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COMBINED TRAJECTORY TRACKING AND PATH FOLLOWING CONTROL FOR DYNAMIC WHEELED MOBILE ROBOTS ⋆

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Abstract: The paper presents a solution to the problem of combined trajectory tracking and path following system design for underactuated mobile robots with non negligible dynamics. This problem is motivated by the practical need to develop control systems for mobile robots that can yield good trajectory tracking performance while keeping some of the properties that are normally associated with path following. The solution described builds on and extends previous work by Hindman and Hauser on so-called maneuver modified trajectory tracking. Simulations with a full nonlinear model of a wheeled mobile robot illustrate the performance of the control system derived.

Keywords: Trajectory Tracking. Path Following Systems. Nonlinear Control. Backstepping Techniques.

1. INTRODUCTION

Over the last few years there has been considerable interest in the design of advanced systems for motion control of autonomous vehicles. The problems of motion control addressed in the literature can be roughly classified into three groups: *point stabilization* - the goal is to stabilize a vehicle at a given target point, with a desired orientation; *trajectory tracking* - the vehicle is required to track a time parameterized reference, and *path following* - the vehicle is required to converge to and follow a desired path, without any temporal specifications. *Point stabilization* of vehicles with nonholonomic constraints presents a true challenge to control system designers since, as pointed out in the celebrated result by Brockett (Brockett, 1983), no smooth (or even continuous) constant state-feedback law will achieve that goal. To overcome this difficulty, two main approaches have been proposed in the literature: smooth time-varying control laws and discontinuous feedback laws; see (Kolmanovsky and McClamroch, 1995) for a comprehensive survey of the field. The *trajectory tracking* problem for fully actuated systems is by now well understood and satisfactory solutions can be found in standard nonlinear control textbooks (Khalil, 1996). However, in the case of underactuated vehicles, that is, when the vehicle has less actuators then state variables to be tracked, the problem is still a very active topic of research. Linearization methods, as well as Lyapunov based control laws have been proposed (Walsh *et al.*, 1994; Canudas de Wit *et al.*, 1993). Applications to underactuated surface vessels can be found in (Godhavn, 1997; Pettersen and Nijmeijer, 1998). *Path following* control has received relatively less attention than the other two problems. See for example (Canudas de Wit *et al.*, 1993; Jiang and Nijmeijer, 1999) and the refer-

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ences therein. Path following algorithms for marine vehicles have been reported in (Encarnação *et al.*, 2000; Encarnação and Pascoal, 2000). The usual underlying assumption in path following control is that the vehicle forward speed is held constant while the controller acts on the vehicle orientation to drive it to the path. Typically, smoother convergence to a path is achieved when compared to the performance obtained with trajectory tracking controllers and the control signals are less likely pushed to saturation (Hindman and Hauser, 1992).

These three classes of problems have traditionally been addressed separately. In fact, Brockett's condition implies that the point stabilization problem cannot be simply solved by designing a trajectory tracking control law and applying it to a trajectory that degenerates into a single point. Conversely, it is not clear how to extend current point stabilization methodologies to solve trajectory tracking problems. In contrast, a paper by Hindman and Hauser (Hindman and Hauser, 1992) that has not received proper attention in the literature shows clearly how to go from a trajectory tracking to a path following controller. In their work, the authors assume that a trajectory tracking controller for a given system is available and that a Lyapunov function is known that yields asymptotic stability of the resulting control system about a desired trajectory. The Lyapunov function captures the distance between the vehicle's posture (position and orientation) and its desired posture along the trajectory as measured in some P-metric, that is,

$$V = \|\chi(t) - \alpha(t)\|_P^2 ,$$

where $\chi(t)$ is the vehicle posture, $\alpha(t)$ is the desired posture, and $\|a\|_P = a^T P a$ with $P > 0$. The key idea in the work of Hindman and Hauser can now be simply explained as follows. To execute a path following maneuver, the vehicle should "look at the closest point on the path" and adopt the posture of an hypothetical vehicle traversing the path at that point as a reference to which it should converge. Thus, instead of feeding a pre-computed desired posture $\alpha(t)$ to the tracking controller, the posture corresponding to the path point that is closest to the vehicle is provided as a reference. The reference posture is formally computed as $\alpha(\pi(\chi))$, where

