Proceedings of the 1998 IEEE/RSJ
Intl. Conference on Intelligent Robots and Systems
Victoria, B.C., Canada • October 1998

A Hybrid System Approach to Motion Control of Wheeled Mobile Robots

Mee-Seub Lim [+], Jinmo Lim [+], Joonhong Lim [+], Sang-Rok Oh [*]

+ : Dept. of Electronic Engineering, Hanyang University, Seoul, Korea.

* : Intelligent System Control Research Center, Korea Institute of Science and Technology.

Abstract

This paper addresses the design of hybrid control systems for the motion control of wheeled mobile robot systems with nonholonomic constraints. The hybrid control system has the 3-layered hierarchical structure: digital automata for the higher process, mobile robot system for the lower process, and the interface as the interaction process between the continuous dynamics and the discrete dynamics. In the hybrid control architecture of mobile robot, the continuous dynamics of mobile robots are modeled by the switched systems. The abstract model and digital automata for the motion control are developed. The motion control tasks for desired-paths with edges and dynamic path following with various initial conditions are investigated as the applications by simulation studies.

1 Introduction

As the applications of mobile robots would be extended to factories, offices and homes, the requirements of tasks would become much higher. For extending of workspace and development of multi-purpose robot system, the wheeled mobile robot (WMR) systems have received much consideration in the last decade. Much research has been carried out in mechanical design, posture detection, visual navigation, path planning, and controller design. The design of control law has been developed to stabilize the motion of mobile robots about either a time-indexed trajectory or a geometric path.

Wheeled mobile robots are typical examples of mechanical systems with nonholonomic constraints. Although navigation and path planning of mobile robots have been investigated extensively over the past decade, the work of dynamic control with nonholonomic constraints is much more recent [3]. As it is known, the two most important requirements of maneuvering are to follow the path by staying as close to the path as possible and to maintain the desired forward velocity. Unfortunately, the state of system with nonholonomic constraints cannot be made asymptotically stable by smooth feedback. Particularly, Zhao [4] proved that the synchro-drive wheel vehicle couldn't be asymptotically stabilized to a single equilibrium point by any continuous feedback control. Sarkar [5] developed a nonlinear feedback of nonholonomic system that realizes the input-output linearization and input-output decoupling. The above methods are, however, limited to the partial control objectives by the choice of output variables and the specific task and the reduced systems can be only small-time locally controllable from the origin [6].

The problem addressed in this paper consist in designing a feedback control law for a nonholonomic car-like vehicle to perform the various task, which are motion planning, trajectory tracking, and dynamic path following. We propose the hybrid dynamical system modeling and motion controls for wheeled mobile robot systems with nonholonomic constraints. A new integrated approach to the global control problem of the locomotion of nonholonomic robot is developed. This hybrid control system has the 3-layered hierarchical structure, which consists of digital automata for the higher process, wheeled mobile robot for the lower process, and the interface for the interaction process between the higher and lower process. The digital automata are designed using the abstract model for the motion characteristics of wheeled mobile robots. The interface is designed which can convert continuous-time signals to symbol space and vice versa. The continuous dynamics as lower process contains the entire continuous-time system with nonholonomic constraints.

2 Hybrid System Models

Hybrid systems are models for networks of digital and continuous devices, in which digital control programs sense and supervise continuous and discrete plant governed by difference or differential equations.

2.1 Nonholonomic System Model

We are interested in the motion control of two-rear-drive wheel vehicle with 2 D.O.F as shown in Fig. 1. It has two driving wheels on an axis that passes through the

0-7803-4465-0/98 $10.00 © 1998 IEEE

vehicle geometric center, they are powered by DC motors and one caster wheel.

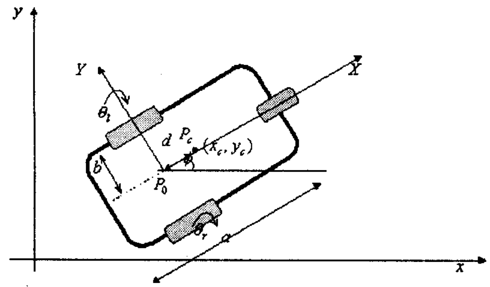


Fig. 1.: Model of two-rear-drive wheel vehicle.

