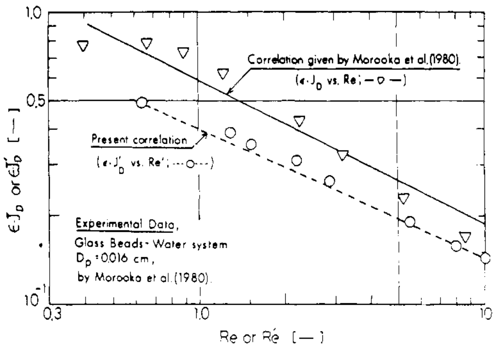

Figure 7. Typical example of comparison between ϵJ_D vs. Re and $\epsilon J_D'$ vs. Re'.

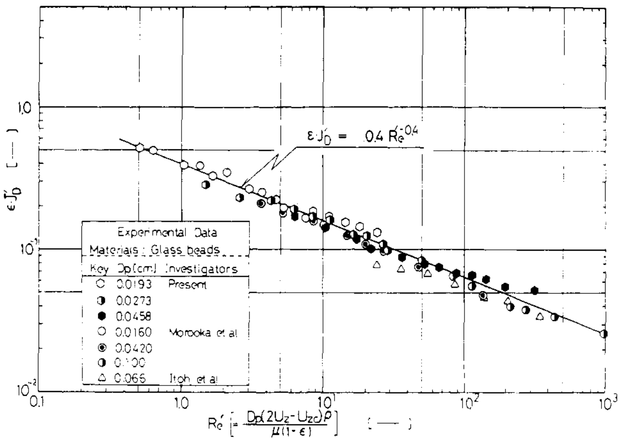

Figure 8. Correlations between the modified Reynolds number, Re', and the modified ϵJ_D factor, $\epsilon J_D'$, for the glass beads–water system.

Nomenclature

D_p = particle diameter
J_D = mass-transfer factor
J_D' = modified mass-transfer factor
k = mass-transfer coefficient
L = length of the electrode
N = total number of particles
N_j = number of particles in the jth state of particles
$P(U_{zj}^r)$ = partition function of particle velocities
Re = Reynolds number
R' = modified Reynolds number
Sc = Schmidt number = k/ν
St = Stanton number, k/U or k/U_{f0}
St' = modified Stanton number
U = average quasi-steady-state velocity of particles corresponding to the average void fraction ϵ during bed expansion
U_{f0} = superficial velocity of the fluidizing medium
U_0 = initial superficial velocity of the fluidizing medium
U_c = minimum value of U
$\bar{U}_z$ = average velocity of particles relative to the upward superficial velocity of the fluidizing medium
U_{zc} = minimum value of U_{zj}
U_{zj} = instantaneous velocity of an individual particle relative to the upward superficial velocity of the fluidizing medium
U_{zj}^r = velocity defined by eq 6
$\bar{U}_{zj}^r$ = average value of U_{zj}^r
U_∞ = superficial velocity of fluid at a new steady state after the stepwise disturbance
W = number of ways
W_A = weight of particles in compartment A
ΔW_A = weight of particles which have passed into compartment B from compartment A

Greek Symbols

$\sigma^2(U_{zj})$ = variance of the instantaneous particle velocity, U_{zj}, defined by eq 8
$\sigma^2(U_{zj})_r$ = variance of the instantaneous particle velocity relative to the average velocity given by eq 10
$\sigma(U_{zj})_r$ = standard deviation of U_{zj}
ϵ = void fraction of the bed

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Simplified Model Predictive Control

Gomatam R. Arulalan and Pradeep B. Deshpande*
Chemical Engineering Department, University of Louisville, Louisville, Kentucky 40292

A new methodology for multivariable control is presented. The method utilizes easily available step responses for implementation. It is easy to understand and implement. It requires a digital computer for implementation, but the memory requirements are modest. Interaction compensation is inherent as in other predictive control methods, and the robustness issues can be addressed in the design stage. Simulation results are presented which show the excellent capability of the method.

The PID controller has been a work horse in process industries for over 40 years. The classical methods of Ziegler–Nichols and Cohen–Coon are industry standards. PID-type controllers are routinely used in SISO (single-input single-output) applications with good results, but success with this type of controller for multivariable systems has been limited.

It is now recognized that the limitations of the PID-type controller can be traced to its characteristics. The PID controller came into existence on the basis of hardware realizability; bellows, orifices in pneumatics and potentiometers, operational amplifiers, resistors, and capacitors in the electronic world could be used to construct it. However, with the evolution of digital control computers, much better designs can be produced without any consideration for hardware realizability. This, in part, has spurred research and development to evolve better control strategies for process systems.

The demands of the times is the second source of incentives. Processes today are much more complex, re-

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quiring a high level of steady-state optimization and good closed-loop control. A major source of complexity in process plants is the existence of interacting multivariable systems which often contain long time delays. Good control of these systems is important since they have a strong influence on smooth operability and overall plant economics. Rising costs of energy and raw materials combined with the availability of powerful, low-cost microprocessors for control have created an additional incentive to evolve better techniques for multivariable process control.

Against this background, two powerful multivariable control techniques were developed independently in the 1970s: one in France, which is now referred to as model algorithmic control (MAC) (Richalet et al., 1978), and the other in the United States, which has been given the name dynamic matrix control (DMC) (Cutler and Ramaker, 1980). At the time, both werre developed on the basis of heuristics. The two methods use a similar approach but utilize different optimization routines for implementation. They take full advantage of the computational power of control computers. Constraint handling is an important facility available in these methods, which was hitherto unavailable with PID-type control systems. Equally important is the fact that these methods need only open-loop step responses for design and no assumptions need be made about the order of the system. Process models are not required for implementation in contrast to traditional methods such as decoupling control. Inverse response characteristics and time delays pose no special problems, but an open-loop unstable plant must be stabilized with conventional feedback before these methods can be applied. To date, these techniques have been applied successfully to such diverse systems as a crude column, fluid catalytic cracker, distillation column, green house, F-16 jet engine, and power plant. A detailed bibliography of references related to these techniques is included for the interested reader at the end of the paper. A monograph describing multivariable control methods is under preparation (Deshpande, 1987).

