Adaptive Control of Mechanical Systems with Classical Nonholonomic Constraints

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Abstract

The position/force control of mechanical systems subject to a set of classical nonholonomic constraints represents an important class of control problems. In this paper a reduced dynamic model, suitable for simultaneous independent motion and force control, is developed by exploiting the physical structure of the systems. Some properties of the dynamic model are obtained to facilitate the controller design. An adaptive control algorithm is therefore derived. Stability analysis shows the stabilization of the manifold.

1 Introduction

The control of mechanical systems with kinematic constraints has received increasing attention and is a topic of great interest. A lot of papers have been published in recent years to deal with the control problem when the kinematic constraints are holonomic constraints [1]-[4]. In contrast, if the kinematic constraints are nonholonomic, control laws developed for holonomic constraints are not applicable; only a few papers have been proposed to address these control issues. In this paper, our discussions are focused on the classical nonholomonic case, and analyses are given from the Lagrangian point of view. As for the Hamiltonian case with other forms of nonholonomic constraints, the reader may refer to [12].

It is well known that in rolling or cutting motions the kinematic constraint equations are classical nonholonomic [10], and the dynamics of such systems is well understood (see, *e.g.* [10]). However, the literature on control with classical nonholonomic constraints is quite recent [5] [7] [8], and the discussion mainly focus on some special examples [11] [13] [14] [15]. Earlier work, which deals with control of nonholonomic systems, is described in [9]. Bloch and McClamroch [5], Bloch *et al.* [7], and Campion *et al.* [8] demonstrated that systems with nonholonomic constraints are always controllable, but cannot be feedback stabilized to a single point with smooth feedback. By using a decomposition transformation and nonlinear feedback, conditions for smooth asymptotic stabilization to an equilibrium manifold are established. d'Andrea-Novel *et al.* [11] and Yun *et al.* [13] showed that the system is linearizable by choosing a proper set of output equations, and then applied respectively their results to the control of wheeled mobile robots and multiple arms. Researchers have also offered both nonsmooth feedback laws [6] [7] [14] and time-varying feedback laws [15] for stabilizing the system to a point. However, it is fair to say that the last two approaches are not yet fully general.

The above mentioned approaches, *e.g.* [5] [7] [8], indeed provide a theoretic framework which can serve as a basis for the study of mechanical systems with nonholonomic constraints; however, all of those results are based on the method of a diffeomorphism and nonlinear feedback (for details, see [16]), which requires a detailed dynamic model and may be sensitive to parametric uncertainties.

In this paper we do not aim at linearizing the system dynamics as in [7], which neglects the physical structure of the system, but instead exploit the physical structure of the systems to develop the robust control law. By assuming complete knowledge of the constraint manifold, and recognizing that the degrees of freedom of mechanical systems decrease due to nonholonomic constraints, a reduced-form equation suitable for motion and force control is derived. Then by exploiting the particular structure of the reduced dynamics, several fundamental properties are obtained to facilitate the controller design. Finally, with the specification of a desired manifold, an adaptive control algorithm is derived, using only the measurements of joint position, velocity, and constraint force. Stability analysis shows the stabilization of the manifold.

2 Model of Robotic System with Classical Nonholonomic Constraints

In this section, we are concerned with mechanical systems whose configuration space is an n-dimensional simply connected manifold $\Re$, and whose dynamics are described, in local coordinates (termed generalized coordinates), by so

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called Euler-Lagrangian formulation as [8][11]

$$D(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) = \mathbf{f} + B(\mathbf{q})\mathbf{u} \tag{1}$$

where $\mathbf{q}$ denotes the n-vector of generalized coordinates; $\mathbf{u}$ denotes the r-vector of generalized control input force; $\mathbf{f}$ denotes the n-vector of constraint forces; $D(\mathbf{q})$ is the $(n \times n)$ symmetric, bounded, positive definite inertia matrix; $C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$ presents the n-vector of centripetal and Coriolis torques; $G(\mathbf{q})$ is the n-vector of gravitational torques; $B(\mathbf{q})$ is an $(n \times r)$ input transformation matrix.

Two simplifying properties should be noted about this dynamic structure.

