# The Miniature Omni-directional Mobile Robot OmniKity-I (OK-I)

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#### Abstract

The structure of OmniKity-I (OK-I), the miniature omni-directional mobile robot, and its kinematics and dynamics models are presented. OK-I has three DOFs in the plane. A conventional two wheeled mobile robot (2-WMR) serves as the base of OK-I for translational motion. The robot body is on top of the base. The third motor controls angular motion of the body as the base turns according to the translational motion. The circular surface connector (CSC) is devised to transfer signals from the base to the body without constraining angular motion of the body. Experimental results show proper omni-directional features of OK-I.

# 1 Introduction

A wheeled mobile robot (WMR) has three coordinates in the plane. Position x, y and robot heading angle  $\theta$  in an absolute frame. An omni-directional WMR can be defined as a WMR that can move along any path that originates from the current posture (x, y,  $\theta$ ) in the configuration space.

For the past decade, many omni-directional robot structures have been suggested. Most of them used specially designed wheels, e.g., universal wheels, Swedish wheels, ball wheels or roller wheels [1]-[5]. These wheels are designed in such a manner that each wheel has two degrees of freedom in motion but only one of them is active-driving. Three or more of these wheels are combined to guarantee the omni-directional motion of the body. Some structures have the advantage that special wheel locations allow slippage detection [1] [5]

In recent years new structures have been introduced. Compared with one degree of active driving of the wheels, two degrees of active-driving concept can be found in [6]-[7]. It is straightforward that a pair of wheels which are active-driving in two dimen-

sion can be combined to make the omni-directional motion. In [7], the velocity factor perpendicular to the robot heading angle is generated by the angular motion of the offset axis. It is known that an offset steered wheel is omni-directional [8]. It is very robust to slippage between the actuators and the wheels and between the wheels and the ground. One point is that a pair of two dimensionally active-driving wheels cost four actuators. Still the redundancy may be compensated by little slippage.

The principle of generating the perpendicular motion factor can be applied in the very same manner to 2-WMR's by modeling a two wheeled mobile robot as an offset steered uni-cycle, not as a conventional unicycle (Figure 1). While the uni-cycle model is active in two dimension in the sense of having active v and  $\omega$ , the offset steered model is in the sense of having active  $v_x$  and  $v_y$ . A big difference between the offset steered uni-cycle and a 2-WMR modeled as an offset steered uni-cycle is that, to be active in two dimension, the former needs to be coupled with another, but the latter does not; a offset steered uni-cycle cannot translate the body with active  $v_x$  and  $v_y$ , but a 2-WMR is sufficient to translate the body. From this, one 2-WMR modeled as an offset steered uni-cycle is an independent active wheel in two dimension in a manner that its  $v_x$  and  $v_y$  are active. In this model the angular velocity of the 2-WMR is dependent on the translational velocities.

In the omni-directional robot OmniKity-I (OK-I), the 2-WMR serves as the base that controls translational motions of the body. To make the robot heading angle independent of the translational motions, the robot body is put on the base with a revolute joint. The base contains two geared DC motors with encoders and a caster. Since the body revolves most of time on the base to keep its desired angular velocity as the base translates, the circular surface connector (CSC) is devised to flow signals from the base to the body where most of the electronic circuitry is

contained without limiting the angular motion of the body.

OK-I is a miniature robot less than 8.5 cm in each side. The name OmniKity comes from Kity, the 1 inch³ micro robot equipped with a CPU, sensors, geared motors, and batteries on-board and specially designed for running in a maze [9]. OK-I is equipped with actuators and control circuitry on-board. Future generations of OK will be with some proximity sensors and on-board cameras for MiroSot, Micro-Robot World Cup Soccer Tournament (www.fira.net), and will be a little bit bigger than the prototype (the first micro omni-directional robot with the ball-wheel mechanism was shown at MiroSot'97 [10]).

