In summary, we have designed a hybrid system that solves the inverted pendulum problem. Global stability is guaranteed with respect to the swing up and balancing control task. For the case of the full state regulation problem we provide stability results which are valid locally. Learning techniques can be use to improve the performance of the hybrid system, and robust/adaptive techniques (e.g., neural networks) can be used to deal with parametric and nonparametric uncertainties.

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A Neural Gain Scheduling Network Controller for Nonholonomic Systems

Jin-Tsong Jeng and Tsu-Tian Lee

***Abstract*— In this paper, we propose a neural gain scheduling network controller (NGSNC) to improve the gain scheduling controller for nonholonomic systems. We have derived the neural networks that can approximate the gain scheduling controller arbitrarily well when the sampling frequency satisfies the Sampling Theorem. We have also shown that the NGSNC is independent of the sampling time. The proposed NGSNC has the following important properties:**

1) **same performance as the continuous-parameter gain scheduling controller;**
2) **less computing time than the continuous-parameter gain scheduling controller;**
3) **good robustness against the sampling intervals;**
4) **straightforward stability analysis.**

We then show that some of nonholonomic systems can be converted to equivalent linear parameter-varying systems. As a result, the NGSNC can stabilize nonholonomic systems.

***Index Terms*— Gain scheduling, neural networks, nonholonomic systems.**

I. Introduction

In this paper, we propose neural gain scheduling networks to improve the gain scheduling techniques for nonholonomic systems. In general, gain scheduling is a technique commonly used in designing controllers for time-varying systems [1]–[6]. Roughly speaking, design of controllers using the gain scheduling technique is as follows:

1) linear time-invariant approximations are obtained;
2) linear time-invariant controllers are designed for each linearized representation of the system at the selected operating points so that the stability and certain performance objectives are achieved;
3) these controllers are then linked together in order to obtain a single controller for the entire range of operation.

Shahruz and Behtash [7] first proposed a new algorithm to design a controller for a linear multi-input multi-output (MIMO) system whose dynamics depend on a time-varying parameter. However, this method can only obtain the piece-wise linear controller. Although continuous-parameter gain scheduling techniques have good performance, these algorithms require much computing time. Hence, we propose a new method which uses neural networks to overcome these problems.

In general, there are two types of connections in neural networks. Neural networks with only feedforward connections are called "feedforward networks," and neural networks with arbitrary connections are "recurrent networks." Since the introduction of the back-propagation learning algorithm by Rumelhart *et al*. [8], several applications of feedforward neural networks have been reported, specifically, in static information processing such as kinematics control, pattern recognition, and function approximation. Mathematically

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speaking, a known continuous mapping on a compact set can be realized by three-layer feedforward neural networks with desired precision [9], [10]. The nonlinear dynamical behavior of recurrent networks is suitable for spatio-temporal information processing. Learning algorithms for recurrent neural networks which employ the steepest decent method to modify weights have been proposed [11], [12]. Several applications of recurrent neural networks have been reported, mainly in dynamic information processing such as control and identification of nonlinear and linear dynamic systems [13]–[15]. For nonholonomic control systems, it is important to define either kinematics models, which are affine nonlinear driftless systems, or dynamic models, which are affine nonlinear systems, for designing the stabilization controllers. The stabilizing methods for nonholonomic systems are highly dependent on modeling techniques [16]. In order to obtain a unified design method in the kinematics model, most nonholonomic systems must be transformed to nonholonomic chained systems. These transformative techniques can be found in [17], [18]. Examples of nonholonomic systems are mobile robots, front-wheel drive carts, a car with n trailers and fire trucks. In particular, the kinematics of a mobile robot, three degrees of freedom with two independently driven wheels, can be transformed into a three-dimensional one chain, single generator chained system. For the n-dimensional one chain single-generator, chained systems, such as front-wheel drive carts and a car with n trailers, a description of the stabilizing controller can be found in [19], [20]. For the m-chain, single-generator chained system, such as a fire truck, a description of the stabilizing controller can be found in [21]–[23]. Because these chained systems are driftless systems, their stability results are all based on a certain Lyapunov function. Moreover, these methods use unsmooth and produce complicated trajectories for the feedback stabilization of nonholonomic systems. Sadegh [24] was the first to apply neural networks to control the dynamic models of a nonholonomic system. However, when a nonholonomic system is modeled as a kinematics model, this method [24] can not be directly used in the design of a controller.

The purpose of this paper is to develop a new neural network controller that not only has the same performance as the continuous-parameter gain scheduling controller, but also requires less computing time than some existing gain scheduling controllers. We show that the proposed neural networks can approximate the gain scheduling controller arbitrarily well when the sampling frequency satisfies the Sampling Theorem. That means the gain scheduling controller can be fully reconstructed by neural networks using the discrete-parameter gain and parameter input as training pairs. Besides, we also show that the NGSNC is independent of the sampling time. As a result, when the sampling time is changed, the controller does not need to be redesigned. In addition, in order to design a controller with pole-assignment, and to overcome the drawbacks of gain scheduling techniques, we use the NGSNC for the kinematics model of nonholonomic systems. In this application, the nonholonomic system is formulated as a linear MIMO parameter-varying system by selecting a suitable nonzero control input. NGSNC is then applied to stabilize the new formulated linear parameter-varying system by pole-assignment. Finally, an illustrative example for stabilization of a car with five trailers is included.

The preceding was an outline of the contents of this paper. The definitions and mathematical preliminaries are presented in Section II. In Section III, the problem statement is described. The main results are given in Section IV. Computer simulations are described in Section V. Finally, conclusions are included in Section VI.

