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DYNAMIC TRACKING OF A WHEELED MOBILE ROBOT

Zhongping Deng and Michael Brady
Robotics Research Group
Department of Engineering Science
University of Oxford
Oxford OX1 3PJ, U.K.

Abstract– Based on ~~the~~ a dynamic model of a wheeled mobile robot (WMR), ~~derived in [8],~~ a dynamic control method is developed so that the WMR tracks a desired trajectory. The full nonlinear dynamics and the kinematic constraints are embodied in the controller, The steering and driving torques are chosen as the control inputs. Examples are presented that demonstrate the effectiveness of the model and the control methods.

1 Introduction

Path planning for nonholonomic mobile robots is currently the subject of active investigation. Although mobile robots have been in use for some time, most attention has been paid to the kinematic modeling and control of such robots[1,4]. Dynamic control of such robots is not only a natural extension of the kinematic approach; it is necessary when the mass of the robot, the load it is carrying and the power needed to drive it have to be considered. In many applications, it is necessary to ensure that a vehicle tracks a commanded trajectory. The dynamics and control of wheeled mobile robots have not been investigated fully, though some early influential work can be recalled:

The Newtonian dynamics of a two-wheeled vehicle were considered in [9] for planning feasible vehicle motions in the plane. The constraints were categorized into path constraints, kinematic constraints and dynamic constraints. Particle dynamics were considered in [6] for motion planning in three dimensions. An optimal control law for a dynamic model of a vehicle for linearized planar motions is presented in [3]. Using the Lagrangian formalism and differential geometry, a general dynamic model is derived for three-wheeled mobile robots with nonholonomic constraints in [2]. A control model for the motion of wheeled mobile robots was developed from the D'Alembert-Lagrangian principle and consideration of the kinematic constraints in [8], the kinematic constraints being treated as part of the control model. Feedback linearization was applied to derive a control law from this model. The torque applied to the steering wheel and the steering angular velocity were the control inputs.

In this paper, based on a dynamic model presented previously, a control method is developed so that the WMR tracks a given trajectory. The full nonlinear dynamics, including the dynamics of the robot and the steering mechanism, as well as nonholonomic kinematic constraints are included in the control design. The control inputs are the steering and drive torques.

The paper is organized as follows: first, the dynamic model of the robot is given. Then the control method is designed in order to control the robot to track a desired trajectory. Next, the nonholonomic kinematic constraints are incorporated in the design of the controller. Finally, the dynamic control inputs, the steering torque and the driving torque, are given and the examples are presented to illustrate the effectiveness of the model and control law.

2 Kinematic constraints

We work with a wheeled mobile robot (WMR)[1,8], which is research version of a fielded, commercial, free-ranging AGV. The WMR consists of a vehicle frame and three wheels, and moves in the horizontal plane. The front wheel is steerable and driven. The rear wheels are idlers. During motion, the plane of each wheel remains vertical and the wheels rotate around their axes. The orientation of the rotation axis of the front wheel can vary with respect to the vehicle frame, and is described by the steering angle α as shown in Fig.1. The rear wheels are fixed with respect to the frame. We assume that the wheels are not deformable and do not slip (perfect rolling) at the contact points with the ground. The notation used in [8] will adopted here.

Cartesian coordinates (x, y) refer to a fixed global reference frame, and reference paths for the frame centre of mass are specified in this frame. The origin of the instantaneously translational (IT) frame (x_1, y_1) is at the mass centre of the vehicle frame and translates with the velocity of the mass centre of the vehicle frame but maintains its axes parallel to the global frame. In this way, the motion of the vehicle is resolved first into a translational motion of the IT frame and then the rotation of the vehicle with respect to IT frame.

