Adaptive Tracking Control of a Nonholonomic Mobile Robot

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***Abstract*—A mobile robot is one of the well-known nonholonomic systems. The integration of a kinematic controller and a torque controller for the dynamic model of a nonholonomic mobile robot has been presented. In this paper, an adaptive extension of the controller is proposed. If an adaptive tracking controller for the kinematic model with unknown parameters exists, an adaptive tracking controller for the dynamic model with unknown parameters can be designed by using an adaptive backstepping approach. A design example for a mobile robot with two actuated wheels is provided. In this design, a new kinematic adaptive controller is proposed, then a torque adaptive controller is derived by using the kinematic controller.**

***Index Terms*—Adaptive backstepping, adaptive tracking control, dynamic model, nonholonomic mobile robot.**

I. Introduction

A mobile robot is one of the well-known systems with nonholonomic constraints, and there are many works on its tracking control [1]–[4]. Their objects are mostly kinematic models, but recently one method for dynamic models has been proposed [5]. This method integrates a kinematic controller and a torque controller for the dynamic model of a nonholonomic mobile robot by using backstepping [6].

The control input of the controller for the kinematic model is generally velocity, but it is more realistic that the input is torque. In [5], a kinematic controller is designed first so that the tracking error between a real robot and a reference robot converges to zero, and secondly a torque controller is designed by using backstepping so that the velocities of a mobile robot converge to the desired velocities, which are given by the kinematic controller designed at the first step.

In this paper, we present a method to design an adaptive tracking controller for the dynamic model of a nonholonomic mobile robot with unknown parameters by adaptive backstepping. The adaptive control methods [7], [8] proposed so far for nonholonomic mobile robots do not consider the model with unknown parameters in its kinematic part, but our method considers the case. We show that there exists an adaptive tracking controller for the dynamic model with unknown parameters if it is possible to design an adaptive tracking controller for the kinematic model with unknown parameters. For an example, we design an adaptive tracking controller of a mobile robot with two actuated wheels. First, we present an adaptive tracking controller for the kinematic model modifying the existing method [9]. Secondly, our main theorem is applied to the dynamic model by using the kinematic adaptive controller and we get a torque controller.

Manuscript received May 28, 1999; revised November 15, 1999. This paper was recommended for publication by Associate Editor J. Laumond and Editor A. De Luca upon evaluation of the reviewers' comments. This paper was presented in part at the International Symposium on Intelligent Robotic Systems, Bangalore, India, January 1998.

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Publisher Item Identifier S 1042-296X(00)08352-X.

II. Kinematics and Dynamics of a Nonholonomic Mobile Robot

Consider the following nonholonomic mobile robot that is subject to m constraints

$$M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) = B(q)\tau + A^T(q)\lambda \tag{1}$$

where $q \in R^n$ is generalized coordinates, $\tau \in R^r$ is the input vector, $\lambda \in R^m$ is the vector of constraint forces, $M(q) \in R^{n\times n}$ is a symmetric and positive-definite inertia matrix, $V(q, \dot{q}) \in R^{n\times n}$ is the centripetal and coriolis matrix, $G(q) \in R^n$ is the gravitational vector, $B(q) \in R^{n\times r}$ is the input transformation matrix, and $A(q) \in R^{m\times n}$ is the matrix associated with the constraints. In the following, we consider the $r = n - m$ case.

The kinematic constraints are assumed to be expressed as

$$A(q)\dot{q} = 0. \tag{2}$$

With respect to the dynamics of mobile robot (1), the following properties are known [10].

Property 1: $M(q)$ is a symmetric and positive-definite matrix.

Property 2: There is a parameter vector $p_0 \in R^{\bar{l}_0}$ on dynamics that satisfies the following equation [11]:

$$M(q)\dot{v} + V(q, \dot{q})v + G(q) = Y_0(q, \dot{q}, v, \dot{v})p_0 \tag{3}$$

where $v \in R^n$ and Y_0 is an $n \times \bar{l}_0$ matrix whose elements consist of known functions.

Property 3: The matrix $\dot{M} - 2V$ is skew-symmetric [12], that is, $x^T(\dot{M} - 2V)x = 0, \forall x \in R^n$.

The nonholonomic mobile robot (1) is transformed to and divided into the following two equations [5]:

$$\dot{q} = S(q)\nu(t) \tag{4}$$

$$\overline{M}(q)\dot{\nu} + \overline{V}(q, \dot{q})\nu + \overline{G}(q) = \overline{B}(q)\tau \tag{5}$$

where $S(q) \in R^{n\times(n-m)}$ spans the null space of $A(q)$ and a full-rank matrix formed by a set of smooth and linearly independent vector fields, $\nu \in R^{n-m}$, $\overline{M} = S^TMS$, $\overline{V} = S^T(M\dot{S} + VS)$, $\overline{G} = S^TG$, and $\overline{B} = S^TB$. The system (4) represents the kinematics of a mobile robot.

The following properties are derived from the previously described Properties 1–3 [5], [13].

Property 1′: $\overline{M}(q)$ is a symmetric and positive-definite matrix.

Property 2′: There is a parametric vector $p_1 \in R^{\bar{l}_1}$ on dynamics that satisfies

$$\overline{M}(q)\dot{\nu} + \overline{V}(q, \dot{q})\nu + \overline{G}(q) = Y_1(q, \dot{q}, \nu, \dot{\nu})p_1 \tag{6}$$

where Y_1 is $(n - m) \times \bar{l}_1$ matrix whose elements consist of known functions.

Property 3′: The matrix $\dot{\overline{M}} - 2\overline{V}$ is skew-symmetric.

In (6), p_1 includes only the parameters on dynamics, not kinematics. The parameters on kinematics are included in Y_1. Now, we assume the structure of the parameters on kinematics.

Assumption II.1: Some parameters in the kinematic part (4) of a mobile robot appear as following:

$$\dot{q} = S(q, \theta)\nu = \sum_{i=1}^{n-m} s_i(q, \theta_i)\nu_i = \sum_{i=1}^{n-m} \left(\sigma_{i0}(q) + \sum_{j=1}^{l_i} \theta_{ij}\sigma_{ij}(q)\right)\nu_i \tag{7}$$

where we let $\theta = [\theta_1, \ldots, \theta_{n-m}]$, $\theta_i = [\theta_{i1}, \ldots, \theta_{il_i}]^T$, $1 \le i \le n-m$ are parametric vectors, and $\sigma_{ij}(q)$, $1 \le i \le n-m$, $0 \le j \le l_i$ are vectors whose elements consist of known functions.

Furthermore, the following Property 2″ is satisfied.

