

Variable Structure Control of Nonholonomic Wheeled Mobile Robot

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Abstract

A variable structure controller(VSC) for a nonholonomic wheeled mobile robot is proposed for tracking desired trajectories. Although the matching condition for VSC is not satisfied due to nonholonomic property of the robot having two drive wheels, the proposed algorithm guarantees bounded tracking errors by choosing the sliding surface gain properly. Yet the robot tracks its given trajectory without error if there is no initial posture and velocity errors. Simulation results show the effectiveness of the proposed controller.

1 Introduction

We propose a variable structure controller(VSC) of a wheeled mobile robot to track the desired trajectory. Regarding the VSC, the readers are referred to [1, 2]. We use the kinematic model which represents the relationship between postures and velocities of the wheeled mobile robot with two drive wheels and an auxiliary wheel [3, 4, 5]. To control the mobile robot, we employed a dynamic model which describes the relationship between applied torques and velocities of the robot derived under four assumptions [4].

Provided that the assumptions are not satisfied(e.g. a flexible tire, existing slippages, traveling mounds or rough grounds, frictions on the motor, etc.), the model may be unreliable. And also the mass or center of gravity of the mobile robot like a loader can be changed while doing a task. This causes model parameter variations. Considering these existing disturbances and the model parameter variations, the VSC is designed to accomplish a stable tracking control of the robot. Computer simulation illustrates the effective-

ness of the proposed scheme.

2 Kinematics

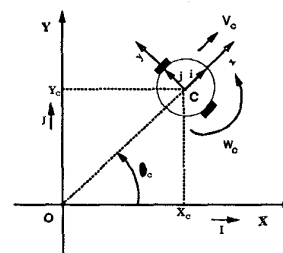


Fig. 1. Coordinate of mobile robot(subscript "c" means "current state").

Consider a mobile robot which is located on a 2-D plane in which a global cartesian coordinate system is defined. The X-Y coordinate is global and the x-y coordinate is attached to the center of robot body as in Figure 1. The robot in the world possesses three degrees of freedom in its relative positioning which are represented by a *posture* \mathbf{p} ,

$$\mathbf{p} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \quad (1)$$

where the heading direction θ is taken counterclockwise from the X-axis to x-axis. The angle θ denotes the orientation of the vehicle or the wheels. The reference posture $\mathbf{p}_r = (x_r \ y_r \ \theta_r)^T$ is a goal posture of the robot and the current posture $\mathbf{p}_c = (x_c \ y_c \ \theta_c)^T$ is its real posture at the present moment.

The robot's motion is controlled by its *linear velocity* v and *angular velocity* ω , which are also functions of

time. The robot's kinematics is defined by a Jacobian matrix $J(\theta)$:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \dot{\mathbf{p}} = J(\theta)\mathbf{q} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \mathbf{q} \quad (2)$$

where $\mathbf{q} = (v \ \omega)^T$.

We define a *robot based error posture* \mathbf{p}_{ec} , which is a transformation of the posture $(\mathbf{p}_r - \mathbf{p}_c)$ with an origin in a local coordinate and a direction of θ_c :

$$\mathbf{p}_{ec} = \begin{pmatrix} x_{ec} \\ y_{ec} \\ \theta_{ec} \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c & 0 \\ -\sin \theta_c & \cos \theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} (\mathbf{p}_r - \mathbf{p}_c) \quad (3)$$

The derivative of error posture, i.e., the velocity of error posture can be described in the following lemma.

Lemma 1

$$\dot{\mathbf{p}}_{ec} = f(\mathbf{p}_{ec}, \mathbf{q}_r, \mathbf{q}_c) = \begin{pmatrix} y_{ec}\omega_c - v_c + v_r \cos \theta_{ec} \\ -x_{ec}\omega_c + v_r \sin \theta_{ec} \\ \omega_{ec} \end{pmatrix} \quad (4)$$

Proof : A similar proof is given in Kanayama *et al.*[5]. Here, \mathbf{q}_c is a velocity at the current posture \mathbf{p}_c and \mathbf{q}_r is a velocity at the reference posture \mathbf{p}_r .

