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Global Exponential Tracking Control of a Mobile Robot System via a PE Condition[1]

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Abstract

This paper presents the design of a differentiable, kinematic control law that utilizes a damped dynamic oscillator with a tunable frequency of oscillation to achieve global asymptotic tracking. Provided the reference trajectory satisfies a mild persistency of excitation condition, we also illustrate how the proposed kinematic controller can be slightly modified to provide global exponential tracking. In addition, we illustrate how the proposed kinematic controller provides for global asymptotic regulation of both the position and orientation of the mobile robot; hence, a unified framework is provided for both the tracking and regulation problem.

1 Introduction

The motion control problem of mechanical systems with nonholonomic constraints has been a heavily researched area due to both the challenging theoretical nature of the problem and its practical importance. One example of a nonholonomic system that has received a large amount of research activity is the wheeled mobile robot (WMR). In recent years, control researchers have targeted the problems of: i) regulating the position and orientation of the WMR to an arbitrary setpoint, ii) tracking a time-varying reference trajectory (which includes the path following problem as a special case [7]), and iii) incorporating the effects of the dynamic model during the control design to provide robustness. With regard to the control of nonholonomic systems, one of the technical hurdles often cited is that the regulation problem cannot be solved via a smooth, time-invariant state feedback law due to the implications of Brockett's condition [5]. To deal with this obstacle, some researchers have proposed controllers that utilize discontinuous control laws, piecewise continuous control laws, smooth time-varying control laws, or hybrid controllers to achieve setpoint regulation (see [19], [21], and the references therein for an in-depth review of the previous work). Specifically, in [4], Bloch et al. developed a piecewise continuous, analytic control structure for locally regulating nonholonomic systems to a setpoint, and in [6], Canudas de Wit et al. constructed a piecewise smooth, local controller to exponentially stabilize a WMR to a setpoint; however, due to the control structure, the orientation of the WMR is not arbitrary. One of the first smooth, time-varying feedback controllers that could be utilized to asymptotically regulate a WMR to a desired setpoint was proposed by Samson in [21]. Smooth, time-varying controllers were also developed for other classes of nonholonomic systems in [8], [20], and [24]. More recently, in [22], Samson developed global asymptotic feedback controllers for a general class of nonholonomic systems in the chain form that stabilize a WMR to a desired posture or a fixed reference-frame path. In [22], Samson also provided a detailed discussion on the convergence issue. In order to overcome the slower, asymptotic response of the previous smooth, time varying controllers, Godhavn et al. [12] and McCloskey et al. [19] constructed continuous feedback control laws with time-periodic terms that locally ρ-exponentially stabilized nonholonomic systems. In [19], McCloskey et al. also illustrated how the dynamic model of a WMR can be incorporated in the control design.

Several controllers have also been proposed for the reference robot tracking problem (i.e., the desired time-varying linear/angular velocity are specified). Specifically, in [16], Kanayama et al. utilized a continuous feedback control law for a linearized kinematic model to obtain local, uniform asymptotic tracking control; whereas, Walsh et al. [25] obtained local exponential stability results for a similar linearized model using a continuous, linear control law. In [14], Jiang et al. also obtained global asymptotic tracking control. In [15], Jiang et al. removed the need for acceleration measurements required in [14] and provided semi-global and global asymptotic tracking solutions for the general chained system form in which the origin was shown to be locally exponential stable. In [11], Escobar et al. illustrated how a field oriented induction motor controller can be redesigned to exponentially stabilize the nonholonomic double integrator control problem (e.g., Heisenberg flywheel); however, the controller exhibited singularities. In [10], Dong et al. utilized the kinematic control proposed in [22] to construct an adaptive control solution for chained nonholonomic systems that yielded global asymptotic tracking. We also note that several researchers (see [1], [7], [15], and the references within) have proposed various controllers for the path following problem.

In this paper, we present a differentiable, kinematic control law that utilizes a damped dynamic oscillator, with a tunable frequency of oscillation, to achieve global asymptotic tracking[2]. Provided the reference trajectory satisfies a mild persistency of excitation (PE) condition, we also illustrate how the proposed kinematic controller provides global exponential tracking. The PE condition is mild in the sense that many reference trajectories satisfy the condition (e.g., a circle trajectory can be exponentially tracked). In addition, we illustrate how the proposed kinematic controller also provides for global asymptotic regulation of both the position and orientation of the mobile robot. In comparison with previous work, the proposed kinematic controller is novel in the respect that: i) a global exponential tracking result is obtained provided certain PE conditions on the reference trajectory are satisfied, ii) a unified scheme is developed which solves both the global exponential tracking and global asymptotic regulation problems (i.e., there is no need to switch between two different controllers), and iii) to the best of our knowledge, this paper represents the first result that illustrates how the excitation of the reference trajectory can be used to improve the transient tracking performance.

The paper is organized as follows. In Section 2, we transform the kinematic model of the WMR into a form which facilitates the subsequent control development. In Section 3, we present the kinematic control law, corresponding closed-loop error system, and stability analysis for the global asymptotic tracking controller. In Section 4, we prove the global exponential tracking result. In Section 5, we illustrate how the proposed controller yields global asymptotic regulation. Concluding remarks are presented in Section 6.

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[2]The structure of the proposed kinematic controller is spawned from the induction motor controller presented in [9].

