Adaptive learning controller for autonomous mobile robots

T.-Y.Kuc, S.-M.Baek and K.Park

Abstract: An adaptive learning controller is proposed for tracking of nonholonomic mobile robots. It consists of an exponential learning control scheme for velocity dynamics and a reference velocity control scheme for the kinematic steering system. In the adaptive learning controller, the velocity dynamics learning control tracks the reference velocity by learning the inverse function of robot dynamics, while the reference velocity controller stabilises the kinematic steering system to the desired reference model even without assuming an ideal velocity servo. It is shown that all the error signals in the learning control system are bounded and the robot motion trajectory converges to the desired trajectory uniformly and asymptotically. The proposed learning controller is applied to a wheeled mobile robot to demonstrate its feasibility and effectiveness.

1 Introduction

In the past two decades, there has been much research on the application of learning control theory to the motion control of robot manipulators. As a result, many learning control schemes were developed to improve the performance of uncertain robot manipulators [1–9]. Although much has been written about the learning control of robot manipulators, there is little about the problem of learning control of nonholonomic mobile robots. Moreover, literature on the integration of nonholonomic kinematic steering and dynamics learning of mobile robot in the presence of uncertainty in system parameters and disturbances is rare. When a learning control theory is applied to mobile robot navigation, some nonlinear feedback control has to be devised together with a learning scheme. The recent work of Oriolo *et al.* [10] also utilises a nonlinear feedback based transformation to convert the mobile robot dynamic system to a one-chain system and construct a learning algorithm for learning of linearised system. The basic idea of this nonlinear feedback control is to generate or learn velocity control inputs which stabilize the closed-loop system for satisfaction of *perfect velocity tracking* [11]. In practice, however, the assumption of perfect velocity tracking does not hold because of the imperfect model knowledge of robot dynamics for nonlinear state transformation and/or lack of ideal velocity servos, weakening the feasibility and robustness of the control method.

The adaptive learning control scheme proposed in this paper counteracts these difficulties by using a velocity dynamics learning controller which achieves asymptotic perfect velocity tracking under the nonideal operating condition with uncertain system parameters, ground frictions, external disturbances, etc. By combining the velocity learning control loop with the outer-loop of kinematic steering system, the uniform asymptotic stabilisation of position trajectory is also achieved in the overall learning control system. Hence, construction of the proposed learning control system consists of two separate steps. First, the inner-loop of velocity learning control is derived for tracking reference velocity from the assumed kinematic steering system. Then, the outer-loop of kinematic steering system is designed for generating the reference velocity trajectory. The two independent control loops are integrated in the closed-loop system and their stability remains unchanged, in despite of the violation of the assumption of perfect velocity tracking. This provides a practical learning control method which emulates an ideal velocity servo by taking into account the explicit dynamics of mobile robot and converting it to a simplified kinematic steering system.

2 Problem formulation

Consider a two-D.O.F. wheeled mobile robot with an auxiliary free wheeling caster given in Fig. 1 and Fig. 2. In the figures, Q and Q_1 represent the centre of the shaft connecting two driving wheels and the centre of gravity of mobile robot, respectively. The kinematics and dynamics of mobile robot are described as follows:

$$\dot{p}_1 = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\theta} \end{bmatrix} = J(\theta)v = \begin{bmatrix} \cos\theta & -h_q \sin\theta \\ \sin\theta & h_q \cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (1)$$

$$D(p)\dot{v} + C(p,\dot{p})v + f(\dot{p}) + d_r + d_n = \tau \quad (2)$$

$$\frac{1}{R}\begin{bmatrix} 1 & 1 \\ L & -L \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \tau \quad (3)$$

where $p = [x\ y\ \theta]^T$, v_1 and v_2 are linear and angular velocities of mobile robot, $D(p)$ is symmetric and positive definite inertia matrix $[D(p) \in R^{2\times 2}]$, $C(p, \dot{p})$ is centripetal and Coriolis force matrix $[C(p, \dot{p}) \in R^{2\times 2}]$, $f(\dot{p})$ is friction

IEE Proceedings online no. 20010150

DOI: 10.1049/ip-cta:20010150

Paper first received 28th February and in revised form 8th August 2000

The authors are with Sung Kyun Kwan University, School of Electrical and Computer Engineering, Chunchun-dong 300, Changan-gu, Suwon, Kyungki-do 440-746, Korea
E-mail: tykuc@yurim.skku.ac.kr

