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Velocity Kinematic Modeling for Wheeled Mobile Robots

Dong Hun Shin and Kyung Hoon Park
Department of Mechanical Engineering, The University of Seoul
shin@uos.ac.kr and athun@hanmail.net

Abstract

This paper presents the velocity kinematic modeling of wheeled mobile robots (WMRs) by applying the matrix coordinate transformation to every pairs of WMRs including wheeled pairs which were dealt with ad-hoc manners in the previous methodologies. A wheeled pair is not replaced with a planar pair and its kinematics is formulated together with those of lower pairs, even though there exists 2 DOF motion at a wheeled pair. The proposed method is implemented successfully to obtain the forward velocity kinematics of a WMR, and it is therefore regarded as a real counterpart to the transformation methodologies widely used for the kinematics of the robot manipulators.

1 Introduction

The matrix coordinate transformation methodology is widely used in the kinematics analysis for the robot manipulators. Generally, it assumes a robot manipulator as a open serial chain of rigid body links connected together by joints, and embeds coordinate systems in each link of a robot manipulator, and then models a robot manipulator kinematically by formulating the relation between these coordinate systems [1], [2], [3]. The Denavit-Hartenberg notation is applied to assign coordinate systems at each joint and the homogeneous transformation matrix is used to describe the relative position and orientation between the coordinate systems embedded in each link [5].

However, this transformation methodology for robot manipulators cannot be directly applied to wheeled mobile robots (WMRs), because of the kinematic characteristics of WMRs compared to those of robot manipulators, as the following :

Firstly, WMRs are multiple-chain mechanisms [6], [8], while robot manipulators are open-chain mechanisms. In other words, the main body link of a WMR is connected to more than one link in parallel and thus the Denavit-Hartenberg notation leads to ambiguous assignments of coordinate systems in a WMR [6], [8].

Secondly, WMRs contain higher pair joints between each wheel and the floor, whereas robot manipulators contain only lower pair joints such as revolute joints and prismatic joints. While the Denavit-Hartenberg notation is used successfully for the lower pair mechanisms, all the matrix notation including the Denavit-Hartenberg notation have not assigned coordinate systems for the general higher pairs successfully.

Considering the characteristics of a WMR, Muir and Neuman proposed the methodology for a WMR kinematics [6], [8]. Muir and Neuman applied the Sheth-Uicker notation [7] that is a modified notation for multiple-chain mechanisms from Denavit-Hartenberg notation to assign coordinate systems in a WMR. But, the problem of a higher pair between each wheel and the floor was detoured : each higher pair of a WMR is replaced with a planar pair. i.e., each wheel is replaced with a cube that slides on the floor surface with the translational and rotational velocity same as that of the wheel center, as in Fig. 1.

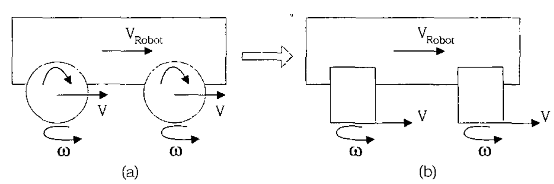

Fig. 1. (a)Wheeled pair and (b)planar pair.

Then, a WMR becomes a robot like a sled whose skids slide on the snow. The velocity of each cube respect to the floor was referred as pseudo velocity which is actually the velocity of a wheel center. Muir obtained the relationship between the robot body velocity and pseudo velocity using coordinate transformation and the Sheth-Uicker notation such as (1), but his formulation skipped over the coordinate system at the contact point between a wheel and the floor. So the wheel motion variables like the angular velocity of a wheel are not included in (1).

$$\dot{P} = \hat{J}_i \cdot \dot{\hat{q}}_i \tag{1}$$

$\dot{P}$: Robot body velocity

$\hat{J}_i$: Pseudo Jacobian

$\dot{\hat{q}}_i$: Pseudo velocity

The wheel motion variables lke angular velocities are included into the kinematic formulation by wheel Jacobian matrix such as (2), which relates the wheel motion variables to the pseudo velocity. However, the wheel Jacobian is not obtained in a systematic approach like the coordinate transformation, but in ad-hoc method.

$$\dot{\hat{q}}_i = W_i \cdot \dot{q}_i \quad (2)$$

W_i : Wheel Jacobian matrix

$\dot{q}_i$: Wheel velocity

This pioneering work by Muir and Neuman was extended to the kinematics for the various type wheeled mobile robot such as AGV with an inclined steering column [9], [10]. However, this approach has the following drawbacks :

1) The wheel is the distinctive and special one among the parts in WMRs. But, all the wheels are replaced with the parts which make a planar pair with the floor, and therefore it turns out to formulate the kinematics of other robots rather than those of real WMRs with the matrix coordinate transformation.
2) The wheel Jacobian describes the most important wheel motion. However, it is obtained in a separate and ad-hoc ways. Therefore, the whole procedure is not considered as systematic and rational one.

In this paper a modified approach is proposed which formulates the kinematics of the whole WMR including the wheel directly with the matrix coordinate transformation. The wheeled pairs are not replaced and their kinematics are formulated by the matrix coordinate transformation, too. Therefore, the proposed is a real counterpart to the matrix coordinate transformation methodology widely employed for the robot manipulators kinematics.

