-loop system (1), (13), (15) exists, is defined for all $t > 0$, satisfies the aim inequality (6) and

$$\lim_{t\to\infty} \|Z(t) - z_d(s(t))\| = 0. \tag{18}$$

4 Motion control based on a dynamic model

In this section, the dynamic model (1), (2) of the vehicle is considered. The peculiarity of this model is the presence of an unmeasured bounded disturbance. Our objective is to design a regulator which is robust with respect to this disturbance. As it was shown in the previous section, the control aim (6), (7) can be achieved if the velocity v and steering angle ϕ are defined by the equations (8) and (15) respectively. However, these variables can not be controlled directly if the dynamic model of the vehicle is used, because the control inputs are the engine torque and steering torque and they affect the accelerations $\dot{v}$ and $\ddot\phi$.

Consider the error functions $e_1(t) = v(t) - v_d(t)$ and $e_2(t) = \phi(t) - \Phi(z(t), \psi(t), s(t), v(t))$. These errors will tend to zero if we ensure the fulfillment of the following reference equations:

$$\begin{aligned}\dot{e}_1 + \gamma_1 e_1 = 0,\\ \ddot{e}_2 + \gamma_2\dot{e}_2 + \gamma_2^\circ e_2 = 0,\end{aligned} \tag{19}$$

where $\gamma_1, \gamma_2, \gamma_2^\circ$ are positive constants. Let us consider a control law

$$u = A(q, \phi, \theta)\, p(t) + b(q, \phi, \theta), \qquad (20)$$

where:

$$\begin{aligned} & p = \mathrm{col}\,(p_1, p_2), \\ & p_1 = \dot{v}_d - \gamma_1 e_1, \\ & p_2 = \ddot{\Phi}\,(z(t), \psi(t), s(t), v(t)) - \gamma_2 \dot{e}_2 - \gamma_2^\circ e_2. \end{aligned} \qquad (21)$$

In the absence of the disturbance in (2), the control law (20) guarantees the fulfillment of the equations (19). Assuming that Φ is a given function of time, the problem of designing a regulator would be solved by means of the Computed Torque Method [17]. However, in the considered case Φ is not a given function of time but the *feedback*. It can be defined by the equation (15). Thus, the equations (20), (21) define the upper level of the two-level hierarchical control law where the lower level involves the kinematic regulator.

Theorem 2 *Suppose that the initial locomotion velocity and steering rate satisfy the inequalities*

$$\begin{aligned} & |v(0) - v_d(0)| \le C_v, \\ & |\dot{\phi}(0)| \le C_\phi \end{aligned} \qquad (22)$$

for some $C_v > 0$, $C_\phi > 0$ respectively, and the initial coordinates of the vehicle satisfy the inequalities (17). Then, for any given ε_z and ε_v there exist small enough ε and C_ψ and large enough parameters $\gamma_\rho, \gamma_\delta, \gamma_1, \gamma_2, \gamma_2^\circ$ such that the aim inequalities (6), (7) are fulfilled on the solution of the closed–loop system (1), (2), (13), (15), (20), (21).

5 Adaptive motion control

Theorem 2 claims that the uniformly bounded additive disturbances can be successfully parried by selecting sufficiently large gains in the control law. However, in order to apply the control law (20), the vector of the vehicle's parameters θ must be known. In practice, the dynamic parameters of the vehicle are not known precisely, and the external disturbances affect the motion. Hence, an estimation of θ must be provided, and a problem of the adaptive control arises.