II. Mathematical Preliminary: Regularization Network

The proposed neural network controller shown in Fig. 1 is one type of regularization network [10]. Let $B = \{(t_i, Y(t_i)) | (t_i, Y(t_i)) \in R \times R^n, t_i = t_{i-1} + \Delta T, \Delta T = t/T_s, i = 1, \cdots, N\}$ be a set of training data, where t_i is the input of nonlinear gain, Y is the output of nonlinear gain, Ts is the sampling time (or interval), and t is the input interval. The regularization approach [10] determines the approximating function f by minimizing the functional

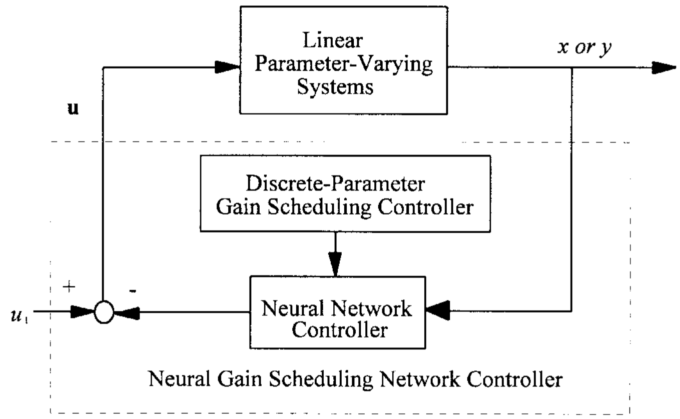

Fig. 1. Proposed structure of NGSNC to improve the gain scheduling controller.

$$H[f] = \sum_{i=1}^{N} (Y(t_i) - f(t_i))^2 + \lambda ||df||_2 \tag{1}$$

where d is a constraint operator, $||\cdot||_2$ (usually a differential operator), is a norm on the function space, and λ is a regularization parameter and positive real number. Minimization of the functional H leads to the associated Euler–Lagrange equation,

$$\hat{d}df(t) = \frac{1}{\lambda} \sum_{i=1}^{N} (Y(t_i) - f(t_i)) \delta(t - t_i) \tag{2}$$

where $\hat{d}$ is the adjoint of the differential operator d, and the right side of (2), coming from the functional derivative with respect to f on H. Equation (2) is a partial differential equation, and it is well known that its solution can be written as the integral transformation with a kernel given by Green's function G of the differential operator $\hat{d}d$. Green's function G satisfies the following distributional differential equation

$$\hat{d}dG(t_i; Y(t)) = \delta(t - Y(t)). \tag{3}$$

Because of the delta functions appearing in (2) the integral transformation becomes a discrete sum. From (3) and (2), we can obtain

$$f(t) = \frac{1}{\lambda} \sum_{i=1}^{N} (Y(t_i) - f(t_i)) G(t; t_i). \tag{4}$$

Let

$$c_i = \frac{1}{\lambda}(Y(t_i) - f(t_i))$$

then

$$f(t) = \sum_{i=1}^{N} c_i G(t; t_i). \tag{5}$$

When (5) is adopted to construct the neural networks, it is called a regularization network.

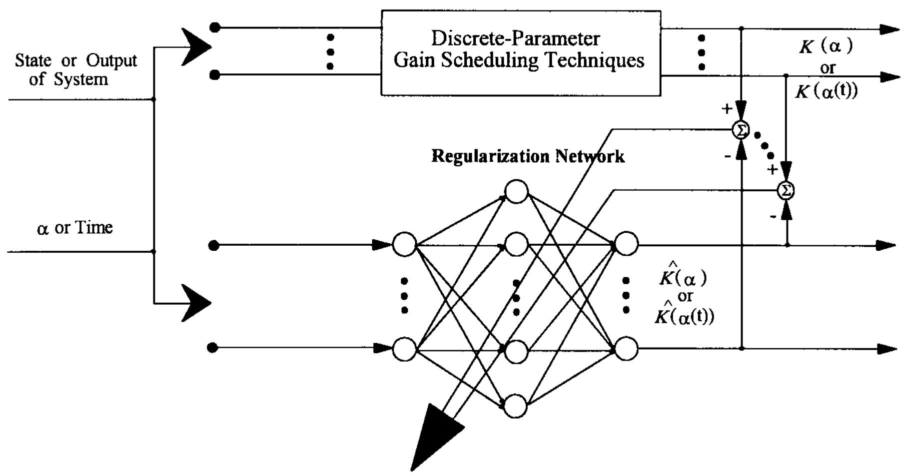

Fig. 2. Structure of neural gain scheduling network.

Proposition 1: [10] For every continuous function f defined on a compact subset of R, and every piece-wise continuous function G which is Green's function of a self-adjoint differential operator, there exists a function $\hat{f}(t) = \Sigma_{i=1}^{N} c_i G(t; t_i)$, such that for all t and any positive ε the following inequality holds:

$$|f(t) - \hat{f}(t)| < \varepsilon, \tag{6}$$

where ε is the error between $f(t)$ and $\hat{f}(t)$. ■

III. Problem Statement

In this paper, we propose a NGSNC to improve the gain scheduling controller for linear parameter-varying systems. Consider a control structure as shown in Fig. 1, where u_1 is exogenous input, u is control input, x represents state, and y is output.

