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Robust Constrained Model Predictive Control using Linear Matrix Inequalities

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Abstract

The primary disadvantage of current design techniques for model predictive control (MPC) is their inability to explicitly deal with model uncertainty. In this paper, we address the robustness issue in MPC by directly incorporating the description of plant uncertainty in the MPC problem formulation. The plant uncertainty is expressed in the time-domain by allowing the state-space matrices of the discrete-time plant to be arbitrarily time-varying and belonging to a polytope. The existence of a feedback control law minimizing an upper bound on the infinite horizon objective function and satisfying the input and output constraints is reduced to a convex optimization over linear matrix inequalities (LMIs). It is shown that for the plant uncertainty described by the polytope, the feasible receding horizon state feedback control design is robustly stabilizing.

1. Introduction

Model predictive control (MPC), also known as receding horizon control, is becoming an increasingly popular control design method in the process industries. It involves solution of an optimization problem on-line (typically a linear or quadratic program) in order to determine optimal future control inputs. The first control move is implemented and at the next sampling time, system measurements are used to resolve the optimization problem.

The principal shortcoming of existing design techniques for MPC is their inability to deal explicitly with plant model uncertainty. Thus, nearly all known formulations of MPC as also the results on stability (e.g., [14],[16]) make the assumption that a single linear time-invariant (LTI) model describes the plant behaviour accurately. Such control systems might perform very poorly when implemented on a physical system which is not exactly described by the model (for example, see [19]). Garcia and Morari [6, 7, 8] suggested that the filter in MAC and IMC can be used to make unconstrained MPC robust. Campo and Morari [4] modified the on-line optimization problem (linear program) for a single plant to a "min-max" problem over a set of plants. Allwright and Papavasiliou [1] have explored the computational issues of this "min-max" algorithm. Genceli and Nikolaou [9] showed how a set of SISO systems can be robustly stabilized, given upper and lower bounds on the plant impulse responses. Zheng and Morari [19] have proposed a model predictive controller for uncertain Finite Impulse Response (FIR) models. Zafiriou [18] has developed necessary/sufficient conditions for robust stability of MPC based on the contraction mapping principle.

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Clearly, what is needed is a computationally inexpensive scheme for robust MPC which allows consideration of a fairly general model uncertainty description and which at the same time guarantees stability for the class of systems described by the uncertainty set.

In the past few years, the important role of Linear Matrix Inequalities (LMIs) in systems and control has increasingly been recognized [3] by researchers. Until recently, only a handful of these LMIs could be solved. The breakthrough came in 1988 when Nesterov and Nemirovski [13] developed interior point methods that apply directly to convex problems involving LMIs. The implication of these developments is that problems whose solutions can be computed numerically with reasonable computational effort must be considered "solved" even though no analytical solutions exist for these problems.

The goal of this paper is to show that the infinite horizon MPC problem with input and output constraints and plant uncertainty can be formulated as a convex optimization problem involving LMIs. The paper is organized as follows: In Section 2, we present some preliminaries. In Section 3, we formulate the robust constrained MPC problem with state-feedback as an LMI problem. We show that a feasible solution of the problem is robustly stabilizing. In Section 4, we present examples illustrating the design procedure and finally, in Section 5, we present concluding remarks and comments on computational issues.