In Section 2 the kinematics of OK-I is presented. Control scheme is descrived in Section 3 and experimental results are reported in Section 4. Concluding remarks follow in Section 5.

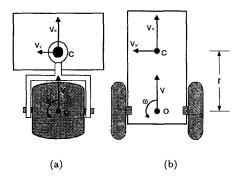


Figure 1: (a) Offset steered uni-cycle model (b) 2-WMR modeled as offset steered uni-cycle

## 2 Kinematic Equations

A conventional 2-WMR is modeled as a uni-cycle as

$$\begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{1}$$

where v and  $\omega$  are the control inputs of the 2-WMR. In (1) point o is assumed to be the center of the 2-WMR. On the other hand, if point c is assumed to be the center, the kinematic equation becomes

$$\begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}. \tag{2}$$

From (2), we have

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}. \tag{3}$$

Since the  $2 \times 2$  matrix on the right-hand side is invertible, the position(x, y) of point c can follow any path as long as r is not zero. It has been noted from the nonholonomic problem point of view that such a point c, r apart from the wheel axis, can be exponentially point stabilizable provided that r is not zero [11]- [12]. This is drawn very naturally from the system equation (3).

A rather interesting form of this equation is

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v \\ r\omega \end{pmatrix}$$
 (4)

from which it is seen that the global translational velocity factors  $v_x$  and  $v_y$  are just the rotational transformation of v, the velocity factor along the robot heading and  $r\omega$ , the velocity factor perpendicular to the robot heading angle as shown in Figure 2. Since v

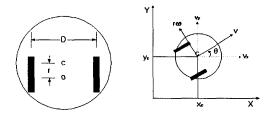


Figure 2: Active-driving wheel in two dimension

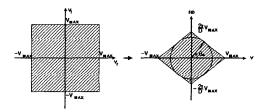


Figure 3: Feasible region of translational velocity

and  $\omega$  are represented by

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{D} & \frac{1}{D} \end{pmatrix} \begin{pmatrix} v_l \\ v_r \end{pmatrix}$$
 (5)

where  $v_l$  and  $v_r$  are the left and right wheel velocities, respectively, the translational velocity is limited to the shaded region as in Figure 3.

Thus the minimum instantaneous translational velocity  $v_m$  is obtained at the angle  $\pm \theta_m$  or  $\pi \pm \theta_m$  to the robot heading

$$v_m = 2V_{MAX} \frac{r}{D} \cdot \sin \theta_m \tag{6}$$

where the angle  $\theta_m$  is a 2-WMR's geometric parameter defined by

$$\theta_m = \tan^{-1} \frac{V_{MAX}}{\frac{2r}{D} \cdot V_{MAX}} = \tan^{-1} \frac{D}{2r}.$$
 (7)

As in (4), the angular velocity of the base  $\omega$  becomes a subordinate variable to the translational velocities  $v_x$  and  $v_y$ . Therefore by putting the body on point c and by introducing an additional control input  $\omega_3$  for the body, the angular velocity  $\omega_B$  of the robot body becomes independently controllable:

$$\omega_B = \omega + \omega_3. \tag{8}$$

Combining (4) and (8) together, the forward kinematic equations are obtained as

$$\begin{pmatrix} v_x \\ v_y \\ w_B \end{pmatrix} = \begin{pmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \\ \omega_3 \end{pmatrix}. \quad (9)$$

The determinant of the above  $3 \times 3$  matrix in (9) is r. Since r is not zero, the inverse kinematics is obtained

$$\begin{pmatrix} v \\ \omega \\ \omega_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\frac{1}{r}\sin\theta & \frac{1}{r}\cos\theta & 0 \\ \frac{1}{r}\sin\theta & -\frac{1}{r}\cos\theta & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ w_B \end{pmatrix} . (10)$$