3 Dynamics

Consider a wheeled mobile robot having two drive wheels and an auxiliary wheel shown in Figure 2. The auxiliary wheel is a kind of caster and its radius is smaller than the drive wheel's. The two drive wheels are cylindrical shape having the same radius and mass. For practical application, it is considered that the center of gravity of the robot is G, not C.

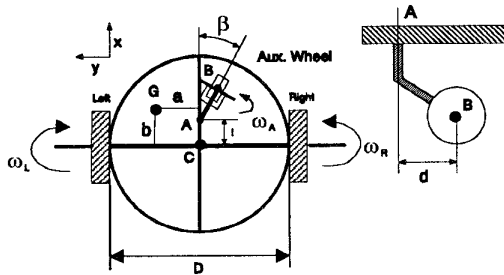


Fig. 2. Wheeled mobile robot model

We introduce the following four practical assumptions to make the modeling problem tractable:

- i. The robot does not contain flexible parts.
- ii. All steering axes are perpendicular to the ground.
- iii. The robot moves on a planar ground.
- iv. The contact between wheels and the ground satisfies the condition of **pure rolling and non-slipping**.

Provided that assumptions are not satisfied (e.g., flexible tire, existing slippage, traveling mound or rough ground, etc.), those phenomena are considered as disturbances or noises.

Using the Lagrangian formalism, dynamic equation can be derived easily (See [4] or [6].) The derived dynamic equation of the mobile robot is represented as follow:

$$\begin{cases} \tau = \mathbf{H}(\beta)\dot{\mathbf{q}}_c + \mathbf{G}(\beta, \mathbf{q}_c) \\ \dot{x}_c = v_c \cos \theta \\ \dot{y}_c = v_c \sin \theta \\ \dot{\theta}_c = \omega_c \\ \dot{\beta} = \frac{1}{d}(\sin \beta v_c - (d + l \cos \beta)\omega_c) \end{cases} \quad (5)$$

where

$$\begin{aligned} \mathbf{q}_c : & \quad \text{linear and angular velocity,} & \mathbf{q}_c \in R^{2 \times 1} \\ \tau : & \quad \text{applied torque,} & \tau \in R^{2 \times 1} \\ \mathbf{H} : & \quad \text{inertia momentum matrix,} & \mathbf{H} \in R^{2 \times 2} \\ \mathbf{G} : & \quad \text{coriolis and centripetal force term,} & \mathbf{G} \in R^{2 \times 1} \end{aligned}$$

Above dynamic equation is similar to the dynamic equations of a robot manipulator and each term in the equation has a similar physical meaning. It can be considered that \mathbf{H} represents the inertia moment matrix and \mathbf{G} represents the coriolis and centripetal terms. The difference is that it has no gravity term due to Assumption ii.

4 The variable structure control of the mobile robot

The inputs to the variable structure controller(VSC) are the desired trajectories of the posture and velocity and its output is an input torque τ to the mobile robot as Figure 3 shows. In the figure, MR represents a mobile robot model derived in the previous section as the dynamic equation. Although

it is difficult to produce desired trajectories because of non-holonomic problem [7], we design a controller under situation that a predesigned path generator produces continuous desired trajectories \mathbf{p}_r , \mathbf{q}_r and $\dot{\mathbf{q}}_r$.

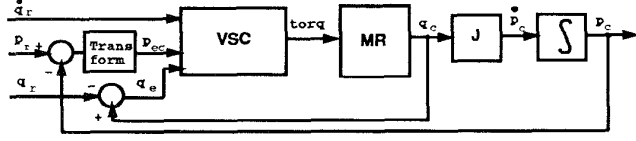


Fig. 3. Control structure of the mobile robot using VSC

The derived dynamic equation has some unconsidered factors, e.g., mass and inertia moment variations due to moving object in the robot, frictions, parameter uncertainties, etc., in the real world. Under these circumstances, the VSC is designed to achieve a stable tracking control of the mobile robot. It is assumed that the velocity \mathbf{q} is measurable by the encoder or tacometer, the matrix \mathbf{H} and the vector \mathbf{G} are known and their physical parameters are bounded. The desired trajectory is represented by bounded time functions in terms of the posture trajectory \mathbf{p}_r , velocity trajectory \mathbf{q}_r and acceleration trajectory $\dot{\mathbf{q}}_r$.