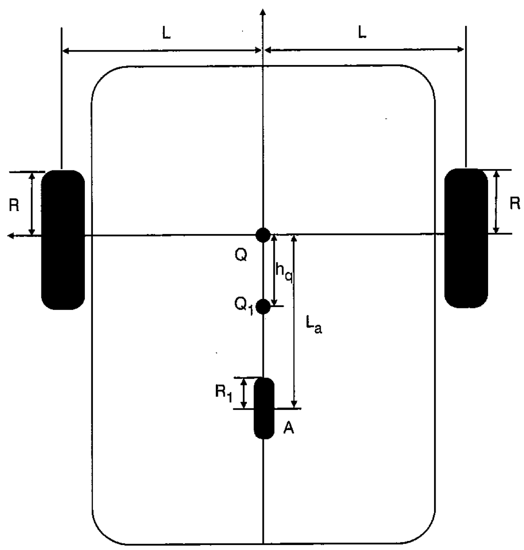

Fig. 1 *Planar mobile robot model*

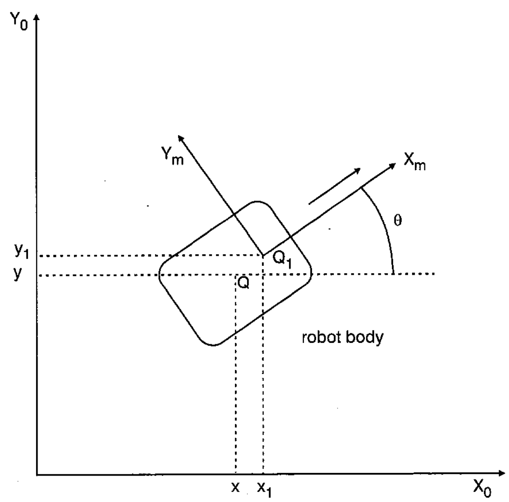

Fig. 2 *Coordinates of mobile robot*

force vector $[f(\dot{p}) \in R^{2\times 1}]$, d_r and d_n are bounded disturbance vectors $[d_r,\ d_n \in R^{2\times 1}]$ and τ is force and torque input vector $[\tau \in R^{2\times 1}]$. u_i's $(i=1,\ 2)$ denote the driving torques applied to wheels of mobile robot, where R and L are the radius of each driving wheel and the distance to the center point Q from each driving wheel in Fig. 1.

In formulating the learning control problem of mobile robot system, the following kinematic and dynamic properties are assumed:

(A1) The mobile robot satisfies the pure rolling and nonslipping condition that $N(\theta)\dot{p}_1 = 0$, where $N(\theta) = [-\sin\theta \quad \cos\theta \quad -h_q]$ which spans the null space of Jacobian $J^T(\theta)$.

(A2) The inertia matrix is bounded such that $0 < \lambda_1 I \leq D \leq \lambda_2 I$, where λ_1 and λ_2 are positive constants for unit matrix I and $\dot{D} - 2C$ is skew-symmetric.

(A3) $D\dot{v} + Cv + f = \Phi(v,\ \dot{v})\rho$, where $\Phi \in R^{2\times m}$ and $\rho \in R^{m\times 1}$ are the known regressor matrix and unknown parameter vector, respectively.

(A4) The disturbance d_r is T-periodic for some finite T and $d_r \in L_{2e}[0,\ T] \cap L_{\infty e}[0,\ T]$. d_n satisfies $d_n \in L_2 \cap L_\infty$ and $|d_n|_\infty \leq d_m$ for unknown d_m.

In (A4), L_p and L_{pe} denote the sets of piecewise continuous vector functions which are bounded in the L_p $(|(\cdot)|_p)$ and extended L_{pe} $(|(\cdot)|_{pe})$ norms, respectively.

Remark: (A3) implies that the friction $f(\cdot)$ includes only the terms with linear parameters. For example, the maximum static friction and viscous friction can be members of friction model $f(\cdot)$. Other nonlinear friction as exponential Stribeck effect can be absorbed in the disturbance term d_n once it is bounded.

Let a desired reference kinematic model of mobile robot

$$\dot{p}_d = \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \\ \dot{\theta}_d \end{bmatrix} = \begin{bmatrix} v_{1d}\cos\theta_d \\ v_{1d}\sin\theta_d \\ v_{2d} \end{bmatrix}, \tag{4}$$

where the subscript d means desired. Define the position tracking error as $\tilde{p} = p_d - p$ for the desired and actual trajectories, p_d and p. Then, using the rotational transformation, the position error can be represented with respect to the mobile robot frame q such that eqn. 5

$$\tilde{q} = R^T(\theta)\tilde{p}, \tag{5}$$

where $\tilde{q} = q_d - q$ and $R(\theta)$ is the rotational matrix. From eqns. 4 and 5, the time derivative of position tracking errors can be written as eqn. 6

$$\dot{\tilde{q}} = A(\tilde{q}_3)v_d + B(\tilde{q})v, \tag{6}$$

where

$$A(\tilde{q}_3) = \begin{bmatrix} \cos\tilde{q}_3 & 0 \\ \sin\tilde{q}_3 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B(\tilde{q}) = \begin{bmatrix} -1 & \tilde{q}_2 \\ 0 & -\tilde{q}_1 \\ 0 & -1 \end{bmatrix}, \text{ and } \tilde{q} = \begin{bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \\ \tilde{q}_3 \end{bmatrix}.$$

Define a smooth reference velocity input and its error as $v_r = [v_{1r} \ \ v_{2r}]^T$ and $\tilde{v} = v_r - v$, respectively. Let $s_r = v_r + a\int_0^t \tilde{v}\, d\eta$ and $s = s_r - v$, where a is positive constant. Further, let $\Phi_r\rho = D(p)\dot{s}_r + C(p,\ \dot{p})s_r + f(\dot{p})$ and $\Phi_d\rho = D(p_d)\dot{v}_d + C(p_d,\dot{p}_d)v_d + f(\dot{p}_d)$, where Φ and ρ are defined in (A3). Then, the dynamic eqn. 2 can be arranged as

$$D(p)\dot{s} + C(p,\dot{p})s = \Phi_e\rho + d_n + l_d - \tau, \tag{7}$$

where $\Phi_e = \Phi_r - \Phi_d$ and $l_d = \Phi_d\rho + d_r$.

With the error eqns. 6 and 7, our control objective is to derive a learning control system which achieves stable kinematic steering and perfect velocity tracking simultaneously. A learning control approach to this control objective is proposed as follows:

(S1) Velocity Learning Control: First, assuming a smooth reference trajectory v_r given for the kinematic steering system, we derive an adaptive learning controller which stabilises the velocity dynamics of error system in eqn. 7 as fast as possible for satisfaction of perfect velocity tracking.