In another important development, Brosilow (1979) and Garcia and Morari (1982, 1985a,b) published a series of papers wherein they showed that MAC and DMC are but two of the several possible manifestations of a single control strategy which they referred to as internal model control (IMC); such classical techniques as dead time compensation and feed forward control could be derived from IMC formulation as well.

The organization of this paper is as follows. We begin by describing IMC concepts and show how the technique is implemented. The implementation of IMC for all but the most simple processes requires the derivation of an approximate process inverse, a step that can become complex. We then describe a new technique, which we refer to as simplified model predictive control (SMPC), that circumvents the difficulties associated with the evaluation of approximate process inverses. The performance of the new algorithm is evaluated through simulation followed by a comparison of SMPC with IMC, DMC, and MAC and finally ending with a discussion of the significance and drawbacks of the proposed method.

Review of Internal Model Control Concepts

The block diagram of a typical sampled data control system is shown in Figure 1. For simplicity, a single-loop system will be considered (Garcia and Morari, 1982) although the IMC concept can also be extended to multivariable systems (Garcia and Morari, 1985a,b). The transfer function, $G_p(z)$, as shown in Figure 1 includes all

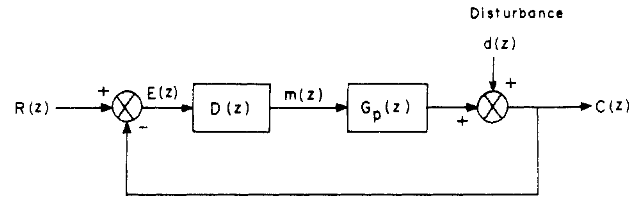

Figure 1. Typical sampled data control system.

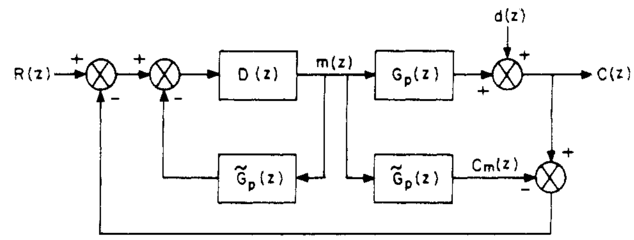

Figure 2. Equivalent model-based control system.

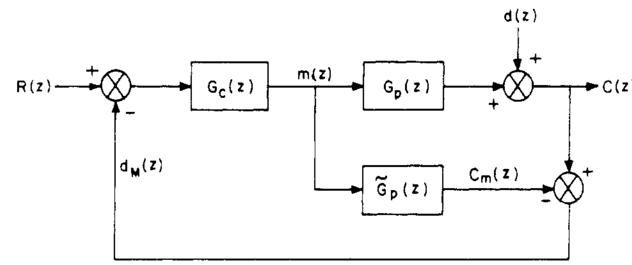

Figure 3. Basic IMC structure.

the elements that are associated with the process block in a control loop. The control algorithm, $D(z)$, may be of the PID type or it may be one of the z-transform-based control algorithms (Deshpande and Ash, 1981).

Now, let us make use of the mathematical model of the process $\tilde{G}_p(z)$, and add the effect of input m upon it to yield a new system shown in Figure 2. It is obvious that the systems shown in Figures 1 and 2 are entirely equivalent. The feedback loop around $D(z)$ can be simplified by using the familiar closed-loop transfer function relationship

$$G_c(z) = \frac{D(z)}{1 + D(z)\tilde{G}_p(z)} \tag{1}$$

The resulting block diagram is shown in Figure 3. Figure 3 depicts the basic IMC structure. One can naturally ask if Figures 1 and 3 are entirely equivalent, what is the advantage of the IMC representation? Garcia and Morari (1982) have shown that the IMC algorithm, $G_c(z)$, is much easier to design than $D(z)$ and that the IMC structure permits us to include robustness as a design objective in a very explicit manner. This can be seen by examining the feedback signal, $d_M(z)$, in Figure 3 which is

$$d_M(z) = [1 + \{G_p(z) - \tilde{G}_p(z)\}G_c(z)]^{-1}d(z) \tag{2}$$

If $G_p(z) = \tilde{G}_p(z)$, that is, the model is perfect, then the feedback signal is simply $d(z)$. Consequently, the control loop is effectively an open-loop system, and therefore stability is not an issue. In the presence of plant-model mismatch, $d_M(z)$ may be suitably modified to obtain robustness.

To design the IMC controller, Garcia and Morari (1982) begin with the following transfer functions derived from Figure 3:

$$m(z) = \frac{G_c(z)}{1 + G_c(z)[G_p(z) - \tilde{G}_p(z)]}[R(z) - d(z)] \tag{3}$$

$$C(z) = d(z) + \frac{G_p(z)G_c(z)}{1 + G_c(z)[G_p(z) - \tilde{G}_p(z)]}[R(z) - d(z)] \tag{4}$$

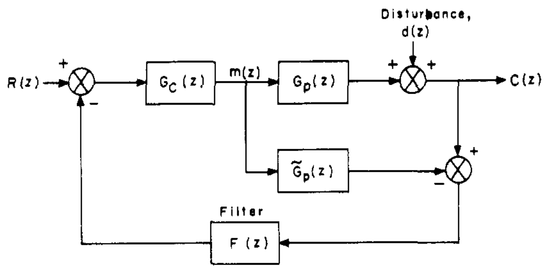

Figure 4. IMC with filter.

For stability the necessary and sufficient condition is that the roots of the following two characteristic equations must lie inside the unit circle in the z plane:

$$\frac{1}{G_c(z)} + [G_p(z) - \tilde{G}_p(z)] = 0 \qquad (5)$$

$$\frac{1}{G_c(z)G_p(z)} + \frac{1}{G_p(z)}[G_p(z) - \tilde{G}_p(z)] = 0 \qquad (6)$$

In the absence of a plant-model mismatch, these equations reduce to

$$\frac{1}{G_c(z)} = 0 \qquad (7)$$

and

$$\frac{1}{G_c(z)G_p(z)} = 0 \qquad (8)$$

Thus, when $G_p(z) = \tilde{G}_p(z)$, the controller and process poles must be inside the unit circle in the z plane. For an open-loop stable plant, therefore, stability (but not necessarily robustness) is ensured for any $G_c(z)$ that is stable.