Here (X,Y) is a world coordinate system, (x,y) is the coordinate system fixed to the cart, P_c is the center of mass of the platform, P_0 is the geometric center with coordinates. And b is the distance between each driving wheel and the axis of symmetry, d is the distance from P_0 to P_c along the x-axis, ϕ heading angle, and v_1, v_2 are velocity of each wheel, respectively. The kinematic model used for this vehicle is given by

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi/2 \\ \sin\phi/2 \\ 2/b \end{bmatrix} v_1 + \begin{bmatrix} \cos\phi/2 \\ \sin\phi/2 \\ -2/b \end{bmatrix} v_2, \tag{1}$$

and the inverse kinematics is given by

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix}. \tag{2}$$

We now consider the dynamics of wheeled mobile robot system with nonholonomic constraints as in Fig.1. The motion equations are given by

$$M(p)\ddot{p} + V(p,\dot{p}) = E(p)\tau - A^T(p)\lambda,\ A(p)\dot{p} = 0, \tag{3}$$

where $p = [x \quad y \quad \theta_l \quad \theta_r]^T$, $M(p)$ is the inertia matrix, $V(p,\dot{p})$ is the vector of position and velocity-dependent forces, $E(p)$ is the input transformation matrix, τ is the control input vector, $A(p)$ is the full rank Jacobian matrix, and λ is the vector of constraints forces, The systems described in equation (3) are not input-state linearizable if at least one of the constraints is nonholonomic. It may be input-output linearizable if a proper set of output equation is chosen [5]. For input-output feedback linearization, we use a differential geometric method. The state equation simplifies to the form of

$$\dot{x} = f_s(x) + g(x)u, \tag{4}$$

where

$$f_s(x) = \begin{bmatrix} S\eta \\ -(S^T MS)^{-1}(S^T M\dot{S}\eta + S^T V) \end{bmatrix},\ u = [\tau_1 \quad \tau_2]^T,$$

$$g(x) = \begin{bmatrix} 0 \\ (S^T MS)^{-1} \end{bmatrix},\ x = (x, y, \theta_l, \theta_r, \omega_l, \omega_r)^T,$$

$\eta = [\omega_l \quad \omega_r]^T$, and $S(p)$ is a null space of $A(p)$.

2.2 Switched System and Motion Control

A general hybrid system [7] is given by

$$H = [Q, \Sigma, A, R], \tag{5}$$

where H is a hybrid state, Q is the set of discrete states, $\Sigma = \{\Sigma_q\}_{q\in Q}$ is the collection of constituent dynamical systems referred to as continuous states and $\Sigma_q = [X_q, \Gamma_q, f_q]$, $A = \{A_q\}_{q\in Q}$ is the collection of switching sets. R is the switching rules and depends on the state X_q. Generally, the continuous systems are described by

$$\dot{x}(t) = f(x(t), u(t)), \tag{6}$$

where $x(t) \in R^n$, is continuous state vector and $u(t) \in R^m$, is control input vector. The effect of discrete components of the state on these continuous components is accounted for by letting f depend on the discrete state,

$$\dot{x}(t) = f_i(x(t), q(t), u(t)),\ q^+(t) = v(x(t), q(t)), \tag{7}$$

where $q \in Q = \{1, 2, ..., N\}$ as discrete states, $v : R^n \times Q \to Q$ is the discrete dynamics. The pair (q, x) ranging over $Q \times R^n$ characterizes the state of the system at any time. A transition of the discrete state from $q = i$ to $q = j \neq i$ is triggered when the continuous state x reaches a given transition set $A_{i,j} \subset R^n$. There is one such set for each ordered pair of distinct indexes from Q. Define the arrival set and departure sets of each transition are by

$$A_i^+ \equiv \bigcup_{j\neq i} A_{j,i}. \tag{8}$$

The arrival set of all values of the continuous state for which a transition into discrete state i can occur from some other discrete state and the departure set of all values of the continuous state for which a transition from discrete state i into some other discrete state can occur.

