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Designing approach on trajectory-tracking control of mobile robot

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Abstract

Based on differential geometry theory, applying the dynamic extension approach of relative degree, the exact feedback linearization on the kinematic error model of mobile robot is realized. The trajectory-tracking controllers are designed by pole-assignment approach. When angle speed of mobile robot is permanently nonzero, the local asymptotically stable controller is designed. When angle speed of mobile robot is not permanently nonzero, the trajectory-tracking control strategy with globally tracking bound is given. The algorithm is simple and applied easily. Simulation results show their effectiveness.

Keywords: Trajectory tracking; Dynamic extension approach; Exact feedback linearization; Globally tracking bound

1. Introduction

Recently, interest in the tracking control of mobile robots has increased with various theoretical and practical contributions being made. Particularly, feedback linearization has attracted a great deal of research interest in recent nonlinear control theory, and some techniques have been employed in mobile robot control. Path tracking problems of several types of mobile robots have been investigated by means of linearizing the static and dynamic state feedback in [1]. The local and global tracking problems via time-varying state feedback based on the backstepping technique have been addressed in [2]. Since the wheel-driven mobile robot has nonholonomic constraints that arise from constraining the wheels of the mobile robot to roll without slipping and the linearized mobile robot with nonholonomic constraints has a controllability deficiency, it is difficult to control them. The point stabilization problem can be regarded as the generation of control inputs to drive the robot from any initial point to target point. The crucial problem in this stabilization question centers on the fact that the mobile robot model does not meet Brockett's [3] well-known necessary smooth feedback stabilization condition, so the mobile robot cannot be stabilized with smooth state feedback, which leads to the limitation in application. Therefore some discrete time-invariant controllers, time-varying controllers and hybrid controllers based on Lyapunov control theories have been proposed in [4]. The global trajectory-tracking problem to reference mobile robot is discussed based on the backstepping technique in [5]. The trajectory-tracking problem to reference mobile robot is discussed based on the terminal sliding-mode technique in [6], but it requires the nonzero speed of rotation. Point stabilization of mobile robot via state-space exact feedback linearization based on dynamic extension approach is proposed in [7]. The point stabilization problem in polar frame can be exactly transformed into the problem of controlling a linear time-invariant system. But its disadvantage is to require the verification of the complex involution. And the point stabilization problem is only discussed but the trajectory tracking is not solved.

In the present paper, the trajectory tracking to reference mobile robot as [5] and [6] is addressed based on dynamic extension approach in [7]. The exact feedback linearization on the kinematic error model of mobile robot is realized. Its proof is simple and different from [7] since the complex process of verifying involution is avoided. By linearization, the nonlinear system is transferred to linear time-invariance system, which is equivalent to two reduced-order linear time-invariance systems that can be controlled easily. If angle speed of mobile robot is permanently nonzero, the local

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asymptotically stable controller is designed. If angle speed of mobile robot is not permanently nonzero, the trajectory-tracking control strategy with globally tracking bound is given. The algorithm is simple and applied easily.

2. Preliminaries and problem formulation

Consider a class of nonlinear systems described as

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

$$y = h(x), \tag{2}$$

where $x \in R^n$, $u \in R^m$, $y = h(x) = (h_1(x), \ h_2(x), \dots, h_m(x))^{\mathrm{T}} \in R^m$ represent the state vector, the input vector and the output vector, respectively. $f(x) \in R^n$, $g(x) = (g_1(x), g_2(x), \dots, g_m(x)) \in R^{n\times m}$ are smooth vector fields.

Definition (Slotine and Li [8] and Feng and Fei [9]). Given $x_0 \in X$, X is an n-dimension differentiable manifold if there exists a neighborhood V of x_0 and integer vector $(r_1, r_2, \dots, r_m)$ such that

(1) $L_{g_j} L_f^k h_i(x) = 0 \quad \forall x \in V, \ 1 \leqslant j \leqslant m,$
$1 \leqslant i \leqslant m, \ 0 \leqslant k \leqslant r_i - 2.$

(2) Matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & \vdots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix}$$

is nonsingular $\forall x \in V$, we say that system (1)–(2) has vector relative degree $(r_1, r_2, \dots, r_m)$ at point x_0.