2. Background

Consider the discrete time-varying linear system

$$x(k+1) = A(k)x(k) + B(k)u(k) \tag{1}$$

$$y(k) = C(k)x(k) \tag{2}$$

which represents the dynamics of the plant under consideration. Here, $u(k) \in \mathcal{R}^{n_u}$ is the manipulated variable, $x(k) \in \mathcal{R}^{n_x}$ is the state of the plant. The plant uncertainty is captured by allowing the state-space matrices $A(k) \in \mathcal{R}^{n_x \times n_x}$, $B(k) \in \mathcal{R}^{n_x \times n_u}$, $C(k) \in \mathcal{R}^{n_y \times n_x}$ to be arbitrarily time-varying and lying in a polytope, i.e., $[A(k) \quad B(k) \quad C(k)] \in \Omega = Co\{[A_1 \quad B_1 \quad C_1], \ldots, [A_L \quad B_L \quad C_L]\}$. In other words, $[A(k) \quad B(k) \quad C(k)] = \Sigma_{i=1}^{L} \alpha_i(k)[A_i \quad B_i \quad C_i]$ where $\alpha_i(k) \geq 0$, $\Sigma_{i=1}^{L} \alpha_i(k) = 1$. This description is also suitable for a nonlinear discrete time-varying system $x(k+1) = f(x(k), u(k), k)$, $y(k) = g(x(k), k)$ if the Jacobian $\left[\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial u} \quad \frac{\partial g}{\partial x}\right] \in \Omega$ (see Liu *et al.* [11] for equivalence of a nonlinear system and a time-varying linear system). Similarly, SISO finite impulse response uncertainty can be translated to such a state-space uncertainty.

2.1. Model Predictive Control

The goal of model predictive control (MPC) is to calculate, at each sampling time k, m manipulated variables $u(k+i|k)$, i=0,1,…, $m-1$, to minimize the quadratic objective $J_p(k) = \Sigma_{i=0}^{p}(x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+i|k))$ subject

to constraints on the state and manipulated variables. Here, Q_1, R are symmetric weighting matrices; $Q_1 > 0$, $R > 0$.
$x(k+i|k)$ is the state of the system at time $k+i$ predicted at time k and $x(k|k)$ is the state of the system measured at time k;
$u(k+i|k)$ is the control move at time $k+i$, computed at time k and $u(k) = u(k|k)$ is the control move implemented at time k;
p is the ouput or prediction horizon;
m is the input horizon or the number of control moves to be computed;
$u(k+i|k) = 0, \quad \forall i \geq m$.
We assume that exact measurements of the state of the system are available at each sampling time k, i.e., $x(k|k) = x(k)$.
The case where $p = \infty$ is referred to as the infinite horizon MPC (IH-MPC) with objective function $J_\infty(k)$. Finite horizon control laws have been known to have poor stability properties even in the nominal case (Bitmead et al. [2]) and require imposition of a terminal state constraint to ensure stability. On the other hand, infinite horizon control laws have been shown to guarantee nominal stability [12, 14]. We will therefore adopt the infinite horizon approach since it guarantees at least nominal stability.

The input constraints we consider in this paper are the Euclidean norm constraint $u(k+i|k)^T u(k+i|k) \leq \alpha_1$ and the ∞ norm constraint $\|u(k+i|k)\|_\infty \leq u_{max} \quad \forall k, i = 0, 1, \ldots, \infty$ and similarly for the output $y(k+i|k)^T y(k+i|k) \leq \alpha_2$ and $\|y(k+i|k)\|_\infty \leq y_{max}$ $\forall k = 0, 1, \ldots, \infty$, $\forall i = 1, 2, \ldots, \infty$. Here, $y(k+i|k)$ is the output at time $k+i$, predicted at time k. Note that the output constraints have been imposed strictly over the future horizon and not at the current time.

2.2. Linear Matrix Inequalities

Definition 1 *A linear matrix inequality is a matrix inequality of the form* $F(x) = F_0 + \Sigma_{i=1}^{l} x_i F_i > 0$ *where* $(x_1, x_2, \ldots, x_l)$ *are the design variables.* $F_i = F_i^T \in \mathcal{R}^{n \times n}$ *are given, and* > 0 *means that the smallest eigenvalue of* $F(x)$ *is positive.*

Definition 2 *An eigenvalue problem (EVP) is an optimization of the following form*

$$\min_{x,\lambda} \quad \lambda \tag{3}$$

$$\text{subject to the LMI constraint } A(x,\lambda) > 0 \tag{4}$$

where $A(x,\lambda)$ *is affine in* (x,λ).

We will use the following result extensively throughout the paper.