It is known [18] that the equation (2) is linear in terms of a suitably selected vector of the parameters $\theta \in \mathbf{R}^N$:

$$\begin{aligned} A(q, \phi, \theta)\, \dot{q} \, + b(q, \phi, \theta) \equiv & \\ & S(q, \dot{q}, \phi)\, \theta + R(q, \dot{q}, \phi), \end{aligned} \qquad (23)$$

where $S(q, \dot{q}, \phi)$ is $(2 \times N)$-matrix, $R(q, \dot{q}, \phi) \in \mathbf{R}^2$. The dimension N of the vector θ depends on the number of uncertain parameters. For instance, mass or moment of inertia of the vehicle and friction coefficients are usually uncertain and can vary during the motion. Although the vector θ is not known precisely, some bounds for variations of its components are known. We suppose that the closed bounded convex set $\Theta \subset \mathbf{R}^N$ is given such that $\theta \in \Theta$. The set Θ establishes a range for variations of the components of θ. It is well known that the matrix $A(q, \phi, \theta)$ is nonsingular for any physically realizable vector of θ. We suppose that this property holds for any $\theta \in \Theta$.

Denote $\tau = \tau(t) \in \mathbf{R}^N$ an estimation vector for the vector of the parameters θ. Then, the equation (2) can be rewritten as

$$A(q, \phi, \tau(t))\dot{q} + b(q, \phi, \tau(t)) = u(t) + \eta(t, \tau(t)), \quad (24)$$

where the error function

$$\begin{aligned} \eta(t, \tau(t)) = \; & w(t) + \\ & S(q(t), \dot{q}(t), \phi(t))(\tau(t) - \theta). \end{aligned} \qquad (25)$$

The equation (24) has the same form as the equation (2), but now its left-hand side depends on the *known* vector $\tau(t)$ instead of the unknown vector θ. The error function $\eta(t, \tau(t))$ has the same role in (24) as the disturbance $w(t)$ in (2). Hence, following the analogy with (24), the control law is defined as

$$u = A(q, \phi, \tau)\, p(t) + b(q, \phi, \tau). \qquad (26)$$

Now, an adaptation algorithm has to be defined, in order to obtain $\tau(t)$.

According to the recursive aim inequalities method [15], the estimate $\tau(t)$ is constructed in such a way that

$$\|\eta(t, \tau(t))\| \le C_\eta, \;\; C_\eta > C_w. \qquad (27)$$

An estimation procedure used is gradient with respect to $\|\eta(t, \tau(t))\|^2$ and includes the dead zone with a threshold C_η:

$$\begin{aligned} \dot{\tau}(t) = -\gamma_\tau \;\, \mathbf{Pr}_{\tau(t)}\, [\, h\,(\|\eta\,(t, \tau(t))\| - C_\eta) \,\cdot & \\ S^T(q, \dot{q}, \phi)\, \eta\,(t, \tau(t))\,], & \end{aligned} \qquad (28)$$

where

$$h(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{otherwise,} \end{cases} \qquad (29)$$

and $\mathbf{Pr}_\tau\,[\,x\,]$ is a projection operator described below. Denote $\partial\Theta$ the boundary surface of the set Θ, and $\mathbf{T}_\tau(\partial\Theta)$ - the tangent space at a point $\tau \in \partial\Theta$ to

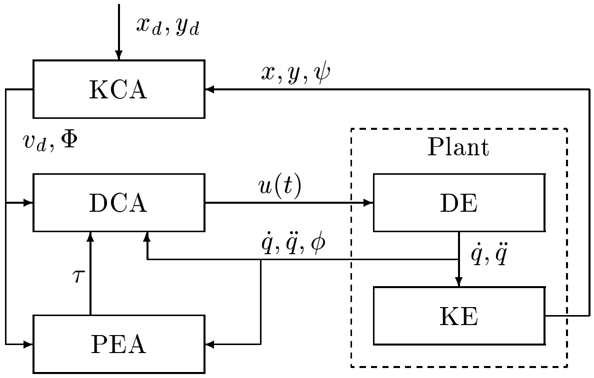

Figure 2: The adaptive control scheme

the surface $\partial\Theta$. The operator $\mathbf{Pr}_\tau\,[x]$ provides the orthogonal projection of the vector x on the tangent space $\mathbf{T}_\tau(\partial\Theta)$ in the case when the point τ belongs to the boundary surface of the set Θ and the vector x is directed outside of the set Θ; otherwise, $\mathbf{Pr}_\tau$ is the identical operator. Thus, assuming that $\tau(0) \in \Theta$ the algorithm (28) guarantees the inclusion $\tau(t) \in \Theta$ for all $t \geq 0$. The error function $\eta(t, \tau(t))$ in the estimation procedure (28) can be evaluated as

$$\eta(t, \tau(t)) = A(q, \phi, \tau(t))\dot{q} + b(q, \phi, \tau(t)) - u(t), \quad (30)$$

where all the variables are available from measurements. Now, we can state the following theorem analogous to the Theorem 2.

