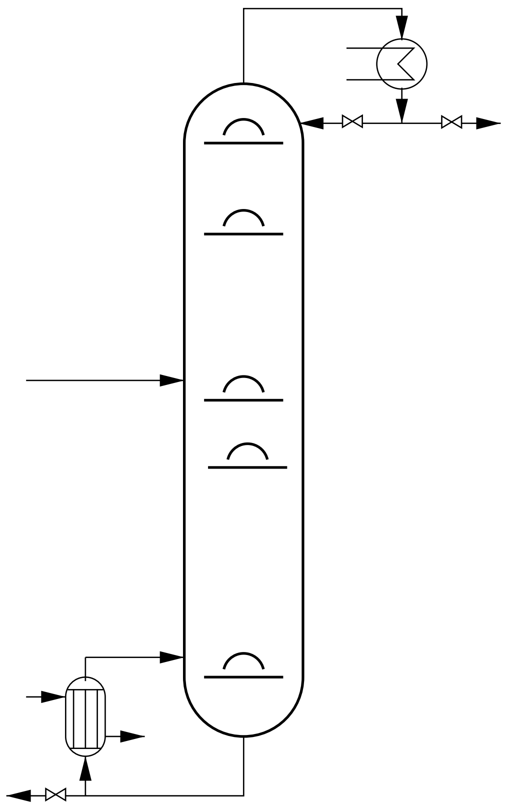
Efficient Algorithms for Nonlinear Model Predictive Control

Moritz Diehl
Interdisciplinary Center for Scientific Computing
University of Heidelberg

with H.G. Bock, J.P. Schlöder, F. Allgöwer, T. Bürner, R. Findeisen, S. Schwarzkopf, I. Uslu

1. Nonlinear Model Predictive Control (NMPC)
2. New Real-Time Optimization Techniques
3. Experiments with a Distillation Column
4. Outlook: Nonlinear Mixed Integer Problems

Differential Algebraic Equation (DAE) Systems



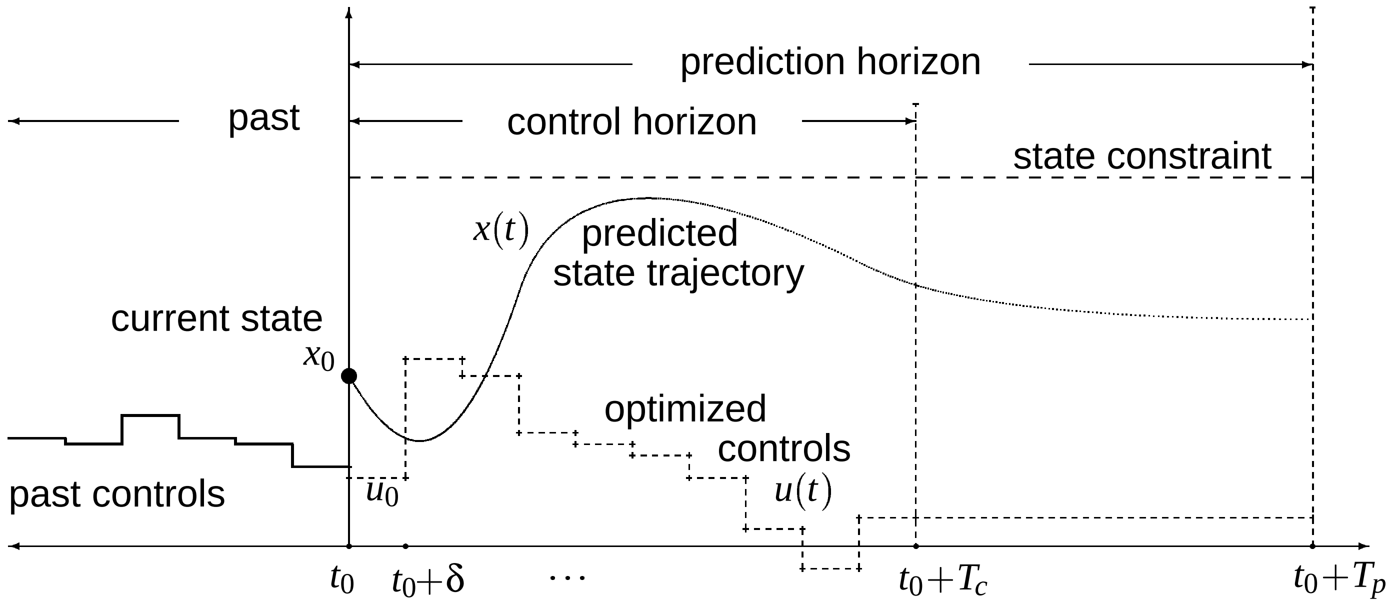
$$\dot{x}(t) = f(x(t), z(t), u(t), p)$$
$$0 = g(x(t), z(t), u(t), p)$$