Lemma 1 (Schur Complements)

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} > 0 \quad \text{where } A = A^T, \quad C = C^T \tag{5}$$

$$\iff \quad C > 0, \quad A - BC^{-1}B^T > 0 \tag{6}$$

$$\text{and} \quad A > 0, \quad C - B^T A^{-1} B > 0 \tag{7}$$

Also, if $A = A^T > 0$, *then*

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \geq 0 \quad \text{where } C = C^T \tag{8}$$

$$\iff C \geq 0, \quad C - B^T A^{-1} B \geq 0 \tag{9}$$

and if $C = C^T > 0$, *then*

$$\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \geq 0 \qquad \text{where } A = A^T \tag{10}$$

$$\iff A \geq 0, \quad A - BC^{-1}B^T \geq 0 \tag{11}$$

Proof: See [15, page 512] □.

The significance of Lemma 1 is that nonlinear matrix inequalities can be translated to equivalent linear matrix inequalities.

3. Model Predictive Control using Linear Matrix Inequalities

With these preliminaries, we can now discuss the problem formulation. We will be considering the infinite horizon MPC (IH-MPC) problem. The motivation for this has been discussed in Section 2.1. We first consider the robust unconstrained IH-MPC problem and then extend the formulation to incorporate input and output constraints.

3.1. Robust Unconstrained IH-MPC

The objective function to be minimized at each samping time k is $J_\infty(k) = \Sigma_{i=0}^{\infty}(x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+i|k))$ by a feedback control law $u(k+i|k) = Fx(k+i|k)$, $\forall i = 0, 1, \ldots, \infty$. An upper bound on $J_\infty(k)$ can be derived as follows: Suppose there exists a function V with $V(0) = 0$ which satisfies $V(x(k+i+1|k)) - V(x(k+i|k)) \leq -[x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+i|k)]$ $\forall k, i = 0, 1, \ldots, \infty$.
For the objective function to be finite, we must have $x(\infty|k) = 0$ and hence, $V(x(\infty|k)) = 0$. Summing up the above equation from $i = 0$ to $i = \infty$, we get $-V(x(k|k)) \leq -J_\infty(k) \Rightarrow J_\infty(k) \leq V(x(k|k))$. This gives an upper bound on our objective function. Suppose, now we choose a quadratic form for the function V, i.e., $V(z) = z^T P z$, $P > 0$. Then, our goal is to synthesize, at each sampling time k, a state-feedback law $u(k+i|k) = Fx(k+i|k)$ $\forall i = 0, 1, \ldots, \infty$ so as to minimize the upper bound $V(x(k|k))$ on $J_\infty(k)$. As is standard in MPC, only the first input $u(k) = u(k|k) = Fx(k|k)$ is implemented, the state $x(k+1)$ is measured and the optimization is repeated at sampling time $k+1$. The following theorem gives us this state-feedback matrix F.

Theorem 1 *Let* $x(k) = x(k|k)$ *be the state of the uncertain polytopic system measured at sampling time* k. *Then, in the absence of input and output constraints, the state-feedback matrix* F *which minimizes the upper bound* $V(x(k|k))$ *on* $J_\infty(k)$ *at sampling time* k *is given by:*

$$F = YQ^{-1} \tag{12}$$

where, $Q > 0$ *and* Y *are the solutions (if they exist) to the following eigenvalue problem (EVP):*

$$\min_{\gamma,Q,Y} \quad \gamma \tag{13}$$

$$\text{subject to} \begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q \end{bmatrix} \geq 0 \tag{14}$$

$$\begin{bmatrix} Q & QA_j^T + Y^T B_j^T & QQ_1^{\frac{1}{2}} & Y^T R^{\frac{1}{2}} \\ A_j Q + B_j Y & Q & 0 & 0 \\ Q_1^{\frac{1}{2}} Q & 0 & \gamma I & 0 \\ R^{\frac{1}{2}} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \ \forall j = 1, 2, \ldots, L \tag{15}$$

Proof: The quadratic function V that we have chosen is $V(z) = z^T P z$, $P > 0$. Defining $Q = \gamma P^{-1}$ and using Lemma 1, minimizing $V(x(k|k))$ is equivalent to

$$\min_{\gamma,Q} \quad \gamma \tag{16}$$

$$\begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q \end{bmatrix} \geq 0 \tag{17}$$