Also illustrated in Fig.1 are the vector $\mathbf{r_c} = [x_c, y_c]^\top$, the position of the mass centre of the vehicle frame expressed in the global coordinates, and the angle θ, which is the orientation of the vehicle relative to the IT frame, as well as to the fixed global reference frame. Generally, θ varies over time. Assume that the robot is rigid. The no-slip condition (perfect rolling) at the contact points can be expressed in terms of the velocity $\dot{\mathbf{r}}_\mathbf{c} = [\dot{x}_c, \dot{y}_c]^\top$ and acceleration $\ddot{\mathbf{r}}_\mathbf{c} = [\ddot{x}_c, \ddot{y}_c]^\top$ of the mass centre of the vehicle frame together with the vehicle angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$. For the front steering and rear wheels, we have the following kinematic constraints for the motion of the wheeled mobile robot[8]:

$$l_r\dot{\theta} = -\dot{x}_c \sin\theta + \dot{y}_c \cos\theta \tag{1}$$
$$r_f\dot{\phi}_f = \dot{x}_c \cos(\theta+\alpha) + \dot{y}_c \sin(\theta+\alpha) + l_f\dot{\theta}\sin\alpha \tag{2}$$
$$l_f\dot{\theta}\cos\alpha = \dot{x}_c \sin(\theta+\alpha) - \dot{y}_c \cos(\theta+\alpha) \tag{3}$$
$$r_w\dot{\phi}_r = \dot{x}_c \cos\theta + \dot{y}_c \sin\theta + d\dot{\theta} \tag{4}$$
$$r_w\dot{\phi}_l = \dot{x}_c \cos\theta + \dot{y}_c \sin\theta - d\dot{\theta} \tag{5}$$

where $\dot{\phi}_f$, $\dot{\phi}_r$ and $\dot{\phi}_l$ is the angular velocity of the front, rear right and left wheel relative to its axis; r_f and r_w is the radius of the

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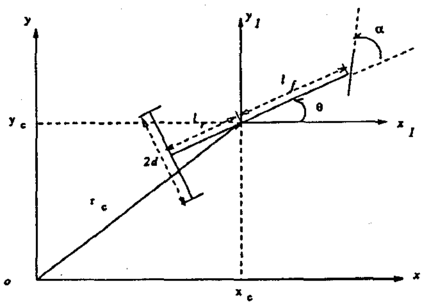

Figure 1: Global (x, y) and IT frame for the vehicle

front and rear wheel; l_f and l_r is the position of the centre of the front and rear wheel relative to the origin of IT frame. From these constraints, we find:

$$
\begin{aligned}
l_r\ddot{\theta} &= -\ddot{x}_c \sin\theta + \ddot{y}_c \cos\theta - \dot{\theta}(\dot{x}_c \cos\theta + \dot{y}_c \sin\theta) && (6)\\
r_f\ddot{\phi}_f &= \ddot{x}_c \cos(\theta+\alpha) + \ddot{y}_c \sin(\theta+\alpha) - l_f\dot{\theta}^2 \cos\alpha \\
&\quad + l_f\ddot{\theta}\sin\alpha && (7)\\
r_f(\dot{\theta}+\dot{\alpha})\dot{\phi}_f &= -\ddot{x}_c \sin(\theta+\alpha) + \ddot{y}_c \cos(\theta+\alpha) \\
&\quad + l_f\dot{\theta}^2 \sin\alpha + l_f\ddot{\theta}\cos\alpha && (8)\\
r_w\ddot{\phi}_r &= \ddot{x}_c \cos\theta + \ddot{y}_c \sin\theta + l_r\dot{\theta}^2 + d\ddot{\theta} && (9)\\
r_w\ddot{\phi}_l &= \ddot{x}_c \cos\theta + \ddot{y}_c \sin\theta + l_r\dot{\theta}^2 - d\ddot{\theta} && (10)
\end{aligned}
$$