Example: Distillation Column (ISR, Stuttgart)

- 82 differential states x
- 122 algebraic states z
- 2 controls u (liquid reflux L_{vol} , heat input Q)
- 2 parameters p (feedflow and -concentration)

Control aim: keep temperatures T_{14}, T_{28} constant, despite disturbances in x and p

Principle of Nonlinear Model Predictive Control



1. Estimate current system state x_0 (and parameters) from measurements.

2. Solve *in real-time* an optimal control problem:

$$\min_{x,z,u} \int_{t_0}^{t_0+T_p} L(x,z,u)\,dt + E\left(x(t_0+T_p)\right) \; s.t. \; \left\{ \begin{array}{rcl} x(t_0) - x_0 & = & 0, \\ \dot{x} - f(x,z,u) & = & 0, \; t \in [t_0, t_0+T_p] \\ g(x,z,u) & = & 0, \; t \in [t_0, t_0+T_p] \\ h(x,z,u) & \geq & 0, \; t \in [t_0, t_0+T_p] \\ r\left(x(t_0+T_p)\right) & \geq & 0. \end{array} \right.$$

3. Implement first control u_0 for time δ at real plant. Set $t_0 = t_0 + \delta$ and go to 1.

Some citations from an NMPC workshop (Ascona, 1998)

“..., there is a daunting challenge: the solution, online, of non-convex optimal control problems.”

(D. Q. Mayne, Imperial College)

“...prohibitively high on-line computational demand...”

(A. Zheng, Univ. of Massachusetts)

“Speed and the assurance of a reliable solution in real-time are major limiting factors in existing applications.”

(S. J. Qin und T. A. Badgwell, Univ. of Texas, Rice Univ.)

Real-Time Optimization Techniques

- **Direct multiple shooting** method (Bock and Plitt, 1981) combines advantages of *simultaneous* solution approach with highly developed DAE solvers.

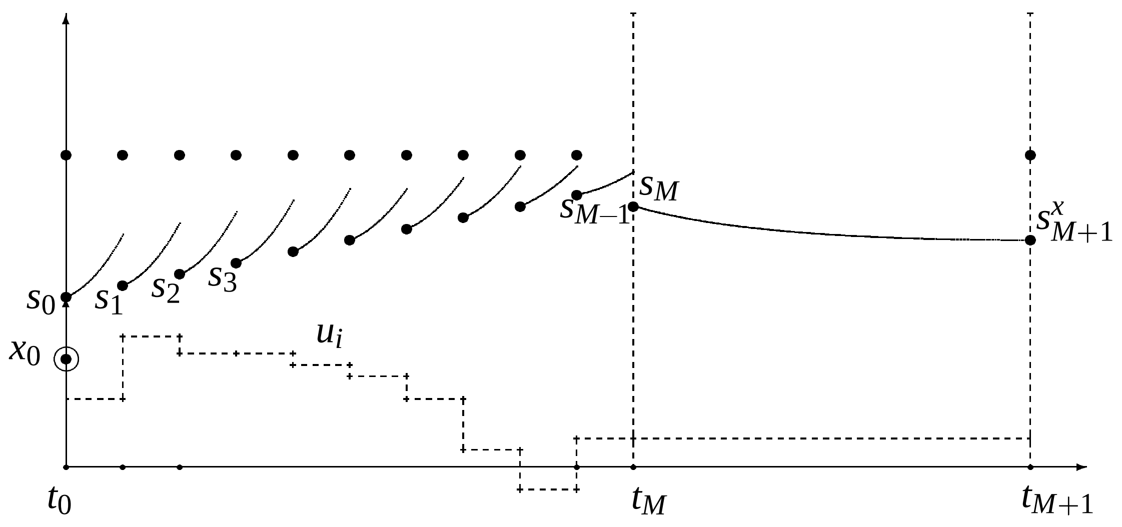
  Leads to a large, but structured NLP.

