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Globally Asymptotically Stable Tracking Control of Mobile Robots

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Abstract

In this paper, we discuss about the design of globally asymptotically stable tracking control law for mobile robot with nonholonomic velocity constraints. The stability of the controller is derived by Lyapunov direct method. And we analyze the system responses according to the variation of control parameters in line tracking problem. It is showed that the output responses represent non-oscillatory property in line tracking. The two-wheeled mobile robot and car-like mobile robot are selected for example and the simulation results represent the effectiveness of our method. And there is a plan to experiment the path following of mobile robot.

1. Introduction

Recently, many researchers investigate various tracking control methods considering the nonholonomic constraints of mobile robots. Y. Kanayama et al. proposed a stable tracking control law using Lyapunov direct method, and mentioned the uniformly asymptotically stable concept using Lyapunov's linearization method[4], and C. Canudas et al. showed the GAS law using Lyapunov direct method and Barbalat's lemma [2], C. Samson and K. Ait-Abderrahim investigated the time varying feedback control[6,7], O. J. Sordalen and C. Canudas de Wit presented a piecewise smooth controller[10], R. M. Murray and S. Sastry proposed chained form for path planning and control[5], G. Walsh et al. presented a linear time varying control from linearization of system about nominal trajectory[11], H. S. Shin et al. proposed a variable structure control scheme[8], B. d'Andrea-Novel et al. investigated the tracking control with stability of a reference trajectory by means of linearizing static and dynamic state feedback [1].

These previous works showed the stable tracking control laws for mobile robots, but few results showed the globally asymptotic stability by means of Lyapunov direct method. The time varying and the discontinuous feedback approaches are more difficult, and they may give rise to erratic or unnatural motions.

In this paper, we consider the nonlinear tracking control problem of mobile robots using Lyapunov theory with globally asymptotic stability. The previous works using Lyapunov direct method considered only the quantity of position error states of control system, but we didn't consider only the position error states but also the posture (orientation) of mobile robot since it was necessary to consider the movable direction of mobile robot under nonholonomic constraint. And we analyze the system responses according to the variation of controller parameters in line tracking problem. We apply the proposed control scheme to the two types of mobile robots ; two-wheeled mobile robot and car-like mobile robot. It is showed that the output responses represent non-oscillatory property in line tracking and effectiveness of our method. And we have a plan to carry out the experiment of path following of two-wheeled mobile robot.

2. Kinematics of Mobile Robots

We assume the mobile robots have nondeformable wheels, furthermore, we suppose that they are moving on a horizontal plane without slip to hold the nonholonomic constraint. The schematic diagram of mobile robots are presented in Figure 1. The difference between the two-wheeled model and car-like model(rear wheels drive) is that the angular velocity of system body can no longer be considered as an independent control variable in car-like model [7].

We formulate the mobile robots with error state equations. Figure 2. represents the concept of error posture, which is a transformation of the reference posture P_r in a local coordinate system with an origin of P_c and an x-axis in the direction of θ_c. This is the difference between P_r and P_c :

$$\mathbf{P}_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} (\mathbf{P}_r - \mathbf{P}_c) \tag{1}$$

And the error state equations of the two-wheeled mobile robot,

$$\dot{x}_e = y_e u_2 - u_1 + v_r \cos\theta_e$$
$$\dot{y}_e = -x_e u_2 + v_r \sin\theta_e \tag{2}$$
$$\dot{\theta}_e = \omega_r - u_2$$

are obtained with simple calculation where $\mathbf{P}_e=[x_e\ y_e\ \theta_e]^T$ is error state vector, $\mathbf{P}_r=[x_r\ y_r\ \theta_r]^T$ is desired state vector, $\mathbf{P}_c=[x_c\ y_c\ \theta_c]^T$ is current state vector, v_r and ω_r are desired forward velocity and rotational velocity respectively and u_1, u_2 are forward velocity and angular velocity respectively(control inputs). For car-like mobile robot,

$$\dot{x}_e = y_e \omega - u_1 + v_r \cos\theta_e$$
$$\dot{y}_e = -x_e \omega + v_r \sin\theta_e \tag{3}$$
$$\dot{\theta}_e = \omega_r - \omega, \quad \omega = \frac{1}{\mathrm{L}}\tan\phi\, u_1$$
$$\dot{\phi} = u_2$$

are given where ϕ is steering angle, L is length of mobile robot, the u_1 and u_2 are forward velocity and steering velocity respectively(control inputs). We use these error state equations in Lyapunov direct method.