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2 Kinematic Problem Formulation

2.1 WMR Kinematic Model

The kinematic model for the so-called kinematic wheel under the nonholonomic constraint of pure rolling and non-slipping is given as follows [19]

$$\dot{q} = S(q)v \tag{1}$$

where $q(t)$, $\dot{q}(t) \in \Re^3$ are defined as

$$q = [x_c \quad y_c \quad \theta]^T \qquad \dot{q} = \begin{bmatrix}\dot{x}_c & \dot{y}_c & \dot{\theta}\end{bmatrix}^T \tag{2}$$

$x_c(t)$, $y_c(t)$, and $\theta(t) \in \Re^1$ denote the linear position and orientation, respectively, of the center of mass (COM) of the WMR, $\dot{x}_c(t)$, $\dot{y}_c(t)$ denote the Cartesian components of the linear velocity of the COM, $\dot{\theta}(t) \in \Re^1$ denotes the angular velocity of the COM, the matrix $S(q) \in \Re^{3\times 2}$ is defined as follows

$$S(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}, \tag{3}$$

and the velocity vector $v(t) \in \Re^2$ is defined as

$$v = [v_1 \quad v_2]^T = \begin{bmatrix}v_l & \dot{\theta}\end{bmatrix}^T \tag{4}$$

with $v_l(t) \in \Re^1$ denoting the linear velocity of the COM of the WMR.

2.2 Control Objective

As defined in previous work (e.g., see [14] and [16]), the reference trajectory is generated via a reference robot which moves according the following dynamic trajectory

$$\dot{q}_r = S(q_r)v_r \tag{5}$$

where $S(\cdot)$ was defined in (3), $q_r(t) = \begin{bmatrix} x_{rc}(t) & y_{rc}(t) & \theta_r(t) \end{bmatrix}^T \in \Re^3$ denotes the desired time-varying position and orientation trajectory, and $v_r(t) = \begin{bmatrix} v_{r1}(t) & v_{r2}(t) \end{bmatrix}^T \in \Re^2$ denotes the reference time-varying linear and angular trajectory. With regard to (5), it is assumed that the signal $v_r(t)$ is constructed to produce the desired motion and that $v_r(t)$, $\dot{v}_r(t)$, $q_r(t)$, and $\dot{q}_r(t)$ are bounded for all time.

To facilitate the subsequent control synthesis and the corresponding stability proof, we define the following transformation

$$\begin{bmatrix} z_1 \\ z_2 \\ w \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \\ -\tilde{\theta}\cos\theta + 2\sin\theta & -\tilde{\theta}\sin\theta - 2\cos\theta & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} \tag{6}$$

where $w(t) \in \Re^1$ and $z(t) = \begin{bmatrix} z_1(t) & z_2(t) \end{bmatrix}^T \in \Re^2$ are auxiliary tracking error variables, $\tilde{x}(t)$, $\tilde{y}(t)$, $\tilde{\theta}(t) \in \Re^1$ denote the difference between the actual Cartesian position and orientation of the COM and the desired position and orientation of the COM as follows

$$\tilde{x} = x_c - x_{rc} \qquad \tilde{y} = y_c - y_{rc} \qquad \tilde{\theta} = \theta - \theta_r. \tag{7}$$

After taking the time derivative of (6), using (1), (2), (3), (4), (5), and (7), we can rewrite the tracking error dynamics in terms of the new variables defined in (6) as follows

$$\begin{aligned} \dot{w} &= u^T J^T z + Az \\ \dot{z} &= u \end{aligned} \tag{8}$$

where $J \in \Re^{2\times 2}$ is an auxiliary skew-symmetric matrix defined as

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \tag{9}$$

and the auxiliary row vector $A(z, v_r, t) \in \Re^{1\times 2}$ is defined as

$$A = \begin{bmatrix} -2v_{r1}\dfrac{\sin(z_1)}{z_1} & 2v_{r2} \end{bmatrix}. \tag{10}$$

The auxiliary variable $u(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T \in \Re^2$ utilized in (8) is used to simplify the transformed dynamics and is explicitly defined in terms of the WMR position and orientation, the WMR linear velocities, and the desired trajectory as follows

$$u = T^{-1}v - \begin{bmatrix} v_{r2} \\ v_{r1}\cos\tilde{\theta} \end{bmatrix} \qquad v = Tu + \Pi \tag{11}$$

where the auxiliary variables $T \in \Re^{2\times 2}$ and $\Pi \in \Re^2$ are defined as follows

$$T = \begin{bmatrix} (\tilde{x}\sin\theta - \tilde{y}\cos\theta) & 1 \\ 1 & 0 \end{bmatrix} \tag{12}$$

and

$$\Pi = \begin{bmatrix} v_{r1}\cos\tilde{\theta} + v_{r2}\left(\tilde{x}\sin\theta - \tilde{y}\cos\theta\right) \\ v_{r2} \end{bmatrix} \tag{13}$$

respectively.

3 Kinematic Control Development

Our control objective is to design a controller for the transformed WMR kinematic model given by (8). To facilitate the subsequent control development, we define an auxiliary error signal $\tilde{z}(t) \in \Re^2$ as the difference between the subsequently designed auxiliary signal $z_d(t) \in \Re^2$ and the transformed variable $z(t)$, defined in (6), as follows

$$\tilde{z} = z_d - z. \tag{14}$$

3.1 Control Formulation

Based on the kinematic equations given in (8) and the subsequent stability analysis, we design the auxiliary signal $u(t)$ as follows

$$u = u_a - k_3 z + u_c \tag{15}$$

where the auxiliary control terms $u_a(t) \in \Re^2$ and $u_c(t) \in \Re^2$ are defined as

$$u_a = k_1 w J z_d + \Omega_1 z_d \tag{16}$$

and

$$u_c = -(I_2 + 2wJ)^{-1}(2wA^T) \tag{17}$$

respectively, the auxiliary signal $z_d(t)$ is defined by the following dynamic oscillator-like relationship

$$\begin{aligned} \dot{z}_d &= \left(k_1\left(w^2 - z_d^T z_d\right) - k_2\right) z_d + J\Omega_2 z_d + \frac{1}{2}u_c \\ \beta &= z_d^T(0) z_d(0), \end{aligned} \tag{18}$$

the auxiliary terms $\Omega_1(t) \in \Re^1$ and $\Omega_2(t) \in \Re^1$ are defined as

$$\Omega_1 = k_1 w^2 + k_1\left(w^2 - z_d^T z_d\right) - k_2 + k_3 \tag{19}$$

and

$$\Omega_2 = k_1 w + w\Omega_1 \tag{20}$$

respectively, k_1, k_2, $k_3 \in \Re^1$ are positive, constant control gains, I_2 represents the standard 2×2 identity matrix, $\beta \in \Re^1$ is a positive constant, and $A(z, v_r, t)$ was defined in (10). Note that it is straightforward to show that the matrix $I_2 + 2wJ^T$ used in (17) is always invertible provided $w(t)$ remains bounded.

