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Tracking Control of a Mobile Robot with Kinematic and Dynamic Constraints*

Heting Xu & Simon X. Yang†
ARIS (Advanced Robotics & Intelligent Systems) Lab
School of Engineering, University of Guelph
Guelph, Ontario, N1G 2W1, Canada
Email: {heting; syang}@uoguelph.ca

Abstract

In this paper, a novel path tracking control method with both kinematic and dynamic constraints is proposed for a nonholonomic mobile robot. By incorporating a biologically inspired shunting model into the conventional bang-bang control technique, the proposed novel tracking controller is capable of generating real-time acceleration commands that can produce smooth, continuous robot velocities, and drive the mobile robot to track the desired trajectories. In the controller design, the accelerations are bounded and the nonholonomic kinematic constraints are satisfied. The stability of the control system and the convergence of tracking errors to zero are proved using a Lyapunov stability theory. The effectiveness of the proposed tracking controller is demonstrated by simulation studies.

1 Introduction

Motion controller should generate a series commands which guide the robot to accurately follow a desired path. Any deviation from the path, either in the form of an offset or orientation is undesirable, since both lead to deviation from the desired robot trajectories. A good controller, thus, becomes important for a stable, efficient path tracking.

Many control strategies has been proposed for the path tracking problem. For example, Jiang [3] proposed a tracking control methodology via time-varying state feedback based on the backstepping technique. Lee and Williams [4] proposed a control method for eliminating the track error quickly by controlling two independent driving wheels at same time. Kanayama [5] presented the decomposition of error between the reference posture and the current posture by PID filtering method. However, most of the previous tracking control approaches mainly focus on path error convergence and system stability. As a result, they may neglect the motion smoothness and dynamic constraints, such as the acceleration bounds which are important factors for avoiding wheel slippage or mechanical shock during the navigation.

In this paper, a novel path tracking controller is presented with the considerations of both kinematics and dynamics constraints. The proposed controller uses a biologically inspired neural dynamics model that is characterized by a shunting equation derived from Hodgkin and Huxley's membrane equation [1], and it is based on the bang-bang control technique to produce bounded real-time acceleration commands that can guide the robot to reach the desired trajectory. Hence, the dynamic constraints and nonholonomic kinematic constraints are fully considered in the controller design. The stability of the control system and the convergence of tracking errors to zero are proved through using a Lyapunov stability theory, which is based on the convergence of the typical bang-bang controller.

The paper is organized as follows. Section 2 provides background information about the tracking control problem, including the nonholonomic mobile robot model and the constraints of a mobile robot. The concept, the algorithm, and the stability analysis are presented in Section 3. In section 4, The simulation results are provided to demonstrate the effectiveness of the proposed controller. The effect of parameters used in the shunting model is discussed in Section 5. Finally, this paper is concluded by highlighting the feature properties of the proposed model.

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†To whom all correspondence should be addressed.

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2 Background

In this section, a typical mobile model is first described. Then the kinematics and error dynamics are presented based. The path tracking problem with consideration of kinematic and dynamic constraints will be finally introduced.

2.1 A Nonholonomic Mobile Robot Model

A mobile robot is located in a two dimensional Cartesian workspace, in which a global coordinate $\{X, O, Y\}$is defined. A local coordinate $\{X', C, Y'\}$ is attached to the robot with the origin at point c, the middle points of two wheels which is the guide point of this mobile robot. A typical mobile model is shown in Fig. 1, where W is the distance between two wheels, and r is the radius of the driving wheel.

