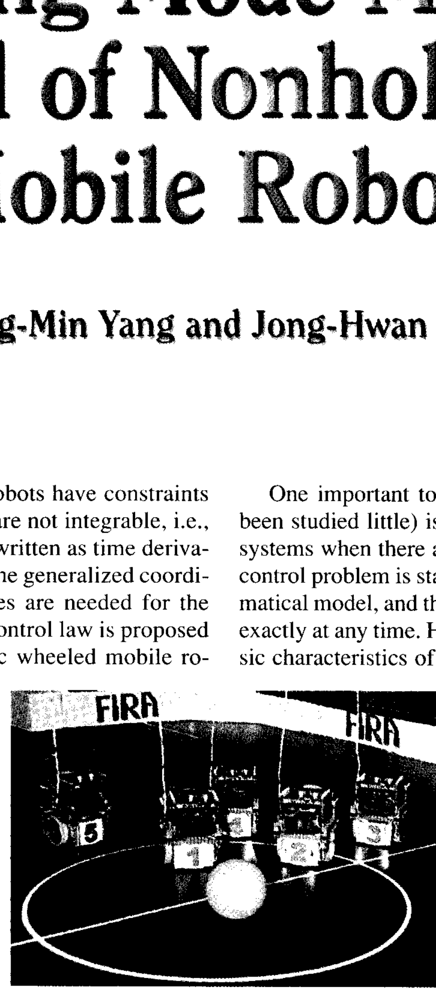
Sliding Mode Motion Control of Nonholonomic Mobile Robots

Jung-Min Yang and Jong-Hwan Kim

As nonholonomic mobile robots have constraints imposed on motions that are not integrable, i.e., the constraints cannot be written as time derivatives of some function of the generalized coordinates, advanced techniques are needed for the tracking control. In this paper a robust control law is proposed for trajectory tracking of nonholonomic wheeled mobile robots. The state variables of the mobile robot are represented in polar coordinates, and the dynamic equation of the system is feedback-linearized by a computed-torque method. A novel sliding mode control law is derived for asymptotically stabilizing the mobile robot to a desired trajectory. It is shown that the proposed scheme is robust to bounded system disturbances. Simulation examples and experimental results are provided to show the effectiveness of the accurate tracking capability and the robust performance of the proposed controller.



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Introduction

Autonomous wheeled mobile robots with inherent velocity constraints are difficult to stabilize to an equilibrium state if the kinematic constraints are nonholonomic. This difficulty is due to the fact that problems in controlling nonholonomic systems are not solvable by methods of linear control theory, and they are not transformable into linear control problems. Due to their richness and difficulty, such nonlinear control problems have resulted in a variety of approaches including various techniques of automatic control [1]-[5]. A survey of recent developments in control of nonholonomic systems is provided in [6].

One important topic that requires much attention (but has been studied little) is the problem of control of nonholonomic systems when there are model uncertainties. In most cases, the control problem is stated in terms of stabilizing a simple mathematical model, and the state variables are supposed to be known exactly at any time. However, taking into account several intrinsic characteristics of nonholonomic systems such as the actual vehicle dynamics, the inertia and power limits of actuators, and the localization errors, their dynamic equations can become uncertain. To overcome such problems, there have been several works on the stabilization problem of nonholonomic systems with model uncertainties [7]-[9]. As a robust control scheme, a few sliding mode control algorithms have been proposed and applied to control problems of nonholonomic systems. Sliding mode control uses a high-speed switching control law to drive the system's state trajectory onto the sliding surface, and to maintain the state trajectory on this surface for all subsequent time [10]. With this methodology, sliding mode control is robust in the sense that the dynamics of the system defined by the sliding surface are unchanged by disturbances and model uncertainties. By proper design of the sliding surface, sliding mode control attains the conventional goals of control such as stabilization, tracking, regulation, etc.

Several control strategies have been proposed in the literature for sliding mode control of nonholonomic systems. Chacal, et al. [11], proposed a sliding mode control that exploits a property named differential flatness of the kinematics of nonholonomic systems. In Guldner, et al. [12], a Lyapunov navigation function

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was proposed to prescribe a set of desired trajectories for navigation of mobile robots to a specified configuration. Sliding mode control was used in the work to guarantee exact tracking of trajectories made by navigation functions. In Shim, et al. [13], a sliding mode control law was proposed in which unicycle-like robots converge to a reference trajectory with bounded errors of position and velocity. The approach was based on dynamic models of nonholonomic mobile robots. Aguilar, et al. [14], presented a path-following feedback controller with sliding mode which is robust to localization and curvature estimation errors for a car-like robot. They used a dynamic extension of the usual kinematic model of a car, in the sense that the curvature is considered as a new state variable.

Though these results are promising, it is difficult to apply sliding mode control to trajectory tracking problems of nonholonomic mobile robots when the reference trajectory is not given in a *closed form*. By closed form, we mean that the reference trajectory can be expressed as a set of output equations so that its dimension is less than that of the state variables. As will be shown later, the mobile robot considered here has two driving wheels. Its configuration is determined by two position variables and one angular variable that is tantamount to the heading direction of the robot. So if the reference trajectory is not given as a closed form, the output state vector should be three-dimensional. In this case, the conventional sliding mode that completely decouples state variables is inapplicable because the dimension of the sliding mode is two, which is determined by the number of control input, but that of the output state vector is three.

In this paper, we propose a new sliding mode control law for solving such a trajectory tracking problem. With the proposed control law, the mobile robot converges to a given reference trajectory with asymptotic stability. We use the dynamic equation of the mobile robot to describe the motion of the robot with bounded disturbances in system dynamics. Specifically, posture variables of the mobile robot represented in *polar coordinates* are used in the proposed control scheme. By means of the computed-torque method, error dynamics of the mobile robot are linearized and the sliding mode control law is applied to stabilize the robot to a reference trajectory and to compensate for existing disturbances. It is shown that the proposed sliding mode control law solves the trajectory tracking problem in which a reference trajectory of non-closed form is given. The proposed algorithm is implemented to control a real nonholonomic wheeled mobile robot that has the same structure as considered in this paper.

The remainder of this paper is organized as follows. In the second section, theoretic formulation of general mechanical systems with nonholonomic constraints is addressed, and the kinematics and dynamics of the mobile robot considered in this paper are derived. In the third section, disturbances in the system dynamics are described and a novel sliding mode control algorithm based on the dynamic model with a disturbance term is proposed. In the fourth section, simulation results are presented to demonstrate numerically the effectiveness of the proposed scheme. In the fifth section, to show the applicability of the proposed control scheme to real systems, experimental results are presented in which the proposed control method is applied to trajectory tracking problems of a nonholonomic wheeled mobile robot. Finally, some concluding remarks are mentioned in the final section.

