Constrained Predictive Control of Robotic Manipulators using On-line Linearized Models

T. Alvarez[*1], F. Tadeo[1], D. Megias[2],
M.Y. El Ghoumari[2], R. Villanova[2], C. Pedret[2], C. de Prada[1]

Abstract

It is well known that robotic systems present physical limitations in the maximum torque that the motors can apply, and its variation rate. Also, safety regulations give strict limitations in the maximum link speed and acceleration. To include these limitations in the robotic control systems this paper proposes the implementation of a Multivariable Constrained Predictive Controller, that solves at each sampling time online the problem of calculating an optimal control sequence in the presence of constraints, using a model of the process. It is proposed to use for prediction a linearized model of the robot at each working point, which makes possible to meet the strict computational limitations of a real-time implementation, without degrading the closed-loop performance. The performance of this approach in the presence of model uncertainties, constraints in the maximum torque, its variation and in the generalized coordinates is studied.

1. Introduction

A central issue in robotic research is the motion control of robotic manipulators. During the last years, many different approaches and control algorithms have been proposed in the literature and proved by simulation and in robotic systems at laboratory scale. If the model is exactly known the feedback linearization technique can be directly applied (Spong and Vidyasagar, 1989), but most practical robotic systems present external disturbances and uncertainties that worsen the performance. To compensate this effect two main strategies have been proposed: the adaptive control strategy and the robust control strategy. Adaptive control of robotic manipulators have been discussed by Ortega and Spong (1989), Slotine and Li

[*] Corresponding Author, Dr. Teresa Alvarez, fax: +34 983 423161, Tlf: +34 983 423566, e-mail: teresa@autom.uva.es
[1] Departamento de Ingeniería de Sistemas y Automática, Facultad de Ciencias, University of Valladolid, 47005 Valladolid - Spain
[2] Automatic Control and Systems Engineering Group, Dept. of Computer Science, Universidad Autónoma de Barcelona, 08193 Bellaterra – Spain

(1989) and Chen et al. (1997). The main drawback of this approach is the difficulty of checking the robustness of the closed-loop system.

On the other hand the robust control strategy for linear (Green and Limebeer, 1995) and non-linear time-invariant systems (van der Schaft, 1992, Isidori and Astolfi, 1992) give optimal disturbance rejection and definite stability test for the feedback systems in the presence of uncertainty. Various authors have successfully applied these ideas to control robotic manipulators at laboratory scale (Liu and Goldenberg, 1996; Jaritz and Spong, 1996; Tzou, 1996).

One of the problems of these approaches is that it is very difficult to consider the limitations in amplitude and rate of control and output signals, which appear in any practical robot control problem. To solve this problem, it is proposed in this paper to use a Constrained Multivariable Predictive Controller for robotic control in the presence of constraints. Predictive controllers use a model of the system to predict the evolution of the control and output signals. Then corrective actions can be applied in each sampling time, taking into account constraints in the signals and disturbances acting in the system (Clarke et al., 1987; Richalet, 1989; Clarke, 1994; Camacho and Bordons, 2000). It is also possible to consider in the prediction the future command signal, which is especially useful in robotic systems, as the robot trajectory is usually pre-calculated. Predictive control then gives a simple solution for solving multivariable problems with constraints, using a possibly nonlinear o time-varying model.

Although there are very few published results on the application of Predictive Control in robotic systems, Predictive Controllers have been previously applied successfully to solve industrial problems in process control engineering (Richalet, 1989; Clarke, 1994; Camacho and Bordons, 2000). In Alvarez et al. (1998), the authors presented a Constrained Multivariable Predictive Controller for robotic control that used a time-invariant model for prediction. In this paper that approach is improved by considering instead of a time-invariant linear model, a time-variant linear model, calculated at each sampling time by linearization of the non-linear physical model of the manipulator, at each working point, using similar ideas as Kothare (1997, 2000) and Pedret *et al*. (2000). Doing so, it is possible to improve the performance of the predictive controller, by improving the accuracy of the predictions. The use of the non-linear physical model for prediction in a robotic system would be infeasible, due to the strict computational demands of a real-time implementation (Ghoumari, 1998, Ghoumari *et al*., 1998, Megías et al., 1999). Botto (1996) and Matko (2000) use a predictive controller with a time-variant model, but they make use of neural nets and fuzzy models, respectively. Nevertheless, if we are working with a robotic system it makes sense to linearize the non-lineal physical model as precise as precise non-linear multivariable models are available (Spong, Vidyasagar, 1989).

2. Constrained Predictive Control

Model Based Predictive Control (MBPC) is a control strategy based on the explicit use of a model to predict the process output over a period of time. At each sampling time the future control signals are calculated by minimization of a cost function, which is usually defined as a weighted combination of tracking errors and control variations. A receding control horizon technique is normally applied: the calculations are repeated every sampling time, to take into account the difference between the predicted state and the measured state.

The controller strategy considered in this paper is a multivariable constrained predictive controller based on the Generalized-Predictive-Controller (GPC: Clarke and Mohtadi, 1987). The basic ideas of GPC are now presented. Then the modifications proposed for real-time industrial applications in robotic systems are discussed.

Process Model

Although a non-linear model can be used for prediction, computational and robustness considerations makes the use of a linear model more adequate when the sampling time is small (Mejias et al., 1998). Let the process be represented by the discrete linear model:

$$A_i\left(q^{-1}\right)y_i(t) = \sum_{j=1}^{N} B_{ij}\left(q^{-1}\right)u_j(t) + \sum_{j=1}^{R} D_{ij}\left(q^{-1}\right)v_j(t) + \frac{T_i}{\Delta}\xi_i(t) \quad \textbf{(1)}$$

where M is the number of inputs, N is the number of system outputs, R is the number of measurable disturbances, A_i, B_{ij}, D_{ij} and T_i are polynomials in the backwards shift operator q^{-1} ($q^{-1}x(t) = x(t-1)$), Δ is the difference operator $\Delta x(t) = x(t) - x(t-1)$, and the ξ_i are uncorrelated white noise signals. The predicted values of the controlled variables $\{\hat{y}_i(t+j)\}_{j=1}^{m}$ are calculated as follows:

$$\hat{y}_i(t+j) = \sum_{j=1}^{M}\sum_{k=1}^{j} \sigma_j g_{ij_k} \Delta u_j(t-k+j) + p_i(t+j) \qquad \textbf{(2)}$$

where g_{ij} are the step responses from the input j to the output i, and p_i is the free response of the output i (that is, the predicted output if the control signal were constant in the future) The coefficients σ_j are binary variables that might be used by the supervisory system to eliminate from the calculation those control signals temporally unavailable for control.

Control signal calculation

The predictive control algorithm calculates the sequence of changes in the control variables $\{\Delta u_i(t+j)\}$, necessary to minimize the distance between the predicted outputs $\{\hat{y}_i(t+j)\}_{j=1}^{m}$ and the corresponding internal reference $\{r_i(t+j)\}_{j=1}^{m}$. This internal reference is usually calculated from the future command signal by low-pass filtering, to obtain a smoother approximation. In order to reduce the computational load only a small number of control variations are considered: $\{\Delta u_i(t+j)\}_{j=0}^{Nu_i-1}$, where Nu_i is called *control horizon*, and normally is much smaller than N.

In a GPC-type formulation this minimum-distance problem is transformed into a constrained optimization problem where a quadratic cost function of the predicted tracking errors and the control signal variations is minimized. The control signal variations $\{\Delta u_i(t+j)\}_{j=0}^{Nu_i-1}$ are the free variables in the optimization:

$$\begin{aligned} &\min_{\Delta u_i(t+j)} J \\ &\text{s.t.} \quad J = \sum_{i=1}^{N} \sum_{j=N1_i}^{N2_i} \eta_i \left(\gamma_i \left(\hat{y}_i(t+j) - r_i(t+j)\right)^2\right) + \sum_{i=1}^{M} \sum_{j=0}^{Nu_i-1} \sigma_i \left(\beta \Delta u_i(t+j)\right)^2 \end{aligned} \qquad \textbf{(3)}$$

where the coefficients γ and β give the relative weight of each prediction error and control variations, while the coefficients σ and η are binary variables that make possible to exclude temporarily a variable in the index.