Let us define the velocity error and acceleration error as

$$\mathbf{q}_e = \mathbf{q}_c - \mathbf{q}_r$$

$$\dot{\mathbf{q}}_e = \dot{\mathbf{q}}_c - \dot{\mathbf{q}}_r$$

and the robot based error posture \mathbf{p}_{ec} as a posture error.

The control objective is to make \mathbf{p}_{ec} and \mathbf{q}_e zero in a disturbance free case or to maintain \mathbf{p}_{ec} and \mathbf{q}_e within boundaries in finite time under bounded disturbances.

4.1 Design of the controller

Let us consider the sliding surface as follow:

$$\mathbf{s} = \mathbf{q}_e + \mathbf{C}\mathbf{p}_{ec} \quad (6)$$

with

$$\mathbf{C} = \begin{bmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \end{bmatrix}$$

where \mathbf{C} is a constant matrix. Let us now construct a control law using above sliding surface as:

$$\tau = \mathbf{H}(\beta)\dot{\mathbf{q}}_r + \mathbf{G}(\beta, \mathbf{q}_c) + \mathbf{H}(\beta)u \quad (7)$$

with

$$u = -k(t)s - k_1(t)\text{sgn}(s) - \mathbf{C}\dot{\mathbf{p}}_{ec} \quad (8)$$

where $\text{sgn}(s)$ is a sign function and $k(t)$ and $k_1(t)$ have positive values.

Practical systems have measurement errors, parameter uncertainties, model uncertainties, etc. These can be considered as input disturbances. Denoting the disturbances as a vector function $\nu(t) \in R^{2 \times 1}$, a real applied input is represented as follows:

$$\tau^* = \tau + \nu(t) \quad (9)$$

By introducing the uncertainty matching condition, we get

$$\nu(t) = \mathbf{H}\eta(t) \quad (10)$$

and

$$u^* = u + \eta(t). \quad (11)$$

It is assumed that there exists a bounded positive constant α which satisfies the following inequality for any 2×1 vector \mathbf{s} which is not equal to 0:

$$\text{A:} \quad \|\mathbf{s}^T \eta(t)\| \leq \alpha \|\mathbf{s}\|.$$

The following lemma is derived using the above control law and the assumption A.

Lemma 2 Consider the mobile robot given by the dynamic equation (5) with the control law (7) and the assumption A. If $k_1(t)$ is given as a value larger than the disturbance bound α , there exists a sliding surface and then \mathbf{s} and $\dot{\mathbf{s}}$ become 0.

Proof: To have a sliding surface, the following sliding condition must be satisfied:

$$\lim_{s \rightarrow 0} \mathbf{s}^T \dot{\mathbf{s}} < 0.$$

Using (7), (9), (10), and (11), we get

$$\begin{aligned} \mathbf{s}^T \dot{\mathbf{s}} &= (\mathbf{q}_e^T + \mathbf{p}_{ec}^T \mathbf{C}^T)(\dot{\mathbf{q}}_e + \mathbf{C}\dot{\mathbf{p}}_{ec}) \\ &= \mathbf{q}_e^T (u^* + \mathbf{C}\dot{\mathbf{p}}_{ec}) + \mathbf{p}_{ec}^T \mathbf{C}^T (u^* + \mathbf{C}\dot{\mathbf{p}}_{ec}) \\ &= \mathbf{s}^T (u^* + \mathbf{C}\dot{\mathbf{p}}_{ec}) \\ &= -k\mathbf{s}^T \mathbf{s} - k_1 \|\mathbf{s}\| + \mathbf{s}^T \eta(t) \\ &\leq -k\mathbf{s}^T \mathbf{s} - (k_1 - \alpha) \|\mathbf{s}\|. \end{aligned} \quad (12)$$

The inequality is satisfied if k_1 is larger than the disturbance bound α . Then the sliding condition is satisfied.