3. Tracking Control law and Stability

In this section, we propose a globally asymptotically stable(GAS) tracking control law from Lyapunov direct method. The derivations of control laws for two-wheeled and car-like mobile robots are almost same, and are obtained directly. We complete the GAS tracking control law with adding a term which represents the orientation of mobile robot to a previous Lyapunov function which considers only the quantity of error states.

Consider the error state equations of two-wheeled mobile robot (2). Following theorem shows the GAS tracking control law for the system.

Theorem 1. For the two-wheeled mobile robot, $\mathbf{P}_e=\mathbf{0}$ is globally asymptotically stable equilibrium point if

$$u_1 = K_x x_e + v_r\cos\theta_e - K_\theta\theta_e u_2$$
$$u_2 = \omega_r + \frac{v_r}{2}\{K_y(y_e + K_\theta\theta_e) + \frac{1}{K_\theta}\sin\theta_e\} \tag{4}$$

where K_x, K_y, K_θ are positive constants and $v_r > 0$.

Proof) Let us select a Lyapunov function candidate like as

$$\mathrm{V}(x_e) = \frac{1}{2}x_e^2 + \frac{1}{2}(y_e + K_\theta\theta_e)^2 + \frac{1}{K_y}(1-\cos\theta_e) \tag{5}$$

Definitely, $\mathrm{V}(\mathbf{x}_e)>0$ at $\mathbf{P}_e\neq\mathbf{0}$ and $\mathrm{V}(\mathbf{x}_e)=0$ at $\mathbf{P}_e=\mathbf{0}$ ($x_e=y_e=\theta_e=0$), consequently $\mathrm{V}(\mathbf{x}_e)$ is positive definite. The time derivative of $\mathrm{V}(\mathbf{x}_e)$,

$$\begin{aligned}
\dot{\mathrm{V}}(\mathbf{x}_e) &= x_e\dot{x}_e + (y_e + K_\theta\theta_e)(\dot{y}_e + K_\theta\dot{\theta}_e) + \frac{1}{K_y}\sin\theta_e\,\dot{\theta}_e \\
&= x_e\dot{x}_e + (y_e + K_\theta\theta_e)\dot{y}_e + \{K_\theta(y_e + K_\theta\theta_e) + \frac{1}{K_y}\sin\theta_e\}\dot{\theta}_e \\
&= x_e(y_e\omega - u_1 + v_r\cos\theta_e) + (y_e + K_\theta\theta_e)(-x_e\omega + v_r\sin\theta_e) \\
&\quad + \{K_\theta(y_e + K_\theta\theta_e) + \frac{1}{K_y}\sin\theta_e\}(\omega_r - \omega) \\
&= -x_e(u_1 - v_r\cos\theta_e + K_\theta\theta_e\omega) + v_r(y_e + K_\theta\theta_e)\sin\theta_e \\
&\quad + \{K_\theta(y_e + K_\theta\theta_e) + \frac{1}{K_y}\sin\theta_e\}(\omega_r - \omega) \\
&= -K_x x_e^2 - \frac{v_r}{2}K_y K_\theta(y_e + K_\theta\theta_e)^2 - \frac{1}{2v_r K_y K_\theta}\sin^2\theta_e
\end{aligned} \tag{6}$$

is negative definite. because $\dot{\mathrm{V}}(\mathbf{x}_e)<0$ at $\mathbf{P}_e\neq\mathbf{0}$ and $\dot{\mathrm{V}}(\mathbf{x}_e)=0$ at $\mathbf{P}_e=\mathbf{0}$ ($x_e=y_e=\theta_e=0$). Therefore, $\mathbf{P}_e=\mathbf{0}$ is globally asymptotically stable equilibrium point [8]. ■

In the previous researches, $\mathrm{V}(\mathbf{x}_e)$ is positive definite but $\dot{\mathrm{V}}(\mathbf{x}_e)$ does not contain all of error states, i.e. $\dot{\mathrm{V}}(\mathbf{x}_e)$ is negative semi-definite. That is to say, it means the locally asymptotically stable concept. Hence, Y. Kanayama et al. mentioned the uniformly asymptotically stable concept using Lyapunov's linearization method[4], and C. Canudas et al. derived the GAS concept using Barbalat's lemma [2]. However, in this paper, we showed GAS control law more simply with the $\dot{\mathrm{V}}(\mathbf{x}_e)$ containing all of error states.