From eqn. (8), we find:

$$
\begin{aligned}
& r_f(\dot{\theta}+\dot{\alpha})\ddot{\phi}_f + r_f(\ddot{\theta}+\ddot{\alpha})\dot{\phi}_f \\
= \; & -x_c^{(3)} \sin(\theta+\alpha) - \ddot{x}_c(\dot{\theta}+\dot{\alpha})\cos(\theta+\alpha) \\
+ \; & y_c^{(3)} \cos(\theta+\alpha) - \ddot{y}_c(\dot{\theta}+\dot{\alpha})\sin(\theta+\alpha) \\
+ \; & 2l_f\dot{\theta}\ddot{\theta}\sin\alpha + l_f\dot{\theta}^2\dot{\alpha}\cos\alpha + l_f\theta^{(3)}\cos\alpha \\
- \; & l_f\ddot{\theta}\dot{\alpha}\sin\alpha \qquad (11)
\end{aligned}
$$

3 Lagrangian dynamics of WMR

There are two main methods to establish the system dynamic equations: the Newton-Euler method[6,9] and the Lagrangian method. We adopt the latter as it enables us to incorporate the nonholonomic constraints easily[7,2].

3.1 Dynamic equations

From the D'Alembert-Lagrangian principle, we have[7,8]

$$
\begin{aligned}
& m\ddot{x}_c + \lambda_1\cos(\theta+\alpha) + \lambda_2\sin(\theta+\alpha) - \lambda_5\sin\theta \\
+ \; & (\lambda_3+\lambda_4)\cos\theta = 0 \\
& m\ddot{y}_c + \lambda_1\sin(\theta+\alpha) - \lambda_2\cos(\theta+\alpha) + \lambda_5\cos\theta \\
+ \; & (\lambda_3+\lambda_4)\sin\theta = 0 \\
& J'\ddot{\theta} + J_{f1}\ddot{\alpha} + \lambda_1 l_f\sin\alpha - \lambda_2 l_f\cos\alpha - \lambda_5 l_r \\
+ \; & (\lambda_3-\lambda_4)d = 0 \\
& I_f\ddot{\phi}_f - \tau_f - \lambda_1 r_f = 0; \qquad I_r\ddot{\phi}_r - \lambda_3 r_w = 0 \\
& I_l\ddot{\phi}_l - \lambda_4 r_w = 0; \qquad J_{f1}(\ddot{\theta}+\ddot{\alpha}) = \tau_\alpha \qquad (12)
\end{aligned}
$$

where $m = m_1+m_f+m_r+m_l$ is the total mass of the vehicle, and m_1, m_f, m_r, m_l denote the masses of the vehicle frame, the front wheel, the right rear wheel and the left rear wheel respectively; $J' = J - 2l_r(2m_r l_r - m_f l_f)$ is a constant determined by the masses of the wheels and their positions with respect to the centre of the vehicle frame; $J = J_1 + m_f l_f^2 + J_{f1} + m_r(l_r^2+d^2) + J_{r1} + m_l(l_r^2 + d^2) + J_{r1}$ is the mass moment of inertia of the whole system about the axis passing through the mass centre of the vehicle frame; J_1 is the mass moment of inertia of the vehicle frame about the axis passing through its mass centre and normal to the xy plane; J_{f1}, J_{r1}, J_{l1} are the mass moments of inertia of the front, right rear and left rear wheel about axes passing through the centres of the wheels in the wheel planes; and I_f, I_r, I_l are the mass moments of inertia of the front, right rear and left rear wheels about axes passing through their mass centre and normal to the wheel planes. Note that the mass centre of the vehicle frame is different from the mass centre of the total system.

The Lagrange multipliers $\lambda_1, \cdots, \lambda_5$ can be interpreted physically as constraint forces[8]. The first two equations represent the dynamic equations projected along the x and y directions. The third is the moment equation with respect to the mass centre of the vehicle frame. We reasonably assume that $I_r = I_l = I$, we can solve for the Lagrange multipliers as described in the next section.