4.2 Analysis of the control law

Using equation (4), the following equation is derived:

$$\begin{bmatrix} \dot{\mathbf{p}}_{ec} \\ \dot{\mathbf{q}}_e \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix} + \begin{bmatrix} f(\mathbf{p}_{ec}, \mathbf{q}_r, \mathbf{q}_c) \\ 0 \end{bmatrix}. \quad (13)$$

Rewriting the above equation as a simple form, we get

$$\dot{\mathbf{X}} = \mathbf{B}u + \mathbf{h} \quad (14)$$

$$\mathbf{s} = \mathbf{L}\mathbf{X} \quad (15)$$

where

$$\mathbf{X} = \begin{bmatrix} \mathbf{p}_{ec} \\ \mathbf{q}_e \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ I_2 \end{bmatrix}$$

$$\mathbf{h} = \begin{bmatrix} f(\mathbf{p}_{ec}, \mathbf{q}_r, \mathbf{q}_c) \\ 0 \end{bmatrix}$$

$$\mathbf{L} = [\mathbf{C} \quad I_2]$$

Since $\dot{\mathbf{s}} = 0$ on the sliding surface, the following equivalent input is derived:

$$\begin{aligned} \dot{\mathbf{s}} &= \mathbf{L}\dot{\mathbf{X}} = \mathbf{L}(\mathbf{B}u + \mathbf{h}) = 0 \\ \Rightarrow u_{eq} &= -(\mathbf{L}\mathbf{B})^{-1}\mathbf{L}\mathbf{h} \end{aligned} \quad (16)$$

Applying equation (16) to equation (14), we get

$$\begin{aligned} \dot{\mathbf{X}} &= (\mathbf{I} - \mathbf{B}(\mathbf{L}\mathbf{B})^{-1}\mathbf{L})\mathbf{h} \\ &= \begin{bmatrix} f(\mathbf{p}_{ec}, \mathbf{q}_r, \mathbf{q}_c) \\ -\mathbf{C}f(\mathbf{p}_{ec}, \mathbf{q}_r, \mathbf{q}_c) \end{bmatrix} \end{aligned} \quad (17)$$

From the above result, we can obtain the following theorem.

Theorem 1 Consider the mobile robot given by the dynamic equation (5) with the control law (7) and the assumption A. If initial errors of \mathbf{p}_{ec} and \mathbf{q}_e are zero, posture and velocity tracking errors of the robot are always 0 even though bounded disturbance exists. Otherwise, the errors remain within the boundary.

Proof : From Lemma 2, the phase trajectory of the robot exists on the sliding surface regardless of the existence of disturbances. And if initial errors are zero, \mathbf{p}_{ec} and \mathbf{q}_e are always 0 based on equation (17). But if those are not zero, $\mathbf{s}(0)$ is not zero. Then \mathbf{s} goes to the sliding surface by Lemma 2 and \mathbf{s} is zero in finite reaching sliding surface time t_h . Then, from equation (17), \mathbf{p}_{ec} is $\mathbf{p}_{ec}(t_h)$ and \mathbf{q}_e is a bounded value relevant to \mathbf{C} and $\mathbf{p}_{ec}(t_h)$. ■

We can accomplish control objectives under bounded disturbances by Theorem 1. In most practical applications, however, we know the initial values since starting velocity of robot is zero and starting posture is given by a user. In this case, we can make

the robot tracking error 0 even under bounded disturbance by proposed control law. On the other hand, in case the initial errors are not zero, the bounded tracking error can be obtained by equation (17) because the asymptotic property of trajectory on the sliding mode is not guaranteed.

4.3 Design of the sliding surface

When the variable structure theory is applied to the robot manipulator or general systems, the state equation always has an \mathbf{X} term unlike equation (17), which gives asymptotic convergence under the assumption that \mathbf{B} and \mathbf{h} satisfy matching condition[2][8]. However, the matching condition is not satisfied in the mobile robot system represented by the dynamic equation (5). Moreover, when we choose a sliding surface, instead of using error and derivative of error having the same orders as in general system, we use \mathbf{q}_e and \mathbf{p}_{ec} whose orders are different. Because the integral of \mathbf{q}_e does not become posture error directly and has no physical meaning. Although it is difficult to show that all the errors always converge to zero on the sliding mode like the VSC of general systems, some error terms can be made zero by choosing \mathbf{C} properly. The following example illustrates how to choose \mathbf{C} .