One important property of predictive controllers is that it is straightforward to include physical and safety limitations in the minimization, by transforming them into mathematical constraints that depend linearly on the control signal variations. For instance, if constraints on the control signal variations, on the magnitude of control signals and on the magnitude of output signals are considered, the optimization problem can be stated as follows:

$$\min_{\Delta u_i(t+j)} J$$

s.t.

$$J = \sum_{i=1}^{N} \sum_{j=N1_i}^{N2_i} \eta_i \left(\gamma_i \left(\hat{y}_i(t+j) - r_i(t+j)\right)^2\right) + \sum_{i=1}^{M} \sum_{j=0}^{Nu_i-1} \sigma_i \left(\beta \Delta u_i(t+j)\right)^2$$

$$D_{m_i} \le \Delta u_i(t+j) \le D_{M_i},\ j = 0,\ldots, Nu_i - 1 \qquad \textbf{(4)}$$

$$U_{m_i} \le u_i(t+j) = u_i(t-1) + \sum_{i=0}^{j} \Delta u_i(t+j) \le U_{M_i},\ j = 0,\ldots, Nu_i - 1$$

$$L_{m_i} \le \hat{y}_i(t+j) \le L_{M_i}$$

GPC Improvements

As the objective of the control technique presented in this paper is the operation in real robotic applications, some additional features have been added to the original GPC formulation. These characteristics are:

1) *Linearization module:*

   The linear model used for prediction is calculated at each sampling time from the linearization and discretization of the non-linear continuous model of the system. Compared with the use of a single linear model, this idea, also used by Ghoumari et al. (1998), Friedland (1996), Ghoumari et al. (1998) and Matko (2000) for LQG, improves the accuracy of the model used for prediction, and, as a consequence, the performance of the overall system. Moreover, the control system is less sensitive to changes in the working point.

   The additional calculations needed to linearize and discretize the model are negligible compared with the calculations necessary to make predictions directly with the non-linear model, so this modifications makes possible to use (an approximation to) the non-linear model, without excessive additional calculations. Mejias et al. (1998) propose a method to linearize and discretize a state-space non-linear model of the plant. In robotic systems there are standard physical model proposed to represent the robotic dynamic behaviour, such as the Lagrange-Euler method (Spong and Vidyasagar, 1989). So we propose to use a general linearization (and discretization) of this standard representation to calculate directly the linear system representation used for prediction by the Predictive Controller, as will be shown by way of an example.

2) *Set-point conditioning module (Prada and Valentín, 1996)*

   If there redundant degrees of freedom, the set-point conditioning module makes possible to assign optimum set-points to the outputs and optimum values to the inputs in steady state.

3) States of the variables

Due to sensor failures, perturbations, changing goals or a system reconfiguration, the number of inputs, outputs and/or measurable perturbations might change. The proposed predictive control approach makes possible to change on-line the number of inputs, outputs and perturbations. For instance, if the number of inputs is smaller than the number of outputs, it is not possible that every output follows a reference. The controller described here allows that some output variables follow a reference trajectory and others not, but all of them remaining within their limits. To take this into account, there have been defined different states for the variable:

i) Control variable states:

auto: the controller calculates its value

manual: its value is fixed by an external source

optimised: its value is calculated by the set-point conditioning module

off: its value is unknown and it is not considered in further calculations

ii) Controlled variable states:

controlled: the variable has a set-point assigned

limited: there is no set-point assigned

off: its value is unknown

iii) Measurable disturbance states:

ok: its value is known

off: its value is unknown

These states are taken into account in the controller structure introducing weighting factors in the quadratic function, the free response and the predicted output, which might change their value according to the state.

4) Ordered set of Constraints(Alvarez and Prada,1997)

Different types of constraints on the manipulated and controlled variables:

i) ***Physical constraints***: limits than comes from the physical characteristics of the plant. They can never be surpassed.

ii) ***Operating constraints***: These limits, more restrictive than the physical ones, correspond to the band within which the variables are expected to lie most of the time.

iii) ***Set-point conditioning limits***. These limits, more restrictive that the operating ones are taken into account if the set-point conditioning module is active.

5) *Constraint Horizon:*

In order to reduce the number of constraints in the optimization problem and to improve the performance of non-minimum-phase systems, constraints on the output variable are only considered in a Constraint Horizon [N3,N4], which is a subinterval of [1,N2]. As N3 can be minor or equal than N1, this allows to reduce the effect of non-minimum phase responses.

$$L_{m_i} \leq \hat{y}_i(t+j) \leq L_{M_i}, \; j = N3_i, \ldots, N4_i \qquad \textbf{(5)}$$

6) *Precalculated Trajectory:*

If the future system reference is known, as is usual in robotic systems, it may be used to calculate the controller internal reference, reducing the response time, by leading the control actions. If it is unknown it is calculated as in the classical GPC (Clarke and Mohtadi, 1987): a filtered step command.

7) *Constraint Handling* (Alvarez and Prada, 1997)*:*

Robotic systems have strict limitations on the control and output signals, which sometimes makes the optimization problem infeasible. A constraint handling procedure is applied if the control problem has no feasible solution. This makes possible for the controller to calculate always a solution, even in the presence of strict limitations.

As it has been explained previously, the objective of model predictive control is to control a process with quadratic objectives subject to general linear constraints. The minimization of the quadratic function gives the sequence of future controls. Although, this is the normal operation algorithm, there are situations where some of the constraints are violated. This problem is known as MBPC Infeasibility. Finding a solution to this problem is harder for MIMO systems than for SISO systems due to the interconnections among variables. The developed strategy (Alvarez and Prada, 1997) consists of two different phases: the initial phase is applied to the manipulated variables and their constraints prior to the calculus of the new sequence of future controls; the second procedure is invoked when the constrained optimization gives no feasible solution and involves all the constraints.

3. Calculation of the Linearized Model

Although the control technique presented in this paper can be applied to any robotic system with soft nonlinearities, for simplicity reasons the method is shown on a generic two-degrees-of-freedom direct-drive robot arm.

Its is well known that in direct-drive robots the coupling among the links is quite significant, prompting the implementation of a multivariable controller (Fu et al. 1988). Also in these robotic systems there are strict physical limitations in the maximum torque that can be applied, and its variation rate. Moreover, as in most mechanical systems, due to safety regulations there are strict limitations in the maximum speed and acceleration that any link can reach. These considerations justify the implementation of a Multivariable Constrained Predictive Controller for robotic systems, such as the one presented in this paper.

To derive the linear model of a robotic systems, this paper proposes to derive first a nonlinear model using the well-known Lagrange-Euler method, and then linearize this model at each sampling time by using a Taylor approximation at the measured working point. The model can then be discretized by using any approximation rule: trapezoidal, bilinear, matching pole-zero, etc. For simplicity reason a trapezoidal approximation rule is used in the example presented in this paper.

4. Example: Direct-drive robot

The dynamic equations of a Two-degrees-of-freedom (2DoF) direct-drive robot can be obtained applying the Lagrange-Euler method (Spong and Vidyasagar, 1989). They can be represented by the general model:

$$
\begin{aligned}
\tau_1 &= f_{l1} + p_1\ddot{q}_1 + p_2\left[\ddot{q}_2\cos(q_2 - q_1) - \dot{q}_2^2\sin(q_2 - q_1)\right] \\
&\quad + p_4\cos(q_1) + p_6\dot{q}_1 \\
\tau_2 &= f_{l2} + p_3\ddot{q}_2 + p_2\left[\ddot{q}_1\cos(q_2 - q_1) + \dot{q}_1^2\sin(q_2 - q_1)\right] \\
&\quad + p_5\cos(q_2) + p_7\dot{q}_2
\end{aligned}
\qquad \textbf{(6)}
$$

Where τ_i is the input torque acting at the ith joint, f_{li} includes the static and low-speed friction and q_i, $\dot{q}_i$, $\ddot{q}_i$ are the ith joint angle, angular velocity and acceleration, respectively.

A discrete-time linearization of this model is presented in the appendix. The obtained discrete time model has the following structure:

$$\begin{pmatrix} \delta q_1 \\ \delta q_2 \end{pmatrix} = \frac{1}{A(q^{-1})} \begin{pmatrix} B_1(q^{-1}) & B_2(q^{-1}) \\ B_3(q^{-1}) & B_4(q^{-1}) \end{pmatrix} \begin{pmatrix} \delta \tau_1 \\ \delta \tau_2 \end{pmatrix} \qquad \mathbf{(7)}$$

It must be pointed out that the polynomials A, B1, B2, B3 and B4, used for prediction, are calculated at each sampling time, at the effective working point given by the most recent measured values of the manipulated ($\tau 1(t)$, $\tau 2(t)$) and controlled variables (q1(t), q2(t)).