Property 2″: There is a parametric vector $p \in R^{\bar{l}}$ on kinematics and dynamics which satisfies

$$\overline{M}(q)\dot{\nu} + \overline{V}(q, \dot{q})\nu + \overline{G}(q) = Y(q, \dot{q}, \nu, \dot{\nu})p \tag{8}$$

where Y is $(n-m) \times \bar{l}$ matrix whose elements consist of known functions and p is a parametric vector which is composed of the elements of p_1 and θ_i. □

III. Adaptive Tracking Control of a Nonholonomic Mobile Robot

In [5], $\nu(t)$ is considered as a control input for the kinematic part (4), and an ideal control input $\nu_c(t)$ is designed to track a reference trajectory. Since the real input of the mobile robot (1) is τ, τ is designed to make $\nu(t) - \nu_c(t) \to 0$ as $t \to \infty$ by using backstepping [6]. But if there exist some unknown parameters in a mobile robot, that is, $S(q)$ has unknown parameters or p is unknown in (8), we cannot design a tracking controller according to [5].

In this paper, it is shown that an adaptive tracking controller can be designed for the dynamic model with unknown parameters if it is possible to design an adaptive tracking controller for the kinematic model with unknown parameters.

Control Objective: Design an adaptive tracking controller for a nonholonomic mobile robot (1), in order that

$$\lim_{t \to \infty} (\overline{q}(t) - q_r(t)) = 0 \tag{9}$$

where $\overline{q}(t) = Cq(t)$, $C \in R^{s \times n}$ and $q_r(t) \in R^s$ is its desired output and differentiable. □

Assumption III.1: An adaptive tracking controller

$$\nu = \nu_c(q, q_r, \hat{a}) \tag{10}$$

$$\dot{\hat{a}}_i = T_i(q, q_r, \hat{a}), \qquad 1 \le i \le k \tag{11}$$

exists for the kinematic model (7), that is, with this controller $\overline{q} \to q_r$ as $t \to \infty$.

And there exists a positive-definite and radially unbounded function V_1 which satisfies

$$\dot{V}_1(q, q_r, \tilde{a}) = \frac{\partial V_1}{\partial q} S\nu_c + \frac{\partial V_1}{\partial q_r}\dot{q}_r + \sum_{i=1}^{k} \frac{\partial V_1}{\partial \hat{a}_i} T_i \le 0 \tag{12}$$

and the signals included in this function are bounded, where $\hat{a}$ is the estimate of an unknown parametric vector $a = [a_1, \ldots, a_k]^T$, which is composed of θ_{ij}, and $\tilde{a} = \hat{a} - a$ is the estimated error. □

The general design method of these adaptive tracking controllers which satisfy Assumption III.1 has not been established so far.

Assumption III.2: $\overline{B}(q)$ in (5) does not include unknown parameters and is nonsingular. □

Assumption III.3: $\partial V_1/\partial q$ does not include unknown parameters. □

Assumption III.2 is easily relaxed by the existing adaptive control technique, if $\overline{B}$ is constant and the sign of each elements is known. Assumption III.3 can be always satisfied by the appropriate selection of V_1.

Now, we provide the following theorem to design an adaptive tracking controller of a mobile robot using adaptive backstepping technique [6]. The adaptive control technique for the dynamic part (5) is based on [10].

Theorem III.1: If Assumptions II.1 and Assumptions III.1–III.3 are satisfied for a nonholonomic mobile robot (1), the following adaptive tracking controller (13)–(16) achieves the control objective: $\overline{q} \to q_r(t \to \infty)$ and the boundedness of the signals included in V_2, which is defined in (17).

$$\tau = \overline{B}^{-1}\left(-K_d\tilde{\nu} + Y_c\hat{p} - \left(\frac{\partial V_1}{\partial q}\hat{S}\right)^T\right) \tag{13}$$

$$\dot{\hat{a}}_i = T_i(q, q_r, \hat{a}), \qquad 1 \le i \le k \tag{14}$$

$$\dot{\hat{\theta}}_i = \Lambda_i\left(\frac{\partial V_1}{\partial q}\sigma_i\right)^T \tilde{\nu}_i, \qquad 1 \le i \le n-m \tag{15}$$

$$\dot{\hat{p}} = -\Gamma Y_c^T \tilde{\nu} \tag{16}$$

where $\hat{\theta} = [\hat{\theta}_1, \ldots, \hat{\theta}_{n-m}]$ is the estimate of θ, $Y_c \equiv Y(q, \dot{q}, \nu_c, \dot{\nu}_c)$, $\hat{S} \equiv S(q, \hat{\theta})$, $\tilde{\nu} = \nu - \nu_c = [\tilde{\nu}_1, \ldots, \tilde{\nu}_{n-m}]$, $\sigma_i = [\sigma_{i1}, \ldots, \sigma_{il_i}]$, $1 \le i \le n-m$, and K_d, Γ, Λ_i, $1 \le i \le n-m$ are symmetric and positive-definite matrices with appropriate dimensions.

V_2 is defined as

$$V_2 = V_1 + \frac{1}{2}\tilde{\nu}^T\overline{M}\tilde{\nu} + \frac{1}{2}\tilde{p}^T\Gamma^{-1}\tilde{p} + \sum_{i=1}^{n-m}\frac{1}{2}\tilde{\theta}_i^T\Lambda_i^{-1}\tilde{\theta}_i \tag{17}$$

with $\tilde{p} = \hat{p} - p$, $\tilde{\theta} = \hat{\theta} - \theta$.

Proof: The derivative of V_2 is

$$\begin{aligned}
\dot{V}_2 &= \frac{\partial V_1}{\partial q} S(\nu_c + \tilde{\nu}) + \frac{\partial V_1}{\partial q_r}\dot{q}_r + \sum_{i=1}^{k}\frac{\partial V_1}{\partial \hat{a}_i} T_i \\
&\quad + \tilde{\nu}^T\left(\frac{1}{2}\dot{\overline{M}} - \overline{V}\right)\tilde{\nu} + \tilde{\nu}^T(\overline{B}\tau - \overline{V}\nu_c - \overline{G} - \overline{M}\dot{\nu}_c) \\
&\quad + \tilde{p}^T\Gamma^{-1}\dot{\hat{p}} + \sum_{i=1}^{n-m}\tilde{\theta}_i^T\Lambda_i^{-1}\dot{\hat{\theta}}_i \\
&= \frac{\partial V_1}{\partial q} S\nu_c + \frac{\partial V_1}{\partial q_r}\dot{q}_r + \sum_{i=1}^{k}\frac{\partial V_1}{\partial \hat{a}_i} T_i + \tilde{p}^T\Gamma^{-1}\dot{\hat{p}} \\
&\quad + \tilde{\nu}^T\left(\overline{B}\tau - Y_c p + \left(\frac{\partial V_1}{\partial q}S\right)^T\right) + \sum_{i=1}^{n-m}\tilde{\theta}_i^T\Lambda_i^{-1}\dot{\hat{\theta}}_i \\
&= \frac{\partial V_1}{\partial q} S\nu_c + \frac{\partial V_1}{\partial q_r}\dot{q}_r + \sum_{i=1}^{k}\frac{\partial V_1}{\partial \hat{a}_i} T_i + \tilde{p}^T\Gamma^{-1}\left(\dot{\hat{p}} + \Gamma Y_c^T\tilde{\nu}\right) \\
&\quad + \sum_{i=1}^{n-m}\tilde{\theta}_i^T\Lambda_i^{-1}\left(\dot{\hat{\theta}}_i - \Lambda_i\left(\frac{\partial V_1}{\partial q}\sigma_i\right)^T\tilde{\nu}_i\right) - \tilde{\nu}^T K_d\tilde{\nu} \\
&= \frac{\partial V_1}{\partial q} S\nu_c + \frac{\partial V_1}{\partial q_r}\dot{q}_r + \sum_{i=1}^{k}\frac{\partial V_1}{\partial \hat{a}_i} T_i - \tilde{\nu}^T K_d\tilde{\nu}
\end{aligned} \tag{18}$$

