Technical Notes and Correspondence

Adaptive Tracking Control of Uncertain Nonholonomic Dynamic System

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***Abstract*—This note considers the tracking problem of nonholonomic dynamic systems with unknown inertia parameters. A new controller is proposed relying on newly defined tracking errors and the passivity property of the nonholonomic dynamic system. The proposed controller ensures that the entire state of the system asymptotically tracks the desired trajectory. Simulation results show effectiveness of the proposed controller.**

***Index Terms*—Adaptive control, nonholonomic system, tracking control, uncertain nonlinear system.**

I. Introduction

In recent years, there has been a growing interest in the design of feedback control laws for nonholonomic systems [14]. It is well-known that a nonholonomic control system cannot be asymptotically stabilized to a rest point by smooth pure-state feedback laws [1] due to Brockett's theorem [2]. However, several approaches have been proposed for stabilizing them [14].

Another important control problem of the nonholonomic system is the tracking problem which is to make the entire state of the closed-loop system track a given desired trajectory. Depending on whether the nonholonomic system is represented by a kinematic or dynamic model, the tracking problem can be classified as either a kinematic tracking problem or a dynamic tracking problem. Several authors have studied the kinematic tracking problem and proposed several controllers. See [6], [11]–[13], and [21] for reference. However, many nonholonomic systems in reality have significant dynamics, and the system inputs are limited torques or forces generated by the physical actuators. Therefore, it will be more realistic to consider the tracking problem of the dynamic models than that of the kinematic models. Additionally, many controllers of the kinematic tracking problem cannot be used directly to control the systems in practice since the control inputs are based on the velocities assumed always achievable instantaneously. To compute the system control inputs, it is assumed that there is "perfect velocity tracking" [13]. This assumption does not hold in practice, since it implies the availability of the full knowledge of the dynamics and the absence of disturbances. Finally, on account of the complexity of the mechanical structure and the likely payload variation from task to task, the inertia parameters of the system are often either inexactly known or not known. This means that there are inevitable uncertainty in the dynamic model of the systems. Recognizing the importance of considering the system dynamics, several researchers have started to pay attention to this problem in recent years. Su [19] studied the dynamic tracking problem of the nonholonomic system with unknown inertia parameters. Chen [5] discussed the dynamic tracking problem with uncertainty using H_∞ techniques. In these two papers the controllers which ensured partial states of the system to track the desired trajectories were proposed. In [9] and [10], the dynamic tracking problem of the nonholonomic systems with uncertainty were discussed. Robust and adaptive controllers were proposed.

In this note, we consider the dynamic tracking problems of a class of the nonholonomic dynamic system with unknown constant inertia parameters. A new controller is proposed with aid of newly defined tracking errors and the passivity property of the dynamic nonholonomic system. To make the tracking problem solvable, it is assumed that the kinematic constraints of the nonholonomic system can be converted into the chained form and that the given desired trajectory satisfies the same nonholonomic constraints as for the system discussed. By the defined tracking errors, a new controller is proposed. Comparing with the existing results, the *contributions* of this paper are as follows. 1) The dynamic tracking problem of a class of nonholonomic systems with unknown constant inertia parameters is solved. The proposed controller ensures the entire state of the system to asymptotically track the desired trajectory. The existing controllers in [4], [5], [19] can only make part of the state but not the entire state tend to the given desired trajectory. Therefore, there is considerable progress made in this paper. 2) A new controller is proposed to solve the dynamic tracking problem of the uncertain nonholonomic systems. The controller proposed in this paper is derived with aid of newly defined tracking errors and the property that the dynamic structure is linear in terms of a suitably selected set of system inertia parameters, Therefore it is different from those in [9] and [10].

II. Problem Statement

Consider the general mechanical system with nonholonomic constraints expressed, in local coordinates (termed generalized coordinates), by the so-called Euler–Lagrangian formulation as [3], [19]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = B(q)\tau + J^T(q)\lambda \tag{1}$$

$$J(q)\dot{q} = 0 \tag{2}$$

where

$q = [q_1, \ldots, q_n]^T$	n-vector of the generalized coordinates;
$M(q)$	$n \times n$ bounded positive-definite symmetric matrix;
$C(q, \dot{q})\dot{q}$	n-vector of centripetal and Coriolis torque;
$G(q)$	n-vector of gravitational torque;
$B(q)$	$n \times r$ input transformation matrix;
τ	r-vector of control input;
$J(q)$	$(n - m - 1) \times n$ full rank matrix;
2	$\leq m + 1 < n$;
r	$\geq m + 1$;
λ	$(n - m - 1)$-vector of Lagrange multiplier which expresses the constraint force;
the superscript T	denotes the transpose.