(S2) Reference Velocity Design: Then, we find a smooth reference velocity trajectory $v_r = z(\tilde{q},\ v_d)$ which steers the kinematic error system in eqn. 6 to zero under the condition $\tilde{v} = 0$ and remains uniformly asmptotically stable when combined with the designed velocity dynamics learning controller of (S1).

3 Fast velocity learning control

Suppose that the desired velocity trajectories of the kinematic model in eqn. 4 is periodic such that $x_d(t)=x_d(t+T)$ for some finite T, where x_d represents p_d, $\dot{p}_d$, or v_d. Then, the velocity learning controller for the uncertain error system in eqn. 7 is constructed as follows.

$$\tau = \Phi_e\hat{\rho} + \Lambda s + \gamma_0 + \gamma_1 + l, \tag{8}$$

where the learning controller accommodates the estimated parameter vector $\hat{\rho}$, feedback input Λs, robust inputs γ_0 and γ_1, and learning input l. The robust inputs γ_0 and γ_1 and feedback gain Λ are defined as $\gamma_0=(1-e^{-\sigma t})\hat{l}_0 sgn(s)$, $\gamma_1=\hat{d}_m sgn(s)$, and $\Lambda=\Lambda_0+(\beta_1/2)\Phi_e\Phi_e^T+[(\beta_2+\beta_3)/2]I$, where $\sigma\geq 0$, β_i's $(i=1, 2, 3)$ are positive constants, and Λ_0 is positive symmetric feedback gain matrix. $\hat{d}_m$ and $\hat{l}_0$ are estimates of d_m and l_0 satisfying $d_m\geq |d_n|_\infty$ and $\hat{l}_0>l_0\geq|\Phi_d\rho|_\infty+|d_r|_{\infty e}+d_m$, respectively.

Substituting the learning controller input eqn. 8 into eqn. 7 and multiplying both sides of the equation by $e^{\sigma t}$ yields after some calculation

$$D\dot{w} + Cw + \tilde{\Lambda}w = e^{\sigma t}(\Phi_e\tilde{\rho} + \delta d_n + \tilde{l} - \gamma_0), \tag{9}$$

where $w=e^{\sigma t}s$, $\tilde{\Lambda}=\Lambda-\sigma D$, $\tilde{\rho}=\rho-\hat{\rho}$, $\delta d_n=d_n-\hat{d}_m sgn(s)$, and $\tilde{l}=l_d-l$. l_d satisfies $l_d(t)=l_d(t+T)$ and called the desired input required for inverse dynamics control of mobile robot.

The following exponential learning rules are used for fast convergence of the passive error system in eqn. 9.

$$\begin{aligned}
&\hat{\rho}(t) = G_\sigma[\hat{\rho}(t-T)] + \beta_1\Phi_e^T s(t) \text{ and}\\
&\hat{\rho}(t) = \int_0^t \beta_1\Phi_r^T s(\eta)d\eta \quad \text{for } t\in[0, T].\\
&\hat{d}_m(t) = G_\sigma[\hat{d}_m(t-T)] + \beta_2|s(t)| \text{ and}\\
&\hat{d}_m(t) = \int_0^t \beta_2|s(\eta)|d\eta \quad \text{for } t\in[0, T].\\
&l(t) = G_\sigma[l(t-T)] + \beta_3 s(t) \text{ and}\\
&l(t) = \Phi_d\hat{\rho}(t) \quad \text{for } t\in[0, T],
\end{aligned} \tag{10}$$

where β_i's $(i=1, 2, 3)$ are positive learning gains and the initial conditions are to assure the boundedness of learning system at the first period of time interval $[0, T]$. $G_\sigma[\cdot]$ is a conditional operator depending on the exponential rate constant σ such that

$$G_\sigma[x] = \begin{cases} \Pr[x], & \text{if } \sigma=0\\ \alpha x, & \text{if } \sigma>0, \end{cases} \tag{11}$$

where $\alpha=e^{-\sigma T}$ forgetting factor and $\Pr[\cdot]$ is projection operator bounding its argument within a compact interval which includes the true or desired value of its argument.

In practice, when the current error $s(t)$ is not available at t, the following learning rules are used instead which replace the current error $s(t)$ with the previous one $s(t-T)$ in eqn. 10.

$$\begin{aligned}
&\hat{\rho}(t) = G_\sigma[\hat{\rho}(t-T)] + \beta_1\Phi_e^T s(t-T) \text{ and}\\
&\hat{\rho}(t) = \int_0^t \beta_1\Phi_r^T s(\eta)d\eta \quad \text{for } t\in[0, T].\\
&\hat{d}_m(t) = G_\sigma[\hat{d}_m(t-T)] + \beta_2|s(t-T)| \text{ and}\\
&\hat{d}_m(t) = \int_0^t \beta_2|s(\eta)|d\eta \quad \text{for } t\in[0, T].\\
&l(t) = G_\sigma[l(t-T)] + \beta_3 s(t-T) \text{ and}\\
&l(t) = \Phi_d\hat{\rho}(t) \quad \text{for } t\in[0, T].
\end{aligned} \tag{12}$$

With the new learning rules in eqn. 12, the learning controller input in eqn. 8 is modified to include the conditional projection operator $G_\sigma[\cdot]$ for learned values of $\hat{\rho}$, $\hat{d}_m$, and l.

Remark: Note that the robust input γ_0 in eqn. 8 reduces to zero when $\sigma=0$.