2 Proposed Method

2.1 Assumption

We begin by introducing the assumptions about the mechanical structure and the motion of a WMR to which our proposed method apply :

a) WMRs consist of rigid links such as body link, steering links, and wheels.
b) WMRs move on a planar surface (floor).
c) The wheel of a WMR rolls on the floor without any translational slip.
d) The wheel of a WMR makes rotational slip at the contact point between each wheel and the floor.

2.2 Wheeled pair

Since WMRs are multiple chain mechanisms, the proposed method uses the Sheth-Uicker notation to assign coordinate systems. Sheth-Uicker notation assigns two coordinate systems at each joint as in Fig. 2, and computes homogeneous transformation matrix.

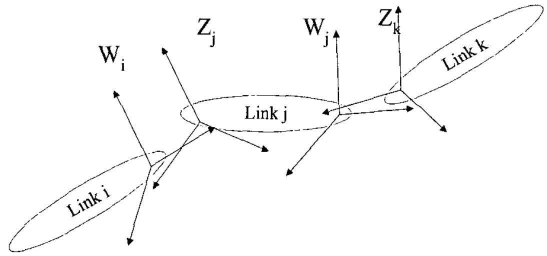

Fig. 2. Sheth-Uicker notation.

But, a wheeled pair is a kind of higher pair where it is known difficult to assign coordinate systems by Sheth-Uicker notation. When a wheel rolls on a floor without translational slip, the point of contact of a wheel with the floor has no velocity.

The point of contact is therefore the instantaneous center of rotation, since all points of the wheel seem to rotate about it [4]. i.e., the wheel seems a revolute pair at the contact point instantaneously.

However, there exists another motion between a wheel and the floor. When a WMR makes turn or a wheel is steered by a steering link, a wheel must rotate(i.e., rotational slip) about the contact point. Thus, following the Sheth-Uicker notation we assigned two coordinate systems at the contact point between a wheel and the floor as in Fig. 3. The coordinate system $\bar{C}$ is attached in the floor and the coordinate system C is embedded in the wheel. C is therefore rotating about X-axis and Z-axis as in Fig. 3, while $\bar{C}$ is stationary[1].

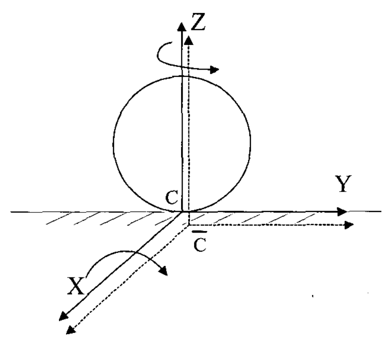

Fig. 3. Motion of wheel at the contact-point.

The coordinate system $\bar{C}$ coincides with C instantaneously and is stationary in a floor, while C moves with a wheel. So, $\bar{C}$ is therefore regarded as an Instantaneous Coincident Coordinate system (ICC) that was introduced in [6], [8], [9], [10] The origins of both coordinate systems are at the contact point. Notice that, as the wheel rolls on the floor, the contact point is renewed. So, the coordinate systems C and $\bar{C}$ assigned at the contact point are valid only for an instant ($\Delta t \rightarrow 0$), and the new coordinate systems will be assigned at the new contact point for the next instant. The transformation matrix describing between two coordinate systems C and $\bar{C}$ will be either of the following :

$$^{\bar{C}}\Theta_C = Rot\,(Z, {}^{\bar{C}}\theta_{zC})\, Rot\,(X, {}^{\bar{C}}\theta_{xC})$$

$$= \begin{bmatrix} \cos^{\bar{C}}\theta_{zC} & -\sin^{\bar{C}}\theta_{zC} & 0 & 0 \\ \sin^{\bar{C}}\theta_{zC} & \cos^{\bar{C}}\theta_{zC} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^{\bar{C}}\theta_{xC} & -\sin^{\bar{C}}\theta_{xC} & 0 \\ 0 & \sin^{\bar{C}}\theta_{xC} & \cos^{\bar{C}}\theta_{xC} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

or,

$$^{\bar{C}}\Theta_C = Rot\,(X, {}^{\bar{C}}\theta_{xC})\; Rot\,(Z, {}^{\bar{C}}\theta_{zC}) \quad (4)$$

The reason for this ambiguity is the fact that the wheel has 2 DOF motion which can not be uniquely separated with 1 DOF motions at the contact point. More in detail, the order of rotations about X-axis and Z-axis cannot be determined when the wheel moves and therefore the resultant matrices are different. This relative motion between a wheel and the floor shows why the Sheth-Uicker notation is not applicable to a higher pair which cannot be simulated as a unique combination of lower pairs. However, the coordinate systems C and $\bar{C}$ coincide with each other when they are set. Moreover, they exist only for an instant. Therefore, both of $^{\bar{C}}\theta_{xC}$ and $^{\bar{C}}\theta_{zC}$ in (3) and (4) are zero and the transformation matrix $^{\bar{C}}\Theta_C$ result in an identity matrix from both of (3) and (4). Similarly, even though the derivation of $^{\bar{C}}\dot{\Theta}_C$ is represented as either of (5) or (6)