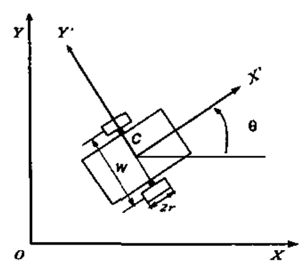

Figure 1: *Model of a typical mobile robot.*

The robot has three degrees of freedom that are represented by a posture $p_c = (x_c, y_c, \theta_c)$, where (x_c, y_c) indicate the spacial position of the robot guide point in the global coordinate system and θ_c is the heading angle of the robot. The kinematics of the robot is given as

$$\begin{aligned} \dot{x}_c &= v_c \cos\theta_c, \\ \dot{y}_c &= v_c \sin\theta_c, \\ \dot{\theta}_c &= \omega_c, \end{aligned} \tag{1}$$

where v_c is the linear (tangential) velocity, ω_c is the angular velocity of the robot guide point c. In the control system, two postures are used: the reference posture $p_t = (x_t, y_t, \theta_t)$ and the current posture $p_c = (x_c, y_c, \theta_c)$. The motion of the target robot can be represented by the target postures and the current posture is real robot position at this moment. So the posture error e_p in the local coordinate is given as

$$\begin{aligned} e_x &= (x_t - x_c)\cos\theta_t + (y_t - y_c)\sin\theta_t, \\ e_y &= -(x_t - x_c)\sin\theta_t + (y_t - y_c)\cos\theta_t, \\ e_\theta &= \theta_t - \theta_c. \end{aligned} \tag{2}$$

This posture error is shown in the following Fig. 2.

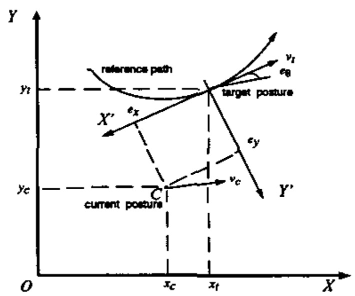

Figure 2: *Posture error*

If the reference velocities are given as (v_t, ω_t), and the current velocities are (v_c, ω_c), based on Eq. (2), the dynamics of error is derived as

$$\begin{aligned} \dot{e}_x &= v_t - v_c\cos e_\theta + \omega_t e_y, \\ \dot{e}_y &= -\omega_t e_x + v_c \sin e_\theta, \\ \dot{e}_\theta &= \omega_t - \omega_c. \end{aligned} \tag{3}$$

2.2 Constraints of a Mobile Robot

When a mobile robot moves in the Cartesian space, it cannot freely reach any point, because it have to meet the kinematic constraint. In addition, in order to avoid wheel slippage or mechanical damage during the mobile robot navigation, it is necessary to smoothly change driving velocity and direction of the mobile robot. This means that the dynamic constraints of the mobile robot should be considered in the design of path tracking controller.

2.2.1 Kinematic Constraint

The kinematic constraint for a mobile robot is referred as the nonholonomic constraint. As we know, the configuration space of a robot is all the possible configurations of a robot. The configuration space of a mechanical system made of rigid bodies is a smooth main-fold. A nonholonomic constraint is generally caused by a rolling contact between two rigid bodies. It expresses that the relative velocity of the two points of contact is zero. Therefore, the nonholonomic constraints states that the robot can only move in the direction normal to the axis of the driving wheels, if the derivatives $\dot{x}$ and $\dot{y}$ exist, θ is not a independent variable which should satisfy the nonholonomic constrain as

$$\dot{y}_c \cos\theta_c - \dot{x}_c \sin\theta_c = 0. \tag{4}$$

Therefore, the degree of freedom of a nonholonomic mobile robot is reduced to two. From the results in controllability theory for nonlinear control systems, any robot, which is subject to a single scalar linear nonholonomic equality constraint (shown in Eq. 4) is fully controllable.