In the system (1) and (2), the constraint (2) is assumed to be completely nonholonomic [17]. Two simplifying properties should be noted about (1) [16], [19].

Property 1: A suitable definition of $C(q, \dot{q})$ makes the matrix $(d/dt)M - 2C$ skew-symmetric [16]. In particular, this is true if the kjth element of the matrix $C(q, \dot{q})$ is defined as

$$c_{kj} = \sum_{i=1}^{n} \frac{1}{2}\left\{\frac{\partial m_{kj}}{\partial q_i} + \frac{\partial m_{ki}}{\partial q_j} - \frac{\partial m_{ij}}{\partial q_k}\right\}\dot{q}_i$$

where m_{ij} is the ijth element of $M(q)$.

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Property 2: There exists a vector a with components depending on mechanical parameters (masses, moments of inertia, etc.), such that $M(q)\dot{\xi} + C(q, \dot{q})\xi + G(q) = Y(q, \dot{q}, \xi, \dot{\xi})a$, where the regressor matrix $Y(q, \dot{q}, \xi, \dot{\xi})$ is a known matrix of q, $\dot{q}$, ξ, and $\dot{\xi}$, a is the inertia parameter vector of the system (1) and (2) [16], [19], ξ and $\dot{\xi}$ are any n-dimensional vector function and its time derivative respectively.

In this note, we assume the inertia parameter vector a is a constant vector and not known. Given n-dimensional desired trajectory $q^d(t)$ which is as differentiable as required and satisfies the nonholonomic constraint

$$J(q^d)\dot{q}^d = 0 \tag{3}$$

the dynamic tracking problem discussed in this paper is defined as finding a control law τ such that

$$\lim_{t\to\infty}(q - q^d) = 0 \quad \lim_{t\to\infty}(\dot{q} - \dot{q}^d) = 0. \tag{4}$$

To solve the tracking problem, we eliminate the Lagrange multiplier λ first. Following [1], [8], let the vector fields $g_1(q), \ldots, g_{m+1}(q)$ form a basis of the null space of $J(q)$. By (2), there exists an $(m+1)$-vector $v = [v_1, \ldots, v_{m+1}]^T$ such that

$$\dot{q} = g(q)v = g_1(q)v_1 + \cdots + g_{m+1}(q)v_{m+1} \tag{5}$$

where $g(q) = [g_1(q), \ldots, g_{m+1}(q)]^T$.

Remark 1: For a given $J(q)$, finding $g(q)$ is not a difficult task. Interest readers may refer to [1], [3].

Differentiating both sides of (5), substituting it into (1) and multiplying both sides from left by $g^T(q)$, one obtains

$$M_1(q)\dot{v} + C_1(q, \dot{q})v + G_1(q) = B_1(q)\tau \tag{6}$$

where

$$\begin{aligned} M_1(q) &= g^T(q)M(q)g(q);\\ C_1(q, \dot{q}) &= g^T(q)(M(q)\dot{g}(q) + C(q, \dot{q})g(q));\\ G_1(q) &= g^T(q)G(q);\\ B_1(q) &= g^T(q)B(q). \end{aligned}$$

The reduced system (5) and (6) describes the motion of the original system. Therefore, the dynamic tracking problem can be considered based on the reduced system (5) and (6) instead of the system (1) and (2). In order to completely actuate the nonholonomic system, $B_1(q)$ is assumed to be a full-rank matrix.