- Iterative solution with a tailored **Sequential Quadratic Programming (SQP)** method.

- **Initial value embedding** of disturbance into linear constraint delivers tangential solution predictor for new x_0.

- **Real-time iterations** use current value of x_0 in each iteration.

Nonlinear Program (NLP) in Direct Multiple Shooting



$$\min_{u_i, s_i^x, s_i^z} \sum_{i=0}^{M} L_i(s_i^x, s_i^z, u_i) + E(s_{M+1}^x) \quad \text{s.t.} \quad \left\{ \begin{aligned} s_0^x - x_0 &= 0 \\ s_{i+1}^x - x_i(t_{i+1}; s_i^x, s_i^z, u_i) &= 0 \\ g(s_i^x, s_i^z, u_i) &= 0 \\ h(s_i^x, s_i^z, u_i) &\geq 0 \\ r(s_{M+1}^x) &\geq 0 \\ (i = 0, 1, \ldots M) & \end{aligned} \right.$$

Sequential Quadratic Programming (SQP)

Nonlinear Program:

$$\min_{w} F(w) \quad \text{s.t.} \quad \begin{cases} G(w) = 0 \\ H(w) \geq 0 \end{cases}$$

Solution iteratively. Start with w^0.

1. Compute functions $F(w^k), G(w^k), H(w^k)$ and derivatives.
2. Obtain step Δw^k as solution of a **Quadratic Program (QP):**

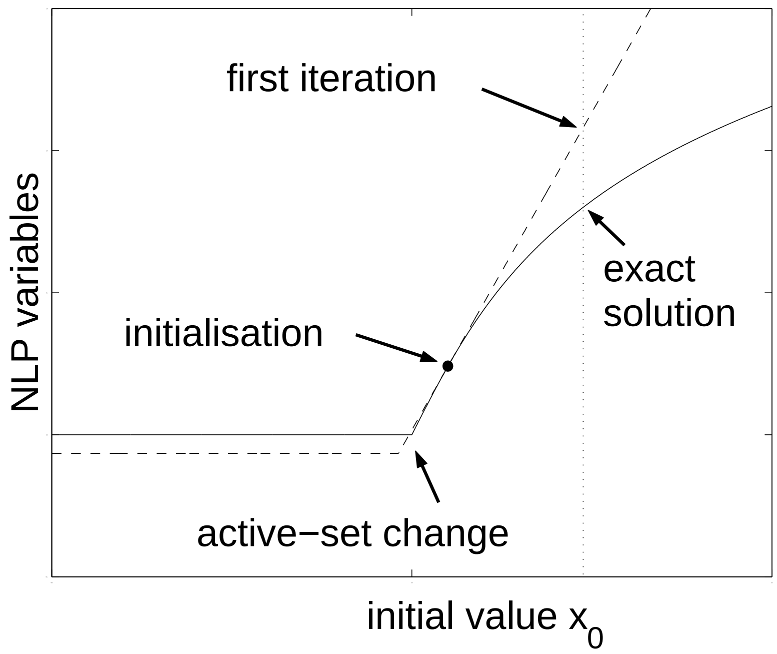
$$\min_{\Delta w} \nabla F(w^k)^T \Delta w + \frac{1}{2} \Delta w^T A^k \Delta w \quad \text{s.t.} \quad \begin{cases} G(w^k) + \nabla G(w^k)^T \Delta w = 0 \\ H(w^k) + \nabla H(w^k)^T \Delta w \geq 0. \end{cases}$$

3. Set $w^{k+1} = w^k + \Delta w^k$, $k = k+1$.

Choice of **Hessian** A^k distinguishes SQP variants:
Exact-Hessian, BFGS-Update, or

Constrained Gauss-Newton: if $F(w) = \frac{1}{2}\|R(w)\|^2$ use $A^k := \frac{\partial R(w^k)}{\partial w}^T \frac{\partial R(w^k)}{\partial w}$

