

## A Kalman Filter to Estimate Direction for Automotive Navigation

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### Abstract

*A Kalman filter algorithm to estimate Direction for automotive navigation is reported. An extended Kalman filter is used to combine a magnetic compass and a rate gyroscope and to compensate for the sensor errors; the result is an optimal estimate for heading direction. A mathematical model for magnetic compass errors caused by body magnetization and the body effect of magnetic material is proposed. An error model of the rate gyroscope is also established. Errors of both sensors are calibrated through a computer simulation. Finally, experimental navigation results are demonstrated.*

### 1 Introduction

The basic technologies of vehicle location for automotive navigation consist of dead-reckoning and map-matching techniques, which date back to studies in 1970's[1]-[4]. Dead-reckoning systems utilize a speedometer to measure vehicle run length and a magnetic compass and/or a gyroscope to detect the vehicle heading. To find the vehicle location a microcomputer in the navigation system integrates the speed vector of the run length and the heading direction. Dead-reckoning can become very erroneous due to the accumulation of sensor errors. The map-matching technique corrects the vehicle location obtained through dead-reckoning by referring to digital road maps in CD-ROMs or IC cards. Map-matching algorithms assume that automobiles run along a street, and they rationalize the correction from off-the-road to on-the-road. Vehicles can often run into such free areas as parking lots, and the road maps can hardly be expected to be well maintained, however. Therefore a successful map-matching technique largely depends on the accuracy of the sensing systems in dead-reckoning.

Presently, commercial automotive navigation systems are widely using GPS (Global Positioning System) receivers to obtain a vehicle location. This satellite radio-navigation system guarantees world-wide and around-the-clock availability with uniform location accuracy. The GPS-based automotive navigation suffers from signal blocking due to tall buildings in urban areas and its position fixes are too noisy to identify an individual

street, however. Thus modern automotive navigation systems tend to utilize both the GPS and dead-reckoning/map-matching system, where directional sensing plays an important role.

Magnetic compass have been used for maritime, aerial and land navigation for a long time. Although the sensor itself has become very accurate and compact for car-navigational use, terrestrial magnetic sensing is still very erroneous because of environmental factors. In urban areas the Earth's magnetic field is distorted a lot due to such artificial structures as buildings, subways, electric power plants and so on. Nearby traffic also generates momentary magnetic noises. Automotive bodies can be a significant error source: the Ferro-magnetic substance of the body can assume a temporary magnetism (body magnetization); and asymmetric body structures yield an anisotropic effect on the magnetic compass installed in the car (body effect). Some commercial automotive navigation systems require the driver to turn the car around to measure the body magnetism and eliminate it[5]-[8].

Rate gyroscopes also have a long history of navigational use, and a gas-rate gyroscope was applied to the first commercial automotive navigation system[9]. Since then many car navigation systems have been built with a rate gyroscope. Gyroscopic azimuth sensing, however, can become very erroneous as time passes because of accumulation of sensing errors. The map-matching technique may reset the diverging gyroscopic error, although it requires a fairly accurate vehicle heading for correct operation.

In this paper we propose a sensor fusion algorithm to find the accurate vehicle heading for automotive navigation. An extended Kalman filter combines a magnetic compass and a rate gyroscope, and compensate for sensor errors to yield an optimal estimate for heading direction. To establish the sensor error model for Kalman filtering, we first summarize the error characteristics of a magnetic compass and a rate gyroscope in an automotive environment. The Kalman filter algorithm is derived from the sensor model. The proposed algorithm is examined on-bench by computer simulation and evaluated in-field by carrying out an experimental automotive navigation.

## 2 Errors of Vehicle Heading Sensors

Flux gate magnetic compasses are widely used in automotive navigation systems to detect a north-south direction. This choice is based on advantage of moderate cost and easy calibration by a micro-computer. Gyroscopes detect a rotation of an automobile and they are hardly affected by noises in the terrestrial magnetism. We summarize the error characteristics of these two sensors in a navigational environment before establishing the sensor fusion technique.

