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Dynamic Feedback Linearization of Nonholonomic Wheeled Mobile Robots

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Abstract

Nonholonomic mechanical systems are known to be in general not stabilizable at equilibrium points by means of smooth state feedback. Nevertheless, smooth time varying laws can solve this stabilization problem.

On the other hand, we show that by means of **dynamic** state feedback, it is possible for 3-wheeled mobile robots to track arbitrary fast trajectories not reduced to equilibrium points.

Section 1 is devoted to some preliminaries about dynamical modelling of nonholonomic mechanical systems. Section 2 particularizes the case of 3-wheeled mobile robots (with free, steering or omnidirectional wheels). In Section 3 we briefly recall the idea of the **dynamic extension algorithm** which leads to full linearization of 3-wheeled mobile robots (with a free or steering wheel). Moreover, dynamic feedback will allow to solve the tracking problem for an omnidirectional mobile robot having less motors than degrees of freedom. This is possible by choosing "output functions" depending on the mass repartition of the robot. This result is quite analogous to the one obtained in [2] for a class of rigid manipulators having less motors than degrees of freedom.

Keywords : Nonholonomic mechanical systems, Mobile robots, Dynamic extension algorithm, Dynamic feedback linearization.

1 Dynamical modelling of nonholonomic mechanical systems

In this section, we consider mechanical systems whose configuration is completely described by a n-vector

$$q = (q_1, \cdots, q_n)'$$

of generalized coordinates and which can be subjected to m kinematic independent **non holonomic** (i.e. non integrable) constraints ($m < n$) of the form :

$$A'(q)\dot{q} = 0 \text{ with } A'(q) = (a_1(q), \cdots, a_m(q))' \qquad (1)$$

where $a_1, \cdots, a_m$ are smooth linearly independent vector fields on $\mathbb{R}^n$. The number of degrees of freedom is usually defined as the difference $n - m$.

Using the Euler-Lagrange formalism, the dynamical behavior of a nonholonomic mechanical system can be written as follows :

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = A(q)\lambda + B(q)u \qquad (2)$$

where $M(q)$ denotes the $n \times n$ inertia matrix (symmetric positive definite), $g(q)$ the n-vector of gravity torques, $C(q, \dot{q})\dot{q}$ the n-vector of centrifugal and Coriolis torques, $A(q)$ is given by (1) and λ is the m-vector of Lagrange multipliers associated to the constraints., Finally, $B(q)$ is a $n \times p$ matrix and u is the p-vector of external forces or torques applied to the system, we will suppose $p = n - m$.

Let us now introduce a set of $n - m$ smooth vector fields $s_1(q), \cdots, s_{n-m}(q)$, such that, denoting the full rank $n \times (n - m)$ matrix $S(q) = (s_1(q), \cdots, s_{n-m}(q))$ we have :

$$A'(q)S(q) = 0 \qquad (3)$$

The constraints (1) and (3) imply the existence of a $(n - m)$-vector $\eta(q, \dot{q})$ satisfying :

$$\dot{q} = S(q)\eta \qquad (4)$$

As explained in [1,4], premultiplying (2) by $S'(q)$ and using (3), we eliminate the Lagrange multipliers to obtain :

$$S'(q)\left[M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q)\right] = S'(q)B(q)u \qquad (5)$$

Then differentiating (4), substituting in (5) and assuming that the $(n-m) \times (n-m)$ matrix $S'(q)B(q)$ has full rank (which is realistic) we can choose a suitable static feedback law u such that system (2) becomes (see for example [1] for details) :

$$\begin{cases} \dot{q} = S(q)\eta \\ \dot{\eta} = v \end{cases} \qquad (6)$$

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The structure of system (6) only depends on the constraints and we have the following results concerning its controllability properties.

