Adaptive Robust Stabilization of Dynamic Nonholonomic Chained Systems

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Abstract

In this paper, the stabilization problem is investigated for dynamic nonholonomic systems with unknown inertia parameters and disturbances. Firstly, the nonholonomic kinematic subsystem is transformed into a skew-symmetric form and the properties of the overall systems are discussed. Then, a robust adaptive controller is presented where adaptive control technique is used to compensate for the parametric uncertainties and sliding mode control is to suppress the bounded disturbances. The controller guarantees the outputs of the dynamic subsystem to track some bounded auxiliary signals which subsequently drive the kinematic subsystem to the origin. Simulation study on the control of a unicycle wheeled mobile robot shows the effectiveness of the proposed approach.

1 Introduction

In recent years, control of chained form and of more general nonholonomic systems has been a very active field of research. Due to Brockett's theorem [3], it is well known that nonholonomic systems with restricted mobility cannot be stabilized to a desired configuration (or posture) via differentiable, or even continuous, pure-state feedback [1]. The design of stabilizing control laws for these systems is a challenging problem which has attracted an ever increasing attention in the control community. A number of approaches have been proposed for the problem, which can be classified as (i) discontinuous time-invariant stabilization [7, 13], (ii) time-varying stabilization [9] and (iii) hybrid stabilization [8].

Most research work on controller design for nonholonomic systems has being focused on the kinematic control problem, where the systems are represented by their kinematic models and velocity acts as the control input. In practice, it is more realistic to formulate the nonholonomic system control problem at the dynamic level, where the torque and force are taken as the control inputs. Different researchers have investigated this problem. Sliding mode control is applied to guarantee the uniform ultimate boundedness of tracking error in [12]. In [4], stable adaptive control is investigated for dynamic nonholonomic chained systems with uncertain constant parameters. Using geometric phase as a basis, control of Caplygin dynamical systems was studied in [1], and the closed-loop system was proved to achieve the desired local asymptotic stabilization of a single equilibrium solution. In [6], a robust sliding mode controller was presented to compensate for the parametric uncertainties and suppress the bounded disturbances in the stabilization of dynamic nonholonomic systems.

In this paper, the stabilization problem is considered for general dynamic nonholonomic systems where the nonholonomic kinematic subsystem can be converted into the chained form, the dynamic subsystem have unknown constant inertia parameters and bounded disturbances. A robust adaptive controller is presented where adaptive control is used to compensate for parametric uncertainties and sliding mode control is to suppress bounded disturbances. The controller guarantees the outputs of the dynamic subsystem (the inputs of the kinematic subsystem) to track some bounded auxiliary signals which subsequently drive the kinematic subsystem to the origin.

2 Dynamics of Nonholonomic Systems

In general, a nonholonomic system having an n-dimensional configuration space with generalized coordinates $q = [q_1, \ldots, q_n]^T$ and subject to $n-m$ constraints can be described by [2]

$$J(q)\dot{q} = 0 \tag{1}$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + \tau_d = B(q)\tau + J^T(q)\lambda \tag{2}$$

where $M(q) \in R^{n\times n}$ is the inertia matrix which is symmetric positive definite, $C(q,\dot{q}) \in R^{n\times n}$ is the centripetal and coriolis matrix, $G(q) \in R^n$ is the gravitation force vector, $B(q) \in R^{n\times r}$ is the input transformation matrix, $\tau \in R^r$ is the input vector of forces

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and torques, $J(q) \in R^{(n-m)\times n}$ is the matrix associated with the constraint, $\lambda \in R^{n-m}$ is the vector of constraint forces, and $\tau_d \in R^n$ denotes bounded unknown disturbances including unstructured unmodeled dynamics.

