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A Unified Discontinuous State Feedback Controller for the Path-Following and the Point-Stabilization Problems of a Unicycle-like Mobile Robot

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Abstract

In the present work, a unified kinematics model is derived for the path-following and the point-stabilization problems of a wheeled mobile robot, using a signed polar representation. The mobile target configuration is represented by the motion of a reference mobile robot which has the same kinematics constraints as the real one. Thereafter, the two problems are solved simultaneously by means of a state feedback control law which is discontinuous at the origin. Finally, simulation results are given to demonstrate the controller's effectiveness.

1. Introduction.

In last years, nonholonomic systems have been extensively studied, in particular wheeled mobile robots (WMR) which are typical examples of such strong nonlinear systems. In fact, the path following (PF) and the point-stabilization (PS) problems of WMR have been recently in the focus of many researchers, and several control schemes have been carried out, separately, for both problems. Various classical state feedback have been proposed for the first problem (PF), (see for example, [8], [10], [11], [12], [13] and [14]). The second problem (PS) is more difficult than the first one, and a deep nonlinear analysis is required. In fact, nonholonomic systems described in cartesian space cannot be stabilized by mean of a smooth time-invariant state feedback control as pointed out by Brockett [5]. To overcome this difficulty, smooth time-varying state feedback controllers have been proposed as an alternative solution (see [9] and [13]). The main drawback of these elegant methods is that they exhibit low rates of convergence and generate oscillating paths. Recently, discontinuous time-invariant control laws, yielding exponential stabilization, have been derived by means of adequate coordinates transformations instead of the traditional cartesian representations. The generated paths are more realistic than those obtained via smooth time-varying methods, (see for example, [1], [2], [3], [4], [6] and [7]). In [3], a locally stabilizing fuzzy-tuned state feedback has been proposed using the polar representation. Using the same idea, a globally stabilizing state feedback has been proposed in [6]. However, this controller allows the vehicle to reach the goal only in forward motion. Thereafter, a linear state feedback, has been proposed in [1] using also the same representation, where local and global stability issues have been discussed. However, this controller suffer from the necessity of using two different models for the forward and the backward motion. In this paper we propose a unified discontinuous control scheme for the path-following and the point-stabilization problems which is an extension of [15]. To this end, a *signed polar representation,* of the configuration error in the basis of the frame linked to the mobile target configuration, is used (see Figure 1). The proposed control law is derived in the general case, where the real WMR tracks a reference WMR, and is still also valid when the target configuration is fixed (i.e. $v_r = 0$ and $\dot{\theta}_r = 0$).

This paper is organized as follows: In section 2, a general kinematics model based on a mobile target configuration tracking is derived for the PF and the PS problems. Section 3 is devoted to the synthesis of a unified control scheme for the two problems. In section 4, some simulation results are given to highlight the effectiveness of the proposed control law. Section 5, concludes the paper.

2. Problem Statement

As shown in Figure 1, the target configuration is represented by a reference WMR with the same kinematics constraints as the real one. $(v, \dot{\theta})$ and $(v_r, \dot{\theta}_r)$ denote, respectively, the linear and rotational velocities of the real and the reference WMR. The variables d and ψ represent the polar coordinates of the position error vector $\overrightarrow{MM_r}$ and the variable γ describes the relative orientation error such that $\gamma = \psi - \theta + \theta_r$.

Our aim is to superpose the real WMR and the reference one by vanishing the position-error vector $\overrightarrow{MM_r}$ and the orientation error $\theta_e = \theta - \theta_r$, (i.e. $d = \psi = \gamma = 0$).

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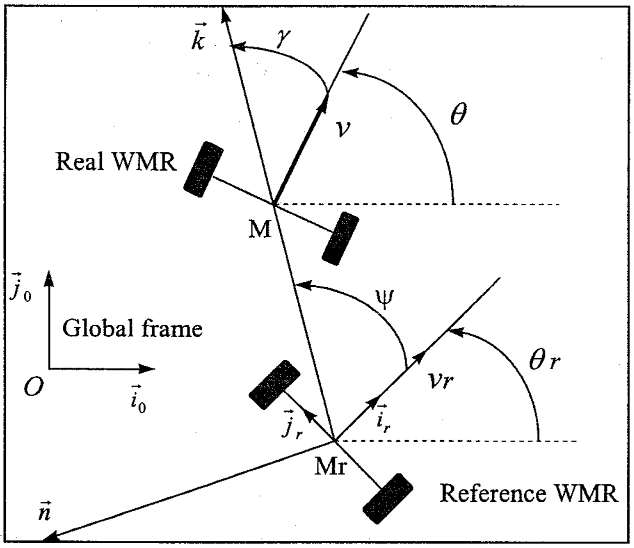

Figure 1. Signed Polar Representation.

