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Adaptive Tracking Control of Nonholonomic Mobile Robots by Computed Torque

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Abstract A computed torque controller for a dynamic model of nonholonomic mobile robots with bounded external disturbance is proposed to treat the adaptive tracking control problem using the separated design method. A velocity controller is first designed for the kinematic steering system to make the tracking error approaching to zero asympotically. Then, a computed torque controller is designed such that the true mobile robot velocity converges to the desired velocity controller. In each step, the controllers are designed independently, and this will simplify the design of controllers. Moreover, the regulation problem and the tracking problem will be treated using the proposed controller. In particular, the mobile robots can globally follow any path such as a straight line, a circle and the path approaching to the origin. Furthermore, the problems of back-into-garage parking and the parallel parking problem can also be solved using the proposed controller. Some interesting simulation results are given to illustrate the effectiveness of the proposed tracking control law.

Keywords: Mobile robots, tracking, time-varying system, nonlinear systems, backstepping

1. Introduction

Nonholonomically constrained mechanical systems, in particular, have attracted significant attention recently [1-7,10-15,18,21,22,25]. Attempts to control such systems are difficult. Even in the case of a simplest kinematic model of mobile robots, it has been found that Brockett's necessary condition [7] does not hold. Consequently, two main research directions have been adopted in recent researches [4,22,25,26]. The first direction, which started from Bloch *et al.* in [4], uses the discontinuous feedback whereas the second one, which was first investigated by Samson in [26], uses the time-varying continuous feedback. In order to obtain faster convergence (e.g., the exponential convergence), an alternative approach was proposed by M'Closkey and Murray in [22] initially and taken up in several studies [22] subsequently. On the other hand, the tracking problem for mobile robots has also attracted as much attention among researches [8,13,19,20,23,24,26]. Despite the apparent advancement of the above methods, there remains however several main restrictions on their applications: (a) They required that one of the tracking line velocity v_r or the angle velocity w_r must not converge to zero. (b) In other cases, for example, the problem of the tracking trajectory approaching to zero remains unsolved. This gives the restriction on applications, and it is impossible to enable a single controller to treat the regulation problem and the tracking problem simultaneously. In practical applications, to solve the problems of back-into garage and parallel parking, a tracking trajectory should be planned to avoid the obstacle and then approaches to the loading dock. Thus, how to drop the restrictions will be interesting.

On the other hand, most of the work reported to date on controlling nonholonomic mechanical systems has concentrated on the kinematic control problem, in which it is assumed that the system velocities are the control inputs and that the system can be adequately represented using the kinematic model. In practical mobile robots, the system model can not be exactly known because of the system uncertainty and external disturbance. Several papers have been published in recent years for this problem [10,11,27]. However, the literature on robustness and adaptive control in presence of uncertainties in the dynamical model of such system is sparse.

To continue this research line, an adaptive control problem will be proposed based on a dynamic model of nonholonomic mobile robots. We use a separating design method to design a computed torque controller. Firstly, a velocity controller for the kinematic steering system will be designed. Secondly, a computed torque controller is designed such that the true mobile robot velocity converges to the desired velocity controller that is deduced from the first step. In each step, the controllers are designed independently, and this will simplify the design of controllers. In particular, the mobile robot can globally follow any path such as a straight line, a circle and the path approaching to the origin using the proposed controllers. Moreover, the problem of back-into-garage parking and the parallel parking problem can also be solved based on our approach. Some simulation results illustrate the effectiveness of the tracking control law.

2. Preliminaries

2.1 A Dynamic Model of Nonholonomic Mobile Robots

Let us consider a nonholonomic system having an n-dimensional configuration space with coordinates $[q_1 \cdots q_n]^T$ and subject to m constraints [10,11].