### 2.1 Magnetic Compass

#### (1) Ideal response of flux gate type compass

Flux gate compasses have two sensor coils wound around a ferrite core. The sensor coils are placed orthogonally and they detect the two horizontal components of the terrestrial magnetism. We denote the two outputs of the sensor coils as  $V_x$  and  $V_y$  and they are expressed as follows:

$$V_x = r H_i \cos \theta \quad (1)$$

$$V_y = r H_i \sin \theta \quad (2)$$

where  $\theta$  is a compass direction,  $H_i$  is horizontal intensity of the magnetic field,  $r$  is gain of the electronic circuit. As seen in Eqs.(1) and (2), the X-Y response of the magnetic compass makes a circle with a radius of  $r H_i$  and a center of zero. The vehicle heading  $\theta$  is expressed as

$$\theta = \tan^{-1} \frac{V_y}{V_x} \quad (3)$$

#### (2) Body magnetization

Because they contain ferro-magnetic substances, automobile bodies can often be magnetized[10]. Pressing during the manufacturing process and passing under DC high voltage electronic at crossings are typical causes of body magnetization. Accordingly body magnetization can be changed during navigational circumstances occasionally. With this change, the center of the X-Y plot circle is shifted to a point which corresponds to the new body magnetization. When an automobile makes a 360-degree-turn, the plot of the two outputs of the magnetic compass is as depicted in Fig.1.

#### (3) Body effect

The asymmetric structure of an automobile body has an effect on the magnetic compass installed in a car. Then the X-Y response of the magnetic compass is no longer a circle[10]. This is because the terrestrial magnetism inside of the automobile is effected by the body. The

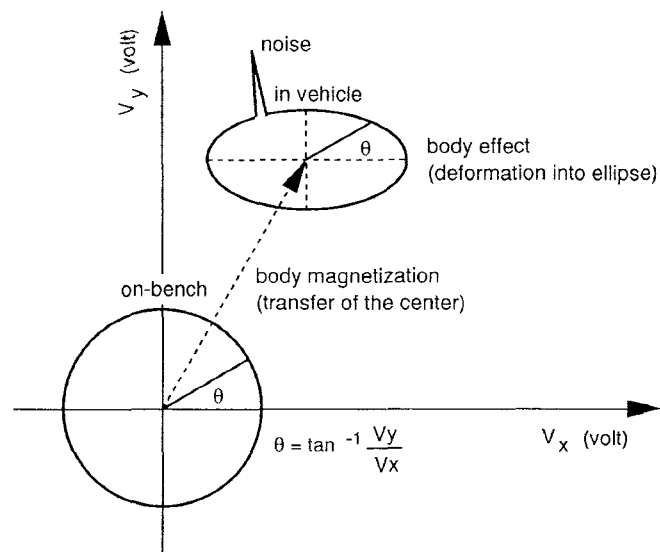


Fig.1 Errors of an automotive magnetic compass

body effect depends on the spot where the compass is installed, for the car body is anisotropic. Thus the direction error caused by the body effect might sometimes be avoidable by utilizing the mounting spot where the body effect is very weak.

#### (4) Magnetic disturbance

The terrestrial magnetic field is occasionally disturbed by noises generated by other passing vehicles or buildings which contain iron material in while driving. The direction measurement is very noisy in urban areas, and its error can be more than 50 degrees in the short term.

#### (5) Variance

The direction of the terrestrial magnetism does not coincide with the true north on a map. This variance varies place-to-place world wide but it can easily be corrected by a variation map.

### 2.2 Rate Gyroscope

Gas rate gyroscopes, vibration gyroscopes and fiber optic gyroscopes have been used for azimuth detection in automotive navigation. Regardless of the operational principle, the gyroscopic output must be integrated to obtain a rotational angle of the vehicle rotation, because the sensors just detect an angular velocity.