Now, the quadratic function V is required to satisfy

$$V(x(k+i+1|k)) - V(x(k+i|k)) \leq -x(k+i|k)^T[Q_1 + F^T R F]x(k+i|k) \quad \forall k, i = 0, 1, \ldots, \infty$$

This is satisfied if

$$(A(k+i) + B(k+i)F)^T P (A(k+i) + B(k+i)F) - P + F^T R F + Q_1 \leq 0 \tag{18}$$

Substituting $P = \gamma Q^{-1}$, applying Lemma 1 and after some algebra and using the fact that $[A(k+i)\ B(k+i)] = \Sigma_{j=1}^{L} \alpha_j(k+i)[A_j\ B_j]$, with $\alpha_j(k+i) \geq 0$, $\Sigma_{j=1}^{L}\alpha_j(k+i) = 1$, we see that (18) is equivalent to

$$\begin{bmatrix} Q & QA_j^T + Y^T B_j^T & QQ_1^{\frac{1}{2}} & Y^T R^{\frac{1}{2}} \\ A_j Q + B_j Y & Q & 0 & 0 \\ Q_1^{\frac{1}{2}} Q & 0 & \gamma I & 0 \\ R^{\frac{1}{2}} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \quad \forall j = 1, 2, \ldots, L \tag{19}$$

$$Q = \gamma P^{-1}, \quad Y = FQ \tag{20}$$

(19), (20), (16) and (17) then give the result. □

Remark 1: It can be shown that in the unconstrained case, this receding horizon state-feedback law reduces to a static state-feedback law.

3.2. Robust Constrained IH-MPC

In this section, we incorporate both input and output constraints into the optimization problem. We will show that the feasible solution to the optimization with input and output constraints is robustly stabilizing.

Before considering constraints, we will first establish the following result on invariant ellipsoids.

Lemma 2 *Suppose* $[A(k)\quad B(k)] \in Co\{[A_1\quad B_1], [A_2\quad B_2], \ldots, [A_L\quad B_L]\}$ *and there exist* P *and* F *such that*

$$(A_j + B_j F)^T P (A_j + B_j F) - P + F^T R F + Q_1 \leq 0 \quad \forall j = 1, 2, \ldots, L$$

Then, if $x(k|k) \in \epsilon_Q = \{z | z^T Q^{-1} z \leq 1\} = \{z | z^T P z \leq \gamma\} \Longrightarrow x(k+i|k) \in \epsilon_Q \quad \forall i = 1, 2, \ldots, \infty$ *where* $Q = \gamma P^{-1} > 0$ *and* $x(k+i+1|k) = (A(k+i) + B(k+i)F)x(k+i|k), \forall i = 0, 1, \ldots, \infty$.

Proof: The proof will be omitted for lack of space □.

3.2.1 Input constraints:

We first consider Euclidean norm constraints on the manipulated variable. Assuming that ϵ_Q is an invariant ellipsoid for the system of predicted states, we have

$$\max_{i \geq 0} \|u(k+i|k)\|_2^2 = \max_{i \geq 0} \|Fx(k+i|k)\|_2^2 \leq \max_{z \in \epsilon_Q} \|YQ^{-1}z\|_2^2 = \max_{x^T x \leq 1} \|YQ^{-\frac{1}{2}}x\|_2^2 = \lambda_{max}(Q^{-\frac{1}{2}} Y^T Y Q^{-\frac{1}{2}}) \leq \alpha_1$$

$$\Longleftrightarrow \begin{bmatrix} \alpha_1 I & Y \\ Y^T & Q \end{bmatrix} \geq 0 \quad \text{using Lemma 1} \tag{21}$$

This is an LMI in Y and Q. Similarly, the spatial ∞-norm constraint can be translated to an LMI constraint as follows:

$$\max_{i \geq 0} \|u(k+i|k)\|_\infty^2 = \max_{i \geq 0} |YQ^{-1}x(k+i|k)|_\infty^2 \leq \max_{z \in \epsilon_Q} |YQ^{-1}z|_\infty^2 \leq \max_j [YQ^{-1}Y^T]_{jj} \leq u_{max}^2$$

$$\Longleftrightarrow \begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \geq 0 \text{ with } \max_i X_{ii} \leq u_{max}^2 \tag{22}$$

These are once again LMIs in Y, X and Q.