3.2 Explicit dynamic solutions

Because eqns. (12) are in closed form for $\lambda_1 - \lambda_5$, τ_f and τ_α, the Lagrange multipliers can be determined when the kinematic parameters are specified. In general, one can solve the algebraic equations numerically. However, we develop analytical solutions here. From eqns. (12), (9) and (10), we have

$$
\begin{aligned}
\lambda_3 &= \tfrac{I}{r_w}\ddot{\phi}_r = \tfrac{I}{r_w^2}(\ddot{x}_c\cos\theta + \ddot{y}_c\sin\theta + l_r\dot{\theta}^2 + d\ddot{\theta}) \\
\lambda_4 &= \tfrac{I}{r_w}\ddot{\phi}_l = \tfrac{I}{r_w^2}(\ddot{x}_c\cos\theta + \ddot{y}_c\sin\theta + l_r\dot{\theta}^2 - d\ddot{\theta})
\end{aligned} \qquad (13)
$$

From the first two lines of eqns. (12), we have

$$
\begin{aligned}
& m(\ddot{x}_c\cos\theta + \ddot{y}_c\sin\theta) + \lambda_1\cos\alpha + \lambda_2\sin\alpha \\
+ \; & \lambda_3 + \lambda_4 = 0 \\
& m[\ddot{x}_c\cos(\theta+\alpha) + \ddot{y}_c\sin(\theta+\alpha)] + \lambda_1 \\
+ \; & (\lambda_3+\lambda_4)\cos\alpha + \lambda_5\sin\alpha = 0 \qquad (14)
\end{aligned}
$$

The two equations are the dynamic equations projected on to the direction of the vehicle motion. Eliminating λ_2, λ_5 from the third equation of the eqns. (14), substituting λ_3 and λ_4 from eqns. (13) and introducing $l_a = l_f + l_r$, which represents the total length of the vehicle, we find:

$$
\begin{aligned}
\lambda_1 \; = \; & -\frac{\ddot{x}_c}{l_a}[l_r m\cos(\theta+\alpha) + c_2\cos\theta\cos\alpha] \\
- \; & \frac{\ddot{y}_c}{l_a}[l_r m\sin(\theta+\alpha) + c_2\sin\theta\cos\alpha] \\
- \; & \ddot{\theta}c_1\frac{1}{l_a}\sin\alpha - \frac{1}{l_a}c_3\dot{\theta}^2\cos\alpha - J_{f1}\ddot{\alpha}\frac{1}{l_a}\sin\alpha \qquad (15)
\end{aligned}
$$

where the constants c_1, c_2 and c_3 are given by:

$$
c_1 = J' + \frac{2I}{r_w^2}d^2; \quad c_2 = l_f m + \frac{2I}{r_w^2}l_a; \quad c_3 = \frac{2I}{r_w^2}l_a l_r
$$

These constants c_i are determined from the system parameters. Again, from the dynamic eqns. (12)

$$
\begin{aligned}
\tau_f &= I_f\ddot{\phi}_f - \lambda_1 r_f \\
\tau_\alpha &= J_{f1}(\ddot{\theta}+\ddot{\alpha}) \qquad (16)
\end{aligned}
$$

In this way, each of the dynamic variables $\lambda_1 - \lambda_5, \tau_f$ can be expressed in terms of the kinematic variables.

4 Dynamic tracking of WMV

4.1 Presentation of tracking problem

If the desired trajectory $\mathbf{r_d}(t) = [x_d(t), y_d(t)]^\top$ is given, the tracking problem is to find the torques $\tau_f(t)$ and $\tau_\alpha(t)$ to ensure that the following relations are satisfied:

$$\lim_{t\to\infty} x_c(t) - x_d(t) = 0; \quad \lim_{t\to\infty} y_c(t) - y_d(t) = 0 \tag{17}$$

where $x_c(t)$ and $y_c(t)$ satisfy the dynamic equations (12) and the kinematic constraints (1)–(5).

Given $\mathbf{r_d}(t) = [x_d(t), y_d(t)]^\top$, and given the initial values $\theta(t_0)$, $\alpha(t_0)$, $\phi_f(t_0)$, $\phi_r(t_0)$ and $\phi_l(t_0)$ of the variables, from the kinematic constraints (1)–(5), their subsequent values can be found by integrating (1)–(5) numerically along the path $[x_d(t), y_d(t)]^\top$. For example, for θ, from (1) we have:

$$l_r\theta_d(t) = l_r\theta(t_0) + \int_{t_0}^{t}(-\dot{x}_d \sin\theta_d + \dot{y}_d \cos\theta_d)dt \tag{18}$$

where subscript d represents the state determined by $\mathbf{r_d}(t)$.