4 Reference velocity generation

Assume an ideal velocity servo, i.e., $v=v_r$, and define the reference velocity v_r as

$$v_r = v_m + v_a, \tag{13}$$

where v_m is obtained from an objective functional and v_a is an auxiliary velocity input. v_m is LS solution of index $L=\min_{v_m}|\dot{\tilde{q}}-A_0\tilde{q}|_2^2$ subject to $\dot{\tilde{q}}=A(\tilde{q}_3)v_d+B(\tilde{q})v_m$, where $|(\cdot)|_2$ denotes l_2 or Euclidian norm of vector $(\cdot)$ and $A_0=\mathrm{diag}(a_{ii})_{3\times 3}$ for negative a_{ii}'s $(i=1, 2, 3)$. The necessary condition $\partial L/\partial v_m=0$ gives

$$\begin{aligned}
v_m &= B^{\#}(A_0\tilde{q} - A(\tilde{q})v_d)\\
&= \begin{bmatrix} -a_{11}\tilde{q}_1 - (a_{22}b_1\tilde{q}_2 + a_{33}b_0\tilde{q}_3)\tilde{q}_2 \\ \quad +v_{1d}(\cos\tilde{q}_3 + b_1b_4\tilde{q}_2\tilde{q}_3) + v_{2d}b_0\tilde{q}_2 \\ -(a_{22}b_1\tilde{q}_2 + a_{33}b_0\tilde{q}_3) + v_{1d}b_1b_4\tilde{q}_3 + b_0v_{2d} \end{bmatrix},
\end{aligned} \tag{14}$$

where $B^{\#}=(B^TB)^{-1}B^T$, $b_0=1/(1+\tilde{q}_1^2)$, $b_1=\tilde{q}_1/(1+\tilde{q}_1^2)$, and $b_4=(\sin\tilde{q}_3)/\tilde{q}_3$.

Choose the auxiliary velocity v_a as

$$v_a = \begin{bmatrix} -v_{2d}b_0\tilde{q}_2 + (-v_{1d}b_3 + v_{2d}b_1)\tilde{q}_3 \\ (a_{22}+a_{33})b_1\tilde{q}_2 + v_{1d}b_0b_4\tilde{q}_2 \end{bmatrix},$$

where

$$b_3 = \frac{\sin\tilde{q}_3}{1+\tilde{q}_1^2}.$$

Applying the reference velocity input eqn. 13 to the error system eqn. 6 yields

$$\dot{\tilde{q}} = A_r q + g(\tilde{q}), \tag{15}$$

where

$$A_r = \begin{bmatrix} a_{11} & v_{2d}b_0 & v_{1d}b_3 - v_{2d}b_1 \\ -v_{2d}b_0 & a_{22}b_2 & a_{33}b_1 + v_{1d}b_0b_4 \\ -v_{1d}b_3 + v_{2d}b_1 & -a_{33}b_1 - v_{1d}b_0b_4 & a_{33}b_0 \end{bmatrix}$$

and

$$g(\tilde{q}) = \begin{bmatrix} (a_{22}b_1 + a_{33}b_1 + v_{1d}b_0b_4)\tilde{q}_2^2 \\ -(a_{22}b_1 + a_{33}b_1 + v_{1d}b_0b_4)\tilde{q}_1\tilde{q}_2 \\ 0 \end{bmatrix}$$

for

$$b_2 = \frac{\tilde{q}_1^2}{1+\tilde{q}_1^2}.$$

Remark: It is observed from eqn. 14 that v_m converges to the desired velocity v_d and the auxiliary velocity v_a to zero as $\tilde{q}\to 0$.

5 Stability of the learning system

With the aid of Lyapunov stability theory, the stability of developed learning system is now analyzed in which not only the two independent controls are convergent but the combined composite learning system shown in Fig. 3 remains uniformly asymptotically stable.

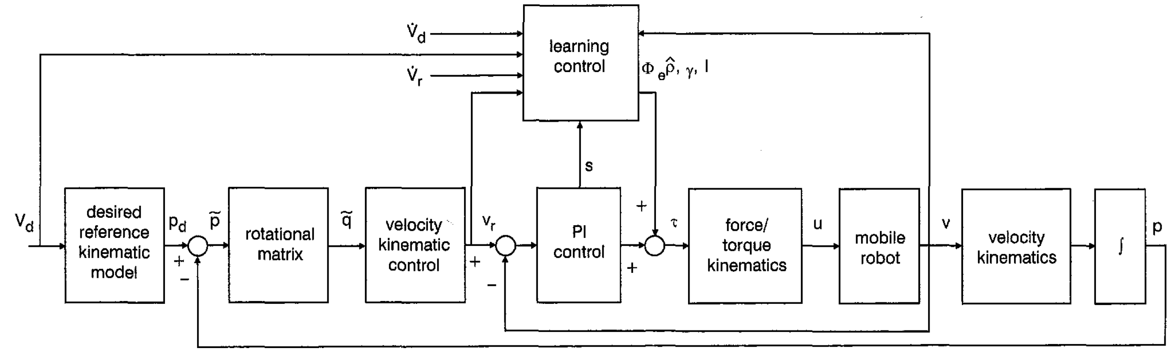

Fig. 3 *Learning controller schematic for tracking of mobile robot*

Lemma 5.1 (*Velocity learning*) If the feedback gain matrix satisfies $\Lambda_0 > \sigma D$, then the dynamic error system (9) using the exponential learning rules (10) or (12) converges as $\lim_{t\to\infty} s = 0$ with rate $e^{-\sigma t}$.