3.2 Error System Development

To facilitate the closed-loop error system development for $w(t)$, we substitute (15) for $u(t)$ defined in (8), add and subtract $u_a^T J z_d$ to the resulting expression, utilize (14), and exploit the skew symmetry of J defined in (9) to rewrite the dynamics for $w(t)$ given by (8), as follows

$$\dot{w} = u_a^T J\tilde{z} - u_a^T J z_d + Az - u_c^T J z \tag{21}$$

where the fact $J^T = -J$ was utilized. Finally, by substituting (16) for only the second occurrence of $u_a(t)$ in (21), substituting (17) for $u_c(t)$, utilizing the skew symmetry of J defined in (9), and the facts that $J^T J = I_2$ and $J^T = -J$, we can obtain the final expression for the closed-loop error system of $w(t)$ as follows

$$\dot{w} = -k_1 w z_d^T z_d + Az + 2wA(I_2 + 2wJ^T)^{-1} J z + u_a^T J\tilde{z}. \tag{22}$$

To facilitate the subsequent stability analysis, we substitute (17) for $u_c(t)$ in (18) to yield the following form for the dynamics of $z_d(t)$

$$\begin{aligned} \dot{z}_d = {} & \left(k_1\left(w^2 - z_d^T z_d\right) - k_2\right) z_d + J\Omega_2 z_d \\ & -(I_2 + 2wJ)^{-1} w A^T. \end{aligned} \tag{23}$$

To determine the closed-loop error system for $\tilde{z}(t)$, we take the time derivative of (14), and then substitute (18) and (8) for $\dot{z}_d(t)$ and $\dot{z}(t)$, respectively, to obtain

$$\dot{\tilde{z}} = \left(k_1\left(w^2 - z_d^T z_d\right) - k_2\right) z_d + J\Omega_2 z_d + \frac{1}{2}u_c - u. \tag{24}$$

After substituting (15) for $u(t)$, and then substituting (16) for $u_a(t)$ in the resulting expression, we can rewrite the expression given by (24) as follows

$$\begin{aligned} \dot{\tilde{z}} = {} & \left(k_1\left(w^2 - z_d^T z_d\right) - k_2\right) z_d + J\Omega_2 z_d - \frac{1}{2}u_c \\ & -k_1 w J z_d - \Omega_1 z_d + k_3 z. \end{aligned} \tag{25}$$

After substituting (19) and (20) for $\Omega_1(t)$ and $\Omega_2(t)$ into (25), respectively, and then using the fact that $JJ = -I_2$, we can cancel common terms and rearrange the resulting expression to obtain

$$\dot{\tilde{z}} = -k_3\tilde{z} + wJ\left[\Omega_1 z_d + k_1 w J z_d\right] - \frac{1}{2}u_c \tag{26}$$

where (14) has been utilized. Finally, we substitute (17) for $u_c(t)$ to determine the final expression for the closed-loop error system of $\tilde{z}(t)$ as follows

$$\dot{\tilde{z}} = -k_3\tilde{z} + (I_2 + 2wJ)^{-1}wA^T + wJu_a \tag{27}$$

where we have used the fact that the bracketed term in (26) is equal to $u_a(t)$ defined in (16).

3.3 Stability Analysis

Theorem 1 *The kinematic controller given by (15), (16), (17), (18), (19), and (20) ensures global asymptotic tracking in the sense that*

$$\lim_{t\to\infty} \tilde{x}(t), \tilde{y}(t), \tilde{\theta}(t) = 0 \tag{28}$$

provided the reference trajectory is selected so that

$$\lim_{t\to\infty} \|v_r\| \neq 0. \tag{29}$$

Proof: To prove Theorem 1, we define the following non-negative, scalar function denoted by $V(w, z_d, \tilde{z}, t) \in \Re^1$ as follows

$$V(t) = \frac{1}{2}w^2 + \frac{1}{2}z_d^T z_d + \frac{1}{2}\tilde{z}^T\tilde{z}. \tag{30}$$

After taking the time derivative of (30) and making the appropriate substitutions from (22), (23), and (27), we obtain the following expression

$$\begin{aligned} \dot{V} &= w[u_a^T J\tilde{z} - k_1 w z_d^T z_d + Az + 2wA(I_2 + 2wJ^T)^{-1}Jz] \\ &\quad + z_d^T[(k_1(w^2 - z_d^T z_d) - k_2)z_d - (I_2 + 2wJ)^{-1}wA^T] \\ &\quad + z_d^T J\Omega_2 z_d + \tilde{z}^T[wJu_a - k_3\tilde{z} + (I_2 + 2wJ)^{-1}wA^T]. \end{aligned} \tag{31}$$

After utilizing the skew symmetry property of J defined in (9), making use of the fact that $J^T = -J$, and cancelling common terms, we can rewrite (31) as follows

$$\begin{aligned} \dot{V} &\leq -k_2 z_d^T z_d - k_3\tilde{z}^T\tilde{z} + wAz - \left[wA(I_2 + 2wJ^T)^{-1}\right] \\ &\quad + \left[wA(I_2 + 2wJ^T)^{-1}(2wJ)\right] z \end{aligned} \tag{32}$$

where (14) has been utilized. Next, after utilizing the fact that $J^T = -J$, we can combine the bracketed terms in (32) as shown below

$$\begin{aligned} \dot{V} &\leq -k_2 z_d^T z_d - k_3\tilde{z}^T\tilde{z} + \\ &\quad wAz - wA\left[(I_2 + 2wJ^T)^{-1}\left(I_2 + 2wJ^T\right)\right] z. \end{aligned} \tag{33}$$