2.2.2 Dynamic Constraint

In addition to the nonholonomic kinematics constraint, the acceleration constraint is also need to be considered in order to avoid slippage during the robot navigation. The linear velocity v_c and the angular velocity ω_c of center c are derived as

$$\begin{aligned} v_c &= \frac{r}{2}(\omega_r + \omega_l), \\ \omega_c &= \frac{r}{w}(\omega_r - \omega_l), \end{aligned} \tag{5}$$

where r is radium of the wheels, and W is the length between wheels, respectively. When considering to avoid slippage and any abrupt change in robot motion, the angular acceleration of each driving wheel is bounded by $\omega_{r,l} \leq \omega_{\max}$. Based on the Eq. (6), the linear and angular accelerations are limited by

$$\begin{aligned} |\alpha_c| &= r\dot{\omega}_{\max} \leq \alpha_{\max}, \\ |\beta_c| &= r\frac{\dot{\omega}_{\max}}{W} \leq \beta_{\max}, \end{aligned} \tag{6}$$

where α_c and β_c are the linear acceleration and angular acceleration of center c, respectively.

2.3 Tracking Problem

The path tracking problem in this paper is to find a control law for accelerations α_c and β_c with consideration of both kinematic and dynamic constraints, which can drive the mobile robot to follow a reference path with position p_t and velocities v_t and ω_t. The acceleration control law for α_c and β_c is

$$\alpha_c = f(e_x, e_y, e_\theta, v_t, \omega_t, \alpha_{\max}), \tag{7}$$

$$\beta_c = g(e_x, e_y, e_\theta, v_t, \omega_t, \beta_{\max}), \tag{8}$$

such that for any arbitrarily large initial tracking errors, the tracking controller is going to converge the posture error to zero. The system architecture of this controller is shown in Fig. 3.

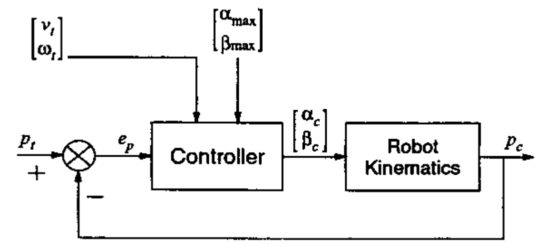

Figure 3: *Schematic diagram of the proposed control system*

The control system is working as follows. First using the Eq. (2) to calculate the errors, and then the proposed controller generates the accelerations of the robot based on the errors and constraints, after that the velocities of the robot can be derived using

$$\begin{aligned} v_c(k+1) &= v_c(k) + \alpha_c dt, \\ \omega_c(k+1) &= \omega_c(k) + \beta_c dt. \end{aligned} \tag{9}$$

The fourth step is to get the robot kinematics by using the Eq. (1), Finally using an integrator, current posture of the robot is derived as follows.

If $\omega_c \neq 0$, the current posture is given as

$$\begin{aligned} x_c(k+1) &= x_c(k) + \frac{v_c}{\omega_c}(\sin\theta_c(k+1) - \sin\theta_c(k)), \\ y_c(k+1) &= y_c(k) - \frac{v_c}{\omega_c}(\cos\theta_c(k+1) - \cos\theta_c(k)), \\ \theta_c(k+1) &= \theta_c(k) + \omega_c dt; \end{aligned} \tag{10}$$

If $\omega_c = 0$, the current posture is given as

$$\begin{aligned} x_c(k+1) &= x_c(k) + v_c \cos\theta_c(k) dt, \\ y_c(k+1) &= y_c(k) + v_c \sin\theta_c(k) dt, \\ \theta_c(k+1) &= \theta_c(k). \end{aligned} \tag{11}$$

3 The Proposed Model

In this section, the time-optimal bang-bang control technique is briefly introduced. Then the concept of landing curve that is used for guiding the robot to land softly on the target is presented. After that, the biologically inspired shunting model that is used for generating smooth velocities is introduced. The proposed control algorithm is presented. Finally the stability and convergence of the proposed control systems are proved using a Lyapunov stability analysis.