Since $J(q) \in R^{(n-m)\times n}$, it is always realizable to find an m rank matrix $S(q) \in R^{n\times m}$ formed by a set of smooth and linearly independent vector fields spanning the null space of $J(q)$, i.e. $S^T(q)J^T(q) = 0$. Since $S(q)$ is formed by a set of smooth and linearly independent vector fields spanning the null space of $J(q)$, define an auxiliary time function $v \in R^m$ such that

$$\dot{q} = S(q)v(t) \tag{3}$$

Equation (3) is also called the kinematic model of nonholonomic systems in the literature. Differentiating equation (3) yields

$$\ddot{q} = \dot{S}(q)v + S(q)\dot{v} \tag{4}$$

Substituting (3) and (4) into equation (2), and then premultiplying by $S^T(q)$, we have the transformed nonholonomic system

$$\dot{q} = S(q)v \tag{5}$$

$$M_1(q)\dot{v} + C_1(q,\dot{q})v + G_1(q) + \tau_{d1} = B_1(q)\tau \tag{6}$$

where $M_1(q) = S^T M(q)S$, $C_1(q,\dot{q}) = S^T(M(q)\dot{S} + C(q,\dot{q})S)$, $G_1(q) = S^T G(q)$, $B_1(q) = S^T B(q)$, $\tau_{d1} = S^T\tau_d$, which is more appropriate for controller design as the constraint λ has been eliminated from dynamic equation (6).

The nonholonomic chained system considered in this paper is m-input, $(m-1)$-chain, single-generator chained form given by [14]

$$\begin{array}{rcl} \dot{x}_1 & = & u_1 \\ \dot{x}_{j,i} & = & u_1 x_{j,i+1}, \quad 2 \le i \le n_j - 1,\ 1 \le j \le m-1 \\ \dot{x}_{j,n_j} & = & u_{j+1} \end{array} \tag{7}$$

Note that, in equation (7), $X = [x_1, X_2, \ldots, X_{m-1}]^T \in R^n$ with $X_j = [x_{j-1,2}, \ldots, x_{j-1,n_{j-1}}]$ $(2 \le j \le m)$ are the states and $u = [u_1, u_2, \ldots, u_m]^T$ are the inputs of the kinematic subsystem.

Assumption 1 *The kinematic model of nonholonomic system given by equation (5) can be converted into chained form (7) by some diffeomorphic state transformations.*

Assume the diffeomorphic state transformations $X = T_1(q)$ and $u = T_2^{-1}(q)v$ can convert equation (5) into the general chained form (7). Accordingly, equation (6) is converted into

$$M_2(X)\dot{u} + C_2(X,\dot{X})u + G_2(X) + \tau_{d2} = B_2(X)\tau \tag{8}$$

where $M_2(X) = T_2^T(q)M_1(q)T_2(q)|_{q=T_1^{-1}(X)}$, $C_2(X,\dot{X}) = T_2^T(q)(C_1(q,\dot{q})T_2 + M_1\dot{T}_2(q))|_{q=T_1^{-1}(X)}$, $G_2(X) = T_2^T(q)G_1(q)|_{q=T_1^{-1}(X)}$, $B_2(X) = T_2^T(q)B_1(q)|_{q=T_1^{-1}(X)}$, $\tau_{d2} = T_2^T(q)\tau_{d1}|_{q=T_1^{-1}(X)}$.

Next, let us further transform the chained form into skew-symmetric chained form for the convenience of controller design. This transformation is a simple extension of the transformation of the one-generation, two-inputs, single-chained system given by Samson [10]. By introducing the skew-symmetric chained form, and via a Lyapunov-like analysis, it is easy to design $U_2 = [u_2, \ldots, u_m]^T$ that globally stabilizes $[X_2, \ldots, X_m]^T$ to the origin, provided that $|u_1|$ and $|\dot{u}_1|$ are bounded, and $u_1(t)$ does not asymptotically tend to zero [10].

The kinematic model of chained form (7) can be written as $\dot{X} = h_1(X)u_1 + h_2 U_2$, where $h_1(X) = [1, x_{1,3}, \ldots, x_{1,n1}, 0, \ldots, x_{m-1,3}, \ldots, x_{m-1,n_{m-1}}, 0]^T$, and $h_2 = [0, \ldots, 0, 1, \ldots, 0, \ldots, 0, 1]^T$.

