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Control and Stabilization of a General Class of Nonholonomic Dynamic Systems

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Abstract

This paper extends results in [1] to a more general class of nonholonomic dynamic systems that are important in applications. These nonlinear control systems are referred to as nonholonomic control systems due to certain nonintegrability assumptions which are made. The class of systems considered in this paper is characterized by general nonlinear base space dynamics that is input-ouput decouplable. Controllability and stabilizability results for this class of nonholonomic control systems are presented.

1. Introduction.

In recent years, there has been considerable attention paid to control problems for various classes of nonholonomic systems. A nice summary of many recent results is available in papers in the recently published book [4]. These classes of problems are interesting because they are nonlinear in a fundamental way; in addition there are numerous interesting areas of application that require nonholonomic models for their proper formulation. Much of the existing literature has been devoted to nonholonomic models which capture only the nonholonomic constraints on the controlled motions; these "kinematic" nonholonomic models do represent the fundamental aspects of all nonholonomic systems. But in addition to inclusion of nonholonomic effects, there are often dynamic effects that are not captured by the kinematic nonholonomic models. Our recently published paper [1] is concerned with nonholonomic models based on a Lagrangian dynamics formalism. In the current paper, we extend the results in [1] to a more general class of nonholonomic control systems which can include dynamics effects which are not necessarily in a Lagrangian form. Such an extension is of particular practical importance if realistic effects, such as actuator dynamics, are to be incorporated into the model.

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2. Control and Stabilization Results

We consider nonlinear control systems of the form

$$\dot{x} = f(x,z) + \sum_{i=1}^{m} g_i(x,z)u_i \quad (1)$$

$$y_i = h_i(x,z), \quad i = 1,\dots,m \quad (2)$$

$$\dot{z} = \sum_{i=1}^{m} k_i(y,z)\dot{y}_i \quad (3)$$

where $u = (u_1,\dots,u_m)$ are the control inputs, $x = (x_1,\dots,x_n)$ are defined to be the base state variables, $y = (y_1,\dots,y_m)$ are defined to be the base output variables, and $z = (z_1,\dots,z_p)$ are defined to be the fiber state variables and f: $XxZ \to R^n$, g_i: $XxZ \to R$, i = 1,...,m, h_i: $XxZ \to R$, i = 1,...,m, k_i: $YxZ \to R$ are C^∞ functions defined on open sets X in R^n,Y in R^m, and Z in R^p each containing the origin. We assume that $f(0,z) = 0$, $h_i(0,z) = 0$, i = 1,...,m for all z in Z and that $m \geq 2$ and that $p \geq 1$. This assumption guarantees that if u = 0, then (0,z) is an equilibrium solution for any z in Z; in particular (0,0) is an equilibrium solution. This is a typical characteristic of nonholonomic control systems: there is a nontrivial manifold of equilibria so that each equilibrium solution, e.g. (0,0), is not an isolated equilibrium solution.

We refer to equations (1), (2) as the base space equations and to equation (3) as the fiber equations. Note that the base space equations have a standard nonlinear form with m inputs and m outputs. The fiber space equations (3) have a special form that is crucial to our subsequent development--that the right hand sides of the equations depend only on the base outputs, their derivatives and the fiber variables. We emphasize that we are interested in the complete nonholonomic control system described by equations (1), (2), (3) where the complete state vector is (x,z) and u is the input vector.

We now make additional assumptions. We first assume that the base space equations are locally input-output decouplable. We make the standard assumption on the existence of a vector relative degree $(r_1,\dots,r_m)$ at (0,0), satisfying $r_1 + r_2 +\dots+ r_m = n$, which guarantees that the base space equations can be written locally in Brunovsky form with a decoupled input-output structure, by using a variable change and feedback involving the base state variables only. If n=m and the vector relative degree $(r_1,...,r_m) = (1,...,1)$, equations (1), (2), (3) are equivalent to the usual nonholonomic kinematic form of the equations. If n=2m and $(r_1,...,r_m) = (2,...,2)$, equations (1), (2), (3) are equivalent to the nonholonomic dynamic system considered in [1] and in [6].

We also assume that equation (3) is completely nonholonomic; this is a nonintegrability condition on the constraints (3) (see [1] or [7]). This is a critical assumption that is central to our subsequent results.

Following our development in [1], we are interested in the following control and stabilization questions: Is there a neighborhood of (0,0) in XxZ such that if (x(0),z(0)) is in this neighborhood then there exists a control function which transfers (x(0),z(0)) to (0,0)? Is there a neighborhood of (0,0) in XxZ and a feedback function such that (0,0) of the resulting closed loop system is locally asymptotically stable? What is the smoothness of any such feedback function? We now provide formal results related to these questions.

Proposition 1. The nonholonomic control system defined by equations (1), (2), (3), under the stated assumptions, is strongly accessible at (0,0).

Proposition 2. The nonholonomic control system defined by equations (1), (2), (3), under the stated assumptions, is small time locally controllable at (0,0).

Proposition 3. The nonholonomic control system defined by equations (1), (2), (3), under the stated assumptions, is not locally asymptotically stabilizable to (0,0) using continuous state feedback.

Proposition 4. Assume the nonholonomic control system defined by equations (1), (2), (3) is analytic; under the stated assumptions, it is locally asymptotically stabilizable to (0,0) using piecewise analytic feedback.

Only an outline of the proofs can be provided. Under the stated assumptions, there is no loss of generality in assuming that equations (1), (2) are in input-output decoupled Brunovsky form. Then the proof of Proposition 1 follows by demonstrating that the standard Lie algebraic rank condition is satisfied. The proof of Proposition 2 is based on a recursive application of a recent result of Coron [3]; this is a generalization of the dynamic extension approach developed in [6]. The proof of Proposition 3 makes use of a well known necessary condition for smooth stabilization in [2] and the proof of Proposition 4 follows directly from Proposition 2 and a result in [5].

3. Example

We briefly mention a modification of an example from [1], the control of a knife edge using steering and pushing inputs. In [1] the control inputs are the pushing force and the steering torque. Here we summarize results in the case that the pushing force and/or steering torque are generated by first order actuator dynamics.

If actuation dynamics for the pushing force only are included, then there is no vector relative degree defined at the origin and the required assumptions are not satisfied. If actuation dynamics for the steering torque only are included, then the vector relative degree is (2,3) at the origin. If actuation dynamics for both the pushing force and the steering torque are included, then the vector relative degree is (3,3) at the origin. In these latter two cases, the resulting control systems are strongly accessible, small time controllable, and asymptotically stabilizable at the origin.

4. Conclusions

We have made the somewhat restrictive assumption that the base space equations (1), (2) are input-output decouplable. It is no doubt possible to relax this assumption in several different way, e.g. if the sum of the relative degree components is less than n.

The theoretical development in this paper extends the development in [1]. The generalization considered here is of practical significance in application of the methodology to a number of engineering control problems that are modeled as nonholonomic dynamic systems. A variety of specific physical problems, using nonholonomic models, have been studied in the literature; references to such applications are cited in [1,4].

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