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Speed Control of Differentially Driven Wheeled Mobile Robots– Tracking and Synchronization

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Abstract – A speed control scheme for a differentially driven wheeled mobile robot is presented in this paper. In the scheme, an compensator for the angular speed is combined with the independent speed control loops of the robot. The controller achieves tracking and synchronization of the speeds of the driving wheels of the robot. The effectiveness of the proposed scheme is verified by the simulation and experimental results.

Keywords – Mobile robot, Speed control

I. INTRODUCTION

Mobile robots are nonholonomic systems which cannot be stabilized with smooth feedbacks [1]. The control of mobile robots has been studied extensively. A good summary of the researches in this area can be found in [2], [3] and [4].

To control a wheeled mobile robot, firstly the desired speed of the robot is derived from its desired trajectory (this process is called *motion control*), then the desired speed is achieved with *speed control* loop for each wheel [5]. Though there are many research results on the motion control of mobile robots, very few were on the speed control which still relies on traditional PID controller designed for each driving wheel. Such a speed controller fails to make the speeds of the wheels synchronize each other (e.g each achieves the desired speed at the same time or with a desired interval). Without this synchronization, the robot tends to depart from its desired trajectory even if the speed tracking of each wheel is achieved. A few approaches were proposed to solve this problem. In [7], a controller was designed to control the speed of the robot instead of the speed of each wheel. The controller relied on the dynamic modeling of the robot. In [6], a proportional (P) controller was proposed which processed the integral of the angular speed tracking error and the speed tracking error of each wheel. In both controllers, the controlled variables are the linear speed and angular speed of the robot. The controllers are complicated and sensitive to the dynamic modeling of the robot.

In this paper, a speed controller for a differentially driven wheeled mobile robot is proposed. In addition to the independent control loop for each wheel, an angular speed compensator is included to enhance synchronization of the motions of the wheels. The controller is still designed around each wheel to keep the simple structure of independent control. Simulation and experimental study are done to verify the effectiveness of the approach.

The paper is organized as follows. In Section 2, the problem formulation is presented. The controller design and its analysis are presented in Section 3. In Section 4, the effectiveness of the controller is verified by the simulation and experimental studies. The conclusion of the work is given in Section 4.

II. PROBLEM FORMULATION

A mobile robot is schematically shown in Figure 1. The speed of the left wheel and the right wheel are denoted by ω_l and ω_r respectively. The linear speed and the angular speed of the robot are denoted by v and ω and their desired values are denoted by v_d and ω_d respectively.

The following kinematic relation exists in the robot system.

$$v = r(\omega_r + \omega_l)/2 \tag{1}$$

$$\omega = r(\omega_r - \omega_l)/2b \tag{2}$$

where $2b$ is the width of the robot and r is the radius of the wheel of the robot. Obviously the linear speed of the robot is related to $\omega_l + \omega_r$ and the angular speed of the robot is related to $\omega_l - \omega_r$. The control objective is to make ω_l and ω_r achieve their desired values ω_{ld} and ω_{rd} respectively.

A typical speed control for each wheel (called *independent control*) is schematically shown in Figure 4. The control inputs are u_l and u_r : the armature voltage of the motors. The compensators (normally of PID type) for the speed control of each motor are denoted by K_l and K_r such that

$$K_l = K_r = k_p + k_i/s + k_d \tag{3}$$

where k_p, k_i and k_d are positive control parameters. Note K_l and K_r are in the form of Laplace transformation. For simplicity without causing ambiguities, variable s is omitted in the Laplace transformation in this paper.

It is obvious that

$$\omega_l = \frac{G_l K_l}{1+G_l K_l}\omega_{ld} \tag{4}$$

$$\omega_r = \frac{G_r K_r}{1+G_r K_r}\omega_{rd} \tag{5}$$

where G_r and G_l are the transfer functions between the armature voltage and the angular speed of each wheel respectively.