Let us consider the following coordinates change

$$\begin{array}{rcl} z_1 & = & x_1 \quad (9) \\ z_{j,2} & = & x_{j,2} \quad (10) \\ z_{j,3} & = & x_{j,3} \quad (11) \\ z_{j,i+3} & = & \rho_{j,i} z_{j,i+1} + L_{h1} z_{j,i+2} \quad (12) \end{array}$$

where $1 \le i \le n_j - 3, 1 \le j \le m-1$, $\rho_{j,i}$ are real positive numbers, and $L_{h1} z_{j,i} = \dfrac{\partial z_{j,i}}{\partial X} h_1(X)$ are the Lie derivatives of $z_{j,i}$ along $h_1(X)$, and the corresponding transformed skew-symmetric chained form system is

$$\begin{array}{rcl} \dot{z}_1 & = & u_1 \quad (13) \\ \dot{z}_{j,2} & = & u_1 z_{j,3} \quad (14) \\ \dot{z}_{j,i+3} & = & -\rho_{j,i+1} u_1 z_{j,i+2} + u_1 z_{j,i+4} \quad (15) \\ \dot{z}_{j,n_j} & = & L_{h1} z_{j,n_j} u_1 + u_{j+1} \quad (16) \end{array}$$

$$M_3(Z)\dot{u} + C_3(Z,\dot{Z})u + G_3(Z) + \tau_{d3} = B_3(Z)\tau \tag{17}$$

where $1 \le j \le m-1, 0 \le i \le n_j - 4$, $M_3(Z) = M_2(X)|_{X=\Psi^{-1}(Z)}$, $C_3(Z,\dot{Z}) = C_2(X,\dot{X})|_{X=\Psi^{-1}(Z)}$, $G_3(Z) = G_2(X)|_{X=\Psi^{-1}(Z)}$, $B_3(Z) = B_2(X)|_{X=\Psi^{-1}(Z)}$, $\tau_{d3} = \tau_{d2}$.

Property 1 *M_3 is symmetric positive definite and bounded. The boundedness of M_3 means that there exist positive scalars μ_1 and μ_2 such that $\mu_1 I \le M_3 \le \mu_2 I$.*

Property 2 *$\dot{M}_3 - 2C_3$ is skew-symmetric, i.e., $X^T(\dot{M}_3 - 2C_3)X = 0,\ \forall X \ne 0$.*

Property 3 *The dynamics can be expressed in the linear-in-parameters form*

$$M_3(Z)\dot{\xi} + C_3(Z,\dot{Z})\xi + G_3(Z) = \Phi(Z,\dot{Z},\xi,\dot{\xi})\theta \tag{18}$$

where $\Phi(Z,\dot{z},\xi,\dot{\xi}) \in R^{m\times m}$ is the known regressor matrix and $\theta \in R^m$ is the unknown parameters vector of system. For any physical system, we know that $||\theta||$ is always bounded.

Assumption 2 *$||\tau_{d3}||$ is bounded by a known scalar, i.e., $||\tau_{d3}|| < \tau_{max}$.*

Assumption 3 *$B(Z)$ is assumed known because it is a function of fixed geometry of the system. Accordingly, $B_3(Z)$ is assume to be known exactly for subsequent discussion.*

3 Controller Design

Consider the nonholonomic systems described by equations (16) and (17). An adaptive sliding mode controller is designed to stabilize the system states Z to the origin. Since $Z = \Psi X$ is of global diffeomorphism, the stabilization problem of X is the same as the stabilization problem of Z.

Define an auxiliary vector $u_d \in R^m$ as

$$u_d = \begin{bmatrix} -k_{u1}z_1 + h(Z_2, t) \\ r_1 - k_{u2}z_{1,n1} + b_2^T \Lambda e_w \\ \vdots \\ r_{m-1} - k_{um}z_{m-1,n_{m-1}} + b_m^T \Lambda e_w \end{bmatrix} \tag{19}$$

where $Z = [z_1, Z_2^T]^T$, k_{uj} $(1 \le j \le m)$, $r_j = -(\rho_{j,n_j-2}z_{j,n_j-1} + L_{h1}z_{j,n_j})u_1, (1 \le j \le m-1)$ with ρ_{j,n_j-2} being positive constants, Λ is a constant matrix and e_w is an error vector to be defined later, $b_j \in R^m$ $(2 \le j \le m)$ with its ith element defined as $b_{j,i} = 1, \forall i = j$ and $b_{j,i} = 0, \forall i \ne j$, and $h(Z_2, t)$ satisfies the following conditions as given in [10]:

Condition 1 *$h(Z_2, t)$ is a function of class $C^{p+1}(p \le 1)$, uniformly bounded with respect to t, with all successive partial derivatives also uniformly bounded with respect to t, and $h(0, t) = 0$.*