3.2.2 Output constraints:

Arguments similar to the ones used for input constraints can be used to show that

$$\begin{bmatrix} \alpha_2 I & C_j A_l Q + C_j B_l Y \\ QA_l^T C_j^T + Y^T B_l^T C_j^T & Q \end{bmatrix} \geq 0 \tag{23}$$

$$\forall j, l = 1, 2, \ldots, L \tag{24}$$

$$\Longrightarrow y(k+i|k)^T y(k+i|k) \leq \alpha_2 \ \forall i = 1, 2, \ldots, \infty$$

and

$$\begin{bmatrix} X & C_j A_l Q + C_j B_l Y \\ QA_l^T C_j^T + Y^T B_l^T C_j^T & Q \end{bmatrix} \geq 0 \tag{25}$$

$$\forall j, l = 1, 2, \ldots, L$$

$$\text{with} \quad \max_j X_{jj} \leq y_{max}^2 \tag{26}$$

$$\Longrightarrow \|y(k+i|k)\|_\infty \leq y_{max} \ \forall i = 1, 2, \ldots, \infty.$$

Note that in both these cases, if C is a constant matrix, then the L^2 LMIs reduce to L LMIs. We now state the following theorem which takes into account input and output constraints and model uncertainty in synthesizing the control action.

Theorem 2 *Let* $x(k) = x(k|k)$ *be the state of the system measured at sampling time* k. *Then, the receding horizon state-feedback matrix* F *which at the sampling time* k *minimizes the upper bound* $V(x(k|k))$ *on* $J_\infty(k)$ *and satisfies the specified input and output constraints is given by*

$$F = YQ^{-1} \tag{27}$$

where, $Q > 0$ *and* Y *are the solutions (if they exist) to the following eigevalue problem (EVP):*

$$\min_{\gamma, Q, Y} \quad \gamma \tag{28}$$

$$\textit{subject to} \quad \begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q \end{bmatrix} \geq 0 \tag{29}$$

$$\begin{bmatrix} Q & QA_i^T + Y^T B_i^T & QQ_1^{\frac{1}{2}} & Y^T R^{\frac{1}{2}} \\ A_i Q + B_i Y & Q & 0 & 0 \\ Q_1^{\frac{1}{2}} Q & 0 & \gamma I & 0 \\ R^{\frac{1}{2}} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \quad i = 1, 2, \ldots, L \tag{30}$$

and either (22) or (21) depending on what input constraint is imposed and either (25,26) or (23) depending on what output constraint is imposed.

Proof: The set of inequalities (30) guarantees that ϵ_Q is an invariant ellipsoid for the future predicted states and hence the arguments used in the translation of the input and output constraints to LMIs hold. The rest of the proof is similar to the proof of Theorem 1. □

Corollary 1 *If the optimization problem in Theorem 2 is feasible at time* k, *then it is feasible for all future times* $t > k$.

Proof: It can be shown that P, F, γ_{min} (or equivalently, Q, Y, γ_{min}) which are optimal solutions at sampling time k are feasible sub-optimal solutions at sampling time $k+1$. This follows from the fact that $\epsilon_Q = \{z | z^T Q^{-1} z \leq 1\} = \{z | z^T P z \leq \gamma\}$ is an invariant ellipsoid for the predicted states $x(k+i|k), i \geq 1$. Thus, at time $k+1$, the optimization has at least one feasible solution and being convex, the problem has a unique optimal solution. □

Corollary 2 *The feasible receding horizon state-feedback law obtained from Theorem 2 is robustly stabilizing.*

Proof: It can be shown using Corollary 1 that $V(x(k)) = x(k)^T P(k) x(k)$ where $P(k)$ is obtained from the optimal solution at time k, is a Lyapunov function for the closed-loop system. □