Remark 1.
By signed polar representation we mean that the error-distance d may be a strictly positive or negative value while ψ belongs to the set $[0,\pi]$. The difference between this representation and the classical one is that the second half of the plane usually described by $\psi \in [\pi, 2\pi]$ and $d > 0$ is substituted by $\psi \in [0,\pi]$ and $d < 0$. This particular representation allows to have positive and negative values for the variable d. This fact will be useful in proposition 1.

2.1 A General Kinematics Model

Using the previous signed polar representation the position-error vector can be written in the mobile frame as follows

$$\overrightarrow{M_r M} = d\,\vec{k} \tag{1}$$

Differentiating (1) with respect to time yields

$$\frac{d\,\overrightarrow{M_r M}}{d\,t} = \dot{d}\,\vec{k} + d\,\frac{d\,\vec{k}}{d\,t} = \dot{d}\,\vec{k} + d\,\dot{\psi}\,\vec{n} \tag{2}$$

Furthermore, we have

$$\frac{d\,\overrightarrow{M_r M}}{d\,t} = \frac{d\,\overrightarrow{OM}}{d\,t} - \frac{d\,\overrightarrow{OM_r}}{d\,t} \tag{3}$$

where

$$\frac{d\,\overrightarrow{OM}}{d\,t} = v\cos\gamma\;\vec{k} - v\sin\gamma\;\vec{n} \tag{4}$$

$$\frac{d\,\overrightarrow{OM_r}}{d\,t} = v_r\cos\psi\;\vec{k} - v_r\sin\psi\;\vec{n} \tag{5}$$

Substituting (2),(4) and (5) in (3), and assuming that $d \neq 0$, one has

$$\begin{aligned} \dot{d} &= v\cos\gamma - v_r\cos\psi \\ \dot{\psi} &= -\frac{v}{d}\sin\gamma + \frac{v_r}{d}\sin\psi \end{aligned} \tag{6}$$

Furthermore, from (Figure 1) one has

$$\gamma = \psi - \theta + \theta_r \tag{7}$$

and

$$\dot{\gamma} = \dot{\psi} - \dot{\theta} + \dot{\theta}_r \tag{8}$$

Finally, we obtain the state representation for the PF and the PS problems

$$\begin{aligned} \dot{d} &= v\cos\gamma - v_r\cos\psi \\ \dot{\psi} &= -\frac{v}{d}\sin\gamma + \frac{v_r}{d}\sin\psi \\ \dot{\gamma} &= -\frac{v}{d}\sin\gamma + \frac{v_r}{d}\sin\psi - \dot{\theta} + \dot{\theta}_r \end{aligned} \tag{9}$$

Setting $u_1 = v - v_r$ and $u_2 = \dot{\theta} - \dot{\theta}_r$, the latter becomes

$$\begin{aligned} \dot{d} &= u_1\cos\gamma - v_r(\cos\psi - \cos\gamma) \\ \dot{\psi} &= -\frac{(u_1 + v_r)}{d}\sin\gamma + \frac{v_r}{d}\sin\psi \\ \dot{\gamma} &= -\frac{(u_1 + v_r)}{d}\sin\gamma + \frac{v_r}{d}\sin\psi - u_2 \end{aligned} \tag{10}$$

Remark 2.
If $v_r = 0$ and $\dot{\theta}_r = 0$, system (10) corresponds to the stabilization model which satisfies the Brockett's necessary condition [5] for the existence of smooth time-invariant state feedback, provided that $d \neq 0$.