Property 1: There exists a p-vector α with components depending on mechanical parameters (masses, moments of inertia, etc.), such that [17]

$$D(\mathbf{q})\dot{\mathbf{v}} + C(\mathbf{q}, \dot{\mathbf{q}})\mathbf{v} + G(\mathbf{q}) = \Phi(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{v}, \dot{\mathbf{v}})\alpha \tag{2}$$

where Φ is a $n \times p$ matrix of known functions of $\mathbf{q}$, $\dot{\mathbf{q}}$, $\mathbf{v}$, $\dot{\mathbf{v}}$; and α is the p-vector of inertia parameters [18].

Property 2: A suitable definition of $C(\mathbf{q}, \dot{\mathbf{q}})$ makes the matrix $(\dot{D} - 2C)$ skew-symmetric [17]. In particular, this is true if the elements of $C(\mathbf{q}, \dot{\mathbf{q}})$ are defined as in [8]

$$C_{ij} = \frac{1}{2}[\dot{\mathbf{q}}^T \frac{\partial D_{ij}}{\partial \mathbf{q}} + \sum_{k=1}^{n}(\frac{\partial D_{ik}}{\partial q_j} - \frac{\partial D_{jk}}{\partial q_i})\dot{q}_k] \tag{3}$$

It should be noted that the first property says that the Lagrangian dynamic equation are linearly parameterizable and the second property is related to the passivity of the mechanical dynamics.

Let consider the situation where kinematic constraints are imposed, which are described by [5] [7] [8] [10] [11] [13]

$$J(\mathbf{q})\dot{\mathbf{q}} = 0 \tag{4}$$

where $J(\mathbf{q})$ is an $(m \times n)$ constraint matrix which is assumed to have full rank m.

The constraint equations (4) are assumed to be classical nonholonomic constraints. Such constraints can arise in many cases, including the case when two surfaces roll against each other [10] [13]. The classical constraints are assumed not integrable. Nonintegrable constraints cannot be reduced to geometric constraints while integrable constraints are essentially geometric constraints (see Neimark and Fufaev [10] for the detailed explanation).

The effect of the constraints can be reviewed as restricting the dynamics to the manifold Ω defined by

$$\Omega = \{(\mathbf{q}, \dot{\mathbf{q}}) | J(\mathbf{q})\dot{\mathbf{q}} = 0\}.$$

It should be noted that since the constraints are nonintegrable, there is, in fact, no explicit restriction on the values of the variables $\mathbf{q}$.

When the nonholonomic constraints (4) are imposed on the mechanical systems (1), the constraint (generalized reaction) forces are given by

$$\mathbf{f} = J^T(\mathbf{q})\lambda \tag{5}$$

where $\lambda \in R^m$ is the associated Lagrangian multipliers [5] [7] [8] [10].

In the following, we denote the constraint matrix $J(\mathbf{q})$ as

$$J^T(\mathbf{q}) = [J_1(\mathbf{q}), \ \dots, J_m(\mathbf{q})].$$

where $J_1, \ \dots, J_m$ are smooth n-dimensional covector fields on $\Re$. Then, the annihilator of the codistribution spanned by the covector fields $J_1, \ \dots, J_m$ is an $(n\text{-}m)$-dimensional smooth nonsingular distribution $\triangle$ on $\Re$. This distribution $\triangle$ is spanned by a set of $(n\text{-}m)$ smooth vector fields $\mathbf{r}_1, \ \dots, \mathbf{r}_{n-m}$:

$$\triangle = span\{\mathbf{r}_1(\mathbf{q}), \ \dots, \mathbf{r}_{n-m}(\mathbf{q})\}$$

which satisfy, in local coordinates, the following relations [8]

$$R^T(\mathbf{q})J^T(\mathbf{q}) = 0 \tag{6}$$

where the full rank matrix $R(\mathbf{q})$ is made up of the vector function $\mathbf{r}_i(\mathbf{q})$:

$$R(\mathbf{q}) = [\mathbf{r}_1(\mathbf{q}), \ \dots, \mathbf{r}_{n-m}(\mathbf{q})].$$

The constraints (4) and (6) imply the existence of a $(n\text{-}m)$-vector $\dot{\mathbf{z}}$ such that

$$\dot{\mathbf{q}} = R(\mathbf{q})\dot{\mathbf{z}}. \tag{7}$$

It should be noted that the $(n\text{-}m)$-vector $\dot{\mathbf{z}}$ represents internal states, so that $(\mathbf{q}, \dot{\mathbf{z}})$ is sufficient to describe the constrained motion.