After noting that the bracketed term in (33) is equal to the identity matrix, we can cancel common terms to obtain the final upper bound for $\dot{V}(t)$ as follows

$$\dot{V} \leq -k_2 z_d^T z_d - k_3\tilde{z}^T\tilde{z}. \tag{34}$$

Based on (30) and (34), we can conclude that $V(t) \in \mathcal{L}_\infty$; thus, $w(t)$, $z_d(t)$, $\tilde{z}(t) \in \mathcal{L}_\infty$. Since $w(t)$, $z_d(t)$, $\tilde{z}(t) \in \mathcal{L}_\infty$, we can utilize (10), (14), (15), (16), (17), (18), (19), (20), (22), (27), and the fact that the reference trajectory is assumed to be bounded to conclude that $A(t)$, $z(t)$, $u(t)$, $u_a(t)$, $u_c(t)$, $\Omega_1(t)$, $\Omega_2(t)$, $\dot{z}_d(t)$, $\dot{w}(t)$, $\dot{\tilde{z}}(t) \in \mathcal{L}_\infty$. Since $\dot{z}_d(t)$, $\dot{\tilde{z}}(t) \in \mathcal{L}_\infty$, we can utilize (14) to show that $\dot{z}(t) \in \mathcal{L}_\infty$ (since $\dot{w}(t)$, $\dot{z}_d(t)$, $\dot{\tilde{z}}(t)$, $\dot{z}(t) \in \mathcal{L}_\infty$, we know that $w(t)$, $z_d(t)$, $\tilde{z}(t)$, and $z(t)$ are uniformly continuous). In order to illustrate that the Cartesian position and orientation signals defined in (1) are bounded, we calculate the inverse transformation of (6) as follows

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\left(\tilde{\theta}\sin\theta + 2\cos\theta\right) & \frac{1}{2}\sin\theta \\ 0 & -\frac{1}{2}\left(\tilde{\theta}\cos\theta - 2\sin\theta\right) & -\frac{1}{2}\cos\theta \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ w \end{bmatrix}. \tag{35}$$

ince $z(t) \in \mathcal{L}_\infty$, it is clear from (7) and (35) that $\tilde{\theta}(t)$, $\theta(t) \in \mathcal{L}_\infty$. Furthermore, from (7), (35), and the fact that $w(t)$, $z(t)$, $\theta(t) \in \mathcal{L}_\infty$, we can conclude that $\tilde{x}(t)$, $\tilde{y}(t)$, $x_c(t)$, $y_c(t) \in \mathcal{L}_\infty$. We can utilize (11), the assumption that the desired trajectory is bounded, and the fact that $\theta(t)$, $u(t)$, $\tilde{x}(t)$, $\tilde{y}(t) \in \mathcal{L}_\infty$, to show that $v(t) \in \mathcal{L}_\infty$; therefore, it follows from (1), (2), (3), and (4) that $\dot{\theta}(t)$, $\dot{x}_c(t)$, $\dot{y}_c(t) \in \mathcal{L}_\infty$. Based on the boundedness of the aforementioned signals, we can take the time derivative of (18) and show that $\ddot{z}_d(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can now be used to show that all remaining signals are bounded. From (14) and (34), it is easy to show that $z_d(t)$, $\tilde{z}(t) \in \mathcal{L}_2$; hence, since $z_d(t)$ and $\tilde{z}(t)$ are uniformly continuous, we can use (14) and a corollary to Barbalat's Lemma [23] to show that $\lim_{t\to\infty} z_d(t)$, $\tilde{z}(t)$, $z(t) = 0$. Next, since $\ddot{z}_d(t) \in \mathcal{L}_\infty$, we know that $\dot{z}_d(t)$ is uniformly continuous. Since we know that $\lim_{t\to\infty} z_d(t) = 0$ and $\dot{z}_d(t)$ is uniformly continuous, we can use the following equality

$$\lim_{t\to\infty} \int_0^t \frac{d}{d\tau}(z_d(\tau))\, d\tau = \lim_{t\to\infty} z_d(t) + \text{Constant} \tag{36}$$

and Barbalat's Lemma [23] to conclude that $\lim_{t\to\infty} \dot{z}_d(t) = 0$. Based on the fact that $\lim_{t\to\infty} z_d(t), \dot{z}_d(t) = 0$, it is straightforward from (17) and (18) to see that $\lim_{t\to\infty} wA^T = 0$. Finally, based on (10) and (29) we can conclude that $\lim_{t\to\infty} w(t) = 0$. The global asymptotic result given in (28) can now be directly obtained from (35). □

4 Global Exponential Tracking Analysis

In the previous section, we utilized a straightforward Lyapunov analysis and Barbalat's Lemma to prove global asymptotic position tracking. Since we have established that all signals in the closed-loop system are bounded, we now illustrate how the nonlinear closed-loop error system formulated in the previous section can be represented as a linear time-varying system as similarly done for closed-loop adaptive control systems (see [17]). This linear time-varying representation allows us to develop a persistency of excitation (PE) condition on the desired reference trajectory that promulgates a global exponential tracking result.

