SHORT HORIZON NONLINEAR MODEL PREDICTIVE CONTROL

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Abstract

This article concerns nonlinear model predictive control of the multivariable, open-loop stable processes whose delay-free part is minimum-phase. The control law is derived by using a discrete-time state-space formulation and the shortest "useful" prediction horizon for each controlled output. This derivation allows to establish the theoretical connections between the derived nonlinear model predictive control law and the discrete-time globally linearizing control, and to deduce the conditions for nominal closed-loop stability under the model predictive control law. Under the nonlinear model predictive controller, the closed-loop system is partially governed by the zero dynamics of the process, which is the nonlinear analog of placing a subset of closed-loop poles at the zeros of a process by a model algorithmic controller.

1. Introduction

Model predictive control (MPC) is an inherently-versatile controller-synthesis methodology. This versatility is due to (i) the optimization-based nature of MPC and (ii) the considerable number of adjustable (tunable) parameters which a model predictive controller can possess. While the optimization-based nature allows to use any desirable performance index, the considerable number of adjustable parameters provides many degrees of freedom to obtain a desirable closed-loop response. In a model predictive controller, model horizon, prediction horizon, control horizon, reference trajectory (set-point filter), penalty matrices in the performance index, and the form of the performance index can be adjustable. Each adjustable item (parameter) has a distinct effect on the closed-loop performance and this effect in general is not "monotonic" and well-understood. The performance of an MPC can be as non-robust and aggressive as a deadbeat controller, or as robust and slow as a "steady-state" controller. Garcia and Morari [5] were the first to study qualitatively the effects of some of the adjustable parameters on the closed-loop performance and to provide some guidelines.

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Many studies have been carried out to investigate closed-loop stability under MPC. So far, significant progress has been made mainly by using the shortest useful (for MPC synthesis) prediction horizon for each controlled output, or very long (infinite) prediction horizons. While the former approach may allow to find an analytical solution to the optimization problem and establish the connections between MPC method and non-MPC methods [5, 14, 15], the latter approach facilitates the proof of closed-loop stability under the MPC, for nonminimum-phase and/or open-loop unstable processes [7, 10].

During the last decade, there has been a growing research interest in developing nonlinear controller design methods for the nonlinear processes operating over wide ranges of operating conditions. The interest has led to substantial progress mainly by using model predictive and geometric approaches. While the former approach [2, 8, 9, 12] is optimization-based and in general leads to a controller without an analytical from, the latter approach is feedback linearization-based and leads to a controller with an analytical form, as in the globally linearizing control (GLC) [13, 14, 15].

This paper deals with the advantages of using short-horizon model predictive control. The use of a short prediction horizon is favorable for several seasons, such as (a) less computational effort needed to solve the minimization problem on-line and possibility of finding an analytical solution, (b) ease of establishing the connections between the MPC and non-MPC methods, and (c) aggressive response of controller. However, such a controller can be as disadvantageous as an analytical (non-MPC) model-based controller. The objectives of this work are:

(i) to derive a nonlinear model predictive control law with the shortest useful prediction horizon for each controlled output.

(ii) to show that the derived model predictive controller is exactly an input-output linearizing (geometric) controller.

Section 2 describes the scope of this study and the formulation of the control problem. Section 3 presents the derivation of the model predictive control law, including the prediction equations and reference trajectories. Section 4 shows that the derived shortest prediction-horizons model predictive controller is exactly a GLC-error feedback controller. Section 5 describes the conditions for nominal closed-loop stability under the nonlinear model predictive controller. In Section 6, the nonlinear model predictive control law is applied to linear processes and shown that the resulting controller is exactly a multivariable model algorithmic (internal model) controller [4, 5, 8, 12].