The predictive control algorithm discussed previously have been implemented in C and linked with the nonlinear simulation of a two-degrees-of-freedom direct-drive robot arm, where the parameters used for simulation are those calculated by Liu and Goldenberg (1996):

$p_1 = 1.05\ \mathrm{kgm}^2$, $p_2 = 0.047\ \mathrm{kgm}^2$, $p_3 = 0.044\ \mathrm{kgm}^2$, $p_4 = 22.84\ \mathrm{kgm}$, $p_5 = 3.94\ \mathrm{kgm}$, $p_6 = 1.88\ \mathrm{Nms/rad}$, $p_7 = 0.29\ \mathrm{Nms/rad}$, $f_{l1} = 8.0\mathrm{Nm}$, $f_{l2} = 4.0\mathrm{Nm}$

These parameters were calculated by identification in nominal conditions, with the robot unloaded. During the operation of the robot, the parameters could vary, so to test the controller robustness the following parameters will be used, which correspond to a payload of 4 Kg:

$p_1 = 1.25\ \mathrm{kgm}^2$, $p_2 = 0.197\ \mathrm{kgm}^2$, $p_3 = 0.194\ \mathrm{kgm}^2$, $p_4 = 28.84\ \mathrm{kgm}$, $p_5 = 6.94\ \mathrm{kgm}$, $p_6 = 1.93\ \mathrm{Nms/rad}$, $p_7 = 0.34\ \mathrm{Nms/rad}$, $f_{l1} = 12.0\mathrm{Nm}$, $f_{l2} = 5.0\mathrm{Nm}$

The model used for prediction is the linearized model of the robot at each working point, using the formulae in the appendix, with a sample-time of 0.05s.

The controller was tuned for good response varying the parameters in the expected variation range. Although the tuning parameters could be selected at each working point, in this case, for simplicity, constant tuning parameters were selected. The parameters were selected to be (for both loops): N2=10, N4=10, Nu=1, N1=1, $T=1-0.98z^{-1}$. To reduce the computational costs the prediction horizon (N2) must be as small as possible. The noise filter T is tuned to improve the controller robustness.

Once the tuning parameters were selected, different experiments were carried out to check the robot performance with the designed controller in different situations. Several desired trajectories were tested in the experiments. The experiments shown in this paper corresponds to the same trajectory, with typical accelerating, uniform speed and decelerating motion segments.

For the desired trajectory, the nominal nonlinear system controlled with the predictive controller with locally linearized models discussed in the paper, the angles and control torques are shown in Figures 1 to 3. The predictive effect is clear in the plots: the control system acts in advance to trajectory changes, modifying the control torques before the desired angle change, compensating the effect, and reducing the total tracking error. The performance of the feedback system is adequate: the controlled system tracks smoothly the reference. The reference tracking properties benefit from using the pre-calculated command trajectory:

To study the feedback system robustness in the presence of model uncertainties the experiments were repeated for different combinations of parameters in the expected uncertainty range. The extreme case, that corresponds to a payload of 4Kg., is shown in Figures 4 to 6. It can be seen that the system performance is maintained despite the parameter variation induced by load changes.

In a different set of experiments the effect of strict constraints was tested. This constraints are frequent in robotic systems due to physical limitations or collision avoidance requirements. Figures 7 to 9 show the simulated effect of introducing the following operating constraint of the joint angles: q1>3.2rad and q2>4.8rad. The plots show that the controller maintains the output variables in the desired range, in spite of the fact that these constraints correspond to the lower trajectory angles: The hard limitations on the joint angles are not exceeded. To obtain this the control effort increases, as shown in Figure 9.

In the experiment depicted in Figure 10 to 12 the effect of saturating control torques was simulated. The control torque in the first joint was limited to be $|\tau_1| \leq 14.8\mathrm{Nm}$ and the variation in each sample time $|\Delta\tau_1| \leq 0.3\mathrm{Nm}$. It can be seen that, as long as the designer includes correctly in the controller the physical constraints, the controller maintains the performance, without causing windup problems in the presence of saturating controllers.

5. Conclusions

This paper has shown that performance and safety of robotic systems can be improved by applying advanced control strategies. Using traditional techniques it is difficult and cumbersome to consider physical and safety limitations in amplitude and rate of control and output signals for multivariable systems. To solve this difficulty, it has been proposed in this paper to use a Constrained Multivariable Predictive Controller. This kind of controllers uses a multivariable model of the robotic system to predict and optimize on-line the evolution of control and output signals. Then, corrective actions can be applied at each sampling time, taking into account constraints in the signals and disturbances acting in the system. It has been proposed to use for prediction a linearized model of the robot at each working point, which has made possible to meet the strict computational limitations of a real-time implementation, without degrading the closed-loop performance.

As an example, the control of a direct-drive two-degrees-of-freedom robot by using a Constrained Multivariable Predictive Controller is addressed. By simulation the improvement in performance and safety of the system that can be obtained when controlling the system with the discussed Predictive Controller has been shown. This improvement has been checked in the presence of hard constraints in control and output signals, an also with different loads.

Demonstration of these techniques for real-time control is being planned. Future research must be done to reduce the complexity and computational time, so MBPC can be implemented to solve difficult industrial robotic problems with many degrees of freedom. The main difficulty comes from computational cost considerations: Although it has been shown that using a linearized model at each working point reduces the computational requirements, further research must be done to minimize the algorithm computational time. In this direction Wright (2000), Rao *et al.* (1999) have proposed novel optimization algorithms for predictive control. Also it may be worth studying the compromise between the complexity of applying different approximation methods versus the computational effort. Another line of research worth studying is the variation of tuning parameters depending on the characteristics of the linearized model.

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Appendix: Derivation of the Discrete-time Linearized Model

A general model for a 2DoF direct-drive robot that relates the input signals (torques t_1 and t_2) with the output signals (angles q_1 and q_2), can be derived using the Euler-Lagrange method:

$$\tau_1 = f_{11} + p_1\ddot{q}_1 + p_2\ddot{q}_2\cos(q_2 - q_1) - p_2[\dot{q}_2]^2\sin(q_2 - q_1) + p_4\cos(q_1) + p_6\dot{q}_1$$
$$\tau_2 = f_{12} + p_3\ddot{q}_2 + p_2\ddot{q}_1\cos(q_2 - q_1) - p_2[\dot{q}_1]^2\sin(q_2 - q_1) + p_5\cos(q_2) + p_7\dot{q}_2$$

If the linearization point is (t_{10}, t_{20}, q_{10}, q_{20}), it is possible to approximate in the neighbourhood of this linearization point substituting $\tau_1 = \tau_{10} + \delta\tau_1$, $\tau_2 = \tau_{20} + \delta\tau_2$, $q_1 = q_{10} + \delta q_1$, $q_2 = q_{20} + \delta q_2$ using the difference variables $\delta\tau_1$, $\delta\tau_2$, δq_1 and δq_2:

$$\tau_{10} + \delta\tau_1 = f_{11} + p_1[\ddot{q}_{10} + \delta\ddot{q}_1] + p_2[\ddot{q}_{20} + \delta\ddot{q}_2]\cos(q_{20} - q_{10} + \delta q_2 - \delta q_1)$$
$$- p_2[\dot{q}_{20} + \delta\dot{q}_2]^2\sin(q_{20} - q_{10} + \delta q_2 - \delta q_1) + p_4\cos(q_{10} + \delta q_1) + p_6[\dot{q}_{10} + \delta\dot{q}_1]$$
$$\tau_{20} + \delta\tau_2 = f_{12} + p_3[\ddot{q}_{20} + \delta\ddot{q}_2] + p_2[\ddot{q}_{10} + \delta\ddot{q}_1]\cos(q_{20} - q_{10} + \delta q_2 - \delta q_1)$$
$$- p_2[\dot{q}_{10} + \delta\dot{q}_1]^2\sin(q_{20} - q_{10} + \delta q_2 - \delta q_1) + p_5\cos(q_{20} + \delta q_2) + p_7[\dot{q}_{20} + \delta\dot{q}_2]$$

As (τ_{10}, τ_{20}, q_{10}, q_{20}) fulfills the nonlinear system equations, it is possible to simplify the above equations by applying:

$$\tau_{10} = f_{11} + p_1\ddot{q}_{10} + p_2\ddot{q}_{20}\cos(q_{20} - q_{10}) - p_2[\dot{q}_{20}]^2\sin(q_{20} - q_{10}) + p_4\cos(q_{10}) + p_6\dot{q}_{10}$$
$$\tau_{20} = f_{12} + p_3\ddot{q}_{20} + p_2\ddot{q}_{10}\cos(q_{20} - q_{10}) - p_2[\dot{q}_{10}]^2\sin(q_{20} - q_{10}) + p_5\cos(q_{20}) + p_7\dot{q}_{20}$$