4.2 Control design

We assume that the vehicle $\mathbf{r_c}$ and $\dot{\mathbf{r}}_\mathbf{c}$ are measurable, that the desired trajectory $\mathbf{r_d}$ of the mass centre is known, and that the time derivatives $\dot{\mathbf{r}}_\mathbf{d}, \ddot{\mathbf{r}}_\mathbf{d}$ and $\mathbf{r}_\mathbf{d}^{(3)}$ exist and are finite. Under these assumptions, from eqns. (12), if:

$$\begin{aligned} & \lambda_1\cos(\theta+\alpha) + \lambda_2\sin(\theta+\alpha) + (\lambda_3+\lambda_4)\cos\theta \\ - & \lambda_5\sin\theta = m[\ddot{x}_d - 2\lambda(\dot{x}_c - \dot{x}_d) - \lambda^2(x_c - x_d)] \\ & \lambda_1\sin(\theta+\alpha) - \lambda_2\cos(\theta+\alpha) + (\lambda_3+\lambda_4)\sin\theta \\ + & \lambda_5\cos\theta = m[\ddot{y}_d - 2\lambda(\dot{y}_c - \dot{y}_d) - \lambda^2(y_c - y_d)] \end{aligned}$$

where $\lambda > 0$. Substituting above equations into eqn. (12), we have the exponentially stable closed-loop dynamics:

$$\begin{aligned} (\ddot{x}_c - \ddot{x}_d) + 2\lambda(\dot{x}_c - \dot{x}_d) + \lambda^2(x_c - x_d) = 0 \\ (\ddot{y}_c - \ddot{y}_d) + 2\lambda(\dot{y}_c - \dot{y}_d) + \lambda^2(y_c - y_d) = 0 \end{aligned}$$

The solutions to the equations are:

$$\begin{aligned} x_c &= x_d + e^{-\lambda(t-t_0)}e_x(t_0) \\ &+ (t-t_0)e^{-\lambda(t-t_0)}[\dot{e}_x(t_0) + \lambda e_x(t_0)] \\ y_c &= y_d + e^{-\lambda(t-t_0)}e_y(t_0) \\ &+ (t-t_0)e^{-\lambda(t-t_0)}[\dot{e}_y(t_0) + \lambda e_y(t_0)] \end{aligned} \tag{19}$$

where $e_x = x_c - x_d$, $e_y = y_c - y_d$. Obviously, the solutions satisfy condition (19). As for the kinematic constraints, substituting the solutions x_c and y_c in eqns (20) into eqns. (1)–(5), and integrating them numerically, we can get the solutions of $\theta, \alpha, \phi_f, \phi_r, \phi_l$. For example, substituting eqn. (21) into eqn. (1), we have:

$$\begin{aligned} & l_r\theta(t) = l_r\theta(t_0) - \int_{t_0}^{t}[\dot{x}_d - \lambda e^{-\lambda(t-t_0)}e_x(t_0) \\ + & (1-\lambda(t-t_0))e^{-\lambda(t-t_0)}(\dot{e}_x(t_0) + \lambda e_x(t_0))]\sin\theta dt \\ + & \int_{t_0}^{t}[\dot{y}_d - \lambda e^{-\lambda(t-t_0)}e_y(t_0) \\ + & (1-\lambda(t-t_0))e^{-\lambda(t-t_0)}(\dot{e}_y(t_0) + \lambda e_y(t_0))]\cos\theta dt \end{aligned}$$

Comparing with eqn. (20), it is easy to find:

$$\lim_{t\to\infty}\theta(t) = \theta_d(t)$$

Similar relations exist to α, ϕ_f, ϕ_r and ϕ_l. Substituting these kinematic quantities, which satisfy the kinematic constraints, into the explicit solutions of the dynamic equations, from eqns (16), we can obtain the inputs τ_f and τ_α, which lead to condition (19) to be satisfied.