4.1 Error System Development

To formulate the nonlinear closed-loop error development given in the previous section as a linear time-varying system, we first define the states of the system, denoted by $x(t) \in \Re^5$, as follows

$$x = \begin{bmatrix} p^T & w \end{bmatrix}^T \tag{37}$$

where the auxiliary signal $p(t) \in \Re^4$ is defined as

$$p = \begin{bmatrix} z_d^T & \tilde{z}^T \end{bmatrix}^T \tag{38}$$

and $w(t)$, $z_d(t)$, and $z(t)$ were defined in (6). In addition, we rewrite the closed-loop dynamics for $w(t)$ into a more convenient form by substituting (16) for only the second occurrence of $u_a(t)$ in (21) and then utilizing (14), the skew symmetry of J defined in (9), and the fact that $J^T J = I_2$ to yield

$$\dot{w} = u_a^T J\tilde{z} - k_1 w z_d^T z_d + A(z_d - \tilde{z}) - u_c^T J(z_d - \tilde{z}). \tag{39}$$

Based on (39) and the definition of $p(t)$ given in (38), we can now obtain a convenient expression for the dynamics of $w(t)$ as follows

$$\dot{w} = B_1^T p \tag{40}$$

where $B_1(t) \in \Re^4$ is defined as

$$B_1 = \begin{bmatrix} -k_1 w z_d^T + A - u_c^T J & u_a^T J - A + u_c^T J \end{bmatrix}^T. \tag{41}$$

In order to express the closed-loop error system for $\tilde{z}(t)$ in a form that facilitates the linear system representation, we substitute (17) into (26) to obtain the following expression

$$\dot{\tilde{z}} = \left(wJ\Omega_1 - k_1 w^2 I_2\right) z_d + \left((I_2 + 2wJ)^{-1}A^T\right) w - k_3\tilde{z}. \tag{42}$$

It is now a straightforward matter to take the time derivative of (38) and make appropriate substitutions from (23) and (42) to express the closed-loop error system for $p(t)$ as follows

$$\dot{p} = A_0 p + B_2 w \tag{43}$$

where the auxiliary terms $A_0(t) \in \Re^{4\times4}$ and $B_2(t) \in \Re^4$ are defined as

$$A_0 = \begin{bmatrix} \left(k_1\left(w^2 - z_d^T z_d\right) - k_2\right) I_2 + J\Omega_2 & 0_{2\times2} \\ wJ\Omega_1 - k_1 w^2 I_2 & -k_3 I_2 \end{bmatrix} \tag{44}$$

and

$$B_2 = \begin{bmatrix} -((I_2 + 2wJ)^{-1}A^T)^T & ((I_2 + 2wJ)^{-1}A^T)^T \end{bmatrix}^T, \tag{45}$$

respectively, $0_{n\times m}$ represents the $n \times m$ zero matrix, and I_2 represents the 2×2 identity matrix. The final linear time-varying representation is obtained by taking the time derivative of (37) and then utilizing (40) and (43) to obtain

$$\begin{aligned} \dot{x} &= A_1 x \\ y &= Cx \end{aligned} \tag{46}$$

where $A_1(t) \in \Re^{5\times 5}$ is defined as

$$A_1 = \begin{bmatrix} A_0 & B_2 \\ B_1^T & 0 \end{bmatrix}, \tag{47}$$

the matrix $C \in \Re^{5\times 5}$ is defined as

$$C = \begin{bmatrix} D & 0_{4\times 1} \\ 0_{1\times 4} & 0 \end{bmatrix}, \tag{48}$$

and the submatrix $D \in \Re^{4\times 4}$ is defined as

$$D = \begin{bmatrix} \sqrt{k_2} I_2 & 0_{2\times 2} \\ 0_{2\times 2} & \sqrt{k_3} I_2 \end{bmatrix}. \tag{49}$$

Remark 1 *In the subsequent exponential stability proof, we will utilize the fact that (34) can be rewritten as*

$$\dot{V} \leq -x^T C^T C x; \tag{50}$$

hence, (50) provides the motivation for the structure of the matrix C defined in (48). In addition, the subsequent stability analysis utilizes the fact that all the signals in the time-varying system given by (46) are bounded as illustrated by Theorem 1. We also require that $B_2(t)$ defined in (45) be differentiable. Based on the proof of Theorem 1, it is straightforward to show that $\dot{B}_2(t)$ is bounded.

4.2 Stability Analysis

Before we state the exponential stability result, we present two lemmas that are used during the proof of the main result.

Lemma 1 *If the reference angular velocity $v_{r2}(t)$ defined in (5) is selected according to the following expression*

$$\int_t^{t+\delta_1} v_{r2}^2(\sigma) d\sigma \geq \xi_1, \tag{51}$$

(i.e., if the reference angular velocity is selected to be persistently exciting (PE)) then the Observability Grammian for the system given in (46), defined as

$$W(t, t+\delta) = \int_t^{t+\delta} \Phi^T(\tau, t) C^T C \Phi(\tau, t) d\tau, \tag{52}$$

satisfies the following inequality

$$W(t, t+\delta) \geq \gamma I_5 \tag{53}$$

for all $t \geq 0$, where $\delta, \delta_1, \xi_1, \gamma \in \Re^1$ are positive constants, $\Phi(\tau, t) \in \Re^{5\times 5}$ denotes the state transition matrix for (46), I_n represents the $n \times n$ identity matrix, and C was defined in (48).

Proof: To prove Lemma 1, we note that a closed-form expression for the state transition matrix of (46) is difficult to find; thus, we employ the fact that the pair $(A_1(t), C)$ of (46) is uniformly observable (UO) if and only if the pair $(A_1(t) - K(t)C, C)$ is UO (see [13] for an explicit proof) where $K(t) \in \Re^{5\times 5}$ is treated as a design matrix. To facilitate the analysis, we construct $K(t)$ to be a continuous, bounded matrix as follows

$$K = \begin{bmatrix} A_0 D^{-1} & 0_{4\times 1} \\ B_1^T D^{-1} & 0 \end{bmatrix}. \tag{54}$$