Now on to the best choice of $G_c(z)$. Equation 4 shows that, in the absence of modeling errors, perfect control can be achieved by setting

$$G_c(z) = \frac{1}{\tilde{G}_p(z)} \qquad (9)$$

The process transfer function, in general, can contain time delays and zeros outside the unit circle. The inversion indicated in eq 9 in such cases can lead to an unrealizable pure predictor or an unstable controller which would violate the stability requirement, eq 7. Therefore, it is not feasible to use the exact process inverse to design $G_c(z)$, in general. Instead, the next best thing to do is to split the model into two parts: one containing all the time delays and zeros outside the unit circle and the other containing the remaining terms of $G_p(z)$, giving a factorization

$$\tilde{G}_p(z) = \tilde{G}_p^{+}(z)\tilde{G}_p^{-}(z) \qquad (10)$$

The controller can then be designed according to

$$G_c(z) = \frac{1}{\tilde{G}_p^{-}(z)} \qquad (11)$$

Equation 13 gives the so-called perfect IMC controller that is based on 100% accurate process models. Robustness in the presence of modeling errors is achieved by inserting a filter in the feedback signal as shown in Figure 4. With the filter in line, the characteristic equation (eq 5) becomes

$$G_c(z)^{-1} + F(z)[G_p(z) - \tilde{G}_p(z)] = 0 \qquad (12)$$

For a given plant-model mismatch, $F(z)$ is selected such that all the roots of eq 12 lie inside the unit circle.

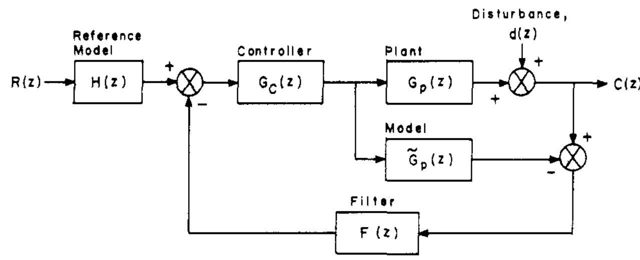

Figure 5. Complete IMC structure.

Now, if a model of the impulse response type, $\tilde{G}(z)$, is used to represent the process, there are several advantages; the order of the process is not at all important, nonminimum-phase characteristics can be easily handled, and parametric modeling is not required. Even with the IR models, the IMC strategy is the same. The first step is to find a stable approximation to the inverse of $\tilde{G}(z)$, and the second step is to design a filter (F) to preserve stability for a given plant-model mismatch. The need to approximate $\tilde{G}(z)^{-1}$ can arise for two reasons: (1) $\tilde{G}(z)$ has roots outside the unit circle and therefore the exact inverse would be unstable, and (2) the use of an exact inverse even when stable often requires excessive control action leading to an oscillatory response of the process between sampling instants, although this problem can be alleviated somewhat if a reference trajectory is employed as shown in Figure 5.

To derive approximate process inverses, Garcia and Morari (1982) formulate a general predictive-type control problem and solve it. In the process they are able to show that dynamic matrix control and model algorithmic control are but special cases of IMC.

The process of determining the process inverses can become tedious. In the following section a new method is presented which retains the advantages of predictive control methods, but since process inverses are not required, the design and implementation is much simpler.

Method

The derivation for single-input single-output systems (Tu and Tsing, 1979) follows. In the next section the algorithm for multivariable systems is developed.

Single-Loop Systems. The block diagram of a typical sampled data control system is shown in Figure 1. The z-transform representation is assumed throughout, but the z-transform operator is omitted in the equations that follow. The method assumes that the system is open-loop stable. The open-loop transfer function of the system is

$$\frac{C}{M} = G_p \qquad (13)$$

The terms are defined in the Nomenclature section. The closed-loop transfer function of the system is

$$\frac{C}{R} = \frac{DG_p}{1 + DG_p} \qquad (14)$$

The normalized response of an open-loop stable process to a unit step change in input is shown in Figure 6. The method is based on the premise that it should always be possible to design a control algorithm that will yield a closed-loop set point response that is at least as good as the normalized open-loop response. Mathematically, this requirement may be stated as

$$\frac{C}{R} = \frac{1}{K_p}G_p \qquad (15)$$

Substituting $C(z)/R(z)$ from eq 15 into eq 14 gives

$$\frac{1}{K_p}G_p = \frac{DG_p}{1 + DG_p} \quad (16)$$

The solution of eq 16 for D is

$$D = \frac{1}{K_p - G_p} \quad (17)$$

Now when Figure 1 is referring to

$$D = \frac{M}{E} \quad (18)$$

Substitution of $D(z)$ fropm eq 18 into eq 17 followed by crossmultiplication and rearrangement yields the result

$$M = \frac{1}{K_p}[E + G_pM] \quad (19a)$$

or

$$M(z) = \frac{1}{K_p}[E(z) + G_p(z)M(z)] \quad (19b)$$

The model of the process transfer function $G_p(z)$ in eq 19b can be represented with the aid of impulse response coefficients, h_i, as (Deshpande, 1985)

$$\tilde{G}(z) = h_1Z^{-1} + h_2Z^{-2} + ... + h_NZ^{-N} \quad (20)$$

Therefore, eq 19b can be written as

$$M(z) = \frac{1}{K_p}[E(z) + (h_1Z^{-1} + h_2Z^{-2} + ... + h_NZ^{-N})M(z)] \quad (21)$$

Inversion of eq 21 in the time domain is

$$M_n = \frac{1}{K_p}E_n + \frac{1}{K_p}(h_1M_{n-1} + h_2M_{n-2} + ... + h_NM_{n-N}) \quad (22)$$

