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AN APPLICATION OF THE EXTENDED KALMAN FILTER FOR INTEGRATED NAVIGATION IN MOBILE ROBOTICS

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Abstract

In this paper the problem of integrated navigation for mobile robotics is addressed. The dynamics of ground robots are nonlinear, as well as measurement devices. For the purposes of position estimation, a dead reckoning problem with a wheeled mobile robot (WMR, for short) is considered using a gyro and two wheel encoders. The extended Kalman filter (EKF, for short) is used to provide an estimate of vehicle state, gyro error dynamics and variables describing the slip effects. The parameters of the error dynamics have been identified and validated off-line by real data. Simulations results have confirmed the efficacy of the proposed approach, by comparing the performances of the EKF with and whitout wheel slip model. The present work is motivated by the target of designing a navigation module for a WMR recently built and still under developement.

1. Introduction

Mobile robotics have been the object of considerable attention in the last years. Mobile robots are becoming more and more diffused and are assuming increasing autonomous capabilities. Researches in this area focused on different topics and, among the others, one of the most important is navigation [1]. Navigation stands for all the techniques, procedures and algorithms used to provide a high-accuracy vehicle position and location. The problem is that of estimating the state of the system with respect to its physical representation (i.e., in term of dynamic equations) and its relationship with the environment. The subject of navigation involves other fields of the robotic applications, such as space and underwater robotics.

Navigation concerns the integration of all the noisy information from the available measurement sensors, for attaining a reliable estimates of position and orientation. Different kinds of sensors may be used, depending on the range of operation: GPS, radar, vision, rate gyro, compass, odometer, accelerometer, pendulum. In the following, we will address the problem of dead reckoning [2], i.e., integrating angular rate and speed measurements, from a solid-state low-cost gyro and wheel encoders. Low-cost gyros are very useful in mobile robotics but are affected by significant drift during on-line operations. An error model have been introduced for improving estimation and its parameter identified [3, 4]. Moreover the slip effects have been modelled for a more precise description of the vehicles's motion [5, 6].

Parameter identification may be coped with according to different frameworks and using different techniques. If nothing is known about the plant, a "black box" approach seems more suitable: a linear or nonlinear model is tuned by means of the available data [7, 8]. If there exist physic laws suited to describe the plant except for some parameters to be determined, the unknown parameters can be gained from the measured inputs and outputs, relying on a state space model [7]. In the case of the mobile robots we considered, the error dynamics have been added to the model and its physical parameters identified off-line by real data and using a least-squares algorithm [3, 4]. Moreover, the slip motion has been modelled to improve the position and orientation estimates. Thus, the problem becomes quite complex: it is required the combined estimation of the original state vector, the error dynamics and the slip effects, by means of the angular rate and wheel speed measurements.

State estimation for stochastic systems is a classical problem that can be solved analytically only under the assumptions of a linear dynamic system, of a linear observation channel and of white Gaussian process and measurement noises. If such assumptions are verified, the solution to the optimal filtering problem can be efficiently computed by the Kalman filter. The Kalman filter gives a linear, unbiased and minimum error variance recursive algorithm to estimate the unknown state of a dynamic process from noisy data taken from a linear measurement channel. The filter requires the knowledge of the second-order statistics of the noise of process being observed and of the measurement noise in order to provide the solution that minimizes the mean square error between the true state and the estimate of state. Otherwise, as in the case of the system describing the dynamics of a WMR, the design of implementable state estimators for nonlinear stochastic systems is a difficult task to be solved [9]. The most widely used nonlinear filter is the EKF, which usually perform well in many applications [10]. The EKF setup implements the Kalman filter by the linearizations of the nonlinear dynamics around the last computed estimation point (see, for instance, [11, 12]). Unfortunately there is a substantial lack of investigation (and so also of results) about the use of the EKF for general nonlinear settings: in these cases, the success of the EKF relies only on the evidence of experimental results.

In Section 2 a brief account of the model for a WMR is given, with particular emphasis on the modelling of the slip effects. In Section 3 we will discuss the problem of the design of a navigation module for a mobile robotic application. The utilization of the EKF for the synthesis of nonlinear estimators is dealt in Section 4. Finally, in Section 5, experimental and simulation results are reported.

2. A model of mobile robot for position estimation

We consider a mobile robot driven by two wheels as shown in Figure 1 and analyze its motion on a planar surface.