$$\pi(\chi) = \arg\min_{\zeta\in\mathbb{R}} \|\chi - \alpha(\zeta)\|_P^2$$

is a projection mapping that computes the trajectory time ζ that minimizes the distance $\|\chi - \alpha(\zeta)\|_P^2$. Under some technical conditions on path shape and path parameterization, it is shown that this strategy actually drives the vehicle to the path. Interestingly enough, the work of Hindman and Hauser goes even one step further. In fact, it shows how to blend into a single control law trajectory tracking and path following behaviors, thus achieving smooth spatial convergence to the trajectory as well as time convergence. This is accomplished by modifying the projection function $\pi(\chi)$ through the addition of a time dependent penalty term to obtain the projection

$$\pi_\lambda(\chi, t) = \arg\min_{\zeta\in\mathbb{R}} \Big[(1-\lambda)^2 \|\chi - \alpha(\zeta)\|_P^2 + \lambda^2 (t-\zeta)^2\Big].$$

The constant $\lambda > 0$ weighs the relative importance of convergence in time over spatial convergence to the path. If $\lambda = 0$ is chosen, pure path following is achieved because $\pi_0(\chi, t) = \pi(\chi)$; if $\lambda = 1$, $\pi_1(\chi, t) = t$ since $\zeta = t$ is always a minimizer of $(t-\zeta)^2$ and the original trajectory tracking controller is recovered. When $0 < \lambda < 1$ the system displays attributes of both schemes.

In this paper the results of Hindman and Hauser are extended to enlarge the class of systems to which their method can be applied. Namely, underactuated vehicles with non negligible dynamics. It is important to point out that, in practical applications, a path control system should only receive as reference inputs kinematic variables such as position and orientation of the vehicle, thus avoiding the need to input any references for the dynamic variables. This calls for a first modification of Hindman and Hauser's strategy. In fact, since the Lyapunov function used by those authors to prove stability of the trajectory tracking control system is also used to define the distance to the path, their methods are intrinsically suitable to vehicles with negligible dynamics only. Backstepping techniques are used in this paper to overcome this problem. A further modification is the explicit introduction of a rotation matrix from inertial to body frame in the control problem formulation. This simplifies considerably the task of control system design in the case of underactuated vehicles. Using the methodology developed, the distance to the path is evaluated by taking into consideration the kinematic variables only (position and orientation).

2. COMBINED TRAJECTORY TRACKING AND PATH FOLLOWING CONTROL

Consider the following general nonlinear system

$$\dot{\chi} = f(\chi) + g(\chi)\xi \tag{1}$$

where $\chi \in \mathbb{R}^n$ is the state and $\xi \in \mathbb{R}^m$ is the control input. The functions $f : D \to \mathbb{R}^n$ and $g : D \to \mathbb{R}^{n+m}$ are smooth in a domain $D \subset \mathbb{R}^n$ that contains $\chi = 0$ and $f(0) = 0$. Let $\alpha(t)$ be a reference trajectory for χ and let $\phi(\chi, \alpha(t))$ be a trajectory tracking control law for (1) such that, with $\xi = \phi(\chi, \alpha(t))$, $\chi \to \alpha$ that is, $\lim_{t\to\infty} (\chi(t) - \alpha(t)) = 0$. Assume that local asymptotic stability of the trajectory tracking system can be proven using the Lyapunov function

$$V_1(\chi,t) = \|M(\chi,t)[\chi-\alpha(t)]\|_P^2, \qquad (2)$$

where $M(\chi,t)$ is a generic non singular matrix and P is a positive definite weighting matrix (notice that V_1 can be simply viewed as the tracking error expressed in a non inertial frame and measured in the $M^T PM$ metric). The importance of introducing an extra matrix M will become clear in Section 3. Assume also that

$$\begin{aligned}\dot V_1(\chi,t) &= 2\Big\langle M[\chi-\alpha(t)], \dot M[\chi-\alpha(t)] + \\ &\qquad\qquad M[\dot\chi-\dot\alpha(t)]\rangle_P \\ &\le -k_1 V_1(\chi,t),\end{aligned}$$

where $\langle a,b\rangle_P = a^T Pb$ is the inner product relative to P and k is a positive scalar. The condition $\dot V_1 \le -k_1 V_1$ can be weakened if one can to prove stability with $\dot V_1$ negative semidefinite by resorting to LaSalle-Yoshizawa or Barbalat's lemmas (Khalil, 1996).