Equation 22, in the absence of modeling errors, will yield a closed-loop set point response that is the same as the normalized open-loop response. Now a heuristic argument is made that it should be possible to speed up the response by introducing a constant in eq 19b to give

$$M(z) = \alpha E(z) + \frac{1}{K_p}\tilde{G}(z)M(z) \quad (23)$$

or equivalently in the time domain

$$M_n = \alpha E_n + \frac{1}{K_p}(h_1M_{n-1} + h_2M_{n-2} + ... + h_NM_{n-N}) \quad (24)$$

Equation 24 is the final form of the algorithm. The term α is the only tuning parameter in the algorithm which can be determined through simulation so that it satisfies a suitable optimization criteria. Some commonly used criteria are

$$\mathrm{IAE} = \int_0^\infty |E| \, \mathrm{d}t \quad (25)$$

and

$$\mathrm{ITAE} = \int_0^t t|E| \, \mathrm{d}t \quad (26)$$

The parameter αK_p varies between 1 and ∞.

Multivariable Systems. The block diagram of a multivariable feedback control system is shown in Figure 7. In this section, vectors and matrices are indicated by boldface letters. The method is illustrated for a 2 × 2 process although it can be readily extended to processes of higher dimensions. It is assumed that the process is open-loop stable.

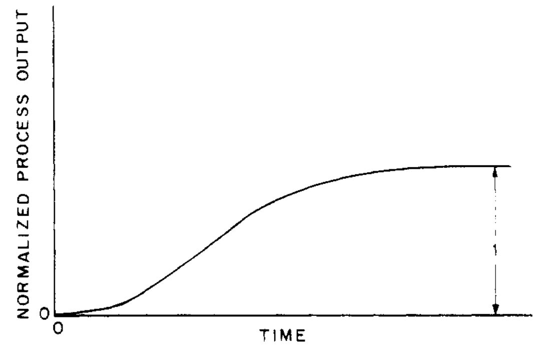

Figure 6. Normalized step response of open-loop stable process.

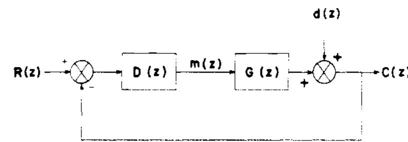

Figure 7. Block diagram of a multivariable control system.

The normalized open-loop response of the multivariable system is

$$\mathbf{C} = \mathbf{G}(\mathbf{K})^{-1}\mathbf{M} \quad (27)$$

where $(\mathbf{K})^{-1}$ = inverse of process gain matrix.

The closed-loop pulse transfer function of the system may be developed as usual by writing out the system of equations indicated in Figure 7. The result is

$$\mathbf{C} = (\mathbf{I} + \mathbf{GD})^{-1}\mathbf{GDR} \quad (28)$$

The z-transform operator has been omitted in eq 27 and 28 to maintain clarity.

The ratio $\mathbf{C}/\mathbf{R}$ does not exist for multivariable systems since $\mathbf{C}$ and $\mathbf{R}$ are matrices. However, it is still possible to define the closed-loop transfer function matrix as (Kuo, 1983)

$$\mathbf{P} = (\mathbf{I} + \mathbf{GD})^{-1}\mathbf{GD} \quad (29)$$

Then, the closed-loop response may be evaluated by the equation

$$\mathbf{C} = \mathbf{PR} \quad (30)$$

Similarly, the normalized open-loop transfer function matrix may be defined as

$$\mathbf{Q} = \mathbf{G}(\mathbf{K})^{-1} \quad (31)$$

Then, the normalized open-loop response may be obtained by

$$\mathbf{C} = \mathbf{QM} \quad (32)$$

Now an argument is made that it should always be possible to design a control algorithm which will give a set point response having the same dynamics as that of the open-loop response. The effect of this argument is that eq 29 and 31 may be equated to give

$$(\mathbf{I} + \mathbf{GD})^{-1}\mathbf{GD} = \mathbf{G}(\mathbf{K})^{-1} \quad (33)$$

Premultiplying each side of eq 33 by $(\mathbf{I} + \mathbf{GD})$ and then postmultiplying by $\mathbf{K}$ yields

$$\mathbf{GDK} = \mathbf{G} + \mathbf{GDG} \quad (34)$$

or

$$\mathbf{DK} = \mathbf{I} + \mathbf{DG} \quad (35)$$

The solution of eq 35 for $\mathbf{D}$ is

$$\mathbf{D} = (\mathbf{K} - \mathbf{G})^{-1} \quad (36)$$

Now from Figure 7

$$\mathbf{M} = \mathbf{DE} \quad (37)$$

Combining eq 36 and 37 gives

$$\mathbf{M} = (\mathbf{K} - \mathbf{G})^{-1}\mathbf{E} \quad (38)$$

Premultiplying each side of eq 38 by (**K** − **G**) and collecting terms gives

$$\mathbf{KM} = \mathbf{E} + \mathbf{GM} \quad (39a)$$

or

$$\mathbf{M} = (\mathbf{K})^{-1}\mathbf{E} + (\mathbf{K})^{-1}\mathbf{GM} \quad (39b)$$

For notational convenience let

$$\mathbf{k} = (\mathbf{K})^{-1} \quad (40)$$

Then eq 39b becomes

$$\mathbf{M} = \mathbf{kE} + \mathbf{kGM} \quad (41)$$

Now the z-transform operator is introduced and the system of equations in eq 41 is expanded to give

$$M_1(z) = k_{11}E_1(z) + k_{12}E_2(z) + k_{11}[G_{11}(z)M_1(z) + G_{12}M_2(z)] + k_{12}[G_{21}(z)M_1(z) + G_{22}(z)M_2(z)] \quad (42a)$$

and

$$M_2(z) = k_{21}E_1(z) + k_{22}E_2(z) + k_{21}[G_{11}(z)M_1(z) + G_{12}M_2(z)] + k_{22}[G_{21}(z)M_1(z) + G_{22}(z)M_2(z)] \quad (42b)$$