The transient responses of ω_l and ω_r affect the robot's trajectory tracking. Assume that the robot is controlled to follow a straight line as shown in Figure 2. Obviously $\omega_{ld} = \omega_{rd}$ in this case. It is very likely that one motor reaches the desired speed faster than another and this causes the robot deviate from the planned straight line. A more feasible approach should take the speed difference of two motors into consideration when designing the speed controller of the robot. In the following section, the design and the analysis of such a controller is presented.

III. SPEED CONTROLLER DESIGN AND ANALYSIS

The proposed controller is schematically shown in Figure 3. The control inputs to the motors (in voltage) are as follows

$$u_l = K_l(\omega_{ld} - \omega_l) + K_a(\omega_{ld} - \omega_{rd} + \omega_r - \omega_l) \tag{6}$$

$$u_r = K_r(\omega_{rd} - \omega_r) - K_a(\omega_{ld} - \omega_{rd} + \omega_r - \omega_l) \tag{7}$$

where K_a is a PID-type compensator such that

$$K_a = k_{pa} + k_{ia}/s + k_{da}s \tag{8}$$

where k_{pa}, k_{ia} and k_{da} are positive control parameters.

Compared with that of independent controller, a compensator K_a is added to process the speed difference $(\omega_l - \omega_r - (\omega_{ld} - \omega_{rd}))$. The output of this compensator is combined with the outputs of each independent controller to generate the input for each motor.

From equations (6) and (6), we have

$$\begin{aligned} \omega_l &= G_{ll}\omega_{ld} + G_{lr}\omega_{rd} \\ \omega_r &= G_{rl}\omega_{ld} + G_{rr}\omega_{rd} \end{aligned} \tag{9}$$

where

$$G_{ll} = \frac{G_l(K_l+K_a)[1+G_r(K_r+K_a)] - G_r G_l K_a^2}{H}$$

$$G_{lr} = \frac{-G_l K_a}{H}$$

$$G_{rl} = \frac{-G_r K_a}{H}$$

$$G_{rr} = \frac{G_r(K_r+K_a)[1+G_l(K_l+K_a)] - G_r G_l K_a^2}{H}$$

$$H = [1+G_l(K_l+K_a)](1+G_r(K_r+K_a)) - G_r G_l K_a^2$$

The dynamic model of the left motor and the right motor can be approximated by a 1st order linear system as shown in Appendix A.

$$G_l = \frac{\omega_l}{u_l} = \frac{k_{ml}}{1+\tau_l s} \tag{10}$$

$$G_r = \frac{\omega_r}{u_r} = \frac{k_{mr}}{1+\tau_r s} \tag{11}$$

Though the relation between the control parameters and the controller performance is not easy to be quantified, some qualitative analysis of the performance of the closed loop controlled system (9) can be made in different situations.

A. when $G_l = G_r$ and disturbance free

In this case, it is assumed that $K_l = K_r = K$. Equation (9) can be re-written as

$$\omega_l = G_{ind}\omega_{ld} - G_{syn}\omega_r \tag{12}$$

$$\omega_r = G_{ind}\omega_{rd} - G_{syn}\omega_l \tag{13}$$

where

$$G_{ind} = \frac{GK(K+2K_a)}{GK(K+2K_a)+K+K_a}$$

$$G_{syn} = \frac{K_a}{GK(K+2K_a)+K+K_a}$$

From equations (12) and (13), it is clear that the speeds of two motors are synchronized through G_{syn}, whereas the speed of each motor is related to its desired value through G_{ind}.