Lemma (Feng and Fei [9]). *The necessary and sufficient condition of exact feedback linearization at x_0 for system (1) is that there exists a neighborhood V of x_0 and smooth real-valued functions $h_i(x) \in V$, $i = 1, 2, \dots, m$, such that system (1)–(2) has vector relative degree $(r_1, r_2, \dots, r_m)$ at the point x_0, and $\sum_{i=1}^{m} r_i = n$.*

The kinematic model of wheel-driven mobile robot as follows:

$$\dot{x} = v\cos\theta,$$
$$\dot{y} = v\sin\theta,$$
$$\dot{\theta} = \omega, \tag{3}$$

where (x, y) is the position of mobile robot and θ is the heading angle. The control variables of mobile robot are the linear velocity v and the angular velocity ω. Here, the trajectory-tracking problem is to track a reference mobile robot with the known posture (x_r, y_r, θ_r) and velocities v_r, ω_r, as shown in Fig. 1. We have the posture error equation of mobile robot [5,6]

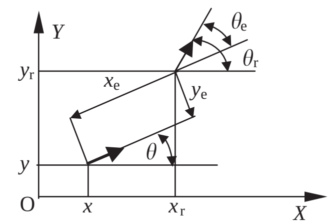

Fig. 1. Posture error coordinate.

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}. \tag{4}$$

Hence we have the posture error difference equations [5,6]

$$\dot{x}_e = \omega y_e - v + v_r \cos\theta_e,$$
$$\dot{y}_e = -\omega x_e + v_r \sin\theta_e,$$
$$\dot{\theta}_e = \omega_e. \tag{5}$$

From above analysis, the trajectory-tracking problem to reference mobile robot can be stated as: find the bounded inputs v and ω so that for an arbitrary initial error, the state (x_e, y_e, θ_e) of system (5) can be held near the origin $(0, 0, 0)$, i.e.

$$\lim_{t\to\infty} \|(x_e, y_e, \theta_e)^{\mathrm{T}}\| = 0. \tag{6}$$

3. Design of trajectory-tracking controllers

It is obvious that system (5) cannot be state-space exact feedback linearization. It cannot also be partial input/output feedback linearization by choosing the outputs $y_1 = y_e$, $y_2 = \theta_e$. The reason is that system (5) has not the relative degree. Actually

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & -x_e \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \tag{7}$$

It is obvious that the decoupling matrix

$$A(x) = \begin{bmatrix} 0 & -x_e \\ 0 & -1 \end{bmatrix}$$

is singular. So system (5) has not the relative degree, but it has a constant rank 1. The reason that system (5) has not the relative degree is that there is only angle velocity ω and not the linear velocity v in the first-order derivative of outputs. So we delay the input ω to appear in the higher derivative of outputs in order to make system (5) obtain the relative degree. Therefor we apply the so-called dynamic extension approach [7] and introduce a new state equation $\dot{\omega}_e = \dot{\omega}_r - \dot{\omega}$. The control inputs of system (5) are modified to $(v, \dot{\omega})$ from (v, ω). Hence we have the following extension state equations:

$$\begin{aligned}
\dot{x}_e &= \omega y_e - v + v_r \cos\theta_e,\\
\dot{y}_e &= -\omega x_e + v_r \sin\theta_e,\\
\dot{\theta}_e &= \omega_e,\\
\dot{\omega}_e &= \dot{\omega}_r - \dot{\omega}.
\end{aligned} \tag{8}$$

Let state variables be $x_1 = x_e$, $x_2 = y_e$, $x_3 = \theta_e$, $x_4 = \omega_e$, then (8) can be written into the form as (1)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} v_r \cos x_3 + x_2(\omega_r - x_4)\\ v_r \sin x_3 - x_1(\omega_r - x_4)\\ x_4\\ \dot{\omega}_r\end{bmatrix} + \begin{bmatrix}-1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}v\\ \dot{\omega}\end{bmatrix}, \tag{9}$$

where the control input is $(v, \dot{\omega})$, hence we have the following Theorem 1.