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q,\dot{q}) = B(q)(\tau + \tau_d) - A(q)^T \lambda \quad (1)$$

where $q = [q_1 \cdots q_n]^T \in \Re^n$ is the generalized coordinates, $M(q) \in \Re^{n\times n}$ is a symmetric, positive definite inertia matrix, $C(q,\dot{q}) \in \Re^{n\times n}$ is the centripetal and coriolis force matrix, $G(q,\dot{q}) \in \Re^{n\times 1}$ is a vector of the gravitational force and frictional force, $\tau \in \Re^{n\times 1}$ is the input vector (applied motor torque), τ_d is the bounded unknown external disturbance, $B(q) \in \Re^{n\times r}$ is the input transformation matrix, $A(q) \in \Re^{m\times n}$

with $0<m<n$ is the full rank matrix associated with the nonholonomic constraints,

$$A(q)\dot{q}=0, \tag{2}$$

and $\lambda \in \Re^{m\times 1}$ is the vector of constraint forces.

Consider the nonholonomic constraint (2). It is not difficult to find an auxiliary vector time function $\mathbf{v}(t)\in\Re^{n-m}$ such that, for all t

$$\dot{q}=S(q)\mathbf{v}(t) \tag{3}$$

where $S(q)$ is a $n\times(n\text{-}m)$ full rank matrix satisfying $S^T(q)A^T(q)=0$ (see [10] for details). The equation (3) is the so-called kinematic equation. On practical applications, the simplest and important example of the nonholonomic mechanical system (1) is a two-driving-wheels mobile robot showing in Fig. 1. This system consists of a vehicle with two driving wheels mounted on the same axis. The motion and orientation of such a system are achieved by independent actuators, e. g., DC motors providing the necessary torque to the rear wheels. Herein, we assume that the reference point lies in the midpoint of the rear wheels and the mobile robot can only move in the direction normal to the axis of the driving wheels (nonholonomic constraints) i.e., the mobile satisfies the conditions of pure rolling and no slipping in wheels, [2,30]. Thus, the following nonholonomic constraint holds

$$\dot{y}\cos\theta-\dot{x}\sin\theta=0 \tag{4}$$

where (x,y) are the Cartesian coordinates of the center of mass of the vehicle, and θ is the angle between the heading direction and the x-axis. Take $q=[x\ \ y\ \ \theta]^T$,

$$A(q)=[-\sin\theta\ \ \cos\theta\ \ 0],\ S(q)=\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix},\ \text{and}\ \ \mathbf{v}=\begin{bmatrix}v\\ w\end{bmatrix}..$$

Then, the kinematic equation (3) is described as

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ w\end{bmatrix} \tag{5}$$

where v and w denote the line velocity and angular velocity, respectively. We multiply the equation (1) by S^T and substituting (3) into (1), then the system (1) now can be transformed into as

$$\overline{M}(q)\dot{\mathbf{v}}+\overline{C}(q,\dot{q})\mathbf{v}+\overline{G}(q)=\overline{\tau}+\overline{\tau}_d \tag{6}$$

where $\overline{M}=S^TMS$, $\overline{C}=S^TCS+S^TM\dot{S}$, $\overline{G}=S^TG$, $\overline{\tau}=S^TB\tau$, and $\overline{\tau}_d=S^TB\tau_d$. In practical systems, the dynamic system (6) has the following properties (see [9,20,28,29]).

Property 1: **(Positive Definiteness)** The matrix $\overline{M}(q)$ is a symmetric positive definite matrix satisfies $\lambda_m I\le\overline{M}(q)$ where λ_m denotes the strictly positive minimum eigenvalue of $\overline{M}$ for all q.

Property 2: **(Skew-Symmetry)** The matrix $\dot{\overline{M}}-2\overline{C}$ is skew-symmetry, i.e., $x^T(\dot{M}-2C)x=0,\ \forall x\in\Re^n$.

This property is simply a statement that the so-called fictitious forces, defined by $C(q,\dot{q})\dot{q}$, do not work on the system (see [9,20,28,29]).

In addition, the following assumptions are given for many systems of practical interest.

Assumption 1: System (6) is linearly parametertrizable (LP), i.e., $\overline{M}(q)\dot{r}+\overline{C}(q,\dot{q})r+\overline{G}(q,\dot{q})=Y(q,\dot{q},r,\dot{r})\Theta$ where Y is an $n\times p_1$ matrix of known function and the unknown parameter Θ is a constant p_1 dimensional vector.