Remark 2: Suppose the output is to track $C^r x^r$ to the origin where $x^r(k+1) = A^r x^r(k)$ is the reference trajectory. Then, straightforward augmentation of the state and state-space matrices (see [10, page 257]) can be used to reduce this problem to the one in Theorem 2. Similarly, for the nominal system to track a constant set-point, the origin of the state-space can be shifted to the desired steady state (see [10, page 504]). This again reduces the problem to that in Theorem 2. Also, for a class of additive state disturbances $v(k)$ with $lim_{k\to\infty}\|v(k)\| = 0$, the feasible control law from Theorem 2 can be shown to be robustly stabilizing. We skip the details for lack of space.

Remark 3: In the absence of plant uncertainty and input and output constraints, it can be shown that the control law in Theorem 2 is identical to the static state feedback law obtained from the discrete-time linear quadratic regulator (LQR) problem [10, page 494].

4. Examples

Example 1:

Consider the nonlinear system described by the following equations:

$$x(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix}$$
$$= \begin{bmatrix} x_2(k) \\ -ax_2(k) - bx_1(k) - cx_1^3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \qquad (31)$$

The Jacobian $\left\{ \frac{\partial f}{\partial x} \ \frac{\partial f}{\partial u} \right\} = \left\{ \begin{bmatrix} 0 & 1 \\ -b - 3cx_1^2 & -a \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

If we confine the state x_1 within some known apriori bounds $|x_1| \le x_{max}$, then it is easy to verify that

$$\frac{\partial f}{\partial x} \in Co \left\{ \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -b - 3cx_{max}^2 & -a \end{bmatrix} \right\} \qquad (32)$$

and thus, $\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial u} \end{bmatrix} \in Co \left\{ \begin{bmatrix} A_1 & B \end{bmatrix}, \begin{bmatrix} A_2 & B \end{bmatrix} \right\}$

where A_1 and A_2 are matrices given in (32) and $B = [0\ 1]^T$. It can be shown that in this case, every trajectory of the original nonlinear system is a trajectory of some time-varying system whose state-space matrices lie within the polytope given above [11]. The requirement $|x_1| \le x_{max}$ translates to a state constraint.

For the purpose of simulation, we use the following parameters: $a = -3, b = 1, c = -1$ and the initial state $x(0) = \begin{bmatrix} 0.5 & -0.2 \end{bmatrix}^T$. Consequently, we can choose $x_{max} = 0.5$. The goal is to stabilize this unstable system from this initial state, given the input constraint $|u| \le 1$. The simulations are shown in Figure 1. Notice that $|x_1| \le x_{max}$, ensuring that the Jacobian indeed lies within the given polytope. The simulation time on a SUN SPARC 10 workstation was about 9 seconds for the plots shown.

Example 2: The following state-space matrices were obtained by linearization of the modelling equations of a non-isothermal, non-adiabatic continuous stirred tank reactor (CSTR) with an *irreversible first-order reaction* (Uppal *et al.* (1974) [17]).

$$A = \begin{bmatrix} 1.0759 & 0.1382 \\ 0.1036 & 1.0068 \end{bmatrix}, \ B = \begin{bmatrix} 0.1036 \\ 0.0051 \end{bmatrix}, \ C = \begin{bmatrix} -3 \\ -6 \end{bmatrix}^T$$

We will concentrate on this linearized model as the plant. The input constraints are $-0.5 \le u \le 0.5$, with the following weights for the IH-MPC objective function, $Q_1 = \begin{bmatrix} 9 & 18 \\ 18 & 36 \end{bmatrix}$, $R = 1$. Figures 1 and 2 show the system responses with the unconstrained static LQR state feedback, the LMI approach of this paper and the quadratic programming (QP) approach [14] (with five control moves). It is noteworthy that the responses with the LMI and QP approach are equally good. Note however that if there is polytopic uncertainty in the A matrix, then the LMI approach can handle the resulting optimization problem. However, the QP approach [14] must be modified to a corresponding "min-max" QP which is currently computationally intractable.