The system (5) and (6) represents a general nonholonomic dynamic system. For this general system, it is hard theoretically to design an effective controller. Therefore, in order to make the dynamic tracking problem solvable, we assume that there exist diffeomorphic transformations

$$\begin{cases} x = [x_1, x_{2,1}, \ldots, x_{n_1,1}, \ldots, x_{2,m}, \ldots, x_{n_m,m}]^T \\ \quad = \phi(q) \\ u = [u_1, \ldots, u_{m+1}]^T = \varphi(q)v \end{cases} \tag{7}$$

such that equation (5) can be converted to the one-generator multi-chain form [22]:

$$\begin{cases} \dot{x}_1 = u_1, \ \dot{x}_{i,j} = u_1 x_{i+1,j} (2 \le i \le n_j - 1) \\ \dot{x}_{n_j,j} = u_{j+1}, \ (1 \le j \le m) \end{cases} \tag{8}$$

with $\sum_{j=1}^m n_j - m + 1 = n$. Therefore, with the transformation (7), the system (5) and (6) can be converted into the following canonical form:

$$\begin{cases} \dot{x}_1 = u_1, \ \dot{x}_{i,j} = u_1 x_{i+1,j} (2 \le i \le n_j - 1) \\ \dot{x}_{n_j,j} = u_{j+1}, \ (1 \le j \le m) \end{cases} \tag{9}$$

$$M_2(x)\dot{u} + C_2(x, \dot{x})u + G_2(x) = B_2(x)\tau \tag{10}$$

where

$$\begin{aligned} M_2(x) &= \varphi^{-T}(q)M_1(q)\varphi^{-1}(q)|_{q=\phi^{-1}(x)};\\ C_2(x, \dot{x}) &= \varphi^{-T}(q)[C_1(q, \dot{q}) - M_1(q)\varphi^{-1}(q)\dot{\varphi}(q)]\varphi^{-1}(q)|_{q=\phi^{-1}(x)};\\ G_2(x) &= \varphi^{-T}(q)G_1(q)|_{q=\phi^{-1}(x)};\\ B_2(x) &= \varphi^{-T}(q)B_1(q)|_{q=\phi^{-1}(x)}. \end{aligned}$$

Remark 2: The necessary and sufficient conditions for the existence of the transformation (7) have been extensively studied. Interested readers may refer to [17], [18], and [22] for calculating the diffeomorphism (7). For most of the nonholonomic wheeled mobile robots the transformation (7) always exists globally or locally and was explicitly given in [7] and [15].

The following properties can be easily proved.

Property 3: $(d/dt)M_2 - 2C_2$ is skew symmetric.

Property 4: For any $(m+1)$-dimensional vector function ξ and its time derivative $\dot{\xi}$, $M_2(x)\dot{\xi} + C_2(x, \dot{x})\xi + G_2(x) = Y_2(x, \dot{x}, \xi, \dot{\xi})a$, where $Y_2(x, \dot{x}, \xi, \dot{\xi})$ is a known matrix of x, $\dot{x}$, ξ, and $\dot{\xi}$.

Next, the above transformations are applied to the given desired trajectory. Since $q^d(t)$ satisfies the constraint (3), it can be easily proved that

$$\dot{q}^d = g(q^d)v^d \tag{11}$$

where $v^d = [v_1^d, \ldots, v_{m+1}^d]^T$ is a suitable time-varying function. Since (5) can be converted to (8) by the transformation (7), there exists the transformation

$$\begin{cases} z = [z_1, z_{2,1} \ldots, z_{n_1,1}, \ldots, z_{2,m}, \ldots, z_{n_m,m}]^T = \phi(q^d) \\ w = [w_1, \ldots, w_{m+1}]^T = \varphi(q^d)v^d \end{cases} \tag{12}$$

such that the system (11) can be converted into

$$\begin{cases} \dot{z}_1 = w_1, \ \dot{z}_{i,j} = w_1 z_{i+1,j} (2 \le i \le n_j - 1) \\ \dot{z}_{n_j,j} = w_{j+1}, \qquad (1 \le j \le m). \end{cases} \tag{13}$$

With the above assumption and transformations, the tracking problem considered in this paper can be rephrased as finding a control law τ of the system (9) and (10) such that

$$\lim_{t\to+\infty}(x - z) = 0, \qquad \lim_{t\to+\infty}(\dot{x} - \dot{z}) = 0 \tag{14}$$

when the inertia parameter vector a is unknown.