$$\begin{aligned} \tau_f &= \ddot{x}_c[c_4\cos(\theta+\alpha) + \frac{r_f}{l_a}c_2\cos\theta\cos\alpha \\ &- \frac{c_5}{l_r}\sin\theta\sin\alpha] + \ddot{y}_c[c_4\sin(\theta+\alpha) \\ &+ \frac{r_f}{l_a}c_2\sin\theta\cos\alpha + \frac{c_5}{l_r}\cos\theta\sin\alpha] + \dot{\theta}^2c_6\cos\alpha \\ &- \frac{c_5}{l_r}\dot{\theta}(\dot{x}_c\cos\theta + \dot{y}_c\sin\theta)\sin\alpha + J_{f1}\ddot{\alpha}\frac{r_f}{l_a}\sin\alpha \\ \tau_\alpha &= J_{f1}(\ddot{\theta} + \ddot{\alpha}) \end{aligned} \tag{20}$$

where the constants c_4, c_5 and c_6 are given by:

$$c_4 = \frac{r_f}{l_a}l_r m + \frac{I_f}{r_f}; \quad c_5 = \frac{I_f}{r_f}l_f + \frac{r_f}{l_a}c_1; \quad c_6 = -\frac{I_f}{r_f}l_f + \frac{r_f}{l_a}c_3$$

These kinematic quatities appeared in the right sides of the above equations come from the equations of the kinematic constraints and they are known. In other words, first, from eqn. (21), we have $x_c(t)$, $y_c(t)$ and their derivatives with respect to time. Then substituting them into the kinematic constraints (1)–(5), and integrating numerically, other kinematic variables, like $\theta, \alpha, \phi_f, \phi_l, \phi_r$, can be found. Finally, the control inputs can be found by eqn. (22).

5 Examples and conclusion

We examine a number of examples, in which we assume that the vehicle geometry and mass parameters are known. In the first example, the desired trajectory is a constant acceleration from rest:

$$x_d(t) = \frac{1}{2}at^2; \qquad y_d(t) = 0.0$$

It is obvious that if $x_c = \frac{1}{2}at^2$, we find the constant control torque τ_f from eqn. (22):

$$\tau_f = a(c_4 + \frac{r_f}{l_a}c_2) = a(r_f m + \frac{I_f}{r_f} + \frac{r_f}{r_w^2}2I)$$

The orientation angle $\theta = 0$, $\alpha = 0$ and $\tau_\alpha = 0$, consistent with elementary physics.

In the second example, the vehicle moves along a circle of radius R. The trajectory is:

$$x_d(t) = R(1-\cos t); \quad y_d(t) = R\sin t \quad 0 \le t \le \pi$$

If $x_c = x_d$; $y_c = y_d$, from eqn. (7) we find:

$$l_r\dot{\theta} = R\cos(\theta + t)$$

This equation has the solution: $\theta = -t + \frac{\pi}{2} + \theta_0$. and the initial condition θ_0 is given by: $\sin\theta_0 = l_r/R$. In other words, the orientation angle of the vehicle has a linear solution when it moves along a semi-circular path. From eqn. (3) the steering angle α satisfies:

$$-l_f \cos\alpha = R\sin(\theta_0 + \alpha)$$

Substituting θ_0, we find: $\tan\alpha = -l_a/(R\cos\theta_0) = const.$ From eqn. (22), we find $\tau_\alpha = 0$. From eqn. (22) and θ, we find: $\tau_f = 0$. Because this is the motion of equal velocity along the semicircle, the applied torque is equal to zero when the work done by the force of the friction is zero. We assume that the motion of the vehicle follows the desired trajectory exactly, otherwise we can not find $\tau_f = 0$.

The dynamic control of a three-wheel, front wheel steering and driving mobile robot has been studied considering its fully non-holonomic constraints. The steering mechanism and the driving dynamics are included in the design of the control method, which keeps the robot tracking a desired trajectory. Robustness of the control law will be studied in the future.

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