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ROBUST MODEL PREDICTIVE CONTROL FOR NONLINEAR SYSTEMS

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Abstract

A technique is introduced for the design of a robustly stabilizing model predictive output feedback controller for continuous time-varying nonlinear constrained systems. The controller is easily implementable and computationally inexpensive. A family of first order approximations of the plant is used for computation of the optimal control moves, which allows the optimization problem to be posed as a quadratic programming problem (QP). Through the use of a "stability constraint" the resulting control algorithm can be made asymptotically stable and is able to handle unknown time-varying disturbances, measurement bias and structural model/plant mismatch.

1. Introduction

Some of the key features contributing to the success of model predictive control (MPC) are:

(1) Process constraints can be incorporated directly into the on-line optimization performed at each time step.

(2) Ability to handle multivariable systems.

(3) Variable structure in the event of faults.

Model predictive control algorithms can be divided into the following three categories (some representative references are given in parentheses):

(1) finite prediction horizon for:
- linear plants [5, 12];
- nonlinear plants [1, 2, 9, 11];

(2) infinite prediction horizon for:
- linear plants [24];
- nonlinear plants [21];

(3) finite prediction horizon with end constraints (also known as "stability constraints") for:
- linear plants [4, 14, 22];
- nonlinear plants [3, 6, 7, 8, 13, 15, 16, 17, 18, 19].

Keerthi and Gilbert [13] and Mayne and Michalska [19] require the controller to solve exactly, at each time instant t, a finite horizon nonlinear control problem with the terminal constraint $x(t+T)=0$, where T is the horizon.

Mayne and Michalska [15] introduce a relaxed version of the stability constraint (SC), $x(t+T) \in W$ (where W is some neighborhood of the origin). Since the terminal constraint has been relaxed, the strategy loses its stabilizing properties inside W. To compensate for this effect, a linear, locally stabilizing controller designed for the linearized system is used inside W. The resulting "hybrid" controller is shown to be globally stabilizing.

All MPC algorithms which use nonlinear ordinary differential equation (ODE) models for prediction of future outputs lead to non-convex optimal control problems. Thus, the optimization may not admit a practical solution. Also, in the "hybrid" type of control law the determination of the set W is not an easy task. For general nonlinear systems this problem is unsolved.

Here these problems are overcome, first, by using linear approximations of the plant so that the optimization step can be formulated as a convex problem and, second, by changing the stability constraint so that the calculation of a final region of attraction is avoided. Furthermore, if the constraint is incorporated into the optimization step as a penalty in the objective function, the optimal control problem can be reduced to a quadratic programming problem.

2. Basic Philosophy of the Controller Design

The MPC algorithm proposed in this paper represents an attempt to combine the best features of the quadratic dynamic matrix control (QDMC) algorithm for nonlinear systems proposed by Garcia in [11] and the nonlinear programming MPC technique with end constraint introduced by Mayne and Michalska in [15, 16, 17, 18, 19]. The control technique introduced here aims for a combination of features like simplicity in the implementation and efficient computation accompanied by a set of conditions which, if satisfied, guarantee certain stability and robustness properties of the closed-loop system.

As in Garcia's work [11], a linearized model of the plant is used for calculation of the optimal control moves and the linearization is updated at the beginning of each sampling interval. The advantage of using a linear model in the computations is that the optimal control problem is convex. On the other hand, the main drawback of Garcia's strategy is that stability cannot be guaranteed since the control parameters are adjusted on a trial and error basis.

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The control technique proposed here incorporates a so-called "stability constraint" into the formulation of the optimal control problem in the same spirit as in [15, 16, 17, 18, 19]. This additional constraint makes the stability and robustness analysis of the control algorithm possible and sufficient conditions on the nonlinearities, disturbances and state estimation errors can be derived under which the closed-loop system is stable.

If the classic quadratic objective function is used, the stability constraint, which is quadratic, can be incorporated as a penalty in the objective function and, in this way, the optimization step is reduced to a simple QP (given that the other constraints are linear).