The linear MIMO parameter-varying system is represented by

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)u(t), \quad x(0) = x_0 \tag{7}$$
$$y(t) = C(\alpha)x(t) \tag{8}$$
$$\alpha = \alpha(t) \tag{9}$$

where the state $x(t) \in R^n$, the input $u(t) \in R^n$, and the output $y(t) \in R^n$; for all $t \geq 0$, the parameter $\alpha = \alpha(t) \in [\alpha_0 \cdots \alpha_n] \equiv I \subset R$; for all $\alpha \in I$ the coefficient matrix $A(\alpha) = [a_{ij}(\alpha)] \in R^{n \times n}$, $B(\alpha) = [b_{ij}(\alpha)] \in R^{n \times n_i}$, and $C(\alpha) = [c_{ij}(\alpha)] \in R^{n_o \times n}$; the number of inputs $n_i \leq n$ and for all $\alpha \in I$ the matrix $B(\alpha)$ is full column rank.

The state feedback control

$$u(t) = -K(\alpha(t))x(t) + u_1(t) \tag{10}$$

to the linear parameter-varying systems for all $t \geq 0$, where for all $\alpha \in I$ the matrix $K(\alpha) \in R^{n_i \times n}$ is the state feedback gain matrix, and for all $t \geq 0$ the input $u_1(t) \in R^{n_i}$ is some exogenous input to the system.

It is assumed that

1) elements of the coefficient matrix A, B, and C are analytic functions of α;
2) parameter α is a continuous and bounded function of t, differentiable almost everywhere with bounded derivative, and is measured for all $t \geq 0$;
3) linear parameter-varying system is completely controllable for all $\alpha \in I$;
4) sampling frequency must satisfy the Sample Theorem.

Our objective is to employ the neural networks to learn the state feedback gain matrix $K(\alpha)$ such that the NGSNC will implement (10).

IV. Main Results

A. Neural Gain Scheduling Network

In order to overcome the drawbacks of the gain scheduling technique, we propose a neural gain scheduling network (NGSN) which is shown in Fig. 2. That is, the proposed structure mixes the gain scheduling technique and a regularization network. For the proposed structure, a state feedback gain matrix is required, and this is computed using the gain scheduling technique as target data. The continuous-time state feedback gain matrix $K(\alpha)$ or $K(\alpha(t))$ can be obtained by the continuous-parameter gain scheduling technique [25]. A matrix $F \in R^{n \times n}$ must be chosen such that $\sigma(F) = \sigma_d$, and that for all $\alpha \in I$, $\sigma(A(\alpha)) \cup \sigma(F) = \phi$, where $\sigma(F)$ denotes all eigenvalues of matrix F, and ϕ is the empty set. In addition, a matrix $\overline{K} \in R^{n_i \times n}$ must be chosen such that the pair $(F, \overline{K})$ is observable. For a given $\alpha \in I$ find the unique solution matrix $T(\alpha) \in R^{n \times n}$ of the Lyapunov matrix equation

$$T(\alpha)F - A(\alpha)T(\alpha) = -B(\alpha)\overline{K}. \tag{11}$$

If $T(\alpha)$ at $\alpha \in I$ is nonsingular, then the gain matrix is

$$K(\alpha) = \overline{K}T^{-1}(\alpha) \tag{12}$$

From the results of the continuous-parameter gain scheduling technique, we know that the computing time is mainly spent in solving the Lyapunov matrix equation in (11) and the matrix inverse in (12). Note that in order to reduce the computing time, the discrete sequence $\alpha_0, \alpha_1, \cdots, \alpha_l, \alpha_{l+1}, \cdots$ can be obtained via the discrete-parameter gain scheduling technique [7]. Let α_s be a point between α_{l-1} and α_l, then $K(\alpha_s)$ can be obtained by linear interpolation as

$$K(\alpha_s) = K(\alpha_{l-1}) + \frac{K(\alpha_l) - K(\alpha_{l-1})}{\alpha_l - \alpha_{l-1}}(\alpha_s - \alpha_{l-1}). \tag{13}$$

Hence, the discrete-parameter gain scheduling technique is used in (13) to approximate the continuous-parameter gain scheduling technique. That is, the discrete-parameter gain scheduling technique uses fewer sampling points than the continuous-parameter gain scheduling technique. However, the transient response and performance are inferior to that of the continuous-parameter gain scheduling technique. Besides, the interval of the discrete-parameter gain scheduling technique is decided via Lemma 1.

Lemma 1 [7]: Consider the linear parameter-varying system (7)–(9). Let the elements of the coefficient matrix A and B be a continuously differentiable function of α, and let the assumption (A3) hold. Let the solution $T(\alpha_l)$ of the Lyapunov matrix (11) for a matrix $\overline{K}$ be nonsingular for a given $\alpha_l \in I$. Let $\varepsilon > 0$, and

$$\alpha_{l+1} = \alpha_l + h \tag{14}$$

where

$$h \equiv \frac{\varepsilon}{\max\{a^*, b^*\}},$$
$$a^* \equiv \max_{1\le i,j\le n} \max_{\alpha\in I} \left|\frac{\partial a_{ij}(\alpha)}{\partial \alpha}\right|, \quad \text{and}$$
$$b^* \equiv \max_{1\le i,j\le n} \max_{\alpha\in I} \left|\frac{\partial b_{ij}(\alpha)}{\partial \alpha}\right|.$$

Then for sufficiently small ε the solution of (11) at α_{l+1} for the same $\overline{K}$ satisfies

$$T^{-1}(\alpha_{l+1}) = T^{-1}(\alpha_l) + O(\varepsilon). \tag{15}$$

■

We can show that both $T(\alpha)$ and $T^{-1}(\alpha)$ are analytic functions of α from Lemma 1. Besides, the state feedback gain matrix is

$$K(\alpha) = \overline{K}T^{-1}(\alpha) \tag{16}$$

Therefore, the state feedback gain matrix $K(\alpha)$ is an analytic function of α. Hence, we can use the regularization neural networks to reconstruct the analytic function by using the discrete-time sequence $\alpha_0, \alpha_1, \cdots, \alpha_l, \alpha_{l+1}, \cdots$.