Differentiating equation (7), we obtain

$$\ddot{\mathbf{q}} = R\ddot{\mathbf{z}} + \dot{R}\dot{\mathbf{z}}. \tag{8}$$

Therefore, the dynamic equation (1), when satisfying the nonholonomic constraint (4), can be rewritten in terms of the internal state variables $\dot{\mathbf{z}}$ as

$$D(\mathbf{q})R(\mathbf{q})\ddot{\mathbf{z}} + C_1(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{z}} + G(\mathbf{q}) = B(\mathbf{q})\mathbf{u} + J^T(\mathbf{q})\lambda \tag{9}$$

where $C_1(\mathbf{q}, \dot{\mathbf{q}}) = D(\mathbf{q})\dot{R}(\mathbf{q}) + C(\mathbf{q}, \dot{\mathbf{q}})R(\mathbf{q})$.

It should be noted that reduced state space is *2n-m* dimensional. The system is described by the n-vector of variables $\mathbf{q}$ and the (n-m)-vector of variables $\dot{\mathbf{z}}$.

Remark: Equation (9) is suitable for control purposes and forms the basis for the subsequent development. This is because the equality constraint equation (4) are embedded into the dynamic equation, resulting in an affine nonlinear system without constraints.

By exploiting the structure of the equation (9), three properties are obtained.

Property 3: The generalized inertia matrix $R^T D(\mathbf{q})R$ is symmetric and positive definite.

Property 4: Define $D_1(\mathbf{q}) = R^T D(\mathbf{q})R$. If $C(\mathbf{q}, \dot{\mathbf{q}})$ is defined as that Property 2 is verified, $(\dot{D}_1 - 2R^T C_1)$ is a skew symmetric matrix.

Proof: Directly, by using the definition of $\dot{D}_1$ and C_1 and by considering the skew symmetry of $(\dot{D} - 2C)$ in property 2.

Property 5: The dynamic structure (9) is linear in terms of the same suitably selected set of inertia parameters as used in Property 1

$$D(\mathbf{q})R(\mathbf{q})\ddot{\mathbf{z}} + C_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{z}} + G(\mathbf{q}) = \Phi_1(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{z}},\ddot{\mathbf{z}})\alpha \quad (10)$$

where Φ_1 is a $(n\times p)$ regressor matrix; α is the p-vector of inertia parameters.

Property 5 may be easily understood by observing that the transformations do not change the linearity in terms of constant parameters α, established for model (1) by Property 1.

The above properties are fundamental for designing the robust force/motion control law.

3 Controller Design for Motion/Force Tracking

It has been proved (see [5] [7] [8]) that the nonholonomic system (1) and (4) cannot be stabilized to a single point using smooth state feedback. It can only be stabilized to a manifold of dimension m due to the existence of m nonholonomic constraints. The objective of stabilizing these systems to a point has been achieved by non-smooth feedback law [6] [7] [14] and time-varying feedback laws [15]. However, it is fair to say that these approaches are not yet fully general. It is worth mentioning that different control objectives may also be pursued, such as stabilization to manifolds of equilibrium points (as opposed to a single equilibrium position) or to trajectories (as long as they do not converge to a point).

By appropriately selecting a set of (n-m)-vector of variables $\mathbf{z}(\mathbf{q})$ and $\dot{\mathbf{z}}(\mathbf{q})$, the objective of the control can be specified as: given a desired $\mathbf{z}_d$, $\dot{\mathbf{z}}_d$, and desired constraint λ_d, determine a control law such that for any $(\mathbf{q}(0),\dot{\mathbf{q}}(0)) \in \Omega$ then $\mathbf{z}(\mathbf{q})$, $\dot{\mathbf{q}}$, and λ asymptotically converge to a manifold Ω_d specified as

$$\Omega_d = \{(\mathbf{q},\dot{\mathbf{q}},\lambda)|\mathbf{z}(\mathbf{q}) = \mathbf{z}_d, \dot{\mathbf{q}} = R(\mathbf{q})\dot{\mathbf{z}}_d, \mathbf{f} = \mathbf{f}_d\}$$

The variables $\mathbf{z}(\mathbf{q})$ can be thought as *n-m* 'output equations' of the nonholonomic system. The choice of $\mathbf{z}(\mathbf{q})$ is, as an example, illustrated in the next section.