To derive the switched system of wheeled mobile robot as in Fig. 1., the equations of motion are given by

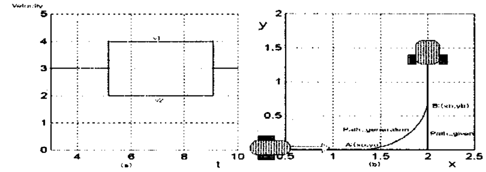
$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ r/2b & -r/2b \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \qquad (9)$$

From equations (1), (2), and (9), we note that the position and orientation of wheeled mobile robot are determined by the difference between v_1 and v_2. At a transition, that is, at a time when the discrete state undergoes change, the state vector is still continuous and initial states reset, though the vector field may change discontinuously. So, the state vectors are switching continuously. The control problems for the position and orientation are the choice of the set of function $v_1(t), v_2(t)$. However, the heading angle ϕ, angular velocity $d\phi/dt$, and position (x, y) do not change discontinuously without stop. The desired actions such as steering, moving (backward, forward), turning (left, right), and stationary are chosen by control modes of digital automata which changes each wheel velocity in an effectively discontinuous manner like as some logical decisions.

Based on these behaviors of wheeled mobile robots, there is continuous switching of mobile robot system due to the difference between $v_1(t)$ and $v_2(t)$ of discrete control input (v_1, v_2) for the continuous state system. Therefore, from the wheeled mobile robot's point of view, the continuous switching systems arise naturally from abstract hybrid systems acting on the navigation and path controls.

The design problems of motion control for mobile robots are to choose the continuous function u_i for each discrete state q, and to define the switching rules for a desired manner. The important requirements of the automobile maneuvering are to follow the path by staying as close to the given path as possible and to maintain the desired velocity and desired heading angle. Consider the following motion of mobile robot systems as in Fig. 2. The desired path with nonlinear edges is given in Fig. 2 and we want to move the mobile robot with the constant forward velocity. In case of the two-wheel-drive mobile robots, mobile robots can not follow the given path and maintain the forward velocity .

In Fig. 2, the mobile robot can not follow the given path with the constant forward velocity because the given path has a nonlinear edge. If the paths consist of lines and circles, the motion control of path may be partitioned to turning motion and straight steering. Based on the abstract motion equation (9), we introduce the switched system for motion control of mobile robots. When the control inputs for each wheel velocity are applied to mobile robots, the trajectory of mobile robot is given by the curved path. The automata diagram of mobile robot for path control is shown in Fig. 3.



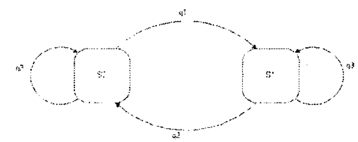


Fig. 2:. The input wheel velocities and trajectory.

Fig. 3 : Digital automata.

3 Hybrid System Controls

The hybrid control approach presented here reduces the difficulties of motion control with nonholonomic constraints for wheeled mobile robot systems and thus allows for its application in a large number of complex task domains and on a wide variety of environments. The hybrid control systems considered here consist of 3-layered hierarchical structure as in Fig. 4.: Digital automata for the higher process by discrete state system, mobile robot system for the lower process by plant, and the interface for the interaction process between the continuous state system and the discrete state system.

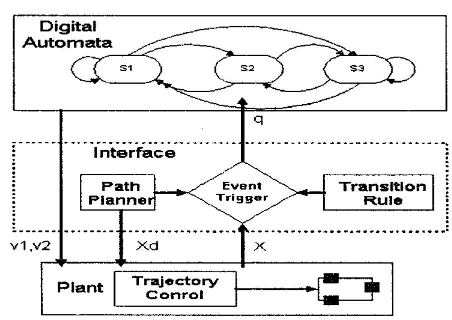


Fig. 4 : Hierarchical structure of hybrid control system.