Assumption 2: The unknown external disturbance $\overline{\tau}_d=\tau_l+F\tau_m$ with $\tau_l\in L_2$ and τ_m satisfies the dynamics $\dot{\tau}_m=E\tau_m$ in which E and F are known matrices. Moreover, assume that there exists a known positive definite matrix P such that $PE+E^TP\le 0$.

2.2 Problem formulation

Here we introduce the states, $\mathbf{x}_1=[x\ \ y\ \ \theta]^T$ and $\mathbf{x}_2=[v\ \ w]^T$, then the dynamic equations (5)-(6) can be rewritten into the state space representation

$$\dot{\mathbf{x}}_1=S(\mathbf{x}_1)\mathbf{x}_2 \tag{7a}$$

$$\dot{\mathbf{x}}_2=\overline{M}^{-1}\left[\overline{\tau}+\overline{\tau}_d-\overline{C}(q,\dot{q})\mathbf{x}_2-\overline{G}(q)\right]. \tag{7b}$$

Here, we assume that the tracking trajectory is generated by a kinematic equation which satisfies the nonholonomic constraint (4). Thus, the tracking trajectory is given as

$$\begin{aligned}\dot{x}_r&=v_r\cos\theta_r\\ \dot{y}_r&=v_r\sin\theta_r\\ \dot{\theta}_r&=w_r.\end{aligned} \tag{8}$$

We also assume that both $v_r(t)$, $w_r(t)$, $\dot{v}_r(t)$, and $\dot{w}_r(t)$ are all bounded, and $v_r(t)$, $w_r(t)$ are both uniformly continuous throughout this paper. Note that in a perfect tracking case, i.e., $x\equiv x_r, y\equiv y_r, \theta\equiv\theta_r$, equation (8) always holds and a smooth reference trajectory can be generated by the result of Kanayama *et al.* [17].

Due to the nonholonomic constraint (4), it is not easily achieved by considering the full systems (7a)-(7b). In order to reduce the order of the system and simplify the design of controllers, we will solve the tracking problem of the kinematic equation (7a) using the velocity as the input signal. Thus, it is not surprising to consider the following velocity control problem.

In this paper, our objective is to find a ***torque controller*** $\overline{\tau}$ that achieves the tracking control of the dynamic system (7a)-(7b), for any given reference signal $(x_r(t),y_r(t),\theta_r(t)$ $v_r(t),w_r(t))$ satisfying the equation (8). Now, the main problems in this paper can be formulated as follows.

Tracking Problem with Velocity Control (TPVC): *To find a control law* $\mathbf{v}=[v\ w]^T$ *such that all states of the kinematic equation (7a), with* $\mathbf{x}_2$ *replaced by* $\mathbf{v}+\mathbf{v}_d$, *are bounded and follow a reference trajectory* $(x_r(t),y_r(t),\theta_r(t))$ *satisfying equation (8), i.e.,* $\lim_{t\to\infty}|x(t)-x_r(t)|=0$, $\lim_{t\to\infty}|y(t)-y_r(t)|=0$ *and* $\lim_{t\to\infty}|\theta(t)-\theta_r(t)|=0$ *where* $\mathbf{v}_d=[v_d\ \ w_d]^T\in L_2$ *is any bounded external disturbance.*

Tracking Problem with Torque Control (TPTC): *To find* *a control law $\bar{\tau}$ such that all states of the dynamic system (7a)-(7b), with external disturbance $\bar{\tau}_d$ satisfying Assumption 2, are bounded and follow a reference trajectory $(x_r(t), y_r(t), \theta_r(t), v_r(t), w_r(t))$ satisfying equation (8).*

Remark 1: To solve the TPTC, it will be divided into two separated steps to solve the TPTC. First, a virtual velocity controller will be proposed to solve the TPVC. Then, a computed torque control input will be designed such that the true velocity functions follow any virtual velocity functions in the L_2 sense.