From equations (12) and (13), we have

$$\omega_l + \omega_r = \frac{GK}{1+GK}(\omega_{ld} + \omega_{rd}) \tag{14}$$

$$\omega_l - \omega_r = \frac{G(K+K_a)}{1+G(K+K_a)}(\omega_{ld} - \omega_{rd}) \tag{15}$$

Compensators K and K_a can be designed using the common linear controller design technique (e.g: Nyquist stability criteria), so that $\omega_l + \omega_r \rightarrow \omega_{ld} + \omega_{rd}$ and $\omega_l - \omega_r \rightarrow \omega_{ld} - \omega_{rd}$. From equations (1) and 2, $v \rightarrow v_d$ and $\omega \rightarrow \omega_d$ accordingly. Clearly the response of $\omega_l - \omega_r$, or the synchronization of the motor speeds can be adjusted through K_a.

By setting $K_a = 0$, we have the responses of $\omega_l + \omega_r$ and $\omega_l - \omega_r$ under the independent control,

$$\omega_l + \omega_r = \frac{GK}{1+GK}(\omega_{ld} + \omega_{rd}) \tag{16}$$

$$\omega_l - \omega_r = \frac{GK}{1+GK}(\omega_{ld} - \omega_{rd}) \tag{17}$$

Except for $\omega_l - \omega_r$, $\omega_l + \omega_r$ is identical to that in equation (14).

B. When $G_l \neq G_r$

In this case, assume that $G_l = G$ and $G_r = G + \Delta$ where Δ represents the difference between G_l and G_r.

From equation (9), we have

$$\omega_l = \frac{G((G+\Delta)K(K+2K_a)+K+K_a)}{H_\Delta}\omega_{ld} - \frac{GK_a}{H_\Delta}\omega_{rd}$$
$$\omega_r = \frac{(G+\Delta)(K+K_a)+G(G+\Delta)(K+2K_a)K}{H_\Delta}\omega_{rd} - \frac{(G+\Delta)K_a}{H_\Delta}\omega_{ld}$$
$$H_\Delta = G(G+\Delta)(K+2K_a)K + (2G+\Delta)(K+K_a)+1$$

The responses of $\omega_l + \omega_r$ and $\omega_l - \omega_r$ can be derived such that

$$\omega_l + \omega_r = \frac{G(K-K_a)+G(G+\Delta)(K+2K_a)K}{H_\Delta}(\omega_{ld}+\omega_{rd}) + \frac{\Delta K + (G+\Delta)K_a}{H_\Delta}\omega_{rd}$$

$$\omega_l - \omega_r = \frac{(G+\Delta)(GK(K+2K_a)+K_a)+G(K+K_a)}{H_\Delta}(\omega_{ld}-\omega_{rd}) - \frac{\Delta K}{H_\Delta}\omega_{rd}$$

Clearly the norm of $1/H_\Delta$ becomes smaller with K_a in the term H_Δ. The effects of the coupling term $\frac{\Delta K}{H_\Delta}\omega_{rd}$ is reduced and the speed synchronization of the motors is improved accordingly.

C. When external disturbances exist

Assume that there are bounded disturbances δ_r and δ_l occuring in ω_r and ω_l respectively as shown in Figure 3. From equation (9) and taking these disturbances into consideration, , we have

$$\omega_l = G_{ll}\omega_{ld} + G_{lr}\omega_{rd} + G_{\Delta ind}\delta_l + G_{\Delta syn}\delta_r \quad (18)$$
$$\omega_r = G_{rl}\omega_{ld} + G_{rr}\omega_{rd} + G_{\Delta syn}\delta_l + G_{\Delta ind}\delta_r \quad (19)$$

where

$$G_{\Delta ind} = 1 + G(K+K_a)/H$$
$$G_{\Delta syn} = GK_a/H$$

with the assumption that $G_l = G_r = G$ and $K_l = K_r = K$.

The responses of $\omega_l + \omega_r$ and $\omega_l - \omega_r$ can be derived such that

$$\omega_l + \omega_r = \frac{GK}{1+GK}(\omega_{ld}+\omega_{rd}) + \frac{1}{1+GK}(\delta_l+\delta_r)$$
$$\omega_l - \omega_r = \frac{G(K+K_a)}{1+G(K+K_a)}(\omega_{ld}-\omega_{rd}) + \frac{1}{1+G(K+K_a)}(\delta_l-\delta_r)$$

Obviously by choosing K_a properly, the effect of the disturbances on the speed of the robot can be reduced.