3.1 Plant

The system to be controlled is modeled as continuous-time system with nonholonomic constraints, this system may be input-output linearizable if a proper set of output equation is chosen. Consider the position control system, where the output equations are functions of only position state variable p as in (3) and (4). This part of hybrid control system contains the entire continuous state space and includes a continuous-time controller. Using the computed-torque PD-control methods in (3) and (4), we can get the feedback control given by

$$\tau=(B_2(x))^{-1}((\ddot{x}_1^d - f_2(x^d)-k_d(\dot{x}_1-\dot{x}_1^d)-k_p(x_1-x_x^d)). \quad (10)$$

where $x_1=[\theta_l \quad \theta_r]^T$, $x_2=[\eta_1 \quad \eta_2]^T$, and $B_2(x)=(S^T MS)^{-1}$. Define $e=x-x^d$ and assume $f_2(x^d)\approx f_2(x)$ and $B_2(x^d)\approx B_2(x)$, then we can get the error equation as

$$\ddot{e}+k_d\dot{e}+k_p e=0. \quad (11)$$

The schematic block diagram of the plant is shown in Fig. 5.

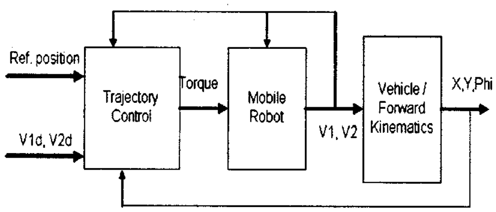


Fig. 5: The schematic diagram for trajectory control of plant as continuous dynamics.

3.2 Digital Control Automata

The digital control automata are a discrete event system that can be modeled as a deterministic automaton. We consider the task that the mobile robot should follow the nonlinear path containing edges with constant velocity. The hybrid automata represent some abstracted motions. This automaton described by a finite set X of real-valued variables, control modes S as continuous flows, and edges E represent discrete jumps and guarded label to another vertices.

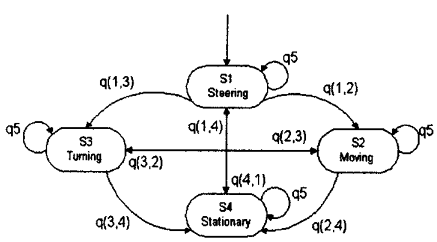


Fig. 6: Digital automata for mobile robot motion control.

The states of the automaton change either instantaneously when a discrete jump occurs or continuously while time elapses. Each automaton of hybrid motion is described as follows.

Automaton S_i;

S_1 : Steering mode, steer the each wheel velocity, (v_1,v_2) to the desired velocity (v_d),
$v_1(t)=v_0+\lambda_1 t$, $v_2(t)=v_0+\lambda_2 t$.

S_2 : Moving mode, drive the each wheel velocity to constant velocity (v_d),
$v_1(t)=v_2(t)=v_d$

S_3 : Turning mode, drive the each wheel velocity to different velocity,
$v_1(t)=\frac{k_1}{k_2}v_2(t)$, $k_1>k_2$ or $k_1<k_2$.

S_4 : Stationary mode, brake the each wheel velocity (v_1,v_2) to the stop, autonomous switching,
$v_1(t)=v_0-\lambda_1 t$, $v_2(t)=v_0-\lambda_2 t$.

Event $q_{i,j}$: the transition occurs when the continuous state vector (x,y,ϕ) intersects the condition set, $A_{i,j}$.

3.3 Interface Control

The hybrid automata and plant communicate with each other indirectly in a hybrid control system because each utilizes a different type of signal. Interface consists of path planner, transition rule, and event trigger.

Path Planner plans the continuous path from the given desired path with nonlinear edges as illustrated in Fig. 7.

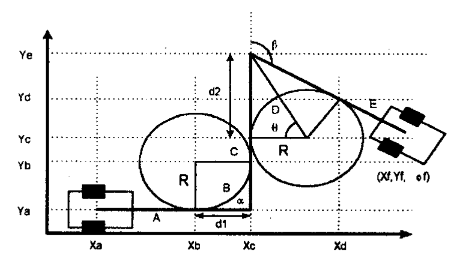


Fig. 7.: Continuous path planning of the given nonlinear path with edges.

In Fig. 7., the turning point $P_t=(x_c-d_1,y_a)$ which occur control mode switching. Here $d=R\tan\alpha$, $R=\frac{p+1}{p-1}b$, where $p=\frac{v_1}{v_2}$, (if $v_1>v_2$) or $p=\frac{v_1}{v_2}$, (if $v_1<v_2$).