To convert the trajectory tracking control law into a combined trajectory tracking and path following control law, let the square of the distance from the path point $(\alpha(\zeta),\zeta)$ to the point (χ,t) be given by the function

$$\begin{aligned}U(\chi,t,\zeta) &= (1-\lambda)^2 V_1(\chi,\zeta) + \lambda^2(t-\zeta)^2 \\ &= (1-\lambda)^2\|M[\chi-\alpha(\zeta)]\|_P^2 + \lambda^2(t-\zeta)^2.\end{aligned}$$

Given $U(\chi,t,\zeta)$, define the mapping

$$\begin{aligned}\pi_\lambda(\chi,t) =& \arg\min_{\zeta\in\mathbb{R}} U(\chi,t,\zeta) = \\ &\arg\min_{\zeta\in\mathbb{R}}\left[(1-\lambda)^2\|M[\chi-\alpha(\zeta)]\|_P^2 + \lambda^2(t-\zeta)^2\right], \qquad (3)\end{aligned}$$

where λ is a positive scalar that weighs the relative importance of time convergence to the trajectory in comparison with spatial convergence to the corresponding path. If $\lambda = 0$, $\pi_\lambda(\chi,t) = \arg\min_{\zeta\in\mathbb{R}} \|M[\chi-\alpha(\zeta)]\|_P^2$ gives the trajectory time ζ that minimizes the distance $\|M[\chi-\alpha(\zeta)]\|_P^2$ and a pure path following controller is obtained; if $\lambda = 1$, $\pi_\lambda(\chi,t) = t$ (since $\zeta = t$ is the minimizer of $(t-\zeta)^2$), yielding the original trajectory tracking controller. When $0<\lambda<1$, the π_λ mapping gives the trajectory time ζ of the path point $(\alpha(\zeta),\zeta)$ that is closest to (χ,t) in the metric induced by $U(\chi,t,\zeta)$, and a mixed trajectory tracking and path following behavior is achieved. Under some mild technical conditions on the reference path shape and parameterization (see (Hindman and Hauser, 1992) for details), $\pi_\lambda(\cdot)$ is unique and continuous.

Note that a necessary condition for $\zeta = \pi_\lambda(\chi,t)$ to be a minimizer of $U(\chi,t,\zeta)$ is that

$$\begin{aligned}&\frac{\partial}{\partial\zeta}U(\chi,t,\zeta) = \\ &2(1-\lambda)^2\left\langle M[\chi-\alpha(\pi_\lambda(\chi,t))], -M\frac{d\alpha}{d\zeta}(\pi_\lambda(\chi,t))\right\rangle_P \\ &-2\lambda^2(t-\pi_\lambda(\chi,t)) = 0. \qquad (4)\end{aligned}$$

Consider now the candidate Lyapunov function

$$\begin{aligned}Y(\chi,t) =& \min_{\zeta\in\mathbb{R}} U(\chi,t,\zeta) = U(\chi,t,\pi_\lambda(\chi,t)) \qquad (5)\\ =& \left[(1-\lambda)^2\|M[\chi-\alpha(\pi_\lambda(\chi,t))]\|_P^2 + \lambda^2(t-\pi_\lambda(\chi,t))^2\right].\end{aligned}$$