$$^{\bar{C}}\dot{\Theta}_C = \frac{d}{dt}Rot(Z, {}^{\bar{C}}\theta_{zC})\cdot Rot(X, {}^{\bar{C}}\theta_{xC}) + Rot(Z, {}^{\bar{C}}\theta_{zC})\cdot \frac{d}{dt}Rot(X, {}^{\bar{C}}\theta_{xC}) \quad (5)$$

$$^{\bar{C}}\dot{\Theta}_C = \frac{d}{dt}Rot(X, {}^{\bar{C}}\theta_{xC})\cdot Rot(Z, {}^{\bar{C}}\theta_{zC}) + Rot(X, {}^{\bar{C}}\theta_{xC})\cdot \frac{d}{dt}Rot(Z, {}^{\bar{C}}\theta_{zC}) \quad (6)$$

$Rot(Z, {}^{\bar{C}}\theta_{zC})$ and $Rot(X, {}^{\bar{C}}\theta_{xC})$ in (5) and (6) are also identity matrix, thus both of (5) and (6) turn out to be

$$^{\bar{C}}\dot{\Theta}_C = \frac{d}{dt}Rot\,(X, {}^{\bar{C}}\theta_{xC}) + \frac{d}{dt}Rot\,(Z, {}^{\bar{C}}\theta_{zC})$$

$$= \begin{bmatrix} 0 & -^{\bar{C}}\omega_{zC} & 0 & 0 \\ ^{\bar{C}}\omega_{zC} & 0 & -^{\bar{C}}\omega_{xC} & 0 \\ 0 & ^{\bar{C}}\omega_{xC} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

Therefore, the relation between the coordinate systems at a wheeled pair is uniquely formulated, even though the pair is a higher pair.

2.3 Coordinate systems in a WMR.

Though a WMR has several wheels and steering links, the velocity kinematic modeling is formulated along the serial chain from a robot body to each wheel separately.

So, Fig. 4 shows a schematic diagram of a WMR with a wheel and the coordinate systems assigned in it.

F is a floor coordinate system, B is a body coordinate system, H is a hip coordinate system, S is a steering coordinate system, A is a wheel axis coordinate system which is fixed to a steering link, W is a wheel coordinate system which is fixed to the wheel center, $\bar{B}$ is an instantaneous coincident coordinate system (ICC) of B, C is the contact-point coordinate system on the wheel, $\bar{C}$ is the contact-point coordinate system on the floor. Notice that, when our assignment of coordinate systems is compared to Muir and Neuman's as in Fig. 5, A and W coordinate systems are added.

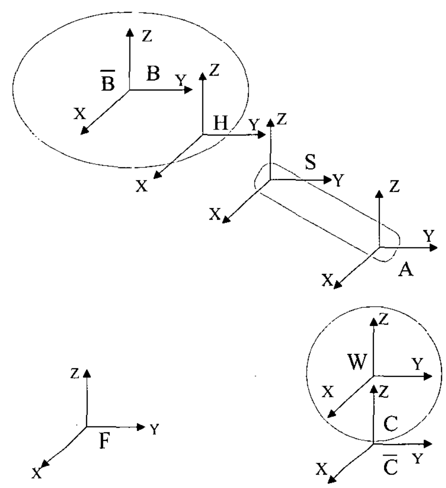

Fig. 4. Coordinate systems in a WMR.

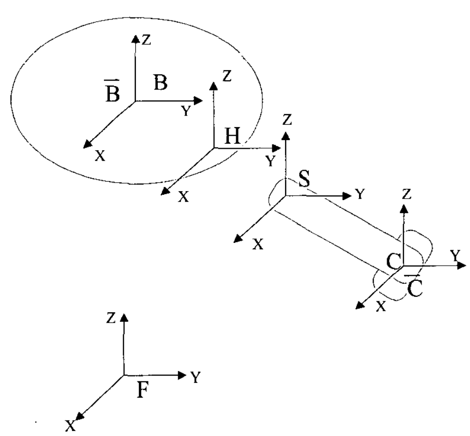

Fig. 5. Muir and Neuman's assignment.

2.4 Transformations

Fig. 6 shows the transformation loop according to coordinate system assignment in Fig. 4.

$$\begin{tikzcd}
\bar{\mathbf{B}} \arrow[r, "\Theta"] & \mathbf{B} \arrow[r, "T"] & \mathbf{H} \arrow[r, "\Theta"] & \mathbf{S} \arrow[d, "T"] \\
 & & & \mathbf{A} \arrow[d, "\Theta"] \\
\mathbf{F} \arrow[uu, "T"] \arrow[r, "T"'] & \bar{\mathbf{C}} \arrow[r, "\Theta"'] & \mathbf{C} & \mathbf{W} \arrow[l, "T"']
\end{tikzcd}$$

Fig. 6. Coordinate transformation loop.

The transformations having only constant terms are represented as T and those containing variable terms are denoted as Θ, which follows the Sheth-Uicker notation.