In the next section, the simulation and the experimental results will be presented to demonstrate the performance of the controller.

IV. SIMULATION

Assume a mobile robot is controlled to follow a straight line with speeds $\omega_l = \omega_r = 1rad/sec$. We will examine the speed responses of the robots in two situations:

1. left and right motor have different dynamic models
2. there are external disturbances

The models of the left motor and right motor are as follows

$$G_l = \frac{30}{1+0.030s}$$
$$G_r = \frac{40}{1+0.020s}$$

The compensators are set as $K_l = K_r = s + 0.3/s$ and $K_a = s + 0.1/s$. Treating the desired speed as the step signal with amplitude 1, the responses of ω_l, ω_r, $\omega_l - \omega_r$ and $\omega_l + \omega_r$ can be obtained with Simulink package. By setting $K_a = 0$, the speed responses under the independent control approach are also obtained.

The various speed responses are plotted from Figures 5 to 8. It can be seen that with the proposed controller ($K_a \neq 0$), $\omega_l - \omega_r$ becomes zero much faster than that for independent control ($K_a = 0$). This improvement can drastically improve the trajectory of the robot.

To verify the performance of the controller under external disturbances, we set the the disturbance to the left motor and the right motor are a regular sine signal such that

$$\delta_l = \delta_r = sin(t) \quad (20)$$

The responses of the controlled system are plotted from Figures 9 to 12. It can be seen that under the disturbances, synchronization of ω_l and ω_r is still maintained while the disturbances greatly affect the responses of the independent control.

V. EXPERIMENTAL STUDY

The controller is tested on a mobile robot developed in our project team. The picture of the robot is shown in Figure 13. Both left and right wheels of the robot are driven by micromotors (2024-006S) from MINIMOTOR SA, Switzerland. The parameters of the motor are $K_t = 6.92 \times 10^{-3}Nm/A$, $R_a = 1.94\Omega$, $J = 2.70 \times 10^{-7}Kgm^2$ and $L_a = 4.5 \times 10^{-4}H$. The mechanical time constant is $11ms$ which is much bigger than the electrical time constant $L_a/R_a = 0.23ms$. The geometrical parameters of the robot are $b = 7.5cm$ and $r = 2.5cm$.

The robot is to be controlled to follow a straight line parallel to the X axis of the coordinate on the plane. The desired speed of the left motor and the right motor are set as $\omega_r = \omega_l = 75rad/s$ or 716 rpm. The original position of the robot is at (0.063, 0.63).

The motion controller is implemented with an Intel 80296 microcontroller. The sampling time for the controller is 2ms. The gains of compensators are $K_l = K_r = 16 + 0.25/s$, and $K_a = 8 + 0.5/s$. A vision system is used to record the trajectories and the velocities of the robot in a sampling interval 16 ms. The trajectory traced by the robot is plotted in Figure 14. The speeds of the motors, the sum and the difference between their speeds are plotted from Figures 15 to 16.

It can be seen from Figure 14 that the robot depart from X-axis direction when independent control is applied ($K_a = 0$). The trajectory is improved greatly under the proposed controller ($K_a \neq 0$). The improvements can also be observed in the speed difference $\omega_l - \omega_r$ in Figure15.

VI. CONCLUSION

In this paper, the speed control of a differentially driven wheeled mobile robot is addressed. A speed controller which combines the speed controller of each motor with an angular speed compensator is proposed. The controller enhances the speed synchronization in addition to the speed tracking. The trajectory of the robot is also improved. The simulation and experimental studies are done to verify the effectiveness of the control approach.