Let M denote the control horizon, let R be the number of steps ahead at which the SC is defined (with $R \leq M$) and, without loss of generality, let the sampling time be equal to 1. Besides, let $x^p(k|k)$ and $x^m(k+R|k)$ denote the states of the plant at $t = k$ and of the linear model at $t = k + R$ using the plant measurement at $t = k$, respectively. Then, the stability constraint can be expressed as:

$$\| x^m(k+R|k) \|_P \leq \alpha \| x^p(k|k) \|_P, \quad \text{with } \alpha \in (0,1) \tag{1}$$

where P is some symmetric, positive definite matrix and $\| x \|_P \doteq \sqrt{x^T P x}$ represents the weighted *2-norm* of the vector $x \in \Re^n$ by P. The authors have chosen to denominate this stability constraint as *contraction constraint*.

The optimal control problem at $t = k$ and $x^p(k|k)$ is solved and only the first control move is implemented. Then the states of the plant are measured (or estimated) and a new optimization problem is solved with the *same* stability constraint and control horizon. This process is repeated until R control steps are implemented, when the stability constraint is finally satisfied by the true states of the model rather than the predicted states as in the preceding steps.

At $t = k + R$ new measurements (or estimates) are obtained and a new stability constraint is defined at $t = k + 2R$ for the next R steps in terms of this new initial condition, $x^p(k+R|k+R)$. Thus, one can easily notice that the only difference between this and Garcia's strategy is the additional state constraint which remains the same and is moved only every R control steps.

By the description of the algorithm one can easily see that if the first optimal control problem (OCP) is feasible, the subsequent $R-1$ control problems are feasible as well since the feasibility of the first optimization implies that there exists a sequence of control moves which satisfy the stability constraint for the given initial condition. The subsequent $R-1$ optimizations represent improvements over this first solution because they take into account the plant measurements (or estimates) at each sampling interval.

3. Description of the System

The time-varying nonlinear systems considered in this paper are described by the following equations:

$$\dot{x}^p(t) = f(x^p(t), u(t), d(t)) \tag{2}$$

$$y^p(t) = g(x^p(t)) + n(t) \tag{3}$$

where $x^p(t) \in \Re^n$ is the vector of states, $u(t) \in \Re^m$ represents the manipulated variables, $d(t) \in \Re^d$ is the vector of unknown disturbances, $y^p(t) \in \Re^r$ is the vector of outputs and $n(t) \in \Re^r$ represents possible measurement bias. Both g and f are assumed to be continuously differentiable functions.

It is assumed that $d(t) \in D$ and $n(t) \in V$, $\forall t \in [0, \infty)$, with the sets D and V being defined as:

$$D \doteq \{d(t) \in L_1^d[0,\infty) \mid \| d(t) \|_1 \leq \epsilon_d, \forall t \in [0,\infty)\}$$

$$V \doteq \{n(t) \in L_1^r[0,\infty) \mid \| n(t) \|_1 \leq \epsilon_n, \forall t \in [0,\infty)\}$$

where $\epsilon_d, \epsilon_n \in [0, \infty)$ are known constant values and $\| . \|_1$ is the *1-norm*.

Besides, the control constraints are given by:

$$U \doteq \{u(t) \in L_1^m(0,\infty) \mid \| u(t) \|_1 \leq \epsilon_u, \forall t \in [0,\infty)\}$$

where $\epsilon_u \in (0, \infty)$ is a known constant value.

4. MPC algorithm

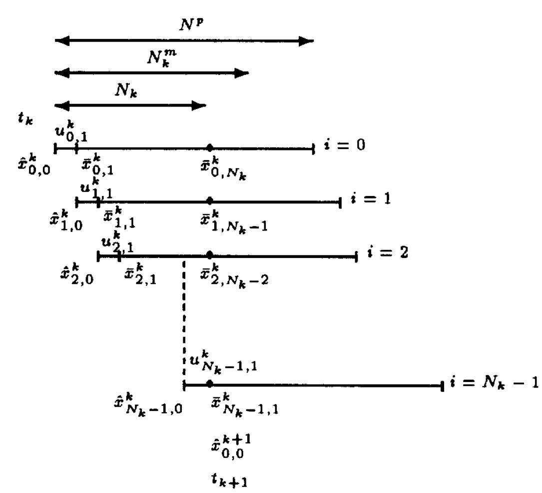

Figure 1: Iteration k of the control algorithm.