The process transfer functions in eq 42 can be represented with the aid of impulse response coefficients as (Deshpande, 1985)

$$G_{ij}(z) = h^1{}_{ij}Z^{-1} + h^2{}_{ij}Z^{-2} + ... + h^N{}_{ij}Z^{-N} \quad (43)$$

with i and j = 1 and 2. Then eq 42a and 42b become

$$M_1(z) = k_{11}E_1(z) + k_{12}E_2(z) + k_{11}[(h^1{}_{11}Z^{-1} + h^2{}_{11}Z^{-2} + ... + h^N{}_{11}Z^{-N})M_1(z) + (h^1{}_{12}Z^{-1} + h^2{}_{12}Z^{-2} + ... + h^N{}_{12}Z^{-N})M_2(z)] + k_{12}[(h^1{}_{21}Z^{-1} + h^2{}_{21}Z^{-2} + ... + h^N{}_{21}Z^{-N})M_1(z) + (h^1{}_{22}Z^{-1} + h^2{}_{22}Z^{-2} + ... + h^N{}_{22}Z^{-N})M_2(z)] \quad (44a)$$

and

$$M_2(z) = k_{21}E_1(z) + k_{22}E_2(z) + k_{21}[(h^1{}_{11}Z^{-1} + h^2{}_{11}Z^{-2} + ... + h^N{}_{11}Z^{-N})M_1(z) + (h^1{}_{12}Z^{-1} + h^2{}_{12}Z^{-2} + ... + h^N{}_{12}Z^{-N})M_2(z)] + k_{22}[(h^1{}_{21}Z^{-1} + h^2{}_{21}Z^{-2} + ... + h^N{}_{21}Z^{-N})M_1(z) + (h^1{}_{22}Z^{-1} + h^2{}_{22}Z^{-2} + ... + h^N{}_{22}Z^{-N})M_2(z)] \quad (44b)$$

Equations 44a and 44b can be inverted into the time domain to give

$$M_1{}^n = k_{11}E_1{}^n + k_{12}E_2{}^n + k_{11}[(h^1{}_{11}M_1{}^{n-1} + h^2{}_{11}M_1{}^{n-2} + ... + h^N{}_{11}M_1{}^{n-N}) + (h^1{}_{12}M_2{}^{n-1} + h^2{}_{12}M_2{}^{n-2} + ... + h^N{}_{12}M_2{}^{n-N})] + k_{12}[(h^1{}_{21}M_1{}^{n-1} + h^2{}_{21}M_1{}^{n-2} + ... + h^N{}_{21}M_1{}^{n-N}) + (h^1{}_{22}M_2{}^{n-1} + h^2{}_{22}M_2{}^{n-2} + ... + h^N{}_{22}M_2{}^{n-N})] \quad (45a)$$

and

$$M_2{}^n = k_{21}E_1{}^n + k_{22}E_2{}^n + k_{21}[(h^1{}_{11}M_1{}^{n-1} + h^2{}_{11}M_1{}^{n-2} + ... + h^N{}_{11}M_1{}^{n-N}) + (h^1{}_{12}M_2{}^{n-1} + h^2{}_{12}M_2{}^{n-2} + ... + h^N{}_{12}M_2{}^{n-N})] + k_{22}[(h^1{}_{21}M_1{}^{n-1} + h^2{}_{21}M_1{}^{n-2} + ... + h^N{}_{21}M_1{}^{n-N}) + (h^1{}_{22}M_2{}^{n-1} + h^2{}_{22}M_2{}^{n-2} + ... + h^N{}_{22}M_2{}^{n-N})] \quad (45b)$$

Equations 45a and 45b, if implemented, will yield closed-loop responses having open-loop dynamics. The algorithm can be speeded up by introducing a matrix of gains α in eq 41 to give

$$\mathbf{M} = \alpha\mathbf{E} + \mathbf{kGM} \quad (46)$$

or equivalently in the time domain

$$M_1{}^n = \alpha_{11}E_1{}^n + \alpha_{12}E_2{}^n + k_{11}[(h^1{}_{11}M_1{}^{n-1} + h^2{}_{11}M_1{}^{n-2} + ... + h^N{}_{11}M_1{}^{n-N}) + (h^1{}_{12}M_2{}^{n-1} + h^2{}_{12}M_2{}^{n-2} + ... + h^N{}_{12}M_2{}^{n-N})] + k_{12}[(h^1{}_{21}M_1{}^{n-1} + h^2{}_{21}M_1{}^{n-2} + ... + h^N{}_{21}M_1{}^{n-N}) + (h^1{}_{22}M_2{}^{n-1} + h^2{}_{22}M_2{}^{n-2} + ... + h^N{}_{22}M_2{}^{n-N})] \quad (47a)$$

and

$$M_2{}^n = \alpha_{21}E_1{}^n + \alpha_{22}E_2{}^n + k_{21}[(h^1{}_{11}M_1{}^{n-1} + h^2{}_{11}M_1{}^{n-2} + ... + h^N{}_{11}M_1{}^{n-N}) + (h^1{}_{12}M_2{}^{n-1} + h^2{}_{12}M_2{}^{n-2} + ... + h^N{}_{12}M_2{}^{n-N})] + k_{22}[(h^1{}_{21}M_1{}^{n-1} + h^2{}_{21}M_1{}^{n-2} + ... + h^N{}_{21}M_1{}^{n-N}) + (h^1{}_{22}M_2{}^{n-1} + h^2{}_{22}M_2{}^{n-2} + ... + h^N{}_{22}M_2{}^{n-N})] \quad (47b)$$

Equation 47 is the final form of the algorithm. The constants α_{11}, α_{12}, α_{21}, and α_{22} are the tuning constants of the algorithm. They are determined by a suitable off-line optimization program that satisfies a suitable performance index, say, of the type

$$\mathbf{I} = \int_0^t |E_1|\,dt + \int_0^t |E_2|\,dt \quad (48)$$

Equation 48 is by no means unique; better performance indexes may be found. For set point changes, the goal of the optimization effort would be to ensure good transient response for one variable, consistent with zero offset, and zero steady-state error for the other.