Lemma 5.2 (*Kinematic steering*) Assume that the desired velocity v_d for reference model (4) is bounded and continuously differentiable and $v_{1d} \neq 0$ or $v_{2d} \neq 0$ for all t. Further, assume that the velocity tracking is complete, i.e., $v = v_r$ for all t. Then, the reference velocity in eqn. 13 stabilises kinematic system uniformly asymptotically in that $\lim_{t\to\infty} \tilde{p} = 0$, provided that the gains are chosen to satisfy the inequalities $a_{ii} < 0$ $(i = 1, 2, 3)$ and $-a_{11}(2a_{11}\bar{a}_{33} + 4\bar{a}_{33}^2 + v_{2d}^2) > v_{1d}^2$ for all t, where $a_{33}b_0 = 2\bar{a}_{33}$ for negative constant $\bar{a}_{33}$.

The brief proofs of Lemma 5.1 and Lemma 5.2 are given in the Appendix.

Theorem 5.1 (*Composite learning*) Suppose that the desired velocity v_d and gains a_{ii} $(i = 1, 2, 3)$ are given as in Lemma 5.2. Further, assume that the feedback gains a and Λ_0 given in Lemma 5.1 satisfy $a \geq 2\sigma$, $4|\bar{a}_{11}\bar{\lambda}_0| \geq 1$ and $4|\bar{a}_{33}\bar{\lambda}_0| \geq 1$, where $a_{11} = 2\bar{a}_{11}$, $a_{33}b_0 = 2\bar{a}_{33}$, and $\lambda_0 = 2\bar{\lambda}_0$ for $\lambda_0 = \lambda_{min}(\Lambda_0) - \sigma\lambda_2$. Then, the reference velocity (13) combined with the learning controller (8) stabilises the learning system asymptotically such that $\lim_{t\to\infty} \tilde{p} = 0$.

Proof: When $\tilde{v} \neq 0$, the error eqn. 15 becomes

$$\dot{\tilde{q}} = A_r\tilde{q} + g(\tilde{q}) - B(\tilde{q})\tilde{v}. \qquad (16)$$

Define the composite Lyapunov functional for error systems in eqns. 9 and 16 as

$$V_c(t) = V_q(t) + V_r(t) + a_0 e^{2\sigma t}\left(\int_0^t \tilde{v}\, d\eta\right)^T\left(\int_0^t \tilde{v}\, d\eta\right), \qquad (17)$$

where V_q and V_r are defined as in the proofs of Lemma 5.2 and Lemma 5.1 and $a_0 = a\lambda_0$.

Differentiating V_q and V_r along the error systems in eqns. 16 and 9 yields, as in Lemma 5.1 and Lemma 5.2, respectively,

$$\dot{V}_c(t) \leq a_{11}\tilde{q}_1^2 + a_{22}b_0\tilde{q}_1^2\tilde{q}_2^2 + a_{33}b_0\tilde{q}_{33}^2 + \tilde{q}_1\tilde{v}_1 + \tilde{q}_3\tilde{v}_2 - \lambda_0 e^{2\sigma t}\left(|\tilde{v}|_2^2 + a(a - 2\sigma)\left|\int_0^t \tilde{v}\, d\eta\right|_2^2\right), \qquad (18)$$

where $w = se^{\sigma t}$ and $s = \tilde{v} + a\int_0^t \tilde{v}\, d\eta$ have been used.

Let $a_{11} = 2\bar{a}_{11}$, $a_{22}b_0 = \bar{a}_{22}$, $a_{33}b_0 = 2\bar{a}_{33}$ and $\lambda_0 = 2\bar{\lambda}_0$ for negative constants $\bar{a}_{ii}$'s $(i = 1, 2, 3)$. Further, let

$$\bar{v}_{11}^T\Gamma_{11}\bar{v}_{11} = [\tilde{v}_1\tilde{q}_1]^T\begin{bmatrix} -\bar{\lambda}_0 e^{2\sigma t} & \frac{1}{2} \\ \frac{1}{2} & \bar{a}_{11} \end{bmatrix}\begin{bmatrix} \tilde{v}_1 \\ \tilde{q}_1 \end{bmatrix} \text{ and}$$

$$\bar{v}_{23}^T\Gamma_{23}\bar{v}_{23} = [\tilde{v}_2\tilde{q}_3]^T\begin{bmatrix} -\bar{\lambda}_0 e^{2\sigma t} & \frac{1}{2} \\ \frac{1}{2} & \bar{a}_{33} \end{bmatrix}\begin{bmatrix} \tilde{v}_2 \\ \tilde{q}_3 \end{bmatrix}.$$

Then, from the gain conditions, it follows that $\bar{v}_{11}^T\Gamma_{11}\bar{v}_{11} \leq 0$, $\bar{v}_{23}^T\Gamma_{23}\bar{v}_{23} \leq 0$ and

$$\dot{V}_c(t) \leq \bar{a}_{11}\tilde{q}_1^2 + \bar{a}_{22}\tilde{q}_1^2\tilde{q}_2^2 + \bar{a}_{33}\tilde{q}_{33}^2 - \lambda_0 e^{2\sigma t}|\tilde{v}|_2^2 + \bar{v}_{11}^T\Gamma_{11}\bar{v}_{11} + \bar{v}_{23}^T\Gamma_{23}\bar{v}_{23} \leq 0. \qquad (19)$$

This confirms boundedness of error signals. Further, the equilibrium point $\tilde{q} = 0$ is asmyptotically stable, since the linearized system of eqn. 16 around $\{\tilde{q}, \tilde{v}\} = \{0, 0\}$, $\dot{\tilde{q}} = A_l\tilde{q} + \tilde{v}_l$, is bounded such that $|\tilde{q}(t)|_2 \leq e^{-\alpha_0 t}|\tilde{q}(0)|_2 + \int_0^t e^{-\alpha_0(t-\eta)}|\tilde{v}_l|_2\, d\eta$ for some positive constant α_0, where A_l is the same as in (20) and $\tilde{v}_l = [\tilde{v}_1 \ \ 0 \ \ \tilde{v}_2]^T$. The details of proof including the boundedness of learning system at the first trial for $[0, T]$ can be established in a similar fashion as Lemma 5.1 and Lemma 5.2. □

Remark: Theorem 5.1 implies that the stability of two independent controls in Lemma 5.1 and Lemma 5.2 is preserved under the imperfect velocity tracking, $v \neq v_r$ for finite t.