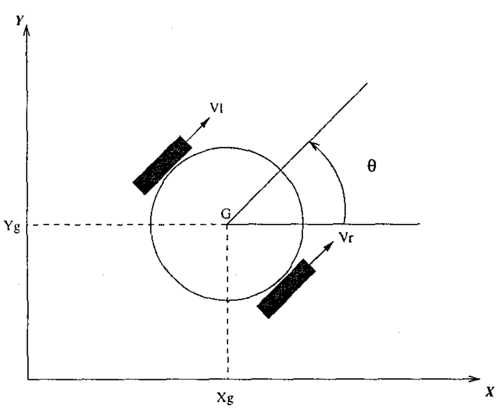

Figure 1: the wheeled mobile robot equipped with the gyro in the fixed coordinate frame

The WMR can be modelled as a nonholonomic dynamic system in which the wheels are assumed to roll and not to slip, but this assumption which takes to a kinematical relationship between the wheel rotation and the position and orientation of the WMR, is not always realistic. Referring to [13], when the rolling conditions are not satisfied, to model the combined effect of rolling and slipping is introduced the tractive force of the wheel. Then the rotational and traslational dynamics of the wheel can be expressed as

$$\begin{cases} F_t = M\ddot{x} \\ \tau = J\ddot{\theta} + F_t r \end{cases} \tag{1}$$

where r and M are respectively the radius and the mass of the wheel, J is the moment of inertia about the wheel center, F_t is the tractive force developed at the wheel contact, $\ddot{x}$ and $\ddot{\theta}$ are the linear and angular acceleration of the wheel center, τ is the torque applied at the wheel axle.

The considered position estimation problem for a two wheel mobile robot is based on the information provided by the gyro and wheels encoders. The gyro provides angular rate information and the data are reliable over long period of time, but even if very small errors affect the inertial sensor informations, when these data are integrated to obtain absolute measurements of orientation, position and velocity, the integrated measurements are characterized by errors that grow with time and distance. Error modelling for solid state gyros has been intensively studied and according to [3, 4] the bias error, characteristic of the gyros, can be expressed in a continuous time domain by the nonlinear, parametric model

$$\varepsilon_{mod}(t) = C_1(1 - e^{-\frac{t}{T}}) + C_2 \tag{2}$$

where $\varepsilon_{mod}(t)$ is the error model for the gyro output when no input is applied and C_1, C_2 and T are parameters which must be determined experimentally. The parametrized model can be expressed by the differential equation

$$\dot{\varepsilon}_{\dot{\phi}}(t) = \frac{C_1 + C_2}{T} - \frac{1}{T}\varepsilon_{\dot{\phi}}(t) \tag{3}$$

with initial conditions $\varepsilon_{\dot{\phi}}(0) = C_2$ and $\dot{\varepsilon}_{\dot{\phi}}(0) = \frac{C_1}{T}$. In discretized form equation (3) becomes

$$\varepsilon_{\dot{\phi}_{k+1}} = \frac{T}{T+T_s}\,\varepsilon_{\dot{\phi}_k} + \frac{T}{T+T_s}(C_1 + C_2) \tag{4}$$

where T_s is the sampling time, $\varepsilon_{\dot{\phi}}(0) = C_2$. Considering the error model for the angular rate observed by the gyro and the rolling/slipping dynamic equations of the wheels, the discrete time dynamic model for the considered mobile robot is expressed by

$$x_{k+1} = x_k + T_s\, v_{x\,k} \tag{5a}$$

$$y_{k+1} = y_k + T_s\, v_{y\,k} \tag{5b}$$

$$\theta_{k+1} = \theta_k + T_s\dot{\theta}_k \tag{5c}$$

$$\dot{\theta}_{k+1} = \dot{\theta}_k + T_s \frac{R}{J_c}(Ft_{r\,k} - Ft_{l\,k}) \tag{5d}$$

$$v_{x\,k+1} = v_{x\,k} + \frac{T_s}{M}(Ft_{r\,k} + Ft_{l\,k})\cos\theta_k \tag{5e}$$

$$v_{y\,k+1} = v_{y\,k} + \frac{T_s}{M}(Ft_{r\,k} + Ft_{l\,k})\sin\theta_k \tag{5f}$$