There is sufficient experience with single-loop systems which indicates that set point designs work adequately well for load changes as well. It is not known whether this experience can be extended to multivariable systems. In any event, the optimization of α's for load changes, utilizing a suitable optimization criteria, poses no special problems as long as load dynamics are known.

The robustness of the algorithm can be enhanced by adding a first-order filter in the feed back path. Excessive ringing of the manipulated variables is to be avoided. This can be accompanied by adding appropriate constraints to the performance index of the type given in eq 48.

Stability Properties of the SMPC Algorithm. If the control system is expressed in the form of a usual sampled data control system shown in Figure 1, then the requirement for loop stability is that the roots of

$$1 + D(z)G_p(z) = 0 \quad (49)$$

must lie inside the unit circle in the z plane. For SMPC the control algorithm, $D(z)$, is given by (see eq 23)

$$D(z) = \frac{\alpha K_p}{K_p - \tilde{G}(z)} \quad (50)$$

There is no requirement, as far as loop stability is concerned, that $D(z)$ must be open-loop stable. Indeed the commonly used PI controller is open-loop unstable. For the examples studied, the open-loop response of the SMPC algorithm is similar to that of a PI control algorithm. The use of either an impulse-response-type model, $\tilde{G}(z)$, or a transfer-function-type model, $G_p(z)$, in eq 50 is correct; however, the latter is more convenient for stability analysis.

The simplified model predictive control algorithm can be examined in the framework of IMC. The use of eq 50 and 1 gives the IMC algorithm

$$G_c(z) = \frac{\alpha K_p}{K_p + (\alpha K_p - 1)\tilde{G}(z)} \quad (51)$$

Table I. Column Data

steady-state operating conditions

stream	flow, lb/min	composition, wt % methanol
distillate	1.18	96
reflux	1.95	96
bottoms	1.27	0.5
feed	2.45	46.5
steam	1.71	

steady-state temperature profile

stream/tray	temp, °F
reflux	151.7
feed	168.0
steam	233.0
condensate	227.5
reboiler	209.6
plate	
1	203.6
2	194.4
3	181.2
4	172.9
5	164.1
6	156.8
7	152.1
8	148.5
condenser	143.9

transfer functions

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.80e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} + \begin{bmatrix} \dfrac{3.8e^{-8.1s}}{14.9s+1} \\ \dfrac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} F(s)$$

Figure 2 then represents the SMPC system as an IMC system.

If we choose to represent the SMPC system as an IMC system shown in Figure 2, the the *loop stability* requirements (see eq 7 and 8) demand that $G_c(z)$ must be open-loop stable. (In all of this discussion, it is emphasized that $G_p(z)$ is always assumed to be open-loop stable and there are no modeling errors.) These concepts are illustrated by an example. Let

$$G_p(z) = \frac{z+b}{z^2-a^2} \tag{52}$$

with $a^2 = 0.5$ and $b = -2$. The substitution of eq 52 into eq 50 gives a $D(z)$ for an SMPC controller that is open-loop unstable, but this does not necessarily make the loop shown in Figure 1 unstable. In fact, this hypothesis can be confirmed by expressing SMPC as an IMC controller. Substituting eq 52 into eq 51 gives after some simplification

$$G_c(z) = \frac{\alpha(z^2-a^2)}{z^2 + (\alpha - 1/K_p)z + [(\alpha - 1/K_p)b - a^2]} \tag{53}$$

According to the IMC stability requirement, the roots of

$$z^2 + (\alpha - 1/K_p)z + [(\alpha - 1/K_p)b - a^2] = 0 \tag{54}$$

must lie inside the unit circle in the z plane. For the particular values of $a^2 = 0.5$ and $b = -2$, a little algebra shows that the controller will be stable for $-1.25 < \alpha < -0.5$. Thus, the example shows that α affects stability, directly.

Implementation and Testing

The algorithm has been tested through simulation on a binary distillation column that separates a mixture of methanol and water into two relatively pure products in a column that is equipped with a total condenser and a reboiler (Wood and Berry, 1973). A schematic of the column and the control system is shown in Figure 8. The goal of the control effort is to maintain the two product

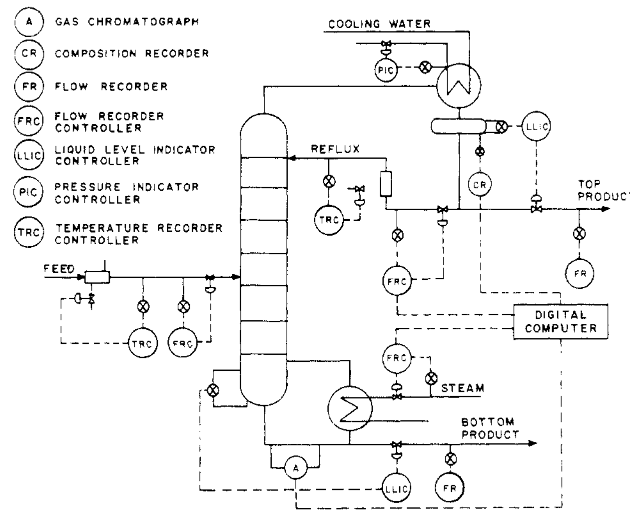

Figure 8. Schematic of a distillation column.