Mobile Robot Model

Consider a mechanical system with n-dimensional generalized coordinates $\mathbf{q}$ subject to m nonholonomic independent constraints that are in the form

$$\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}, \tag{1}$$

where $\mathbf{A}(\mathbf{q})$ is an $m \times n$ full-rank matrix. Let $s_1(\mathbf{q}),\ldots,s_{n-m}(\mathbf{q})$ be a set of smooth and linearly independent vector fields in $\mathcal{N}(\mathbf{A})$, the null space of $\mathbf{A}(\mathbf{q})$, i.e.,

$$\mathbf{A}(\mathbf{q})s_i(\mathbf{q}) = \mathbf{0}, \quad i = 1,\ldots,n-m.$$

Let $\mathbf{S}(\mathbf{q}) = [s_1(\mathbf{q})\cdots s_{n-m}(\mathbf{q})]$ be the full-rank matrix made up of these vectors such that

$$\mathbf{A}(\mathbf{q})\mathbf{S}(\mathbf{q}) = \mathbf{0} \tag{2}$$

and Δ be the distribution spanned by these vector fields as

$$\Delta = \mathrm{span}\{s_1(\mathbf{q}),\cdots,s_{n-m}(\mathbf{q})\},$$

then follows $\dot{\mathbf{q}} \in \Delta$.

We are now concerned with the dynamic equation of the mechanical system with nonholonomic constraints (1), which is described by an Euler-Lagrangian formulation [15], [16] as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{E}(\mathbf{q})\boldsymbol{\tau} - \mathbf{A}^T(\mathbf{q})\boldsymbol{\lambda}, \tag{3}$$

where $\mathbf{M}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{V}(\mathbf{q},\dot{\mathbf{q}})$ is the centripetal and Coriolis force term, $\mathbf{E}(\mathbf{q})$ is the $n \times r$ full-rank input transformation matrix, $\boldsymbol{\tau}$ is the r-dimensional control input vector, $\mathbf{A}(\mathbf{q})$ as in (1) is the $m \times n$ Jacobian matrix and $\boldsymbol{\lambda}$ is the m-vector of constraint forces. For simplicity of analysis, we assume $r = n - m$ in this paper.

Eqs. (1) and (2) imply the existence of an $(n-m)$-dimensional velocity vector $\mathbf{z} = [z_1,\cdots,z_{n-m}]^T$ such that

$$\dot{\mathbf{q}} = \mathbf{S}(\mathbf{q})\mathbf{z}. \tag{4}$$

It should be noted that $\mathbf{z}$ represents internal states so that $(\mathbf{q},\mathbf{z})$ is sufficient to describe the constraint motion. Differentiating (4), we obtain

$$\ddot{\mathbf{q}} = \mathbf{S}\dot{\mathbf{z}} + \dot{\mathbf{S}}\mathbf{z}. \tag{5}$$

Substituting the expression for $\ddot{\mathbf{q}}$ into (3) and premultiplying by $\mathbf{S}^T$, we have

$$\mathbf{S}^T(\mathbf{M}\mathbf{S}\dot{\mathbf{z}} + \mathbf{M}\dot{\mathbf{S}}\mathbf{z} + \mathbf{V}) = \mathbf{S}^T\mathbf{E}\boldsymbol{\tau}. \tag{6}$$

Note that since $\mathbf{S} \in \mathcal{N}(\mathbf{A})$, $\mathbf{S}^T\mathbf{A}^T\boldsymbol{\lambda}$ vanishes in this equation. (6) is suitable for control purposes because the nonholonomic constraints (1) are embedded into this dynamic equation.

Multiplying by $(\mathbf{S}^T\mathbf{E})^{-1}$, (6) is reduced to

$$\mathbf{H}(\mathbf{q})\dot{\mathbf{z}} + \mathbf{G}(\mathbf{q},\mathbf{z}) = \boldsymbol{\tau}, \tag{7}$$

where $\mathbf{H}(\mathbf{q}) = (\mathbf{S}^T\mathbf{E})^{-1}\mathbf{S}^T\mathbf{M}\mathbf{S}$ is the $(n-m)\times(n-m)$ matrix, $\mathbf{G}(\mathbf{q},\mathbf{z}) = (\mathbf{S}^T\mathbf{E})^{-1}\mathbf{S}^T(\mathbf{M}\dot{\mathbf{S}}\mathbf{z} + \mathbf{V})$ is the $(n-m)\times m$ matrix and

$\boldsymbol{\tau} \in R^{(n-m)\times m}$ is the input torque vector. If the system is a wheeled mobile robot, the torque vector is applied to driving wheels.

The mobile robot considered here is of a unicycle type, shown in Fig. 1. The robot body is of symmetric shape and the center of mass is at the geometric center C of the body. It has two driving wheels fixed to the axis that passes through C, and one passive centered orientable wheel. The two fixed wheels are controlled independently by motors, and the passive wheel prevents the robot from tipping over as it moves on the plane. In this paper, we assume that the motion of the passive wheel can be ignored in the dynamics of the mobile robot.

Fig. 2 shows the position of the robot represented in the world $X-Y$ coordinate and the polar coordinate with the assumption that their origins are identical. The local coordinate $X_0 - Y_0$ is fixed to the robot with C as the origin. (x_c, y_c) is the position of the geometric center C in the world $X-Y$ coordinates and θ_c is the angle between the X-axis and the X_0-axis representing the heading direction. (x_c, y_c) can be also expressed as (ρ_c, ϕ_c) in polar coordinates. v_c denotes the linear velocity of the robot in the direction of the X_0-axis and ω_c the angular velocity.

As we are concerned with the posture of the robot, $\mathbf{q}$ is defined as

$$\mathbf{q} = [x_c, y_c, \theta_c]^T \text{ or } [\rho_c, \phi_c, \theta_c]^T.$$

The mobile robot has the nonholonomic constraint that the driving wheels *purely roll* and *do not slip*, written as

$$\dot{y}_c \cos\theta_c - \dot{x}_c \sin\theta_c = 0.$$

For $n = 3$ and $m = 1$, $\mathbf{z}$ becomes two dimensional. We choose v_c and ω_c as the internal state variables such that

$$\mathbf{z} = [v_c, \omega_c]^T.$$

Therefore, the derivation of (4) is:

$$\begin{pmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta}_c \end{pmatrix} = \begin{pmatrix} \cos\theta_c & 0 \\ \sin\theta_c & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_c \\ \omega_c \end{pmatrix}. \tag{8}$$

From (8) and the property of $\rho_c = \sqrt{x_c^2 + y_c^2}$ and $\phi_c = \tan^{-1}(\frac{y_c}{x_c})$, $\dot{\mathbf{q}}$ is derived in polar coordinates as:

$$\dot{\mathbf{q}} = \begin{pmatrix} \dot{\rho}_c \\ \dot{\phi}_c \\ \dot{\theta}_c \end{pmatrix} = \begin{pmatrix} v_c \cos(\phi_c - \theta_c) \\ -\frac{v_c}{\rho_c} \sin(\phi_c - \theta_c) \\ \omega_c \end{pmatrix}, \tag{9}$$

which corresponds to (4), where

$$\mathbf{S}(\mathbf{q}) = \begin{pmatrix} \cos(\phi_c - \theta_c) & 0 \\ -\frac{1}{\rho_c} \sin(\phi_c - \theta_c) & 0 \\ 0 & 1 \end{pmatrix}.$$

In the sliding mode controller design, the polar coordinate shall be used.