Legend:

1. $\bar{x}^k(t) \doteq$ are the states obtained from the linearization of the system at $(\hat{x}^k_{0,0}, u^k_{0,0})$.
2. $\hat{x}^k(t) \doteq$ are the estimates of the states of the plant, $x^{p,k}(t)$.
3. $\bullet \Rightarrow$ marks the location of the stability constraint for $t \in [t_k, t_{k+1}]$.

DATA:

1. $t_0 \doteq$ initial time for computations.
2. $T \doteq$ sampling interval.
3. $N_{max} \doteq$ maximum number of control moves allowed.
4. $N^p \doteq$ number of prediction steps ($N^p \geq N_{max}$).
5. $\delta^* \doteq$ estimation horizon for computation of the estimate of the states at t_0.
6. the input variable history during the time interval $[t_0 - \delta^*, t_0]$, $u(t) \doteq u_{[t_0-\delta^*, t_0]}$.
7. the corresponding output variable history during the time interval $[t_0 - \delta^*, t_0]$, $y^p(t) \doteq y^p_{[t_0-\delta^*, t_0]}$.
8. estimate of the plant states at $t = t_0 - \delta^*$, $\hat{x}(t_0 - \delta^*)$.

STEP 0:

1. Set $k = 0$.
2. Using $\hat{x}(t_0 - \delta^*)$, $u_{[t_0-\delta^*,t_0]}$, $y^p_{[t_0-\delta^*,t_0]}$ and the following observer equations,

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{f}(\hat{x}(t), u(t)) + \\ &+ \hat{K}(t)[y^p(t) - \hat{y}(t)] \end{aligned} \quad (4)$$

$$\hat{y}(t) = \hat{g}(\hat{x}(t)), \quad (5)$$

with $\hat{x}(t) \in \Re^n, \hat{y}(t) \in \Re^r, \hat{K}(t) \in \Re^{n\times r}$, $\hat{f}: \Re^n \times \Re^m \rightarrow \Re^n$ and $\hat{g}: \Re^n \rightarrow \Re^r$, compute the estimate of the plant states at $t = t_0$, $\hat{x}^0_{0,0} \doteq \hat{x}(t_0)$.

The stability properties of the observer represented by equations (4) and (5) were explored by de Oliveira and Morari in [8]. This will not be discussed here due to the lack of space.

3. Define $u^0_{0,0} \doteq u(t_0)$.

STEP 1:

1. Select the desired rate of contraction of the states of the linear model at iteration k, $\alpha_k \in (0,1)$; the weight of the contraction constraint in the objective function, $\beta_k \in (0, \infty)$, and the bounds on the manipulated variable at iteration k, $\{u^k_{min}, u^k_{max}\} \in \Re^m$, with $\epsilon_u \geq \max\{\| u^k_{min} \|_1, \| u^k_{max} \|_1\}$.
2. Set $i = 0$.

STEP 2: At $t = t_k + iT$ solve the following "Free-end optimal control problem", $OCP(\hat{x}^k_{i,0}, t_k + iT)$:

Objective function:

$$\begin{aligned} min_{u^k_i(t)} \quad & \left\{ \int_{t_k+iT}^{t_k+(i+N^p)T} J^k_i(t)dt \right\} + \\ + \quad & \beta_k \| \bar{x}^k_i(t_k + N_k T) \|^2_{P_k} \end{aligned} \quad (6)$$

with:

$$J^k_i(t) \doteq \| x^k_i(t) \|^2_{Q_k} + \| u^k_i(t) \|^2_{R_k} + \| q^k_i(t) \|^2_{S_k} \quad (7)$$

where P_k, Q_k, R_k, S_k are symmetric, positive definite weighting matrices; N_k is the number of moves ahead of the initial condition $\hat{x}^k_{0,0}$ where the stability constraint is located and $u^k_i(t) = u^k_i(t_k + (i + N^m_k)T)$, $\forall t \in [t_k + (i + N^m_k)T, \ t_k + (i + N^p)T]$. N^m_k is the control horizon satisfying $N_k \leq N^m_k \leq N_{max} \leq N^p$, $\forall k \in N$.