Remark: If $\mathbf{z}_d$, $\dot{\mathbf{z}}_d$ are zero and $\mathbf{f}_d$ free, then $\Omega_d = \{(\mathbf{q},\dot{\mathbf{q}})|\mathbf{z}(\mathbf{q}) = 0, \dot{\mathbf{q}} = 0\}$ is the equilibrium manifold defined in [7]. If $\mathbf{q} = [(\mathbf{q}^1)^T, (\mathbf{q}^2)^T]$ is a partition, where $\mathbf{q}^1 \in R^{n-m}$, $\mathbf{q}^2 \in R^m$, and $\mathbf{z}(\mathbf{q}) = \mathbf{q}^1$, $\mathbf{z}_d = \dot{\mathbf{z}}_d = 0$, and $\mathbf{f}_d$ free, then $\Omega_d = \{(\mathbf{q},\dot{\mathbf{q}})|\mathbf{q}^1 = 0, \dot{\mathbf{q}}^1 = 0, \dot{\mathbf{q}}^2 = 0\}$ is the equilibrium manifold in [5]. If $\dot{\mathbf{z}}(\mathbf{q}) = R^T\mathbf{q}$, $\dot{\mathbf{z}}_d = 0$, and λ_d free, then $\Omega_d = \{(\mathbf{q},\dot{\mathbf{q}})|R^T\mathbf{q} = 0\}$ is the invariant set in [8]. Therefore, compared with [5] [7] [8], we extend the stabilization problem to the tracking problem, including tracking of the contact force $\mathbf{f}$.

In the following, we define

$$\mathbf{e}_z = \mathbf{z} - \mathbf{z}_d \quad (11)$$

$$\mathbf{e}_\lambda = \lambda - \lambda_d \quad (12)$$

$$\dot{\mathbf{z}}_r = \dot{\mathbf{z}}_d - \Lambda\mathbf{e}_z \quad (13)$$

where Λ is a positive definite matrix whose eigenvalues are strictly in the right-hand complex plane.

Defining α as a p-vector, containing the unknown elements in the suitably selected set of equivalent dynamic parameters, then the linear parameterizability of the dynamics (Property 5) leads to

$$D(\mathbf{q})R(\mathbf{q})\ddot{\mathbf{z}}_r + C_1(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{z}}_r + G(\mathbf{q}) = \Phi_1(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{z}}_r,\ddot{\mathbf{z}}_r)\alpha \quad (14)$$

where Φ_1 is the $(n\times p)$ regressor matrix.

An adaptive control law is defined as

$$B\mathbf{u} = \Phi_1(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{z}}_r,\ddot{\mathbf{z}}_r)\hat{\alpha} - KR\mathbf{s} - J^T\lambda_c \quad (15)$$

$$\dot{\hat{\alpha}} = -\Gamma\Phi_1^T R\mathbf{s} \quad (16)$$

where Φ_1 is defined in (14); $\hat{\alpha}$ is an estimate of α; R is defined in (6); K and Γ are positive definite matrices; the vector s, which can be thought of as a sliding surface, is defined as

$$\mathbf{s} = \dot{\mathbf{e}}_z + \Lambda\mathbf{e}_z; \quad (17)$$

the force term λ_c is defined as

$$\lambda_c = \lambda_d - K_\lambda\mathbf{e}_\lambda; \quad (18)$$

where K_λ is a positive definite diagonal matrix of force control feedback gains.

The above controller consists of two parts. The first part provides the input torques for achieving desired 'output' and internal state tracking. The second part provides the desired force tracking.

The following theorem can be stated:

Theorem: Consider the mechanical system described by (1) and (4), using the control law (15) and (16), then the following holds for any $(\mathbf{q}(0),\dot{\mathbf{q}}(0)) \in \Omega$:

i): $\mathbf{e}_z \in L_2^{n-m} \cap L_\infty^{n-m}$ and $\dot{\mathbf{e}}_z \in L_2^{n-m}$, $\mathbf{e}_z \to 0$ as $t \to \infty$.

ii): If $\ddot{\mathbf{z}}_d$, $\dot{\mathbf{z}}_d$, $\mathbf{z}_d$, and $\mathbf{f}_d$ are bounded functions and R and $\dot{R}$ are bounded matrices, then $\mathbf{f} - \mathbf{f}_d$ is bounded and inversely proportional to the norm of the matrix $(K_\lambda + I)$.