From Fig. 6, the relative position transformation from $\bar{B}$ to B is expressed as

$$ {}^{\bar B}\Theta_B = {}^F T_{\bar B}^{-1}\,{}^F T_{\bar C}\,{}^{\bar C}\Theta_C\,{}^W T_C^{-1}\,{}^A\Theta_W^{-1}\,{}^S T_A^{-1}\,{}^H\Theta_S^{-1}\,{}^B T_H^{-1} \tag{8}$$

Differentiating (8) with respect to time results in

$$\begin{aligned} {}^{\bar B}\dot\Theta_B = {}&{}^F T_{\bar B}^{-1}\,{}^F T_{\bar C}\,{}^{\bar C}\dot\Theta_C\,{}^W T_C^{-1}\,{}^A\Theta_W^{-1}\,{}^S T_A^{-1}\,{}^H\Theta_S^{-1}\,{}^B T_H^{-1} \\ &+{}^F T_{\bar B}^{-1}\,{}^F T_{\bar C}\,{}^{\bar C}\Theta_C\,{}^W T_C^{-1}\,{}^A\dot\Theta_W^{-1}\,{}^S T_A^{-1}\,{}^H\Theta_S^{-1}\,{}^B T_H^{-1} \\ &+{}^F T_{\bar B}^{-1}\,{}^F T_{\bar C}\,{}^{\bar C}\Theta_C\,{}^W T_C^{-1}\,{}^A\Theta_W^{-1}\,{}^S T_A^{-1}\,{}^H\dot\Theta_S^{-1}\,{}^B T_H^{-1} \end{aligned} \tag{9}$$

Since ${}^F T_{\bar C}$ and ${}^F T_{\bar B}^{-1}$ in (9) are not available, we obtain an alternative expression for ${}^F T_{\bar C}$ and ${}^F T_{\bar B}^{-1}$ with known transformation matrices from (8) as the following.

$$ {}^F T_{\bar B}^{-1}\,{}^F T_{\bar C} = {}^{\bar B}\Theta_B\,{}^B T_H\,{}^H\Theta_S\,{}^S T_A\,{}^A\Theta_W\,{}^W T_C\,{}^{\bar C}\Theta_C^{-1} \tag{10}$$

Substituting (10) into (9), and eliminating the unity transformation matrices ${}^{\bar B}\Theta_B$, ${}^{\bar C}\Theta_C$ and ${}^{\bar C}\Theta_C^{-1}$, (9) becomes (11)

$$\begin{aligned} {}^{\bar B}\dot\Theta_B = {}&{}^B T_H\,{}^H\Theta_S\,{}^S T_A\,{}^A\Theta_W\,{}^W T_C\,{}^{\bar C}\dot\Theta_C\,{}^W T_C^{-1}\,{}^A\Theta_W^{-1}\,{}^S T_A^{-1}\,{}^H\Theta_S^{-1}\,{}^B T_H^{-1} \\ &+{}^B T_H\,{}^H\Theta_S\,{}^S T_A\,{}^A\Theta_W\,{}^A\dot\Theta_W^{-1}\,{}^S T_A^{-1}\,{}^H\Theta_S^{-1}\,{}^B T_H^{-1} \\ &+{}^B T_H\,{}^H\Theta_S\,{}^H\dot\Theta_S^{-1}\,{}^B T_H^{-1} \end{aligned} \tag{11}$$

The velocity matrix of the robot body ${}^{\bar B}\dot\Theta_B$ is computed by defining transformation matrices according to the type of a WMR.

3 Example : Velocity kinematics of a tricycle typed WMR

In this section, we apply the method derived in section 2 to the forward kinematic modeling of a tricycle typed WMR which consists of a conventional steered wheel and two non-steered wheels. The velocity matrices for each wheel are obtained respectively and then a forward kinematic solution is computed.

3.1 Conventional steered wheel

We suppose that a conventional steered wheel is connected to a robot body link by a steering link as in Fig. 7. It is therefore noted that the schematic diagram of Fig. 4 and transformation graph of Fig. 6 can be directly used for the mechanism of Fig. 7.

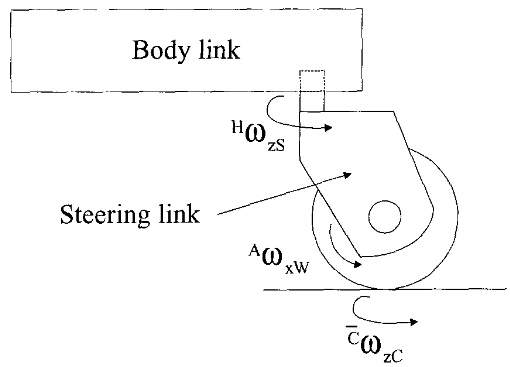

Fig. 7. Conventional steered wheel.