Discrete version:

$$\begin{aligned} \min_{(u^k_{i,1}, \ldots, u^k_{i,N^m_k})} & \{ \sum_{j=1}^{N_p} [\| x^k_{i,j} \|^2_{Q_k} + \| u^k_{i,j} \|^2_{R_k} + \\ + & \frac{\| \Delta u^k_{i,j} \|^2_{S_k}}{T^2} \} + \beta_k \| \bar{x}^k_{i,N_k-i} \|^2_{P_k} \end{aligned} \quad (8)$$

where $\Delta u^k_{i,j} \doteq u^k_{i,j} - u^k_{i,(j-1)}$, $j = 1, \ldots, N^m_k$ and $\Delta u^k_{i,j} = 0, \forall j \in [N^m_k + 1, \ldots, N^p]$.

Trajectory Constraints:

$$\delta \dot{x}^k_i(t) = A^k_i \delta x^k_i(t) + B^k_i \delta u^k_i(t) \quad (9)$$

$$q^k_i(t) = \dot{u}^k_i(t) \quad (10)$$

and

$$\dot{\bar{x}}^k_i(t) - \hat{x}^k_{0,0} = A^k_0(\bar{x}^k_i(t) - \hat{x}^k_{0,0}) + B^k_0(u^k_i(t) - u^k_{0,0}) \quad (11)$$

where:

$$A^k_i \doteq \frac{\partial f}{\partial x}(\hat{x}^k_{i,0}, u^k_{i,0}, 0)$$

$$B^k_i \doteq \frac{\partial f}{\partial u}(\hat{x}^k_{i,0}, u^k_{i,0}, 0)$$

Discrete version:

$$\delta x^k_{i,(j+1)} = \Phi^k_i \delta x^k_{i,j} + \Gamma^k_i \delta u^k_{i,(j+1)} \quad (12)$$

$$q^k_{i,(j+1)} = \frac{u^k_{i,(j+1)} - u^k_{i,j}}{T} \quad (13)$$

and

$$\dot{\bar{x}}^k_{i,j} - \hat{x}^k_{0,0} = A^k_0(\bar{x}^k_{i,j} - \hat{x}^k_{0,0}) + B^k_0(u^k_{i,j} - u^k_{0,0}) \quad (14)$$

where $j \in [0, N^p - 1]$, $\delta x^k_{i,j} \doteq x^k_{i,j} - \hat{x}^k_{i,0}$, $\delta u^k_{i,(j+1)} \doteq u^k_{i,(j+1)} - u^k_{i,0}$, $x^k_{0,0} \doteq \bar{x}^k_{0,0} \doteq \hat{x}^k_{0,0}$, $x^k_{i,0} \doteq \hat{x}^k_{i,0}$, $\bar{x}^k_{i,0} \doteq \bar{x}^k_{(i-1),1}$, and the matrices Φ^k_i and Γ^k_i are given by:

$$\Phi^k_i \doteq e^{A^k_i T}$$

$$\Gamma^k_i \doteq \int_0^T e^{A^k_i \tau} B^k_i d\tau$$

Control Constraints:

$$u^k_{min} \leq u^k_i(t) \leq u^k_{max}, \quad (15)$$

Discrete version:

$$u^k_{min} \leq u^k_{i,j} \leq u^k_{max}, \quad j = 1, \ldots, N^m_k \quad (16)$$

Time Constraints:

$t \in [t_k + iT, t_k + (i + N^p)T], i \in [0, N_k - 1], k \in N.$

The set of matrices $\{P_k\}_{k\in N}$ satisfies the following Lyapunov equations:

$$\begin{aligned}(A^k_{cl,0})^T P_k &+ P_k A^k_{cl,0} = -\hat{Q}_k \doteq -[Q_k + \\ &+ (K_k)^T R_k K_k + (A^k_{cl,0})^T (K_k)^T \\ & S_k \; K_k \; A^k_{cl,0}]\end{aligned} \tag{17}$$

where $A^k_{cl,0} \doteq A^k_0 + B^k_0 K_k$ and the feedback gains $\{K_k\}_{k\in N}$ are chosen so that $\{A^k_{cl,0}\}_{k\in N}$ have all their eigenvalues in the left half plane.

Since the objective function (6) is used as a Lyapunov function for the linear closed-loop system, the derivation of equation (17) follows directly from the form of this function.

Result of $OCP(\hat{x}^k_{i,0}, t_k + iT)$: For the feasible number of moves $N_k \leq N_{max}$, the optimal discrete control sequence $(u^k_{i,1}, \ldots, u^k_{i,N^m_k})$ is obtained.