where we used the following equation:

$$\begin{aligned}
\overline{M}\dot{\tilde{\nu}} &= \overline{M}(\dot{\nu} - \dot{\nu}_c) \\
&= \overline{B}\tau - \overline{V}\nu - \overline{G} - \overline{M}\dot{\nu}_c \\
&= \overline{B}\tau - \overline{V}(\tilde{\nu} + \nu_c) - \overline{G} - \overline{M}\dot{\nu}_c.
\end{aligned} \tag{19}$$

From Assumption III.1, (17), and (18), the signals included in V_2 are bounded. Because $\dot{\tilde{\nu}}$ is proved to be bounded, $\ddot{V}_2 \in L_\infty$. From Barbalat's lemma [14], [6], we can show $\nu(t) \to \nu_c(t)$ as $t \to \infty$. Therefore, the equation $\dot{q} = S\nu = S(\tilde{\nu} + \nu_c) = S\nu_c + S\tilde{\nu}$ shows $\overline{q}(t) \to q_r(t)$ as $t \to \infty$, since $\tilde{\nu} \to 0$ as $t \to \infty$ and the kinematic model of a mobile robot satisfies Assumption III.1 ■

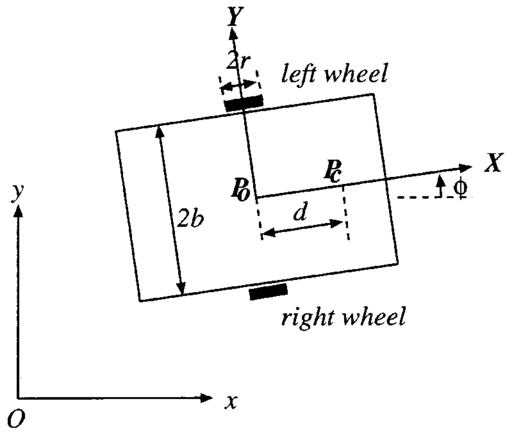

Fig. 1. Mobile robot with two actuated wheels.

Remark III.1: Because adaptive control is applied to treat unknown parameters in the kinematic part, it is more important to consider the dynamic part properly, that is, our proposed model-based controller is better than a high-gain feedback controller to treat the dynamics. As is generally known, the adaptive control system designed for the kinematics may be unstable if there exists the error $\tilde{\nu}$ [15].

IV. Mobile Robot with Two Actuated Wheels

In this section, we consider a mobile robot with two actuated wheels as an example which the theorem can be applied to. An adaptive tracking controller is designed for the kinematic model and the dynamic model, and some simulation results are provided.

A. Model of a Mobile Robot with Two Actuated Wheels

We consider the mobile robot with two actuated wheels, which is shown in Fig. 1 [16].

With regard to the mobile robot shown in Fig. 1, $2b$ is the width of the mobile robot and r is the radius of the wheel. $O-xy$ is the world coordinate system and P_0-XY is the coordinate system fixed to the mobile robot. P_0 is the origin of the coordinate system P_0-XY and the middle between the right and left driving wheels. The center of mass of the mobile robot is P_c, which is on the X-axis, and the distance from P_0 to P_c is d. For the later description, m_c and m_w are the mass of the body and wheel with a motor, I_c, I_w, and I_m are the moment of inertia of the body about the vertical axis through P_c, the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively.

The configuration of the mobile robot can be described by five generalized coordinates

$$q=[x,\,y,\,\phi,\,\theta_r,\,\theta_l]^T \tag{20}$$

where $(x,\,y)$ are the coordinates of P_0, ϕ is the heading angle of the mobile robot, and θ_r, θ_l are the angles of the right and left driving wheels.

We assume the wheels roll and do not slip. Then, there exist three constraints; the velocity of P_0 must be in the direction of the axis of symmetry and the wheels must not slip

$$\dot{y}\cos\phi-\dot{x}\sin\phi=0 \tag{21}$$

$$\dot{x}\cos\phi+\dot{y}\sin\phi+b\dot{\phi}=r\dot{\theta}_r \tag{22}$$

$$\dot{x}\cos\phi+\dot{y}\sin\phi-b\dot{\phi}=r\dot{\theta}_l. \tag{23}$$

These constraints can be rewritten in the form

$$A(q)\dot{q}=0 \tag{24}$$

where

$$A(q)=\begin{bmatrix}\sin\phi & -\cos\phi & 0 & 0 & 0\\ \cos\phi & \sin\phi & b & -r & 0\\ \cos\phi & \sin\phi & -b & 0 & -r\end{bmatrix}. \tag{25}$$

Equations (4) and (5) can be written as the following:

$$\dot{q}=S(q)\nu(t) \tag{26}$$

$$\overline{M}(q)\dot{\nu}+\overline{V}(q,\,\dot{q})\nu=\overline{B}(q)\tau \tag{27}$$

where $S(q)$ is selected as

$$S(q)=\begin{bmatrix}\frac{r}{2}\cos\phi & \frac{r}{2}\cos\phi\\ \frac{r}{2}\sin\phi & \frac{r}{2}\sin\phi\\ \frac{r}{2b} & -\frac{r}{2b}\\ 1 & 0\\ 0 & 1\end{bmatrix} \tag{28}$$

and $\overline{M}$, $\overline{V}$, $\overline{B}$ are expressed as

$$\overline{M}=\begin{bmatrix}\frac{r^2}{4b^2}(mb^2+I)+I_w & \frac{r^2}{4b^2}(mb^2-I)\\ \frac{r^2}{4b^2}(mb^2-I) & \frac{r^2}{4b^2}(mb^2+I)+I_w\end{bmatrix}$$

$$\overline{V}=\begin{bmatrix}0 & \frac{r^2}{2b}m_c d\dot{\phi}\\ -\frac{r^2}{2b}m_c d\dot{\phi} & 0\end{bmatrix}$$

$$\overline{B}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{29}$$

and $\tau=[\tau_r,\,\tau_l]^T$ consists of motors' torques τ_r and τ_l, which act on the right and left wheels, respectively, and let $m=m_c+2m_w$, $I=m_c d^2+2m_w b^2+I_c+2I_m$.