3. Control Law Synthesis

In this section, we derive a unified feedback control law to solve simultaneously the PF and the PS problems.
One has the following result:

Proposition 1. *Consider system (10) under the following control law:*

$$\begin{cases} u_1 = \dfrac{1}{\cos\gamma}\big(-k_1 d + v_r(\cos\psi - \cos\gamma)\big) \\ u_2 = k_2\gamma - (k_3\psi + \gamma)(\dfrac{u_1 + v_r}{d})\dfrac{\sin\gamma}{\gamma} + \dfrac{v_r}{d}\sin\psi \end{cases} \tag{11}$$

where k_1, k_2 and k_3 are positive parameters, and $v_r(t) = \dfrac{\psi}{\sin\psi}(v_{rd}(t) - k_4 d)$, where $k_4 = 0$ in the point-stabilization case and $k_4 > 0$ in the path-following case. Assume that $v_{rd}(t)$ is bounded and does not change its sign, *and assume that* $\gamma \neq \frac{\pi}{2} + k\pi$ *(*$k = 1,2,\cdots$*) and* $d(0) \neq 0$. *Then*

Case 1: *path-following* $(v_{rd}(t) \neq 0)$

(i) The equilibrium point $(d = 0, \psi = 0, \gamma = 0)$ *of the closed-loop system (10)-(11) is exponentially stable provided that* $Sign(v_{rd}(t)) = -Sign(d(0))$.
(ii) The control variable u_1 *is bounded and tends to zero, and hence* $v \to v_{rd}$ *when* $t \to \infty$.

Case 2: *point-stabilization* $(v_{rd}(t) = \dot{\theta}_r(t) = 0$ *and* $k_4 = 0)$

(i)The equilibrium point $(d = 0, \psi = 0, \gamma = 0)$ *of the closed-loop system (10)-(11) is asymptotically stable.*
(ii) The control variables u_1 *and* u_2 *are bounded and tend asymptotically to zero provided that one of the following conditions is fulfilled*

$k_1 > 0,\ k_3 > 0$ *and* $2k_1\sqrt{k_3} \leq k_2 < k_1(1 + k_3)$,
or
$k_1 > 0, k_3 > 1$ *and* $2k_1 < k_2 < 2k_1\sqrt{k_3}$.

■

Proof. Let us consider the following Lyapunov candidate function:

$$V(d,\psi,\gamma) = \frac{1}{2}(d^2 + \gamma^2 + k_3\psi^2) \tag{12}$$

Its time derivative along the trajectories of the system (10) is

$$\dot{V}(d,\psi,\gamma) = d\big(u_1 \cos\gamma - v_r(\cos\psi - \cos\gamma)\big) + (\gamma + k_3\psi)\left(-\frac{(u_1 + v_r)}{d}\sin\gamma + \frac{v_r}{d}\sin\psi\right) - \gamma\, u_2 \tag{13}$$

Substituting (11) in (13), yields

$$\dot{V}(d,\psi,\gamma) = -k_1 d^2 - k_2\gamma^2 + k_3\frac{v_r}{d}\psi\sin\psi \tag{14}$$

Substituting v_r by $\frac{\psi}{\sin\psi}(v_{rd}(t) - k_4 d)$ leads to

$$\dot{V}(d,\psi,\gamma) = -k_1 d^2 - k_2\gamma^2 - k_4 k_3\psi^2 + k_3\frac{v_{rd}}{d}\psi^2 \tag{15}$$

Case 1: path-following case$(v_{rd}(t) \neq 0)$

Substituting u_1 in the first equation of (10) by its expression given in (11), we obtain $\dot{d} = -k_1 d$. This means that, if $d(0) \neq 0$, the variable d decays exponentially to zero with never crossing $d = 0$, i.e. $sign(d) = sign(d(0))$ for all $t \geq 0$. Thus, from (15), it is clear that if $Sign(v_{rd}(t)) = -Sign(d(0))$, then there exist α, such that

$$\dot{V}(d,\psi,\gamma) \leq -k_1 d^2 - k_2\gamma^2 - k_4 k_3\psi^2 \leq -\alpha\, V(d,\psi,\gamma),$$

where α can be taken as $2\inf(k_1, k_2, k_4)$. Therefore, the equilibrium point $(d = 0, \psi = 0, \gamma = 0)$ of the closed-loop system (10)-(11) is exponentially stable. Finally, the convergence of $v_r \to v_{rd}$ and $v - v_r \to 0$ becomes obvious.