Example Let us choose \mathbf{C} as follow:

$$\begin{aligned} \mathbf{C} &= \begin{bmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \end{bmatrix} \\ &= \begin{bmatrix} c_1 & c_2 & 0 \\ 0 & 0 & c_6 \end{bmatrix} \end{aligned}$$

The following equations are then derived from the fact that \mathbf{s} and $\dot{\mathbf{s}}$ are 0 on sliding mode:

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} \dot{v}_e + c_1\dot{x}_{ec} + c_2\dot{y}_{ec} \\ \dot{\omega}_e + c_6\dot{\omega}_e \end{bmatrix} = 0$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} v_e + c_1x_{ec} + c_2y_{ec} \\ \theta_{ec} + c_6\theta_{ec} \end{bmatrix} = 0$$

Now if $c_6 > 0$, we know that ω_e and θ_{ec} converge to zero asymptotically. If we assume that $|\theta_{ec}| < \epsilon_\theta$, we get

$$\begin{aligned} \cos(\theta_{ec}) &= 1 - \epsilon_1 \\ \sin(\theta_{ec}) &= \epsilon_2 \end{aligned}$$

where $0 < \epsilon_1, \epsilon_2 \ll 1$.

Rewriting $\dot{s}_1 = 0$ using equation (4),

$$\dot{v}_e + c_1(y_{ec}\omega_e - v_e + v_r \cos(\theta_{ec})) + c_2(-x_{ec}\omega_e + v_r \sin(\theta_{ec})) = 0$$

and substituting above $\cos(\theta_e)$ and $\sin(\theta_e)$ to the above equation, we get

$$\dot{v}_e - c_1v_e + c_1\gamma = 0$$

where

$$\begin{aligned}\gamma &= (y_{ec} - \frac{c_2}{c_1}x_{ec})w_c - (\epsilon_1 - \frac{c_2}{c_1}\epsilon_2)v_r \\ &\simeq (y_{ec} - \frac{c_2}{c_1}x_{ec})w_c \\ &\leq |y_{ec} - \frac{c_2}{c_1}x_{ec}| |w_c|\end{aligned}$$

Thus,

$$\begin{aligned}\dot{v}_e - c_1 v_e &\leq c_1 |\gamma| \\ \Rightarrow v_e &\leq -|\gamma| + v_e(0)e^{c_1 t} \leq -|\gamma| + |v_e(0)| \text{ if } c_1 \leq 0\end{aligned}$$

Therefore, θ_{ec} and ω_e converge to zero asymptotically.

5 Computer Simulations

We simulated the proposed control law to demonstrate its effectiveness. In a simulation program, kinematics and dynamics described in Section 2 and 3 are used as the mobile robot model. A specification of the model is shown in Table 1.

Table 1. Parameter values used in the simulation

Notation	value	explanation
M	20 Kg	mass of body
D	0.5 m	diameter of body
l	0.25 m	position of auxiliary wheel
d	0.05 m	length of auxiliary wheel rod
r	0.07 m	radius of drive wheel
r_A	0.02 m	radius of auxiliary wheel
m	0.2 Kg	mass of drive wheel
m_A	0.1 Kg	mass of auxiliary wheel
a	0.05 m	x position of center of mass
b	0.05 m	y position of center of mass

Figure 4 shows desired posture and velocity trajectories used in this simulation. These include various types of locomotion like straight, curve, etc.

The sliding surface is chosen as follows:

$$\begin{aligned}C &= \begin{bmatrix} c_1 & c_2 & c_3 \\ c_1 & -c_2 & -c_3 \end{bmatrix} \\ &= \begin{bmatrix} -100 & -100 & -100 \\ -100 & 100 & 100 \end{bmatrix}\end{aligned}$$

Figure 5 shows simulation results of posture and velocity errors using $k = 0$ and $k_1 = 1$, respectively, in the case of no disturbance and under disturbance which is random noise of mean 0.005 and variance 0.005. Regardless of disturbance, all the errors remain

within the small boundary. Torque applied to right-hand and left-hand wheels for the case of Figure 5 is plotted in Figure 6. As we can expect, there is a chattering phenomenon because of discontinuous function *sign*. It should be noted that the system under disturbance requires more torque than disturbance-free system. If the bound of disturbance becomes larger, the system with $k_1 = 1$ becomes unstable. So we simulated it with $k_1 = 10$ to show the stability against larger disturbance. The results are shown in Figure 7. Although the random noise of mean 0.01 and variance 0.01 is applied to the system, the errors remain within some boundaries which are larger than those in Figure 5. Figure 8 shows errors when initial errors are not zero. It shows all the errors remain within the small boundary.