Initial Value Embedding



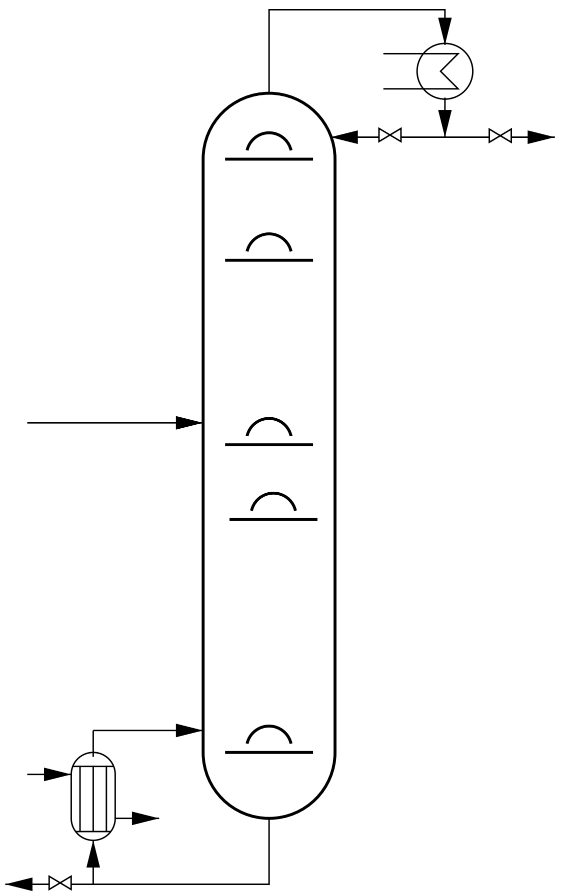
- first iteration is tangential predictor for exact solution (for exact Hessian SQP)
- also valid for active set changes
- derivative can be computed *before* x_0 is known: first iteration nearly without delay

Real-time iterations: do not iterate to convergence – iterate, *while* problem is changing!

1. **Preparation Step (long):**
   Linearize system at current iterate, presolve components of quadratic program.
2. **Feedback Step (short):**
   When new x_0 is known, solve QP and implement control u_0 immediately. Complete SQP iteration. Go to 1.

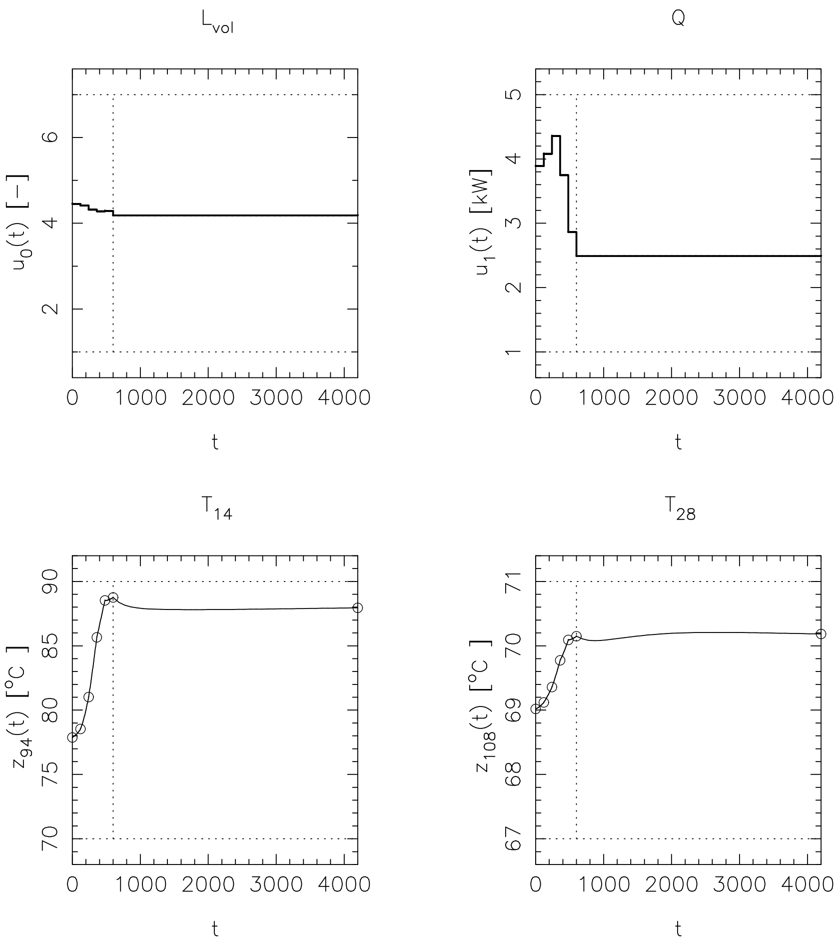
- minimal cycle-duration (as **one** SQP iteration)
- negligible feedback delay ($\approx$ 1 % of cycle)
- nevertheless fully nonlinear optimization