Differentiating Y with respect to time gives

$$\begin{aligned}\dot Y(\chi,t) =\ & 2(1-\lambda)^2\Big\langle M[\chi-\alpha(\pi_\lambda(\chi,t))], \\ & \dot M[\chi-\alpha(\pi_\lambda(\chi,t))] + \\ & M\left[\dot\chi - \frac{\partial\alpha}{\partial\zeta}(\pi_\lambda(\chi,t))\frac{d\pi_\lambda}{dt}(\chi,t)\right]\Big\rangle_P + \\ & 2\lambda^2(t-\pi_\lambda(\chi,t))\left(1-\frac{d\pi_\lambda}{dt}(\chi,t)\right) \\ =\ & 2(1-\lambda)^2\Big\langle M[\chi-\alpha(\pi_\lambda(\chi,t))], \\ & \dot M[\chi-\alpha(\pi_\lambda(\chi,t))] + \\ & M\left[\dot\chi - \frac{\partial\alpha}{\partial\zeta}(\pi_\lambda(\chi,t))\right]\Big\rangle_P + \\ & 2\left(1-\frac{d\pi_\lambda}{dt}(\chi,t)\right)\Big[(1-\lambda)^2\Big\langle M[\chi-\alpha(\pi_\lambda(\chi,t))], \\ & M\frac{d\alpha}{d\zeta}(\pi_\lambda(\chi,t))\Big\rangle_P - \lambda^2(t-\pi_\lambda(\chi,t))\Big].\end{aligned}$$

From (4), the second term of this equality is zero, hence

$$\begin{aligned}\dot Y(\chi,t) &= (1-\lambda)^2\dot V_1(\chi,\pi_\lambda(\chi,t)) \\ &\le -(1-\lambda)^2 k_1 V_1(\chi,\pi_\lambda(\chi,t)) \le 0.\end{aligned}$$

To conclude asymptotic time convergence to the path one must resort to Barbalat's Lemma (Khalil, 1996) since $\dot Y$ is not negative definite in $t-\zeta$. Assuming that the first and second derivatives of $\alpha(\zeta)$ (the vehicle's velocity and its acceleration along the reference path, respectively) are bounded, it easy to prove that $\ddot Y$ is bounded. Thus, $\dot Y$ is uniformly continuous. Since $\dot Y \le 0$ and Y is bounded bellow by zero, $Y(\chi,t)\to Y_\infty$ as $t\to\infty$, where Y_∞ is positive and finite. Now, Barbalat's Lemma allows for the conclusion that $\dot Y(\chi,t)\to 0$ as $t\to\infty$. This implies that $\chi\to\alpha(\pi_\lambda(\chi,t))$ as $t\to\infty$. Using (4), it follows that $\pi_\lambda(\chi,t)\to t$ as $t\to\infty$. Therefore, $\chi\to\alpha(t)$ as $t\to\infty$ that is, the system tracks the reference trajectory asymptotically.

In order to apply the method described to the problem of motion control of autonomous vehicles, some modifications are required. This is due to the fact that even though the equations of motion of these vehicles exhibit a clear coupling between kinematics and dynamics, the specification of the trajectories to be tracked should include kinematic variables only. For example, in the case of a dynamic wheeled mobile robot (WMR) a reference trajectory may simply include the desired evolution of the center of mass (and possibly the attitude) of that vehicle. In this case, however, the methodology proposed by Hindman and Hauser still provides a powerful tool to obtain a path following controller for the WMR, assuming a trajectory tracking control law exists and its stability has been proved using Lyapunov theory. In fact, the work of Hindman and Hauser provides a clear recipe to suitably modify the Lyapunov function for trajectory tracking so as to yield a new Lyapunov function for path following stability.

Suppose for example that (1) captures the kinematics of a vehicle and that (after a suitable coordinate transformation) the dynamics can be cast in the form

$$\dot{\xi} = u, \tag{6}$$

where u is the actual input vector. Suppose also that a stabilizing trajectory tracking controller $\xi = \phi(\chi, \alpha(t))$ for (1) and a corresponding Lyapunov function $V_1(\chi, t)$ have been found. The vehicle dynamics can be included in the control design through a backstepping step (Krstić *et al.*, 1995) as follows.