III. Controller Design

In order to facilitate the controller design, let $y = [y_1, y_{2,1}, \ldots, y_{n_1,1}, \ldots, y_{2,m}, \ldots, y_{n_m,m}]^T = x - z$, $\alpha = [\alpha_1, \alpha_{2,1}, \ldots, \alpha_{n_1,1}, \ldots, \alpha_{2,m}, \ldots, \alpha_{n_m,m}]^T$ and

$$\begin{cases} \alpha_1 = \alpha_{2,j} = 0, \\ \alpha_{3,j} = -k_{2,j}y_{2,j}w_1^{2l-1}, \\ \alpha_{4,j} = -(y_{2,j} - \alpha_{2,j}) - k_{3,j}(y_{3,j} - \alpha_{3,j})w_1^{2l-1} \\ \qquad + \dfrac{1}{w_1}\displaystyle\sum_{i=0}^{0}\dfrac{\partial\alpha_{3,j}}{\partial w_1^{[i]}}w_1^{[i+1]} + \sum_{i=2}^{2}\dfrac{\partial\alpha_{3,j}}{\partial y_{i,j}}y_{i+1,j}, \\ \quad \vdots \\ \alpha_{n_j,j} = -(y_{n_j-2,j} - \alpha_{n_j-2,j}) - k_{n_j-1,j}(y_{n_j-1,j} \\ \qquad -\alpha_{n_j-1,j})w_1^{2l-1} + \dfrac{1}{w_1}\displaystyle\sum_{i=0}^{n_j-4}\dfrac{\partial\alpha_{n_j-1,j}}{\partial w_1^{[i]}}w_1^{[i+1]} \\ \qquad + \displaystyle\sum_{i=2}^{n_j-2}\dfrac{\partial\alpha_{n_j-1,j}}{\partial y_{i,j}}y_{i+1,j}, \ (1 \le j \le m) \end{cases}$$

where $l = \max\{n_j,\, 1 \le j \le m\} - 2$, $w_1^{[i]}$ means ith derivative of w_1 (i.e., $w_1^{[i]} = d^i w_1/dt^i$), $k_{i,j}(2 \le i \le n_j - 1,\, 1 \le j \le m)$ are positive constants.

Remark 3: By the definitions of $\alpha_{i,j}(3 \le i \le n_j,\, 1 \le j \le m)$, it can be proved that $\alpha_{i,j}(4 \le i \le n_j,\, 1 \le j \le m)$ do not contain the term $1/w_1$ after expanding each terms in $\alpha_{i,j}$. As an example, by simple calculation, $\alpha_{4,j} = -(y_{2,j} - \alpha_{2,j}) - k_{3,j}(y_{3,j} - \alpha_{3,j})w_1^{2l-1} - (2l-1)k_{2,j}y_{2,j}w_1^{2l-3}\dot{w}_1 - k_{2,j}w_1^{2l-1}y_{3,j}$, which does not contain $1/w_1$. Therefore, $\alpha_{i,j}(3 \le i \le n_j,\, 1 \le j \le m)$ are well defined when w_1 goes through zero or is zero.

Define the tracking error

$$\begin{aligned} e &= [e_1,\, e_{2,1},\, \ldots,\, e_{n_1,1},\, \ldots,\, e_{2,m},\, \ldots,\, e_{n_m,m}]^T \\ &= x - z - \alpha \end{aligned} \tag{15}$$

one has the following Lemma.

Lemma 1: If $w_1^{[i]}(0 \le i \le \max\{n_j,\, 1 \le j \le m\} - 2)$ are bounded, $\lim_{t\to\infty} e = 0$ and $\lim_{t\to\infty} \dot{e} = 0$ imply that (14) holds.

Poof: For 1, $1 \le j \le m$, since $\alpha_1 = \alpha_{2,j} = 0$, it is obvious that $\lim_{t\to\infty} e_{i,j} = 0$ and $\lim_{t\to\infty} \dot{e}_{i,j} = 0 (1 \le i \le 2)$ imply that $\lim_{t\to\infty} y_{i,j} = 0$ and $\lim_{t\to\infty} \dot{y}_{i,j} = 0 (1 \le i \le 2)$. Since w_1 is bounded, $\alpha_{3,j}$ and $\dot{\alpha}_{3,j}$ converge to zero. Furthermore, $y_{3,j} = e_{3,j} + \alpha_{3,j}$ and $\dot{y}_{3,j}$ converge to zero. Noting that $\alpha_{3,j}$ and $\dot{\alpha}_{3,j}$ converge to zero, $\alpha_{4,j}$ and $\dot{\alpha}_{4,j}$ converge to zero. Furthermore, $y_{4,j}$ and $\dot{y}_{4,j}$ converge to zero. Similarly, we can prove that $y_{i,j}$ and $\dot{y}_{i,j}(5 \le i \le n_j)$ converge to zero. Therefore, (14) holds. ■

Let $\hat{a}$ be the estimated value of a, $\hat{M}_2$, $\hat{C}_2$, and $\hat{G}_2$ be the values of M_2, C_2 and G_2 corresponding to the estimated parameter vector $\hat{a}$. With aid of the defined tracking errors, the following theorem can be proved.