$$\dot{\varphi}_{L\,k+1} = \dot{\varphi}_{L\,k} + \frac{T_s}{J_L}(u_{L\,k} - Ft_{l\,k} r) \tag{5g}$$

$$\dot{\varphi}_{R\,k+1} = \dot{\varphi}_{R\,k} + \frac{T_s}{J_R}(u_{R\,k} - Ft_{r\,k} r) \tag{5h}$$

$$v_{c\,k+1} = v_{c\,k} + \frac{T_s}{M}(Ft_{r\,k} + Ft_{l\,k}) \tag{5i}$$

$$e_{k+1} = e_k \frac{T}{T+T_s} + \frac{T_s}{T+T_s}(C_1 + C_2) \tag{5j}$$

where $k = 0, 1, \ldots$, x and y are the cartesian coordinates of the WMR center of mass with respect to the fixed coordinate system, θ and $\dot{\theta}$ are the angular position and velocity of the WMR with respect to the fixed frame, v_x and v_y are the center of mass velocity components with respect to the fixed coordinate system, $\dot{\varphi}_L$ and $\dot{\varphi}_R$ are the rotational speeds of the left and right wheel respectively, v_c is the center of mass velocity in the motion direction, e is the error model of the gyro rate, T is the time constant of the gyro error model, C_1 and C_2 are constant parameters of the error model of the gyro, M is the WMR mass, R is the WMR radius, J_c is the WMR moment of inertia with respect to the z axes of the fixed frame, m is the mass of every wheel, r is the radius of a wheel, J_L and J_R are the moments of the inertia of the left and right wheel respectively, u_L and u_R are the torques acting on the left and right wheel, Ft_{li} and Ft_{ri} are the forces that the left and right wheel respectively develope at the contact point.

3. Design of a navigation module for a mobile robotic application

The objective of the Navigation Module for a mobile robotic application is to obtain a reliable estimation of the position and of the attitude of a vehicle in its operational environment. Basically two different approaches are reported in the literature: one of the two is based on the Kalman filter [14], the other makes use of the extended Kalman filter [1]. The difference between the two frameworks stems from the utilization of a linear kinematic model for the first one, while, for the second one, the EKF is required due to the nonlinear dynamic model which is referred to [15]. Clearly, if we rely on nonlinear model, is much more difficult to tune the performances of the filter in term of frequency requirements, and, from the viewpoint of the theory, there are no theoretical results about the convergency properties. However, from the practical point of view, the use of the all available information in term of dynamical equations is well suited and may be useful the reduce the number of sensors, like, for instance, accelerometers. The design of a navigation module influnces also the performances of the controller for what the band requirements are concerned [16].

4. The EKF state and parameter estimator

The most widespread and reliable state estimator for nonlinear systems is the extended Kalman filter, which has been used for a lot of real-world engineering applications [10]. The success of the EKF has emerged despite of its poor theoretical framework: only very few results about the convergence properties of the EKF are available, most of them in the deterministic setting.

The EKF is the Kalman filter of an approximate model of the nonlinear system linearized around the last estimate, supposing all the stochastic processes Gaussian [9, 11]. Anyway, the real model of the plant is described by nonlinear equations and the mean and the covariances computed by the EKF are only approximations of the real conditioned mean and covariance, which in general are non-Gaussian for a nonlinear systems. EKF formulas differ from the Kalman ones [12]: in a linear Gaussian setting, it is feasible to perform the *off-line* computation of the covariances, and so is it for the gains. In the nonlinear case, the computation of mean and covariance can be attained only *on-line*, because the EKF setup implements the Kalman filter by the linearizations of the nonlinear dynamics around the last computed estimation point [11].

The EKF procedure can be applied to the previous nonlinear model (5) [11, 2], to provide an estimate of the state variables as well as of the variables expressing the slipping effects or the error of the gyro rate. The measurement channel represents the gyro and the wheel encoders, which give the angular velocity and the rotational speed of the two wheels.