3. Controller Design and Stability Analysis

In this section, a novel separated design method will be proposed in the following. The first step, the nonholonomic tracking problem is simplified by neglecting the vehicle dynamics (7b) and considering only the steering kinematic equation (7a) in which $\mathbf{x}$ is replaced by $\mathbf{v} + \mathbf{v}_d$, where $\mathbf{v}$ is the virtual controller and $\mathbf{v}_d \in L_2$ is the artificial disturbance. Then, we solve the TPVC. The second step is to compute a torque control $\bar{\tau}$ for the dynamic equation (7b) such that the true velocity $\mathbf{x}_2$ follows the any desired velocity $\mathbf{v}$ in the L_2 sense. If we let $\mathbf{v}_d = \mathbf{x}_2 - \mathbf{v}$, then it can be seen that the TPTC is also solved in this case. A block diagram of the design configuration is shown in Fig. 2. Note that, in our approach, it does not need to consider the whole states of dynamic system (7a)-(7b) simultaneously. The velocity controller and the torque controller are designed independently to solve the associated tracking problems. Thus, it simplifies the design of controllers.

3.1 The Velocity Control

There are several approaches to solve the TPVC in present literatures in which the tracking problem and the regulation problem were studied separately. In many cases, it needs to consider the regulation problem and the tracking problem in the same time, e. g., the parallel-parking problem. In this subsection, a general controller will be proposed to treat the TPVC without any further assumptions.

First, consider the steering kinematic equation (7a) or (5) and the reference system (8). Using the local frame, we then define the error coordinates by (see [16])

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}. \tag{9}$$

Therefore, the tracking error model is obtained as

$$\begin{aligned} \dot{x}_e &= w y_e - v + v_r \cos\theta_e \\ \dot{y}_e &= -w x_e + v_r \sin\theta_e \\ \dot{\theta}_e &= w_r - w. \end{aligned} \tag{10}$$

For convenience, new coordinates and inputs are chosen as

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta_e \\ y_e \\ -x_e \end{bmatrix}, \quad \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = \begin{bmatrix} w_r - w \\ v - v_r \cos x_0 \end{bmatrix}. \tag{11}$$

Then, system (11) can be rewritten as (see [22])

$$\dot{x}_0 = u_0 \tag{12a}$$

$$\dot{x}_1 = (w_r - u_0) x_2 + v_r \sin x_0 \tag{12b}$$

$$\dot{x}_2 = -(w_r - u_0) x_1 + u_1. \tag{12c}$$

Let us define two positive definite functions $V_1 = (x_1^2 + x_2^2)/2$, $V_2 = \frac{1}{2}\bar{x}_0^2$, and a new state $\bar{x}_0$ as follows:

$$\bar{x}_0 = x_0 + \frac{\varepsilon h x_1}{1 + (x_0^2 + x_1^2 + x_2^2)^{1/2}} = x_0 + \frac{\varepsilon h x_1}{1 + \Lambda^{1/2}} \tag{13}$$

where $h(t) = 1 + \gamma\cos(t - t_0)$, $0 < \gamma < 1$, $0 < \varepsilon < 1/2(1+\gamma)$, $\Lambda = x_0^2 + x_1^2 + x_2^2$, and t_0 will be specified in the proof of Theorem 1. Note that if $(\bar{x}_0, x_1, x_2)$ converges to zero, then the TPVC will be solved. Now, we are in a position to present the following theorem.

Theorem 1: *Consider the kinematic equation (7a) with bounded external disturbance $\mathbf{v}_d \in L_2$ where $\mathbf{x}_2$ is replaced by $\mathbf{v} + \mathbf{v}_d$ with $\mathbf{v} = [v, w]$. Then the TPVC can be solved using the following control laws*

$$v = -k_0 x_2 + v_r \cos x_0 \tag{14a}$$

$$w = w_r + \frac{\beta(x_1, x_2, t)}{\alpha(x_1, x_2, t)} + k_1 \bar{x}_0 \tag{14b}$$

where $k_0, k_1 > 0$,

$$\alpha(x_0, x_1, x_2, t) = 1 + \frac{\varepsilon h x_2}{1 + \Lambda^{1/2}} + \frac{\varepsilon h x_1 x_0}{\left[1 + \Lambda^{1/2}\right]^2 \Lambda^{1/2}}$$

$$\beta(x_0, x_1, x_2, t) = \varepsilon\Big[\frac{\dot{h} x_1 + h w_r x_2 + h v_r \sin x_0}{1 + \Lambda^{1/2}} - \frac{h x_1}{(1 + \Lambda^{1/2})^2 \Lambda^{1/2}} (x_1 v_r \sin x_0 - k_0 x_2^2)\Big]. \qquad \blacksquare$$

Remark 2: If $v_r(t)$ and $w_r(t)$ are only bounded, the same result still hold. However, it needs deeper stability analysis. We omit its proof here. In Section 4, an interesting simulation will be given to show that the proposed controller can be applied to this case well.