B. Adaptive Control of the Kinematic Model

We design an adaptive tracking controller for the kinematic part (26) modifying the method proposed by Kanayama *et al.* [9].

First, we consider ν as a control input and construct the adaptive control system for the following kinematic model:

$$\frac{d}{dt}\begin{bmatrix}x\\ y\\ \phi\\ \theta_r\\ \theta_l\end{bmatrix}=\begin{bmatrix}\frac{r}{2}\cos\phi & \frac{r}{2}\cos\phi\\ \frac{r}{2}\sin\phi & \frac{r}{2}\sin\phi\\ \frac{r}{2b} & -\frac{r}{2b}\\ 1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} \tag{30}$$

where ν_1 and ν_2 represent the angular velocities of right and left wheels.

We focus on only three states x, y, ϕ, except θ_r and θ_l. The relationship between v, w, and ν_1, ν_2 is the following:

$$\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix}=\begin{bmatrix}\frac{1}{r} & \frac{b}{r}\\ \frac{1}{r} & -\frac{b}{r}\end{bmatrix}\begin{bmatrix}v\\ w\end{bmatrix} \tag{31}$$

where v is the straight line velocity and w is the angular velocity of the mobile robot at the point P_0.

Substituting (31) for (30), we get the ordinary form of a mobile robot with two actuated wheels

$$\frac{d}{dt}\begin{bmatrix}x\\ y\\ \phi\end{bmatrix}=\begin{bmatrix}\cos\phi & 0\\ \sin\phi & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ w\end{bmatrix}. \tag{32}$$

The various design methods for this system (32) have already been proposed. Our method is based on the method [9] whose objective is tracking on a reference robot shown in Fig. 2.

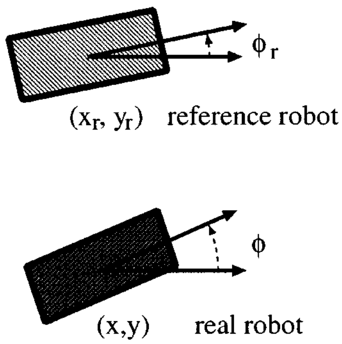

Fig. 2. Reference robot and real robot.

The kinematics of the reference robot is given as

$$\frac{d}{dt}\begin{bmatrix} x_r \\ y_r \\ \phi_r \end{bmatrix} = \begin{bmatrix} \cos\phi_r & 0 \\ \sin\phi_r & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_r \\ w_r \end{bmatrix} \tag{33}$$

where x_r, y_r, and ϕ_r are the configure of the reference robot, and v_r, w_r are its reference inputs.

We define e_1, e_2, e_3 as following:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_r - x \\ y_r - y \\ \phi_r - \phi \end{bmatrix}. \tag{34}$$

e_1, e_2, e_3 describe the difference of position and direction of the reference robot from the real robot. The inputs v, w, which make e_1, e_2, e_3 converge to zero, are given by the following [9], [5]:

$$\begin{aligned} v_f &= v_r \cos e_3 + K_1 e_1 \\ w_f &= w_r + v_r K_2 e_2 + K_3 \sin e_3 \end{aligned} \tag{35}$$

where K_1, K_2, K_3 are positive constants.

We can easily confirm that e_1, e_2, e_3 satisfy

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = v\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + w\begin{bmatrix} e_2 \\ -e_1 \\ -1 \end{bmatrix} + \begin{bmatrix} v_r\cos e_3 \\ v_r \sin e_3 \\ w_r \end{bmatrix}. \tag{36}$$

We define V_0 as

$$V_0 = \frac{1}{2}(e_1^2 + e_2^2) + \frac{1-\cos e_3}{K_2} \tag{37}$$

then, the derivative of V_0 satisfies the following inequality:

$$\dot{V}_0 = e_1\dot{e}_1 + e_2\dot{e}_2 + \dot{e}_3\frac{\sin e_3}{K_2} = -K_1 e_1^2 - \frac{K_3\sin^2 e_3}{K_2} \le 0. \tag{38}$$

If the parameters in kinematics (30), r and b, are unknown, we cannot choose the inputs as (35) because of the relationship (31) between v, w and ν_1, ν_2. Hence, we design an adaptive controller to attain the control objective by using the estimates of r and b.

By using ν_1 and ν_2, (36) is transformed to

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \nu_1\begin{bmatrix} -\frac{r}{2} + \frac{r}{2b}e_2 \\ -\frac{r}{2b}e_1 \\ -\frac{r}{2b} \end{bmatrix} + \nu_2\begin{bmatrix} -\frac{r}{2} - \frac{r}{2b}e_2 \\ \frac{r}{2b}e_1 \\ \frac{r}{2b} \end{bmatrix} + \begin{bmatrix} v_r\cos e_3 \\ v_r\sin e_3 \\ w_r \end{bmatrix} \tag{39}$$

where we set

$$a_1 = \frac{1}{r} \quad \text{and} \quad a_2 = \frac{b}{r} \tag{40}$$

and it is assumed that we know a positive constant δ which satisfies $a_2 \ge \delta$ noticing $a_2 > 0$.

Then, ν_1 and ν_2 are chosen as the following:

$$\begin{aligned} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} &= \begin{bmatrix} \hat{a}_1 & \hat{a}_2 \\ \hat{a}_1 & -\hat{a}_2 \end{bmatrix}\begin{bmatrix} v_f \\ w_f \end{bmatrix} & (41) \\ &= \begin{bmatrix} a_1 + \tilde{a}_1 & a_2 + \tilde{a}_2 \\ a_1 + \tilde{a}_1 & -a_2 - \tilde{a}_2 \end{bmatrix}\begin{bmatrix} v_f \\ w_f \end{bmatrix}. & (42) \end{aligned}$$

Therefore

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} &= \left(1 + \frac{\tilde{a}_1}{a_1}\right)v_f\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \\ &\quad + \left(1 + \frac{\tilde{a}_2}{a_2}\right)w_f\begin{bmatrix} e_2 \\ -e_1 \\ -1 \end{bmatrix} + \begin{bmatrix} v_r\cos e_3 \\ v_r\sin e_3 \\ w_r \end{bmatrix}. \end{aligned} \tag{43}$$

We define V_1 as

$$V_1 = V_0 + \frac{1}{2\gamma_1 a_1}\tilde{a}_1^2 + \frac{1}{2\gamma_2 a_2}\tilde{a}_2^2 \tag{44}$$

with positive constants γ_1, γ_2.

The derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= e_1\dot{e}_1 + e_2\dot{e}_2 + \frac{\dot{e}_3\sin e_3}{K_2} + \frac{\tilde{a}_1}{\gamma_1 a_1}\dot{\hat{a}}_1 + \frac{\tilde{a}_2}{\gamma_2 a_2}\dot{\hat{a}}_2 \\ &= e_1\left(-K_1 e_1 - \frac{\tilde{a}_1}{a_1}v_f\right) + e_2 v_r \sin e_3 \\ &\quad + \left(-K_2 e_2 v_r - K_3 \sin e_3 - \frac{\tilde{a}_2}{a_2}w_f\right)\frac{\sin e_3}{K_2} \\ &\quad + \frac{\tilde{a}_1}{\gamma_1 a_1}\dot{\hat{a}}_1 + \frac{\tilde{a}_2}{\gamma_2 a_2}\dot{\hat{a}}_2 \\ &= \dot{V}_0 + \frac{\tilde{a}_1}{\gamma_1 a_1}\left(\dot{\hat{a}}_1 - \gamma_1 e_1 v_f\right) + \frac{\tilde{a}_2}{\gamma_2 a_2}\left(\dot{\hat{a}}_2 - \gamma_2\frac{w_f \sin e_3}{K_2}\right). \end{aligned} \tag{45}$$

Now, the parameter update rules are chosen as

$$\begin{aligned} \dot{\hat{a}}_1 &= \gamma_1 e_1 v_f \\ \dot{\hat{a}}_2 &= \gamma_2\frac{w_f\sin e_3}{K_2} + f(\hat{a}_2) \end{aligned} \tag{46}$$

where

$$f(\hat{a}_2) = \begin{cases} 0, & \hat{a}_2 > \delta \\ \left(1 - \frac{\hat{a}_2}{\delta}\right)^2 (f_0^2 + 1), & \hat{a}_2 \le \delta \end{cases} \tag{47}$$

with $f_0 = (\gamma_2 w_f \sin e_3)/K_2$.

Then

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + \frac{\tilde{a}_2}{\gamma_2 a_2}\left(\dot{\hat{a}}_2 - \gamma_2\frac{w_f\sin e_3}{K_2}\right) \\ &= \dot{V}_0 + \frac{\tilde{a}_2}{\gamma_2 a_2}f(\hat{a}_2) \\ &= \dot{V}_0 + \frac{\hat{a}_2 - a_2}{\gamma_2 a_2}f(\hat{a}_2). \end{aligned} \tag{48}$$

When $\hat{a}_2 > \delta$, the second term of the right-hand side of (48) is zero. When $\hat{a}_2 \le \delta$, the second term is less than zero because $f(\hat{a}_2) \ge 0$, $\hat{a}_2 - a_2 \le \delta - a_2 \le 0$. Therefore, it is shown that $\dot{V}_1 \le 0$. Also, from (43), $\dot{e}_1$ and $\dot{e}_3$ are bounded since $\tilde{a}_1$ and $\tilde{a}_2$ are bounded. After all, $\ddot{V}_1$ is bounded. Barbalat's lemma shows that $\dot{V}_1 \to 0$ as $t \to \infty$, that is, $e_1 \to 0$ and $\sin e_3 \to 0$. Furthermore, (35) and (41) show that ν_1 and

ν_2 are bounded. If we want to avoid $e_3 \to \pm\pi$, one sufficient condition is that the initial value satisfies $V_1(0) < (2/K_2)$.

From (43), the derivative of $\dot{e}_3$ is

$$\begin{aligned}\ddot{e}_3 =& -\left(1+\frac{\tilde{a}_2}{a_2}\right)\dot{w}_f - \frac{\dot{\hat{a}}_2}{a_2}w_r + \dot{w}_r \\ =& -\left(1+\frac{\tilde{a}_2}{a_2}\right)(\dot{w}_r + \dot{v}_r K_2 e_2 + v_r K_2 \dot{e}_2 + K_3 \dot{e}_3 \cos e_3) \\ & - \frac{\dot{\hat{a}}_2}{a_2} w_f + \dot{w}_r. \end{aligned} \tag{49}$$

Since $\sin e_3 \to 0$ as $t \to \infty$, e_3 goes to some finite number. Since $\tilde{a}_2$, e_2, $\dot{e}_2$, $\dot{e}_3$, w_f are bounded, $\ddot{e}_3$ is bounded if we choose v_r, w_r, $\dot{v}_r$, $\dot{w}_r$ to be bounded. Barbalat's lemma shows $\dot{e}_3 \to 0$ as $t \to \infty$. From the equation

$$\dot{e}_3 = -\left(1+\frac{\tilde{a}_2}{a_2}\right)(w_r + v_r K_2 e_2 + K_3 \sin e_3) + w_r \tag{50}$$

we can show $-(\hat{a}_2/a_2)v_r K_r e_2 \to 0$ if we choose $w_r \to 0$, because $e_1 \to 0$, $\sin e_3 \to 0$ as $t \to \infty$.

Finally, we show $\hat{a}_2 > 0$.

If $\hat{a}_2 \le ((2-\sqrt{2})/2)\delta < \delta$, the following is satisfied:

$$\begin{aligned}\dot{\hat{a}}_2 &= f_0 + \left(1-\frac{\hat{a}_2}{\delta}\right)^2 (f_0^2+1) \ge f_0 + \frac{1}{2}(f_0^2+1) \\ &= \frac{1}{2}(f_0+1)^2 \ge 0. \end{aligned} \tag{51}$$

From this inequality, we can see $\hat{a}_2 \ge ((2-\sqrt{2})/2)\delta > 0$.

Therefore, we can obtain the following theorem.

Theorem IV.1: If we choose the control inputs as (41) and the parameter update rules as (46) for the kinematic model (30) of a mobile robot with unknown parameters r and b, the closed-loop signals are bounded. If we choose the reference inputs such that v_r does not go to zero and w_r goes to zero, that is, the reference path is a straight line, then $x \to x_r$, $y \to y_r$, $\phi \to \phi_r$.

Remark IV.1: In Theorem IV.1, we assume that w_r goes to zero, that is, if the reference trajectory is not a straight line, the tracking errors do not converge to zero. Recently, we got some results [17], [18] that resolve this difficulty.