3.1 Time-optimal Bang-Bang Control

If the control is bang-bang, the wheels accelerations can switch from their upper limit to their low limit. A bang-bang control for a double integrator system is represented by

$$\begin{aligned} v_s &= v_t - v_c + (2a_{\max}|x_t - x_c|)^{1/2}\mathrm{sign}(x_t - x_c), \\ \alpha_c &= \alpha_{\max}\mathrm{sign}(v_s), \end{aligned} \tag{12}$$

where x_c, v_c, α_c are position, velocity and acceleration of the robot, respectively, and v_s is a switching condition of the acceleration. The bang-bang control is asymptotically stable, the errors converges to the origin if the acceleration is bounded by $\alpha_{\max}$ [6].

3.2 Landing Curve

In order to satisfy the nonholonomic constraints, the landing curve is employed. This landing curve provides the heading angle and velocity values that guide the robot could softly land on the target path. Here, cubic spiral is chosen, because it provides an more optimal smooth path than other curves like clothoids. The landing curve is designed as follows: its origin O_p is on the target line, the curve is defined and satisfied the following boundary conditions as

$$y_p = \pm c_e {x_p}^3, \tag{13}$$

where if $x_p = 0, y'_p = 0$, and $y''_p = 0$, and c_e is a positive constant, the sign can be chosen according to e_y, in the following discussion, it is assumed to be positive.

3.3 Shunting Model

The shunting model was first proposed by Grossberg to understand the real-time adaptive behavior of individuals to complex and dynamic environmental contingencies [2]. In this paper the shunting model inspired by biological neuron system is employed to improve the tracking performance by substituting the sign function in the bang-bang control, since the output of shunting model is changed smoothly. The model is given as follows

$$\frac{d\zeta}{dt} = -A\zeta + (B - \zeta)S^+ - (D + \zeta)S^- \tag{14}$$

where A, B, D are positive constants, S^+ is excitatory input and S^- is the inhibitory input. The outputs are guaranteed to stay in a region $[-D, B]$ for any excitatory and inhibitory inputs.

3.4 Proposed Control Algorithm

This control law is developed based on bang-bang control by means of shunting model and landing curve with consideration of dynamic constraints. It includes both proportional feedback and derivative term of the offset to guarantee system stability. The control law consists of two parts: linear velocity control and steering control law.

3.4.1 Linear Velocity Control

There is no nonholonomic constraint for the linear velocity, so the change of linear velocity is given by

$$\begin{aligned} v_s &= \dot{e}_x + (2a_{\max}|e_x|)^{1/2}v_{s1}, \qquad (15) \\ \frac{dv_{s1}}{dt} &= -Av_{s1} + (B - v_{s1})S^+ - (D + v_{s1})S^-, \end{aligned}$$

where the positive input S^+ is defined as $S^+ = \max(e_x, 0)$, while negative input S^- is defined as $S^- = \max(-e_x, 0)$. From Eq. 15, consider the acceleration dynamic constraint, the linear acceleration in continuous values is derived as

$$\alpha_c = \begin{cases} \alpha_{\max}, & \text{if } v_s/dt \geq \alpha_{\max}, \\ -\alpha_{\max}, & \text{if } v_s/dt < \alpha_{\max}, \\ v_s/dt, & \text{otherwise.} \end{cases}$$

3.4.2 Steering Control Law

Regarding the steering control law, we first need to determine the values of heading angle and angular velocity of the landing curve because of the nonholonomic constraint, and then the robot is supposed to follow the landing curve. The heading angle θ_p and angular velocity ω_p of landing curve are given by

$$\begin{aligned} \theta_p &= \theta_t + \mathrm{atan}\left[3c_e(\frac{e_y}{c_e})^{2/3}\right], \\ \omega_p &= \omega_t + \frac{2(\frac{e_y}{c_e})^{-1/3}e_y}{1 + \tan(\theta_p - \theta_t)^2}. \end{aligned} \tag{16}$$