Theorem 1: Consider the system (9) and (10) with unknown constant inertia parameter vector a. If z and $z_1^{[i]}(1 \le i \le \max\{n_j,\, 1 \le j \le m\} - 1)$ are bounded and $\lim_{t\to\infty} \inf |w_1(t)| > 0$, the control law

$$\tau = B_2^{\#}(x)[\hat{M}_2(x)\dot{\eta} + \hat{C}_2(x,\, \dot{x})\eta + \hat{G}_2(x) - K_p(u - \eta) - \Lambda] \tag{16}$$

$$\begin{aligned} \dot{p} = &- k_0 p - k_1 e_1 \\ &- \sum_{s=1}^{m} \left[\sum_{i=2}^{n_s - 1} e_{i,s} x_{i+1,s} - \sum_{j=3}^{n_s} e_{j,s} \sum_{i=2}^{j-1} \frac{\partial \alpha_{j,s}}{\partial y_{i,s}} x_{i+1,s} \right] \end{aligned} \tag{17}$$

and the adaptive law

$$\dot{\hat{a}} = -\Gamma^{-1} Y_2^T(x,\, \dot{x},\, \eta,\, \dot{\eta})(u - \eta) \tag{18}$$

ensure that (14) holds, that p asymptotically converges to zero, and that $\hat{a}$ is bounded, where $\#$ is any left inverse, $\eta = [\eta_1,\, \ldots,\, \eta_{m+1}]^T$ and $\Lambda = [\Lambda_1,\, \ldots,\, \Lambda_{m+1}]^T$ with

$$\begin{cases} \eta_1 = w_1 + p \\ \eta_{j+1} = w_{j+1} - e_{n_j - 1,j} w_1 - k_{n_j,j} e_{n_j,j} \\ \qquad + \displaystyle\sum_{i=0}^{n_j - 3} \frac{\partial \alpha_{n_j,j}}{\partial w_1^{[i]}} w_1^{[i+1]} \\ \qquad + w_1 \displaystyle\sum_{i=2}^{n_j - 1} \frac{\partial \alpha_{n_j,j}}{\partial y_{i,j}} y_{i+1,j},\ (1 \le j \le m) \\ \Lambda_1 = k_1 e_1 + \displaystyle\sum_{s=1}^{m} \left[\sum_{i=2}^{n_s - 1} e_{i,s} x_{i+1,s} \right. \\ \qquad \left. - \displaystyle\sum_{j=3}^{n_s} e_{j,s} \sum_{i=2}^{j-1} \frac{\partial \alpha_{j,s}}{\partial y_{i,s}} x_{i+1,s} \right] \\ \Lambda_{j+1} = e_{n_j,j},\ (1 \le j \le m) \end{cases}$$

and the control parameters $k_0 > 0$, $k_1 > 0$, $k_{i,j} > 0 (2 \le i \le n_j,\, 1 \le j \le m)$, K_p and Γ are positive–definite matrices.

Proof: Let $\tilde{u} = u - \eta = [\tilde{u}_1,\, \tilde{u}_2,\, \ldots,\, \tilde{u}_{m+1}]^T$, $\tilde{a} = \hat{a} - a$, the closed-loop system of (9) and (10) and (16)–(18) can be written as