5. Simulation and experimental results

As far as the experimental results is concerned, the identification of the error model (4) has been performed using real data and a least-squares algorithm. Validation has been carried out by evaluating the sample autocorrelation of the residuals [3, 4], which is depicted in Figure 2. The rest of the research concerns only simulations. The previous identified and validated error model became part of (5), which has been the basis of extensive simulations. Basically, the objective was to design a reliable navigation system, and, toward this end, an evaluation of the improvement due to a more complex setup with the slip model has been accomplished. Measurements have been generated with model (5), an two EKF, with and without slip modelling, used for position estimation. The RMS errors over 100 runs are depicted in Figure 3 for x, in Figure 4 for y and θ. A simulation of the behavior of the estimates with and without slip modelling is shown in Figures 5, 6 and 7.

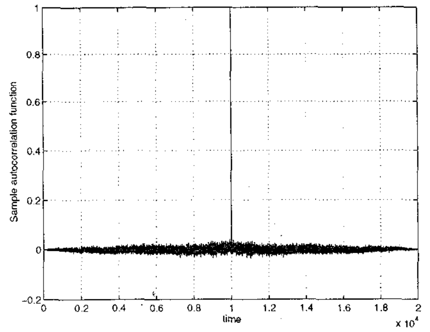

Figure 2: sample autocorrelation function for x coordinate

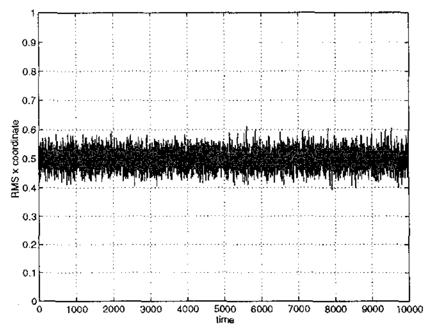

Figure 3: behavior of the RMS error for x coordinate

6. Conclusions

This paper reports the first results of our current research in mobile robotics. First step concerned the subject of control, now we focus on navigation. In the future, we will pay attention on the integration of control and navigation. The results of the application of the EKF for navigation are enough encouraging. The choice of a more accurate model with slip effects has been demonstrated crucial to attain a precise position estimation. Future researches will be devoted to improve both estimation algorithm by means of simulations and the implementation of the navigation module for a WMR.

References

[1] H. F. Durrant-White, "Where am I?", *Industrial Robot*, vol. 21, no. 2, pp. 11–15, 1994.

[2] K. Komoriya and E. Oyama, "Position estimation of a mobile robot using optical fiber gyroscope (OFG)", in *IROS '94, International Conference on Intelligent Robots and Systems. Advanced Robotic Systems and the Real World*, Monaco, Germany, September 1994, vol. 1, pp. 143–149.

[3] B. Barshan and H. F. Durrant-White, "Orientation estimate for mobile robots using gyroscopic information", in *IROS '94, International Conference on Intelligent Robots and Systems. Advanced Robotic Systems and the Real World*, Monaco, Germany, September 1994, vol. 3, pp. 1867–1874.

[4] B. Barshan and H. F. Durrant-White, "Inertial navigation systems for mobile robots", *IEEE Trans. on Robotics and Automation*, vol. 11, pp. 328–342, 1995.

[5] S. Senini, F. Flinders, and W. Oghanna, "Dynamic simulation of wheel-rail interaction for locomotive traction studies", in *1993 IEEE/ASME Joint Railroad Conference*, Pittsburgh, USA, 1993, pp. 27–34.

[6] F. Gustafsson, "Estimation and change detection of tire–road friction using the wheel slip", in *Proceedings of the 1996 IEEE International Symposium on Computer-Aided Control System Design*, 1994, pp. 99–104.

[7] L. Ljung and T. Söderström, *Theory and Practice of Recursive Identification*, MIT Press, Boston, 1983.

[8] T. Söderström and P. Stoica, *System Identification*, Prentice Hall, New York, 1989.

[9] A. H. Jazwinski, *Stochastic Processes and Filtering Theory*, Academic Press, New York, 1970.

[10] H. W. Sorenson, *Kalman Filtering Theory and Application*, IEEE Press, 1990.

[11] A. Gelb, *Applied Optimal Estimation*, M.I.T. Press, 1974.

[12] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*, Prentice Hall, New York, 1979.

[13] R. Balakrishna and A. Ghosal, "Modeling of slip for wheeled mobile robots", *IEEE Trans. on Robotics and Automation*, vol. 11, pp. 126–132, 1995.

[14] P. Oliveira, A. Pascoal, and C. Silvestre, "Guidelines to the design of a navigation system for an autonomous underwater vehicle", Tech. Rep. 1-94, Institute for Systems and Robotics, Instituto Superior Técnico, Lisbon, Portugal, 1994.