Therefore, the dynamics of the robot angular velocity is obtained as

$$\begin{aligned} \omega_s &= (\omega_p - \omega_c) + (2\omega_{\max}|\theta_p - \theta_c|)^{1/2}\omega_{s1}, \qquad (17) \\ \frac{\omega_{s1}}{dt} &= -A\omega_{s1} + (1 - \omega_{s1})S^+ - (1 + \omega_{s1})S^-, \end{aligned}$$

where S^+ is equal to $\theta_p - \theta_c$ if $\theta_p > \theta_c$, otherwise 0. S^- is equal to $-(\theta_p - \theta_c)$, if $\theta_p < \theta_c$, otherwise, $S^- = 0$. The angular acceleration is derived as

$$\beta_c = \begin{cases} \beta_{\max}, & \text{if } \omega_s/dt \geq \beta_{\max}, \\ -\beta_{\max}, & \text{if } \omega_s/dt < \beta_{\max}, \\ \omega_s/dt, & \text{otherwise.} \end{cases}$$

3.5 Stability Analysis

For the driving velocity control, since we choose constant B and D are equal to 1, that means, the output of v_{s1} will stay at ± 1. Under this condition, the control law related to the bang-bang control and the stability of the bang-bang control guaranteed that the tangential path error e_x converges to zero [6]. Meanwhile in order to show that e_y and e_θ converge to zero too, a Lyapunov function is defined as

$$V = \frac{1}{2}{e_y}^2 + \frac{1}{2}{e_\theta}^2, \tag{18}$$

so the derivative of V is given as

$$\dot{V} = e_y \dot{e}_y + e_\theta \dot{e}_\theta. \tag{19}$$

From Eq. (3), $\dot{V}$ becomes

$$\dot{V} = e_y(v_c \sin e_\theta - \omega_t e_x) + e_\theta(\omega_t - \omega_c), \tag{20}$$

base on above results $\theta_c \to \theta_p$ and $e_x \to 0$, eventually,

$$e_y \dot{e}_y = \frac{-e_y \tan(\theta_p - \theta_t)}{(1 + \tan(\theta_p - \theta_t)^2)^{1/2}}. \tag{21}$$

Since

$$\tan(\theta_p - \theta_t) = 3c_e(\frac{e_y}{c_e})^{2/3} sign(e_y), \tag{22}$$

Eq. (21) becomes

$$e_y \dot{e}_y = \frac{-3e_y v_c c_e(\frac{e_y}{c_e})^{2/3}\text{sign}(e_y)}{(1 + \tan(\theta_p - \theta_c)^2)^{1/2}} < 0, \tag{23}$$

because $c_e > 0$ and $e_y \text{sign}(e_y) > 0$. It's same for the second term of Eq. (20), $\omega_c \to \omega_p$, the result becomes

$$e_\theta \dot{e}_\theta = \frac{-2e_\theta v_c \sin e_\theta(\frac{e_y}{c_e})^{-1/3}\text{sign}(e_y)}{(1 + \tan(\theta_p - \theta_c)^2)^{1/2}} < 0, \tag{24}$$

because $(e_y/c_e)^{-1/3}\text{sign}(e_y) > 0$ and $e_\theta \sin e_\theta > 0$ if $0 < |e_\theta| < \frac{\pi}{2}$. From the results of Eq.s (23) and (24), $\dot{V} < 0$, if and only if for $e_y = 0$ and $e_\theta = 0$, the $\dot{v} = 0$. Therefore, e_y and e_θ converge to zero. The system is stable.

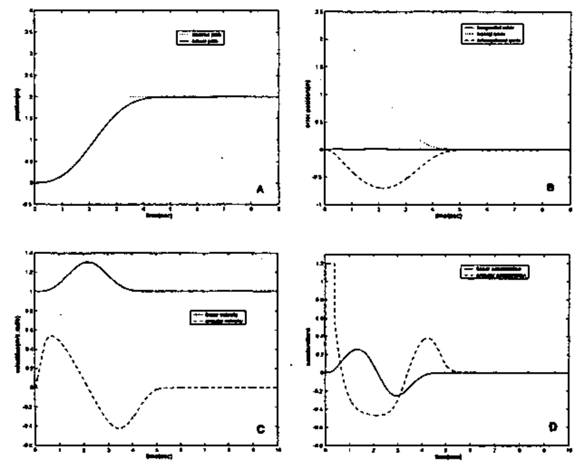

Figure 4: *Tracking a straight line.* A: *Tracking performance ;* B: *Tracking errors;* C: *Velocities of the robot;* D: *Accelerations of the robot.*

4 Simulation Studies

This section provides two simulation studies to demonstrate the effectiveness of the proposed tracking control algorithm. The mobile robot is first to tracking a simple straight line, then a path of two straight lines jointing with a half circle.