This substitution gives

$$\delta\tau_1 = p_1\delta\ddot{q}_1 + p_2\cos(q_{20} - q_{10})\delta\ddot{q}_2 + p_2\ddot{q}_{20}\sin(q_{20} - q_{10})[\delta q_2 - \delta q_1]$$
$$- p_2 2\dot{q}_{20}\sin(q_{20} - q_{10})\delta\dot{q}_2 - p_2[\dot{q}_{20}]^2\cos(q_{20} - q_{10})[\delta q_2 - \delta q_1] - p_4\sin(q_{10})\delta q_1 + p_6\delta\dot{q}_1$$

$$\delta\tau_2 = p_3\delta\ddot{q}_2 + p_2\cos(q_{20} - q_{10})\delta\ddot{q}_1 - p_2\ddot{q}_{10}\sin(q_{20} - q_{10})[\delta q_2 - \delta q_1]$$
$$- p_2 2\dot{q}_{10}\sin(q_{20} - q_{10})\delta\dot{q}_1 - p_2[\dot{q}_{10}]^2\cos(q_{20} - q_{10})[\delta q_2 - \delta q_1] - p_5\sin(q_{20})\delta q_2 + p_7\delta\dot{q}_2$$

Applying the Laplace transform to these equation gives:

$$\delta\tau_1 = p_1 s^2\delta q_1 + p_2\cos(q_{20} - q_{10})s^2\delta q_2 - p_2\ddot{q}_{20}\sin(q_{20} - q_{10})\delta q_1 + p_2\ddot{q}_{20}\sin(q_{20} - q_{10})\delta q_2$$
$$- p_2 2\dot{q}_{20}\sin(q_{20} - q_{10})s\delta q_2 + p_2[\dot{q}_{20}]^2\cos(q_{20} - q_{10})\delta q_1 - p_2[\dot{q}_{20}]^2\cos(q_{20} - q_{10})\delta q_2$$
$$- p_4\sin(q_{10})\delta q_1 + p_6 s\delta q_1$$

$$\delta\tau_2 = p_3 s^2\,\delta q_2 + p_2\cos(q_{20} - q_{10})s^2\,\delta q_1 - p_2\ddot{q}_{10}\sin(q_{20} - q_{10})\,\delta q_2 + p_2\ddot{q}_{10}\sin(q_{20} - q_{10})\,\delta q_1$$
$$- p_2 2\dot{q}_{10}\sin(q_{20} - q_{10})s\,\delta q_1 - p_2[\dot{q}_{10}]^2\cos(q_{20} - q_{10})\,\delta q_2 + p_2[\dot{q}_{10}]^2\cos(q_{20} - q_{10})\,\delta q_1$$
$$- p_5\sin(q_{20})\,\delta q_2 + p_7 s\,\delta q_2$$

These equations can be expressed as:
$$\begin{pmatrix}\delta\tau_1 \\ \delta\tau_2\end{pmatrix} = \begin{pmatrix}A_1(s) & A_2(s) \\ G_1(s) & G_2(s)\end{pmatrix}\begin{pmatrix}\delta q_1 \\ \delta q_2\end{pmatrix}$$

where

$$A_1(s) = a_{11}s^2 + a_{12}s + a_{13}$$
$$A_2(s) = a_{21}s^2 + a_{22}s + a_{23}$$
$$G_1(s) = g_{11}s^2 + g_{12}s + g_{13}$$
$$G_2(s) = g_{21}s^2 + g_{22}s + g_{23}$$
$$a_{11} = p_1$$
$$a_{12} = p_6$$
$$a_{13} = -p_2\ddot{q}_{20}\sin(q_{20} - q_{10}) + p_2[\dot{q}_{20}]^2\cos(q_{20} - q_{10}) - p_4\sin(q_{10})$$
$$a_{21} = p_2\cos(q_{20} - q_{10})$$
$$a_{22} = -2p_2\dot{q}_{20}\sin(q_{20} - q_{10})$$
$$a_{23} = p_2\ddot{q}_{20}\sin(q_{20} - q_{10}) - p_2[\dot{q}_{20}]^2\cos(q_{20} - q_{10})$$

$$g_{11} = p_2 \cos(q_{20} - q_{10})$$
$$g_{12} = -2p_2\dot{q}_{10}\sin(q_{20} - q_{10})$$
$$g_{13} = p_2\ddot{q}_{10}\sin(q_{20} - q_{10}) - p_2[\dot{q}_{10}]^2 \cos(q_{20} - q_{10})$$
$$g_{21} = p_3$$
$$g_{22} = p_7$$
$$g_{23} = -p_2\ddot{q}_{10}\sin(q_{20} - q_{10}) + p_2[\dot{q}_{10}]^2 \cos(q_{20} - q_{10}) - p_5\sin(q_{10})$$

Then the continous matrix transfer function calculated from linearization is:

$$\begin{pmatrix} \delta q_1 \\ \delta q_2 \end{pmatrix} = \frac{1}{D}\begin{pmatrix} G_2 & -A_2 \\ -G_1 & A_1 \end{pmatrix}\begin{pmatrix} \delta\tau_1 \\ \delta\tau_2 \end{pmatrix}$$

where

$$D = A_1G_2 - G_1A_2 = d_1s^4 + d_2s^3 + d_3s^2 + d_4s + d_5$$
$$d_1 = a_{11}g_{21} - a_{12}g_{11}$$
$$d_2 = a_{12}g_{21} + a_{11}g_{22} - a_{22}g_{11} - a_{21}g_{12}$$
$$d_3 = a_{12}g_{22} + a_{11}g_{23} + a_{13}g_{21} - a_{22}g_{12} - a_{21}g_{13} - a_{23}g_{11}$$
$$d_4 = a_{12}g_{23} + a_{13}g_{22} - a_{22}g_{13} - a_{23}g_{12}$$
$$d_5 = a_{13}g_{23} - a_{23}g_{13}$$

Finally, using the trapezoidal approximation: $s = \frac{1 - z^{-1}}{T}$, the discrete time model is:

$$\begin{pmatrix} \delta q_1 \\ \delta q_2 \end{pmatrix} = \frac{1}{A}\begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix}\begin{pmatrix} \delta\tau_1 \\ \delta\tau_2 \end{pmatrix}$$

where

$$a_1 = \frac{d_1}{T^4} + \frac{d_2}{T^3} + \frac{d_3}{T^2} + \frac{d_4}{T} + d_5$$
$$a_2 = -4\frac{d_1}{T^4} - 3\frac{d_2}{T^3} - 2\frac{d_3}{T^2} - \frac{d_4}{T}$$
$$a_3 = 6\frac{d_1}{T^4} + 3\frac{d_2}{T^3} + \frac{d_3}{T^2}$$
$$a_4 = -4\frac{d_1}{T^4} + \frac{d_2}{T^3}$$
$$a_5 = \frac{d_1}{T^4}$$
$$b_{11} = \frac{g_{21}}{T^2} + \frac{g_{22}}{T} + g_{23}; \quad b_{12} = -2\frac{g_{21}}{T^2} - \frac{g_{22}}{T}; \quad b_{13} = \frac{g_{21}}{T^2}$$
$$b_{21} = -\frac{a_{21}}{T^2} - \frac{a_{22}}{T} - a_{23}; \quad b_{22} = 2\frac{a_{21}}{T^2} + \frac{a_{22}}{T}; \quad b_{23} = -\frac{a_{21}}{T^2}$$
$$b_{31} = -\frac{g_{11}}{T^2} - \frac{g_{12}}{T} - g_{13}; \quad b_{32} = 2\frac{g_{11}}{T^2} + \frac{g_{12}}{T}; \quad b_{33} = -\frac{g_{11}}{T^2}$$
$$b_{41} = \frac{a_{11}}{T^2} + \frac{a_{12}}{T} + a_{13}; \quad b_{42} = -2\frac{a_{11}}{T^2} - \frac{a_{12}}{T}; \quad b_{43} = +\frac{a_{11}}{T^2}$$