6 Conclusion

In this paper, we have proposed the variable structure controller of the wheeled mobile robot to track the desired trajectory. Although it is difficult to prove asymptotic stability on the sliding mode because the mobile robot used in this paper does not satisfy matching conditions and the design of the sliding surface is different from that of general systems, we can get that some error terms converge to zero by choosing sliding surface gain C properly and the others remain within the boundaries, regardless of changing model parameters and disturbances.

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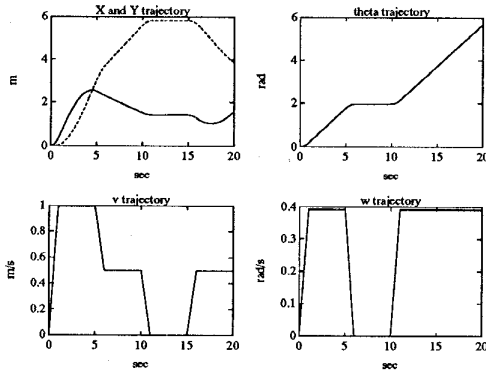


Fig. 4. Desired trajectories for simulation(dashed line is y_r .)

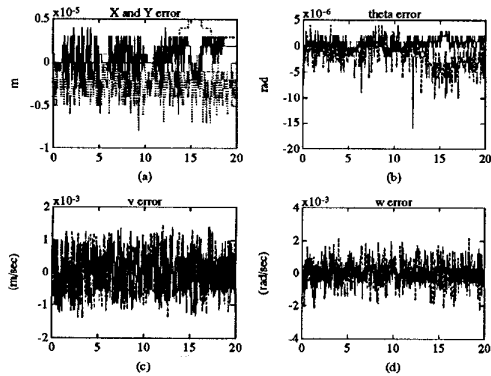


Fig. 5. Errors with $k = 0$ and $k_1 = 1$ (In (a), solid line is x_{ec} and dashed line is y_{ec} for no disturbance, and dotted line is x_{ec} and dashdot line is y_{ec} under disturbance. In (b), (c) and (d), solid line:disturbance free case, dotted line:existing disturbance case.)

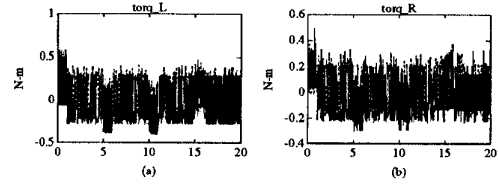


Fig. 6. Torque in the case of $k = 0$ and $k_1 = 1$ (solid line:disturbance free case, dotted line:existing disturbance case).

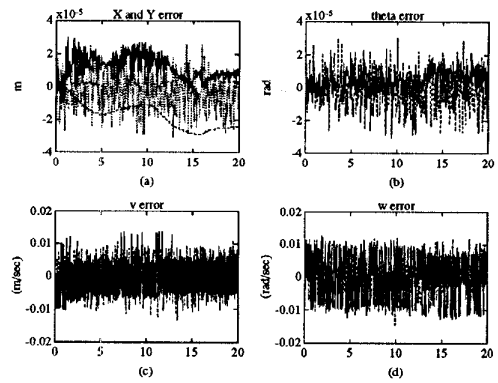


Fig. 7. Errors with $k = 0$ and $k_1 = 10$ (In (a), solid line is x_{ec} and dashed line is y_{ec} for no disturbance, and dotted line is x_{ec} and dashdot line is y_{ec} under disturbance. In (b), (c) and (d), solid line:disturbance free case, dotted line:existing disturbance case.)