Proposition 1 *[5] Under the assumption of nonholonomy, system (6) is Small Time Locally Controllable at $(q, 0)$, namely the attainable set from any equilibrium point $(q, 0)$ contains a neighborhood of $(q, 0)$.*

In contrast with the linear case, the links between controllability and stabilizability are not clear for nonlinear dynamics. In spite of the STLC property, from Brockett's necessary conditions for smooth feedback stabilizability, we have the following negative result concerning nonholonomic mechanical systems :

Proposition 2 *[4] Any equilibrium point $(q, 0)$ of system (6) cannot be made asymptotically stable by a smooth state feedback law.*

Nevertheless, stabilizing **time varying** control laws have been obtained for such robots (see for example [13,8,12]).
In the next section we will study a particular class of nonholonomic mechanical systems, made of 3-wheeled mobile robots.

2 Description and modelling of 3-wheeled mobile robots

Mobile robots constitute a typical example of nonholonomic systems (see e.g. [3,13,1,6,11]). We consider here a robot moving on an horizontal plane, constituted by a rigid trolley equipped with non deformable wheels. The contact between the wheels and the ground satisfies the conditions of **pure rolling and non slipping**. The motion of the robot is achieved by actuators which provide torques acting on the rotation and/or the orientation of the axis of some of the wheels.
Consider an inertial reference frame $\{0, I_1, I_2\}$ in the plane of motion. Define a reference point Q on the trolley, and a basis $\{x_1, x_2\}$ attached to the trolley. The position of the trolley in the plane is completely specified by the following 3 variables :

- x , y : the coordinates of the reference point Q in the inertial frame,
- θ : the orientation of the basis $\{x_1, x_2\}$ with respect to the inertial basis.

We define the vector ξ as :

$$\xi = (x \;\; y \;\; \theta)' \tag{7}$$

We can characterize the position of a particular wheel by the following set (see Fig. 1 and [1] for details) :

$$\{R_i \, , \, l_i \, , \, d_i \, , \, \alpha_i \, , \, \gamma_i \, , \, \beta_i \, , \, \phi_i \; ; \; i = 1 \, , \cdots , \, 3\}$$

Denoting $R(\theta)$ the (3×3) orthogonal rotation matrix :

$$R(\theta) = \begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8}$$

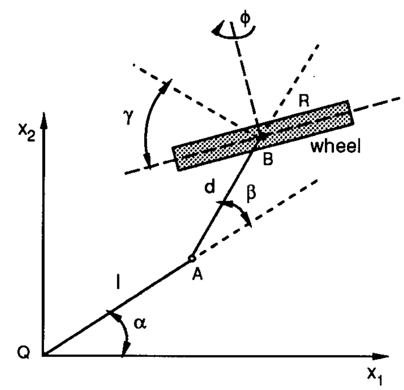

Figure 1: Characterization of a wheel

with the previous description of the wheels, it becomes easy to explicit the pure rolling and non slipping conditions, namely :

- The pure rolling conditions, i.e. the fact that the component of the velocity of the contact point of the wheel with the ground in the plane of the wheel is zero, can be written :

$$J_1(\beta)R(\theta)\dot{\xi} + J_2\dot{\beta} + J_3\dot{\phi} = 0 \tag{9}$$

with :

$$\begin{cases} (J_1)_{i1}(\beta) = -sin(\alpha_i + \beta_i + \gamma_i) \\ (J_1)_{i2}(\beta) = cos(\alpha_i + \beta_i + \gamma_i) \\ (J_1)_{i3}(\beta) = d_i cos\gamma_i + L_i cos(\beta_i + \gamma_i) \, , \; i = 1, 3 \\ J_2 = \begin{pmatrix} d_1 cos\gamma_1 \\ d_2 cos\gamma_2 \\ d_3 cos\gamma_3 \end{pmatrix} \\ J_3(i,i) = R_i \, , \; i = 1, \cdots, 3 \\ J_3(i,j) = 0 \, , \text{ if } i \neq j \end{cases} \tag{10}$$