Theorem 1. *Under outputs $y_1 = x_2$, $y_2 = x_3$, and angle velocity $\omega \neq 0$, system (9) can be linearized exactly.*

Proof. We differentiate the output equations $y_1 = x_2$, $y_2 = x_3$, then we have

$$\dot{y}_1 = \dot{x}_2 = v_r \sin x_3 - x_1(\omega_r - x_4), \tag{10}$$

$$\ddot{y}_1 = \dot{v}_r \sin x_3 + 2v_r x_4 \cos x_3 - v_r\omega_r \cos x_3 - x_2(\omega_r - x_4)^2 + (\omega_r - x_4)v - x_1\dot{\omega}, \tag{11}$$

$$\dot{y}_2 = \dot{x}_3 = x_4, \tag{12}$$

$$\ddot{y}_2 = \dot{\omega}_r - \dot{\omega}. \tag{13}$$

From (11) and (13), we have the decoupling matrix

$$A(x) = \begin{bmatrix}\omega_r - x_4 & -x_1\\ 0 & -1\end{bmatrix}. \tag{14}$$

From (14) we have

$$\det(A(x)) = -(\omega_r - x_4) = -\omega \neq 0. \tag{15}$$

Hence under outputs $y_1 = x_2$, $y_2 = x_3$, and angle velocity $\omega \neq 0$, system (9) has the relative degree $(r_1, r_2) = (2, 2)$ and $r_1 + r_2 = 4 = n$. Using lemma in Section 2, there exists the local diffeomorphism so that system (9) can be linearized exactly.

The local diffeomorphism and transformed states are defined as follows:

$$\begin{aligned}
\xi_1 &= y_1 = x_2,\\
\xi_2 &= \dot{y}_1 = L_f y_1 = v_r \sin x_3 - x_1(\omega_r - x_4),\\
\xi_3 &= y_2 = x_3,\\
\xi_4 &= \dot{y}_2 = L_f y_2 = x_4
\end{aligned} \tag{16}$$

the input transformations are defined as follows:

$$\begin{aligned}
\lambda_1 &= \dot{v}_r \sin x_3 + 2v_r x_4 \cos x_3 - v_r\omega_r \cos x_3 - x_2(\omega_r - x_4)^2 + (\omega_r - x_4)v - x_1\dot{\omega},\\
\lambda_2 &= \dot{\omega}_r - \dot{\omega}.
\end{aligned} \tag{17}$$

Using (10)–(13), (16) and (17), system (9) can be transformed into a linear time-invariant system:

$$\dot{\xi} = \begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}\xi + \begin{bmatrix}0&0\\1&0\\0&0\\0&1\end{bmatrix}\lambda, \tag{18}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix}\xi, \tag{19}$$

where $\xi = [\xi_1, \xi_2, \xi_3, \xi_4]^T$ is the new state vector. $\lambda = [\lambda_1, \lambda_2]^T$ is the new control input.