C. Adaptive Control of the Dynamic Model

From above sections, the mobile robot with two actuated wheels satisfies Assumptions II.1 and Assumptions III.1–III.3. Therefore, we can design an adaptive tracking controller for the dynamic model (26) and (27) from Theorem III.1. According to Theorem III.1, we design an adaptive controller and perform some simulations.

From Theorem III.1, the adaptive tracking controller for the dynamic model is

$$\begin{aligned}\tau =& -k_d \begin{bmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{bmatrix} + \begin{bmatrix}\dot{\nu}_{1c} & \dot{\nu}_{2c} & \dot{\phi}\nu_{2c} \\ \dot{\nu}_{2c} & \dot{\nu}_{1c} & -\dot{\phi}\nu_{1c}\end{bmatrix}\begin{bmatrix}\hat{p}_1\\ \hat{p}_2\\ \hat{p}_3\end{bmatrix} \\ & - \begin{bmatrix}\hat{\theta}_{11}\left(\frac{\partial V_1}{\partial x}\cos\phi + \frac{\partial V_1}{\partial y}\sin\phi\right) + \hat{\theta}_{12}\frac{\partial V_1}{\partial \phi} \\ \hat{\theta}_{21}\left(\frac{\partial V_1}{\partial x}\cos\phi + \frac{\partial V_1}{\partial y}\sin\phi\right) - \hat{\theta}_{22}\frac{\partial V_1}{\partial \phi}\end{bmatrix} \end{aligned} \tag{52}$$

$$\frac{d}{dt}\begin{bmatrix}\hat{p}_1\\ \hat{p}_2\\ \hat{p}_3\end{bmatrix} = -\gamma \begin{bmatrix}\dot{\nu}_{1c} & \dot{\nu}_{2c} \\ \dot{\nu}_{2c} & \dot{\nu}_{1c} \\ \dot{\phi}\nu_{2c} & -\dot{\phi}\nu_{1c}\end{bmatrix}\begin{bmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{bmatrix} \tag{53}$$

$$\frac{d}{dt}\begin{bmatrix}\hat{\theta}_{11}\\ \hat{\theta}_{12}\end{bmatrix} = \lambda \begin{bmatrix}\frac{\partial V_1}{\partial x}\cos\phi + \frac{\partial V_1}{\partial y}\sin\phi \\ \frac{\partial V_1}{\partial \phi}\end{bmatrix}\tilde{\nu}_1 \tag{54}$$

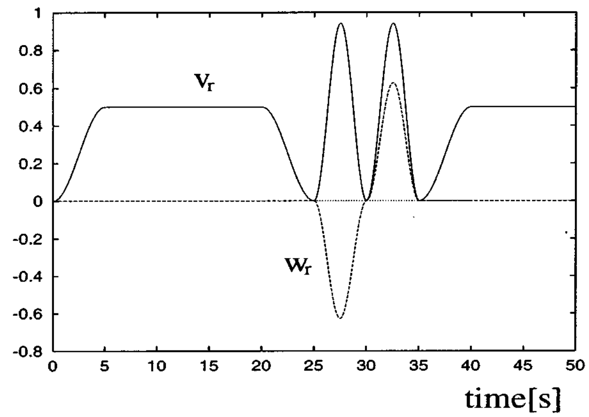

Fig. 3. Reference inputs v_r, w_r.

$$\frac{d}{dt}\begin{bmatrix}\hat{\theta}_{21}\\ \hat{\theta}_{22}\end{bmatrix} = \lambda \begin{bmatrix}\frac{\partial V_1}{\partial x}\cos\phi + \frac{\partial V_1}{\partial y}\sin\phi \\ -\frac{\partial V_1}{\partial \phi}\end{bmatrix}\tilde{\nu}_2 \tag{55}$$

where $p_1 = (r^2/4b^2)(mb^2+I) + I_w$, $p_2 = (r^2/4b^2)(mb^2 - I)$, $p_3 = (r^2/2b)m_c d$, and $\hat{p}_1$, $\hat{p}_2$, $\hat{p}_3$ are the estimates, and

$$\begin{aligned}\frac{\partial V_1}{\partial x} &= -e_1\cos\phi + e_2\sin\phi \\ \frac{\partial V_1}{\partial y} &= -e_1\sin\phi - e_2\cos\phi \\ \frac{\partial V_1}{\partial \phi} &= -\frac{\sin e_3}{K_2}.\end{aligned}$$

Moreover, $\dot{\hat{\theta}}$ is defined by (46) and ν_c is given as (41).

D. Simulation Results

In this section, we perform a computer simulation on the dynamic model of a mobile robot by using the adaptive tracking controller which was designed in the previous section. In this simulation, physical parameters and design parameters are $a = 2$, $b = 0.75$, $d = 0.3$, $r = 0.15$, $m_c = 30$, $m_w = 1$, $I_c = 15.625$, $I_w = 0.005$, $I_m = 0.0025$, $K_1 = K_2 = K_3 = k_d = 5$, $\gamma = \lambda = 5$. The initial values of the estimated parameters are about 1/10—four times the real values.

The reference inputs v_r, w_r are chosen as following:

$$\begin{aligned}
0 \le t < 5: &\quad v_r = 0.25\left(1-\cos\frac{\pi t}{5}\right) \\
&\quad w_r = 0 \\
5 \le t < 20: &\quad v_r = 0.5 \\
&\quad w_r = 0 \\
20 \le t < 25: &\quad v_r = 0.25\left(1+\cos\frac{\pi t}{5}\right) \\
&\quad w_r = 0 \\
25 \le t < 30: &\quad v_r = 0.15\pi\left(1-\cos\frac{2\pi t}{5}\right) \\
&\quad w_r = -v_r/1.5 \\
30 \le t < 35: &\quad v_r = 0.15\pi\left(1-\cos\frac{2\pi t}{5}\right) \\
&\quad w_r = v_r/1.5 \\
35 \le t < 40: &\quad v_r = 0.25\left(1+\cos\frac{\pi t}{5}\right) \\
&\quad w_r = 0 \\
40 \le t: &\quad v_r = 0.5 \\
&\quad w_r = 0.
\end{aligned}$$

Fig. 3 shows the reference inputs.

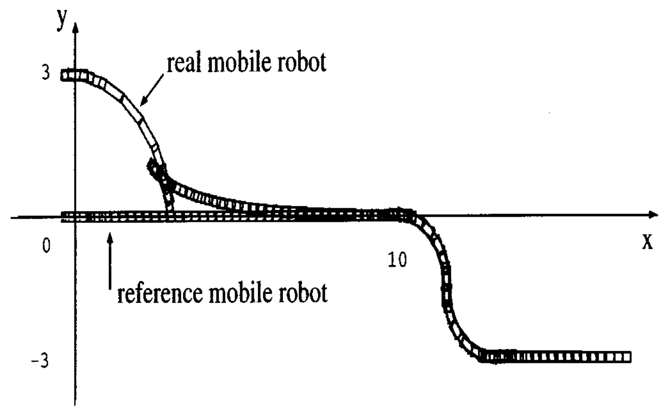

Fig. 4. Simulation result $x - y$.