Replacing v and  $\omega$  by  $v_l$  and  $v_r$ , we have

$$\begin{pmatrix} v_l \\ v_r \\ w_3 \end{pmatrix} = J^{-1} \begin{pmatrix} v_x \\ v_y \\ w_B \end{pmatrix}$$
 (11)

where

$$J^{-1} = \left( \begin{array}{ccc} \cos\theta + \frac{D}{2r}\sin\theta & \sin\theta - \frac{D}{2r}\cos\theta & 0 \\ \cos\theta - \frac{D}{2r}\sin\theta & \sin\theta + \frac{D}{2r}\cos\theta & 0 \\ \frac{1}{r}\sin\theta & -\frac{1}{r}\cos\theta & 1 \end{array} \right).$$

As the base revolves below the body during most of the operation, ordinary wires cannot be used for transferring the signals from the base to the body. For OK-I, the circular surface connector is devised to flow signals without restricting the revolute motion of the body. In the next section the structure of OK-I and the control scheme are described.

### 3 Control Scheme and Prototype

The control loop of OK-I is twofold. One is for the low level PID control of the wheels and body. There are three loops of this type as shown in Figure 4. In the figure subscript d means *desired* and the value with index (k-1) is of one sampling interval before. Three dedicated PID controllers are used which

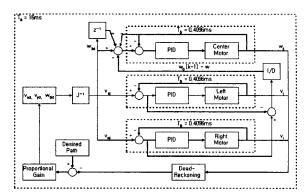


Figure 4: Control loop

have the 0.4096 ms sampling interval. The other is the outer loop for angular speed control of the body. The error,  $e_{w_3}$ , is fed back to calculating  $w_{3d}$  to keep the body's angular velocity at every 16 ms (the outer loop sampling interval):

$$e_{w_3} = (w_{3d}[k-1] - w_3) - (w_d[k-1] - w).$$
 (12)

The onboard CPU calculates the desired velocity of each motor as per the desired path at every sampling time of 16 ms. It is highly desired that the base has little odometry error to the ground as the body has to keep the angular velocity of  $w_B$ .

The prototype OK-I, sized 7.5 cm  $\times$  7.5 cm  $\times$  8.5 cm, has a 16 bit controller 80C196KC as a main CPU with 32KB RAM and ROM, three PID controllers on the body and three geared motors with encoders and a caster on the base. Owing to the three dedicated PID controllers, the main CPU is fully devoted to dead-reckoning and calculating desired velocities for the given path. The maximum wheel speed  $V_{MAX}$  is 40 cm/s and r and D are 1 cm and 5.9 cm, respectively. Figure 5 shows the prototype standing by Kity. The space in the left of the body is reserved for batteries, but for the moment power is provided from an external power supply.

The circular surface connector in Figure 6 is implemented with the printed circuit board on the base and 10 commuters on the body. It flows motor and encoder

signals of the base wheels to the body where most circuitries are located. There are ten signals from the two base wheels: two power lines for the encoders, two pairs of the encoder signals  $(A_l, B_l, A_r, B_r)$  and two pairs of the motor signals  $(M_l+, M_l-, M_r+, M_r-)$ . Since CSC can flow up to only several hundreds of miliamperes, for large scale robots the signals sent though this connecter should be changed. For example, instead of being supplied power from the body, the base would be provided with a separate battery. In this case the revolute axis can serve as a ground connector between the body and the base circuits. And the direct motor signals (M+, M-) would be replaced by a PWM and a direction signal that are in much less current. By putting supporters around the circular surface connectors, the OK-I is expected to carry a high payload.