Sliding Mode Controller

Problem Statement

It is supposed that a feasible desired trajectory for the mobile robot is prespecified by an open-loop path planner. The problem is to design a robust controller so that the mobile robot follows the desired trajectory. A reference trajectory is defined as $\mathbf{q}_r(t) = [\rho_r(t), \phi_r(t), \theta_r(t)]^T$ in which $\rho_r(t)$, $\phi_r(t)$ and $\theta_r(t)$ are piecewise smooth functions. As was mentioned in the introduction, it is not necessary that $\mathbf{q}_r(t)$ be expressed as a closed form, such as $\rho_r = \psi(\phi_r)$ where $\psi(\phi_r)$ is a closed form function of ϕ_r. $\mathbf{z}_r(t) = [v_r(t), \omega_r(t)]^T$ is defined to be the velocity vector derived from $\mathbf{q}_r(t)$.

Practical robotic systems have input disturbances, such as measurement errors, parameter and model uncertainties, etc. Let us denote the disturbances as a vector $\boldsymbol{\tau}_d \in R^{2\times 1}$. The real dynamic equation of the mobile robot with the disturbance term is represented as follows:

$$\mathbf{H}(\mathbf{q})\dot{\mathbf{z}} + \mathbf{G}(\mathbf{q}, \mathbf{z}) + \boldsymbol{\tau}_d = \boldsymbol{\tau}, \tag{10}$$

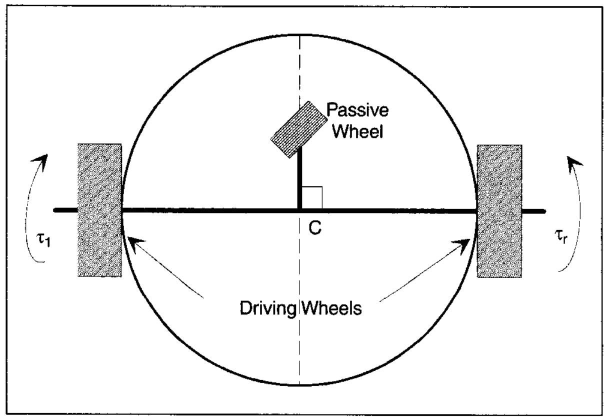

Fig. 1. Top view of the mobile robot.

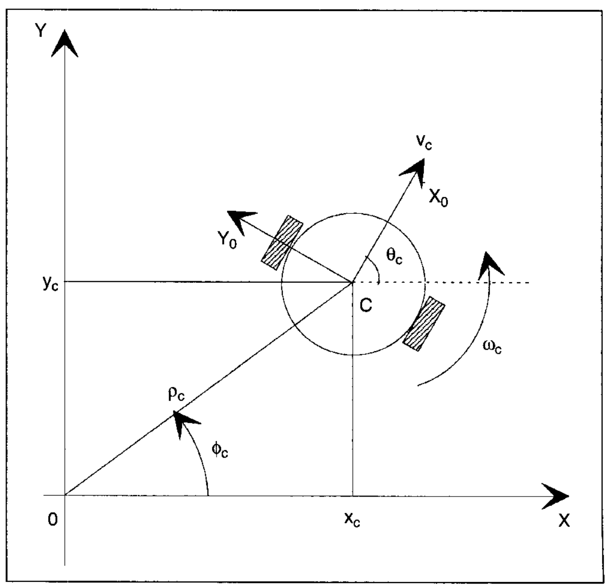

Fig. 2. Coordinates of the mobile robot.

where $\boldsymbol{\tau} = [\tau_l, \tau_r]^T$ is the torque vector applied to the left and right driving wheels. It is assumed that the disturbance vector can be expressed as a multiplier of matrix $\mathbf{H}(\mathbf{q})$, or it satisfies *the matching condition*, and has a known boundary:

$$\begin{aligned}\boldsymbol{\tau}_d &= \mathbf{H}(\mathbf{q})\mathbf{f},\\ \mathbf{f} &= [f_1, f_2]^T, |f_1| \le f_{m_1}, |f_2| \le f_{m_2},\end{aligned}$$

where f_{m_1} and f_{m_2} are upper bounds of disturbances.

Before designing the sliding mode, the following assumptions about the trajectory of the robot are given:

i) Without loss of generality, the range of the angular variables of the robot is set to be between $-\pi$ and π:

$$\begin{aligned}-\pi &< \phi_r \le \pi,\\ -\pi &< \phi_c \le \pi,\\ -\pi &< \theta_r \le \pi,\\ -\pi &< \theta_c \le \pi.\end{aligned}$$

ii) Both the reference trajectory and the real trajectory do not cross the origin of the world coordinates after starting from the initial position. Also, the initial position of the reference trajectory is always set to be the origin:

$$\begin{aligned}\rho_r(t) &= 0, \quad t = 0,\\ \rho_r(t) &> 0, \quad t > 0,\\ \rho_c(t) &> 0, \quad t > 0.\end{aligned} \tag{11}$$

iii) The heading angle of the robot and the angle coordinate cannot be perpendicular with each other:

$$\left| |\phi_c(t) - \theta_c(t)| - (2m+1)\frac{\pi}{2} \right| \ge \alpha, \ \forall t, \tag{12}$$

where $m = 0$ or 1 and α is a positive constant.

Assumption (iii) implies that the mobile robot should not have a posture whose heading direction is tangential to any circle drawn around the origin of the world coordinates.

Let us define the error of the robot posture in polar coordinates as

$$\begin{aligned}\rho_e &= \rho_c - \rho_r,\\ \phi_e &= \phi_c - \phi_r,\\ \theta_e &= \theta_c - \theta_r.\end{aligned}$$

Sliding Mode Design

From the dynamic equations (7) and (9), we can see that the position variables ρ_c and ϕ_c are dependent on the two internal state variables v_c and ω_c, but θ_c is dependent only on ω_c. As a matter of fact, from the inherent property of the considered mobile robot, the position of the robot and the heading direction are determined by the linear velocity and the angular velocity, respectively. If we use the conventional sliding surfaces which decouple the internal states, such as $\omega_e + \gamma\theta_e$ (where $\omega_e = \dot{\theta}_e$ and $\gamma > 0$), ρ_e and ϕ_e become unaffected by θ_c in the controlled system and, consequently, unable to converge to zero simultaneously. Therefore, in this paper we propose a new sliding mode in which convergence of θ_e leads to convergence of both ρ_c and ϕ_e.