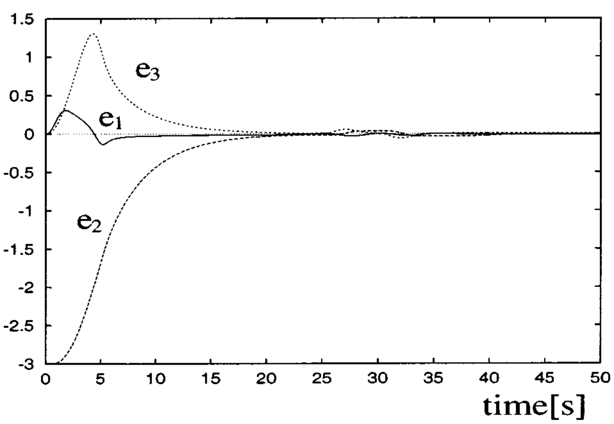

Fig. 5. Tracking errors e_1, e_2, e_3.

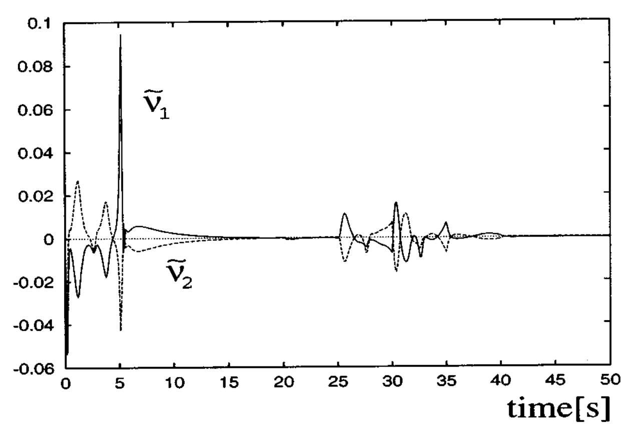

Fig. 6. Errors between ideal and real value: $\tilde{\nu}_1$, $\tilde{\nu}_2$.

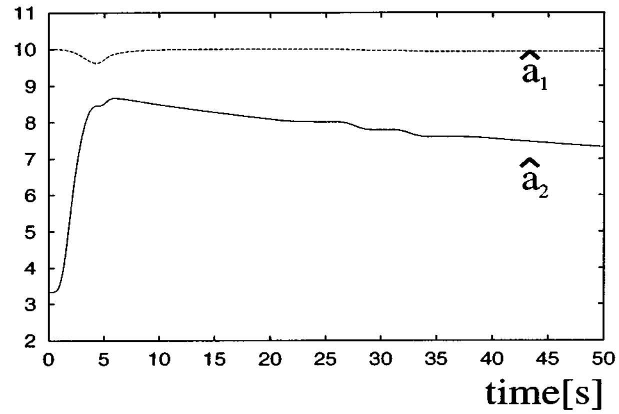

Fig. 7. Estimated parameters $\hat{a}_1$, $\hat{a}_2$.

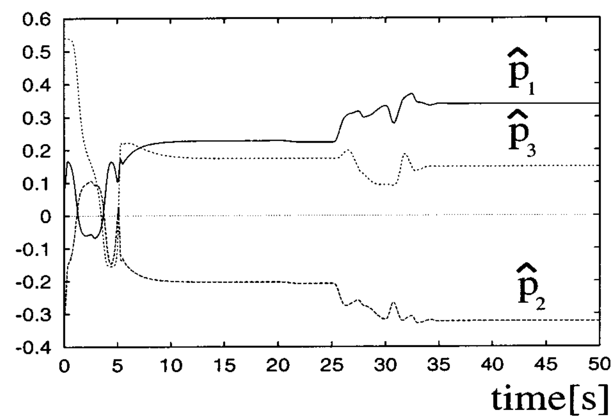

Fig. 8. Estimated parameters $\hat{p}_1$, $\hat{p}_2$, $\hat{p}_3$.

The simulation results are shown in Figs. 4–8. From these simulation results, we can confirm the usefulness of Theorem III.1 and the limitation of Theorem IV.1. The control performance is good when w_r is close to zero, but the performance becomes bad as w_r is far from zero.

V. Conclusion

In this paper, we proposed a design method of an adaptive tracking controller for a nonholonomic mobile robot with unknown parameters. It was proved that an adaptive tracking controller for the dynamic model can be designed by using adaptive backstepping if an adaptive tracking controller for the kinematic model exists. As one example, we designed an adaptive controller of a mobile robot with two actuated wheels and provided some simulation results. In future works, the class of systems which satisfy Assumption II.1 should be clarified and the design method of an adaptive tracking controller for the kinematic model written in Assumption III.1 should be established.

References

[1] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for a nonholonomic mobile robot," in *Proc. IEEE/RSJ Int. Workshop Intelligent Robots and Systems*, 1991, pp. 1236–1241.

[2] C. Samson and K. Ait-Abderrahim, "Feedback control of a nonholonomic wheeled cart in cartesian space," in *Proc. IEEE Int. Conf. Robotics and Automation*, 1991, pp. 1136–1141.

[3] Y. Nakamura and S. Savant, "Nonholonomic motion control of an autonomous underwater vehicle," in *Proc. IEEE/RSJ Int. Workshop Intelligent Robots and Systems*, 1991, pp. 1254–1259.

[4] M. Sampei, T. Tamura, T. Itoh, and M. Nakamichi, "Path tracking control of trailer-like mobile robot," in *Proc. IEEE/RSJ Int. Workshop Intelligent Robots and Systems*, 1991, pp. 193–198.

[5] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot: backstepping kinematics into dynamics," in *Proc. 34th IEEE Conf. Decision Control*, 1995, pp. 3805–3810.

[6] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design*. New York: Wiley, 1995.

[7] Y. Chang and B. Chen, "Adaptive tracking control design of nonholonomic mechanical systems," in *Proc. 35th IEEE Conf. Decision Control*, 1996, pp. 4739–4744.

[8] S. V. Gusev, I. A. Makarov, I. E. Paromtchik, V. A. Yakubovich, and C. Laugier, "Adaptive motion control of a nonholonomic vehicle," in *Proc. 1998 IEEE Int. Conf. Robotics and Automation*, 1998, pp. 3285–3290.

[9] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for an autonomous mobile robot," in *Proc. IEEE Int. Conf. Robotics and Automation*, 1990, pp. 384–389.