6 Experiment

To demonstrate its feasibility and effectiveness, the presented adaptive learning control scheme has been applied to velocity and position tracking of wheeled mobile robot. Fig. 4 shows the schematic diagram of the mobile robot system used in the experiment. In the adaptive learning controller, the estimated linear parameters include mass, inertia, coefficients of Coriolis force and viscous friction, and the maximum static friction. The bounded nonlinear friction as Stribeck effect and other external disturbances are absorbed in the estimate of disturbance bound d_m. The control parameters are set to $\Lambda_0 = 30$, $\beta_1 = 0.6$, $\beta_2 = 0.6$, $\beta_3 = 0.6$, $a = 3$, $\bar{a}_{11} = -10$, $\bar{a}_{22} = -10$, $\bar{a}_{33} = -10$, and $\alpha = 0.95$. In practice, the positive learning gains β_i's $(i = 1, 2, 3)$ are less than unity due to sensitivity of learning signal and the forgeting factor α is in between 0.95 and 1. Then, the exponential rate constant is given as $\sigma = -\ln\alpha/T$, since $\alpha = e^{-\sigma T}$. Although a good amount of experimental data has been obtained, only a few figures are given below owing to page limitation. In Fig. 5 and Fig. 6, the tracking performances of PID and learning controllers are compared with each other when the robot follows a circle trajectory. In the Figures, the maximum position and orientation errors of the two controllers are (50 mm, 16 mm), and (0.04 rad, 0.004 rad), respectively. In the learning controller, the estimated parameters remain bounded for all the periods

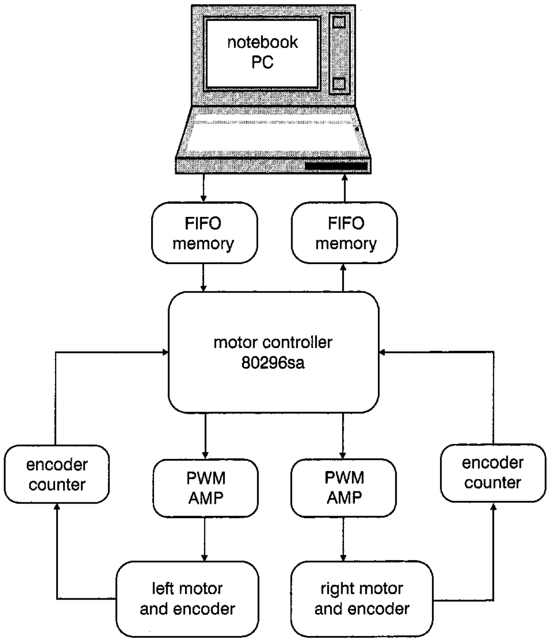

Fig. 4 *Schematic diagram of mobile robot system*

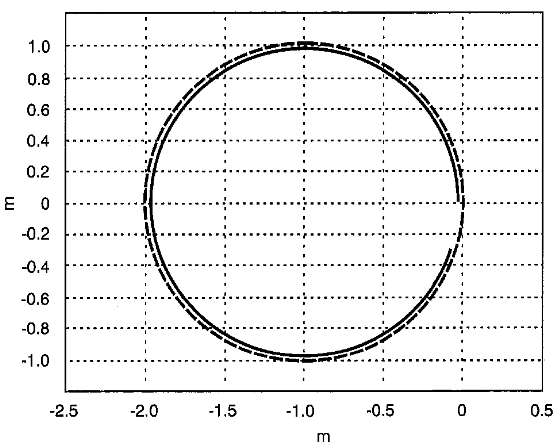

Fig. 5 *Response of PID controller to a circle trajectory*
---- desired
—— actual

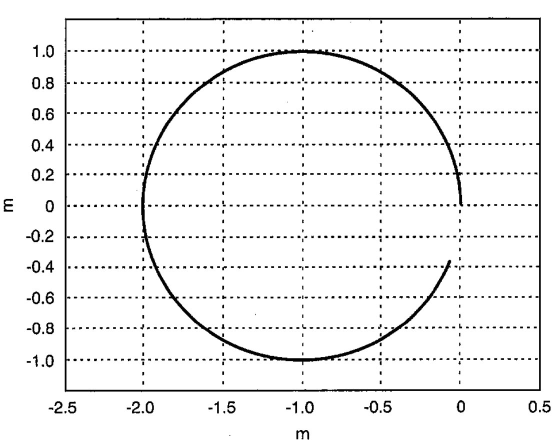

Fig. 6 *Response of learning controller to a circle trajectory*
---- desired
—— actual