Case 2: point-stabilization case $(v_{rd}(t) = 0$ and $k_4 = 0)$
In this case, one can apply LaSalle's theorem, since the closed-loop system is time-invaiant. Since $v_r = 0$, one can conclude, from (15), that d and γ tend to zero when t tends to infinity. Thus, $\dot{\gamma}$ tends also to zero. From (11), $u_1 \to 0$ and $u_2 \to -u_1\frac{\sin\gamma}{d}$. Furthermore, from (10), $u_2 \to -k_3 u_1\frac{\psi}{d}$. Therefore, $-u_1\frac{\sin\gamma}{d} \to -k_3 u_1\frac{\psi}{d}$ which means that $k_3\psi$ tends $\sin\gamma$ when when t tends to infinity. Thus, the asymptotic convergence of ψ to zero immediately follows. Now, to guarantee the boundedness and the convergence to zero of the control variable u_2, one must ensure the boundedness and the convergence to zero of the ratios $\frac{\psi}{d}$ and $\frac{\gamma}{d}$. We have said previously that if $d(0) \neq 0$, the variable d decays exponentially to zero with never crossing $d = 0$. Thus, we have just to ensure that the rates of convergence of ψ and γ are greater than the rate of convergence of d in the neighborhood of the equilibrium point. To this end, let us consider the closed loop system (10)-(11), in the neighborhood of the equilibrium point

$$\begin{aligned} \dot{d} &= -k_1\, d \\ \dot{\psi} &= k_1\,\gamma \\ \dot{\gamma} &= -k_2\gamma - k_1 k_3\,\psi \end{aligned} \tag{16}$$

From the latter, it appears clearly that the error-distance d converges to zero as $(e^{-k_1 t})$, and γ, ψ converge to zero with a rate of convergence at least equal to $(e^{-\lambda t})$, where $(-\lambda)$ is the real part of the dominant pole of the following subsystem:

$$\begin{pmatrix}\dot{\psi}\\ \dot{\gamma}\end{pmatrix} = \begin{pmatrix} 0 & k_1 \\ -k_1 k_3 & -k_2 \end{pmatrix}\begin{pmatrix}\psi\\ \gamma\end{pmatrix} \tag{17}$$

Now, it is possible to conclude that the vehicle reaches the goal asymptotically to the X-axis (i.e. γ and ψ converge to zero faster than d) if $\lambda > k_1$, which leads to the conditions given in (ii).

■

There are few remarks that should be pointed out:

Remark 3.
In the path following case, we have not the proof for the boundedness and the convergence to zero of the control

variable u_2 which can be large when d tends to zero. To overcome this drawback, one can introduce a saturation for the variable d, by vanishing the control variable u_1 when d is sufficiently small. Indeed, the control variable u_1, in (11), can be modified as follows

$$u_1 = \begin{cases} \dfrac{1}{\cos\gamma}\left(-k_1 d + v_r(\cos\psi - \cos\gamma)\right) & \text{if } \ |d| > \varepsilon \\ 0 & \text{if } \ |d| \le \varepsilon \end{cases}$$

This modification ensures asymptotic stability for the state variables ψ and γ and practical stability for the state variable d in an arbitrary small domain defined by $|d| \le \varepsilon$, $\varepsilon > 0$. In this case, the boundedness and the convergence to zero of the control variable u_2 is then ensured.

Remark 4.
The assumption $d(0) \neq 0$ is not very restrictive, as it is always possible to apply an open loop control, for an arbitrary small period of time, to drive the vehicle away from $d = 0$ and then switch to the feedback (11).

Remark 5.
If we want to track a desired path in forward motion ($v_{rd}(t) > 0$), we have to set $d(0) < 0$, and if we want to track a desired path in backward motion ($v_{rd}(t) < 0$), we have to set $d(0) > 0$.

4.Simulation Results

In this section, to highlight the effectiveness of the proposed control law, we present some simulation results for path-following and point-stabilization problems of a unicycle-like mobile robot.

4.1 Point-Stabilization Case

In this case, v_r and $\dot{\theta}_r$ are both equal to zero, and the desired final configuration is the origin.
the control parameters have been chosen as $k_1 = 0.5$, $k_2 = 1.5$ and $k_3 = 3$. Figure 2 and Figure 3 show, respectively, the asymptotic convergence of the state variables towards zero and the vehicle motion, in the parking maneuver, starting from ($d = 2$, $\psi = \frac{\pi}{2}$, $\gamma = \frac{\pi}{2}$).
The presented simulation results bring into evidence the smoothness of the generated paths under the proposed control law.