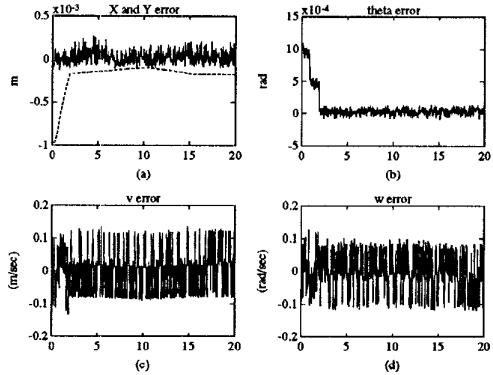


Fig. 8. Errors in case that initial points are not zero($x(0)$, $y(0)$ and $\theta(0)$ are all 0.0001 :dashed line is y_{ec} .)

A Four-wheeled Robot to Pass over Steps by Changing Running Control Modes

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Abstract

We have developed a robot called a 'variable structure type four-wheeled robot' which, with a simple structure, has ability to pass over a step. This robot has only one degree of freedom to change the body's structure and can run on four wheels (4-wheeled mode) or on two wheels like a wheeled inverted pendulum (2-wheeled mode). For the realization of stepping up and down, there are three difficult problems to be solved: stable control in 2-wheeled state, transfer from 4- to 2-wheeled mode (swing-up), and transfer from 2- to 4-wheeled mode (landing). The control method we have developed for a wheeled inverted pendulum is used to control the 2-wheeled state using the signal of a rate gyroscope. In this paper, we mainly discuss control methods for transfers from 4- to 2-wheeled mode and from 2- to 4-wheeled mode. Especially in the transfer from 4- to 2-wheeled state, we make effective use of the driving torque of the motor which change the body's structure. In the transfer from 2- to 4-wheeled state, we have investigated a control method for soft landing to minimize the touch-down impact of the robot which is non-holonomic system. By combining our control methods, we have realized an experiment of passing over a step successfully.

1. Introduction

There are many types of locomotion, wheeled, legged, tracked, etc. Generally, wheeled locomotion is superior to legged and tracked locomotion in running ability (energy efficiency, running speed, etc.) on a flat plane. However, the latter are superior to the former in the ability to pass over obstacles. Therefore, mobile robots with the merits of many types of locomotion are developed.

The aim of our research is the development of a wheeled robot with ability to adapt to unknown rough roads.

Many studies of wheeled robots with ability to pass over obstacles whose height is larger than the radius of the wheel have been carried out in Japan. T. Ohmichi et al.[1] and N. Kimura et al.[2] developed leg-wheel robots with a wheel at the end of each leg. E. Nakano et al.[3] also developed a leg-wheel robot but it has two wheels and four legs separately. S. Hirose et al.[4] developed a robot with an articulated body made of several body segments, each

having a wheel. These robots have many degrees of freedom and consequently have complicated structures. They all use movement with statical stability to climb up a step. Yamafuji et al.[5] developed a wheeled inverted pendulum robot with supporting wheels. It can raise itself but cannot go over a step.

We have developed a wheeled robot with simple structure and ability to pass over steps (we call it a variable structure type four-wheeled robot). To reduce the degrees of freedom, we stabilize the robot in 2-wheeled state, which is statically unstable state, using the control method[6] for a wheeled inverted pendulum which we already proposed. Yamafuji's robot[5], uses an external sensor, i.e., a potentiometer with a contacting rod, to detect the inclination angle of the body and it is difficult to realize fast running on a rough road and passing over steps.

In this paper, we use two running modes : 4-wheeled mode in which four wheels contact the ground and 2-wheeled mode in which two wheels contact the ground. We mainly discuss control methods for changing the running mode from 4- to 2-wheeled mode and from 2- to 4-wheeled mode. We also experimentally demonstrate passing over a step whose height is larger than the radius of the wheel.

2. The structure of the variable structure type four-wheeled robot

Fig.1 shows the structure of the variable structure type four-wheeled robot.

This robot is made of two wheeled inverted pendulums connected with a pantograph mechanism. Two DC servo motors which we call the wheel motor, are attached to the lower part of the body for driving the front and rear wheels and one servo motor, the structure motor, is attached to the pantograph beam for changing the structure of the robot. A rotary encoder is attached to each motor for sensing the rotation angle of each motor. A rate gyroscope is attached on the beam for sensing the angular velocity of the body inclination. This robot is designed so that the center of gravity of the body can go beyond the front or rear wheel axle by driving the structure motor and thus make the robot statically unstable.