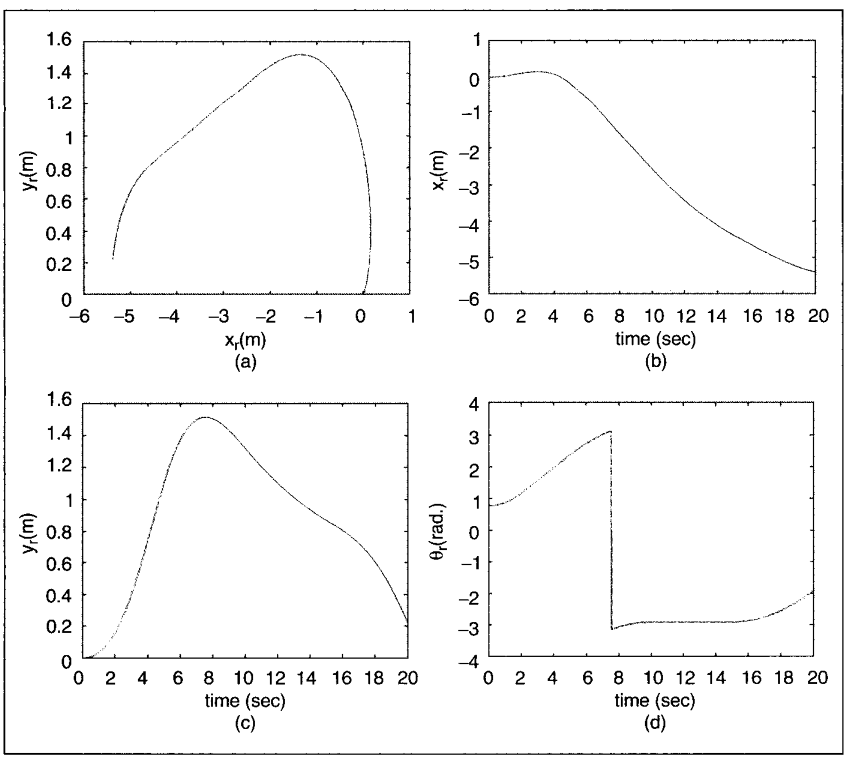

Fig. 3. Reference trajectory $\mathbf{q}_r$: (a) x_r-y_r plot, (b) x_r, (c) y_r, and (d) θ_r.

Let us define the sliding mode $\mathbf{s} = [s_1, s_2]^T$ as

$$\begin{aligned}s_1 &= \dot{\rho}_e + \gamma_1\rho_e,\\ s_2 &= \dot{\theta}_e + \gamma_2\theta_e + \operatorname{sgn}(\theta_e)|\phi_e|,\end{aligned} \tag{13}$$

where γ_1 and γ_2 are positive constant parameters and $\operatorname{sgn}(\cdot)$ is the sign function.

Theorem 1 If the sliding mode (13) is asymptotically stable, then ρ_e, ϕ_e and θ_e converge asymptotically to zeros.

Proof: See Appendix, I.

As a feedback-linearization method, the control input is defined by the computed-torque method [16] as

$$\boldsymbol{\tau} = \mathbf{H}(\mathbf{q})\dot{\mathbf{z}}_r + \mathbf{G}(\mathbf{q}, \mathbf{z}) + \mathbf{H}(\mathbf{q})\mathbf{u}, \tag{14}$$

where $\mathbf{u} \equiv [u_1, u_2]^T$ is a control law which determines error dynamics. Applying the proposed control law (14), the dynamic equation (10) reduces to

$$\dot{\mathbf{z}} + \mathbf{f} = \dot{\mathbf{z}}_r + \mathbf{u}. \tag{15}$$

In the following theorem, we propose the control law $\mathbf{u}$ which stabilizes the sliding mode.

Theorem 2 Let us define a robust control law as

$$
\begin{aligned}
u_1 &= \frac{1}{\cos(\phi_c - \theta_c)}\{-Q_1 s_1 - P_1\,\mathrm{sgn}(s_1) - \gamma_1 \dot{\rho}_e \\
&\quad + \dot{v}_r \cos(\phi_r - \theta_r) + R(\mathbf{q}_r, \mathbf{z}_r) - R(\mathbf{q}, \mathbf{z})\} - \dot{v}_r, \\
u_2 &= -Q_2 s_2 - P_2\,\mathrm{sgn}(s_2) - \gamma_2 \dot{\theta}_e - \mathrm{sgn}(\theta_e)\frac{d}{dt}|\phi_e|,
\end{aligned} \tag{16}
$$

where $R(\mathbf{q}_r, \mathbf{z}_r)$ and $R(\mathbf{q}, \mathbf{z})$ are

$$
\begin{aligned}
R(\mathbf{q}_r, \mathbf{z}_r) &= \frac{v_r^2}{\rho_r}\sin^2(\phi_r - \theta_r) + v_r\omega_r \sin(\phi_r - \theta_r), \\
R(\mathbf{q}, \mathbf{z}) &= \frac{v_c^2}{\rho_c}\sin^2(\phi_c - \theta_c) + v_c\omega_c \sin(\phi_c - \theta_c)
\end{aligned}
$$

and Q_i and P_i, $i = 1,2$, are constant positive values. Then, the above control law stabilizes the sliding mode (13) if the following conditions hold:

$$
Q_i \geq 0, P_i \geq f_{m_i}, i = 1,2. \tag{17}
$$

Proof: See Appendix, II.

It is noted that u_1 is well-defined from (11) and (12). The form of the proposed control law (16) is similar to that of the reaching law method [17]. Specifically, predividing by $\cos(\phi_c - \theta_c)$ is included in u_1 for deriving the acceleration term of ρ_e. Boundedness of u_1 is guaranteed by (12). Since $\cos(\phi_c - \theta_c) = 0$ when $|\phi_c - \theta_c| = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, there might be cases where u_1 becomes large if α in (12) is around zero. However, such a drawback is compensated for by the property that using u_1 reduces the effect of disturbance f_1 that should be eliminated in the sliding surface (see Appendix, III).

Because of the term $\mathrm{sgn}(\theta_e)|\phi_e|$, the behavior of the sliding surface s_2 is highly affected by θ_e when θ_e is around zero. Suppose θ_e fluctuates from negative to positive around zero at some instant. In this case, Δs_2 is affected by the change of $|\phi_e|$. This characteristic might cause chattering. For more precise design and to overcome chattering, it may be necessary to use a continuous function instead of $\mathrm{sgn}(\cdot)$, such as, for instance, the saturation function.