APPENDIX A: MODEL OF A DC MOTOR [5]

For a general DC motor, it is well known that its dynamic model with the armature voltage u being the input and the angular velocity ω_m being the output is expressed as

$$\frac{\omega_m(s)}{u(s)} = \frac{K_t}{L_a J s^2 + (BL_a + JR_a)s + BR_a + K_t^2} \quad (21)$$

where R_a and L_a are the armature resistance and inductance respectively, J is the motor inertia, B is the viscous coefficient and K_t is the torques constant.

For a DC motor, the following assumptions are normally held

$$L_a/R_a \ll J/B$$
$$K_t^2 \gg R_a B$$

and the 2nd order model (21) is thus simplified to a 1st order model

$$\frac{\omega(s)}{u(s)} = \frac{k_m}{1 + \tau_m s} \quad (22)$$

where

$$\tau_m = (R_a J + BL_a)/K_t^2, \qquad k_m = 1/K_t$$

$\tau = JR_a/K_t^2$ is the time constant of the motor.

References

[1] R. W. Brockett, R. S. Millan and H. J. Sussmann, Eds, Birkhauser, Boston, 1983.

[2] J.-P. Laumond (ed.), *Robot motion planning and control*, Springer,1998

[3] I. Kolmanovsky and N. H. McClamroach, "Developments in nonholonomic control problems", IEEE Contr. Syst. Mag., Vol.15, pp. 20-36, Dec, 1995.

[4] I. J. Cox and C. T. Wilfong, *Autonomous robot vechiles", Springer-Verlag, 1990.*

[5] S. Meshkat, *Advanced Motion Control*, Intertec Communication INC – Ventura, CA, 1988.

[6] J. L. Jones, B. A. Seiger and A. M. Flynn, *Mobile Robots: Inspriration to Implementation*, A K Peters, Natic, Massachusetts, 1999

[7] D. otten, "Building MITEEE Mouse III", Circuit Cellar Ink, pp. 32–51, 1990

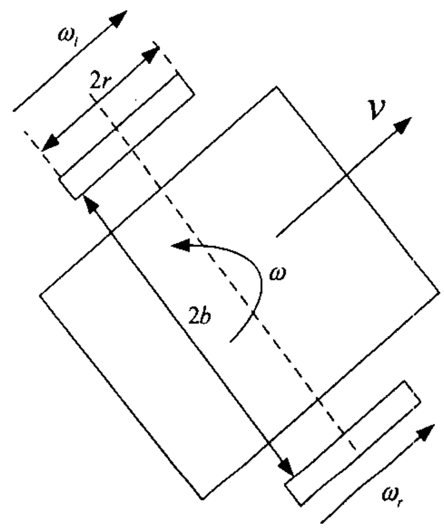

Fig. 1. Diffeentially driven wheeled mobile robot

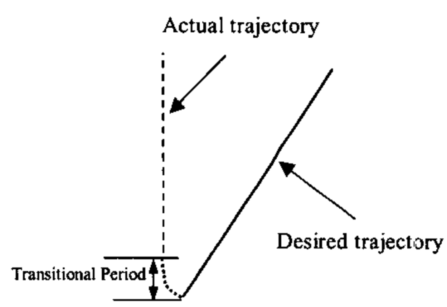

Fig. 2. Speed mismatch of two motors causes the robot deviate from the planned trajectory