Theorem 1: Assume that assumption (A4) holds. If the state feedback gain matrix $K(\alpha)$ is a continuous function of α, then the NGSN can approximate $K(\alpha)$ arbitrarily well.

Proof: It can be proved directly via Proposition 1. ■

When assumptions (A1)–(A4) hold and the gain matrix K is a function of α, we propose Algorithm 1 given below to compute and learn the state feedback gain matrix. Algorithm 1 adopts the discrete-parameter data to reconstruct the continuous function by regularization neural networks. The proposed algorithm is as follows:

Algorithm 1:

Step 1: Solve (11) for $T(\alpha_0)$ and compute $K(\alpha_0)$ from (12). Let $l = 1$.

Step 2:

$$I_{l-1} \equiv [\alpha_{l-1}\alpha_{l-1} + \varepsilon]$$
$$a^* \equiv \max_{1\le i,j\le n} \max_{\alpha\in I_{l-1}} \left|\frac{\partial a_{ij}(\alpha)}{\partial \alpha}\right|,$$
$$b^* \equiv \max_{1\le i,j\le n} \max_{\alpha\in I_{l-1}} \left|\frac{\partial b_{ij}(\alpha)}{\partial \alpha}\right|$$

If $\max\{a^*_{l-1}, b^*_{l-1}\} \le 1$, **then** $h_{l-1} = \varepsilon$;

otherwise

$$h_{l-1} \equiv \frac{\varepsilon}{\max\{a^*_{l-1}, b^*_{l-1}\}}$$

$$\alpha_l = \alpha_{l-1} + h_{l-1}$$

77 Solve (11) for $T(\alpha_l)$

If $T(\alpha_l)$ is singular, **then** set $\alpha_l = \alpha_l'$ where

$\alpha_{l-1} < \alpha_l' < \alpha_l$, and **go to 77**.

Step 3: Compute $K(\alpha_l)$ from (12).

Step 4: **If** $\alpha_l < \alpha_n$, **then** $l = l + 1$ and go to Step 2; **else** Solve (11) for $T(\alpha_n)$ and compute $K(\alpha_n)$ from (12).

Step 5: Use the neural networks $\hat{K}(\alpha)$ to learn the state feedback gain matrix $K(\alpha)$.

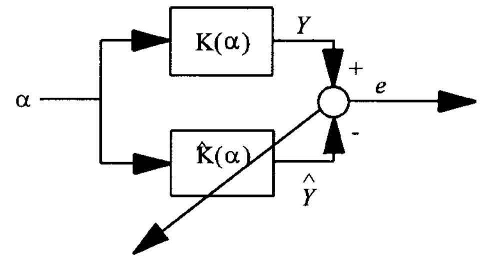

Fig. 3. Simplification of Fig. 1 when α is used as input.

Step 6: **If** $K(\alpha) \approx \hat{K}(\alpha)$ for all input, **then stop; otherwise** go to Step 5. ■

In Algorithm 1, a basic property of regularization neural networks is used to replace the linear interpolation in the discrete-parameter gain scheduling technique. This basic property in regularization is that neural networks can generalize (interpolate and extrapolate) the training information to similar situations. Hence, Steps 1–4 in Algorithm 1 are used to generate $K(\alpha_l)$ for all l, and Steps 5–6 are used to learn the state feedback gain matrix $K(\alpha)$. Thus, the proposed method can overcome the drawbacks of the piece-wise linear controller obtained via the discrete-parameter gain scheduling technique.

When the parameter α is a time-variant case, Algorithm 1 cannot be directly used. Hence, in order to overcome this drawback of Algorithm 1, the following developments are required.

Lemma 2 [26]: Let (S, d_S), (U, d_U), and (W, d_W) be metric spaces. Let f: $S \to U$ and g: $f(S) \to W$ be functions, and let q be the composite function defined on S by the equation

$$q(s) = g(f(s)) \quad \text{for all } s \text{ in } S.$$

If f is a continuous function at s and if g is continuous at $f(s)$, then q is continuous at s. ■

From Lemma 1, we have shown that $K(\alpha)$ is an analytic function. In addition, assumption (A2) implies that $\alpha(t)$ is also an analytic function. Therefore, using Lemma 2, we can show that the composite functions $K(\alpha(t))$ is continuous. Hence, when $K(\alpha(t))$ is a continuous function of t, we can use regularization neural networks to reconstruct the continuous functions using the discrete-time sequence $t_0, t_1, \cdots, t_l, t_{l+1}, \cdots$. Thus, we have the following Theorem 2.

Theorem 2: Assume that the assumption (A4) holds. If the state feedback gain matrix $K(\alpha(t))$ is a continuous function of t, then the NGSN will approximate $K(\alpha(t))$ arbitrarily well.