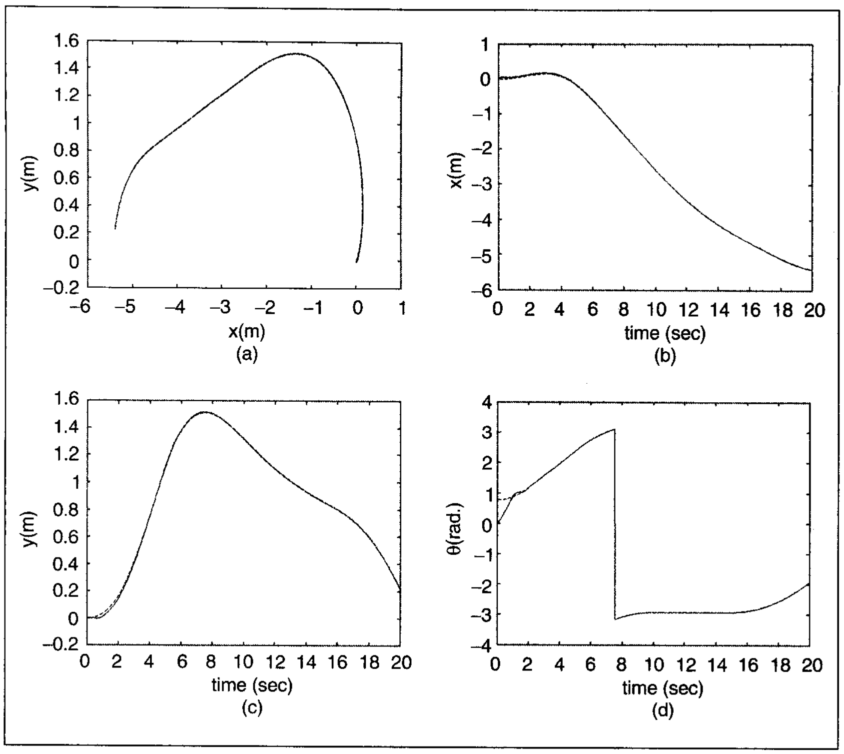

Fig. 4. Robot trajectory $\mathbf{q}$*: (a)* $x - y$ *plot, (b)* x_c*, (c)* y_c *and (d)* θ_c*.*

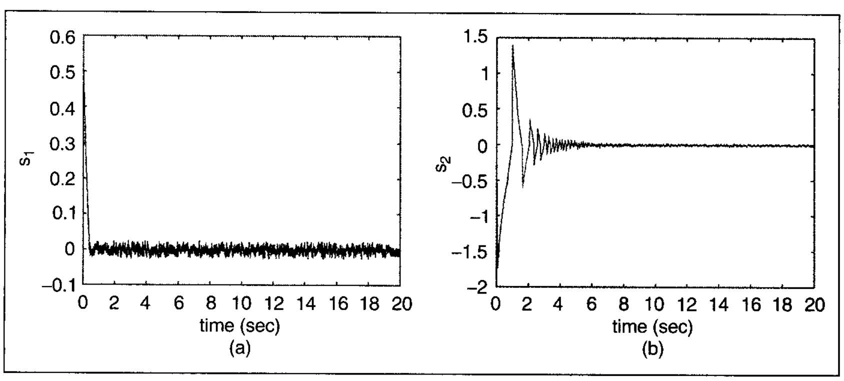

Fig. 5. Sliding mode: (a) s_1 *and (b)* s_2*.*

Simulation Example

To show the effectiveness of the proposed sliding mode control law numerically, computer simulations were carried out on the trajectory tracking problem of a nonholonomic wheeled mobile robot. The mobile robot is assumed to have the same structure as in Fig. 1. For obtaining the dynamic equation, parameter values of the mobile robot are given in Table 1. Input disturbances were defined as Gaussian random noises with mean 0 and variance 0.2. The upper bounds were predetermined as $f_{m_1} = 0.3$(N) and $f_{m_2} = 0.4$(N). The parameters of sliding modes were held constant during the simulation: $Q_1 = Q_2 = 1, P_1 = P_2 = 1$, and $\gamma_1 = \gamma_2 = 2$. It is certain that Q_i and P_i satisfy the stabilization condition (17).

The reference trajectory was given as in Fig. 3. As shown in the figure, $\mathbf{q}_r$ is not a closed form where each of the three elements is given independently. Also, the posture variables were represented in $X - Y$ coordinates. This means that the desired trajectory of mobile robots is generated by a path planner in the frame of the $X - Y$ coordinates, which is often the case. In applying the sliding mode control algorithm, the reference trajectory was transformed such that the position of the robot is expressed by polar coordinates ρ_r and ϕ_r.

It was assumed there were no initial errors of position variables, or $x_e(0) = y_e(0) = 0$, but there existed a non-zero value of the initial error of the heading direction. Such a situation of the initial condition is often experienced in the locomotion of mobile robots in the real world. The initial error of the heading direction was set to be $\theta_e(0) = \frac{\pi}{4}$(rad). It was also assumed that there were no initial velocities, that is to say, the robot was stationary at the initial time.

Fig. 4 presents the robot's cartesian trajectory and the heading direction where dashed lines are reference values and solid lines are actual values of the robot. It could be noted that the robot converges asymptotically to the reference trajectory in terms of both position and orientation. Fig. 5 shows reaching transients of the sliding mode. As expected, although some overshoot that comes from the initial error exists in the early state of s_2, the stabilization of sliding surfaces is attained in the steady state. This demonstrates the invariant property of the system with respect to the perturbation. The results in Figs. 4 and 5 show that the control objectives were achieved successfully.

Experiments

Description of Control System

To demonstrate the effectiveness and applicability of the proposed method, a real-time implementation of the control strategy was developed for a mobile robot named MIRO made for the purpose of playing robot-soccer games [18]-[21]. Fig. 6 shows the feature of the mobile robot the size of which is 7.5 cm × 7.5 cm × 7.5 cm. It has the same structure as in Fig. 1, with two driving wheels and one passive centered orientable wheel. Specifically, in the MIRO robot a *rolling-ball* is attached as the substitution of the passive wheel. The use of the rolling-ball would be desirable because it is omnidirectional so that its motion forces hardly impact on the dynamics of the mobile robot, complying with our assumption. The CPU of the robot is an AT89C52 microprocessor running at 24 MHz, and DC motors with LM629 motion controllers are mounted on the driving wheels. The LM629 chip is used as a velocity controller which receives only a reference velocity as the input signal and implicitly produces torque according to the reference velocity via the L298 driver chip. The LM629 chip makes it impossible to access torque directly in the control procedure. Therefore, in the experiments we generated control inputs as velocity terms by integrating **u** from (15). The maximum speed of the robot is about 1.46 m/sec.