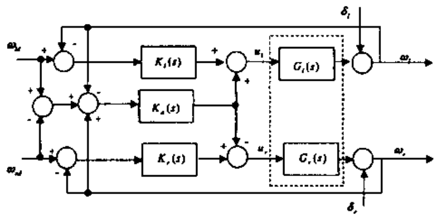

Fig. 3. Controller Structure used in Our Design

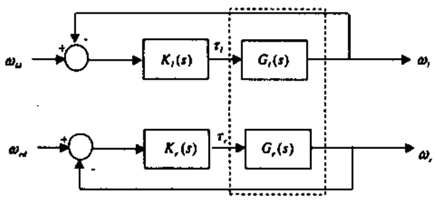

Fig. 4. Controller structure–Independent Control

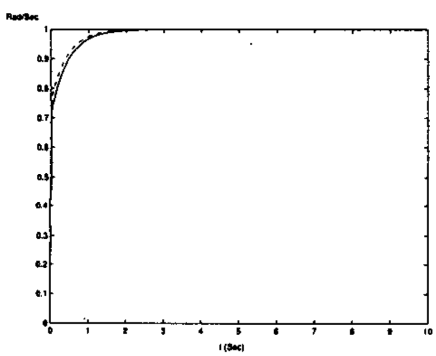

Fig. 5. Left motor speed ω_l (Solid: when $K_a \neq 0$, Dashed : when $K_a = 0$)

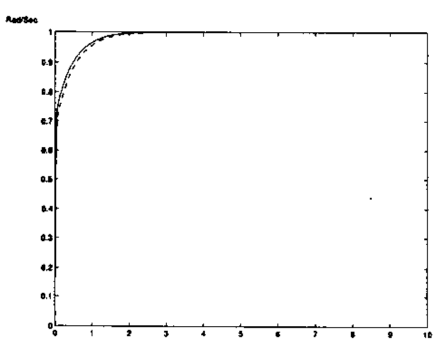

Fig. 6. Right motor speed ω_r (Solid: when $K_a \neq 0$, Dashed : when $K_a = 0$)

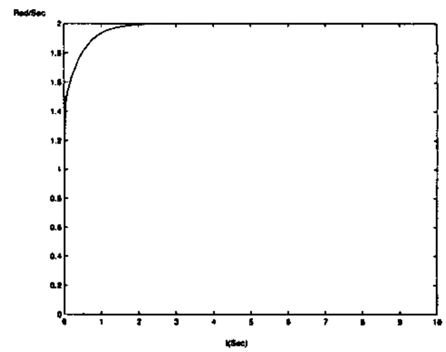

Fig. 7. $\omega_l \quad \omega_r$ (Solid: when $K_a \neq 0$, Dashed : when $K_a = 0$)

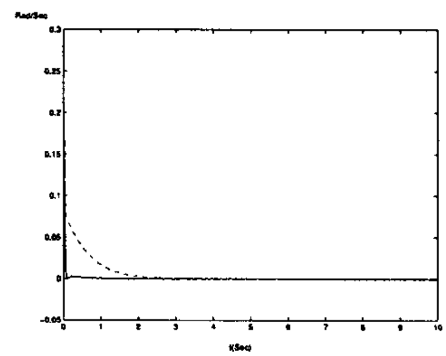

Fig. 8. $\omega_l - \omega_r$ (Solid: when $K_a \neq 0$, Dashed : when $K_a = 0$)

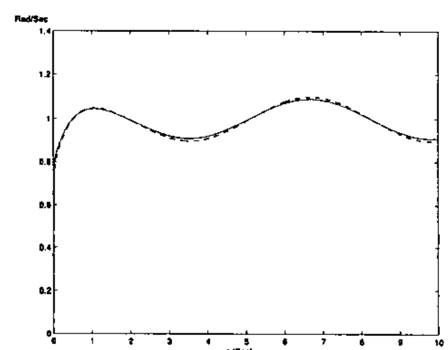

Fig. 9. Left motor speed ω_l under disturbances (Solid: when $K_a \neq 0$, Dashed : when $K_a = 0$)

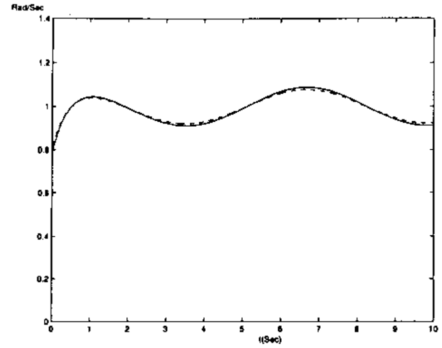

Fig. 10. Right motor speed ω_r under disturbances (Solid: when $K_a \neq 0$, Dashed : when $K_a = 0$)