Condition 2 *There is a time-diverging sequence t_i, and a positive continuous function $\alpha(.)$, such that $||Z_2|| \le l > 0 \longrightarrow \sum_{j=1}^{j=p}(\frac{\partial^j h}{\partial t^j})^2 \le \alpha(l) > 0, \forall i \in N$, where N denote the set of the natural number.*

Control law τ and adaptive law of the parameters will be designed later to make the outputs of the dynamic subsystem (the inputs of the kinematic system) u tend to the auxiliary signals u_d. As it has been shown in [10], when u_1 tends to u_{d1}, $u_1 Z_2$ and $\dot{Z}_2$ converge to zero, the definition of $h(Z_2, t)$ will guarantee Z goes to zero as well.

Define the following signals $e_u = u - u_d$, $\dot{e}_w = e_u$, and $u_r = u_d - \Lambda e_w$. The switching function is selected as

$$s = \dot{e}_w + \Lambda e_w \tag{20}$$

where Λ is the constant matric whose eigenvalues are all in the right-half of complex plane.

In the presence of disturbance, adaptive control alone cannot guarantee the kinematic subsystem to track the auxiliary vector u_d exactly, as a consequence, stability of the whole system is not guaranteed [4]. To solve this problem, sliding mode control is introduced to suppress disturbance in order to guarantee the asymptotical stability of the closed-loop system. The parametric adaption is drived by the filter error s instead of output error e_u of the dynamic subsystem as in [4], the design of auxiliary vector u_d and controller τ is different and more involoved to compensate for the complex terms in stability analysis of the dynamic nonholonomic system.

From Property 3, we have

$$M_3(Z)\dot{u}_r + C_3(Z, \dot{Z})u_r + G_3(Z) = \Phi(Z, \dot{Z}, u_r, \dot{u}_r)\theta \tag{21}$$

where θ is the unknown parametric vector and Φ is the regressor matrix of known kinematic functions.

Combining (17) and (21), and noting $\dot{s} = \dot{u} - \dot{u}_r$, we have

$$M_3(Z)\dot{s} + C_3(Z, \dot{Z})s = B_3\tau - \tau_{d3} - \Phi(Z, \dot{Z}, u_r, \dot{u}_r)\theta \tag{22}$$

Consider robust control law defined by

$$\tau = B_3^+(Z)[\Phi\hat{\theta} - csgn(s) - \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|sgn(s)] \tag{23}$$

where $\hat{\theta}$ is the estimate of θ, $c = \tau_{max} + \varepsilon$ with $\varepsilon > 0$ and B_3^+ is the left inverse of B_3 defined as $B_3^+ = B_3^T(B_3B_3^T)^{-1}$.

Substituting (23) into (22), the closed-loop error equation is given by

$$\begin{aligned} M_3\dot{s} \;=\; & \Phi\tilde{\theta} - csgn(s) - \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|sgn(s) \\ & -C_3 s - \tau_{d3} \end{aligned} \tag{24}$$

Remark 1 *The term $\Phi\hat{\theta}$ is to solve the problem of parametric uncertainties using adaptive techniques. For the bounded disturbance τ_{d3}, it is suppressed by sliding mode control, $-csgn(s)$. The last term $-\sum_{j=1}^{m-1}\frac{1}{\prod_{i=1}^{n_j-2}\rho_{j,i}}|z_{j,n_j}|sgn(s)$ is necessary to compensate complex terms in stability proof of the dynamic nonholonomic system. Note that the last two terms in (23) will bring in chattering to stable system since discontinuous surfaces exist.*

Theorem 1 *For nonholonomic system described by (16) and (17), and control law (23). If the parameter estimates are updated as*

$$\dot{\hat{\theta}} = -\Gamma^{-1}\Phi^T s \tag{25}$$

where Γ is a symmetric positive define constant matrix, then Z is globally asymptotically stabilizable at the origin $Z = 0$.