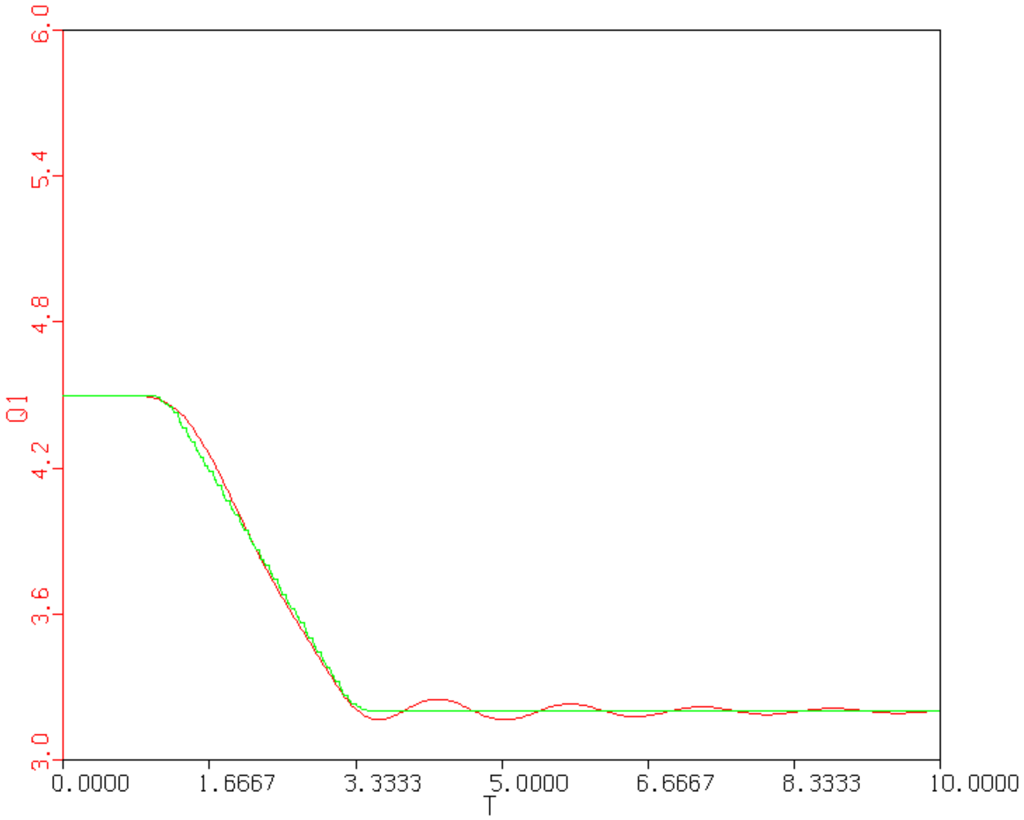


Fig. 1: Q1 with the nonlinear nominal system and time-varying models

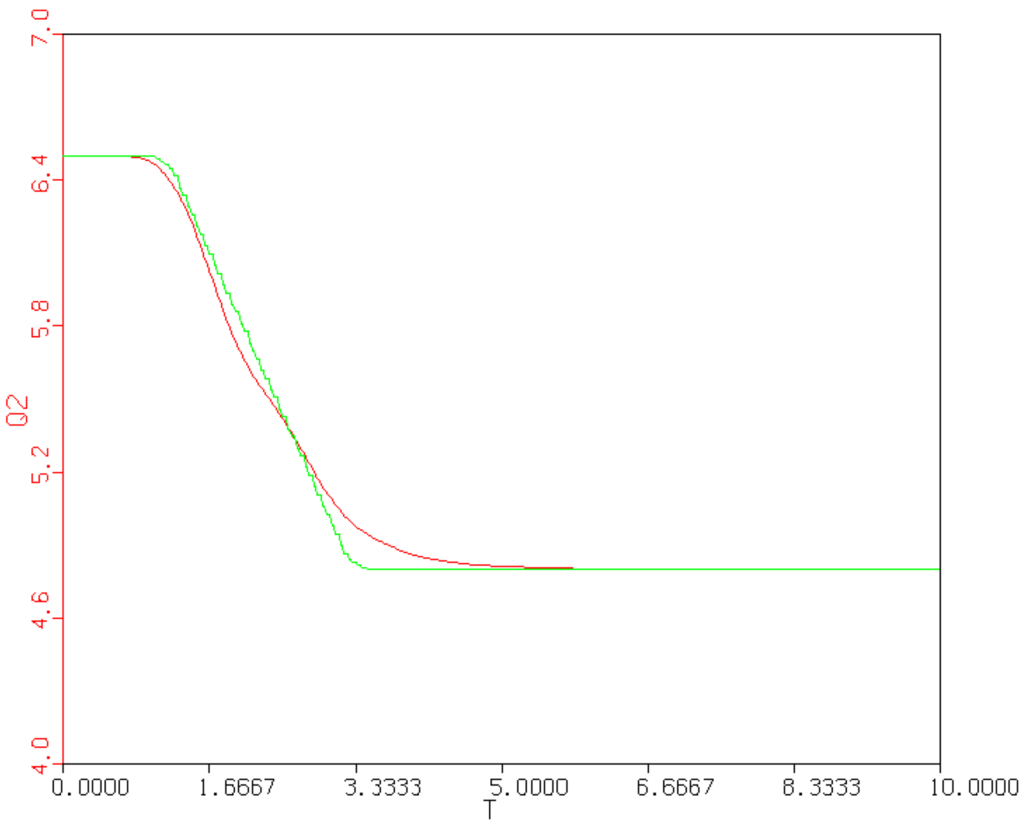


Fig. 2: Q2 with the nonlinear nominal system and time-varying models

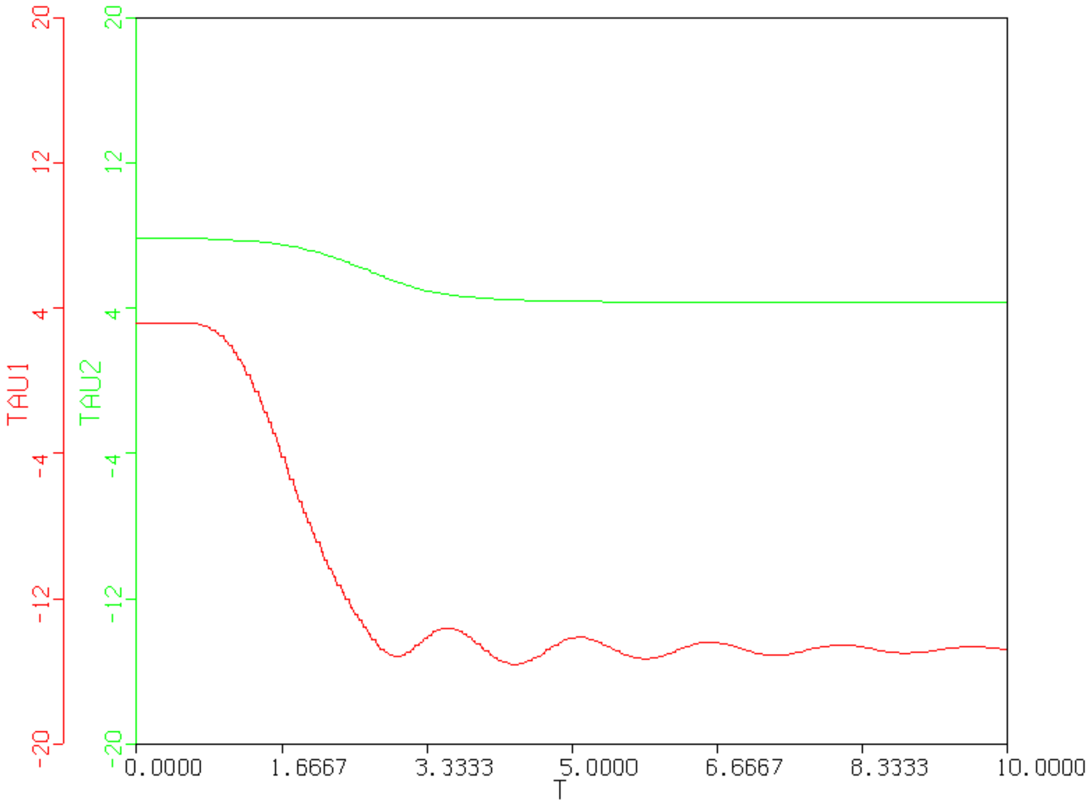


Fig. 3: Control torques with the nonlinear nominal system and time-varying models