The main gyroscope errors we focus are a bias error and a scale error[11]. A bias error is defined as the finite output at the zero input and a scale error represents the error in a proportionality coefficient of the gyroscope input to the output. We denote the input angular velocity as  $\Omega_i$ , and the gyroscope output as  $\Omega_o$ , then we can express the output characteristics as follows:

$$\Omega_o = (1 + e) \Omega_i + b \quad (4)$$

where  $e$  is the scale error and  $b$  is the bias error.

Assuming that  $e = 0$  and bias  $b$  is constant, then we obtain the rotation angle  $\theta_o$  of the automobile by integrating the gyroscope output as follows:

$$\theta_o = \int_0^T (\Omega_i + b) dt = \theta_i + bT \quad (5)$$

where  $T$  is an integration time. Obviously the bias error is accumulated with time, and it rotates the vehicle trace after driving a long time. Since the scale error is proportional to the rotation, the direction error is significant when the automobile makes turns at intersections or on winding roads.

### 3 Sensor Fusion by Extended Kalman Filter

To obtain an absolute direction accurate enough for automotive navigation we propose a sensor fusion technique based on an extended Kalman filter. To use the filtering scheme we first build dynamic model of the vehicle direction and sensor errors, then get measurement characteristics of a magnetic compass.

#### 3.1 Sensor Fusion

The main idea behind sensor fusion here is to correct errors of both the gyroscope and magnetic compass using information from each other. This is possible because the two sensors differ in their operational principle and, then in error characteristics. To produce a repetition algorithm for the sensor fusion, we apply an extended Kalman filter which gives stochastically optimal estimates for a dynamical system. In other words we need dynamics of estimates (vehicle direction, sensor errors) and relationships between the estimates and measurements (sensor data) for Kalman filtering.

#### 3.2 Gyroscope Bias Error Dynamics

A scale error causes a significant direction error when a rotation is large enough. A gyroscope with a 1% scale error, which is moderate in automotive navigation, corresponds to a direction error of 0.9% when an automobile turns in a right angle at an intersection. A large rotation, however, triggers a correction in the map matching process occasionally. Moreover, accuracy of the road direction in a database is a few degrees. Accordingly we neglect a scale error as a minor factor for a direction error.

A large range of temperature shifts in an automobile is one of the main reasons for a bias error of gyroscopes for automotive navigation. If accompanying electronic device are conventional, their off-set voltage, which is

susceptible to the ambient temperature, determines the bias error in practice. The change rate of temperature in the automobile or its trunk, where the gyroscope is located is relatively slow compared to that of vehicle direction, so we use the following model for gyroscope error  $b(n)$ .

$$b(n+1) = b(n) \quad (6)$$

#### 3.3 Vehicle Direction Dynamics

We measure the automobile direction using the gyroscope, which we modeled above, for a short time, and we obtain a difference equation model for the vehicle direction  $\theta(n)$  as follows.

$$\theta(n+1) = \theta(n) + (\Omega_o(n) - b(n))T \quad (7)$$

#### 3.4 Body Magnetization Dynamics

Body magnetization of the automobile body first occurs in the manufacturing process, and it changes in size and direction occasionally when the vehicle passes through areas where large magnetism exists locally such as at crossing of electrified railroads and so on. We, however, consider it constant piecewise with respect to time, then we obtain the model for its components  $mx(n)$  and  $my(n)$ .

$$mx(n+1) = mx(n) \quad (8)$$

$$my(n+1) = my(n) \quad (9)$$

#### 3.5 Body Effect Dynamics

The body effect in the vicinity of the compass is determined by the distribution of ferro-magnetic substances in the automobile, that is, the vehicle shape. Hence, we assume that it is rarely changed. We denote components of the body effect as  $rx(n)$  and  $ry(n)$ , then the model for the body effect is expressed as follows:

$$rx(n+1) = rx(n) \quad (10)$$

$$ry(n+1) = ry(n) \quad (11)$$

#### 3.6 Measurement Characteristics

A static magnetic field in the vicinity of the compass consists of body magnetization, terrestrial magnetism and the terrestrial magnetism induction. We denote the horizontal vector of the terrestrial magnetism as  $H_i$  and compass output vector as  $H_o$ , then the output characteristics are expressed as

$$H_o = H_i + \mu H_i + H_v \quad (12)$$

where  $\mu$  is magnetic permeability at the point where the compass is mounted, and  $H_v$  is a vector for body magnetization. Accordingly the two outputs of the compass can be expressed as follows:

$$V_x = H_i \cos \theta + a H_i \cos \theta + b H_i \sin \theta + m_x \quad (13)$$

$$V_y = H_i \sin \theta + c H_i \cos \theta + d H_i \sin \theta + m_y \quad (14)$$

where

$$\mu = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad H_v = \begin{bmatrix} m_x \\ m_y \end{bmatrix}.$$

Since the distribution of iron material is not uniform in the vehicle, permeability  $\mu$  varies with the compass location. If we put a magnetic compass on the line where the distance between both sides of the automobile is equal, then  $b$  and  $c$  are almost zero because ferro-magnetic substances distributed in the automobile can be considered symmetrical with respect to the line. From the above arguments we obtain the following output characteristics of the magnetic compass

$$V_x = r_x H_i \cos \theta + m_x \quad (15)$$

$$V_y = r_y H_i \sin \theta + m_y \quad (16)$$

where

$$r_x = 1 + a, \quad r_y = 1 + b.$$

### 3.7 The Dynamical System for Kalman Filtering

The dynamics and compass output characteristics we mentioned above are summarized into a dynamical system as:

$$\begin{bmatrix} \theta(n+1) \\ b(n+1) \\ m_x(n+1) \\ m_y(n+1) \\ r_x(n+1) \\ r_y(n+1) \end{bmatrix} = \begin{bmatrix} 1 & -T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta(n) \\ b(n) \\ m_x(n) \\ m_y(n) \\ r_x(n) \\ r_y(n) \end{bmatrix} + \begin{bmatrix} T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Omega_o \quad (17)$$

$$V_x(n) = r_x(n) H_i \cos \theta(n) + m_x(n) \quad (18)$$

$$V_y(n) = r_y(n) H_i \sin \theta(n) + m_y(n) \quad (19)$$

where  $n = 1, 2, 3 \dots$ .

Using extended Kalman filtering for the dynamical system, we estimate the direction of the vehicle and calibrate the two sensors in terms of the bias error of the gyroscope, components of the body magnetization and the body effect in real time. We set the sampling time  $T$  as 0.1s assuming that the frequency of vehicle rotation is under 5Hz.

## 4 Off-Board Examination

We built a trial system for automotive navigation and data logging. Based on the logged raw data and the modified data to simulate some kinds of sensor errors, we examined the proposed correction technique to reduce sensor errors.

### 4.1 Trial System

The rotation of the automobile was measured by a fiber optic gyroscope and its accuracy for angular velocity was 0.02 deg/s, which is appropriate for automotive navigation. The north-south direction was measured by a flux gate magnetic compass, which was mounted at the center of the roof to avoid a more complicated body effect which can generated in the vehicle. The run length