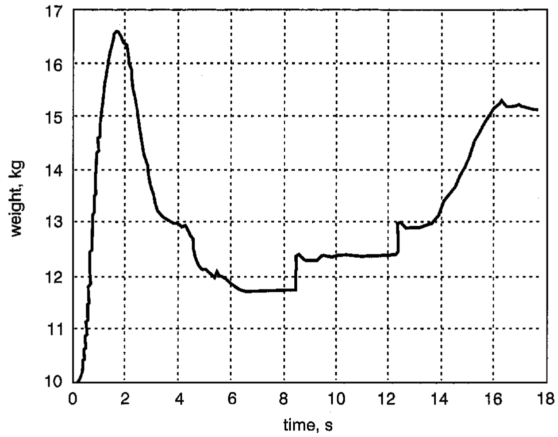

Fig. 7 *Estimated mass parameter*

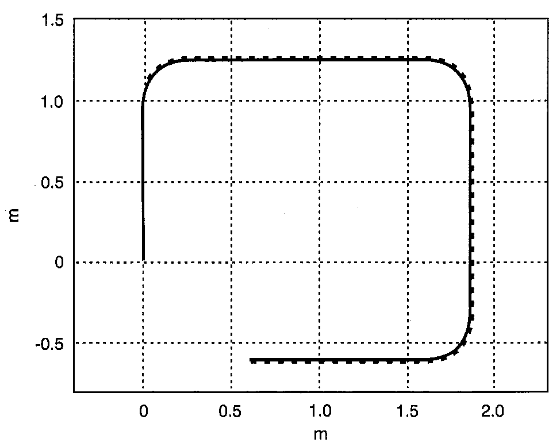

Fig. 8 *Response of PID controller to a rhombus trajectory*
---- desired
—— actual

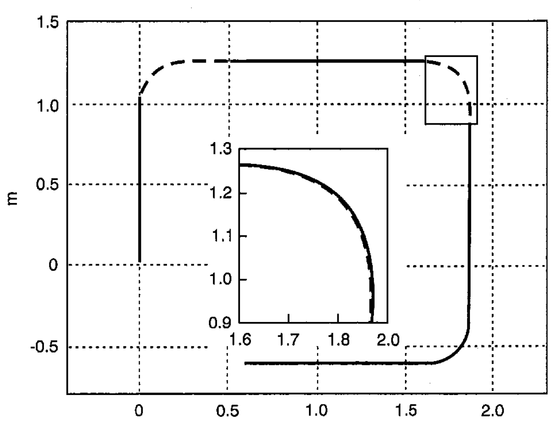

Fig. 9 *Response of learning controller to a rhombus trajectory*
---- desired
—— actual

of learning operation, where the estimate of largest parameter (mass) is shown in Fig. 7. Figs. 8 and 9 show the tracking performances of PID and learning controllers with rhombus trajectories. In the Figures, the increases of tracking errors due to frictional disturbances are seen near at the corners where the direction of motion and velocity changes. Nevertheless, it is observed from the figures that the tracking error of learning controller is much smaller than that of conventional PID controller. Throughout the experiment, the proposed learning controller has demonstrated good performance of nonlinear disturbance rejection and robustness to uncertain parameters.

7 Conclusion

A stable learning control approach for tracking of nonholonomic mobile robots has been developed which works under the uncertain operating conditions for system parameters and disturbances. The proposed learning controller does not require the assumption of perfect velocity tracking due to usage of a velocity learning control scheme. A practical design procedure for stable reference velocity input is also derived for uniform asymptotic convergence of the kinematic steering system. The composite learning controller which combines the position steering control with the velocity learning controller is shown to be uniformly and asymptotically stable through theoretical proof and experiment with a wheeled mobile robot.

8 Acknowledgment

This work was supported in part under the Brain Tech 21 program of MIST from BSRC.

9 References

1 ARIMOTO, S., KAWAMURA, S., and MIYASAKI, F.: 'Bettering operation of robots by learning', *J. Robot. Syst.*, 1984, pp. 123–140
2 ATKESON, C.G., and MCINTYRE, J.: 'Robot trajectory learning through practice'. Proceedings of the IEEE Conference on *Robotics and automation*, 1986, San Francisco, California, U.S.A., pp. 1737–1742
3 BONDI, P., CASALINE, G., and GAMBARDELLA, L.: 'On the iterative learning control theory for robotic manipulators', *IEEE J. Robot. Autom.*, 1988, **4**, (1), pp. 14–22
4 KANEKO, K., and HOROWITZ, R.: 'Repetitive and adaptive ontrol of robot manipulators with velocity estimation', *IEEE Trans. Robot. Autom.*, 1997, **13**, (2), pp. 204–216
5 KUC, T.-Y., NAM, K., and LEE, J.S.: 'An iterative learning control of robot manipulators', *IEEE Trans. Robot. Autom.*, 1991, **7**, (6), pp. 836–841
6 MILLER, W.T. III, GRANZ, F.H., and KRAFT, L.G.: 'Application of general learning algorithm to the control of robotic manipulators', *Int. J. Robot. Res.*, 1987, **6**, (2), pp. 84–98
7 PARK, P.H., KUC, T.-Y., and LEE, J.S.: 'Adaptive learning control of uncertain robotic systems', *Int. J. Control*, 1996, **65**, (5), pp. 725–744
8 QU, Z., DORSEY, J., DAWSON, D.M., and JOHNSON, R.W.: 'Linear learning control of robot motion', *J. Robot. Syst.*, 1993, **10**, (1), pp. 123–140
9 SAAB, S.S.: 'On the P-type learning control', *IEEE Trans. Autom. Control*, 1994, **39**, (11), pp. 2298–2302
10 ORIOLO, G., PANZIERI, S., and ULIVI, G.: 'An iterative learning controller for nonholonomic mobile robots', *Int. J. Robot. Res.*, 1998, **17**, (9), pp. 954–970
11 KANAYAMA, Y., KIMURA, Y., MIYAZAKI, F., and NOGUCHI, T.: 'A stable tracking control method for an autonomous mobile robot'. Proceedings of IEEE International Conference on *Robotics and automation*, 1990, pp. 384–389
12 VIDYASAGAR, M.: 'Nonlinear systems analysis' (Prentice-Hall, Inc., Englewood Cliffs, N.J., 1978)