Proof: Based on equation (17), using equations (9), (14), (15), and after some calculations, the following is obtained:

$$DR\dot{\mathbf{s}} = \Phi_1\tilde{\alpha} - KR\mathbf{s} - C_1\mathbf{s} - J^T(\lambda_c - \lambda), \quad (19)$$

where $\tilde{\alpha} = \hat{\alpha} - \alpha$. According to equation (6), the above equation becomes

$$D_1\dot{\mathbf{s}} = R^T\Phi_1\tilde{\alpha} - R^TKR\mathbf{s} - R^TC_1\mathbf{s}. \quad (20)$$

Thus, we define a positive function for system (20) as

$$V = \frac{1}{2}(\mathbf{s}^TD_1\mathbf{s} + \tilde{\alpha}^T\Gamma^{-1}\tilde{\alpha}) \quad (21)$$

A simple calculation shows that along solutions of (20)

$$\begin{aligned} \dot{V} &= \mathbf{s}^T(R^T\Phi_1\tilde{\alpha} - R^TKR\mathbf{s}) + \mathbf{s}^T(\frac{1}{2}\dot{D}_1 - R^TC_1)\mathbf{s} \\ &\quad + \tilde{\alpha}^T\Gamma^{-1}\dot{\tilde{\alpha}} \\ &= -\mathbf{s}^TR^TKR\mathbf{s} \le 0 \end{aligned} \quad (22)$$

where we have used Property 6 to eliminate the term $s^T(\frac{1}{2}\dot{D}_1 - C_1)s$. This shows that $s \in L_2^{n-m} \cap L_\infty^{n-m}$ and $\tilde{\alpha} \in L_\infty^p$. This in turn implies, using (17), $e_z \in L_2^{n-m} \cap L_\infty^{n-m}$ and $\dot{e}_z \in L_2^{n-m}$, $e_z \to 0$ as $t \to \infty$ by resorting to the following lemma:

Lemma: Let the closed-loop transfer function $H(s)$ be exponentially stable and strictly proper. Then if the control $u \in L_2$, then the output $y = H(t) * u \in L_2 \cap L_\infty$, and $y \in L_2$, y is continuous and $y(t) \to 0$ as $t \to \infty$ where $H(t)$ is the corresponding impulse response of $H(s)$ and * is the convolution operator.

Hence i) of Theorem has been proven.

Since s, e_z, and $\dot{e}_z$ are bounded, it follows that $\dot{z}$, $\dot{z}_r$, and $\ddot{z}_r$ are all bounded. Now, assume the matrix R and $\dot{R}$ to be bounded matrices and also $\ddot{z}_d$, $\dot{z}_d$, z_d, and λ_d to be bounded functions, then, all signals on the right side of (20) are bounded and one can conclude that $\dot{s}$ and therefore $\ddot{z}$ are bounded. Substituting the control (15) and (16) into reduced order dynamic model (9) yields

$$\begin{aligned} J^T(\lambda - \lambda_c) &= [-\Phi_1(q, \dot{q}, \dot{z}, \ddot{z})\tilde{\alpha} + KRs] \\ &= \sigma(q, \dot{q}, \dot{z}, \ddot{z}, \dot{z}_r, \ddot{z}_r) \end{aligned} \tag{23}$$

where σ is a bounded function. Thus

$$J^T e_\lambda = (K_\lambda + I)^{-1}\sigma,$$

and the force tracking error $(f - f_d)$ are bounded and can be adjusted by changing the feedback gain K_λ. Thus, the theorem is proved. ∇.

4 Simulated Example

A simplified model of a vertical wheel rolling withou slipping on a plane surface, as given in details in [5] [10], is used to verify the validity of the control approach outlined in this paper.

The dynamic model can be expressed as [5] [10]

$$\begin{aligned} m\ddot{x} &= \lambda_1 \\ m\ddot{y} &= \lambda_2 \\ I_\theta \ddot{\theta} &= -\lambda_1 R cos\phi - \lambda_2 R sin\phi + u_1 \\ I_\phi \ddot{\phi} &= u_2 \end{aligned} \tag{24}$$

where x, y are coordinates of the point of contact of the wheel on the plane, θ is a rotation angle of the wheel due to rolling, measured from a fixed reference, ϕ is a heading angle of the wheel, measured from the x-axis, m is the mass of the robot, and I_θ and I_ϕ are inertial moments, R is the radius of the wheels, u_1 is the control torque about the rolling axis of the wheel and u_2 is the control torque about the vertical axis through the point of contact. For simplicity, we set R = 1.