Based on the definition of $K(t)$ given in (54), we have

$$A_1 - KC = \begin{bmatrix} 0_{4\times 4} & B_2 \\ 0_{1\times 4} & 0 \end{bmatrix}; \tag{55}$$

hence, the state transition matrix for the pair $(A_1(t) - K(t)C, C)$, denoted by $\Phi_1(\tau, t) \in \Re^{5\times 5}$, can be calculated as follows

$$\Phi_1 = \begin{bmatrix} I_4 & \int_t^\tau B_2(\sigma) d\sigma \\ 0_{1\times 4} & 1 \end{bmatrix}. \tag{56}$$

Given the following definition for the Observability Grammian for the pair $(A_1(t) - K(t)C, C)$

$$W_1(t, t+\delta_2) = \int_t^{t+\delta_2} \Phi_1^T(\tau, t) C^T C \Phi_1(\tau, t) d\tau, \tag{57}$$

we can substitute (48) and (56) into (57) to calculate the following expression

$$W_1(t, t+\delta_2) = \int_t^{t+\delta_2} \begin{bmatrix} D^T D & D^T D \int_t^\tau B_2(\sigma) d\sigma \\ \int_t^\tau B_2^T(\sigma) d\sigma D^T D & \int_t^\tau B_2^T(\sigma) d\sigma D^T D \int_t^\tau B_2(\sigma) d\sigma \end{bmatrix} d\tau \tag{58}$$

where $\delta_2 \in \Re^1$ is a positive constant.

To facilitate further analysis, we note that $\int_t^{t+\delta_1} B_2^T(\sigma) B_2(\sigma) d\sigma$ can be rewritten as follows

$$\int_t^{t+\delta_1} B_2^T(\sigma) B_2(\sigma) d\sigma = 2 \left[\int_t^{t+\delta_1} \frac{1}{(1+4w^2)^2} \left((-2v_{r1} \tfrac{\sin(z_1)}{z_1} + 4wv_{r2})^2 + (4wv_{r1} \tfrac{\sin(z_1)}{z_1} + 2v_{r2})^2 \right) d\sigma \right]. \tag{59}$$

After some algebraic manipulation, we note that (59) can be simplified to the following expression

$$\int_t^{t+\delta_1} B_2^T(\sigma) B_2(\sigma) d\sigma = 8 \int_t^{t+\delta_1} \frac{1}{(1+4w^2)} \left(v_{r1}^2 \left(\frac{\sin(z_1)}{z_1} \right)^2 + v_{r2}^2 \right) d\sigma. \tag{60}$$

Next, since $w(t)$, $z(t)$, $v_r(t) \in \mathcal{L}_\infty$, we can select positive constants $\zeta_1, \gamma_1 \in \Re^1$ such that $\int_t^{t+\delta_1} B_2^T(\sigma) B_2(\sigma) d\sigma$ can be upper bounded as follows

$$\int_t^{t+\delta_1} B_2^T(\sigma) B_2(\sigma) d\sigma \geq \zeta_1 \int_t^{t+\delta_1} v_{r2}^2 d\sigma \geq \gamma_1 \tag{61}$$

where the assumption given in (51) was utilized. Given the definition for $W_1(t, t+\delta_2)$ in (58), the fact that $B_2(t)$ and $\dot{B}_2(t)$ are bounded, and the fact that $\int_t^{t+\delta_1} B_2^T(\sigma) B_2(\sigma) d\sigma$ satisfies (61), we can apply Lemma 13.4 in [17] to (58) to show that there exists some positive constant $\gamma_2 \in \Re^1$ such that

$$W_1(t, t+\delta_2) \geq \gamma_2 I_5; \tag{62}$$

hence, the pair $(A_1(t) - K(t)C, C)$ is UO. Since the pair $(A_1(t) - K(t)C, C)$ is UO, then the pair $(A_1(t), C)$ is UO (see Lemma 4.8.1 in [13] for an explicit proof); hence, by the definition of uniform observability (see [2]), the result given in (53) can now be directly obtained.

Lemma 2 *Let $V(x, t) \in \Re^1$ be a continuously differentiable function such that*

$$\gamma_a \|x\|^2 \leq V(x, t) \leq \gamma_b \|x\|^2 \tag{63}$$

$$\dot{V}(x, t) \leq 0 \tag{64}$$

and

$$\int_t^{t+\delta} \dot{V}(\phi(x, t, \tau), \tau) d\tau \leq -\gamma_c V(x, t) \quad \text{for } t \geq 0 \tag{65}$$

where γ_a, γ_b, γ_c, $\delta \in \Re^1$ are some positive constants, and $\phi(x, t, \tau)$ denotes the solution of the system that starts at (x, t). If (63), (64), and (65) hold globally, then the system is globally exponentially stable in the sense that

$$\|x(t)\| \leq \alpha_0 \exp(-\beta_0 t) \tag{66}$$

for some positive constants α_0, $\beta_0 \in \Re^1$.

Proof: See Theorem 4.5 in [17].

Remark 2 *Note that since (66) is an exponential envelope originating at α_0 which need not be proportional to $\|x(0)\|$, the result does not adhere to the standard definition of global exponential stability (see the discussion in [21] and [17]); however, for any initial condition, $x(t)$ exponentially converges to zero.*

Theorem 2 *The position and orientation tracking errors defined in (7) are globally exponentially stable in the sense that*

$$|\tilde{x}(t)|, |\tilde{y}(t)|, \left|\tilde{\theta}(t)\right| \leq \alpha_1 \exp(-\beta_1 t) \tag{67}$$

for some positive scalar constants α_1 and β_1, provided the reference angular velocity satisfies (51).