Proof: It can be proved directly via Proposition 1. ■

Now, we shall show that the NGSN is independent of the sampling interval when the training pairs are $(\alpha, K(\alpha))$. Fig. 2 can be simplified as Fig. 3, where α is the input of nonlinear gain, Y is the output of nonlinear gain, $\hat{Y}$ is the output of NGSN, and e is the error between Y and $\hat{Y}$. The sampling interval must satisfy the Sample Theorem. From Fig. 3, we have

$$\begin{aligned} e(n) &= Y(n) - \hat{Y}(n) \\ &= K(\alpha_l) - \hat{K}(\alpha_l) \end{aligned} \tag{17}$$

where $\alpha_l = \alpha_{l-1} + h_{l-1}$, $l = 0, 1, 2, \cdots$

According to Theorem 1 the NGSN will approximate $K(\alpha)$ arbitrarily well. Therefore, we have

$$|K(\alpha) - \hat{K}(\alpha)| < \sum_{n=1}^{N} e^2(n) = \varepsilon \tag{18}$$

and

$$K(\alpha_l) = \hat{K}(\alpha_l) + \varepsilon/N \tag{19}$$

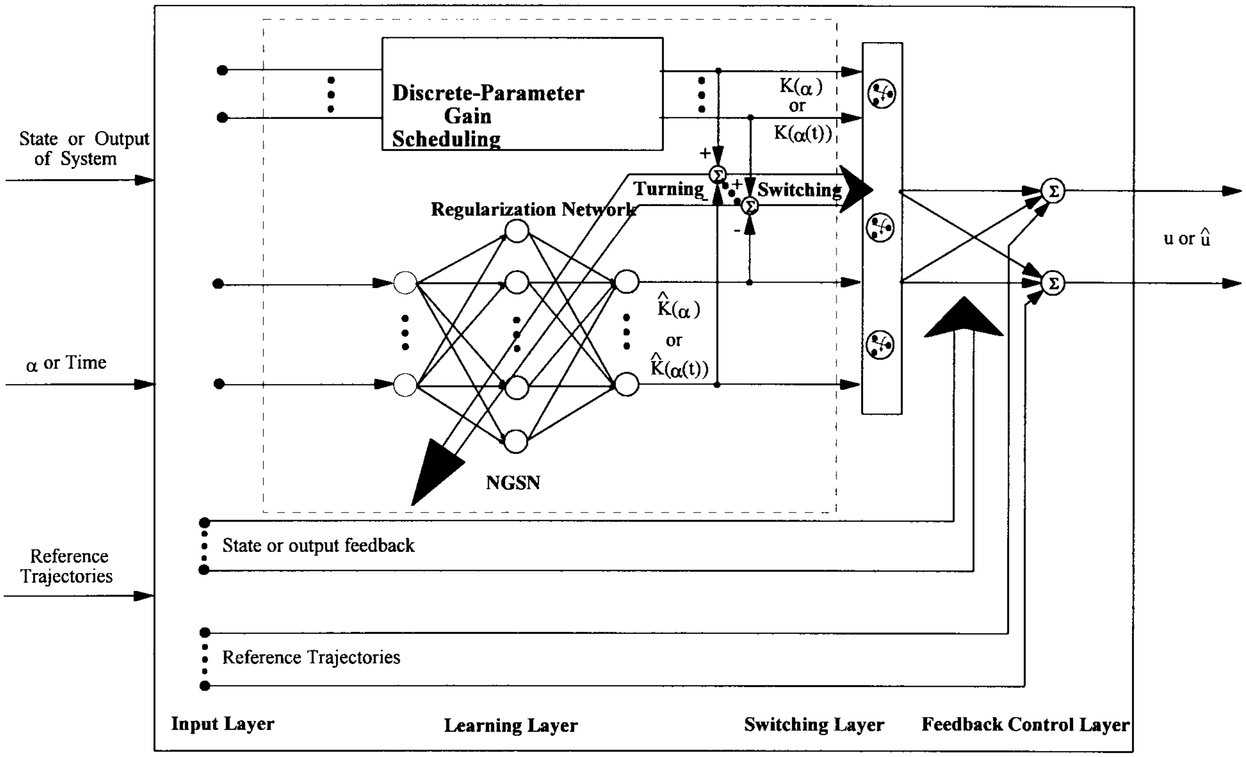

Fig. 4. Structure of the NGSNC.

where

α_l constant in the interval $I = [0, \alpha]$;
N number of sample points;
ε arbitrary small constant.

Therefore, we conclude that

$$K(\alpha) \approx \hat{K}(\alpha), \quad \text{for all input}$$

or

$$||\hat{K} - K||_\infty < \varepsilon, \quad \text{for all input}. \tag{20}$$

That is, $\hat{K}$ converges to K uniformly on α. Since there is no restriction on the step size of the NGSN output, we can conclude that the $\hat{K}$ is independent of the sampling interval.

B. Neural Gain Scheduling Networks Controller

In this section, we shall describe the structure and functions of the proposed NGSNC. The structure of the neural gain scheduling network controller is shown in Fig. 4, where u is the output of NGSNC while the regularization network of NGSNC is learning; $\hat{u}$ is the output of NGSNC when the regularization network has been learned successfully; $K(\alpha)$ and $K(\alpha(t))$ are the feedback gain matrix of discrete-parameter gain scheduling techniques; and $\hat{K}(\alpha)$ and $\hat{K}(\alpha(t))$ are the output of regularization networks.

Fig. 4 shows that there are mainly four layers in this proposed NGSNC. That is, the proposed NGSNC consists of an input layer, a learning layer, a switching layer, and a feedback control layer. Firstly, the input signals of the input layer include state or output of a linear parameter-varying system, time or α (a parameter of a linear parameter-varying system), and reference trajectories. Secondly, the learning layer mainly uses a regularization network to ensure that the gain scheduling techniques can be fully reconstructed by neural networks using the discrete-parameter gain and parameter-input as training pairs. Thirdly, the switching layer is used to determine if the learning should be stopped or not. We apply a three-way switch to perform this function. When the error feedback between K and $\hat{K}$ is large, the output of NGSNC is u and the regularization network of NGSNC is learning. When the error feedback is very small, it means that the gain scheduling techniques have been fully reconstructed by the regularization neural network. Thus, the three-way switch turns on to $\hat{u}$. Finally, the feedback control layer implements state feedback control as given in (10). From (10), the state x can be regarded as the weight of NGSNC, and the reference trajectories can be regarded as the bias of NGSNC. That is

$$u = -Kx + u_1 = KW + b \tag{21}$$

where

W $= -x$;
b $= u_1$;
W weight of the feedback control layer;
b bias of feedback control layer.