- The **non slipping** conditions, i.e. the fact that the component of the velocity of the contact point, orthogonal to the plane of the wheel is zero, are of the form :

$$C_1(\beta)R(\theta)\dot{\xi} + C_2\dot{\beta} = 0 \tag{11}$$

where C_1 and C_2 are given by :

$$\begin{cases} (C_1)_{i1}(\beta) = cos(\alpha_i + \beta_i + \gamma_i) \\ (C_1)_{i2}(\beta) = sin(\alpha_i + \beta_i + \gamma_i) \\ (C_1)_{i3}(\beta) = d_i sin\gamma_i + L_i sin(\beta_i + \gamma_i) \, , \; i = 1, 3 \\ C_2 = \begin{pmatrix} d_1 sin\gamma_1 \\ d_2 sin\gamma_2 \\ d_3 sin\gamma_3 \end{pmatrix} \end{cases} \tag{12}$$

We note that these constraints (9)-(11) are in the general form of kinematical constraints (1).

2.1 The case of a robot with a free wheel

We use a lower index notation to identify the quantities relative to each wheel. We consider here the particular case where the 2 front wheels (index 2 and 3) have a fixed orientation but the two motors provide the torques for their rotation, while the orientation of wheel 1 is varying, wheel 1 being self-aligning. The reference point Q is the center of the segment B_2B_3 (see Fig. 2). The basis vector x_1 is aligned with B_2B_3. The geometric characteristics are :

$$\begin{cases} R_1 = R_2 = R_3 = R \ , \ \ l_1 = l_2 = l_3 = L \\ d_1 = d \ , \ d_2 = d_3 = 0 \\ \alpha_1 = \dfrac{3\pi}{2} \ , \ \alpha_2 = 0 \ , \ \alpha_3 = \pi \\ \beta_1 = \beta \ , \ \beta_2 = \beta_3 = 0 \ \ , \ \ \gamma_1 = \dfrac{\pi}{2} \ , \ \gamma_2 = \gamma_3 = 0 \end{cases}$$

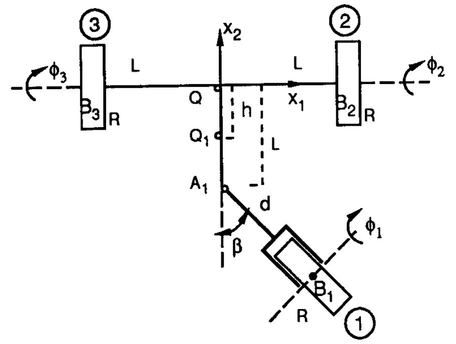

Figure 2: A robot with a free wheel

The robot motion is then completely described by the following vector of 7 generalized coordinates :

$$q(t) = (x \ y \ \theta \ \beta \ \phi_1 \ \phi_2 \ \phi_3)' \tag{13}$$

Using (9)-(10) and (11)-(12)the **pure rolling** and **non slipping** conditions can be deduced. We easily check that the matrix $A(\beta,\theta)$ has rank 5, consequently this robot has 2 degrees of freedom. Moreover, using the notations of section 1, we can compute the matrix $S(q)$ defined in (3) (see [1]). The two actuators providing the rotation of wheels 2 and 3, it is then easy to check that the input matrix $S'(q)B(\beta)$ has full rank and we can apply the general method described in section 1, to obtain the form (6) which can be reduced in this particular case, noticing that $\dot\beta$ and $\dot\phi$ are uniformly bounded provided that η is bounded :

$$\begin{cases} \dot x = -\eta_1 sin\theta \\ \dot y = \eta_1 cos\theta \\ \dot\theta = \eta_2 \\ \dot\eta_1 = v_1 \\ \dot\eta_2 = v_2 \end{cases} \tag{14}$$

2.2 The case of a robot with a steering wheel

The description of the wheels is quite analogous to that of Section 2.1, except for wheel 1 which is described as follows (see Fig. 4) :

$$R_1 = R,\ l_1 = L,\ d_1 = 0,\ \alpha_1 = \frac{3\pi}{2},\ \beta_1 = \beta,\ \gamma_1 = 0.$$