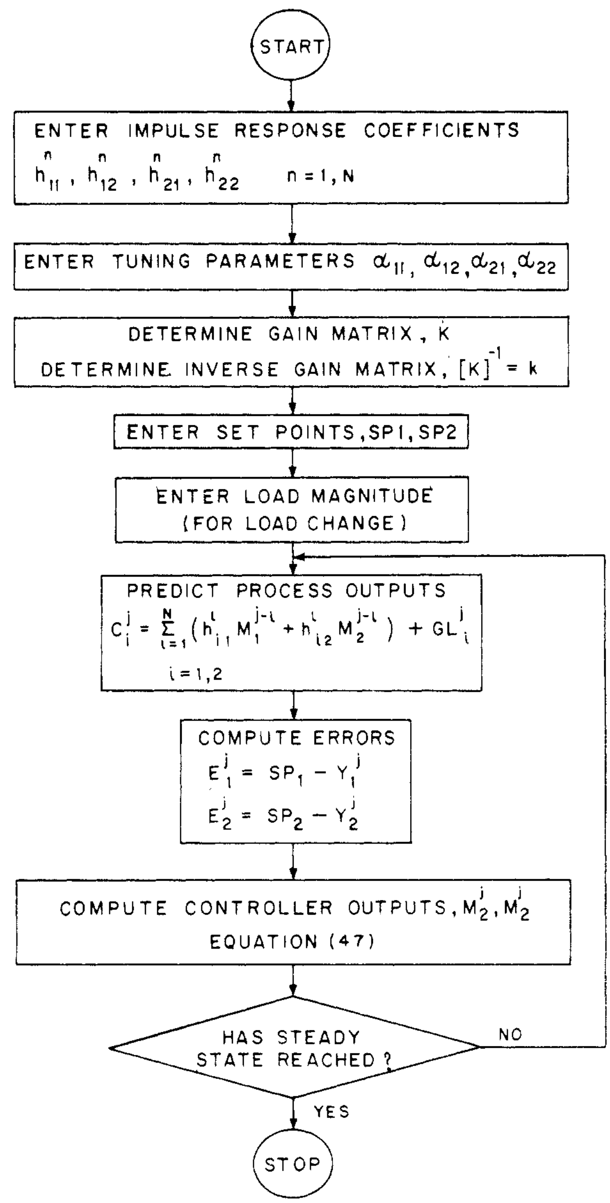

Figure 9. Flow chart of simplified model predictive control strategies.

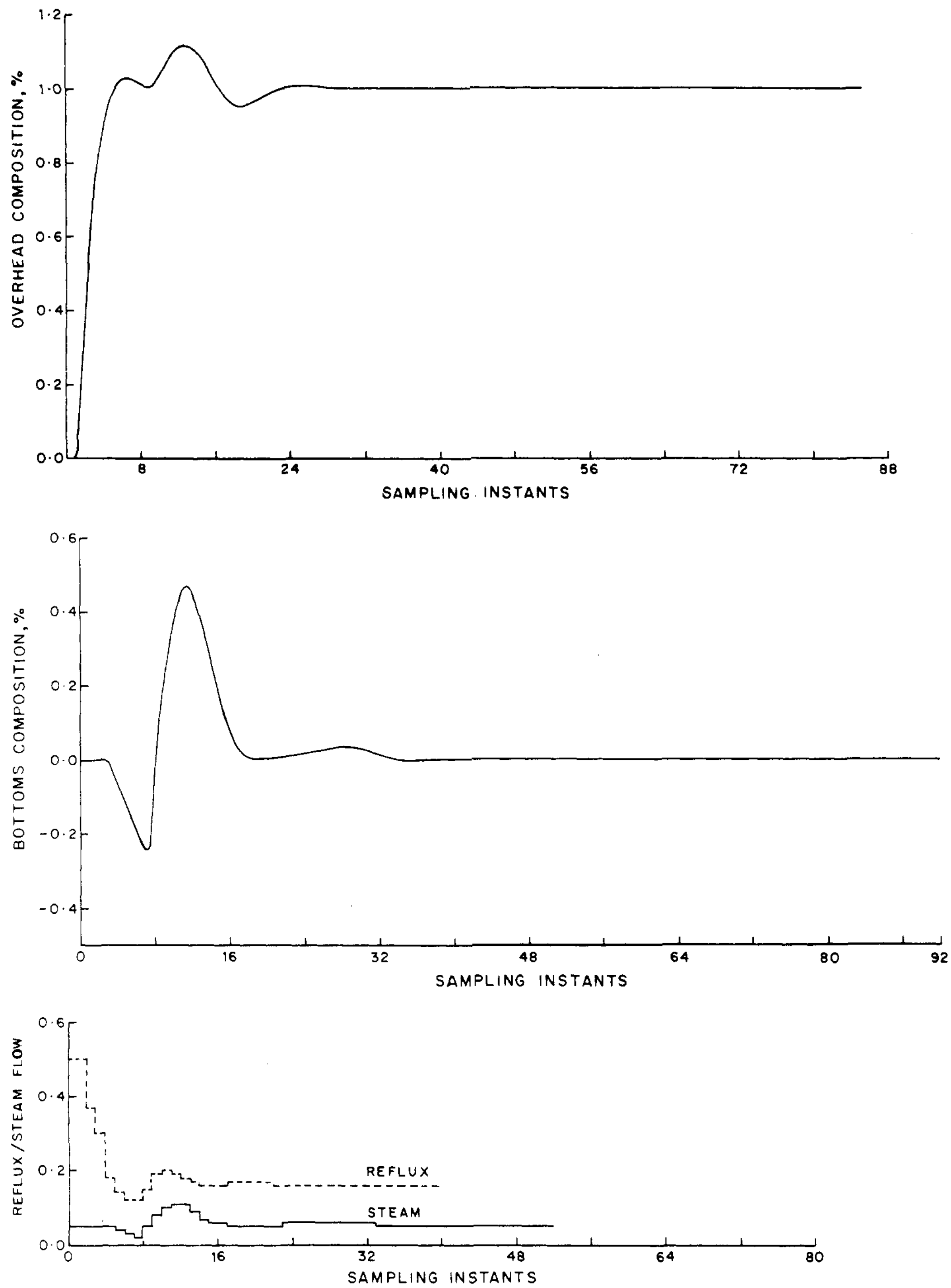

Figure 10. (a, top) Response of overhead composition for a set point change in overhead composition. (b, middle) Response of bottoms composition for a set point change in overhead composition. (c, bottom) Manipulated variable moves associated with overhead composition set point change.

compositions at the set point in the presence of disturbances.

During the course of their study, Wood and Berry (1973) determined open-loop transfer functions by pulse testing. These transfer functions and the steady-state operating data are shown in Table I. Open-loop step responses determined from these transfer functions are used as inputs to SMPC.