Further

$$\hat{u} = \hat{K}W + b. \tag{22}$$

From Algorithm 1, we show that the NGSNC can overcome the linear interpolation problem for the gain scheduling controller under the proposed structure. From [3], [4], and [7], the stability can be guaranteed for the gain scheduling techniques. At the same time, we mixed the gain scheduling technique and the neural network to propose a NGSNC in this paper. It turns out that when assumption (A4) holds, the proposed NGSNC has the same stability result as that of [3], [4], and [7]. Proposition 1 guaranteed that the regularization network can approximate the smooth function arbitrary well. Examples of regularization networks are backpropagation (Bp) and radial basis function network (RBF).

C. Nonholonomic Systems and NGSNC

In this section, we show that the kinematics model of a nonholonomic system can be represented as a linear MIMO parameter-varying system. In order to obtain a unified design method, the kinematics model of a nonholonomic system must be transformed to a chained system [17]–[18]. In this paper, we consider the m-chain, single-generator chained system [22] represented by (23), shown at the bottom of the page, where $n_1, n_2, \cdots, n_m$ are dimensions of an m-chain, single-generator chained system, respectively. Equation (23) can be rewritten as two systems

$$\text{system 1:} \qquad \dot{X} = A(u_0)X + BU \tag{24}$$

$$\text{system 2:} \qquad {}^0\dot{x}_0 = u_0 \tag{25}$$

where $X = [{}^1x_0 \cdots {}^1x_{n_1} \cdots {}^mx_0 \cdots {}^mx_{n_m}]^T$, $U = [u_1 \cdots u_m]^T$, and $A(u_0)$ as well as B can be easily obtained from (23). When u_0 in system 2 is properly chosen and is represented by $u_0 = \alpha(t) \neq 0$, the system becomes a linear MIMO parameter-varying system as follows:

$$\begin{aligned} \dot{X} &= A(u_0)X + BU \\ u_0 &= \alpha(t). \end{aligned} \tag{26}$$

The controllability matrix Q is

$$Q = [B \quad AB \quad \cdots \quad A^pB], \tag{27}$$

with

$$\det(Q) = u^{n_1(n_1+1)+\cdots+n_m(n_m+1)/2}. \tag{28}$$

As long as $u_0 \neq 0$, the controllability matrix Q is of full rank, and gain scheduling techniques can be applied to stabilize the above chained system.

It is easy to see that as long as $u_0 \neq 0$, the three assumptions (A1)–(A3) hold and the system in (26) can be stabilizing by gain scheduling techniques. Besides, when the assumption (A4) holds, the nonholonomic system can also be stabilizing by the NGSNC. Moreover, the control input u_0 can be determined easily.

Let u_0 be an exponential function. Taking the Laplace transformation of (25), we have

$${}^0x_0(s) = \frac{{}^0x_0(0) + u_0(s)}{s}. \tag{29}$$

It is assumed that state ${}^0x_0$ has zero steady state error; therefore

$$\lim_{t\to\infty} {}^0x_0(t) = \lim_{s\to 0} {}^0x_0(0) + u_0(s) = 0 \tag{30}$$

and $u_0(0) = -{}^0x_0(0)$. Thus, $u_0(s)$ has the form

$$u_0(s) = -\frac{{}^0x_0(0)k_\beta}{s + k_\beta} \tag{31}$$

and hence

$$u_0(t) = -{}^0x_0(0)k_\beta e^{-k_\beta t} \tag{32}$$

where ${}^0x_0(0) \neq 0$ and k_β is a design constant. When, ${}^0x_0(0) = 0$ one can change its initial value by properly choosing different reference coordinates such that the new ${}^0x_0(0)$ will be nonzero.

Remark 1: From [2] and [3], the control input u of the gain scheduling techniques for a nonlinear system is similar to (10). Hence, without loss of generality, the NGSNC can also be applied to nonlinear systems. Besides, one of the purposes of using gain scheduling techniques is the ability to design the controller with pole-assignment. Therefore, the NGSNC can also achieve pole-assignment for a time-varying or nonlinear system.

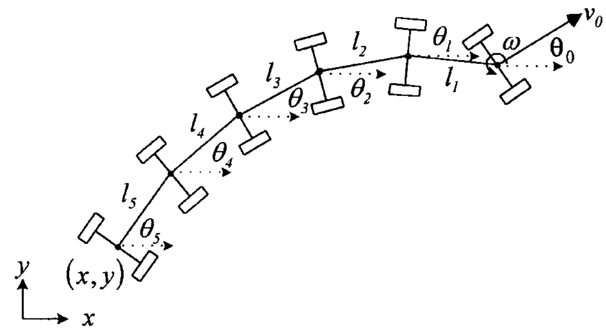

Fig. 5. Configuration of a car with five trailers.