10 Appendix

Proof of Lemma 5.1:

(Boundedness of initial condition) Let $V_0(t) = (1/2)s^T(t)Ds(t) + (1/2\beta_1)\tilde{\rho}^T\tilde{\rho} + (1/2\beta_2)\tilde{d}_m^2$. Differentiating $V_0(t)$ along the error equation (9) without multiplying $e^{\sigma t}$ and using the initial conditions for learning rules (10), we obtain $\dot{V}_0(t) \leq -s^T\Lambda_0 s + |d_r||s|$. This implies the boundedness of s since $\dot{V}_0(t)$ is negative if $|s| \geq (|d_r|_{\infty e})/(\lambda_0)$, where $|d_r|_{\infty e}$ and $\lambda_0 = \lambda_{min}(\Lambda_0)$ represent the $L_{\infty e}$ norm of d_r and the minimum eigenvalue of Λ_0, respectively.

(Convergence of learning control) Let $\pi\tilde{x}(t) = x_d(t) - G_\sigma[x(t)]$ for $\sigma = 0$ and $\pi\tilde{x}(t) = x_d(t) - e^{\sigma t}x(t)$ for $\sigma > 0$, where $\{x_d, x\}$ denotes $\{l_d, l\}$, $\{\rho, \hat{\rho}\}$, or $\{d_m, \hat{d}_m\}$. Then, the learning rules in eqn. 10 can be rearranged in the form $\tilde{x}(t) = \pi\tilde{x}(t-T) - \beta_i h_i(s(t))$ for $\sigma = 0$ and $\pi\tilde{x}(t) = \pi\tilde{x}(t-T) - \beta_i e^{\sigma t}h_i(s(t))$ for $\sigma > 0$, where $h_i(s)$ is some function of s for $i = 1, 2, 3$.

Let $V_r(t) = (1/2)w^T Dw + \int_{t-T}^{t} [(1/2\beta_1)\pi\tilde{\rho}^T\pi\tilde{\rho} + (1/2\beta_2)|\pi\tilde{d}_m|^2 + (1/2\beta_3)\pi\tilde{l}^T\pi\tilde{l}]d\eta$. Differentiating $V_r(t)$ along the error system (9) with the learning rules yields after some calculation $\dot{V}(t) \leq -w^T(t)\bar{\Lambda}w(t) \leq 0$, where the relations $w^T\delta d_n \leq (d_m - \hat{d}_m)|w|_2$ and $|\tilde{l}|_2^2 \geq |\pi\tilde{l}|_2^2$ for $\sigma = 0$ have been used. This implies the L_2 and L_∞ boundedness of error signals. Further, we have $s \to 0$ asymptotically when $\sigma = 0$ and $s \to 0$ exponentially when $\sigma > 0$, since $\dot{s}$, $w \in L_\infty$. □

Proof of Lemma 5.2: Define a Lyapunov function candidate of the position error system in eqn. 15 as $V_q(t) = (1/2)\tilde{q}^T(t)\tilde{q}(t)$. Differentiating V_q along the error system and observing that $A_r = \mathrm{diag}(A_r) + \mathrm{skew}(A_r)$ and $\tilde{q}^T g = 0$, we obtain $\dot{V}_q(t) = \tilde{q}^T\mathrm{diag}(A_r)\tilde{q} = a_{11}\tilde{q}_1^2 + \bar{a}_{22}\tilde{q}_1^2\tilde{q}_2^2 + 2\bar{a}_{33}\tilde{q}_3^2 \leq 0$, where $a_{22}b_0 = \bar{a}_{22}$ and $a_{33}b_0 = 2\bar{a}_{33}$ for negative constants $\bar{a}_{22}$ and $\bar{a}_{33}$. This implies that $\tilde{q} = 0$ is stable equilibrium point of the error system. Now, consider the linearised model of eqn. 15 around $\tilde{q} = 0$.

$$\dot{\tilde{q}} = A_l\tilde{q}, \tag{20}$$

where

$$A_l = \begin{bmatrix} a_{11} & v_{2d} & 0 \\ -v_{2d} & 0 & -v_{1d} \\ 0 & v_{1d} & 2\bar{a}_{33} \end{bmatrix}.$$

Note that the matrix A_l and its time derivative are bounded, since the gains $\{a_{11}, \bar{a}_{33}\}$ and desired trajectories $\{v_d, \dot{v}_d\}$ are bounded. Further, the real parts of eigenvalues of A_l are negative, since the characteristic polynomial of A_l reads $|sI - A_l| = s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3 = 0$, where $\alpha_1 = -(a_{11} + 2\bar{a}_{33}) > 0$, $\alpha_2 = (2a_{11}\bar{a}_{33} + v_{2d}^2) > 0$, and $\alpha_3 = (v_{1d}^2 - 2\bar{a}_{33}v_{2d}^2) > 0$. The Routh-Hurwitz criterion indicates that if $\alpha_1\alpha_2 - \alpha_3 = -a_{11}(2a_{11}\bar{a}_{33} + v_{2d}^2) - 4a_{11}\bar{a}_{33}^2 - v_{1d}^2$ is positive, then the real parts of eigenvalues are negative as it is from the gain conditions. Hence, the error system in eqn. 15 is uniformly asymptotically stable [12]. □