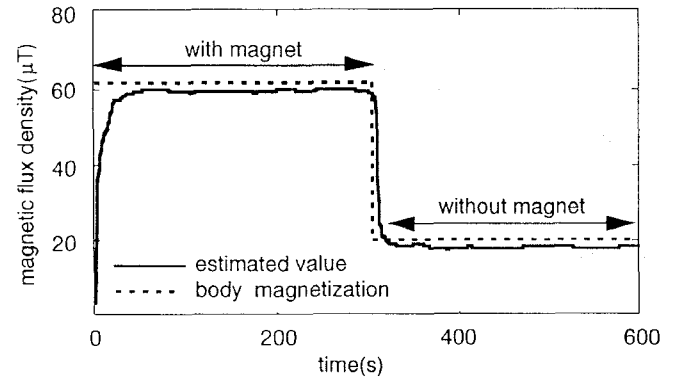


Fig.2 Estimation of body magnetization

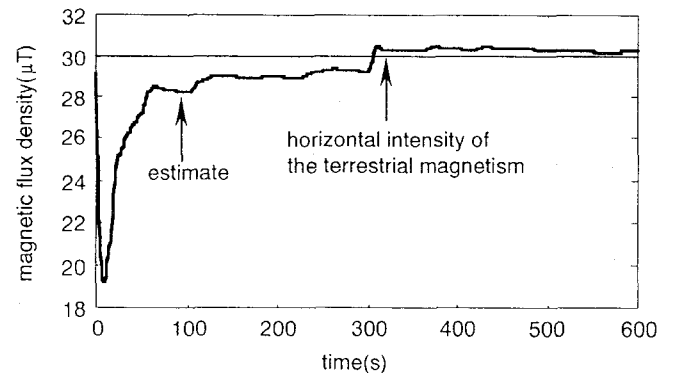


Fig.3 Estimation of terrestrial magnetization

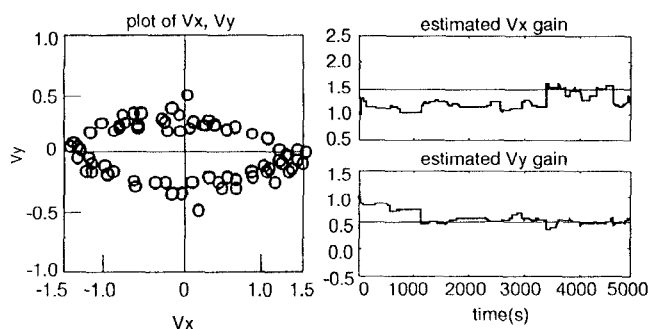


Fig.4 Estimation of body effect

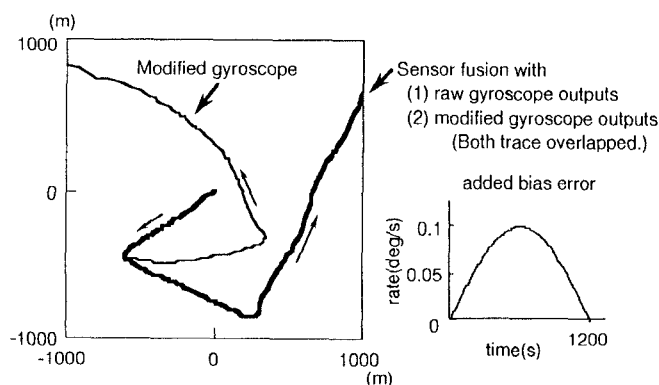


Fig.5 Driving traces(bias error modified)

was measured by a conventional speedometer. The sensor outputs such as rotation, direction and run length were stored in a PC.

## 4.2 Magnetic Compass Correction

Corrections of the body magnetism and the body effects were examined here. To simulate an occasional change of body magnetization inside of the automobile, we put a small magnet near the mounted magnetic compass, then removed it. We changed the magnetism 300s after the data acquisition was started. The estimated value successfully followed the change of the body magnetization as shown in Fig.2. In this case the trace of the two outputs of the magnetic compass happened to be a almost circle, and this meant that the body effect was negligibly small. Then intensity of the terrestrial magnetism was easily obtained from the components of the estimated body magnetization. The value agreed with the horizontal intensity in Tokyo, Japan ( $30\mu T$ ) where the data were collected as shown in Fig.3.

The body effect which we formulated earlier can be simulated by deliberately changing gains of the two outputs of the magnetic compass. Fig.4 plots the compass outputs. They were deformed into an ellipse. For the gains,  $V_x = 1.5$  times the original value and  $V_y =$  one-half half the original value, appropriate estimates of the body effect were obtained.