Theorem 3 *Suppose that the initial locomotion velocity and steering rate satisfy the inequalities (22) and the initial coordinates of the vehicle satisfy the inequalities (17) with small enough ε and $\tau(0) \in \Theta$. Then, for any given ε_z and ε_v there exist small enough ε and C_ψ and large enough parameters γ_s, γ_δ, γ_1, γ_2, γ_2°, γ_τ such that the aim inequalities (6), (7) are fulfilled on the solution of the closed-loop system (1), (2), (13), (15), (20), (26).*

The proposed adaptive control scheme is shown in Fig. 2. The inner loop involves: Kinematic Control Algorithm (KCA), Dynamic Control Algorithm (DCA) and Parameters Estimation Algorithm (PEA). The essence of these blocks is given by the equations (15), (26) and (28) respectively. The kinematic equations (1) (block KE) and dynamic equations (2) (block DE) form the plant model.

Note, that considered in [18] controller for a rigid link manipulator (holonomic system) has a similar inner loop structure. Our control scheme differs from this type of a classic controller because of the kinematic control law of the feedback form with respect to the coordinates of the vehicle.

6 Experiments

The experiments have been performed on an automatic car designed on the base of a LIGIER electric car shown in Fig. 3. This vehicle can either be manually driven as a car, or move autonomously [19, 20]. To allow autonomous motions, the car is equiped with a control unit based on a Motorola VME162-CPU board and a transputer net. The sensor unit of the car consists of ultrasonic range sensors (Polaroid 9000) and a linear CCD-camera. The steering wheel servo-system is equiped with a direct current motor and an optical encoder to measure the steering angle. The locomotion servo-system of the vehicle is equiped with an asynchronous motor and two optical encoders at the rear wheels to provide data on the locomotion velocity.

Figure 3: A LIGIER electric car

An operation of the developed controller is illustrated by Fig. 4 where the desired path is given as the (x, y)-points obtained by discretization of the functions

$$\begin{cases} x_d(s) = s, \\ y_d(s) = 1.0 - \cos s/3.0. \end{cases} \quad (31)$$

The vehicle starts at (0, 0), speeds up to 1 m/s and follows the desired path given by (31).

7 Conclusion

Three consequently complicated problems of the motion stabilization of a nonholonomic vehicle to a desired trajectory were considered. The control scheme developed has a two-level architecture and is based on the kinematic and dynamic models of the vehicle. On the first level, the kinematic model is used and the control law stabilizes the motion of the vehicle to the

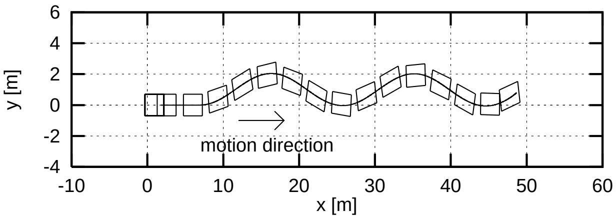

Figure 4: An example of the path following

desired trajectory. On the second level, the dynamic model is used and the control law stabilizes the first level feedback. The robustness of the motion control with respect to the bounded unmeasured disturbances was investigated. The adaptive control scheme was proposed to stabilize the motion while the uncertain dynamic parameters of the vehicle.

The proposed algorithm stabilizes the motion to a desired trajectory given analitically. This approach can also be applied to control the autonomous vehicle in an automatic convoy where the autonomous vehicle has to follow the leading vehicle in a prescribed distance. In this case, the relative coordinates of a target point of the leading vehicle are used as the desired coordinates x_d, y_d.

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