The constant transformation matrices in (11) are defined to be

$$ {}^B T_H = \begin{bmatrix} \cos{}^B\theta_{zH} & -\sin{}^B\theta_{zH} & 0 & {}^B Dx_H \\ \sin{}^B\theta_{zH} & \cos{}^B\theta_{zH} & 0 & {}^B Dy_H \\ 0 & 0 & 1 & {}^B Dz_H \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$ {}^S T_A = \begin{bmatrix} \cos{}^S\theta_{zA} & -\sin{}^S\theta_{zA} & 0 & {}^S Dx_A \\ \sin{}^S\theta_{zA} & \cos{}^S\theta_{zA} & 0 & {}^S Dy_A \\ 0 & 0 & 1 & {}^S Dz_A \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$ {}^W T_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos{}^W\theta_{xC} & -\sin{}^W\theta_{xC} & 0 \\ 0 & \sin{}^W\theta_{xC} & \cos{}^W\theta_{xC} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -R \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

where R is the radius of the wheel. The variable transformation matrices in (11) are defined to be

$$ {}^H\Theta_S = \begin{bmatrix} \cos{}^H\theta_{zS} & -\sin{}^H\theta_{zS} & 0 & 0 \\ \sin{}^H\theta_{zS} & \cos{}^H\theta_{zS} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$ {}^A\Theta_W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos{}^A\theta_{xW} & -\sin{}^A\theta_{xW} & 0 \\ 0 & \sin{}^A\theta_{xW} & \cos{}^A\theta_{xW} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

$$
{}^{\bar{B}}\Theta_B = \begin{bmatrix} \cos^B\theta_{zB} & -\sin^B\theta_{zB} & 0 & {}^BDx_B \\ \sin^B\theta_{zB} & \cos^B\theta_{zB} & 0 & {}^BDy_B \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}
$$

$$
{}^{\bar{C}}\Theta_C = \begin{bmatrix} \cos^{\bar{C}}\theta_{zC} & -\sin^{\bar{C}}\theta_{zC} & 0 & 0 \\ \sin^{\bar{C}}\theta_{zC} & \cos^{\bar{C}}\theta_{zC} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^{\bar{C}}\theta_{xC} & -\sin^{\bar{C}}\theta_{xC} & 0 \\ 0 & \sin^{\bar{C}}\theta_{xC} & \cos^{\bar{C}}\theta_{xC} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}
$$

Differentiating variable transformation matrices with respect to time, and simplifying with ${}^{\bar{B}}\theta_B = {}^{\bar{C}}\theta_{zC} = {}^{\bar{C}}\theta_{xC} = 0$, we obtain

$$
{}^{\bar{B}}\dot{\Theta}_B = \begin{bmatrix} 0 & -{}^{\bar{B}}\omega_{zB} & 0 & {}^{\bar{B}}Vx_B \\ {}^{\bar{B}}\omega_{zB} & 0 & 0 & {}^{\bar{B}}Vy_B \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{19}
$$

$$
{}^{\bar{C}}\dot{\Theta}_C = \begin{bmatrix} 0 & -{}^{\bar{C}}\omega_{zC} & 0 & 0 \\ {}^{\bar{C}}\omega_{zC} & 0 & -{}^{\bar{C}}\omega_{xC} & 0 \\ 0 & {}^{\bar{C}}\omega_{xC} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}
$$

$$
{}^{H}\dot{\Theta}_S = \begin{bmatrix} -{}^H\omega_{zS}\sin{}^H\theta_{zS} & -{}^H\omega_{zS}\cos{}^H\theta_{zS} & 0 & 0 \\ {}^H\omega_{zS}\cos{}^H\theta_{zS} & -{}^H\omega_{zS}\sin{}^H\theta_{zS} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}
$$

$$
{}^{H}\dot{\Theta}_S^{-1} = \begin{bmatrix} -{}^H\omega_{zS}\sin{}^H\theta_{zS} & {}^H\omega_{zS}\cos{}^H\theta_{zS} & 0 & 0 \\ -{}^H\omega_{zS}\cos{}^H\theta_{zS} & -{}^H\omega_{zS}\sin{}^H\theta_{zS} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{22}
$$

$$
{}^{A}\dot{\Theta}_W = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -{}^A\omega_{xW}\sin{}^A\theta_{xW} & -{}^A\omega_{xW}\cos{}^A\theta_{xW} & 0 \\ 0 & {}^A\omega_{xW}\cos{}^A\theta_{xW} & -{}^A\omega_{xW}\sin{}^A\theta_{xW} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}
$$

$$
{}^{A}\dot{\Theta}_W^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -{}^A\omega_{xW}\sin{}^A\theta_{xW} & {}^A\omega_{xW}\cos{}^A\theta_{xW} & 0 \\ 0 & -{}^A\omega_{xW}\cos{}^A\theta_{xW} & -{}^A\omega_{xW}\sin{}^A\theta_{xW} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{24}
$$