Practical Realization



- Parameter estimation using dynamic experiments
- Online state estimation with Extended Kalman Filter variant, using only 3 temperature measurements to infer all 82 system states
- Implementation of estimator and optimizer on Linux Workstation.
- Communication with Process Control System via FTP all 10 seconds.
- Self-synchronizing processes.

Optimization Problem for Distillation Column



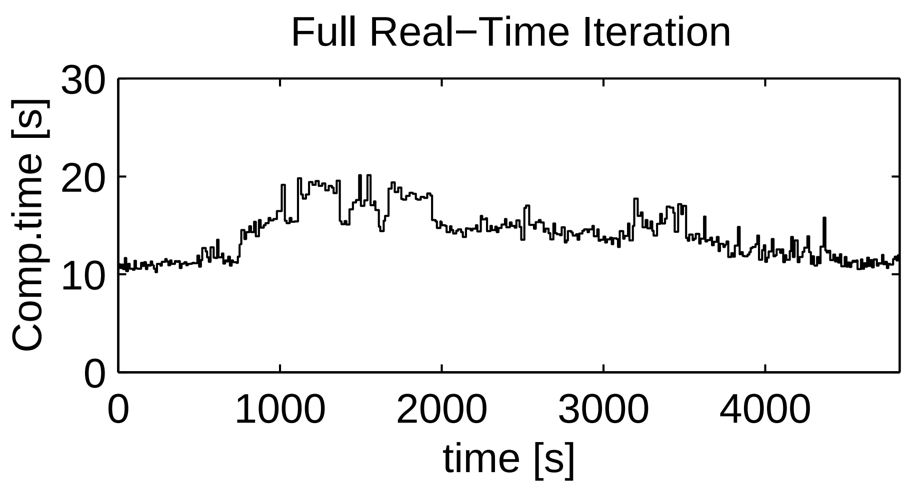
- Least squares objective:

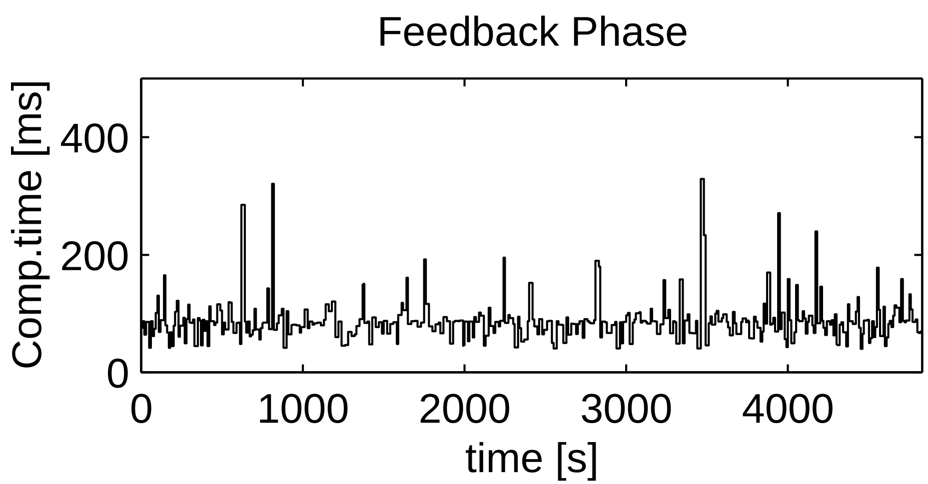
$$\int_{t_0}^{t_0+T_p} \left\| \begin{array}{c} T_{14}(t) - T_{14}^{\mathrm{ref}} \\ T_{28}(t) - T_{28}^{\mathrm{ref}} \end{array} \right\|_2^2 + \varepsilon \left\| \begin{array}{c} L_{\mathrm{vol}}(t) - L_{\mathrm{vol}}^{\mathrm{ref}} \\ Q(t) - Q^{\mathrm{ref}} \end{array} \right\|_2^2 dt$$