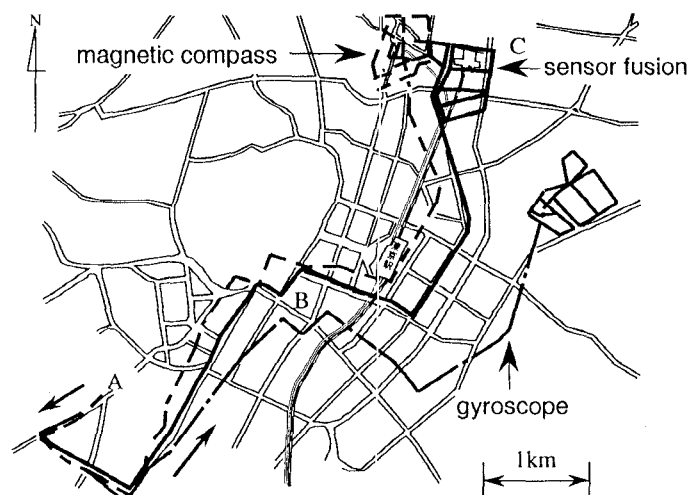


Fig.6 Navigational results

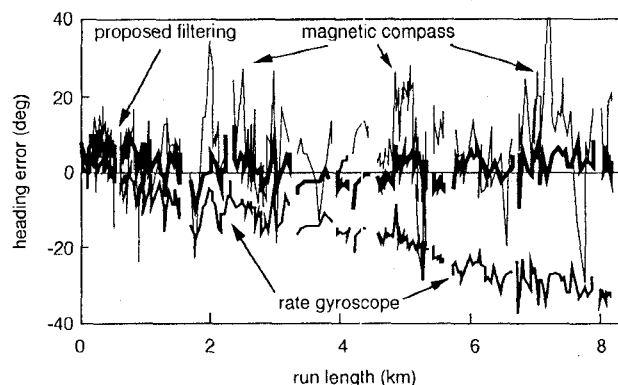


Fig.7 Vehicle heading error

## 4.3 Gyroscope Bias Error Correction

We added a sine wave to the gyroscope output, the wave period was 40 minutes and its amplitude was  $0.1 \text{ deg/s}$  as shown in Fig.5. These corresponded to a case in which the bias error of the gyroscope is gradually shifted according to, for example, an ambient temperature change. The automobile trace for the proposed filtering was not influenced by the bias error change. Hence the modified bias error was considered to be successfully corrected. On the other hand the trace from a modified gyroscope alone was clearly degraded.

## 5 Experimental Results

We examined the trial navigation system in downtown areas of Tokyo, Japan. The experimental vehicle started from point A and reached C through B in 50 minutes as shown in Fig.6. On this route the magnetic environment was deteriorated because of magnetic noise generated by much traffic and many buildings. Furthermore, there were electric railroad crossings and subway systems which might cause changes of the vehicle body magnetization. The trace of the automobile was obtained from

autonomous inertial navigation, without a map matching correction, to emphasize the performance of the sensor fusion.

Navigation results with the sensor fusion almost followed the route. Their direction errors were always under 10 deg as shown in Fig.7, and the remaining errors would be easily corrected by a map matching procedure. We evaluated the heading errors by reference to road directions in digital road maps only when the automobile runs almost straight. So the heading errors in Fig.7 are discontinuous at some points where the vehicle turned at crossings or curves. Using an gyroscope alone to measure a direction, we obtained the rotated trace for accumulation of bias error. The location results by a magnetic compass alone were not accurate enough for correction by a map database, and the maximum direction error went over 40 deg.

## 6 Conclusion

An extended Kalman filter which combines a magnetic compass and a gyroscope was proposed. The characteristics of sensor errors and outputs were discussed. The bias error of the gyroscope was corrected and the magnetic compass was calibrated in terms of body magnetization and body effect. An optimal estimate for heading direction was obtained. Accuracy of the vehicle direction obtained in navigational experiments was always within 10 deg.

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