3.2 The Torque Control

In the previous discussion, the velocity controller (the virtual controller) (14a)-(14b), denoted by $\mathbf{x}_2^*$, are designed to achieve the tracking problem of steering kinematic equation (7a). Subsequently, the system (7b) is considered in order to compute the torque controller. Here, we rewrite it as

$$\dot{\mathbf{x}}_2 = \overline{M}^{-1}\left[\bar{\tau} + \bar{\tau}_d - \overline{C}(q, \dot{q})\mathbf{x}_2 - \overline{G}(q)\right]. \tag{15}$$

Define a Lyapunov-like function as

$$W = \tfrac{1}{2}(\mathbf{x}_2 - \mathbf{x}_2^*)^T M (\mathbf{x}_2 - \mathbf{x}_2^*). \tag{16}$$

Using the *Properties 1-2* and *Assumptions 1-2*, the derivation of W along the trajectories of the dynamic system (7b) becomes

$$\begin{aligned}\dot{W} &= \tfrac{1}{2}(\mathbf{x}_2-\mathbf{x}_2^*)^T\left[\dot{\overline{M}}(\mathbf{x}_2-\mathbf{x}_2^*)+2\overline{M}(\dot{\mathbf{x}}_2-\dot{\mathbf{x}}_2^*)\right] \\ &= (\mathbf{x}_2-\mathbf{x}_2^*)^T\left[\tfrac{1}{2}\dot{\overline{M}}(\mathbf{x}_2-\mathbf{x}_2^*)+\overline{\tau}+\overline{\tau}_d-\overline{C}(q,\dot{q})\mathbf{x}_2-\overline{G}(q)-\overline{M}\dot{\mathbf{x}}_2^*\right] \\ &= (\mathbf{x}_2-\mathbf{x}_2^*)^T\left[-Y(q,\dot{q},x_2^*,\dot{x}_2^*)\Theta+\overline{\tau}+F\tau_m+\tau_l\right]\end{aligned} \quad (17)$$

Because Θ and τ_m are unknown parameters, it is necessary to construct an estimator to estimate these parameters. Using the certainly equivalence principles and the gradient method, the TPTC can be solved in the following theorem.

Theorem 2: *Consider the dynamic system (7a)-(7b) of nonholonomic mobile robot (1) with unknown parameters Θ and external bounded disturbance τ_d. Suppose Assumptions 1-2 hold. Then the TPTC can be solved using the feedback law*

$$\overline{\tau} = Y(q,\dot{q},x_2^*,\dot{x}_2^*)\hat{\Theta} - F\hat{\tau}_m - K_p(\mathbf{x}_2-\mathbf{x}_2^*) \quad (18)$$

with parameter update laws

$$\dot{\hat{\Theta}} = -(\Gamma^T)^{-1}Y(q,\dot{q},x_2^*,\dot{x}_2^*)^T(\mathbf{x}_2-\mathbf{x}_2^*) \quad (19)$$

$$\dot{\hat{\tau}}_m = E\hat{\tau}_m + P^{-1}F^T(\mathbf{x}_2-\mathbf{x}_2^*)^T \quad (20)$$

where $K_p \in \Re^{l\times l}$, $\Gamma \in \Re^{2\times 2}$ are any positive definite matrices, and $\mathbf{x}_2^*$ is the virtual velocity controller given in (14a)-(14b). ■

Recently, there are several velocity controllers for solving the tracking control problem of kinematic systems [13,20,26]. Base on our approach, the velocity controller and the torque controller are designed independently. Then, a computed torque controller is design such that the true mobile robot's velocity converges to any desired velocity controller. So, the proposed method is useful for other velocity controllers that were presented in literatures.