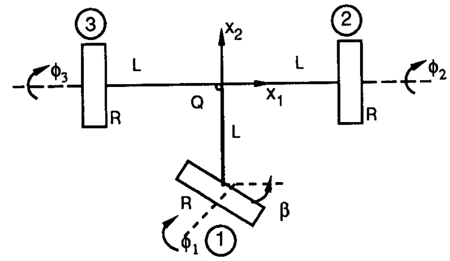

Figure 3: A robot with a steering wheel

The robot motion is also described by the following vector of 7 generalized coordinates :

$$q(t) = (x \ y \ \theta \ \beta \ \phi_1 \ \phi_2 \ \phi_3)' \tag{15}$$

The **pure rolling** and **non slipping** conditions can be obtained as previously and it is easy to check that this robot has also 2 degrees of freedom and to compute the corresponding matrix $S(q)$. Moreover, the 2 motors providing both orientation and rotation of wheel 1, the matrix $S'(q)B(\beta)$ has full rank and we can apply once more the general method described in section 1, to obtain the representation (6) which can be written in reduced form in this particular case, noticing that $\dot\phi_i, i = 1, \cdots 3$, are uniformly bounded provided that η is bounded :

$$\begin{cases} \dot x = -Lsin\beta sin\theta\eta_1 \\ \dot y = Lsin\beta cos\theta\eta_1 \\ \dot\theta = cos\beta\eta_1 \\ \dot\beta = \eta_2 \\ \dot\eta_1 = v_1 \\ \dot\eta_2 = v_2 \end{cases} \tag{16}$$

2.3 The case of a robot with omnidirectional wheels

Several full size realizations of such robots have been built, two typical examples being the 4-wheels URANUS robot (see [11]) and the 3-wheels UCL robot (see [6]) studied here. The perfect mobility of this kind of robots comes from the particular feature of the wheels which are made up of free rollers and for which only pure rolling constraints have to be considered. The geometric characteristics are :

$$\begin{cases} R_i = R \ , \ \ l_i = L \ , \ \ d_i = \beta_i = \gamma_i = 0 \ , \ i = 1,3 \\ \alpha_1 = \dfrac{\pi}{3} \ , \ \alpha_2 = \pi \ , \ \alpha_3 = \dfrac{5\pi}{3} \end{cases}$$

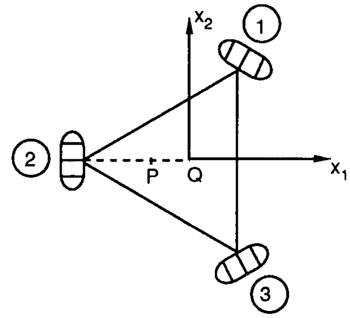

Figure 4: A robot with omnidirectional wheels

The robot motion is then described by the following vector of 6 generalized coordinates :

$$q(t) = (x \ y \ \theta \ \phi_1 \ \phi_2 \ \phi_3)' \tag{17}$$

Using once again (9)-(10), the **pure rolling** conditions can be deduced. It is easy to check that the 3x6-matrix $A(\theta)$ associated with the constraints has rank 3. Consequently, the robot has 3 degrees of freedom.
We assume that the wheels are identical and that the point Q in Figure 5 is the center of mass of the unique body made of the robot and the 3 wheels, with mass m and inertia I around the axis Qx_3 orthogonal to (x_1, x_2). This body is then described by the $\xi = (x, y, \theta)'$ coordinates, the position ϕ_i $i = 1, \cdots, 3$, of the wheels satisfying the pure rolling constraints. The dynamical behavior of this body takes consequently the following form (see [6]) :

$$MR(\theta)\ddot{\xi} = -J_1' J_3^{-1} u \tag{18}$$

with :

$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix}$$

$J_1' J_3^{-1}$ being invertible, it is easy to check that the 3-dimensional control vector u can be chosen to obtain a linear decoupled behavior of the form :

$$\ddot{\xi} = v \tag{19}$$

Consequently system (18) is linearizable via **static** feedback when each wheel or degree of freedom is directly controlled. In the next section we will examine the case when only 2 degrees of freedom in system (18) are controlled, this situation could occur when a motor breaks down. In that case linearization will be yet possible by means of **dynamic** feedback.