Proof: For the convenience of proof, define the following two functions:

$$V_1(s,\tilde{\theta}) = \frac{1}{2}s^T M_3 s + \frac{1}{2}\tilde{\theta}^T \Gamma \tilde{\theta} \tag{26}$$

$$V_2(Z_2) = \sum_{j=1}^{m-1} \frac{1}{2}[z_{j,2}^2 + \frac{1}{\rho_{j,1}} z_{j,3}^2 + \ldots + \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} z_{j,n_j}^2] \tag{27}$$

where $\tilde{\theta} = \hat{\theta} - \theta$. Because θ is a constant vector, $\dot{\hat{\theta}} = \dot{\tilde{\theta}}$. The derivative of V_1 along equation (24) is given as

$$\begin{aligned}
\dot{V}_1 &= s^T M_3 \dot{s} + \frac{1}{2}s^T \dot{M}_3 s + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \\
&= s^T(\Phi\tilde{\theta} - \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} |z_{j,n_j}| sgn(s) - \tau_{d3} \\
&\quad -csgn(s)) + s^T(\frac{1}{2}\dot{M}_3 - C_3)s + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}}
\end{aligned} \tag{28}$$

Since $\dot{M}_3 - 2C_3$ is skew-symmetric, we have

$$\begin{aligned}
\dot{V}_1 &= s^T(\Phi\tilde{\theta} - \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} |z_{j,n_j}| sgn(s) \\
&\quad -csgn(s) - \tau_{d3}) + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}}
\end{aligned} \tag{29}$$

The time derivative of V_2 is given by

$$\begin{aligned}
\dot{V}_2(Z_2) &= \sum_{j=1}^{m-1} [z_{j,2}\dot{z}_{j,2} + \frac{1}{\rho_{j,1}} z_{j,3}\dot{z}_{j,3} + \ldots \\
&\quad + \frac{1}{\prod_{i=1}^{nj-2} \rho_{j,i}} z_{j,n_j}\dot{z}_{j,n_j}]
\end{aligned} \tag{30}$$

Substituting (16) into (30), we have

$$\begin{aligned}
\dot{V}_2 &= \sum_{j=1}^{m-1} [\frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} z_{j,n_j}((\rho_{j,n_j-2} z_{j,n_j-1} \\
&\quad + L_{h1} z_{j,n_j}) u_1 + u_{j+1})]
\end{aligned} \tag{31}$$

Combining (28) and (31) yields

$$\begin{aligned}
\dot{V} &= s^T(\Phi\tilde{\theta} - \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} |z_{j,n_j}| sgn(s) \\
&\quad -csgn(s) - \tau_{d3}) + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \\
&\quad + \sum_{j=1}^{m-1} [\frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} z_{j,n_j}((\rho_{j,n_j-2} z_{j,n_j-1} \\
&\quad + L_{h1} z_{j,n_j}) u_1 + u_{j+1})]
\end{aligned} \tag{32}$$

By adding $u_{d_{j+1}} - u_{d_{j+1}}$, equation (32) becomes

$$\begin{aligned}
\dot{V} &= s^T(\Phi\tilde{\theta} - \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} |z_{j,n_j}| sgn(s) \\
&\quad -csgn(s) - \tau_{d3}) + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \\
&\quad + \sum_{j=1}^{m-1} [\frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} z_{j,n_j}((\rho_{j,n_j-2} z_{j,n_j-1} \\
&\quad + L_{h1} z_{j,n_j}) u_1 + u_{d_{j-1}} + u_{j+1} - u_{d_{j-1}})]
\end{aligned} \tag{33}$$

Noting that $\dot{e}_{w_{j+1}} = u_{j+1} - u_{d_{j-1}}$, and substituting (19) into equation (33), we have

$$\begin{aligned}
\dot{V} &= s^T[\Phi\tilde{\theta} - \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} |z_{j,n_j}| sgn(s) - \tau_{d3} \\
&\quad -csgn(s)] + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} + \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} z_{j,n_j} \\
&\quad [\dot{e}_{w_{j+1}} - k_{u_{j+1}} z_{j,n_j} + b_{j+1}^T \Lambda e_w] \\
&= (\tilde{\theta}^T \Phi^T s + \tilde{\theta}^T \Gamma \dot{\hat{\theta}}) - \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} k_{u_{j+1}} z_{j,n_j}^2 \\
&\quad -s^T(csgn(s) + \tau_{d3}) - \sum_{j=1}^{m-1} s^T \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} |z_{j,n_j}| sgn(s) \\
&\quad + \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} z_{j,n_j} (\dot{e}_{w_{j+1}} + b_{j+1}^T \Lambda e_w)
\end{aligned} \tag{34}$$