When we consider the physical meaning of the derivation of control law, the Lyapunov function (5) does not include only the absolute value of each error state but also the posture of mobile robot. The term ($y_e + K_\theta\theta_e$) represents the posture of mobile robot. The more mobile robot face to desired position, the less the value of ($y_e + K_\theta\theta_e$) increases. Consequently, the proposed control law drive the mobile robot to face to desired position. Therefore, we complete the GAS tracking control law with considering the posture of mobile robot.

For car-like mobile robot, consider the error state equations (3). Following theorem shows the GAS tracking control law for the system.

Theorem 2. For the car-like mobile robot, $\mathbf{P}_e=\mathbf{0}$ is globally asymptotically stable equilibrium point if

$$u_1 = K_x x_e + v_r\cos\theta_e - K_\theta\theta_e\omega$$
$$\omega = \omega_r + \frac{v_r}{2}\{K_y(y_e + K_\theta\theta_e) + \frac{1}{K_\theta}\sin\theta_e \tag{7}$$

where K_x, K_y, K_θ are positive constants and $v_r > 0$.

We can obtain the control input u_2 from differentiation of ω using following relationship;

$$\omega = \frac{1}{L}\tan\phi\, u_1 \tag{8}$$
$$\dot{\phi} = u_2$$

The proof of theorem 2 is same with that of theorem 1.

4. Response Analysis and Simulation

Response Analysis

In this section, we analyze the output response according to the controller design parameters, K_X, K_Y, K_θ when we apply the proposed GAS control law. In order to simplify the analysis, we consider only line tracking situation at a constant velocity v_r and ω_r=0. The analysis is carried out using the eigenvalues of system matrix of close-loop system linearized at $x_e=y_e=\theta_e=0$.

When we substitute (4) for (2), the error state equations of closed-loop system,

$$\dot{x}_e = -K_x x_e + (y_e + K_\theta\theta_e)\left[\omega_r + \frac{v_r}{2}\left\{K_y(y_e + K_\theta\theta_e) + \frac{1}{K_\theta}\sin\theta_e\right\}\right]$$
$$\dot{y}_e = -x_e\left[\omega_r + \frac{v_r}{2}\left\{K_y(y_e + K_\theta\theta_e) + \frac{1}{K_\theta}\sin\theta_e\right\}\right] + v_r\sin\theta_e$$
$$\dot{}_e = -\frac{v_r}{2}\left\{K_y(y_e + K_\theta\theta_e) + \frac{1}{K_\theta}\sin\theta_e\right\} \tag{9}$$

are derived. And

$$\mathbf{A} = \begin{bmatrix} -K_r & 0 & 0 \\ 0 & 0 & v_r \\ 0 & -\frac{v_r K_y}{2} & -\frac{v_r}{2}(K_y K_\theta + \frac{1}{K_\theta}) \end{bmatrix} \tag{10}$$

is the system matrix of linearized system of (9). The eigenvalues of the system matrix **A**, $\lambda_{1,2,3}$ are obtained as follows;

$$\lambda_1 = -K_x$$
$$\lambda_{2,3} = \frac{v_r}{4}\left[-(K_y K_\theta + \frac{1}{K_\theta}) \pm \{(K_y K_\theta + \frac{1}{K_\theta})^2 - 8K_y\}^{\frac{1}{2}}\right] \tag{11}$$

We can analyze the transient response of closed-loop system from the location of complex poles, since the location of complex poles represents the damping ratio, ζ, of transient response near the reference trajectory. The calculation of the location of poles proves that the poles are always in the area not exceeding ±45° in the left-half s-plane with no reference to the variation of parameters . That is, $\zeta \le$ cos45° is guaranteed, and $\zeta \ge 0.707$. It means that the transient response of line tracking always converges without oscillation regardless of gain parameters. Therefore, we need not be careful to choose the gain parameters to reduce oscillation or overshoot.