V. Illustrated Examples

All simulations were done using Matlab. Consider a car with five trailers, as shown in Fig. 5, as an example of a six-dimensional one chain signal-generator, chained systems [23], where v_0 is the tangential velocity, ω is the angular velocity of the car, l_i is the distance from the wheels of trailer i to the wheels of trailer $i-1$, with $i = 2, \cdots, 5$, θ_i is the orientation angle of trailer i with respect to the x-axis, with $i = 1, \cdots, 5$. A kinematic model of a car with 2 DOF pulling five trailers can be written as

$$\begin{aligned} \dot{x} &= v_5 \cos\theta_5 \\ \dot{y} &= v_5 \sin\theta_5 \\ \dot{\theta}_5 &= \frac{v_4}{l_5}\sin(\theta_4 - \theta_5) \\ \dot{\theta}_i &= \frac{v_{i-1}}{l_i}\sin(\theta_{i-1} - \theta_i), \qquad i = 1, \cdots, 5 \\ &\vdots \\ \dot{\theta}_1 &= \frac{v_0}{l_1}\sin(\theta_0 - \theta_1) \\ \dot{\theta}_0 &= \omega \end{aligned} \tag{33}$$

where $v_i = v_0 \prod_{j=1}^{i} \cos(\theta_{j-1} - \theta_j)$. The initial condition vector is denoted by $[x, \theta_0, \cdots, \theta_5, y]$. The car with five trailers can be represented as a one-chain, signal-generator, chained system [23] as follows:

$$\begin{bmatrix} {}^1\dot{x}_0 \\ {}^1\dot{x}_1 \\ {}^1\dot{x}_2 \\ \vdots \\ {}^1\dot{x}_5 \\ {}^1\dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ u_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & u_0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & u_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & u_0 & 0 \end{bmatrix} \begin{bmatrix} {}^1x_0 \\ {}^1x_1 \\ {}^1x_2 \\ \vdots \\ {}^1x_5 \\ {}^1x_6 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u_1$$

$${}^1\dot{x}_0 = u_0. \tag{34}$$

$$\begin{array}{lllll} {}^0\dot{x}_0 = u_0 & {}^1\dot{x}_0 = u_1 & {}^2\dot{x}_0 = u_2 & \cdots, & {}^m\dot{x}_0 = u_m \\ & {}^1\dot{x}_1 = {}^1x_0u_0 & {}^2\dot{x}_1 = {}^2x_0u_0 & \cdots, & {}^m\dot{x}_1 = {}^mx_0u_0 \\ & \vdots & \vdots & \ddots & \vdots \\ & {}^1\dot{x}_{n_1} = {}^1x_{n_1-1}u_0 & {}^2\dot{x}_{n_2} = {}^2x_{n_2-1}u_0 & \cdots, & {}^m\dot{x}_{n_m} = {}^mx_{n_m-1}u_0, \end{array} \tag{23}$$

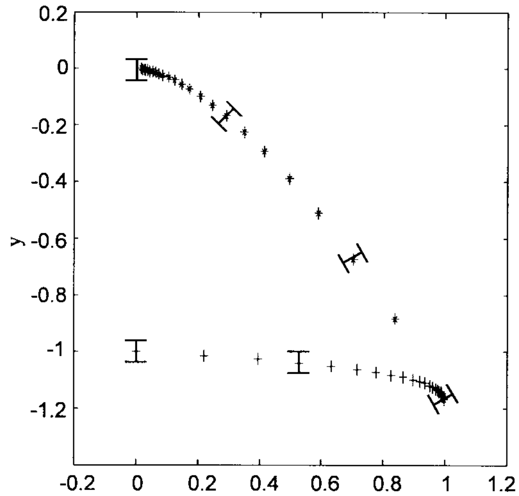

Fig. 6. Display of the sampling point of the trajectory for the car with five trailers converging to origin from (0, 0, 0, 0, 0, 0, 0, -1) under the discrete-parameter gain scheduling techniques.

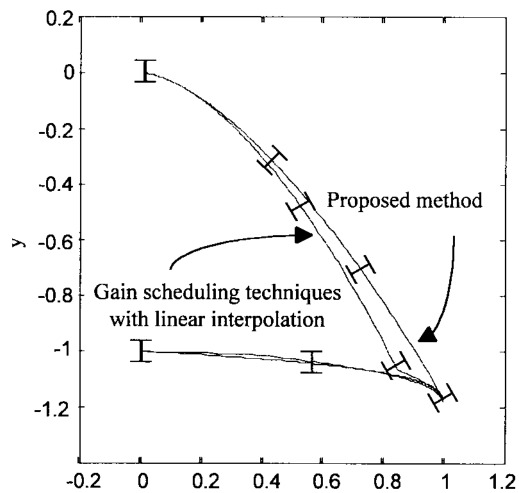

Fig. 7. Display of the trajectory of the car with five trailers converging to origin from (0, 0, 0, 0, 0, 0, 0, -1) via the proposed method and the discrete-parameter gain scheduling techniques with linear interpolation. (The sampling points are 48.)

The control input u_0 is selected such that

$$u_0 = \begin{cases} -1e^{-0.5t} & 0 < t \leq 2, \\ e^{-0.35(t-6)} & 2 < t. \end{cases} \tag{35}$$

The elements of the coefficient matrix A and B are analytic functions of u_0. The parameter u_0 is a continuous and bounded function of t, differentiable almost everywhere with bounded derivative, and is measured for all $t \geq 0$. The linear parameter-varying system is completely controllable for all $u_0 \in I(t)$. Hence, this linear parameter-varying system can be solved by NGSNC. We consider controll a car with five trailers stabilization at the origin point (0, 0, 0, 0, 0, 0, 0, 0) from (0, 0, 0, 0, 0, 0, 0, -1). In this paper, the sample points are 48 as the target data for the proposed method. Fig. 6 shows the sampling point of the trajectory for the car with five trailers under the discrete-parameter gain scheduling techniques. In this case, we show the discrete-trajectory via using the sampling gain and mark the last trailer with some "H" on the figure. In general, a car can not jump between two discrete sampling points based on discrete-gain. Hence, an interpolation, NGSNC or discrete-parameter gain scheduling techniques with linear interpolation, between two sampling points is needed to calculate all gain for the continuous trajectory. Consequently, Fig. 7 displays the trajectory of the car with five trailers converging to origin from (0, 0, 0, 0, 0, 0, 0, -1) via the proposed method and the discrete-parameter gain scheduling techniques with linear interpolation. We observe that the proposed method does provide a smoother converging trajectory than the discrete-parameter gain scheduling techniques with linear interpolation from Fig. 7. Finally, Fig. 8 displays the trajectories of the car with five trailers converging to origin via the proposed method for four different initial conditions, (0.5, 0, 0, 0, 0, 0, $-(\pi/6)$, -1), (-0.5, 0, 0, 0, 0, 0, $-(\pi/4)$, 1), (0.5, 0, 0, 0, 0, 0, $(\pi/3)$, 0.5), and (-0.5, 0, 0, 0, 0, 0, $(\pi/4)$, -0.6), respectively.