Consider the change of variables $z = \xi - \phi$. The variable z can be interpreted as the difference between the *virtual control variable* ξ and the *virtual control law* ϕ. Thus, if z tends to zero, the system approaches the manifold correspondent to the "kinematic system". This motivates the choice of

$$V_2 = Y(\chi, t) + \frac{1}{2} z^T z$$

as a candidate Lyapunov function. Differentiating V_2 with respect to time yields

$$\begin{aligned} \dot{V}_2 = & \frac{\partial Y}{\partial \chi}(\chi)\left[f(\chi) + g(\chi)\phi\right] + \\ & \frac{\partial Y}{\partial \chi}(\chi) g(\chi) z + z^T\left(u - \dot{\phi}\right) \\ \leq & -(1-\lambda)^2 k_1 V_1(\chi, \pi_\lambda(\chi, t)) + \\ & \frac{\partial Y}{\partial \chi}(\chi) g(\chi) z + z^T\left(u - \dot{\phi}\right). \end{aligned}$$

Setting

$$u = \dot{\phi} - g^T(\chi)\frac{\partial Y}{\partial \chi}^T(\chi) - K_2 z,$$

where K_2 is a positive definite matrix, it follows that

$$\dot{V}_2 \leq -(1-\lambda)^2 k_1 V_1(\chi, \pi_\lambda(\chi, t)) - z^T K_2 z \leq 0.$$

Using the same arguments as before, time convergence to the path can be concluded.

3. APPLICATION TO A DYNAMIC WHEELED MOBILE ROBOT

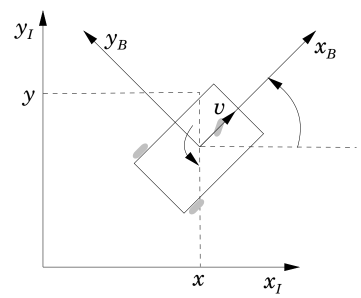

Fig. 1. Weeled mobile robot: relevant variables.

This section describes the application of the combined trajectory tracking and path following control law derived in the previous section to the control of a wheeled mobile robot (WMR) with non negligible dynamics.

The WMR equations of motion are (see Figure 1)

$$\dot{x} = v\cos\psi \tag{7}$$

$$\dot{y} = v\sin\psi \tag{8}$$

$$\dot{\psi} = \omega \tag{9}$$

$$\dot{v} = \frac{F}{m} \tag{10}$$

$$\dot{\omega} = \frac{N}{I_z} \tag{11}$$

where the triplet (x, y, ψ) describes the position and the orientation of the robot position relative to the inertial frame, and v and ω are the linear and angular velocities of the vehicle, respectively. The symbol m denotes the vehicle mass, while I_z is its moment of inertia about the z_B axis. Finally, the force and the torque that act on the robot are denoted by F and N, respectively. Equations (7)-(9) are the kinematic relations for the WMR and thus $\chi = (x, y, \psi)^T$. Equations (10)-(11) capture the robot dynamics. Using the input transformation

$$u = \left(\frac{F}{m}, \frac{N}{I_z}\right)^T$$

and defining $\xi = (v, \omega)^T$, the vehicle dynamics can be written in the form $\dot{\xi} = u$.

In (Canudas de Wit *et al.*, 1993), the authors propose a kinematic trajectory tracking controller for (7)-(9) using v and ω as *virtual controls* as follows. Let $\alpha(t) = (x_r(t), y_r(t), \psi_t(t))^T$ be a reference trajectory for χ such that

$$\begin{aligned} \dot{x}_r &= v_r\cos\psi_r \\ \dot{y}_r &= v_r\sin\psi_r \\ \dot{\psi}_r &= \omega_r \\ \dot{v}_r &= \frac{F_r}{m} \\ \dot{\omega}_r &= \frac{N_r}{I_z}. \end{aligned} \tag{12}$$

The tracking error is expressed in the body frame as

$$e = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \psi_r - \psi \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}.$$

Differentiating e with respect to time,

$$\dot{e} = \begin{bmatrix} v_r\cos e_3 - v + e_2\omega \\ v_r\sin e_3 - e_1\omega \\ \omega_r - \omega \end{bmatrix}$$

is obtained. Consider the candidate Lyapunov function

$$V_1 = \frac{c_1}{2}\left(e_1^2 + e_2^2\right) + \frac{1}{2}e_3^2 = \|M(\chi - \alpha(t))\|_P^2$$

with $P = diag\left(\frac{c_1}{2}, \frac{c_1}{2}, \frac{1}{2}\right)$ and c_1 a positive scalar. The matrix

$$M=\begin{bmatrix} {}^{B}R_I & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where ${}^{B}R_I$ is the rotation matrix from inertial to body frame, relates the tracking error in the inertial frame with the same error in the body frame. Differentiating V_1 with respect to time and setting

$$\xi=\phi=\begin{bmatrix} v \\ \omega \end{bmatrix}=\begin{bmatrix} v_r\cos e_3+c_2e_1 \\ \omega_r+c_1e_2v_r\dfrac{\sin e_3}{e_3}+c_3e_3 \end{bmatrix},\quad(13)$$