3 Full linearization by means of dynamic state feedback

We consider a dynamical system given in general state space form :

$$\dot{X} = f(X) + \sum_{i=1}^{m} g_i(X) u_i \tag{20}$$

where the state X is in $\mathbb{R}^n$, the input u in $\mathbb{R}^m$. The vector fields f and g_i are smooth.
We can allow static state feedback laws of the form :

$$u(X) = \alpha(X) + \beta(X)v \tag{21}$$

α being a smooth m-vector of the state, β a non singular $m \times m$ matrix of the state and v an auxiliary input. β being non singular u is also called a **regular** static state feedback.
Necessary conditions for (20) to be completely linearizable by diffeomorphism and feedback can be found e.g. in [10].
Nevertheless, full linearization can possibly be realized by considering more general dynamic feedback laws of the form :

$$\begin{cases} u = \alpha(X, \zeta) + \beta(X, \zeta) v \\ \dot{\zeta} = a(X, \zeta) + b(X, \zeta) v \end{cases} \tag{22}$$

We will obtain such a dynamic feedback through the choice of m suitable "output functions"

$$Y_i = h_i(X) \ , \ i = 1, \cdots, m \tag{23}$$

We use on system (20),(23) the dynamic extension algorithm (see for example [9]). Namely, the idea of this algorithm is to delay some "combinations of inputs" simultaneously affecting several outputs, via the addition of integrators, perhaps to enable other inputs to act in the meanwhile and therefore perhaps to obtain an extended decoupled system of the form :

$$Y_k^{\rho_k} = w_k \ , \ k = 1, \cdots, m \tag{24}$$

Y_k^i denotes the i-th derivative of Y_k w.r.t. time, ρ_k is called the relative degree of Y_k and w_k denotes the new auxiliary inputs. Moreover, in order to get **full** linearization we must have for the n_e-dimensional extended system :

$$\sum_{i=1}^{m} \rho_i = n_e \tag{25}$$

where n_e is the dimension of the original system plus the number of added integrators.
The problem of finding sufficient conditions for dynamic feedback linearization is quite open. Nevertheless, a simple necessary condition for a system to be dynamically feedback linearizable in the neighborhood of an equilibrium point X_0 is given in [7] :

Proposition 3 *If a system is dynamically feedback linearizable in the neighborhood of an equilibrium point X_0, then its tangent linearization at X_0 is controllable, in the linear sense.*

3.1 The case of a robot with a free wheel

We are interested in this section to the reduced form (14) describing the dynamical behavior of a robot with a free wheel. We have proved (see for example [1,5]) that only partial linearization via static feedback was possible for system (14). The stability of the non linearized subsystem, and particularly the angle θ, could not be ensured for general reference trajectories.
Moreover, it is an easy computation to check that the tangent linearization of system (14) is not controllable at any equilibrium point $X_0 = (x_0, y_0, \theta_0, 0, 0)'$.
Nevertheless, the following result shows that system (14) is generically dynamic feedback linearizable. This can give a method to track trajectories in x, y, and θ coordinates.