The perspective view of the robot system with the vision system, the host computer, the communication system, and the pitch is shown in Fig. 7. The vision system estimates the posture information of the robot and transmits it to the host computer. It consists of a TMC-7 CCD camera with the resolution of 320 × 240 pixels and a Dooin Multimedia 7 image grabber with the processing rate of 30 frames/ sec. Since the localization of the robot is made only by the vision system, the processing rate of the vision system limits the sampling time for the controller. In the experiments the sampling time is set to be 33 ms, which is identical to the pro-

Table 1. Parameter values of the mobile robot.

Parameter	Value
mass of the robot body	20 Kg
diameter of the robot body	0.5 m
distance of the auxiliary wheel from C	0.25 m
radius of the drive wheel	0.07 m
radius of the auxiliary wheel	0.02 m
mass of the drive wheel	0.2 Kg
mass of the auxiliary wheel	0.1 Kg

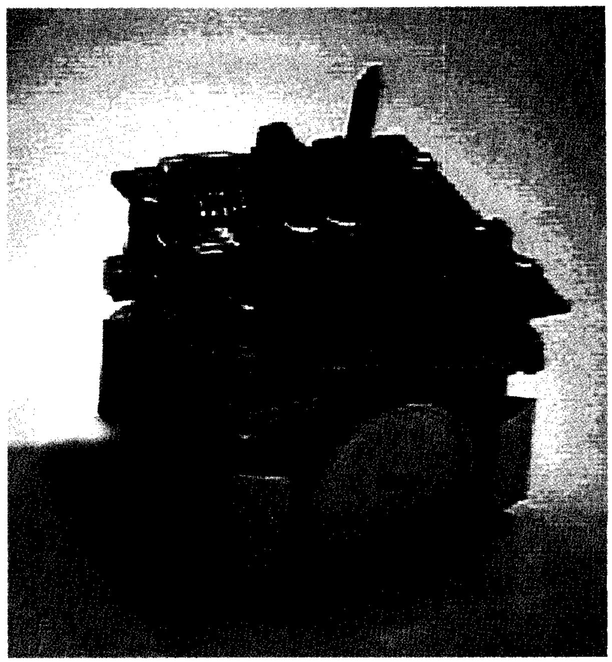

Fig. 6. MIRO robot.

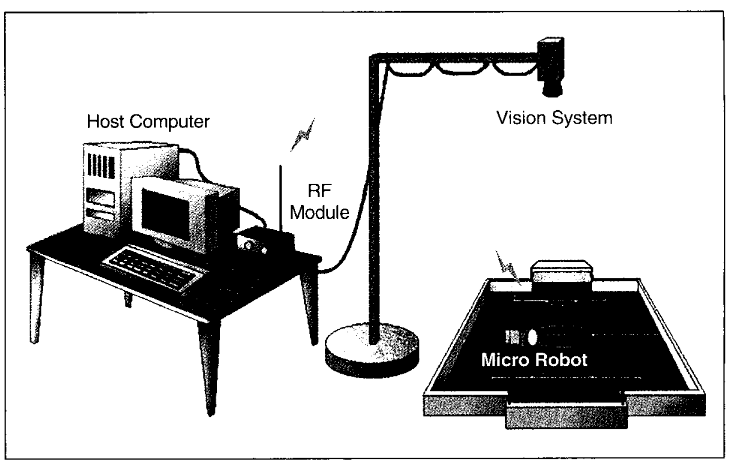

Fig. 7. Overview of the robot system.

cessing rate. The host computer is a Pentium processor with 133 MHz. Control commands are generated from the host computer and sent to the robot through the communication system, which is a radio frequency (RF) module running at the speed of 9600 bps. The software for implementing the control algorithm was developed in Visual C++ 1.5. The experiments were carried out on the MiroSot pitch [18], the size of which is 130cm × 90cm.

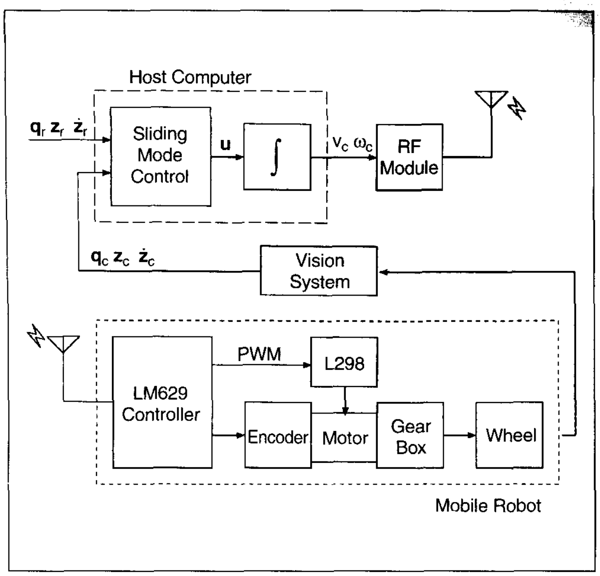

Fig. 8. Schematic diagram of the robot control system.

Fig. 8 is the schematic diagram of the robot control system. While there exist known or unknown external disturbances in the hardware of the control system, we focused our concern on the following disturbances in the experiments.

1) *Integration error*: As shown in Fig. 8, control inputs should be transformed by an integrator into velocity values for the LM629 motion controller. In this process, intrinsic errors from integration lead to perturbation of the control input **u**.

2) *Noise in control signal*: The control commands are sent to the mobile robot as the RF signal, which is inevitably contaminated with added noises from the outer environment. In the experiments, the upper bounds of the noises were set heuristically.

In addition to these, it is noted that there exist disturbances in signals of the posture information from the vision system. As was mentioned above, the position and orientation of the robot are estimated only visually without any dead-reckoning capability. Therefore, there are measurement errors in the posture data which stem from the time delay of the vision system or inaccuracy of its processed images.

Experimental Results

In order to validate the applicability of the proposed control scheme, experiments have been performed on the trajectory tracking problem. The mobile robot was commanded to track a time-varying reference trajectory on the pitch with initial posture errors. Two kinds of reference trajectory, straight-line and curve motion, were planned in the frame of $X-Y$ coordinates. As in the simulation example, reference trajectories were transformed into polar coordinates before generating control commands.