5. Conclusions

We have presented a new approach to robust model predictive control using linear matrix inequalities. It allows consideration of a wide variety of problems including regulation, trajectory tracking, set-point tracking (nominal case) and disturbance rejection, subject to input and output constraints. More importantly, for an infinite horizon objective function [14, 12], with an explicit plant uncertainty description, the on-line minimization has to be modified to a "min-max" problem which is currently computationally intractable. With our scheme, we can resolve this problem by minimizing an upper bound on the objective function. The on-line optimization can be solved extremely efficiently using polynomial-time algorithms. (see LMI Lab [5])

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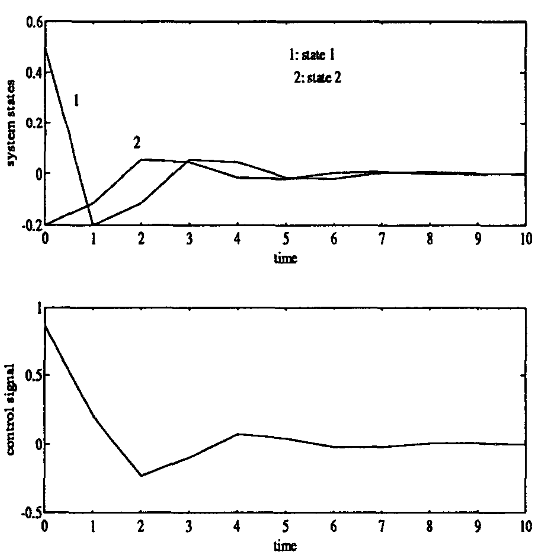

Figure 1: State response and control for the nonlinear system using LMI-based MPC. The input constraint is $-1 \leq u \leq 1$

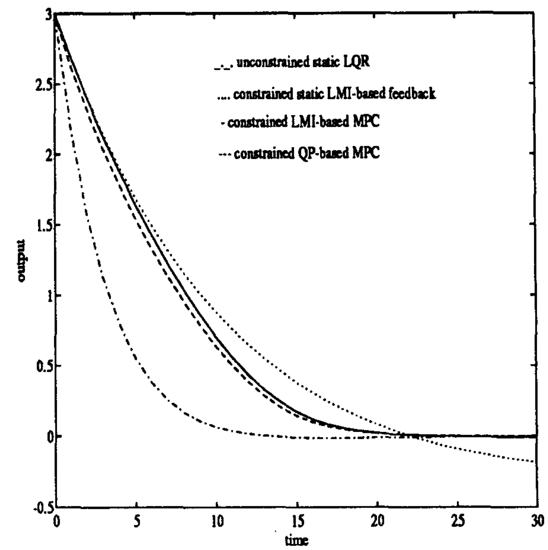

Figure 2: Comparison of outputs using the LQR static state feedback, LMI-based static state feedback, LMI-based MPC and QP-based (five step) MPC approach. The input constraint is $-0.5 \leq u \leq 0.5$

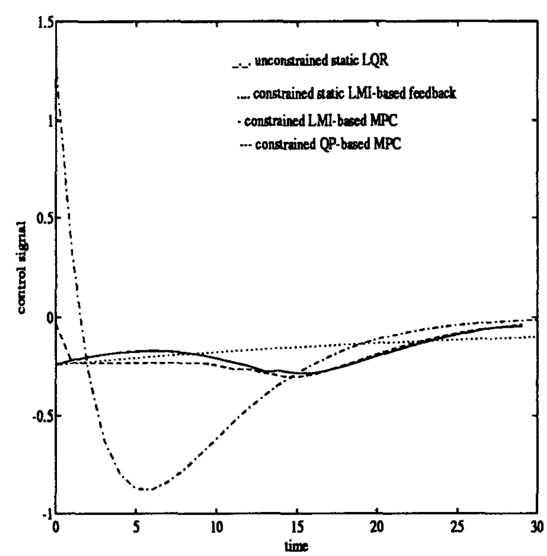

Figure 3: Comparison of control signals corresponding to Figure 2