Substituting defined transformation matrices into (11) and simplify with ${}^A\theta_{xW} = -{}^W\theta_{xC}$. The first term of the right hand side in (11) is computed as

$$
\begin{bmatrix} 0 & -{}^{\bar{C}}\omega_{zC} & {}^{\bar{C}}\omega_{xC}\sin^B\theta_{zA} & (a) \\ {}^{\bar{C}}\omega_{zC} & 0 & -{}^{\bar{C}}\omega_{xC}\cos^B\theta_{zA} & (b) \\ -{}^{\bar{C}}\omega_{xC}\sin^B\theta_{zA} & {}^{\bar{C}}\omega_{xC}\cos^B\theta_{zA} & 0 & (c) \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{25}
$$

where

$$
\begin{aligned}
(a) &= {}^{\bar{C}}\omega_{zC}({}^BDy_H + {}^SDx_A\sin{}^B\theta_{zS} + {}^SDy_A\cos{}^B\theta_{zS}) \\
&\quad + {}^{\bar{C}}\omega_{xC}(-{}^SDz_A\sin{}^B\theta_{zA} + R\sin{}^B\theta_{zA} - {}^BDz_H\sin{}^B\theta_{zA}) \\
(b) &= -{}^{\bar{C}}\omega_{zC}({}^BDx_H + {}^SDx_A\cos{}^B\theta_{zS} - {}^SDy_A\sin{}^B\theta_{zS}) \\
&\quad + {}^{\bar{C}}\omega_{xC}({}^SDz_A\cos{}^B\theta_{zA} - R\cos{}^B\theta_{zA} + {}^BDz_H\cos{}^B\theta_{zA}) \\
(c) &= {}^{\bar{C}}\omega_{xC}({}^BDx_H\sin{}^B\theta_{zA} - {}^BDy_H\cos{}^B\theta_{zA} + {}^SDx_A\sin{}^S\theta_{zA} \\
&\quad - {}^SDy_A\cos{}^S\theta_{zA})
\end{aligned}
$$

The second term of the right hand side in (11) is computed as

$$
\begin{bmatrix} 0 & 0 & -{}^A\omega_{xW}\sin^B\theta_{zA} & (d) \\ 0 & 0 & {}^A\omega_{xW}\cos^B\theta_{zA} & (e) \\ {}^A\omega_{xW}\sin^B\theta_{zA} & -{}^A\omega_{xW}\cos^B\theta_{zA} & 0 & (f) \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}
$$

where

$$
\begin{aligned}
(d) &= {}^A\omega_{xW}({}^BDz_H\sin{}^B\theta_{zA} + {}^SDz_A\sin{}^B\theta_{zA}) \\
(e) &= {}^A\omega_{xW}(-{}^BDz_H\cos{}^B\theta_{zA} - {}^SDz_A\cos{}^B\theta_{zA}) \\
(f) &= {}^A\omega_{xW}(-{}^BDx_H\sin{}^B\theta_{zA} + {}^BDy_H\cos{}^B\theta_{zA} - {}^SDx_A\sin{}^S\theta_{zA} \\
&\quad + {}^SDy_A\cos{}^S\theta_{zA})
\end{aligned}
$$

The third term of the right hand side in (11) is computed as

$$
\begin{bmatrix} 0 & {}^H\omega_{zS} & 0 & -{}^H\omega_{zS}{}^BDy_H \\ -{}^H\omega_{zS} & 0 & 0 & {}^H\omega_{zS}{}^BDx_H \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{27}
$$

simplifying with (25), (26), (27), and ${}^{\bar{C}}\omega_{xC} = {}^A\omega_{xW}$, (11) is expressed as

$$
{}^{\bar{B}}\dot{\Theta}_B = \begin{bmatrix} 0 & {}^H\omega_{zS} - {}^{\bar{C}}\omega_{zC} & 0 & A \\ -{}^H\omega_{zS} + {}^{\bar{C}}\omega_{zC} & 0 & 0 & B \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{28}
$$

where

$$
\begin{aligned}
A &= {}^{\bar{C}}\omega_{zC}({}^BDy_H + {}^SDx_A\sin{}^B\theta_{zS} + {}^SDy_A\cos{}^B\theta_{zS}) \\
&\quad + {}^A\omega_{xW}R\sin{}^B\theta_{zA} - {}^H\omega_{zS}{}^BDy_H \\
B &= -{}^{\bar{C}}\omega_{zC}({}^BDx_H + {}^SDx_A\cos{}^B\theta_{zS} - {}^SDy_A\sin{}^B\theta_{zS}) \\
&\quad - {}^A\omega_{xW}R\cos{}^B\theta_{zA} + {}^H\omega_{zS}{}^BDx_H
\end{aligned}
$$

By introducing ${}^BDy_A$ and ${}^BDx_A$ as in (29) and (30),

$$
{}^BDy_H + {}^SDx_A\sin{}^B\theta_{zS} + {}^SDy_A\cos{}^B\theta_{zS} = {}^BDy_A \tag{29}
$$

$$ {}^{B}Dx_{H} + {}^{S}Dx_{A}\cos{}^{B}\theta_{zS} - {}^{S}Dy_{A}\sin{}^{B}\theta_{zS} = {}^{B}Dx_{A} \tag{30} $$

A and B in (28) can be simplified and ${}^{\bar{B}}\dot{\Theta}_{B}$ is written as (31) from (19) and (28),

$$ \begin{bmatrix} 0 & -{}^{B}\omega_{zB} & 0 & {}^{B}Vx_{B} \\ {}^{\bar{B}}\omega_{zB} & 0 & 0 & {}^{B}Vy_{B} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & {}^{H}\omega_{zS} - {}^{\bar{C}}\omega_{zC} & 0 & A \\ -{}^{H}\omega_{zS} + {}^{\bar{C}}\omega_{zC} & 0 & 0 & B \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{31} $$