4.1 Tracking a Straight Line

The proposed tracking controller is first applied to tracking a horizontal straight line. In this simulation, the dynamic constraints are chosen as $a_{\max} = 0.3m/sec^2$, $\beta_{\max} = 1.2rad/sec^2$. The target velocities are chosen as $v_t(0) = 1m/s$, $\omega_t(0) = 0rad/s$. The initial error posture is $e_x(0) = 0m$, $e_y(0) = 1m$, $e_\theta = 0rad$. The initial velocities of the robot are $v_c(0) = 1m/s$, and $\omega_c(0) = 0rad$. Fig. 4A shows the real time tracking performance of the robot. The dynamic tracking errors are shown in Fig. 4B. Fig.s 4C and 4D give the dynamic velocities and accelerations during the robot navigation. From the results, it shows that the robot starts from a initial position and then is trying to track the desired line. It takes a short time to catch up the desired line and land on the desired path smoothly. The position errors are converged to zero quickly. It shows that the proposed controller has very smooth and quick tracking response.

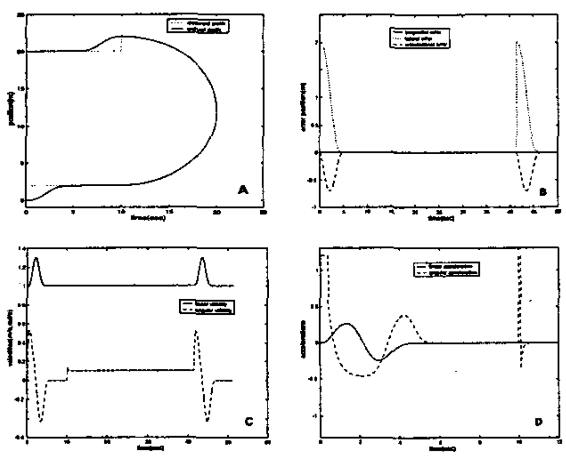

Figure 5: *Tracking a path of two straight lines jointing with a half circle.* A: *Tracking performance;* B: *Tracking errors;* C: *Velocities of the robot;* D: *Accelerations of the robot.*

4.2 Tracking a Circular Path

The proposed algorithm is also can be applied to a more complicated path. In Fig. 5, the simulation results for following a path that consists of two straight lines and a half circle are shown in Fig. 5. All the conditions are same as in previous case, except $\omega_t = 0.1rad/s$. From Fig. 5, it shows that the robot also starts from a initial position with large offset from the desired path. It can track the desired path smoothly and the tracking errors are also converge to zero quickly. Meanwhile all the dynamic constraints are satisfied.

5 Conclusion

A path tracking algorithm for nonholonomic mobile robot is proposed based on previous works by employing the landing curve and shunting model. This method is derived with fully consideration of acceleration and nonholonomic constraints. Hence, the tracking controller using this algorithm can generate limited acceleration commands that produce smooth robot velocities, by which the robot is capable of tracking the desired trajectories. To deal with the nonholonomic problem which makes it impossible to design the time-optimal path tracking controller, the proposed algorithm is not strictly time-optimal. However, by employing the shunting model, the proposed controller can guarantee the real-time and smooth tracking performance action. The stability analysis and a series of simulation results show that the method proposed is efficient and stable. This method is mainly hardware-independent, hence can be applied easily to various kinds of mobile robots with two driving wheels.

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