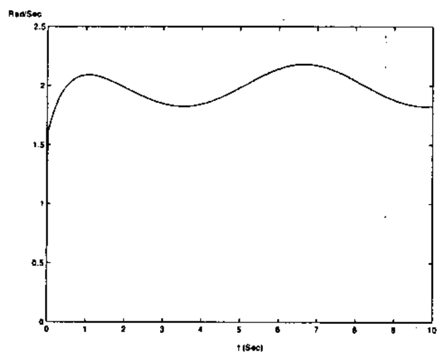

Fig. 11. ω_l ω_r under disturbances (Solid: when $K_a \neq 0$, Dashed : when $K_a = 0$)

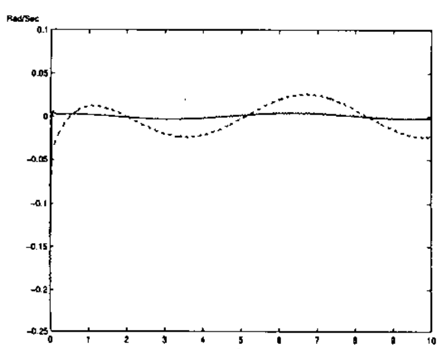

Fig. 12. $\omega_l - \omega_r$ under disturbances (Solid: when $K_a \neq 0$, Dashed : when $K_a = 0$)

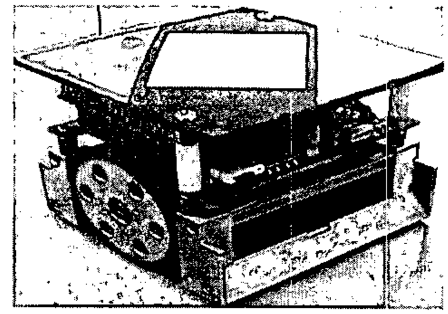

Fig. 13. Mobile Robot

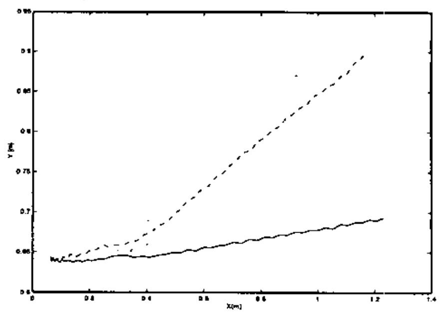

Fig. 14. Trajectory of the robot (Solid: when $K_a \neq= 0$, Dashed : when $K_a = 0$)

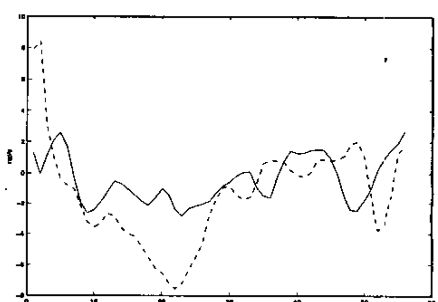

Fig. 15. Speed difference $\omega_l - \omega_r$ (Solid: when $K_a \neq= 0$, Dashed : when $K_a = 0$)

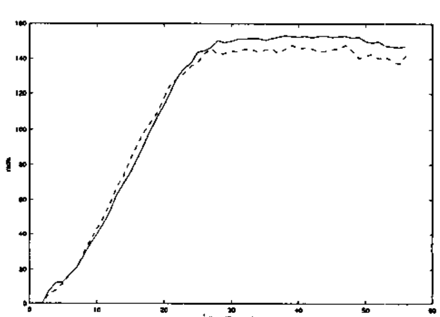

Fig. 16. Speed sum ω_l ω_r (Solid: when $K_a \neq= 0$, Dashed : when $K_a = 0$)