4. Simulation results

In this section, we consider a two-driving-wheels mobile robot as shown in Fig. 1. The surface friction and the gravitational force are neglected here, i.e., $C(q,\dot{q}) = G(q,\dot{q}) = 0$. Thus, $Y(q,\dot{q},r,\dot{r}) = \dot{r} = diag(\dot{v},\dot{w})$ and $\Theta = \lfloor M \quad I \rfloor'$, where M and I represent the mass and inertia of the vehicle, respectively. Thus, it is clear that *Properties 1-2* and *Assumption 1* hold. We now illustrate the use of the proposed feedback controller (14a)-(14b) and (18-20) on a vehicle with two driving wheels, in which the control inputs of vehicle are the torque of two wheels, denotes by τ_l and τ_r. Then, the relationship for $\overline{\tau}$ and $\tau = \lfloor \tau_l \quad \tau_r \rfloor'$ can be stated as

$$\overline{\tau} = \begin{bmatrix}\overline{\tau}_1\\ \overline{\tau}_2\end{bmatrix} = \begin{bmatrix}\frac{1}{r}(\tau_l+\tau_r)\\ \frac{R}{r}(\tau_l-\tau_r)\end{bmatrix} \text{ or } \tau = \begin{bmatrix}\tau_r\\ \tau_l\end{bmatrix} = \begin{bmatrix}\frac{r}{2}(\overline{\tau}_1+\frac{\overline{\tau}_2}{R})\\ \frac{r}{2}(\overline{\tau}_1-\frac{\overline{\tau}_2}{R})\end{bmatrix} \quad (22)$$

where r and R represent the radius of a wheel and the distance between two wheels. For the sake of simulation, we assume that M=10 and I=5.

The following simulations are given by MATLAB. Here we suppose that the disturbance is described as

$$\overline{\tau}_d = \begin{bmatrix}10+\sin(3t)+10e^{-t}\sin(t)\\ 10+\cos(3t)+10e^{-t^2}\end{bmatrix}.$$

Hence

$$E = \begin{bmatrix}0 & 3 & 0\\ -3 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \quad F = \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}, \quad \tau_l = \begin{bmatrix}10e^{-t}\sin(t)\\ 10e^{-t^2}\end{bmatrix}.$$

More precisely, according to Theorem 2 we can solve the TPTC for a circle tracking trajectory and a straight line tracking trajectory. Note that the TPTC problem can also be solved if the tracking velocities w_r and v_r are only bounded although a complete stability analysis is not given here. Interesting simulation are also given for the cases of back-into-garage parking or the parallel parking problem. The corresponding tracking trajectories are described as follows subsequently.

Case 1: (Straight-line) $x_r(t) = 10t/\sqrt{2}$, $y_r = 10t/\sqrt{2}$, $\theta_r = \pi/4$,

$v_r = 10$, and $w_r = 0$

Case 2: (Back-into-garage parking)

$$x_r(t) = \begin{cases}15-3t & 0\le t<5\\ 0 & otherwise\end{cases}, \quad y_r = \begin{cases}4 & 0\le t<10\\ 4-(t-10) & 10\le t<14\\ 0 & 14\le t\end{cases},$$

$$\theta_r = \begin{cases}\pi & 0\le t<5\\ \pi+\frac{t-5}{10}\cdot\pi & 5\le t<10\\ \frac{3\pi}{2} & 10\le t\end{cases} \quad v_r = \begin{cases}3 & 0\le t<5\\ 0 & 5\le t<10\\ 1 & 10\le t<14\\ 0 & 14\le t\end{cases},$$

$$w_r = \begin{cases}0 & 0\le t<5\\ \frac{\pi}{10} & 5\le t<10\\ 0 & 10\le t\end{cases}$$