Substituting adaptive law (25) into (34) yields

$$\begin{aligned}
\dot{V} &= -\sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} k_{u_{j+1}} z_{j,n_j}^2 - s^T(csgn(s) + \tau_{d3}) \\
&\quad - \sum_{j=1}^{m-1} s^T \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} |z_{j,n_j}| sgn(s) \\
&\quad + \sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} z_{j,n_j} (\dot{e}_{w_{j+1}} + b_{j+1}^T \Lambda e_w)
\end{aligned} \tag{35}$$

From the definition of (20), we have $\dot{e}_{w_{j+1}} + b_{j+1}^T \Lambda e_w = s^T b_{j+1}$ which lead to

$$\begin{aligned}
\dot{V} &= -\sum_{j=1}^{m-1} \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} k_{u_{j+1}} z_{j,n_j}^2 - s^T(csgn(s) + \tau_{d3}) \\
&\quad + \sum_{j=1}^{m-1} s^T \frac{1}{\prod_{i=1}^{n_j-2} \rho_{j,i}} (-|z_{j,n_j}| sgn(s) + z_{j,n_j} b_{j+1})
\end{aligned}$$

Since $s^T(-|z_{j,n_j}| sgn(s) + z_{j,n_j} b_{j+1}) \leq 0$ and $-s^T(csgn(s) + \tau_{d3}) = -(\tau_{max} s^T sgn(s) + s^T \tau_{d3}) - \varepsilon s^T sgn(s) \leq -\varepsilon s^T sgn(s) \leq 0$, $\dot{V} \leq 0$ is obtained. Accordingly, s and Z_2 are bounded in the sense of Lyapunov. Then $s \in L_1^m \cap L_\infty^m$. From equation (24), since $C_3 s$, $B_3\tau - \Psi\theta$ and τ_{d3} are all bounded, $\dot{s} \in L_\infty^m$. Using the fact that $s \in L_1^m \cap L_\infty^m$ and $\dot{s} \in L_\infty^m$, then from Corollary of Baralart's theory [5], s tends to zero as $t \to \infty$. For all the eigenvalues of matrix Λ are in the right-half complex plane, e_w and e_u will go to zero when $t \to \infty$. The input vector u of the kinematic subsystem will converge to the auxiliary vector u_d as $t \to \infty$.

Next, let us prove the asymptotic stability of the closed-loop system. The first equation of the controlled system is $\dot{z}_1 = -k_{u1}z_1 + h(Z_2,t) + e_{u1}$. From Condition 1 of $h(Z_2,t)$, $h(Z_2,t)$ is uniformly bounded. In addition, with e_{u1} tend to zero, the above equation is a stable linear system subjected to the bounded additive perturbation $h(Z_2,t) + e_{u1}$. Therefore, $z_1(t)$ is also bounded uniformly.

Because z_1 and $h(Z_2,t)$ are bounded, it is clear that u_{d1} is bounded from (19). Together with e_u and e_w converge to zero , u_1 is bounded. Since u_1 and Z_2 are bounded, e_u and e_w go to zero, u_{dj} and u_j ($2 \leq j \leq m$) are bounded. Under condition that Z_2, u_1 and u_j ($2 \leq j \leq m$) are bounded, it is clear that $\dot{z}_{j,n_j}$ and $\dot{z}_{j,i}$, ($1 \leq j \leq m-1, 2 \leq u \leq n_j - 1$) are bounded.

In the following, let us show that $u_{d1}Z_2$ tends to zero. For $1 \leq j \leq m-1$, consider $\dot{z}_{j,n_j} = L_{h1}z_{j,n_j}u_1 + u_{j+1} = -\rho_{j,n_j-2}z_{j,n_j-1}u_{d1} + (-k_{u2}z_{j,n_j} - g_{j+1}^T\Lambda e_w + e_{u_{j+1}} - \rho_{j,n_j-2}z_{j,n_j-1}e_{u1})$. Since $\dot{z}_{j,n_j-1}$, u_{d1}, z_{j,n_j-1}, $\dot{u}_{d1}$ are all bounded, $u_{d1}z_{j,n_j-1}$ is uniformly continuous. For z_{j,n_j}, e_u and e_w tend to zero, the rest part of $\dot{z}_{j,n_j}$ tend to zero. From the extended version of Barbalat's Lemma [10], $u_{d1}z_{j,n_j-1}$ tend to zero.