The nonholonomic constraints are written as

$$\begin{aligned} \dot{x} &= \dot{\theta} cos\phi \\ \dot{y} &= \dot{\theta} sin\phi \end{aligned} \tag{25}$$

The matrix $J(q)$ is therefore defined as

$$J(q) = \begin{bmatrix} 1 & 0 & -cos\phi & 0 \\ 0 & 1 & -sin\phi & 0 \end{bmatrix},$$

where $q = [x\ y\ \theta\ \phi]^T$, $\lambda = [\lambda_1\ \lambda_2]^T$. The constraint forces are $f = J(q)^T\lambda$. The 'outputs' are chosen as

$$z(q) = [\theta\ \phi]^T$$

The matrix $R(q)$ defined in (6) is chosen as:

$$R = \begin{bmatrix} cos\phi & 0 \\ sin\phi & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

so the relation $\dot{q} = R(q)\dot{z}$ is satisfied.

The desired manifold Ω_d is chosen as

$$\Omega_d = \{(q, \dot{q}, \lambda) | z(q) = 0, \dot{q} = 0, \lambda = 1\}.$$

The robust control law (15) with (16) is used so that $q = [x\ y\ \theta\ \phi]^T$, and λ approach Ω_d.

The unknown parameters α in (14) is chosen as $\alpha = [m,\ I_\theta,\ I_\phi]^T$, then, the regressor matrix defined in (14) can be written as

$$\Phi_1(q, \dot{q}, \dot{z}_r, \ddot{z}_r) = \begin{bmatrix} \ddot{\theta}_r cos\phi - \dot{\theta}_r sin\phi & 0 & 0 \\ \ddot{\theta}_r sin\phi + \dot{\theta}_r cos\phi & 0 & 0 \\ 0 & \ddot{\theta}_r & 0 \\ 0 & 0 & \ddot{\phi}_r \end{bmatrix}$$

The true values of α are m=1, $I_\theta = 1$, and $I_\phi = 1$. The two tunable parameters Λ_1 and Λ_2 are chosen as $\Lambda_1 = 5$, $\Lambda_2 = 5$. The control gain K and the force control gain K_λ are chosen as $K = diag(1,\ 1,\ 20,\ 20)$, $K_\lambda = diag(1,\ 1)$.

The initial values of λ_1 and λ_2 are chosen as $\lambda_1(0) = \lambda_2(0) = 0.5$. The initial positions and velocities of robot are chosen as

$$x(0) = 0,\ y(0) = 0,\ \theta(0) = 45°,\ \phi(0) = 45°$$

$$\dot{x}(0) = 0,\ \dot{y}(0) = 0,\ \dot{\theta}(0) = 0,\ \dot{\phi}(0) = 0.$$

Using the controller (15) with (16), the results of the simulation are shown in Figs. 1-8. Fig. 1 shows the trajectory of θ, Fig. 2 shows the trajectory of ϕ, Figs. 3-6 show the trajectories of $\dot{x}$, $\dot{y}$, $\dot{\theta}$, and $\dot{\phi}$. Fig. 7 and 8 shows the tracking error of λ_1 and λ_2. These results verify the validity of the proposed algorithm.

5 Conclusion

In this paper, the issue of appropriate control of position and constraint force is addressed for a class of nonholonomic mechanical systems. By specifying an 'output' function vector, a reduced dynamic model, suitable for simultaneous force and motion control, is established. An adaptive control formulation is then proposed, ensuring that a system with m nonholonomic constraints can be stabilized to an m-dimensional desired manifold. However, the definition of the desired manifold depends on the specific choice of 'output' function vector, which is related to the form of the constraint equations and the dynamic system. One choice is demonstrated via a simple simulation example. It should be noticed that the 'output' function vector may or may not be physically motivated. Given the 'output equations', the proposed control law indeed provides a convenient solution for the robust force and motion control of nonholonomic systems. A simple example has been used to illustrate the methodology developed in this note, and simulation results are satisfactory.