Experimental results for the straight-line tracking are shown in Figs. 9 and 10. Fig. 9 is the result where there were relatively small initial posture errors of 7.09 cm, 1.72 cm and -0.38 radian, for x_e, y_e and θ_e, respectively. Fig. 10 is the result in which the robot started tracking with large initial errors of 52.58 cm, 16.62 cm and -3.09 radian. As we can see, the mobile robot eventually approached the reference trajectory with asymptotic stability. In particular, as shown in Fig. 10(a), if the robot was located with large initial posture errors, there was a time of transition during which the tracking error of the heading direction was stabilized through forward and backward motion of the robot. Such a motion ascribes to the constraint of the robot's trajectory that the heading direction should not be tangential to a circle around the origin of the world coordinates. Fig. 11 is the result of the curve tracking with initial errors of 11.20

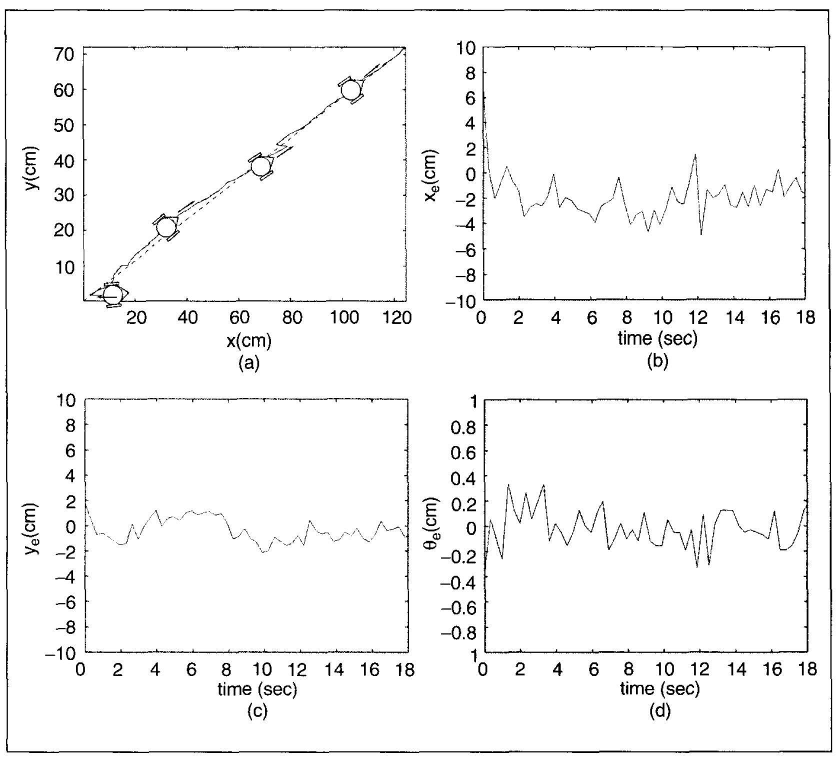

Fig. 9. Straight-line motion with initial errors of (7.09 cm, 1.72 cm, -0.38 radian): (a) $x-y$ plot (dashed line is the reference trajectory), (b) x_e, (c) y_e, and (d) θ_e.

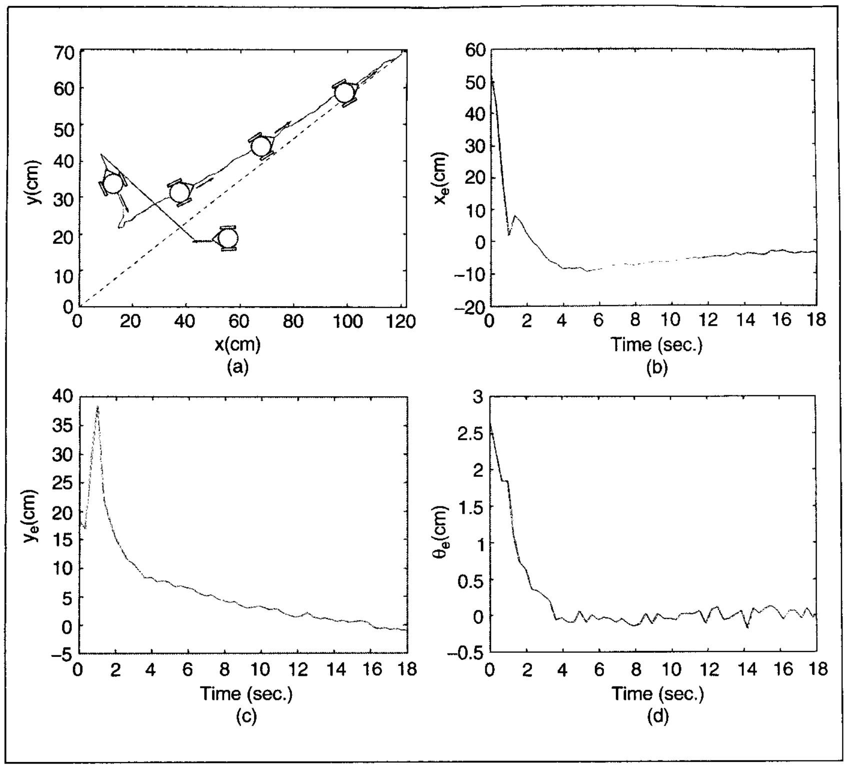

Fig. 10. Straight-line motion with initial errors of (52.58 cm, 16.62 cm, -3.09 radian): (a) $x-y$ plot (dashed line is the reference trajectory), (b) x_e, (c) y_e, and (d) θ_e.

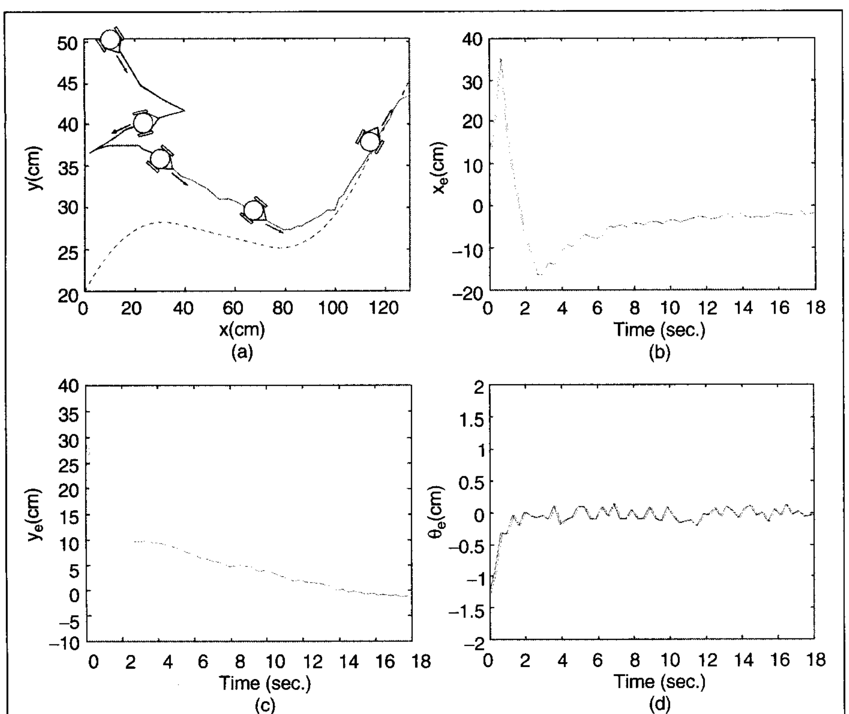

Fig. 11. Curve motion with initial errors of (11.20 cm, 28.92 cm, -1.31 radian): (a) $x-y$ plot (dashed line is the reference trajectory), (b) x_e, (c) y_e, and (d) θ_e.