<u>**STEP 3**</u>: Check if the desired rate of contraction is obtained, i.e.,

$$\| \bar{x}^k_{i,N_k - i} \|_{P_k} \leq \alpha_k \| \hat{x}^k_{0,0} \|_{P_k} \tag{18}$$

If (18) is satisfied, proceed to **STEP 4**. Otherwise, repeat **STEP 2** with a bigger value of β_k.

<u>**STEP 4**</u>: Apply the first element of the discrete sequence of optimal control moves, $u^k_{i,1}$, to the plant during the interval $[t_k + iT, t_k + (i+1)T]$ and measure the corresponding output response at $t = t_k + (i+1)T$, $y^{p,k}_{(i+1),0}$.

<u>**STEP 5**</u>:

1. Using $\hat{x}^k_{i,0}, u^k_{i,1}$ and $y^{p,k}_{(i+1),0}$, compute the estimate of the states of the plant at $t = t_k + (i+1)T$, $\hat{x}^k_{(i+1),0}$.
2. Set $i = i + 1$.
3. Define $u^k_{i,0} \doteq u^k_{(i-1),1}$. While $i < N_k$, go back to **STEP 2**.

<u>**STEP 6**</u>: Set $k = k + 1$. Go back to **STEP 1**.

<u>Remark 1:</u> Ideally, N_k should be the smallest number of moves for which feasibility of $OCP(\hat{x}^k_{0,0}, t_k)$ holds and the contraction constraint is satisfied (which is called the optimal horizon). However, in practice, one does not need to be so rigorous, i.e., N_k may be larger than the optimal horizon as long as the stability conditions which will be derived later hold.

<u>Remark 2:</u> Note that because all constraints are linear and the objective function is quadratic, the optimal control problem is a QP.

5. Basic Assumptions

The basic assumptions on the plant are:

A(1) (A^k_i, B^k_i) is a stabilizable pair for all $i \in [0, N_k - 1]$, $\forall k \in N$.

A(2) (A^k_i, C^k_i) is a detectable pair for all $i \in [0, N_k - 1]$, $\forall k \in N$.

A(3) The observer is designed so that the following conditions are satisfied:

$$f(\hat{x}^k_{i,0}, u^k_{i,0}, 0) = \hat{f}(\hat{x}^k_{i,0}, u^k_{i,0}) \tag{19}$$

$$A^k_i = \frac{\partial \hat{f}}{\partial x}(\hat{x}^k_{i,0}, u^k_{i,0}) \tag{20}$$

$$B^k_i = \frac{\partial \hat{f}}{\partial u}(\hat{x}^k_{i,0}, u^k_{i,0}) \tag{21}$$

$$C^k_i = \frac{\partial \hat{g}}{\partial x}(\hat{x}^k_{i,0}) \tag{22}$$

$$i \in [0, N_k - 1], \; \forall k \in N.$$

A(4) It is assumed that there exist constants $\rho_k \in (0, \infty)$ such that for all $\hat{x}^k_{0,0} \in B_{\rho_k} \doteq \{x \mid \| x \|_{P_k} \leq \rho_k\}$, the sequence of optimization problems $\{OCP(\hat{x}^k_{0,0}, t_k)\}_{k\in N}$ is feasible and the contraction constraint is satisfied.

Due to the fact that $OCP(\hat{x}^k_{0,0}, t_k)$ is subjected to control constraints and the contraction constraint on the state variables, the larger the horizon N_k the bigger the chance that all constraints are satisfied.

Thus assumption **A(4)** means that for $\hat{x}^k_{0,0} \in B_{\rho_k}$ there exists a horizon N_k long enough (but smaller than N_{max}) so that all the state, trajectory and control constraints are satisfied.