$$\begin{cases} \dot{e}_1 = p + \tilde{u}_1 \\ \dot{e}_{2,j} = e_{3,j} w_1 - k_{2,j} e_{2,j} w_1^{2l} + (p + \tilde{u}_1) x_{3,j} \\ \dot{e}_{3,j} = e_{4,j} w_1 - e_{2,j} w_1 - k_{3,j} e_{3,j} w_1^{2l} \\ \qquad + (p + \tilde{u}_1) \left(x_{4,j} - \dfrac{\partial \alpha_{3,j}}{\partial y_{2,j}} x_{3,j} \right) \\ \qquad \vdots \\ \dot{e}_{n_j - 1,j} = e_{n_j,j} w_1 - e_{n_j - 2,j} w_1 - k_{n_j - 1,j} e_{n_j - 1,j} w_1^{2l} \\ \qquad + (p + \tilde{u}_1)(x_{n_j,j} - \displaystyle\sum_{i=2}^{n_j - 2} \frac{\partial \alpha_{n_j - 1,j}}{\partial y_{i,j}} x_{i+1,j}) \\ \dot{e}_{n_j,j} = -k_{n_j,j} e_{n_j,j} - e_{n_j - 1,j} w_1 + \tilde{u}_{j+1} - (p + \tilde{u}_1) \\ \qquad \cdot \displaystyle\sum_{i=2}^{n_j - 1} \frac{\partial \alpha_{n_j,j}}{\partial y_{i,j}} x_{i+1,j}, \quad (1 \le j \le m) \\ M_2(x)\dot{\tilde{u}} = Y_2(x,\, \dot{x},\, \eta,\, \dot{\eta})\tilde{a} - C_2(x,\, \dot{x})\tilde{u} - K_p \tilde{u} - \Lambda \\ \dot{\tilde{a}} = -\Gamma^{-1} Y_2^T(x,\, \dot{x},\, \eta,\, \dot{\eta})\tilde{u} \\ \dot{p} = -\mu_2 p - \Lambda_1. \end{cases} \tag{19}$$

Let

$$V = \tfrac{1}{2} \left(p^2 + k_1 e_1^2 + \tilde{u}^T M_2 \tilde{u} + \tilde{a}^T \Gamma \tilde{a} \right) + \tfrac{1}{2} \sum_{j=1}^{m} \sum_{i=2}^{n_j} e_{i,j}^2.$$

Differentiating V along (19) yields

$$\begin{aligned} \dot{V} = &-k_0 p^2 - \sum_{j=1}^{m} \sum_{i=2}^{n_j - 1} k_{i,j} e_{i,j}^2 w_1^{2l} \\ &- \sum_{j=1}^{m} k_{n_j,j} e_{n_j,j}^2 - \tilde{u}^T K_p \tilde{u}. \end{aligned} \tag{20}$$

Therefore, V is nonincreasing and converges to a limiting value $V_{\lim} \ge 0$. Furthermore, p, e, $\tilde{u}$, and $\tilde{a}$ are bounded. By assumption in the theorem, x are bounded. In view of (19), $\dot{p}$, $\dot{e}$, $\dot{\tilde{a}}$ and $\dot{\tilde{u}}$ are bounded. Thus, p, e, and $\hat{a}$ are uniformly bounded. Next, we prove that p, e, and $\dot{e}$ tend to zero. Since $\ddot{V}$ is bounded, $\dot{V}$ is uniformly continuous. Noting (10), by Lemma 1 in [8], $\dot{V}$ tends to zero. Therefore, p, $w_1^l e_{i,j}(2 \le i \le n_j - 1,\, 1 \le j \le m)$, $e_{n_j,j}(1 \le j \le m)$, and $\tilde{u}$ tend to zero. Furthermore, $\dot{e}$ and $e_{i,j}(2 \le i \le n_j - 1,\, 1 \le j \le m)$ tend to zero due to $\lim_{t\to\infty} \inf |w_1(t)| > 0$.

Differentiating $w_1^l p$, yields

$$\begin{aligned} \frac{d}{dt}(w_1^l p) &= -k_1 w_1^l e_1 + l w_1^{l-1} \dot{w}_1 p - k_0 w_1^l p \\ &\quad - w_1^l \sum_{s=1}^{m} \left[\sum_{i=2}^{n_s - 1} e_{i,s} x_{i+1,s} - \sum_{j=3}^{n_s} e_{j,s} \sum_{i=2}^{j-1} \frac{\partial \alpha_{j,s}}{\partial y_{i,s}} x_{i+1,s} \right] \end{aligned}$$

where the first term is uniformly continuous, the other terms tend to zero. Noting that $w_1^l p$ converges to zero, by application of Lemma 1 in [8], $d/dt(w_1^l p)$ converges to zero. Furthermore, $w_1^l e_1$ converges to zero. Therefore, e_1 tends to zero. By Lemma 1, (14) holds. ■

Remark 4: In the controller (16)–(18), η is an internal reference signal of u, Λ is a cross term to prevent the peaking-phenomenon of the overall system [20]. $\hat{a}$ is the estimated value of a and it does not necessarily converge to a. The control parameters are k_0, k_1, $k_{i,j}(2 \le i \le n_j,\, 1 \le j \le m)$, K_p and Γ. It is only necessary that these parameters are positive or positive–definite.