The nonlinear system (9) is transformed into the linear time-invariant system (18)–(19) by the state and input transformations (16)–(17). For the linear time-invariant system (18)–(19), we can apply the known linear control method such as pole-assignment method to implement control, hence we have the following Theorem 2. □

Theorem 2. *Assuming angle velocity $\omega \neq 0$, when system (9) is controlled by controller (20a), it has the local asymptotic stability. If $\omega \neq 0$ cannot be satisfied, system (9) is controlled by controllers (20a) and (20b) alternately, it has the globally bounded tracking to reference mobile robot with the known posture (x_r, y_r, θ_r) and velocities v_r, ω_r.*

$$\begin{bmatrix}v\\ \dot{\omega}\end{bmatrix} = \begin{cases} -\dfrac{1}{\omega_r - x_4}\begin{bmatrix}(\dot{v}_r \sin x_3 + 2v_r x_4 \cos x_3 - v_r\omega_r \cos x_3\\ -x_2(\omega_r - x_4)^2 - \lambda_1) + x_1(\lambda_2 - \dot{\omega}_r),\\ (\omega_r - x_4)(\lambda_2 - \omega_r)\end{bmatrix}, & |\omega| \geqslant \varepsilon, \quad (20a)\\[2ex] -\dfrac{1}{\varepsilon}\begin{bmatrix}(\dot{v}_r \sin x_3 + 2v_r x_4 \cos x_3 - v_r\omega_r \cos x_3\\ -x_2\varepsilon^2 - \lambda_1) + x_1(\lambda_2 - \dot{\omega}_r),\\ \varepsilon(\lambda_2 - \omega_r)\end{bmatrix}, & |\omega| < \varepsilon, \quad (20b)\end{cases}$$

where ε is a given arbitrary small positive number. The new control inputs $\lambda_1 = k_{11}\xi_1 + k_{12}\xi_2$, $\lambda_2 = k_{21}\xi_3 + k_{22}\xi_4$, where k_{i1}, k_{i2}, $i = 1, 2$ are the parameters that *make the matrix* $(A + BK_i)$ *stable.* A, B, K_i, $i = 1, 2$ *are, respectively,*

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$K_i = \begin{bmatrix} k_{i1} & k_{i2} \end{bmatrix}, \quad i = 1, 2. \tag{21}$$

Proof. Under angle velocity $\omega \neq 0$, from Theorem 1, system (9) can be transformed into linear time-invariant system (18)–(19). It is clear that system (18)–(19) is completely controllable and completely observable. It contains two reduce-order independent subsystems

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \lambda_1, \tag{22a}$$

$$\begin{bmatrix} \dot{\xi}_3 \\ \dot{\xi}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_3 \\ \xi_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \lambda_2, \tag{22b}$$

It is well known that linear time-invariant systems (22) can be well controlled via pole-assignment approach. So let $\lambda_1 = k_{11}\xi_1 + k_{12}\xi_2$, $\lambda_2 = k_{21}\xi_3 + k_{22}\xi_4$. The coefficients $k_{11}, k_{12}, k_{21}, k_{22}$ are chosen such that the two closed-loop subsystems (22) are exponentially stable [9]. Namely, the equilibrium point $\xi = 0$ of system (18) is exponentially stable. From the state transformation (16) we have that $\xi = 0$ means $x_1 = x_2 = x_3 = x_4 = 0$, namely the equilibrium point $(x_e, y_e, \theta_e, \omega_e) = (0, 0, 0, 0)$ is local asymptotically stable. And using (17) yields (20a), further we have $v \to v_r$.

If angle velocity $\omega \neq 0$ cannot be satisfied, to guarantee the realization of control, we choose the suitable small positive number ε that can be specified artificially according to the need of practice, so that when $|\omega| \geqslant \varepsilon$, controller (20a) is used, when $|\omega| < \varepsilon$, controller (20b) is used. Hence controllers (20a) and (20b) are used alternately, which can guarantee the bounded tracking to reference mobile robot with the known posture (x_r, y_r, θ_r) and velocities v_r and ω_r.

From (20) we see that the control inputs v and $\dot{\omega}$ are bounded as long as v_r and $\dot{\omega}_r$ are bounded. □

Remark 1. For system (8), angle velocity $\omega \neq 0$, otherwise, if $\theta_e \neq 0$ or $\theta_e = 0, \omega_r \neq 0$ or $\theta_e = 0, \omega_r = 0, y_e \neq 0$, the mobile robot cannot implement the tracking. If $\theta_e = 0$, $\omega_r = 0$ and $y_e = 0$, designing $v = v_r - kx_e$, $k < 0$ can realize the tracking.