2. Scope and Control Problem Formulation

The focus of this study is on the nonlinear, square (equal number of inputs and outputs), multivariable processes described by a mathematical model of the form

$$\left\{ \begin{array}{rcl} x(k+1) & = & \Phi(x(k), u(k)) \\ y(k) & = & h(x(k)) \end{array} \right. \tag{1}$$

where $x = [x_1 \cdots x_n]^T$, $u = [u_1 \cdots u_m]^T$, and $y = [y_1 \cdots y_m]^T$ denote the vectors of state variables, manipulated inputs, and controlled outputs respectively, all in deviation variable form. It is assumed that: $x \in X \subset \mathbb{R}^n$ and $u \in U \subset \mathbb{R}^m$, where X and U are connected open sets which contain the origin; $\Phi(x,u)$ is an analytic vector function on $X \times U$; $h(x)$ is an analytic vector function on X; for each controlled output y_i, there is a positive integer ℓ such that $y_i(k+\ell)$ explicitly depends on a present manipulated input move $u_j(k)$ (each controlled output is affected at least by a manipulated input).

The control problem is to derive a nonlinear MPC law based on a process model of the form of Eq.1. The derived control law uses the shortest useful prediction horizon for each controlled output, should be applicable to open-loop stable nonlinear processes, and should be robust to model errors.

3. Nonlinear Model Predictive Controller Synthesis

This section describes the derivation of the shortest prediction-horizons MPC law. The derived control law has: (a) prediction horizons of one sampling period beyond the minimum deadtimes, (b) control horizons of one, (c) a quadratic performance index, and (d) a set of linear reference trajectories. As we will see, it is indeed a multivariable, nonlinear model algorithmic controller.

3.1. Output Prediction

For a process with a mathematical model of the form of Eq.1, the future values of each controlled output y_i can be calculated by using the model. In so doing, it is convenient to define first the smallest deadtime between each controlled output y_i and the manipulated input vector u; that is, the smallest number of sampling periods after which a manipulated input affects the controlled output y_i. This is exactly what is known as the relative order of the controlled output y_i, in geometric control.

Definition 1 [11]: For a system of the form of Eq.1, the relative order of an output y_i with respect to the manipulated input vector u is the smallest integer r_i for which $y_i(k+r_i)$ depends explicitly on the control move $u(k)$. If such an integer does not exist, we say that $r_i = \infty$. Equivalently, the relative order r_i is the smallest integer for which:

$$h_i \circ \underbrace{\Phi \circ \cdots \circ \Phi}_{r_i \text{ times}}$$

depends explicitly on u. Here the notation $\circ$ represents the usual composition of functions in x: e.g., $\Phi(x,u) \circ \Phi(x,u) = \Phi(\Phi(x,u),u)$.

Remark 1: Using the chain rule, it can be shown that for a system of the form of Eq.1, the relative order of an output y_i with respect to the manipulated input vector u is the smallest integer r_i for which

$$\left[\frac{\partial h_i(x)}{\partial x}\right] \left[\frac{\partial \Phi(x,u)}{\partial x}\right]^{r_i-1} \left[\frac{\partial \Phi(x,u)}{\partial u}\right] \not\equiv [0 \ \cdots \ 0]$$

As a result of the definition of the relative orders $r_1, \cdots, r_m$, we can define the notation

$$\begin{array}{rcl} h_i^0(x) & \triangleq & h_i(x) \\ h_i^\ell(x) & \triangleq & h_i^{\ell-1}(\Phi(x,u)), \qquad \ell = 1, \cdots, r_i - 1 \end{array}$$

Using this notation and the definition of the relative order r_i, we see that

$$\frac{\partial}{\partial u} h_i^{r_i-1}(\Phi(x,u)) \not\equiv [0 \ \cdots \ 0]$$

and

$$\begin{array}{rl} y_i(k+\ell) = h_i^\ell(x(k)), & \ell = 0, \cdots, r_i - 1 \\ y_i(k+r_i) = h_i^{r_i-1}(\Phi(x(k), u(k))) & \end{array} \tag{2}$$

Thus, the relative order r_i is the smallest number of sampling periods after which a manipulated input u_j affects the output y_i; it is the shortest useful prediction horizon for the output y_i. If a prediction horizon of less than r_i is used for the output y_i, the predicted value of the output y_i will be independent of the present control move $u(k)$; such a short prediction horizon cannot be used to derive an MPC law.