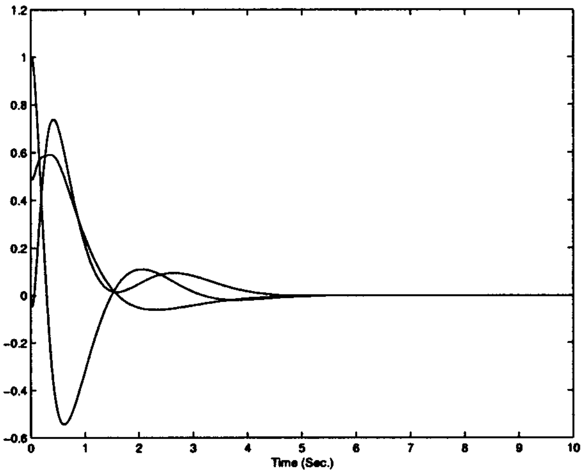

Fig. 1. Response of $(q - q^d)$.

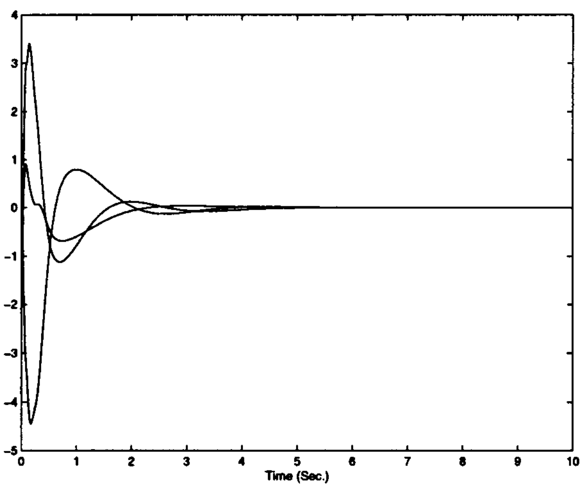

Fig. 2. Response of $(\dot{q} - \dot{q}^d)$.

Remark 5: The assumption that z is bounded can be relaxed to a) $z_{i,j}(2 \le i \le n_j, 1 \le j \le m)$ are bounded; b) if one of $M_2(x)$, $C_2(x, \dot{x})$ and $G_2(x)$ is unbounded on x_1, z_1 is assumed to be bounded. If one of $M_2(x)$, $C_2(x, \dot{x})$ and $G_2(x)$ is unbounded on $x_{2,j}$, $z_{2,j}$ is assumed to be bounded, for $j = 1, 2, \ldots, m$, respectively. The assumptions about z are easily satisfied, since in most situations the given desired trajectory is bounded and as differentiable as required by choice. The assumption that $\lim_{t\to\infty} \inf |w_1(t)| > 0$ is made due to the nonholonomic constraints and the uncertainty.

Remark 6: The controller (16)–(18) can make x and $\dot{x}$ asymptotically tend to z and $\dot{z}$ respectively. Therefore, the dynamic tracking problem of the nonholonomic system with unknown inertia parameters is solved. There are only limited reports on the dynamic tracking problem of the nonholonomic system with uncertainty. The controllers proposed in [5], [19], [9] cannot solve the dynamic tracking problem defined in this paper. The controller (16)–(18) is different from the controller proposed in [10]. In this paper the controller is proposed based on newly defined tracking errors. Additionally, the controller (16)–(18) is superior to the controller in [10] in that the converging rate of the tracking errors can be directly regulated by the control parameters.

IV. Simulation

In this section, we apply the result obtained in Section III to the tracking problem of a wheeled mobile robot discussed. This robot is constituted by a rigid trolley equipped with two co-axle driving wheels and one passive wheel (see [8], [9] for details). The dynamics of the system can be written as (1) and (2) with $q = [x_Q, y_Q, \theta]^T$, $M(q) = \mathrm{diag}[m_r, m_r, I_Q]$, $C(q, \dot{q}) = 0$, $G(q) = 0$, $B(q) = 1/R[col\{-\sin\theta, \cos\theta, L\}, col\{-\sin\theta, \cos\theta, -L\}]$ and $J(q) = [\cos\theta, \sin\theta, 0]$, where $x_Q, y_Q, \theta, m_r, I_Q, R$, and L are defined in [9], the inertia parameter vector $a = [m_r, I_Q]^T$ is assumed to be unknown.