Remark 2. For simplification, note (21) and (22), we can choose the same feedback gain $K_1 = K_2$ to stabilize systems (22a) and (22b) at the same time. So the computation burden is reduced.

4. Simulation research

We implement the simulation research to verify the effectiveness of tracking controllers (20a) and (20b). In the simulation, the parameters such as the initial conditions, desired velocities and feedback gains are listed in Table 1. We choose the same feedback gain for two time-invariant systems (22a) and (22b) and set $\varepsilon = 0.01$. Figs. 2–5 show the responses of the trajectory-tracking control of mobile robot. Figs. 2 and 3 show the simulation results that mobile robot follows the straight lines, where controllers (20a) and (20b) are used alternately in order to guarantee the bounded tracking to reference mobile robot with $\omega_r = 0$. From Figs. 2 and 3 we see that the performance of tracking is good and bounded; though x_e has a bias from zero before $t = 5$ s, it approximates near zero after $t = 5$ s. Figs. 4 and 5 show the simulation results that mobile robot follows the curves. From Figs. 4 and 5 we see that the performance of tracking is better. And all controllers are bounded, which guarantees the realization of control.

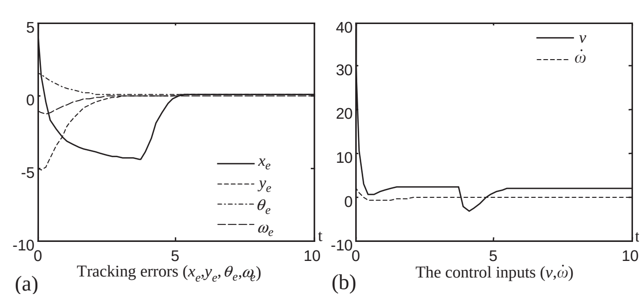

Fig. 2. Tracking to reference mobile robot with constant velocity $(v_r, \omega_r) = (2, 0)$: (a) tracking errors $(x_e, y_e, \theta_e, \omega_e)$ and (b) control inputs $(v, \dot{\omega})$.

Table 1
Parameters of simulations

Simulation results	Initial values of $(x_e, y_e, \theta_e, \omega_e)$	Desired velocities (v_r, ω_r)	Feedback gains $(k_{11}, k_{12}, k_{21}, k_{22})$
Fig. 2	$(5, -5, \pi/2, -1)$	$(2, 0)$	$(-4, -4, -4, -4)$
Fig. 3	$(-5, 5, -\pi/2, 1)$	$(10\sin(2t + \pi/6), 0)$	$(-4, -4, -4, -4)$
Fig. 4	$(1, -1, \pi/2, 0.5)$	$(2, 1)$	$(-4, -4, -4, -4)$
Fig. 5	$(5, -5, \pi/2, 0.5)$	$(10\sin(2t + \pi/6), 1)$	$(-4, -4, -4, -4)$

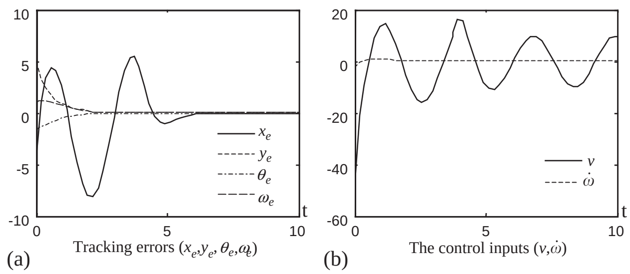

Fig. 3. Tracking to reference mobile robot with time-varying velocity $(v_r, \omega_r) = (10\sin(2t + \pi/6), 0)$: (a) tracking errors $(x_e, y_e, \theta_e, \omega_e)$ and (b) control inputs $(v, \dot{\omega})$.