Case 3: (Parallel parking)

$$x_r(t) = \begin{cases}2a\cos(c(t+\frac{\pi}{4c})) & 0\le t\le\frac{\pi}{2c}\\ -a\sqrt{2} & \frac{\pi}{2c}\le t\end{cases},$$

$$y_r(t) = \begin{cases}b\sin(2c(t+\frac{\pi}{4c})) & 0\le t\le\frac{\pi}{2c}\\ -b & \frac{\pi}{2c}\le t\end{cases},$$

$$\theta_r(t) = \begin{cases}\pi-\tan^{-1}\left(\frac{b\cos(2c(t+\pi/4c))}{a\sin(c(t+\pi/4c))}\right) & 0\le t\le\frac{\pi}{2c}\\ \pi & \frac{\pi}{2c}\le t\end{cases},$$

$$v_r(t) = \begin{cases}2c\sqrt{a^2\sin^2(c(t+\frac{\pi}{4c}))+b^2\cos^2(2c(t+\frac{\pi}{4c}))} & 0\le t\le\frac{\pi}{2c}\\ 0 & \frac{\pi}{2c}\le t\end{cases}$$

$$w_r(t) = \begin{cases}\dfrac{abc\cos(c(t+\frac{\pi}{4c}))\cos(2c(t+\frac{\pi}{4c}))+2\sin(2c(t+\frac{\pi}{4c}))}{(a\sin(c(t+\frac{\pi}{4c}))^2+(b\cos(2c(t+\frac{\pi}{4c}))^2} & 0\le t\le\frac{\pi}{2c}\\ 0 & \frac{\pi}{2c}\le t\end{cases}$$

where c=0.1 is a constant relating the velocity and a=5, b=4. Fig. 3 shows the tracking trajectory for the parallel parking problem.

Several simulations are shown in Figs. 3-5 for above cases, respectively. Moreover, for simplicity, the positive matrices $P = I_{3\times3}$, $\Gamma = I_{3\times3}$, and $K_p = diag(k_2,k_3)$ are chosen. The initial conditions $\mathbf{x}(0)$ and the corresponding parameters of controllers are chosen as

Case 1:Initial condition: $\begin{bmatrix} 2 & 0 & \pi & 0 & 0 \end{bmatrix}$.

Parameters: $k_0 = 4,\ k_1 = 4,\ k_2 = 10,\ k_3 = 8,$

$\varepsilon = \frac{1}{3},\ \gamma = 0.5,\ t_0=0.$

Case 2:Initial condition: $\begin{bmatrix} 17 & 7 & 0 & 0 & 0 \end{bmatrix}$.

Parameters: $k_0 = 4,\ k_1 = 4,\ k_2 = 20,\ k_3 = 10,$

$\varepsilon = \frac{1}{3},\ \gamma = 0.5,\ t_0=0.$

Case 3:Initial condition: $\begin{bmatrix} 10 & 8 & \pi & 0 & 0 \end{bmatrix}$.

Parameters: $k_0 = 0.4,\ k_1 = 1.5,\ k_2 = 20,\ k_3 = 5,$

$\varepsilon = \frac{1}{3},\ \gamma = 0.5,\ t_0=0.$

In the TPVC problem associated with the approaching to zero case (e. g., cases 2 and 3) has remained unsolved in the present literatures except for our previous paper [20]. However, this research effort has shown that the problem can also be solved using the proposed controller (14a)-(14b). Moreover, based on our approach, the assumptions (one of v_r and w_r does not converge to zero) in [13] are unnecessary. In case 3, $v_r(t)$ and $w_r(t)$ are only bounded and our assumptions for the velocity do not hold because of the discontinuity of tracking velocity (there is a jump in the figures of $v_r(t)$ and $w_r(t)$). For this reason, a small tracking error occurs in this case (see Fig. 4(b)). However, Fig. 4-5 demonstrates the ability of our approach even if the continuous assumption does not hold. From simulation results, we observe that the proposed controller improves the response compare with our previous result [20]. However, the controller presented here is more complex than the controller in literature [20]. Additionally, the designed coefficients can be chosen according to the specified performance (torque effort or transient response *etc*.).

5. Conclusion

In this paper, the adaptive tracking problem of nonholonomic mobile robots with bounded external disturbance was studied using the torque controller. We have proposed a novel separated design method that simplified the design of controllers. A new velocity controller was proposed for solving the regulation problem and the tracking problem for the velocity control problem. Additionally, a computed torque controller was also presented such that the mobile robot velocity converges to the any desired velocity. Base on our approach, it did not need to consider the whole system simultaneously, and only the reduced-order subsystems were considered independently in every stage. Moreover, mobile robots can globally follow any path such as a straight line, a circle and the path approaching to the origin using the proposed controller. When apply to the problems of back-into-garage and parallel parking, it was also shown that a satisfactory tracking result can be obtained. Future research may toward to the tracking control with the slipping occurred.