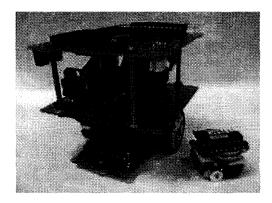


Figure 5: OK-I

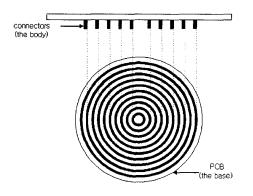


Figure 6: The circular surface connector (CSC)

### 4 Experimental Results

For testing omni-directional mobility and deadreckoning precision, two pen drawings are performed by OK-I (Figure 7). One is for the translational motion (Figure 8) and the other is for the angular motion while translating (Figure 9). In both tests, no external locating sensors are used. Since the path following

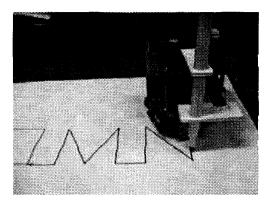


Figure 7: Pen drawing by OK-I

is purely based on the kinematic equation, the speed along the path is limited to 1 cm/s. Owing to the 2-WMR characteristics of the base, simple proportional feedback to the two wheels is enough for making the robot trajectory asymptotically close to the desired path. The height of each letter is 5 cm, the width of

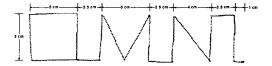


Figure 8: Translational motion

letters 'O' and 'M' is 5 cm, and that of 'N' is 4 cm. The last letter 'I' is completed with a 1 cm horizontal line. All the letters are placed 2.5 cm apart. So the path consisting of 16 line segments sums up approximately 73.6 cm traveling distance and 15 turnings. The initial base angle is 180 degrees (or facing to the left side of the page), and the maximum translational velocity along the path is 1 cm/s. Of the total 73.6 cm traveling with 15 turnings, the final distance error was less than 3 mm all the time. As one of the wheels changes the direction twice when drawing the angles in the figure, the back-lash caused most of the odometry error. Ticks on the middle of some straight

lines are due to the back-lash, where the base turns less or stops while the body still keeps turning to the opposite direction. Error modeling techniques or attaching encoders on the wheel axis will be considered to reduce the back-lash errors in the future OK series.

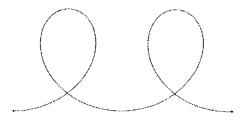


Figure 9: Angular motion with translation

The angular motion of OK-I is quite smooth once the base heads towards the direction of the traveling. To demonstrate smooth angular motion even in the case when the base is angled with the traveling direction, the initial angle of the base is 90 degrees to the direction of the traveling as in Figure 9. The translational velocity along the path keeps 1 cm/s and the angular velocity is 36 deg./s. During the 20 cm (or 20 s) traveling, smooth angular motion of the body was obtained even when the base was turning to the direction of traveling at the very initial stage. In this test, the distance between the left and right cross points measured 10 cm with less than 0.5 mm error.

#### 5 Conclusions

The omni-directional mobile robot OmniKity-I (OK-I) was developed and its kinematic equation was derived. By introducing two dimensionally active-driving wheel concept to the conventional 2-WMR, it can track any desired path. The 2-WMR serves the base of the structure and by putting the robot body on the center of the base, the robot can follow any path in the configuration space  $(x, y, \theta)$ .

The circular surface connector (CSC) was devised so that ten signals (two power lines for encoders, two pairs of encoder signals (A, B), two pairs of motor signals (M+, M-)) can flow between the body and the base without restricting the revolute motion of the body.

OK-I, the prototype, showed proper omnidirectional mobility by drawing various sharp angles and smooth circles. Similarities of the base to the conventional 2-WMR make the control of path following or point stabilization even simple. Still it is critical that the base should have little slippage so that the body can keep the desired angular motion with respect to the ground. The 2-WMR base takes the advantage of little slippage compared with other specially designed omni-directional wheels.

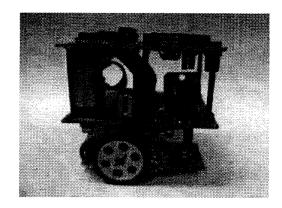


Figure 10: Right side of OK-I

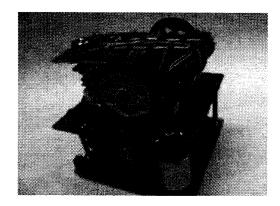


Figure 11: Bottom view of OK-I

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