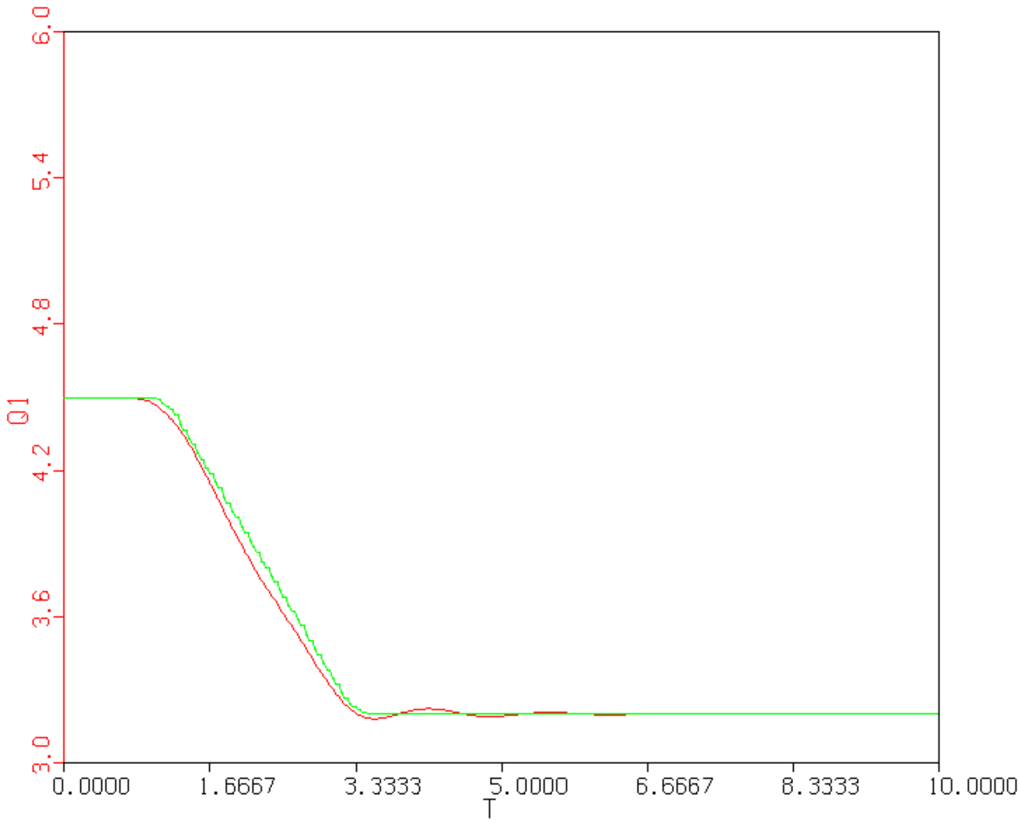


Fig. 4: Q1 in the presence of uncertainty in the model

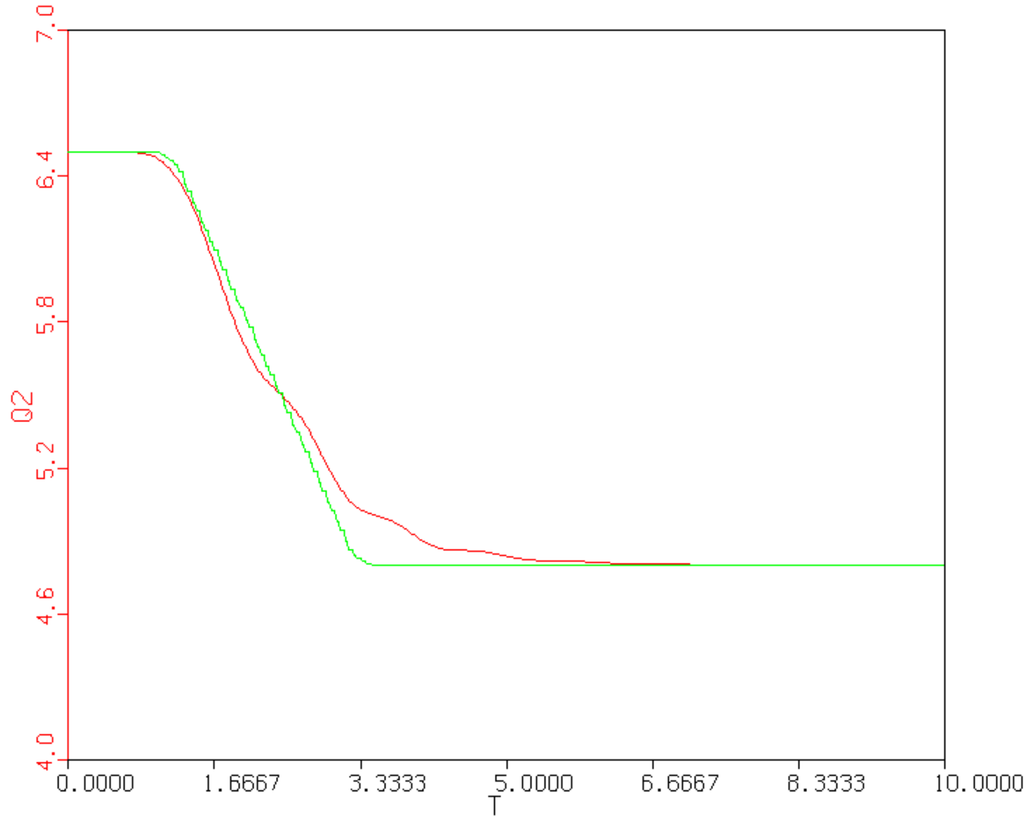
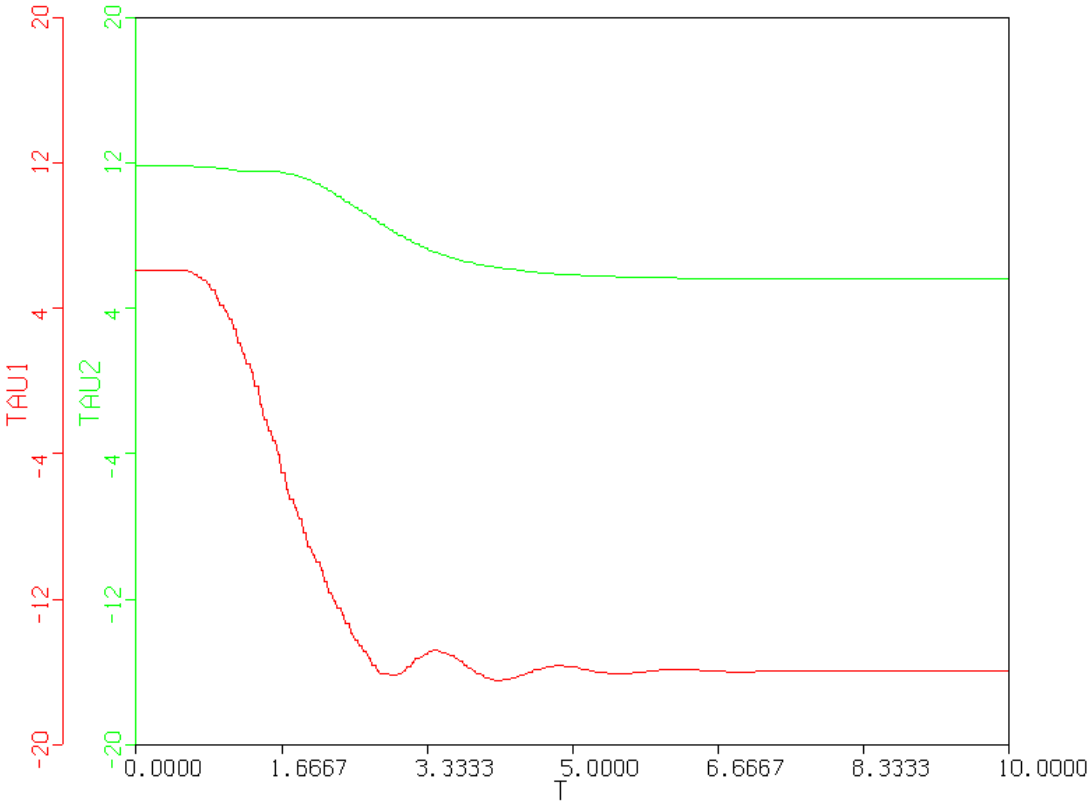