Simulation Results

We carry out the simulation about the application of the proposed control laws to the two types of mobile robots. Firstly we observe the response characteristics for variation of controller parameters of the two-wheeled mobile robot case. It is shown that it is possible to design a different settling time in spite of same transient response form. And subsequently, the tracking responses of mobile robots are presented. The reference velocity v_r=5m/s (=18km/h), and we observe the response for 10 seconds. We restrict the control inputs as follows ;

$|u_1| \le$ 10 m/s (=36 km/h)
$|u_2| \le$ 180°/s for two-wheeled mobile robot.
$\le$ 90°/s for car-like mobile robot.

Firstly, we design two parameter sets. The first set is K_X=8, K_Y=0.4, K_θ=0.6 and second set is K_X=8, K_Y=1, K_θ=0.4. Both of parameter sets make the damping ratio, ζ, be almost 1, critical damping. The difference of two sets is just weighting, that is to say, second set has more weighting on y_e-axis position to approach faster. The simulation results on the variation of parameters are shown in Figure 3 and 4 respectively. They show the non-oscillatory responses and the results of above intention. In Figure 4, the settling time is a little faster because we give more weighting to y_e-axis position to approach faster. Hence, we can design the different approaching characteristics with same transient characteristics near reference trajectory.

And we simulate the two types of mobile robots for line tracking and sine wave tracking cases to verify the performance of proposed control scheme. The parameter set for the car-like mobile robot is K_x=8, K_y=0.14, K_θ=1.1. and L=1. Figure 3 to 7 present the line tracking and sine wave tracking responses of the mobile robots. They present asymptotic stable tracking performance and non-oscillatory responses. When we ignore the limits of the control inputs, we can observe that the tracking of various trajectories is always possible for variation of parameters. However, when we restrict the control input as above, it is necessary to give attention to choose the controller parameters. As above results, the proposed GAS controllers for the mobile robots guarantee the stable response and convergence to the reference trajectory without oscillation.

And we prepare the experiment of path following of two-wheeled mobile robot using R/F control system and vision system as presented in Figure 8. The two-wheeled mobile robot is small size which is cubical type with 7.5 × 7.5 × 7.5 cm. The mobile robot consists of control board, R/F receiver and actuators. We design the control board using 80c196 microprocessor, and use the mini DC servo motor for actuator with encoder and gear reduction. The experimental results will be given in a forthcoming paper.

5. Conclusion

In this paper, we proposed the globally asymptotically stable(GAS) tracking control law using Lyapunov direct method. The GAS tracking control law is completed by considering not only the absolute amount of position error state but also the orientation of mobile robots. Consequently, the proposed control law drives the mobile robot to face to desired position. And it is shown that the proposed method guarantees the non-oscillatory output response. And, the simulation results show the stable convergence responses of two-wheeled and car-like mobile robots.

In the further study, we have a plan to carry out the experiment of path following of two-wheeled mobile robot using R/F control system and vision system.

6. References

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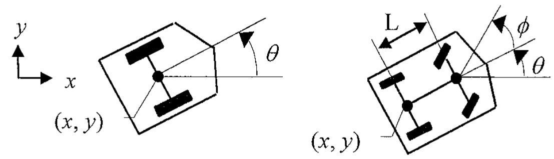

(a) two-wheeled mobile robot (b) car-like mobile robot
Fig. 1. The schematic diagram of 2 types mobile robots

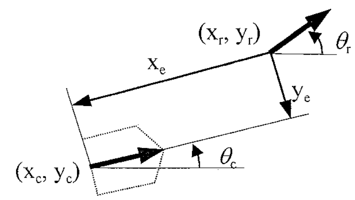

Fig. 2. Error posture of mobile robot

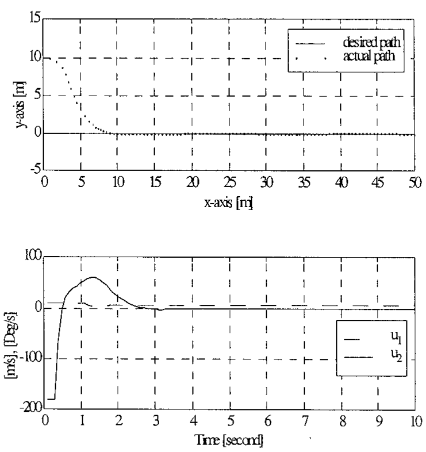

Fig. 3. Line tracking of two-wheeled mobile robot
(K_x=8, K_y=0.4, K_θ=0.6)