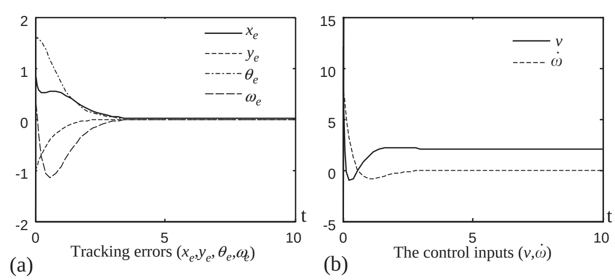

Fig. 4. Tracking to reference mobile robot with constant velocity $(v_r, \omega_{r,}) = (2, 1)$: (a) tracking errors $(x_e, y_e, \theta_e, \omega_e)$ and (b) control inputs $(v, \dot{\omega})$.

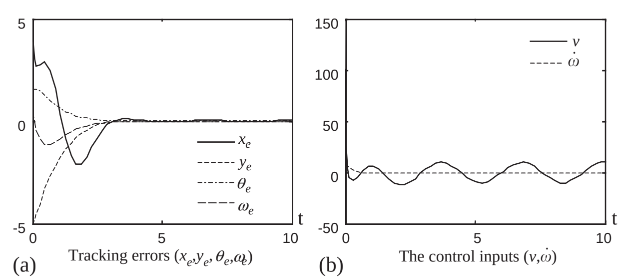

Fig. 5. Tracking to reference mobile robot with time-varying velocity $(v_r, \omega_r) = (10\sin(2t + \pi/6), 1)$: (a) tracking errors $(x_e, y_e, \theta_e, \omega_e)$ and (b) control inputs $(v, \dot{\omega})$.

5. Conclusion

In practice, to make the mobile robot obtain some postures and velocities, we can assume a reference mobile robot with these postures and velocities, and consider the trajectory-tracking problem to reference mobile robot. In this paper, the trajectory-tracking problem to reference mobile robot is addressed based on dynamic extension approach. The exact feedback linearization on the kinematic error model of mobile robot is realized. The nonlinear system is transferred to two reduced-order linear time-invariance systems that can be controlled easily. The following control is realized, i.e. if angle speed of the mobile robot is permanently nonzero, the local asymptotically stable controller is designed. If angle speed of the mobile robot is not permanently nonzero, the trajectory-tracking control strategy with globally tracking bound is given. The approaches are simple and efficient.

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References

[1] D'Andrea-Novel B, Campion G, Bastin G. Control of nonholonomic wheeled mobile robots by state feedback linearization. Int J Robot Res 1995;14(6):543–59.
[2] Zhongping Jiang, Henk Nijmeijer. Tracking control of mobile robots: a case study in backstepping. Automatica 1997;33(7): 1393–9.
[3] Brockett RW. Asymptotic stability and feedback stabilization. In: Differential geometric control theory. Boston: Birkhauser; 1983. p. 181–91.
[4] Kolmanovsky I, McClamroch NH. Developments in nonholonomic control problems. IEEE Control System 1995;15(6):20–36.
[5] Wu Wei-Guo, Chen Hui-Tang, Wang Yue-Juan. Global trajectory tracking control of mobile robots. Acta Autom. Sin. 2001;27(3): 326–31.
[6] Li Shi-Hua, Tian Yu-Ping. Tracking control of mobile robot. Control and Decision 2000;15(5):626–8.
[7] KyuCheol Park, Hakyoung Chun, Jang Gyulee. Point stabilization of mobile robots via state-space exact feedback linearization. Robotics Comput Integ Manuf 2000;16:353–63.
[8] Slotine JJ, Li W. Applied nonlinear control. Englewood Cliffs, NJ: Prentice-Hall; 1991.
[9] Feng CB, Fei SM. Analysis and design of nonlinear control system, 2nd ed. Beijing: Publishing House of Electronics Industry; 1998.