A(5) Let the reachable set of states X be defined by:

$$\begin{aligned} X &\doteq \{x^{p,k}(t), \hat{x}^k(t) \text{ and } x^k(t) \in \Re^n \mid \\ x^{p,k}(t) &\doteq x^p(t, t_k, x^{p,k}_{0,0}, u^k(t), d^k(t)), \\ \hat{x}^k(t) &\doteq \hat{x}(t, t_k, \hat{x}^k_{0,0}, u^k(t), 0) \text{ and} \\ x^k(t) &\doteq x(t, t_k, \hat{x}^k_{0,0}, u^k(t), 0), \\ & \quad t \in [t_k, t_{k+1}], \hat{x}^k_{0,0} \in B_{\rho_k}, u^k(t) \in U, \\ & \quad d^k(t) \in D; \forall k \in N\} \end{aligned} \tag{23}$$

Then it is assumed that there exist Lipschitz constants $L_{1,k}, L_{2,k}, L^g_k \in [0, \infty)$ and modeling bounds $\gamma_k \in [0, \infty)$ so that for $x^k(t), x^{p,k}(t)$, $\hat{x}^k(t) \in X; u^k(t) \in U$ and $d^k(t) \in D$, the following bounds hold:

A(5.1)

$$\begin{aligned}\| A^k_0 \bar{x}^k_i(t) + B^k_0 u^k_i(t) \|_{P_k} &\leq L_{1,k}(\| \bar{x}^k_i(t) \|_{P_k} + \\ &+ \| u^k_i(t) \|_1)\end{aligned}$$

A(5.2)

$$\| f(x_i^{p,k}(t), u_i^k(t), d_i^k(t)) - f(\bar{x}_i^k(t), u_i^k(t), 0) \|_{P_k} \leq$$

$$\leq L_{2,k}(\| x_i^{p,k}(t) - \bar{x}_i^k(t) \|_{P_k} + \| d_i^k(t) \|_1)$$

A(5.3)

$$\| g(x_i^{p,k}(t)) - g(\hat{x}_i^k(t)) \|_{P_k} \leq L_k^g(\| e_i^k(t) \|_{P_k})$$

with $e_i^k(t) \hat{=} x_i^{p,k}(t) - \hat{x}_i^k(t)$.

A(5.4)

$$\begin{aligned} \| f(\bar{x}_i^k(t), u_i^k(t), 0) \|_{P_k} &\leq \gamma_k(\| \bar{x}_i^k(t) \|_{P_k} + \\ &+ \| u_i^k(t) \|_1) \end{aligned}$$

for $t \in [t_k + iT, t_k + (i+1)T]$, $i \in [0, N_k - 1]$, $\forall k \in N$.

6. Analysis of the algorithm

The proofs of the following theorems can be found in de Oliveira and Morari [8] and will not be shown here due to lack of space.

Theorem 1 *Let $\rho_k \in (0,\infty)$ and $L_{1,k}$, $L_{2,k}$, L_k^g, $\gamma_k \in [0,\infty)$ satisfy assumptions* **A(4)**, **A(5.1)**, **A(5.2)**, **A(5.3)** *and* **A(5.4)**, *respectively. If $\hat{x}_{0,0}^k \in B_{\rho_k}$, then there exist $\lambda_{1,k}$, $\lambda_{2,k}$, $\lambda_{3,k} \in [0,\infty)$ such that:*

$$\begin{aligned} \| x_{0,0}^{p,(k+1)} - \bar{x}_{0,0}^{k+1} \|_{P_k} &\leq \lambda_{1,k} \| \hat{x}_{0,0}^k \|_{P_k} + \lambda_{2,k} + \\ &+ \lambda_{3,k} \| e_{0,0}^k \|_{P_k}, \qquad (24) \\ & \qquad \forall k \in N \end{aligned}$$

with $\lambda_{1,k} \to 0$ as $\gamma_k \to 0$ and $\lambda_{2,k} \to 0$ as $\gamma_k, \epsilon_d, \epsilon_n \to 0$.

Theorem 2 *Using the results from* **Theorem 1** *and the Contraction Mapping Principle, if the modeling bound γ_k, the state estimation error at t_k, $e_{0,0}^k$, and the combined effect of disturbances and measurement bias, ϵ_k, satisfy the following bounds,*

1. $\gamma_k < \bar{\gamma}_k$.

2. $\| e_{0,0}^k \|_{P_k} \leq \bar{e}_k$.