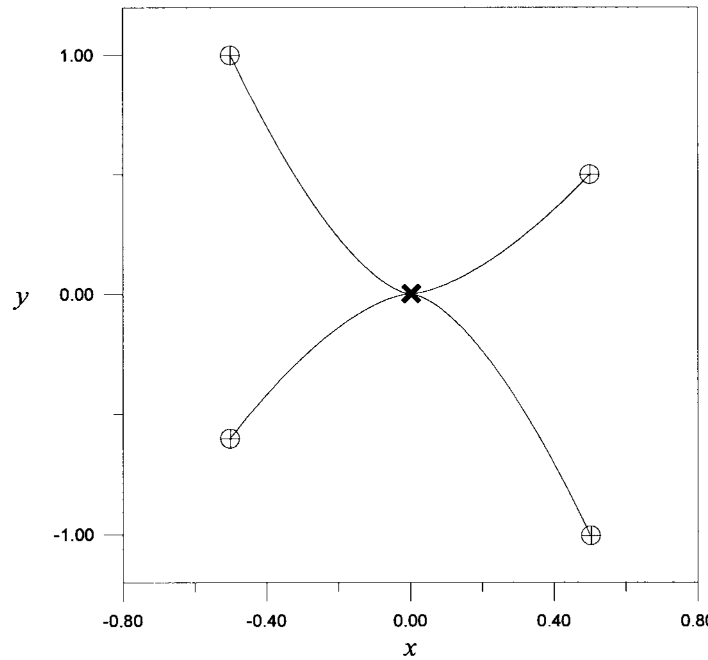

Fig. 8. Display of the trajectories of the car with five trailers converging to origin from four different initial conditions via the proposed method.

VI. Conclusions

In this paper, we have developed a new method using a neural network to improve the gain scheduling techniques for linear parameter-varying systems. In addition, we have also developed two modified algorithms and proposed a NGSNC to overcome the drawbacks of conventional gain scheduling techniques. As a result, the proposed NGSNC has the same performance as the continuous-parameter gain scheduling controller, uses less computing time than the continuous-parameter gain scheduling controller, and exhibits good robustness against the sampling interval. Because the conventional gain scheduling technique is simple and practical, and the proposed NGSNC is superior to conventional gain scheduling techniques, our proposed method has more advantages in application. That is, the proposed method can be easily extended to control other systems based on the conventional gain scheduling technique.

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Competitiveness of Manufacturing Firms: An Application of Fuzzy Weighted Average

Chiang Kao and Shiang-Tai Liu

***Abstract*—In order to attain a competitive edge in the world marketplace, manufacturing companies must produce high-quality products at low cost. To achieve this, many companies have noticed that in addition to technology, management also plays a vital role. Following some previous studies, this paper devises a competitiveness index based on automation technology and manufacturing management for manufacturing firms. Since most measures are described subjectively by linguistic terms, a fuzzy set approach is adopted, in that the fuzzy measures from different criteria are weighted by their corresponding importance, which are also represented by fuzzy values. The fuzzy weighted average approach produces results which are more informative. To distinguish the superiority of the competitive power of different firms, a fuzzy ranking method is applied. As an illustration, the competitiveness indexes of 15 machinery firms in Taiwan are calculated and ranked.**

I. Introduction

A trend in world trade is "trade without barrier." This trend has been especially prevalent since the establishment of the World Trade Organization (WTO). Using tariff and taxes to protect domestic market from the intrusion of foreign products will no longer be possible. Pressure from the global competition is conceivable in the coming years. This pressure mandates manufacturing industry to manufacture high quality products at low cost to meet the needs of customers. It does not require extensive analysis to reveal that competitiveness is the basis for successful operation for a company and the backbone of the economic development of a country.

Improving competitiveness should be a national challenge. It behooves all company managers to do their utmost to achieve this goal. In recent years, there has been increasing concern in many countries about the decline in competitiveness and the long run impacts that this can have on their ability to compete in the world market. Cleveland *et al.* [11] and a number of authors [21], [27], [28] have suggested the concept of production competence in order to assess the level of support that manufacturing provides for the strategic objectives of a company. Azzone and Rangone [3] find that such a measure of production competence only provides a snapshot of manufacturing performance with a short term perspective and propose a framework of manufacturing competence based on manufacturing critical capabilities and infrastructural resources, which is able to consider both short term results and long term performance. This paper discusses the competitiveness of manufacturing firms from another point of view.

It has long been recognized by many leading Western companies that manufacturing technology is a major determinant of the efficiency of the manufacturing process and are using it to retain their competitive edge [25]. The application of complex mechanical, electronic, and computerized systems in the operation and control of production has achieved higher production rates with improved product quality. The case studies conducted by Arze and Svensson

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