(c_2 and c_2 are positive scalars) yields

$$\dot{V}_1=-c_1c_2e_1^2-c_3e_3^2\leq 0.$$

Assuming boundedness of $v_r(t)$, $\omega_r(t)$, and their time derivatives, and using Barbalat-like arguments, the authors show that, with the control law (13), $\chi\to\alpha$ as $t\to\infty$.

The system is thus in the form (1), (6) with $\chi=(x,y,\psi)^T$, $\xi=(v,\omega)^T$, $f(\chi)=0$, and $g(\chi)=\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$; ϕ is given by (13). The method developed in Section 2 can now be applied to derive a combined trajectory tracking and path following control law for the WMR.

The resulting control system was simulated for a WMR with $m=10\,\mathrm{Kg}$ and $I_z=10\,\mathrm{Kgm}^2$. The controller parameters were set to $c_1=1$, $c_2=c_3=1.5$, and $K_2=diag(1,1)$. The reference path to be followed was derived from equations (12) by setting $v_r=1\,\mathrm{ms}^{-1}$ and $\omega_r(\zeta)=0.1\sin(0.04\pi\zeta)\,\mathrm{rads}^{-1}$. Three different situations where simulated: i) pure trajectory tracking ($\lambda=1$), ii) pure path following ($\lambda=0$), and iii) combined trajectory tracking and path following ($\lambda=0.1$, thus placing greater emphasis on path following than on trajectory tracking). Figures 2 and 3 refer to the pure trajectory tracking case. From the $x-y$ trajectory in Figure 2 one can see that the vehicle turns back in its attempt to be at the given reference point at the prescribed time. Figure 3 shows that the time errors converge quickly to zero. In the pure path following case (Figures 4 and 5), the approach to the path is very smooth. However, Figure 5 shows that the time errors do not converge to zero and that the vehicle always stays ahead of the reference vehicle. In the case of combined trajectory tracking and path following, the vehicle converges quickly to the desired path and only then will it react to achieve zero trajectory tracking error. This is clearly shown in Figures 6 and 7.

It is interesting to compare the control law here derived with the path following controller for a WMR also presented in (Canudas de Wit *et al.*, 1993) for the particular case where the reference path is a straight line. Suppose that $\lambda=0$,

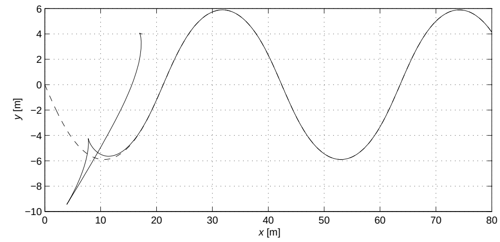

Fig. 2. WMR $x-y$ trajectory and reference path - pure trajectory tracking case.

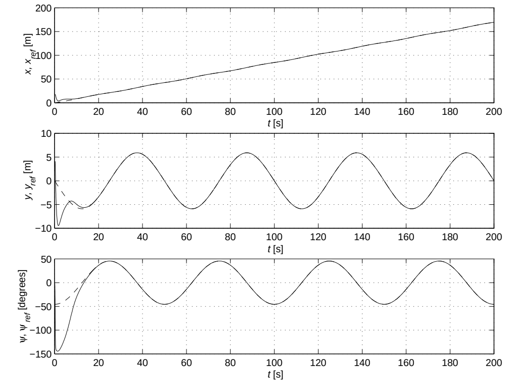

Fig. 3. Time errors $\chi(t)-\alpha(t)$ - pure trajectory tracking case.