cm, 28.92 cm, and -1.31 radian, for x_e, y_e and θ_e, respectively. Although there were forward and backward fluctuations along the X direction until about 6 s, all the three posture variables converged to desired trajectories. It is confirmed that the proposed sliding mode control law was successfully used for the purpose of trajectory tracking of the nonholonomic mobile robot in the presence of external disturbances.

Conclusion

A novel control algorithm to achieve trajectory tracking of mobile robots with nonholonomic constraints was proposed. Our approach consists of a computed-torque method for feedback-linearization of the dynamic equation and sliding mode control to realize robust trajectory tracking of the error dynamics of the system. The major contributions of this paper lie in the establishment of a novel tracking control scheme for mobile robots with two control inputs to asymptotically stabilize to a desired trajectory consisting of three posture variables. In particular, the proposed control scheme has the ability to solve a trajectory tracking problem when the reference trajectory is not given in a closed form.

Until now, in the class of applications of sliding mode control algorithm for nonholonomic systems, this problem had been unsolved. It was shown that, by applying the sliding mode control, the controlled behavior of the mobile robot is robust against initial condition errors and input disturbances which stem from measurement error and communication noise. The proposed algorithm was verified by numerical simulations and implemented on a vision-based mobile robot system. The experimental results show good tracking performance of the robot in the presence of model uncertainties and external disturbances.

Appendix

1) Proof of Theorem 1: If s_1 converges to zero, trivially ρ_e converges to zero. If s_2 converges to zero, in steadystate it becomes $\dot{\theta}_e = -\gamma_2\theta_e - \text{sgn}(\theta_e)|\phi_e|$. Since $|\phi_e|$ is always bounded, the following relationship between θ_e and $\dot{\theta}_e$ holds:

$$\theta_e > 0 \Rightarrow \dot{\theta}_e < 0,$$
$$\theta_e < 0 \Rightarrow \dot{\theta}_e > 0.$$

Therefore, the equilibrium state of θ_e is asymptotically stable. Finally, it can be known from s_2 that convergence of θ_e and $\dot{\theta}_e$ leads to convergence of $|\phi_e|$ to zero.

II) Proof of Theorem 2: The second derivative of ρ_c is calculated as

$$\begin{aligned}\ddot{\rho}_c &= (v_c \cos(\phi_c - \theta_c))' \\ &= \dot{v}_c \cos(\phi_c - \theta_c) + \frac{v_c^2}{\rho_c}\sin^2(\phi_c - \theta_c) + v_c\omega_c \sin(\phi_c - \theta_c) \\ &= \dot{v}_c \cos(\phi_c - \theta_c) + R(\mathbf{q}, \mathbf{z}).\end{aligned}$$

Therefore, $\ddot{\rho}_e$ is expressed as

$$\begin{aligned}\ddot{\rho}_e &= \ddot{\rho}_c - \ddot{\rho}_r \\ &= \dot{v}_c \cos(\phi_c - \theta_c) + R(\mathbf{q}, \mathbf{z}) - \dot{v}_r \cos(\phi_r - \theta_r) - R(\mathbf{q}_r, \mathbf{z}_r).\end{aligned} \tag{18}$$

Substituting the expression for $\ddot{\rho}_e$ into u_1, we have

$$u_1 = \frac{1}{\cos(\phi_c - \theta_c)}(-Q_{1}s_1 - P_1 \text{sgn}(s_1) - \gamma_1\dot{\rho}_e - \ddot{\rho}_e) + \dot{v}_c - \dot{v}_r. \tag{19}$$

The first low of (15) is

$$\dot{v}_c + f_1 = \dot{v}_r + u_1. \tag{20}$$

From (19) and (20),

$$\ddot{\rho}_e = -Q_1 s_1 - P_1 \text{ sgn}(s_1) - \gamma_1 \dot{\rho}_e - f_1 \cos(\phi_c - \theta_c). \tag{21}$$

Similarly, $\ddot{\theta}_e$ is expressed as

$$\ddot{\theta}_e = -Q_2 s_2 - P_2 \text{sgn}(s_2) - \gamma_2\dot{\theta}_e - \text{sgn}(\theta_e)\frac{d}{dt}|\phi_e| - f_2. \tag{22}$$

Define $V = \frac{1}{2}\mathbf{s}^T\mathbf{s}$ as a Lyapunov function candidate. Then, from (21) and (22), $\dot{V}$ is derived as

$$\begin{aligned}\dot{V} &= s_1\dot{s}_1 + s_2\dot{s}_2 \\ &= s_1(\ddot{\rho}_e + \gamma_1\dot{\rho}_e) + s_2(\ddot{\theta}_e + \gamma_2\dot{\theta}_e + \text{sgn}(\theta_e)\frac{d}{dt}|\phi_e|) \\ &= s_1(-Q_1 s_1 - P_1\text{sgn}(s_1) - f_1\cos(\phi_c - \theta_c)) \\ &\quad + s_2(-Q_2 s_2 - P_2\text{sgn}(s_2) - f_2) \\ &= -\mathbf{s}^T Q\mathbf{s} - (P_1|s_1| + f_1 s_1\cos(\phi_c - \theta_c)) - (P_2|s_2| + f_2 s_2).\end{aligned} \tag{23}$$

For $\dot{V}$ to be negative semidefinite, it is sufficient to choose Q_i and P_i such that $Q_i \geq 0, P_i \geq f_{m_i}, i = 1,2$.

III) Reduction of the disturbance effect: From (23), the real boundary of disturbance f_1 that should be eliminated in the sliding surface is $f_1\cos(\phi_c - \theta_c)$, instead of f_1. Thus, the design parameter P_1 can be chosen in the following range:

$$P_1 \geq f_1 \cos(\phi_c - \theta_c).$$

This implies that the effect of disturbance f_1, which should be eliminated in the sliding surface, decreases, as does the value of $\cos(\phi_c - \theta_c)$.

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Sliding Mode Motion Control of Nonholonomic Mobile Robots

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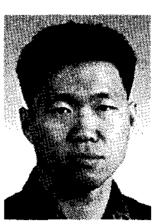

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