Fig. 5: Q2 in the presence of uncertainty in the model



Control Fig. 6: torques with uncertainty in the system and time-varying models

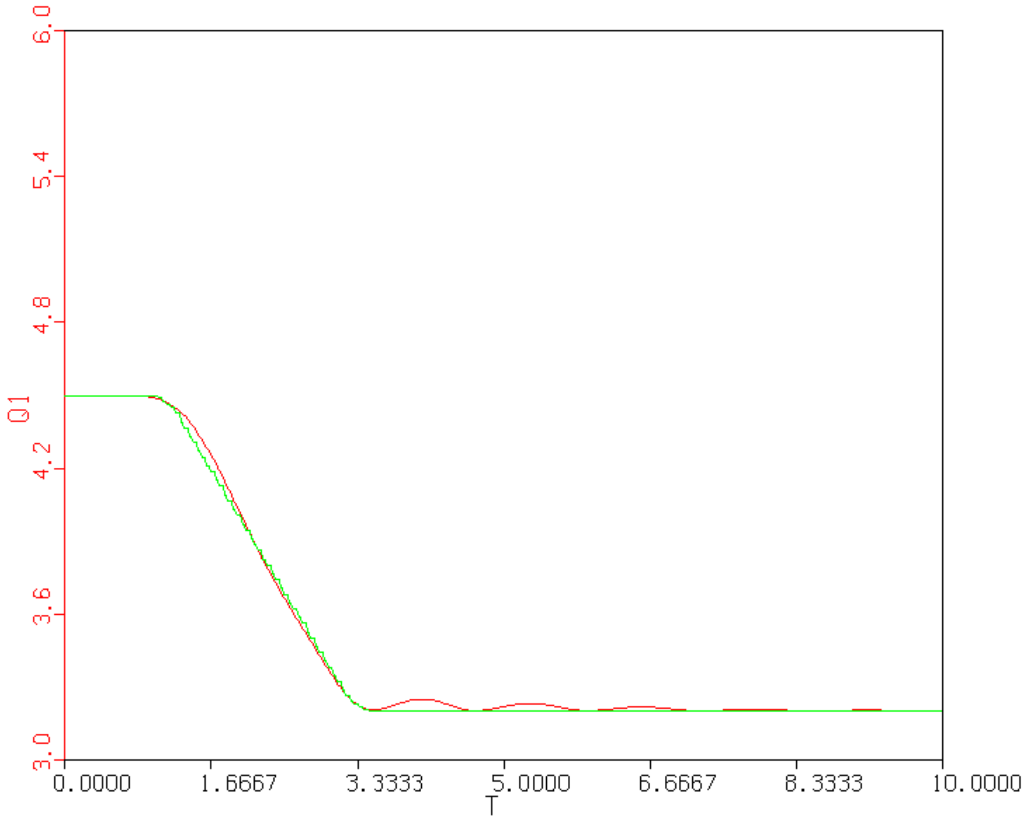


Fig. 7: Q1 with time-varying models and hard limitation on the output

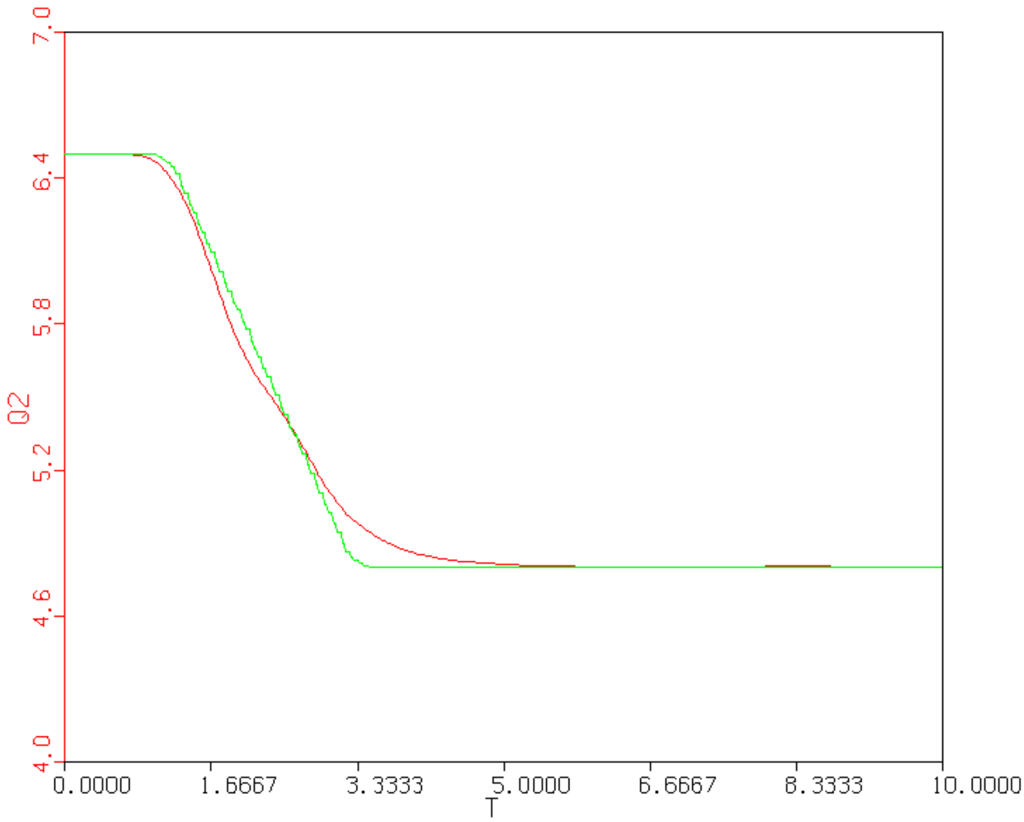


Fig. 8: Q2 with time-varying models and hard limitation on the output

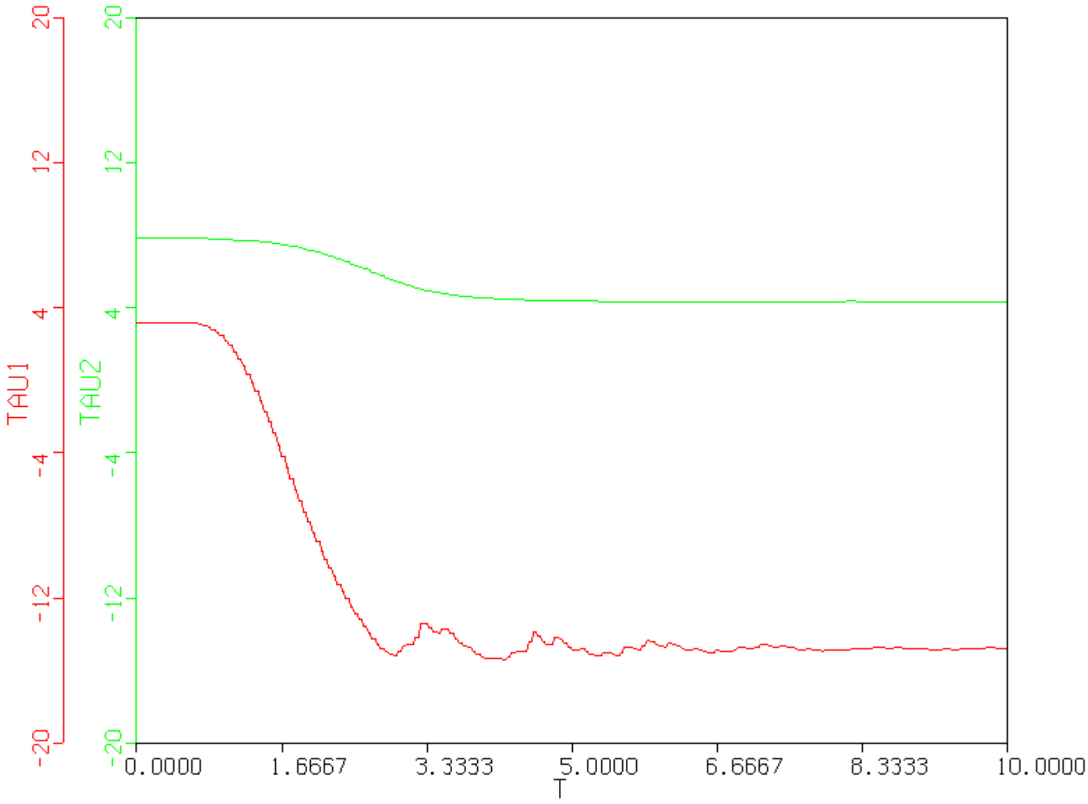


Fig. 9: Control torques with time-varying models and hard limitation on the output