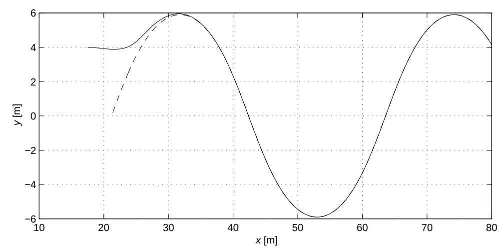

Fig. 4. WMR $x-y$ trajectory and reference path - pure path following case.

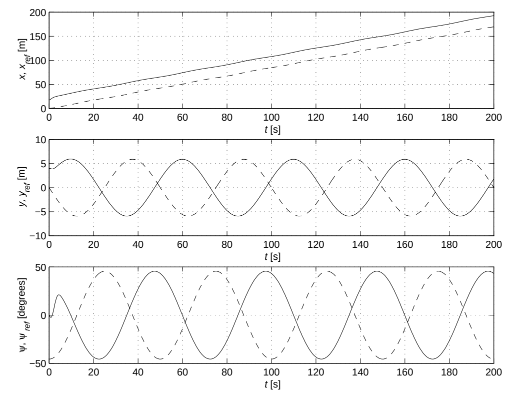

Fig. 5. Time errors $\chi(t)-\alpha(t)$ - pure path following case.

i. e., a pure path following behavior is desired, and that P is a diagonal matrix with $P_{11}=P_{22}$. In this case, $\pi_0(\chi,t)=\pi(\chi)$ will always return the trajectory time ζ that minimizes the Euclidean distance $\sqrt{(x-x_r(\zeta))^2+(y-y_r(\zeta))^2}$ since $\psi_r(\zeta)$ is constant and the M^TPM metric

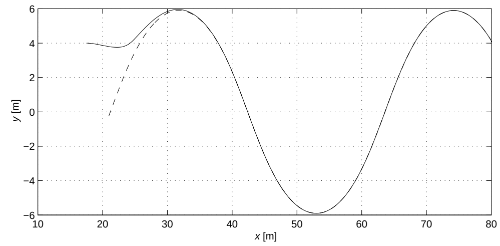

Fig. 6. WMR $x - y$ trajectory and reference path - combined trajectory tracking and path following.

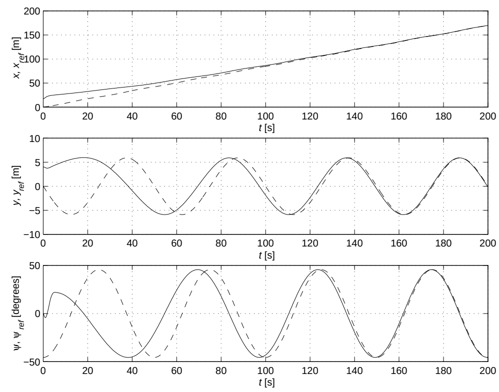

Fig. 7. Time errors $\chi(t) - \alpha(t)$ - combined trajectory tracking and path following.

coincides with the P metric in what regards the first two components of the vectors. Assuming, without loss of generality, that the reference path is aligned with the x_I axis of the inertial frame, $\alpha(\pi(\chi))$ will be the orthogonal projection of (x, y) onto x_I. Thus,

$$e(t) = M(0, -y, -\psi)^T, \forall_{t>0}.$$

Substituting the error expression into the control signal (13) and since $\omega_r = 0$ for a straight line,

$$\xi = \begin{bmatrix} v_r \cos\psi - c_2 y \sin\psi \\ -c_1 y v_r \dfrac{\sin\psi}{\psi} \cos\psi - c_3\psi \end{bmatrix} \quad (14)$$

is obtained. Comparing the path following control law proposed in (Canudas de Wit *et al.*, 1993) for the case where the reference path curvature is null with (14), shows that the angular velocity command in the latter contains an additional term in $\cos\psi$. Furthermore, while the forward velocity v is assumed to be constant in (Canudas de Wit *et al.*, 1993), the control design here derived requires that v be controlled as in (14).

4. CONCLUSIONS

The paper presented a solution to the problem of combined trajectory tracking and path following for dynamic underactuated mobile robots. The solution described builds on and extends previous work by Hindman and Hauser on so-called maneuver modified trajectory tracking. The performance of the control system derived was assessed in simulation with the dynamic model of a wheeled mobile robot.

Further work will address the problem of robust stabilization against parameter uncertainty.

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