For 3$\leq i \leq n_j - 1$ and 1$\leq j \leq m-1$, consider

$$\begin{aligned}\frac{d}{dt}(u_{d1}^2 z_{j,i}) &= u_{d1}^2(-\rho_{j,i-2}u_1 z_{j,i-1} + u_1 z_{j,i+1}) \\ &\quad + z_{j,i}\frac{d}{dt}(u_{d1})^2 \\ &= -\rho_{j,i-2}u_{d1}^3 z_{j,i-1} + (-\rho_{j,i-2}u_{d1}^2 e_{u1} z_{j,i-1} \\ &\quad + u_{d1}^2 u_1 z_{j,i+1} + z_{j,i}\frac{d}{dt}(u_{d1})^2)\end{aligned}$$

Because $\frac{d}{dt}u_{d1}^3 z_{j,i-1} = u_{d1}^3 \ \dot{z}_{j,i-1} + 3u_{d1}^2\dot{u}_{d1}z_{j,i-1}$ and $\dot{z}_{j,i-1}$, u_{d1}, $z_{j,i-1}$, $\dot{u}_{d1}$ are all bounded, $z_{j,i-1}u_{d1}^3$ is uniformly continuous. Since e_{u1}, $z_{j,i+1}$ and $z_{j,i}$ tend to zero, the rest items of $\frac{d}{dt}(u_{d1}^2 z_{j,i})$ tend to zero. Due to the extended version of Barbalat's Lemma [10], we have $u_{d1}z_{j,i} \to 0$ when $t \to \infty$.

From the above analysis, we know that $u_{d1}Z_2$ tend to zero. Because $u_{d1}Z_2$, z_{j,n_j}, e_u and e_w tend to zero, $u_{dj}(2 \leq j \leq m)$ tend to zero, together with $u_{d1}Z_2$ and e_u tend to zero, $\dot{Z}_2$ tend to zero as well. Since e_u tends to zero, $\dot{z}_1$ tends to u_{d1}, together with $u_{d1}Z_2$ and $\dot{Z}_2$ tend to zero, it is proved that for the skew-symmetric form and definition of u_{d1}, $Z \to 0$ as $t \to \infty$ [10]. Then the feedback control law (23) asymptotically stabilize Z to zero, the theorem is proved. **Q.E.D.**

4 Simulation

Consider a unicycle wheeled mobile robot moving on a horizontal plane, whose dynamics are given by

$$\dot{x} = v_1 cos\theta,\ \dot{y} = v_1 sin\theta,\ \dot{\theta} = v_2 \tag{36}$$

$$M_1(q)\dot{v} + C_1(q)v + G_1 + \tau_{d1} = B_1\tau \tag{37}$$

where $M_1 = [m_0,\ 0;\ 0,\ I_0]$, $C_1 = 0$, $G_1 = 0$, $B_1 = 1/R[1,\ 1;\ L, -L]$, $v = [v_1, v_2]^T$ with v_1 and v_2 denoting the linear and angular velocity respectively, R is the radius of the wheels and $2L$ is the length of the axis of the fixed wheels. Using the following change of coordinates $X = T_1(q)$ [15], i.e.,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ sin\theta & -cos\theta & 0 \\ cos\theta & sin\theta & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

together with the change of input $u = T_2^{-1}(q)v$, i.e., $u_1 = v_2$, $u_2 = v_1 - v_2 x_2$, and the transform matrix $\Psi = I$ in this special case, system (36) and (37) is converted to

$$\begin{array}{rcl}\dot{z}_1 & = & u_1 \\ \dot{z}_2 & = & z_3 u_1 \\ \dot{z}_3 & = & u_2 \\ M_3(Z)\dot{u} & + & C_3(Z,\dot{Z})u + G_3(Z) + \tau_{d3} = B_3\tau\end{array} \tag{38}$$

where $M_3(Z) = [z_2^2 m_0 + I_0, z_2 m_0;\ z_2 m_0, m_0]$, $C_3(Z,\dot{Z}) = [z_2\dot{z}_2 m_0, 0;\ m_0\dot{z}_2, 0]$, $B_3(Z) = 1/R[z_2 + L, z_2 - L;\ 1, 1]$, $G_3 = 0$, and

$$M_3(Z)\dot{u}_r + C_3(Z,\dot{Z})u_r = \Phi(Z,\dot{Z},u_r,\dot{u}_r)\theta$$

where the inertia parameter vector $\theta = [m_0, I_0]^T$ and

$$\Phi(Z,\dot{Z},u_r,\dot{u}_r) = \begin{bmatrix} z_2^2\dot{u}_{r1} + z_2\dot{u}_{r2} + z_2\dot{z}_2 u_{r1} & \dot{u}_{r1} \\ z_2\dot{u}_{r1} + \dot{u}_{r2} + \dot{z}_2 u_{r1} & 0 \end{bmatrix} \tag{39}$$