- control horizon 10 min
- prediction horizon 10 h
- stiff DAE model with 204 state variables

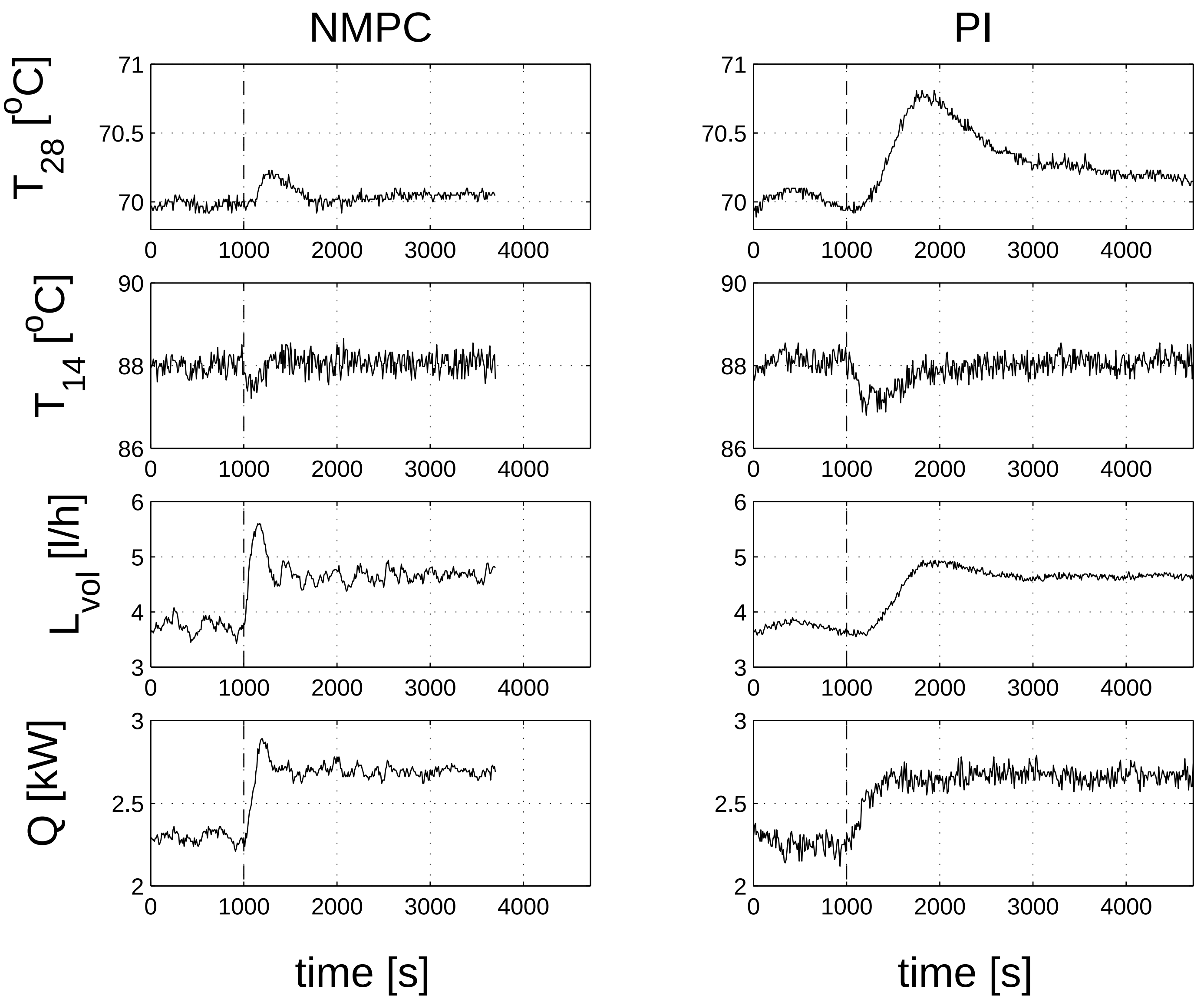
Solution for new initial value x_0 every 30 seconds required...