Proof: To prove Theorem 2, we define the non-negative function $V_2(x) \in \Re^1$ as follows

$$V_2(x,t) = \frac{1}{2}x^T x \tag{68}$$

where $x(t)$ was defined in (37). Based on (30), (34), (37), (38), (48), (49), (68), and the proof for Theorem 1, the time derivative of (68) can be expressed as follows

$$\dot{V}_2(x,t) \leq -x^T C^T C x \tag{69}$$

where C was defined in (48). After integrating (69), we obtain the following expression

$$\int_t^{t+\delta} \dot{V}_2(\phi(x,\tau,t),\tau)d\tau \leq -x^T[\int_t^{t+\delta} \Phi^T(\tau,t)C^T C\Phi(\tau,t)d\tau]x \tag{70}$$

where we have use the fact that $\phi(x,\tau,t)$, which denotes the solution of the solution to the linear system defined in (46) that starts at (x,t), can be expressed as follows [3]

$$\phi(x,\tau,t) = \Phi(\tau,t)x(t), \tag{71}$$

and $\Phi(\tau,t)$ denotes the state transition matrix for (46). After noting that the bracketed term in (70) is equal to $W(t,t+\delta)$, defined in (52), we can utilize (53), (68), Lemma 1, and (51) show that

$$\int_t^{t+\delta} \dot{V}_2(\phi(x,\tau,t),\tau)d\tau \leq -2\gamma V_2(x,t). \tag{72}$$

From (68), (69), and (72), it is clear that the conditions given in Lemma 2 are globally satisfied; hence,

$$\|x(t)\| \leq \alpha_2 \exp(-\beta_2 t) \tag{73}$$

where α_2 and $\beta_2 \in \Re^1$ are positive constants. The global exponential result given in (67) can now be directly obtained from (14), (35), (37), and (38). □

Remark 3 *Some examples of persistently exciting reference trajectories include: i)* $v_{r2} \neq 0$, *ii)* $\lim_{t\to\infty} v_{r2} = c_1 \neq 0$ *(e.g., a circle trajectory can be exponentially tracked) and iii)* $v_{r2} = \sin(t)$. *In addition, we note that since* $\lim_{t\to\infty} z(t) = 0$ *(see Theorem 1), then there exists some time, denoted by* t_p, *such that*

$$|z_1(t)| < \frac{\pi}{2} \qquad \forall t > t_p; \tag{74}$$

thus,

$$\frac{\sin(z_1)}{z_1} > \frac{2}{\pi} \qquad \forall t > t_p. \tag{75}$$

Based on (75) and (60), we can rewrite (61) as follows

$$\int_t^{t+\delta_1} B_2^T(\sigma)B_2(\sigma)d\sigma \geq \frac{4\zeta_1}{\pi^2}\int_t^{t+\delta_1}(v_{r1}^2 + v_{r2}^2)d\sigma \;\; \forall t > t_p; \tag{76}$$

hence, if the reference condition given in (51) is modified as given below

$$\int_t^{t+\delta_1} \|v_r(\sigma)\|^2 d\sigma \geq \xi_1 \qquad \forall t > t_p. \tag{77}$$

(i.e., if either the linear or angular reference velocity is selected to be PE), then

$$|\tilde{x}(t)|, |\tilde{y}(t)|, \left|\tilde{\theta}(t)\right| \leq \alpha_1 \exp(-\beta_1 t), \qquad \forall t > t_p. \tag{78}$$

The inequality given in (78) indicates that if (77) is satisfied, then there is some time during the transient during which the asymptotic tracking result becomes an exponential tracking result; hence,many different types of geometric trajectories can be exponentially tracked (e.g., circles, lines, sinusoids, etc.,).

5 Setpoint Extension

Many of the previously proposed tracking controllers do not reduce to the regulation problem because of technical restrictions placed on the reference trajectory similar to that given in (29). In this section, we illustrate how the kinematic tracking controller proposed in the previous section also ensures global asymptotic position and orientation regulation provided k_2 given in (18) and (19) is set equal to zero. Since this new control objective is now targeted at the regulation problem, the desired position and orientation vector, denoted by $q_r = \begin{bmatrix} x_{rc} & y_{rc} & \theta_r \end{bmatrix}^T \in \Re^3$ and originally defined in (5), is now assumed to be an arbitrary desired constant vector. Based on the fact that q_r is now defined as a constant vector, it is straightforward to see that $v_r(t)$, given in (5), and consequently $A(z, v_r, t)$ and $u_c(t)$ defined in (10) and (17), respectively, are now set to zero for the regulation control problem. As a result of the new control objective, we also note that the auxiliary variable $u(t)$ originally defined in (11), is now defined as follows

$$u = T^{-1}v \qquad v = Tu \tag{79}$$

where the matrix T was defined in (12).

5.1 Stability Analysis

Theorem 3 *The kinematic controller given by (15), (16), (18), (19), and (20) with* $k_2 = 0$, *ensures global asymptotic regulation in the sense that*

$$\lim_{t\to\infty} \tilde{x}(t), \tilde{y}(t), \tilde{\theta}(t) = 0 \tag{80}$$

where the position and orientation setpoint errors were defined in (7).

Proof: To prove Theorem 3, we take the time derivative of the non-negative, function given in (30), and then substitute (22), (27), and (18) in the resulting expression (where $A(z, v_r, t)$, $u_c(t)$, and k_2 are all equal to zero for the regulation problem) and follow the proof of Theorem 1 to obtain the following expression

$$\begin{aligned}\dot{V} \;\; = \;\; & w\left[u_a^T J\tilde{z} - k_1 w z_d^T z_d\right] + \tilde{z}^T\left[wJu_a - k_3\tilde{z}\right] \\ & + z_d^T\left[k_1(w^2 - z_d^T z_d)z_d + J\Omega_2 z_d\right].\end{aligned} \tag{81}$$

After utilizing the skew symmetry property of J defined in (9), making use of the fact that $J^T = -J$, and then cancelling common terms, we can upper bound $\dot{V}(t)$ of (81) as follows

$$\dot{V} = -k_1\|z_d\|^4 - k_3\|\tilde{z}\|^2. \tag{82}$$

Based on the same arguments as given for the proof of Theorem 1, we can show that all signals remain bounded during closed-loop operation, and that $\lim_{t\to\infty} z_d(t), \tilde{z}(t), z(t) = 0$. Since (30) is a positive, radially unbounded function with a negative semi-definite time derivative as shown in (82), we can also conclude that $\lim_{t\to\infty} V(w, z_d, \tilde{z}, t) = c_1$ where $c_1 \in \Re^1$ is a constant. Furthermore, since $\lim_{t\to\infty} z_d(t), \tilde{z}(t) = 0$, it is straightforward from (30) that $\lim_{t\to\infty} w^2 = c_2$ where $c_2 \in \Re^1$ is a non-negative constant.