Proposition 4 *By considering x and y as output functions, system (14) is fully dynamic feedback linearizable.*

Proof :
It is an easy computation from (14) to check that the only input appearing in $\ddot{x}$ and $\ddot{y}$ is v_1. So applying the dynamic extension algorithm, we delay v_1, i.e. we introduce a new input w_1 such that :

$$\dot{v}_1 = w_1 \tag{26}$$

Then computing $x^{(3)}$ and $y^{(3)}$, the extended system with extended state vector

$$\bar{X} = (x,\, y,\, \theta,\, \eta_1,\, \eta_2,\, v_1)'$$

is linearizable by static state feedback $w_1(\bar{X})$ and $v_2(\bar{X})$, and diffeomorphism :

$$\xi = (x,\, y,\, \dot{x},\, \dot{y},\, \ddot{x},\, \ddot{y})' \tag{27}$$

since :

$$\left\{ \begin{array}{l} x^{(3)} = -w_1 sin\theta - v_2\eta_1 cos\theta - 2\eta_2 v_1 cos\theta + \eta_2^2\eta_1 sin\theta \\ y^{(3)} = w_1 cos\theta - v_2\eta_1 sin\theta - 2\eta_2 v_1 sin\theta - \eta_2^2\eta_1 cos\theta \end{array} \right. \tag{28}$$

We can remark that the static linearizing feedback law is singular when $det \begin{pmatrix} -sin\theta & -\eta_1 cos\theta \\ cos\theta & -\eta_1 sin\theta \end{pmatrix}$ is zero, namely when $\eta_1 = \dot{y}cos\theta - \dot{x}sin\theta$ is zero. We conclude that dynamic linearization is impossible at any equilibrium point since η_1 is obviously zero. This is not surprising from proposition 3 since the tangent linearization of (14) is not controllable at any equilibrium point $X_0 = (x_0, y_0, \theta_0, 0, 0)'$ as previously mentioned.
On the other hand, dynamic feedback linearization is possible for every reference trajectory such that η_1 is not zero. □
This example has also been considered in [14].

3.2 The case of a robot with a steering wheel

We are interested in this section to the reduced form (16) describing the dynamical behavior of a robot with a steering wheel. This system is also only partially linearizable via static feedback and its tangent linearization is not controllable at any equilibrium point $X_0 = (x_0, y_0, \theta_0, \beta_0, 0, 0)'$.
Nevertheless, the following result shows that system (16) is generically dynamic feedback linearizable, and as in the case of the robot with a free wheel, it will be possible to track suitable reference trajectories.

Proposition 5 *By considering x and y as output functions, system (16) is fully dynamic feedback linearizable.*

Proof :
It is an easy computation from (16) to check that the only input appearing in $\ddot{x}$ and $\ddot{y}$ is v_1. So applying the dynamic extension algorithm, we delay v_1, namely we introduce a new input w_1 such that :

$$\dot{v}_1 = w_1 \tag{29}$$

Then computing $x^{(3)}$ and $y^{(3)}$, another combination of inputs w_2 appears, namely :

$$w_2 = \eta_1 cos\beta v_2 + sin\beta w_1 \tag{30}$$

We delay w_2

$$\dot{w}_2 = w_3 \tag{31}$$

Considering now w_1 and w_3 as input variables, and computing $x^{(4)}$ and $y^{(4)}$, we show easily that the extended system with extended state vector $\bar{X} = (x,\, y,\, \theta,\, \beta,\, \eta_1,\, \eta_2,\, v_1,\, w_2)'$ is linearizable by static state feedback $w_1(\bar{X})$ and $w_3(\bar{X})$, and diffeomorphism :

$$\xi = (x,\, y,\, \dot{x},\, \dot{y},\, \ddot{x},\, \ddot{y},\, x^{(3)},\, y^{(3)})' \tag{32}$$

since, denoting $c\theta = cos\theta$ and $s\theta = sin\theta$:

$$\left\{ \begin{array}{l} x^{(4)} = w_1(-a(\bar{X})s\theta + b(\bar{X})c\theta) - Lw_3 s\theta + f_1(\bar{X}) \\ y^{(4)} = w_1(a(\bar{X})c\theta + b(\bar{X})s\theta) + Lw_3 c\theta + f_2(\bar{X}) \\ \eta_1 \neq 0 \;\;,\;\; \beta \neq \frac{\pi}{2} \bmod \pi \end{array} \right. \tag{33}$$