3. $\epsilon_k \hat{=} \epsilon_{d,k} + \epsilon_{n,k} < \bar{\epsilon}_k$.

where $\bar{\gamma}_k$, $\bar{\epsilon}_k$ and $\bar{e}_k$ are functions of $L_{1,k}$, $L_{2,k}$, α_k, N_k, ϵ_u, ρ_k, L_k^g, $\hat{K}_k$, P_k, then $\{\hat{x}_{0,0}^k\}_{k \in N} \in B_{\rho_k}$. This means that the sequence of optimization problems $OCP(\hat{x}_{i,0}^k, t_k + iT), i \in [0, N_k - 1]$ is well-defined for all $k \in N$. Besides, the residual error of the control algorithm is smaller than $\hat{\rho}$, i.e.,

$$\lim_{k \to \infty} \| \hat{x}_{0,0}^k \|_{P_k} < \hat{\rho} \qquad (25)$$

where $\hat{\rho}$ defines a small neighborhood of the origin. The expression for $\hat{\rho}$ in terms of the parameters afore mentioned can be found in [8].

Theorem 3 *If there exists a finite number $\bar{k} \in N$, so that γ_k and $e_{0,0}^k$ satisfy the following conditions,*

(1) *if $k < \bar{k}$: same bounds as in* **Theorem 2**.

(2) *if $k \geq \bar{k}$:*

1. $\gamma_k < \min\{\bar{\gamma}_k, \hat{\gamma}_k\}$.

2. $\| e_{0,0}^k \|_{P_k} \leq \min\{\bar{e}_k, \hat{e}_k\}$.

where the bounds $\hat{\gamma}_k$ and $\hat{e}_k$ are functions of the parameters given in **Theorem 2** *and the additional parameter $a_k \in [\lambda_{1,k} + \alpha_k, 1), \forall k \geq \bar{k}$, and the disturbances and measurement bias approach their nominal values asymptotically then the closed-loop system is asymptotically stable to the origin.*

7. Examples

Example 1 (Bioreactor System)

The first example is a continuous bioreactor with substrate inhibition studied in [10]:

$$\dot{x}(t) = (\mu(t) - D(t))x(t) \qquad (26)$$

$$\dot{s}(t) = (s_f - s(t))D(t) - \frac{\mu(t)x(t)}{y} \qquad (27)$$

where:

$$\mu(t) = \frac{\mu_{max} s(t)}{k_m + s(t) + k_1 s^2(t)}$$

Simulation Results

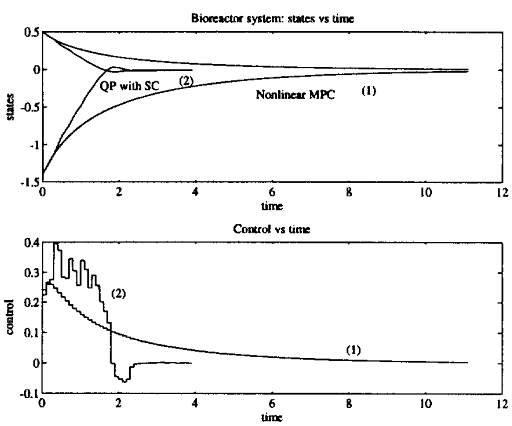

Figure 2: Simulation of a step in the setpoint from a stable to a saddle point.

Example 2 (Nonholonomic System)

The second example is the model of a car given by:

$$\dot{x} = cos(\theta)v \qquad (28)$$

$$\dot{y} = sin(\theta)v \qquad (29)$$

$$\dot{\theta} = w \qquad (30)$$

More details on this system can be found, e.g., in [20, 23, 25]. Here the results obtained with the proposed MPC are compared to the smooth controller in [25].

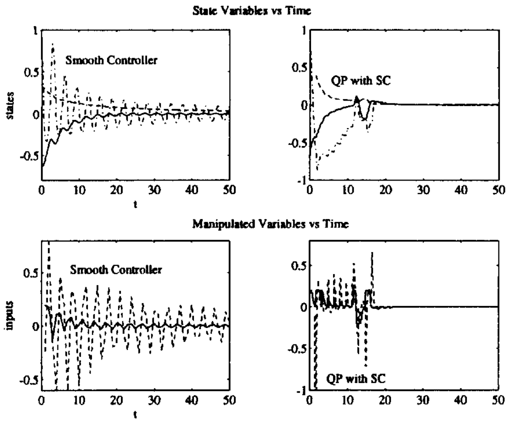

Figure 3: Simulation results of the car example.

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