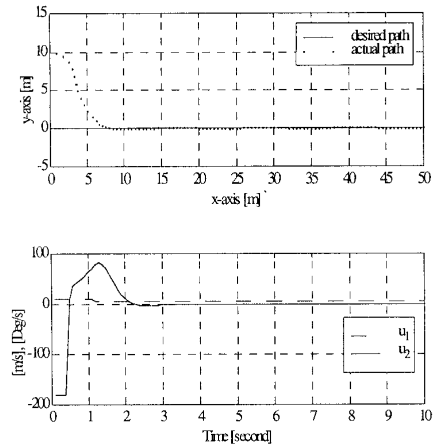

Fig. 4. Line tracking of two-wheeled mobile robot (K_x=8, K_y=1, K_θ=0.4)

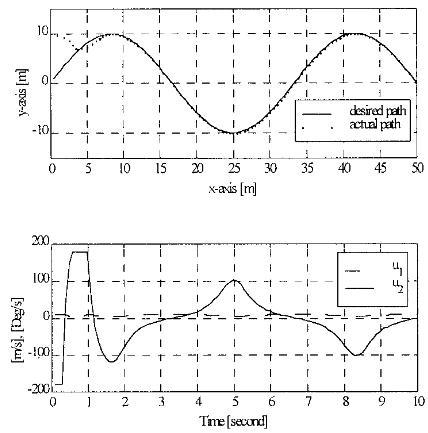

Fig. 5. Sine wave tracking of two-wheeled mobile robot

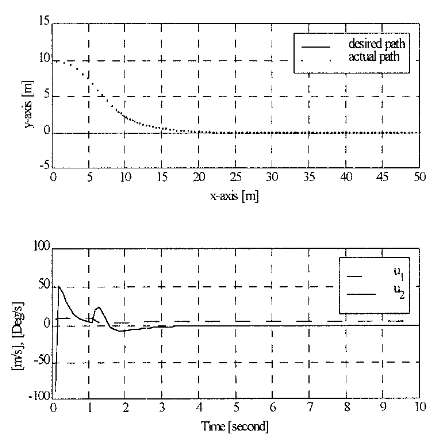

Fig. 6. Line tracking of car-like mobile robot (K_x=8, K_y=0.14, K_θ=1.1)

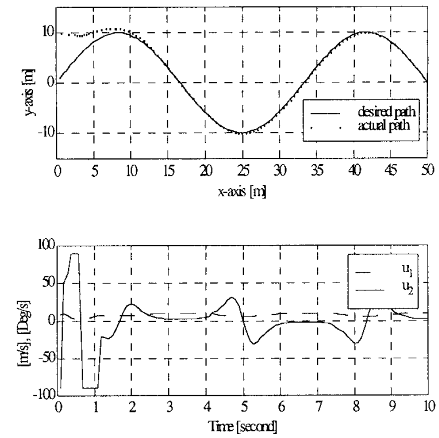

Fig. 7. Sine wave tracking of car-like mobile robot (K_x=8, K_y=0.14, K_θ=1.1)

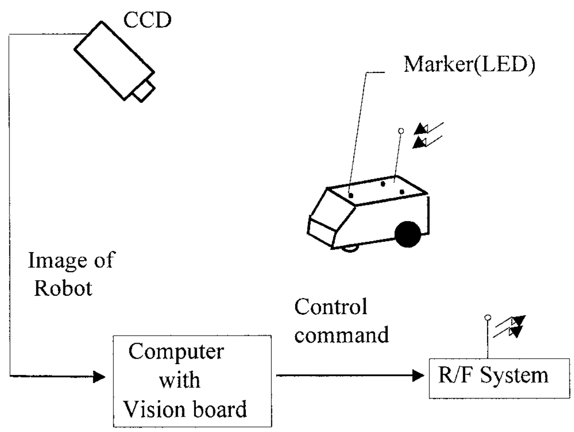

Fig. 8. The feature of experiment for path following of the two-wheeled mobile robot