From (31), the velocity kinematics for the conventional steered wheel is expressed as

$$ \begin{bmatrix} {}^{\bar{B}}Vx_{B} \\ {}^{\bar{B}}Vy_{B} \\ {}^{\bar{B}}\omega_{zB} \end{bmatrix} = \begin{bmatrix} R\sin{}^{B}\theta_{zA} & {}^{B}Dy_{A} & -{}^{B}Dy_{H} \\ -R\cos{}^{B}\theta_{zA} & -{}^{B}Dx_{A} & {}^{B}Dx_{H} \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} {}^{A}\omega_{xW} \\ {}^{\bar{C}}\omega_{zC} \\ {}^{H}\omega_{zS} \end{bmatrix} \tag{32} $$

3.2 Conventional non-steered wheel

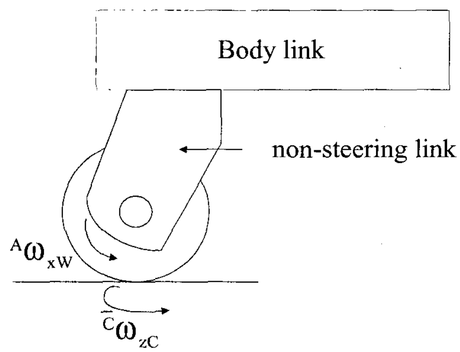

Fig . 8. Conventional non-steered wheel.

Since a conventional non-steered wheel makes the steering transformation matrix ${}^{H}\Theta_{S}$ to be constant, velocity matrix (11) becomes simpler as the following.

$$ {}^{B}\dot{\Theta}_{B} = {}^{B}T_{H}\,{}^{H}\Theta_{S}\,{}^{S}T_{A}\,{}^{A}\Theta_{W}\,{}^{W}T_{C}\,{}^{\bar{C}}\dot{\Theta}_{C}\,{}^{W}T_{C}^{-1}\,{}^{A}\Theta_{W}^{-1}\,{}^{S}T_{A}^{-1}\,{}^{H}\Theta_{S}^{-1}\,{}^{B}T_{H}^{-1} + {}^{B}T_{H}\,{}^{H}\Theta_{S}\,{}^{S}T_{A}\,{}^{A}\Theta_{W}\,{}^{A}\dot{\Theta}_{W}^{-1}\,{}^{S}T_{A}^{-1}\,{}^{H}\Theta_{S}^{-1}\,{}^{B}T_{H}^{-1} \tag{33} $$

For simplicity, if we set ${}^{H}\Theta_{S}$ is an identity matrix, then ${}^{B}T_{H}\,{}^{H}\Theta_{S}\,{}^{S}T_{A} = {}^{B}T_{A}$, and ${}^{\bar{C}}\omega_{xC} = {}^{A}\omega_{xW}$. So, (33) becomes (34)

$$ \begin{aligned} {}^{\bar{B}}\dot{\Theta}_{B} &= {}^{B}T_{A}\,{}^{A}\Theta_{W}\,{}^{W}T_{C}\,{}^{\bar{C}}\dot{\Theta}_{C}\,{}^{W}T_{C}^{-1}\,{}^{A}\Theta_{W}^{-1}\,{}^{B}T_{A}^{-1} + {}^{B}T_{A}\,{}^{A}\Theta_{W}\,{}^{A}\dot{\Theta}_{W}^{-1}\,{}^{B}T_{A}^{-1} \\ &= \begin{bmatrix} 0 & -{}^{\bar{C}}\omega_{C} & 0 & R\,{}^{A}\omega_{xW}\sin{}^{B}\theta_{zA} + {}^{B}Dy_{A}\,{}^{\bar{C}}\omega_{zC} \\ {}^{\bar{C}}\omega_{C} & 0 & 0 & -R\,{}^{A}\omega_{xW}\cos{}^{B}\theta_{zA} - {}^{B}Dx_{A}\,{}^{\bar{C}}\omega_{zC} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{34} $$

From (19) and (34), the velocity kinematics for the conventional non-steeered wheel is expressed as

$$ \begin{bmatrix} {}^{\bar{B}}Vx_{B} \\ {}^{\bar{B}}Vy_{B} \\ {}^{\bar{B}}\omega_{zB} \end{bmatrix} = \begin{bmatrix} R\sin{}^{B}\theta_{zA} & {}^{B}Dy_{A} \\ -R\cos{}^{B}\theta_{zA} & -{}^{B}Dx_{A} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{A}\omega_{xW} \\ {}^{\bar{C}}\omega_{zC} \end{bmatrix} \tag{35} $$

3.3 Forward Kinematics

Using the velocity kinematics for a conventional steered wheel and two conventional non-steered wheels, we compute forward kinematics solution about a tricycle type WMR as in Fig. 9 and Fig. 10. The conventional steered wheel noted with subscript 3 in Fig. 9 is a castor wheel which is neither actuated nor sensed. Two conventional non-steered wheels are supposed to be actuated and sensed. The radius of a conventional steered wheel is R_1 and that of two non-steered wheels is R respectively. Since the parameters in (32) are computed as