The auxiliary vector in the control is

$$u_d = \begin{bmatrix} -k_{u1}z_1 + h(Z_2,t) \\ -\rho_1 z_2 u_1 - k_{u2}z_3 + \Lambda_{21}e_{w1} + \Lambda_{22}e_{w2} \end{bmatrix}$$

where $h(Z_2,t) = (z_2^2 + z_3^2)sint$. It is easy to see that the selected $h(Z_2,t)$ satisfy the requested Conditions 1 and 2.

In the simulation, the parameters of the system are assumed to be $m_0 = I_0 = 1.0$, $R = 0.1$, $L = 1.0$, $k_{u1} = 0.2$, $k_{u2} = 1.0$, $\hat{\theta} = [0.5, 0.5]^T$, $\rho_1 = 1.0$, $c = 0.5$, $\Lambda = I$, $\Gamma = 10I$ and disturbances τ_{d3_1} and τ_{d3_2} are random number in the range [-0.1,0.1]. Simulation results are shown in Figure 1 and 2. From Figure 1, we can see that the responses of states x_1, x_2 and x_3 of the chained form asymptotically tend to zero. From Figure 2, the control sequence τ_1 and τ_2 tend to zero as well. As shown in Figure 3, the estimate of the parameters $\hat{m}_0$ and $\hat{I}_0$ are all bounded. The results of the simulation verify the validity of proposed algorithm.

5 Conclusion

In this paper, stabilization of dynamic chained systems has been investigated with unknown constant inertia parameters and disturbances. Feedback control design and analysis have been performed via explicit Lyapunov techniques which apply naturally once the original chained form has been transformed into an equivalent, skew-symmetric chained form. The proposed scheme has also been used to control an unicycle wheeled mobile robot to show its effectivity.

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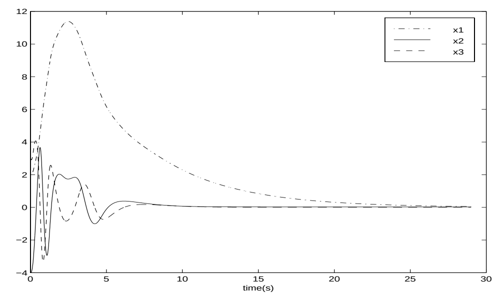

Figure 1: Responses of state x_1, x_2 and x_3.

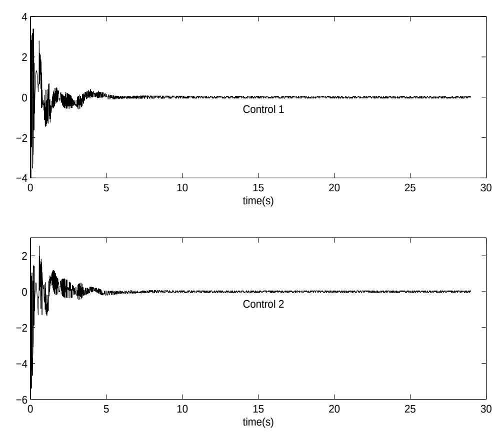

Figure 2: Control signals τ_1 and τ_2.

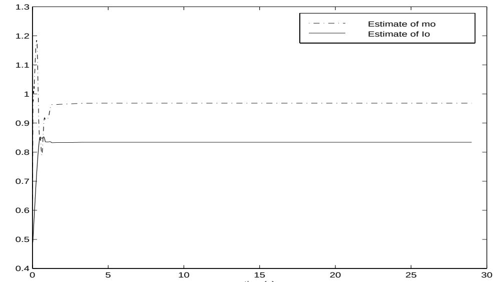

Figure 3: Response of the estimated parameter $\hat{\theta}$.