Given desired trajectory $q^d = [2\cos t, 2\sin t, t]^T$, we can design the tracking controller by application of results in Sections II and III. Let $g_1(q) = [\sin q_3, \cos q_3, 0]^T$ and $g_2 = [0, 0, 1]^T$, the system (1) and (2) can be converted into (5) and (6). By the transformation $x_1 = q_3$, $x_{2,1} = q_1 \cos q_3 + q_2 \sin q_3$, $x_{3,1} = -q_1 \sin q_3 + q_2 \cos q_3$, $u_1 = v_2$, $u_2 = v_1 - (q_1 \cos q_3 + q_2 \sin q_3)v_2$, the system can be converted to (9) and (10) with $m = 1$ and $n_1 = 3$. By notations of Section III, the tracking controller can be easily obtained. In the simulation, we assume the real inertia parameter vector $a = [1, 1]^T$, $R = 0.25$, and $L = 0.5$. Let the initial condition $q(0) = [3, 0, 0.5]^T$ and $\dot{q}(0) = [0, 0, 0]^T$, we choose the control parameters $k_0 = k_1 = k_{2,1} = k_{3,1} = 5$, $K_p = \mathrm{diag}[5, 5]$, $\Gamma = \mathrm{diag}[1, 1]$, $p(0) = 0$, and $\hat{a}(0) = [0.5, 0.5]^T$. The simulation is done by Matlab. Responses of $(q - q^d)$ and $(\dot{q} - \dot{q}^d)$ are shown in Figs. 1 and 2, respectively. It should be noted that p converges to zero and that $\hat{a}$ is bounded which are omitted here for space limit. Effectiveness of the controller is verified.

V. Conclusion

This paper deals with the tracking problem of the nonholonomic dynamic system with unknown inertia parameters. A new tracking controller is proposed. The proposed controller ensures that the configuration state asymptotically tracks the desired trajectory.

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Some Algebraic Aspects of the Strong Stabilizability of Time-Delay Linear Systems

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***Abstract*—This note considers some algebraic aspects of the output feedback strong stabilizability of time-delay linear systems. A sufficient condition for the existence of a stable finite-dimensional stabilizing compensator for a single-input–single-output (SISO) time-delay system is given. An illustrative example is also given, and two open problems are posed.**

***Index Terms*—multidimensional (n-D) system, stabilization, strong stabilizability, time-delay system.**

I. Introduction

Stability and stabilization of time-delay systems have been extensively studied using various approaches, such as the analytic and algebraic methods, and more recently the linear matrix inequality (LMI) method, in both the frequency domain and the time domain (ee, e.g., [1], [5], [4], and the literature cited therein).

In the frequency domain formulation, a linear single-input–single-output (SISO) system without time delays (conventionally referred to as a finite-dimensional system) can be described by a rational transfer function, and can be treated adequately by powerful polynomial algebraic methods. However, a time-delay system involves exponential function terms, and it is hence more difficult to apply classical polynomial algebraic methods.

Nevertheless, many researchers have made fruitful efforts at solving control problems in time-delay systems in the algebraic framework. There are two representative approaches within the algebraic framework. One of them is to explore directly the abstract algebraic structure of the rings generated by certain kinds of rational and exponential functions in a single variable in the frequency domain. This approach aims at obtaining exact solutions for time-delay systems and is an attractive direction for future research. See, for example, [8]. Another approach is to assume first the exponential term as an independent variable and apply purely algebraic and algebraic geometric methods to obtain useful results which are subsequently interpreted back into the time-delay system formulation. Although the latter usually can only obtain weak results due to its strong assumption, it can still yield very useful results just because of its simplicity and elegance. See, e.g., [9] for an interesting result by such a method. Closely related to this approach is the study of multidimensional (n-D) systems which tries to establish a unified formulation of the analysis and control of linear systems possessing more than one independent variables [2], [3]. Note that a one-dimensional (1-D) system is conventionally referred to as a finite dimensional system, in the sense that the number of state variables is finite in some state-space realization.

In this note, we apply a recent result on strong stabilizability of linear n-D systems to time-delay linear systems. Specifically, we present a

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