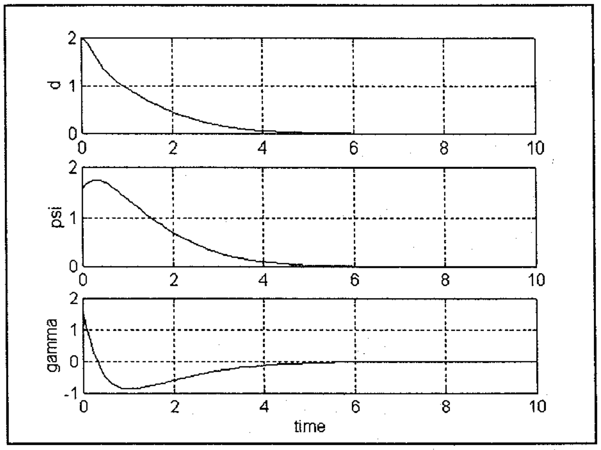

Figure 2. Timeplots of the state variables

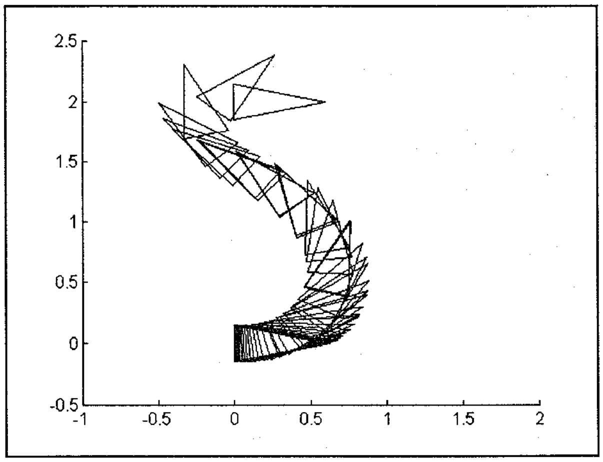
Figure 3. Vehicle motion in the parking maneuver.

4.2 Path Following Case

In this case, the desired trajectory is a circle described by the motion of the WMR ($v_{rd} = 1\, m/s$ and $\dot{\theta}_r = 0.33\, rd/s$). Figure 4 and Figure 5 show, respectively, the asymptotic convergence of the state variables to zero and the vehicle motion starting from ($d = -1$, $\psi = \frac{\pi}{6}$, $\gamma = \frac{\pi}{6}$).

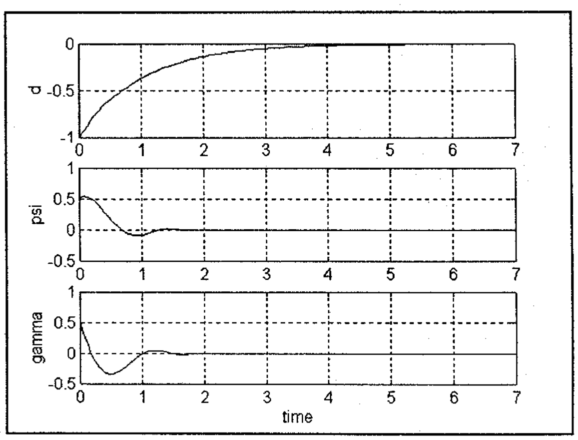

Figure 4. Time plots of the state variables

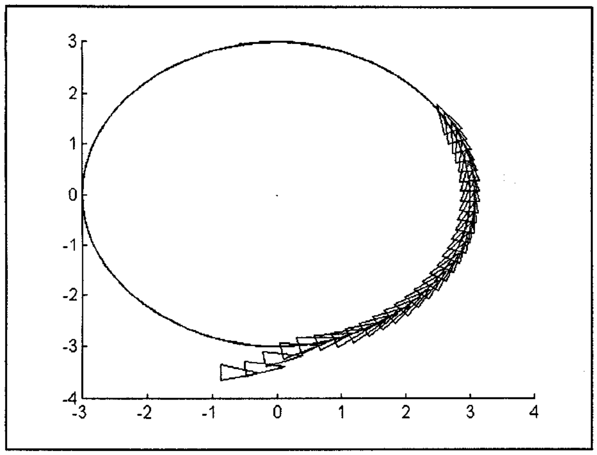

Figure 5 Circular path tracking, starting from the configuration $(-1, \pi/6, \pi/6)$

5. Conclusion

In this paper, a general kinematics model based on a mobile target configuration tracking has been derived for the path-following and the point-stabilization problems of a nonholonomic mobile robot. Thereafter, a discontinuous state feedback controller has been proposed to solve simultaneously the two problems. The discontinuity involved in this controller is not very restrictive, since we have just to avoid $d = 0$ at the initial time by applying, if necessary, an open loop control for an arbitrary small period of time to drive the vehicle away from $d = 0$ and then switch to the state feedback (11).

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