And the path planner plans the dynamic path with variable initial conditions as in Fig. 8. The initial conditions P_r and center of circle C_r are given as follows:,

$$C_r=(x_0+r\cos(\phi_0+\pi/2),y_0+r\sin(\phi_0+\pi/2)),$$
$$P_r=(x_0,y_0,\phi_0).$$

The heavy line in Fig. 8 is a planned path for dynamic path following. The event trigger is a decision-maker for a switching of control mode. The discrete events $q_{i,j}$, where i is a current control mode and j is another control mode to switch, change the control mode in digital controller or stay on current control mode. The decision-making procedures are as follows:

1) Compare the states of plant output at time t_k to the states of planned path at time t_{k+1}.
2) The vectors of tangential component respect to two state vectors are equal or not?
3) Decision of the next control mode j as shown in Fig. 9.

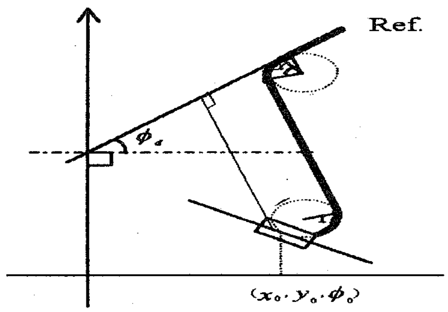


Fig. 8.: Path planning for dynamic path-following with the various initial conditions.

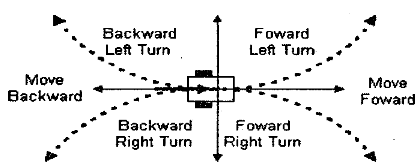


Fig. 9.: Geometric structure of event trigger.

4 Applications

This section describes the results of some applications for hybrid system modeling and control of wheeled mobile robots presented above. Two tasks are considered such as the path control for a given path with nonlinear and dynamic path following with various initial conditions such as various initial positions and heading angles.

4.1 Parallel Parking

The given path consists of line (A-C-E-G) and some edges, as in Fig.10.

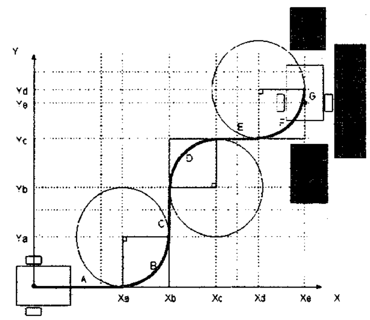
First, path planner plans a continuous-path without edges and defines the turning points (P_t).

Second, define the sequence of control modes in hybrid automata and the transition conditions-arrival and departure sets.

The configurations of parallel parking task are given by

$v_d = 3\ m/\sec$, $v_0 = 0$, $P_0 = (0,0,0°)$, $P_f = (x_e, y_e, 90°)$,

where P_0 is initial (x, y, ϕ) and P_0 is final (x, y, ϕ) and v_0 is an initial velocity of mobile robot. The turning motion is restricted to the forward turning, and



$\lambda_1 = \lambda_2 = 1.0$, $k_1/k_2 = 0.5$ or $k_2/k_1 = 0.5$.

Fig. 10.: Parallel parking task.

The outputs of hybrid automata controller for a parallel parking and the heading angle of plant are shown in Fig. 11.

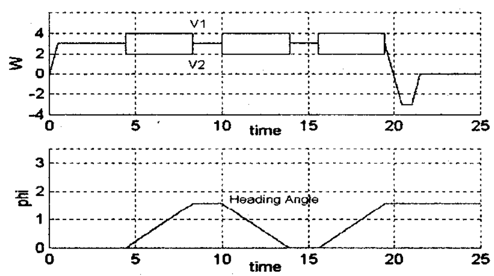


Fig. 11.: The digital automata controller state and heading angle of mobile robot for parallel parking task.

The path following results using hybrid control is given in Fig. 12.