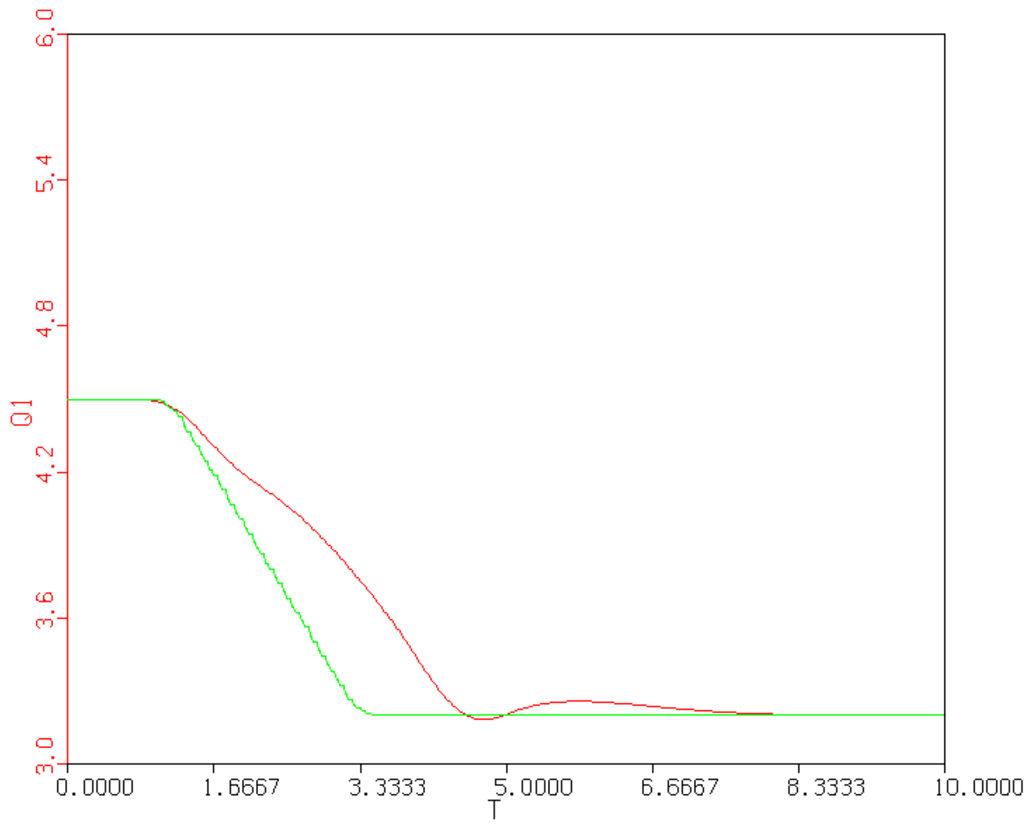


Fig. 10: Q1 with time-varying models and hard limitation on $\tau 1$ and $\Delta\tau 1$

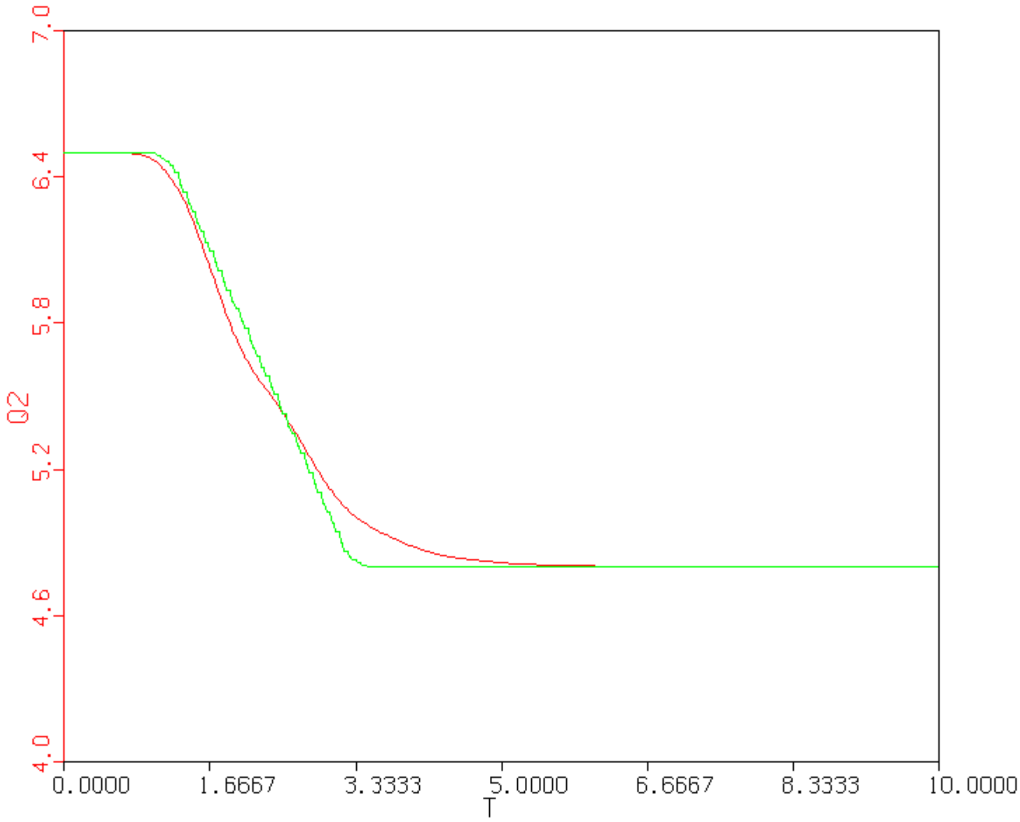


Fig. 11: Q2 with time-varying models and hard limitations on $\tau 1$ and $\Delta\tau 1$

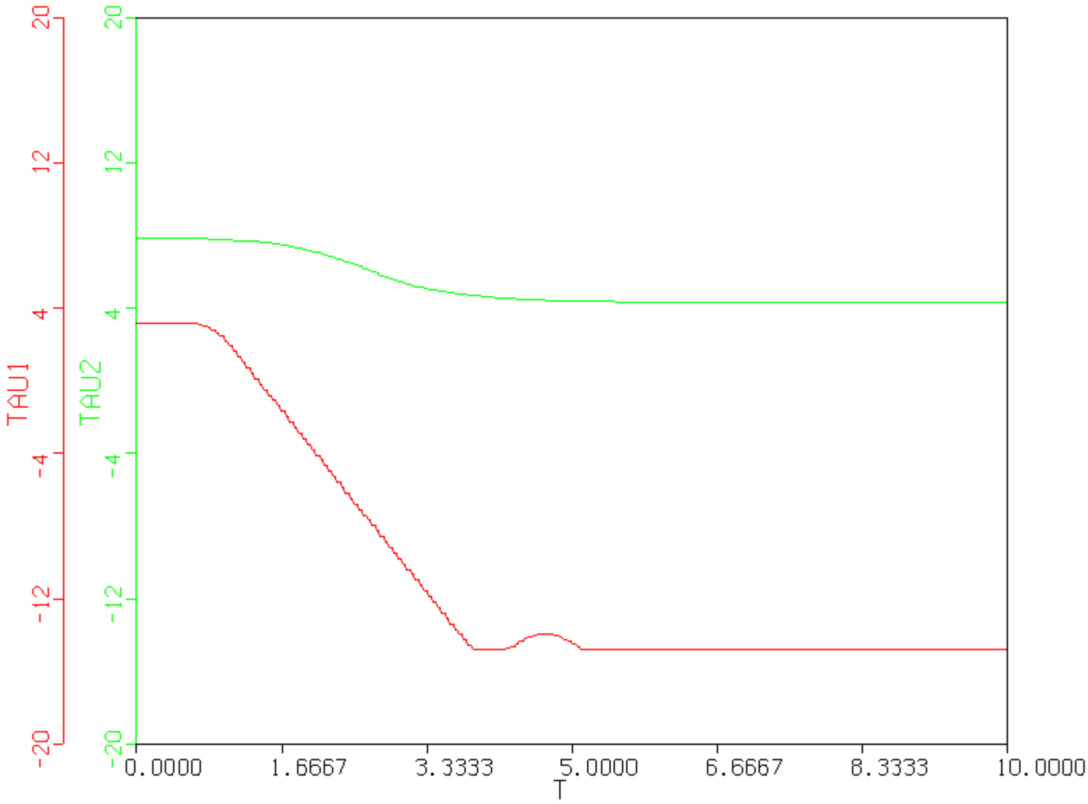


Fig. 12: Control torques with time-varying models and hard limitations on $\tau 1$ and $\Delta\tau 1$