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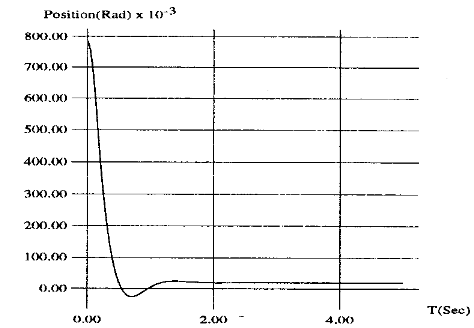

Figure 1: Position trajectory of θ.

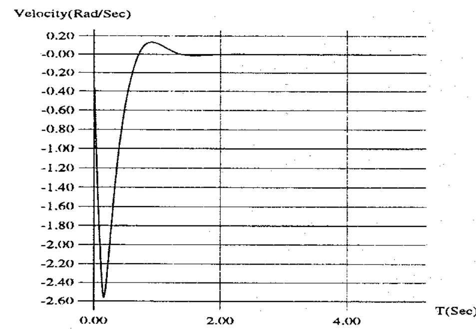

Figure 5: Velocity of θ.

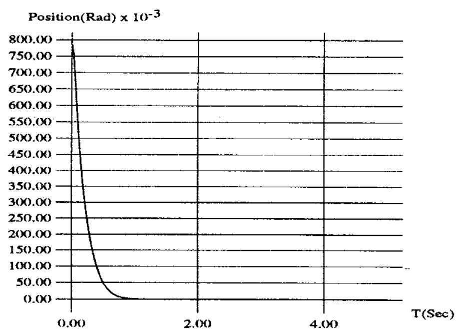

Figure 2: Position trajectory of ϕ.

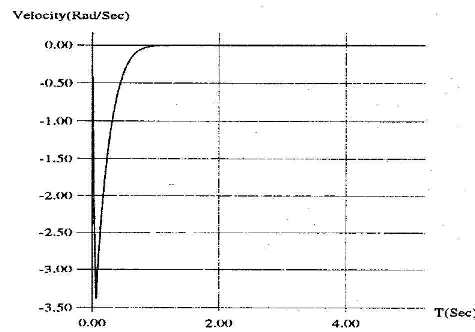

Figure 6: Velocity of ϕ.

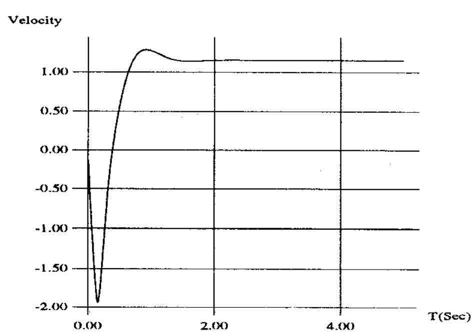

Figure 3: Velocity of x.

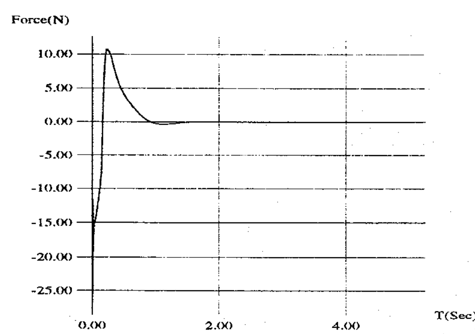

Figure 7: Contact force λ_1.

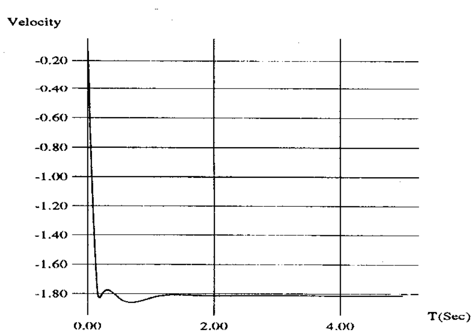

Figure 4: Velocity of y.

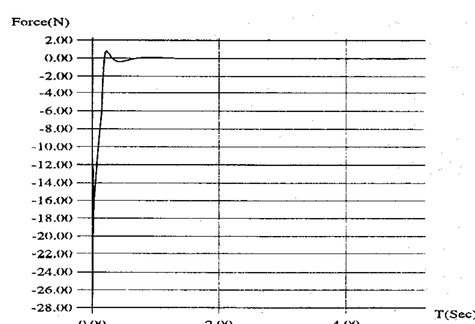

Figure 8: Contact force λ_2.