If a system output y_i does not have a finite relative order (r_i is not finite), this means that none of the manipulated inputs $u_1, \cdots, u_m$ affect the output y_i (y_i is not controllable). Throughout this paper, it is assumed that all the relative orders are finite and $\partial[h_i^{r_i-1}(\Phi(x,u))]/\partial u \neq [0 \cdots 0]$ on $X \times U$, $i = 1, \cdots, m$.

Remark 2: In the case that a discrete-time nonlinear system of the form of Eq.1 with a finite relative order r_i describes the exact sampled-data representation of a continuous-time process, $(r_i - 1)\Delta t$ represents the smallest deadtime between manipulated inputs and the controlled output y_i, whereas the additional delay Δt is the time delay due to the sampling. Here Δt represents the sampling period.

After specifying the smallest useful prediction horizon for each controlled output, we need to derive a prediction equation for each output y_i, to predict the future values of the output y_i up to r_i sampling periods ahead. This can be achieved simply by using the relations of Eq.2:

$$\begin{aligned} y_i(k+\ell) &= h_i^{\ell}(x(k)), \quad \ell = 1, \cdots, r_i - 1 \\ y_i(k+r_i) &= h_i^{r_i-1}(\Phi(x(k), u(k))) \end{aligned}$$

The future *changes* in each output y_i are then calculated from:

$$\begin{aligned} y_i(k+\ell) - y_i(k) &= h_i^{\ell}(x(k)) - h_i(x(k)), \\ &\qquad \ell = 1, \cdots, r_i - 1 \\ y_i(k+r_i) - y_i(k) &= h_i^{r_i-1}(\Phi(x(k), u(k))) - h_i(x(k)) \end{aligned}$$

where x is obtained by using a full-order open-loop state-observer (i.e., by on-line simulation of the process model). Because a full-order open-loop state-observer can only be used for open-loop stable processes, the use of an observer of this type limits the class of processes which the derived controller will be applicable to. If the measured output signal $\bar{y}_i(k)$ is added to the above predicted changes, we obtain the following "closed-loop" prediction equations for the output y_i:

$$\begin{aligned} \hat{y}_i(k+\ell) &\triangleq \bar{y}_i(k) + h_i^{\ell}(x(k)) - h_i(x(k)), \\ &\qquad \ell = 1, \cdots, r_i - 1 \\ \hat{y}_i(k+r_i) &\triangleq \bar{y}_i(k) + h_i^{r_i-1}(\Phi(x(k), u(k))) - h_i(x(k)) \end{aligned} \tag{3}$$

where $\hat{y}_i(k+\ell)$ is the predicted value of the output $\bar{y}_i$, ℓ sampling periods ahead. Equation 3 confirms the interpretation of the relative order r_i as the minimum number of sampling periods after which the output y_i is affected by a manipulated input u_j.

3.2. Reference Trajectories: Setpoint Filters

Reference trajectory of an output y_i denoted by y_{d_i}, is the trajectory which the model predictive controller will try to force the output y_i to follow in the absence of any constraints and penalties on the controller action. It depends on the output set-points $y_{sp_1}, \cdots, y_{sp_m}$, and has a set of tunable parameters which allow to adjust the shape of each reference trajectory and thus the aggressiveness of the controller action. Because we intend to derive a shortest prediction-horizons MPC law, we have to use reference trajectories (setpoint filters) to give some robustness to the resulting model predictive controller. Otherwise, the controller will be exactly an output deadbeat controller [6], which is known for its poor robustness.

For each controlled output y_i, a linear reference trajectory y_{d_i}, the same as in the model algorithmic control, is defined: it is linearly related to the set-points according to

$$y_{d_i}(k+r_i) = y_{sp_i}(k) + \sum_{j=1}^{m} \alpha_{ij} \left[\hat{y}_j(k+r_j-1) - y_{sp_j}(k)\right] \tag{4}$$

where α_{ij}s are tunable scalar parameters. According to Eq.4, the value of the reference trajectory y_{d_i} at r_i sampling periods ahead, is set to the sum of the present value of the ith set-point and a weighted sum of the mismatches between each set-point and $(r_i - 1)$-sampling-periods-ahead predicted value of its corresponding output. In a vector form, the reference trajectories are governed by