We can remark that the static linearizing feedback law will be singular when $\eta_1 sin\beta cos\beta$ is zero. Once more, we conclude that dynamic feedback linearization is impossible at any equilibrium point since η_1 is obviously zero. This is not surprising from proposition 3 since the tangent linearization of (16) is not controllable at any equilibrium point $X_0 = (x_0, y_0, \theta_0, \beta_0, 0, 0)'$ as previously mentioned.
On the other hand, dynamic linearization is possible for every reference trajectory such that $\eta_1 sin\beta cos\beta$ is not zero. □

3.3 The case of an omnidirectional robot with less motors than degrees of freedom

Without loss of generality we can suppose that wheel 2 is not controlled. System (18) then becomes :

$$\left\{ \begin{array}{l} \ddot{\xi} = \frac{1}{R} R'(\theta) \begin{pmatrix} \frac{1}{m} & 0 \\ 0 & \frac{1}{2m} \\ 0 & \frac{L}{I} \end{pmatrix} v \;\; \text{where} \\ v_1 = \frac{\sqrt{3}}{2}(u_1 - u_3) \\ v_2 = u_1 + u_3 \end{array} \right. \tag{34}$$

The following proposition is then satisfied :

Proposition 6 *By considering h_1 and h_2 as "output functions" :*

$$\left\{ \begin{array}{l} h_1 = x - d cos\theta \\ h_2 = y - d sin\theta \ \ with \\ d = \frac{I}{2mL} \end{array} \right. \tag{35}$$

system (34) is fully dynamic feedback linearizable.

Proof :
Computing $\ddot{h}$ we obtain :

$$\begin{array}{l} \ddot{h} = \begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix} w_1 \\ \text{where } \ w_1 = \frac{1}{mR} v_1 + d\dot{\theta}^2 \end{array} \tag{36}$$

Differentiating $\ddot{h}$ twice and delaying w_1 twice :

$$\left\{ \begin{array}{l} \dot{w}_1 = \psi_1 \\ \dot{\psi}_1 = w_2 \end{array} \right. \tag{37}$$

we obtain :

$$\begin{array}{l} h^{(4)} = v_2 \frac{L}{RI} w_1 \begin{pmatrix} -sin\theta \\ cos\theta \end{pmatrix} + w_2 \begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix} \\ + 2\psi_1 \dot{\theta} \begin{pmatrix} -sin\theta \\ cos\theta \end{pmatrix} - \dot{\theta}^2 w_1 \begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix} \end{array} \tag{38}$$

Considering w_2 and v_2 as input variables we achieve full linearization for the extended system with extended state vector

$$\bar{X} = (x,\, y,\, \theta,\, \dot{x},\, \dot{y},\, \dot{\theta},\, w_1,\, \psi_1)'$$

by static state feedback $w_2(\bar{X})$ and $v_2(\bar{X})$, and diffeomorphism :

$$\xi = (h_1,\, h_2,\, \dot{h}_1,\, \dot{h}_2,\, \ddot{h}_1,\, \ddot{h}_2,\, h_1^{(3)},\, h_2^{(3)})' \tag{39}$$

the linearizing feedback law is singular when w_1 is zero, from (36) the linearization is also not possible at equilibrium points $X_0 = (x_0,\, y_0,\, \theta_0,\, 0,\, 0,\, 0,\, 0,\, 0)'$. Once more this is not surprising since the tangent linearization of system (34) is not controllable when the tangent linearization of system (18) (with each wheel controlled) is controllable and therefore system (18) is static feedback linearizable as previously mentioned. □

Remark 1 *h_1 and h_2 are in fact the cartesian coordinates in the reference frame $\{0, I_1, I_2\}$ of a point P on the x_1 axis, P having coordinates $(-d, 0)$ in the (x_1, x_2) basis attached to the trolley, see Fig. 4.*

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