In order to facilitate further analysis, we define the following non-negative function $v_0(z_d, t) \in \Re^1$ as follows

$$v_0 = \tfrac{1}{2}z_d^T z_d \qquad v_0(0) = \tfrac{\beta}{2} \tag{83}$$

where β was defined in (18). Based on (83) and the fact that $\lim_{t\to\infty} z_d(t) = 0$, it is straightforward that $\lim_{t\to\infty} v_0(t) = 0$; hence, for all $\varepsilon_1 > 0$,

$$v_0(t) < \varepsilon_1 \qquad \forall t > T_{o1}(\varepsilon_1) \tag{84}$$

where ε_1, $T_{o1}(\varepsilon_1) \in \Re^1$ are positive constants. After taking the time derivative of $v_0(t)$ given in (83), we have

$$\dot{v}_0 = -4k_1 v_0^2 + 2k_1 w^2 v_0 \tag{85}$$

where (18) and the skew symmetry property of J defined in (9) have been used. After dividing (85) by v_0^2 and then integrating the resulting equation, we have

$$\tfrac{1}{v_0(t)} - \tfrac{1}{v_0(0)} = \int_0^t 4k_1 d\sigma - \int_0^t \tfrac{2k_1 w^2(\sigma)}{v_0(\sigma)}d\sigma. \tag{86}$$

Based on the fact that

$$\int_0^t \tfrac{2k_1 w^2(\sigma)}{v_0(\sigma)}d\sigma \geq 0, \tag{87}$$

we can rearrange (86) to obtain the following upper bound for $v_0(t)$ as

$$v_0(t) \geq \frac{1}{\frac{1}{v_0(0)} + 4k_1 t}. \tag{88}$$

Now, we use (88), (84), the structure of (85), and the fact that $\lim_{t\to\infty} v_0(t) = 0$ to prove by contradiction that $\lim_{t\to\infty} w^2(t) =$

0. To facilitate the proof by contradiction, we assume that $\lim_{t\to\infty} w^2(t) = c_2 > 0$; hence, for all $\varepsilon_2 > 0$,

$$\left|w^2 - c_2\right| < \varepsilon_2 \qquad \forall t > T_{o2}(\varepsilon_2) \tag{89}$$

where ε_2, $T_{o2}(\varepsilon_2) \in \Re^1$ are positive constants. If we select $\varepsilon_1 = \frac{c_2}{8}$ and $\varepsilon_2 = \frac{c_2}{2}$, then from (88), (84), and (89), we have that

$$0 \leq v_0 < \tfrac{c_2}{8} \qquad \forall t > T_{01}(\tfrac{c_2}{8}) \tag{90}$$

and

$$\tfrac{c_2}{2} < w^2 < \tfrac{3c_2}{2} \qquad \forall t > T_{02}(\tfrac{c_2}{2}). \tag{91}$$

Furthermore, if we select $T_0 \in \Re^1$ as

$$T_0 = \max(T_{01}(\frac{c_2}{8}), T_{02}(\frac{c_2}{2})) \tag{92}$$

then from (85), (90), and (91), we can conclude that $\dot{v}_0(t)$ is non-negative as shown below

$$\dot{v}_0(t) \geq -4k_1 v_0 \left(\frac{c_2}{8}\right) + 2k_1 \left(\frac{c_2}{2}\right) v_0 = \frac{1}{2} k_1 c_2 v_0 \geq 0 \qquad \forall t > T_0. \tag{93}$$

Now, note that from (88), we have that

$$v_0(T_0) \geq \frac{1}{\frac{1}{v_0(0)} + 4k_1 T_0}. \tag{94}$$

In addition, we note that

$$v_0(t) = v_0(T_0) + \int_{T_0}^{t} \dot{v}_0(\sigma) d\sigma \tag{95}$$

which can be lower bounded as follows

$$v_0(t) \geq v_0(T_0) \tag{96}$$

as a result of (93). Based on (94) and (96), we can conclude that

$$v_0(t) \geq \frac{1}{\frac{1}{v_0(0)} + 4k_1 T_0}; \tag{97}$$

however, (97) is a contradiction to the fact that $\lim_{t\to\infty} v_0(t) = 0$. Since the assumption that $\lim_{t\to\infty} w^2(t) = c_2 > 0$ leads to a contradiction, we can conclude that $\lim_{t\to\infty} w^2(t) = 0$; hence, $\lim_{t\to\infty} w(t) = 0$. Finally, since $\lim_{t\to\infty} z_d(t), \tilde{z}(t), w(t) = 0$, the global asymptotic result given in (80) can now be directly obtained from (35). □

6 Conclusion

In this paper, we have presented a differentiable, kinematic control law for mobile robots. The proposed kinematic controller is novel in that: i) global exponential tracking was obtained provided certain PE conditions on the reference trajectory are satisfied, ii) a unified scheme was developed which solves both the global exponential tracking and the global asymptotic regulation problems, and iii) we have illustrated how the excitation of the reference trajectory can be used to improve the transient tracking performance. The proposed kinematic controller can be easily extended to the dynamic control problem (e.g., we can develop an adaptive controller that fosters global asymptotic tracking despite parametric uncertainty in the mobile robot dynamic model). It should also be noted that in addition to the WMR problem, the kinematic portion of the proposed controller can be applied to other nonholonomic systems (see [4] for examples). Future work will involve experimental comparisons with other kinematic controllers and the implementation of an adaptive controller.

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