$$\begin{bmatrix} y_{d_1}(k+r_1) \\ \vdots \\ y_{d_m}(k+r_m) \end{bmatrix} = \{I_m - \alpha\} y_{sp}(k) + \alpha \begin{bmatrix} \hat{y}_1(k+r_1-1) \\ \vdots \\ \hat{y}_m(k+r_m-1) \end{bmatrix} \tag{5}$$

where $\alpha = [\alpha_{ij}]$ is an $m \times m$ matrix which is chosen such that all of its eigenvalues lie inside the unit circle. As we will see, this is a necessary condition for closed-loop stability. The reference trajectory y_{d_i} can be made independent of the set-points y_{sp_j}, $j = 1, \cdots, m$, $j \neq i$, by setting the non-diagonal entries of the matrix α zero.

3.3. Optimization Problem - Control Law

Let us first consider a quadratic performance index of the form

$$\min_{u(k)} \left\{ \sum_{j=1}^{m} \theta_j \left[y_{d_j}(k+r_j) - \hat{y}_j(k+r_j)\right]^2 \right\} \tag{6}$$

where $\theta_1, \cdots, \theta_m$ are positive tunable parameters (weights on the deviations of predicted outputs from their reference trajectories). In this case, the controller action tries to minimize a weighted sum of the squared mismatches between each predicted output $\hat{y}_i$ and its corresponding reference trajectory (y_{d_i}), r_i sampling periods ahead.

In the absence of any constraints, the minimum is obviously zero:

$$\begin{bmatrix} y_{d_1}(k+r_1) - \hat{y}_1(k+r_1) \\ \vdots \\ y_{d_m}(k+r_m) - \hat{y}_m(k+r_m) \end{bmatrix} = 0,$$

and the minimizing $u(k)$ is therefore the solution of the system of nonlinear algebraic equations (using Eqs.3 and 5)

$$\begin{bmatrix} h_1^{r_1-1}(\Phi(x(k), u(k))) \\ \vdots \\ h_m^{r_m-1}(\Phi(x(k), u(k))) \end{bmatrix} = (I_m - \alpha)\left[e(k) + h(x(k))\right] + \alpha \begin{bmatrix} h_1^{r_1-1}(x(k)) \\ \vdots \\ h_m^{r_m-1}(x(k)) \end{bmatrix} \tag{7}$$

where the error $e = y_{sp} - \bar{y}$. The above control law is in general implicit and nonlinear in u, and therefore the solution for u may not be unique. To address this non-uniqueness problem, we need to impose another condition on the process model. This additional condition can be expressed in terms of the geometric notion of characteristic matrix defined below.

Definition 2 [11]: Consider a discrete-time system of the form of Eq.2, and assume that each output y_i possesses a finite relative order r_i. The $m \times m$ matrix

$$\mathcal{C}(x,u) \triangleq \begin{bmatrix} \frac{\partial}{\partial u} h_1^{r_1-1}(\Phi(x,u)) \\ \vdots \\ \frac{\partial}{\partial u} h_m^{r_m-1}(\Phi(x,u)) \end{bmatrix}$$

is called the characteristic matrix of the system described by Eq.1.

Throughout this article, it will be assumed that $\mathcal{C}(x,u)$ is nonsingular on $X \times U$. This assumption can always be guaranteed, as long as $det\,[\mathcal{C}(0,0)] \neq 0$, by appropriately defining the sets X and U.

For a process model of the form of Eq.1 with nonsingular characteristic matrix $\mathcal{C}(x,u)$, as a consequence of the implicit function theorem, the set of algebraic equations

$$\begin{bmatrix} h_1^{r_1-1}(\Phi(x,u)) \\ \vdots \\ h_m^{r_m-1}(\Phi(x,u)) \end{bmatrix} = v \qquad (8)$$

is locally solvable for the manipulated input vector u. The corresponding implicit function will be denoted by

$$u = \Psi_o(x, v) \qquad (9)$$

and will be assumed to be well-defined and unique on $X \times h(X)$.