Computation Times During Application

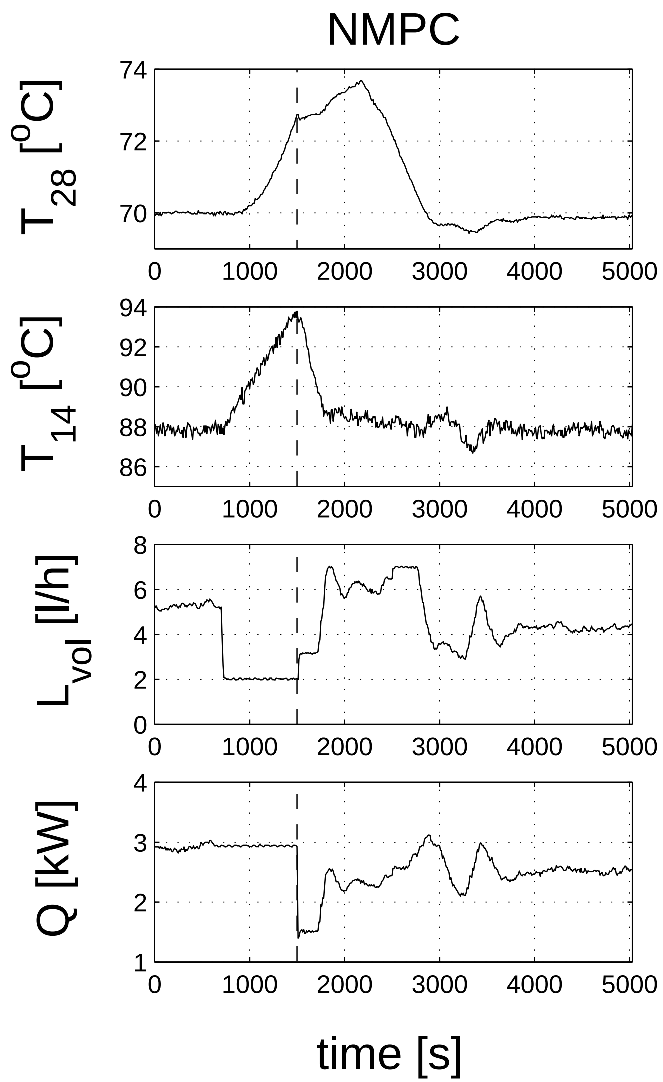




Feedflow Change by 20%: Transient Phase (Comparison with PI-Controller)