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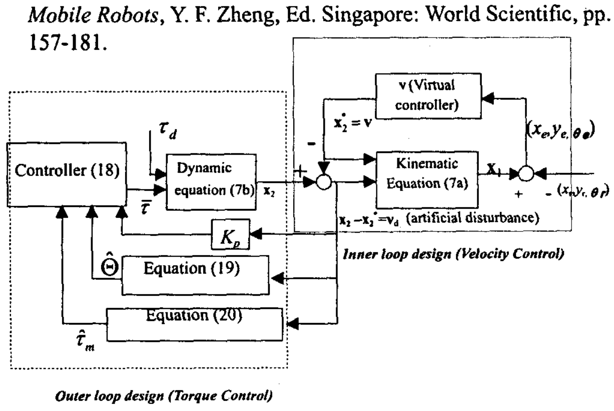

Figure 2: A block diagram of the designed configuration

Figs.3-5: (a) Motion of the vehicle in the (x,y) plane; (b) Tracking error (x: solid line; y: dashed line; θ : dash-dotted line); (c) Control signal (τ_r : solid line; τ_l : dashed line); (d) Estimated parameters (ˆ : solid line; $\hat{I}$: dashed line).

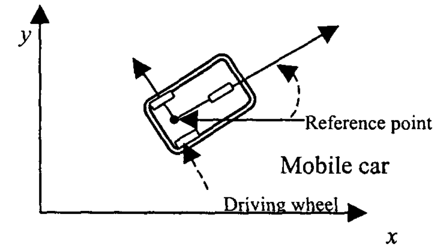

Figure 1: A two-driving-wheels mobile robot

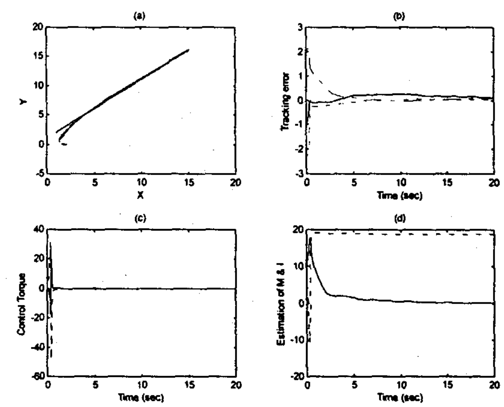

Figure 3: Simulation result for a straight-line tracking trajectory.

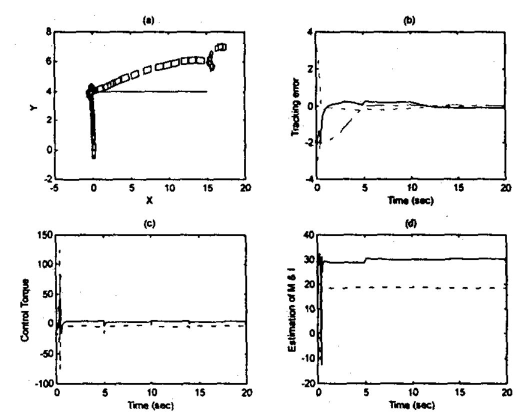

Figure 4: Simulation result for the case of back-into-garage parking:

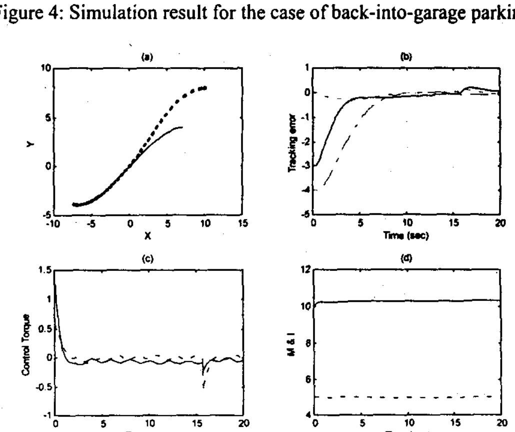

Figure 5: Simulation result for the case of parallel parking problem.