If the characteristic matrix of the process model is nonsingular, using the definition of Ψ_o (Eqs.8 and 9), the solution to the system of algebraic equations of Eq.7 takes the form

$$u(k) = \Psi_o\left(x(k),\ (I_m - \alpha)\left[e(k) + h(x(k))\right] + \alpha \begin{bmatrix} h_1^{r_1-1}(x(k)) \\ \vdots \\ h_m^{r_m-1}(x(k)) \end{bmatrix} \right) \qquad (10)$$

which is also the solution to the minimization problem of Eq.6. Thus, the nonlinear MPC law is given by Eq.10, where $x(k)$ is obtained again by simulating the process model, $x(k+1) = \Phi(x(k), u(k))$; it is the dynamic error-feedback controller:

$$\begin{aligned} x(k+1) &= \Phi\left\{ x(k), \Psi_o\left(x(k),\ (I_m - \alpha)\left[e(k) + h(x(k))\right] + \alpha \begin{bmatrix} h_1^{r_1-1}(x(k)) \\ \vdots \\ h_m^{r_m-1}(x(k)) \end{bmatrix} \right) \right\} \\ u(k) &= \Psi_o\left(x(k),\ (I_m - \alpha)\left[e(k) + h(x(k))\right] + \alpha \begin{bmatrix} h_1^{r_1-1}(x(k)) \\ \vdots \\ h_m^{r_m-1}(x(k)) \end{bmatrix} \right) \end{aligned} \qquad (11)$$

In this case because there is no constraint, with a proper initialization of process model, the controller forces every controlled output to follow its reference trajectory exactly; that is, the outputs will evolve according to the same linear dynamics which governs the reference trajectories:

$$\begin{bmatrix} y_1(k+r_1) \\ \vdots \\ y_m(k+r_m) \end{bmatrix} - \alpha \begin{bmatrix} y_1(k+r_1-1) \\ \vdots \\ y_m(k+r_m-1) \end{bmatrix} = \{I_m - \alpha\}\, y_{sp}(k) \qquad (12)$$

This implies that the model predictive controller indices a linear input-output behavior to the closed-loop system; the model predictive controller is an input-output linearizing controller. Indeed, it is exactly a GLC-error feedback controller [13].

Remark 3: A more general nonlinear model predictive controller can be derived by solving the following minimization problem:

$$\min_{u(k)} \left\{ \sum_{j=1}^{m} \theta_j \left[y_{d_j}(k+r_j) - \hat{y}_j(k+r_j) \right]^2 + \sum_{j=1}^{m} \rho_j u_j^2(k) \right\} \qquad (13)$$

where $\rho_1, \cdots, \rho_m$ are positive tunable parameters (weights on the magnitudes of manipulated inputs), subject to the input constraints $u_{l_\ell} \leq u_\ell(k) \leq u_{h_\ell}$, $\ell = 1, \cdots, m$. In this case, control law is the solution to the minimization problem of Eq.13 subject to the input constraints

$$u_{l_\ell} \leq u_\ell(k) \leq u_{h_\ell}, \qquad \ell = 1, \cdots, m$$

where $y_{d_\ell}(k+r_\ell)$ and $\hat{y}_\ell(k+r_\ell)$ are given by Eqs.3 and 5, and x is obtained by simulating the process model, $x(k+1) = \Phi(x(k), u(k))$.

In contrast to the single-input single-output case [15], the constrained minimization problem of Remark 3, in general, does not have an analytical solution; it should be solved numerically on-line to calculate the exact optimal controller action. An approximate analytical solution to the above constrained minimization problem however can be obtained by calculating the unconstrained controller action (i.e., solution to the unconstrained minimization problem) and then clipping the unconstrained controller action or scaling down the unconstrained controller action while maintaining its direction [1]. In the special case that

- a rearrangement of manipulated inputs or controlled outputs makes the characteristic matrix of the process model diagonal (process is strongly or completely input-output decoupled), and
- the matrix α is chosen to be diagonal,

the exact solution to the constrained minimization problem is obtained simply by calculating and then clipping the unconstrained controller action.