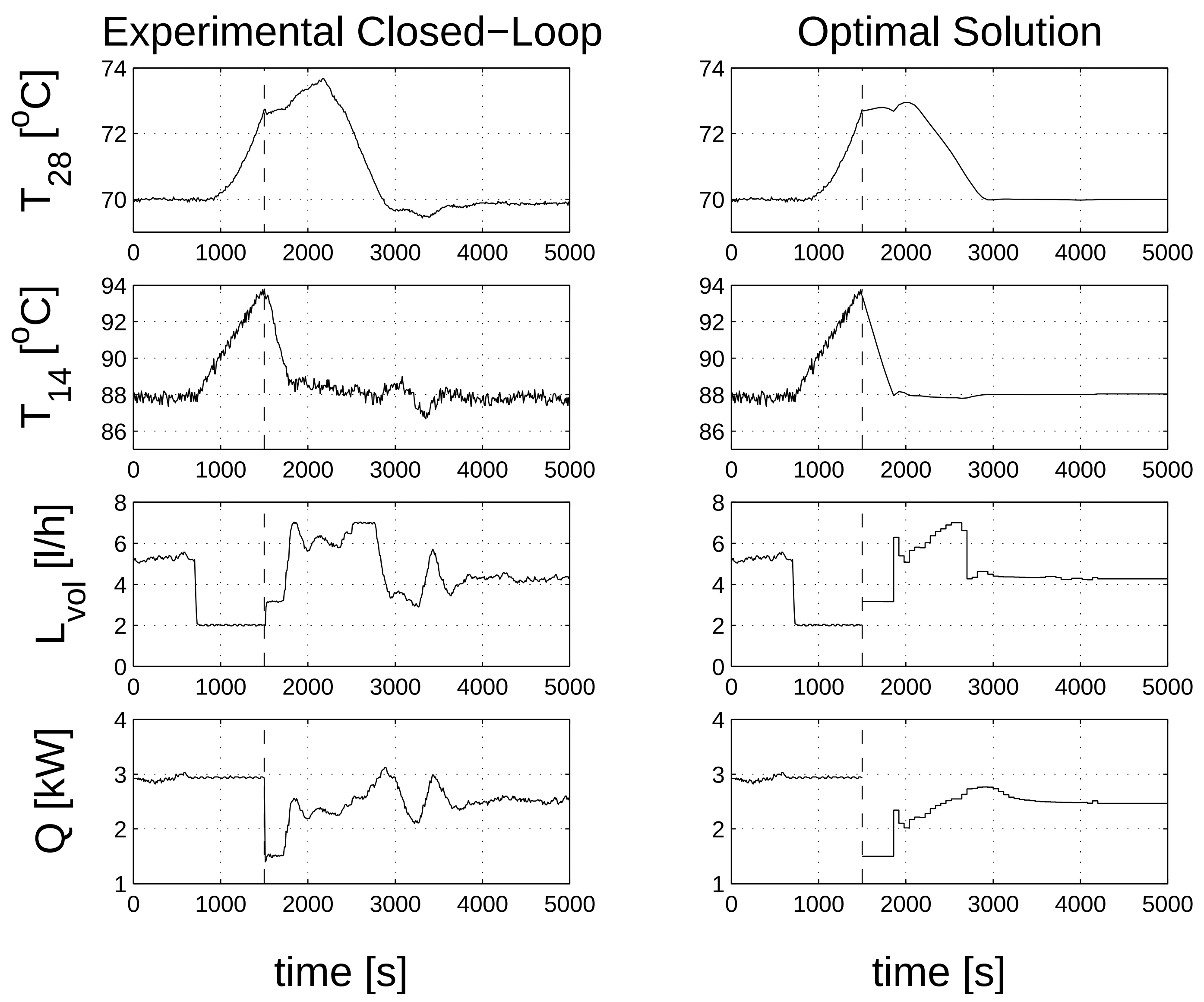


Large Disturbance: Heating-up of Column, then NMPC



- NMPC only starts at $t = 1500$ s
- PI-controller not implementable, as disturbance too large (valve saturation)
- NMPC: at start control bound active $\Rightarrow T_{28}$ rises further
- Disturbance attenuated after half an hour

A Posteriori Comparison with Theoretically Optimal Solution



Preliminary Conclusions

New real-time optimization techniques make NMPC with large scale DAE models and short timescales possible.

They are characterized by:

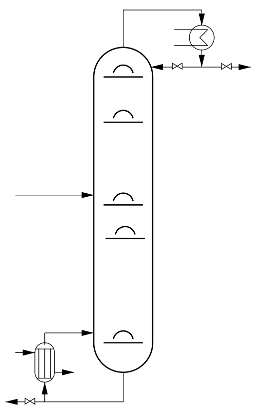
- **direct multiple shooting** with **Gauss-Newton** approach
- **initial value embedding** to deliver tangential predictor
- **real-time iterations** to have minimal cycle times
- negligible feedback delay

Contractivity of algorithm is proven.
Successfully tested on a real distillation column.

Outlook: Nonlinear Mixed Integer Problems

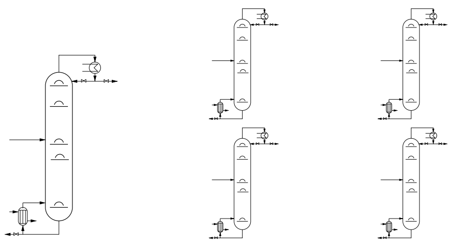
Practical problems often involve **switching** functions and **integer** variables

E.g. for distillation column:



- model change when liquid on tray evaporates,
- reflux valve closes when inflow in condenser too low, ...
- choice of feedtray variable,
- discrete heating: off, or full power, ...

Or coupled processes:



- where to connect?
- what schedule for products?

New techniques for Nonlinear Mixed Integer Programming required!