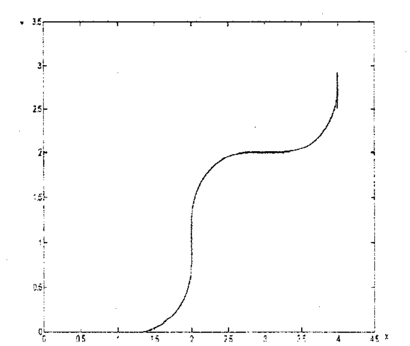


Fig. 12: Trajectory of mobile robot

In this example, Fig.11 represent the robot configuration at the discrete state transition of the hybrid control system and Fig. 12 show that the trajectory of the robot's point P_0

4.2 Dynamic Path Following

The results of dynamic path following with variable initial conditions are in Fig. 13.

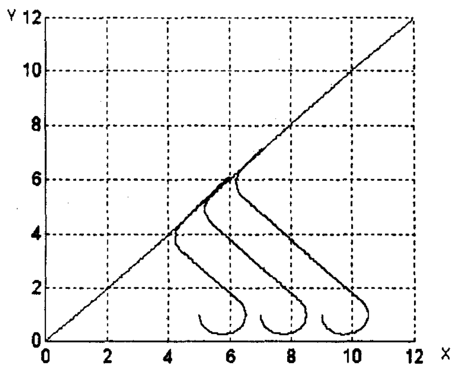


Fig. 13: Dynamic Path following of the path with various initial conditions.

The desired path (P_d) is given and the initial conditions (P_{a0} , P_{b0} , P_{c0}) of mobile robot are as follows,

$$P_d : y = x, \; v_d = 5m/\sec, \; v_0 = 0.0,$$
$$P_{a0} = (5,1,-\pi/2), P_{b0} = (7,1,-\pi/2), P_{c0} = (9,1,-\pi/2).$$

In dynamic path-following problem, the hybrid control system shows the robustness to the various initial configurations.

5. Concluding Remarks

In this paper, we proposed the hybrid control systems for motion control of wheeled mobile robot systems with nonholonomic constraints. A new integrated approach to the global control problem of the locomotion of nonholonomic robot is developed. This hybrid control system has the 3-layered hierarchical structure: digital automata for the higher process, mobile robot system for the lower process, and the interface for the interaction process between the continuous dynamics and the discrete dynamics. The continuous dynamics of mobile robots are modeled by the switched systems and the abstracted motion and digital automata for the path control are developed. The tasks of path planning, trajectory tracking, and dynamic path following are evaluated in the proposed hybrid control architecture. The path planning and tracking of the desired-paths with edges and dynamic path-following with various initial conditions are investigated as the applications by the computer simulations.

Our new integrated approach for the locomotion of nonholonomic mobile robot system has solved the global control problems that has not addressed in the literature before. The simulation studies showed the robustness of the proposed controller to the motion planing and control.

References

[1] Hans. S, Witsenhausen, " A Class of Hybrid-State Continuous-time Dynamic Systems," IEEE Trans. On Automatic Control,11(12), pp. 161-167, 1996.

[2] Michael S. Branicky, Vivek Bokar, and Sanjoy K. Mitter, "A Unified Framework for Hybrid Control," In Cohen and Quadrat, pp. 352-358, 1994.

[3] A.Bloch and N.H. McClamroch, "Control of Mechanical Systems with Classical Nonholonomic Constraints and Calculation of Joint Driving Force," Proc. of 28th Int. Conf. on Decision and Control, pp. 201-205, 1990.

[4] Y. Zhao and S.L. ReMent, "Kinematics, Dynamics, and Control of Wheeled Mobile Robots," Proc. of Int. Conf. Robotics and Automation, pp. 91-96, 1992.

[5] N. Sarkar, X. Yun, and V. Kumar, "Control of Mechanical Systems with Rolling Constraints : Application to Dynamic Control of Mobile Robots," Int. J. of Robotics Research, vol. of 13, no. 1, 1994.

[6] A. Bloch and N.H. McClamroch, "Controllability and Stability Properties of a Nonholonomic Control System," Proc. of 29th Int. Conf. on Decision and Control, pp. 1312-1314.

[7] Lucio Taverini, "Differential Automata and their Discrete Simulators" in *Nonlinear Analysis, Theory, Methods , and Applications*, 11(6), pp. 665-683, 1987.