In the absence of the input constraints, however the minimization problem of Remark 3 has an analytical solution: the minimizing $u(k)$ is the solution to the system of the nonlinear algebraic equations

$$[diag\{\rho_i\}]\,u(k)+[C(x(k),u(k))]^T \bullet \left\{ \begin{bmatrix} h_1^{r_1-1}(\Phi(x(k),u(k))) \\ \vdots \\ h_m^{r_m-1}(\Phi(x(k),u(k))) \end{bmatrix} - (I_m-\alpha) \bullet [e(k)+h(x(k))] - \alpha \begin{bmatrix} h_1^{r_1-1}(x(k)) \\ \vdots \\ h_m^{r_m-1}(x(k)) \end{bmatrix} \right\} = 0$$

where $x(k)$ is obtained by simulating the process model, $x(k+1)=\Phi(x(k),u(k))$, which is assumed to be at least locally asymptotically open-loop stable. In this case, with $\rho_j \neq 0$, $j=1,\cdots,m$, because the controller action is penalized, it remains bounded and also cannot force the controlled outputs to follow the linear reference trajectories. Thus, this controller is not input-output linearizing. The theoretical properties of the controller are unknown at this point.

4. Connections with Geometric Control

In the previous section, it was pointed out that when there is no constraint and process model is properly initialized, the model predictive controller induces the linear input-output response described by Eq.12, to the closed-loop system; the model predictive controller is an input-output linearizing controller. Indeed, it is exactly a GLC-error feedback controller [13].

This equivalence implies that an unconstrained MPC law with the shortest useful prediction horizon for each output is exactly a geometric controller. In other words, the GLC-error feedback controller is an unconstrained nonlinear MPC law with (a) prediction horizons of one sampling period beyond the minimum deadtimes, (b) control horizons of one, (c) linear reference trajectories, and (d) a quadratic performance index without any penalties on controller action. More precisely, the geometric controller is an unconstrained, nonlinear model algorithmic controller. Furthermore, this equivalence indicates that under the nonlinear model predictive controller, the closed-loop system is partially governed by the zero dynamics of the process, which is the nonlinear analog of placing a subset of closed-loop poles at the finite zeros of a process by a linear model algorithmic controller.

5. Closed-Loop Properties Under the MPC Law

Because the derived MPC law and the geometric controller are identical, the conditions for closed-loop stability under the two control laws are the same. The conditions for closed-loop stability under a GLC-error feedback controller are known [13], and thus so are the stability conditions for the derived MPC.

5.1. Closed-Loop Stability

In the absence of any constraints and any penalties on the controller action, the closed-loop system under the model predictive controller will be input-output stable, if the matrix α is chosen such that all of its eigenvalues lie inside the unit circle. For an input-output stable closed-loop system, the local internal closed-loop stability will be ensured, if the process is locally hyperbolically minimum-phase and locally asymptotically open-loop stable.

5.2. Input-Output Decoupling

If α is chosen to be a diagonal matrix ($\alpha = diag\{\alpha_i\}$), the error-feedback controller of Eq.11 will induce the completely input-output decoupled, first-order-plus-deadtime response

$$y_i(k+r_i)-\alpha_i y_i(k+r_i-1) = (1-\alpha_i) y_{sp_i}(k), \quad i=1,\cdots,m$$

to the closed-loop system.

6. Application to Unconstrained Linear Processes

This section deals with the application of the shortest prediction-horizons MPC law of Eq.11 to unconstrained linear processes. As we will see, the resulting linear controller is exactly a multivariable model algorithmic (internal model) controller [4, 5, 8, 12].

Consider linear multivariable processes described by a discrete-time state-space model of the form:

$$\begin{cases} x(k+1) &= Ax(k)+Bu(k), \qquad x(0)=0, \quad u(0)=0 \\ y(k) &= Cx(k) \end{cases} \tag{14}$$

where A, B, and C are matrices of dimensions $n{\times}n$, $n{\times}m$, and $m{\times}n$ respectively. This is a special case of the system of Eq.1: $\Phi(x(k),u(k)) = Ax(k)+Bu(k)$; $h(x(k)) = Cx(k)$. The input-output behavior of the system of Eq.14 can be represented by the z-domain matrix transfer function:

$$G(z) = C\,(zI_n - A)^{-1}\,B \tag{15}$$

Applying Definitions 1 and 2 to the system of Eq.14, we see that:

- the relative order r_i is the smallest integer for which $c_iA^{r_i-1}B \neq [0 \cdots 0]$, where c_i is the ith row of the matrix C;
- for each output y_i, the following relations hold:

$$\begin{aligned} h_i^{\ell}(x) &= c_iA^{\ell}x, \quad \ell = 0,\cdots,r_i-1 \\ h_i^{r_i-1}(\Phi(x,u)) &= c_iA^{r_i}x + c_iA^{r_i-1}Bu \end{aligned} \tag{16}$$

- the characteristic matrix

$$\mathcal{C} = \begin{bmatrix} c_1A^{r_1-1}B \\ \vdots \\ c_mA^{r_m-1}B \end{bmatrix} = \lim_{z\to\infty} \left[diag\{z^{r_i}\}\, G(z)\right],$$

which is assumed to be nonsingular.

In this linear case, the function Ψ_o, which was defined implicitly as the solution of Eq.8, has the simple explicit-form expression $\Psi_o(x,v) = \mathcal{C}^{-1}(v - \mathcal{B}x)$, where $\mathcal{B} = [c_1A^{r_1} \cdots c_mA^{r_m}]^T$.

Using the algebraic identity

$$diag\{z^{r_i}\}\, C(zI_n - A)^{-1}B = \mathcal{B}(zI_n - A)^{-1}B + \mathcal{C}$$

the matrix transfer function of Eq.15 can be recast as

$$G(z) = diag\left\{z^{-r_i}\right\}\left[\mathcal{B}(zI_n - A)^{-1}B + \mathcal{C}\right]$$

which provides a factorization of the matrix transfer function $G(z)$ into two parts: (i) a pure delay part $diag\left\{z^{-r_i}\right\}$, and (ii) a "delay-free" part [if $\mathcal{C}$ is nonsingular]

$$H(z) = \mathcal{B}(zI_n - A)^{-1}B + \mathcal{C} \tag{17}$$

For this case, the model predictive controller of Eq.11 takes the form:

$$\begin{aligned} x(k+1) &= \left\{A - B\mathcal{C}^{-1}\left[\mathcal{B} - \alpha\mathcal{D} - (I_m - \alpha)\,\mathcal{C}\right]\right\}x(k) \\ &\qquad + B\mathcal{C}^{-1}(I_m - \alpha)e(k) \\ u(k) &= -\mathcal{C}^{-1}\left[\mathcal{B} - \alpha\mathcal{D} - (I_m - \alpha)\,\mathcal{C}\right]x(k) \\ &\qquad + \mathcal{C}^{-1}(I_m - \alpha)e(k) \end{aligned} \tag{18}$$

where $\mathcal{D} = \left[c_1A^{r_1-1} \cdots c_mA^{r_m-1}\right]^T$. The system of Eq.18 is a minimal-order state-space realization of the z-domain transfer function

$$u(z) = [H(z)]^{-1}\left[I_m - \alpha z^{-1} - (I_m - \alpha)\, diag\left\{z^{-r_i}\right\}\right]^{-1} \bullet (I_m - \alpha)\, e(z) \tag{19}$$

where $H(z)$ is given by Eq.17. This error-feedback controller has integral action and is exactly a multivariable model algorithmic controller and also a multivariable internal model controller [4] for a requested closed-loop response of the form

$$y(z) = diag\left\{z^{-r_i}\right\}\left(I_m - \alpha z^{-1}\right)^{-1}(I_m - \alpha)\, y_{sp}(z)$$

If α is chosen to be a diagonal matrix ($\alpha = diag\{\alpha_i\}$), the controller of Eq.19 simplifies to

$$u(z) = [H(z)]^{-1}\, diag\left\{\frac{1 - \alpha_i}{1 - \alpha_i z^{-1} - (1 - \alpha_i)\, z^{-r_i}}\right\} e(z)$$

This error-feedback controller is a minimal-order state-space realization of an internal model controller with a diagonal filter [4], which induces the completely input-output decoupled, closed-loop response

$$y_i(z) = z^{-r_i}\frac{1 - \alpha_i}{1 - \alpha_i z^{-1}}y_{sp_i}(z), \quad i = 1,\cdots,m.$$

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