[15] J. Vaganay and M. J. Aldon, "Attitude estimation for a vehicle using inertial sensors", *Control Engineering Practice*, vol. 2, no. 2, pp. 281–287, 1994.

[16] J. Guldner and V. I. Utkin, "Tracking the gradient of artificial potential fields: sliding mode control for mobile robots", *Int. J. Control*, vol. 63, no. 3, pp. 417–432, 1996.

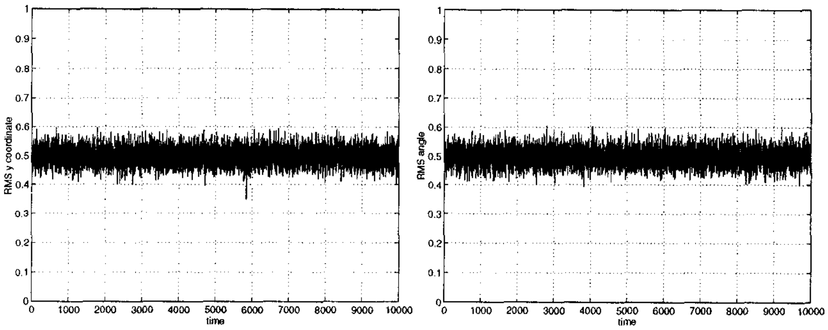

Figure 4: behavior of the RMS errors for y coordinate and θ angle

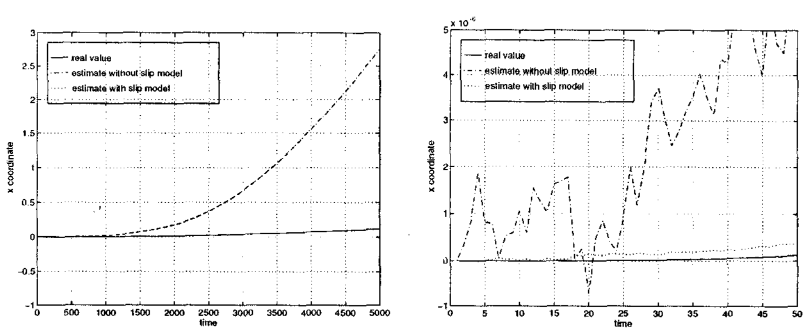

Figure 5: behavior of the x coordinate and its estimates

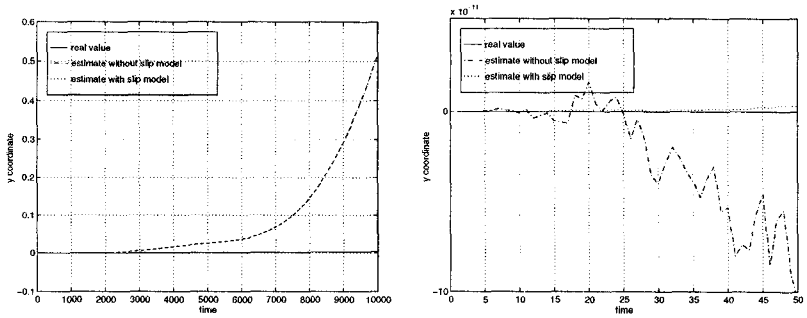

Figure 6: behavior of the y coordinate and its estimates

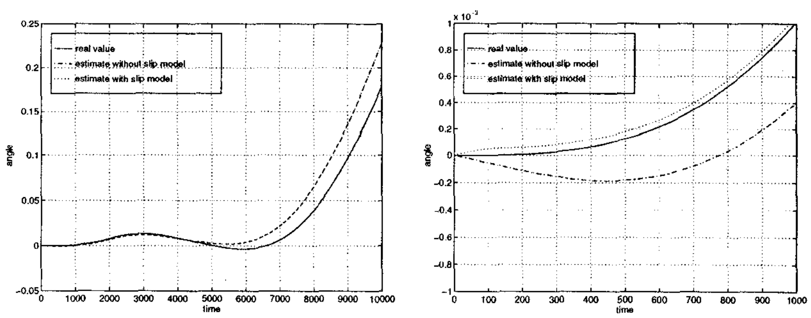

Figure 7: behavior of the angle θ and its estimates