${}^{B}Dy_{A3} = -l_b - l_c\cos{}^{H}\theta_{zS}$, ${}^{B}Dy_{H} = -l_b$, ${}^{B}Dx_{H} = 0$

${}^{B}Dx_{A3} = l_c\sin{}^{H}\theta_{zS}$, ${}^{B}\theta_{zA3} = {}^{H}\theta_{zS}$

the velocity kinematics of the conventional steered wheel is obtained as

$$ \begin{bmatrix} {}^{\bar{B}}Vx_{B} \\ {}^{\bar{B}}Vy_{B} \\ {}^{\bar{B}}\omega_{zB} \end{bmatrix} = \begin{bmatrix} R_1\sin{}^{H}\theta_{zS} & -l_b - l_c\cos{}^{H}\theta_{zS} & l_b \\ -R_1\cos{}^{H}\theta_{zS} & -l_c\sin{}^{H}\theta_{zS} & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} {}^{A3}\omega_{xW3} \\ {}^{\bar{C}3}\omega_{zC3} \\ {}^{H}\omega_{zS} \end{bmatrix} \tag{36} $$

For conventional non-steered wheels, the parameters in (35) are ${}^{B}Dy_{A1} = {}^{B}Dy_{A2} = 0$, ${}^{B}Dx_{A1} = l_a$, ${}^{B}Dx_{A2} = -l_a$ ${}^{B}\theta_{zA1} = {}^{B}\theta_{zA2} = 0$ and therefore the velocity kinematics (35) of the lower wheel(noted with subscript 1 in Fig. 9) is

$$ \begin{bmatrix} {}^{\bar{B}}Vx_{B} \\ {}^{\bar{B}}Vy_{B} \\ {}^{\bar{B}}\omega_{zB} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -R & l_a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{A1}\omega_{xW1} \\ {}^{\bar{C}1}\omega_{zC1} \end{bmatrix} \tag{37} $$

and that of the upper wheel(subscript 2 in Fig.9) is

$$ \begin{bmatrix} {}^{\bar{B}}Vx_{B} \\ {}^{\bar{B}}Vy_{B} \\ {}^{\bar{B}}\omega_{zB} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -R & -l_a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{A2}\omega_{xW2} \\ {}^{\bar{C}2}\omega_{zC2} \end{bmatrix} \tag{38} $$

Since the actuated and sensed variables are supposed with ${}^{A1}\omega_{xW1}$ and ${}^{A2}\omega_{xW2}$, the forward velocity kinematics solution for the robot is computed as the following.

$$ \begin{bmatrix} {}^{B}Vx_{B} \\ {}^{B}Vy_{B} \\ {}^{B}\omega_{zB} \end{bmatrix} = \frac{R}{2l_a} \begin{bmatrix} 0 & 0 \\ -l_a & -l_a \\ 1 & -1 \end{bmatrix} \begin{bmatrix} {}^{A1}\omega_{xW1} \\ {}^{A2}\omega_{xW2} \end{bmatrix} \tag{39} $$

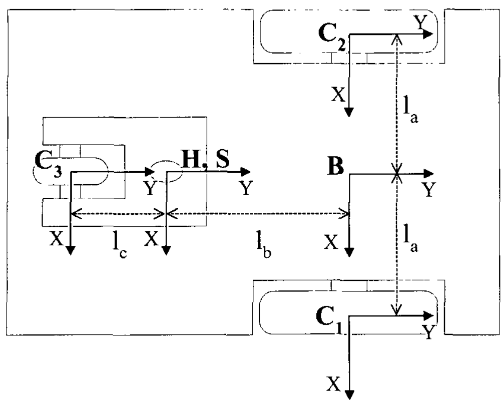

Fig. 9 Top view of the tricycle WMR.

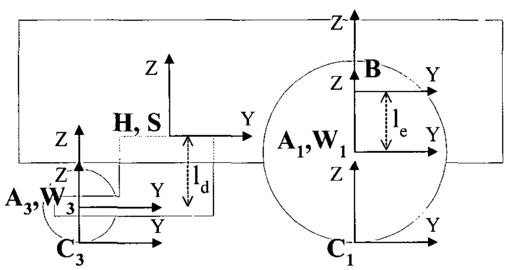

Fig. 10 Side view of the tricycle WMR.

4 Conclusion

In this paper, a revision of Muir and Neuman's method was proposed, which applies the matrix coordinate transformation to the velocity kinematics modeling for WMRs. While the previous method replaces a wheeled pair of WMRs with a planar pair and obtains its kinematics in an ad-hoc manner, the proposed method models the kinematics of a wheeled pair itself. We assigned the coordinate systems at a wheeled pair by Sheth-Uicker notation and found that the relation between the coordinate systems can be formulated uniquely, even though there exists 2 DOF motion at the pair which is referred to as a higher pair. Consequently, we analyzed the kinematics of WMRs with homogeneous transformation matrices between all the adjacent pair joints of WMRs. The proposed is therefore more systematic and rational than the previous, whereas the proposed requires rather complex computation. The proposed method is implemented to the forward velocity kinematics of a tricycle typed WMR and the successful results showed its usefulness.

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