[10] J. E. Slotine and W. Li, "On the adaptive control of robot manipulators," *Int. J. Robot. Res.*, vol. 6, no. 3, pp. 49–59, 1987.
[11] H. Mayeda, K. Osuka, and A. Kanagawa, "A new identification method for serial manipulator arm," in *Proc. 9th IFAC World Congress*, 1984, pp. 2429–2434.
[12] S. Arimoto and F. Miyazaki, "Stability and robustness of PID feedback control for robot manipulators of sensory capability," in *Robotics Research*, M. Brady and R. P. Paul, Eds. Cambridge, MA: MIT Press, 1984, pp. 783–799.
[13] C. Su and Y. Stepanenko, "Robust motion/force control of mechanical systems with classical nonholonomic constraints," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 609–614, Mar. 1994.
[14] W. Li and J. Slotine, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.
[15] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Englewood Cliffs, NJ: Prentice-Hall, 1996.
[16] N. Sarkar, X. Yun, and V. Kumar, "Control of mechanical systems with rolling constraints: Application to dynamic control of mobile robots," *Int. J. Robot. Res.*, vol. 13, no. 1, pp. 55–69, 1994.
[17] H. Wang, T. Fukao, and N. Adachi, "Adaptive tracking control of nonholonomic mobile robots: A backstepping approach," in *Proc. 1998 Japan–USA Symp. Flexible Automation*, 1998, pp. 1093–1096.
[18] ——, "An adaptive tracking control approach for nonholonomic mobile robot," in *Proc. 1999 IFAC World Congress*, 1999, pp. 509–515.

New Potential Functions for Mobile Robot Path Planning

S. S. Ge and Y. J. Cui

***Abstract*—This paper first describes the problem of goals nonreachable with obstacles nearby when using potential field methods for mobile robot path planning. Then, new repulsive potential functions are presented by taking the relative distance between the robot and the goal into consideration, which ensures that the goal position is the global minimum of the total potential.**

***Index Terms*—GNRON problem, new repulsive potential function, potential field.**

I. Introduction

The potential field method has been studied extensively for autonomous mobile robot path planning in the past decade [1]–[16]. The basic concept of the potential field method is to fill the robot's workspace with an artificial potential field in which the robot is attracted to its goal position and is repulsed away from the obstacles [1]. This method is particularly attractive because of its mathematical elegance and simplicity. However, it has some inherent limitations. A systematic criticism of the inherent problems based on mathematical analysis was presented in [3], which includes the following: 1) trap situations due to local minima; 2) no passage between closely spaced obstacles; 3) oscillations in the presence of obstacles; and 4) oscillations in narrow passages. Besides the four problems mentioned above, there exists an additional problem, goals nonreachable with obstacles nearby (GNRON). In most of the previous studies, the goal position is set relatively far away from obstacles. In these cases, when the robot is near its goal position, the repulsive force due to obstacles is negligible, and the robot will be attracted to the goal position by the attractive force. However, in many real-life implementations, the goal position needs to be quite close to an obstacle. In such cases, when the robot approaches its goal, it also approaches the obstacle nearby. If the attractive and repulsive potentials are defined as commonly used [2]–[4], the repulsive force will be much larger than the attractive force, and the goal position is not the global minimum of the total potential. Therefore, the robot cannot reach its goal due to the obstacle nearby.

Manuscript received August 31, 1999; revised June 15, 2000. This paper was recommended for publication by Associate Editor J. Ponce and Editor V. Lumelsky upon evaluation of the reviewers' comments. This paper was presented in part at the Third Asian Control Conference, Shanghai, China, July 4–7, 2000.

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Publisher Item Identifier S 1042-296X(00)09775-5.

To overcome this problem, the repulsive potential functions for path planning are modified by taking into account the relative distance between the robot and the goal. The new repulsive potential function ensures that the total potential has a global minimum at the goal position. Therefore, the robot will reach the goal finally. Note that we are not trying to tackle the common local minima problems due to obstacles between the robot and the goal. We shall restrict our attention to the formulation and solution of the GNRON problem only.

This paper is organized as follows. In Section II, the cause of the GNRON problem is analyzed after the introduction of the potential field methods. Section III presents the new repulsive potential function and its properties. In Section IV, the relationship between scaling parameters of the potential functions is presented. In Section V, safety issues of the new potential functions are discussed, and a control system directly making use of the new potentials is also suggested. Simulation results are presented in Section VI to show the problems of the conventional potential field method and the effectiveness of the new method.

II. Potential Field Method and GNRON Problem

For simplicity, we assume that the robot is of point mass and moves in a two-dimensional (2-D) workspace. Its position in the workspace is denoted by $\mathbf{q} = [x \quad y]^T$.

Different potential functions have been proposed in the literature. The most commonly used attractive potential takes the form [1]–[3]

$$U_{\mathrm{att}}(\mathbf{q}) = \frac{1}{2}\,\xi\rho^m(\mathbf{q},\,\mathbf{q}_{\mathrm{goal}}) \tag{1}$$

where ξ is a positive scaling factor, $\rho(\mathbf{q},\,\mathbf{q}_{\mathrm{goal}}) = \|\mathbf{q}_{\mathrm{goal}} - \mathbf{q}\|$ is the distance between the robot $\mathbf{q}$ and the goal $\mathbf{q}_{\mathrm{goal}}$, and $m = 1$ or 2. For $m = 1$, the attractive potential is conic in shape and the resulting attractive force has constant amplitude except at the goal, where U_{att} is singular. For $m = 2$, the attractive potential is parabolic in shape. The corresponding attractive force is then given by the negative gradient of the attractive potential

$$\mathbf{F}_{\mathrm{att}}(\mathbf{q}) = -\nabla U_{\mathrm{att}}(\mathbf{q}) = \xi(\mathbf{q}_{\mathrm{goal}} - \mathbf{q}) \tag{2}$$

which converges linearly toward zero as the robot approaches the goal. One commonly used repulsive potential function takes the following form [1]:

$$U_{\mathrm{rep}}(\mathbf{q}) = \begin{cases} \dfrac{1}{2}\,\eta\left(\dfrac{1}{\rho(\mathbf{q},\,\mathbf{q}_{\mathrm{obs}})} - \dfrac{1}{\rho_0}\right)^2, & \text{if } \rho(\mathbf{q},\,\mathbf{q}_{\mathrm{obs}}) \le \rho_0 \\ 0, & \text{if } \rho(\mathbf{q},\,\mathbf{q}_{\mathrm{obs}}) > \rho_0 \end{cases} \tag{3}$$

where η is a positive scaling factor, $\rho(\mathbf{q},\,\mathbf{q}_{\mathrm{obs}})$ denotes the minimal distance from the robot $\mathbf{q}$ to the obstacle, $\mathbf{q}_{\mathrm{obs}}$ denotes the point on the