A flow chart for implementing SMPC is shown in Figure 9. The matrix of controller gains $\underline{\alpha}$ is determined through an off-line optimization program such that the objective function in eq 48 is satisfied. The optimization program is based on a random search procedure that is described elsewhere (Ralston et al., 1985). The process gain matrix and its inverse and the controller gains are shown in Table II. All computer work has been carried out on the DEC 10/90 systems.

Figure 10, parts a and b, shows the responses of the product compositions for a change in the set point of the overhead composition. The manipulated variable moves for this test are shown in Figure 10c. Figure 11 presents similar information for a set point change in bottoms composition and Figure 12 for a load upset in feed flow to the column. The load responses shown in Figure 12 are somewhat oscillatory although the magnitude of the oscillations is small. The oscillations can be reduced, albeit at the expense of ISE, by selecting the elements of the α matrix that are smaller than α_{ISE}.

Table II. Controller Parameters

process gain matrix and inverse

$$\mathbf{K} = \begin{bmatrix} 12.8 & -18.9 \\ 6.6 & -19.4 \end{bmatrix} \qquad \mathbf{K}^{-1} = \begin{bmatrix} 0.15698 & -0.15294 \\ 0.05341 & -0.10358 \end{bmatrix}$$

		tuning parameters, α				
type of change	magnitude of change	α_{11}	α_{12}	α_{21}	α_{22}	min IAE
set point (distillate comp.)	1%	0.5004	−0.2907	0.0509	−0.230	7.406
set point (bottoms comp.)	1%	0.5418	−0.2463	0.1717	−0.2298	8.109
load (feed flow)	0.34 lb/min	1.0580	−0.0822	0.1564	−0.2640	7.17

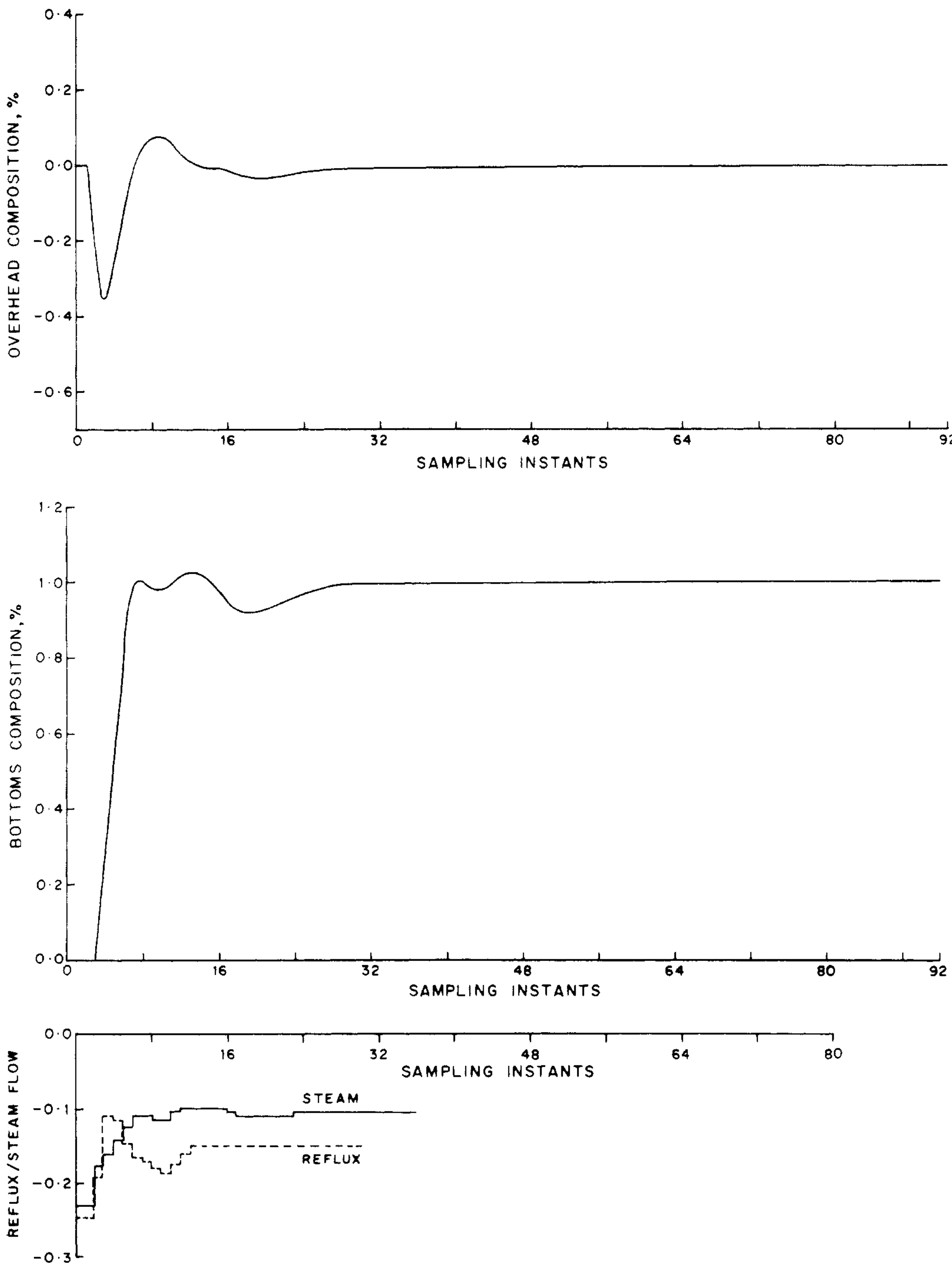

Figure 11. (a, top) Response of overhead composition for a set point change in bottoms composition. (b, middle) Response of bottoms composition for a set point change in bottoms composition. (c, bottom) Manipulated variable moves associated with bottom composition set point change.

For the foregoing sample, closed-loop simulation results with PID-type control are not available. However, a related paper (Arulalan and Deshpande, 1986a) compares the performance of SMPC with DMC and PID (with and without decoupling) for a 2 × 2 blending process, and the results confirm the excellent capabilities of SMPC. The interested reader may consult the paper for details.

Discussion

Figures 10–12 show good closed-loop responses con-

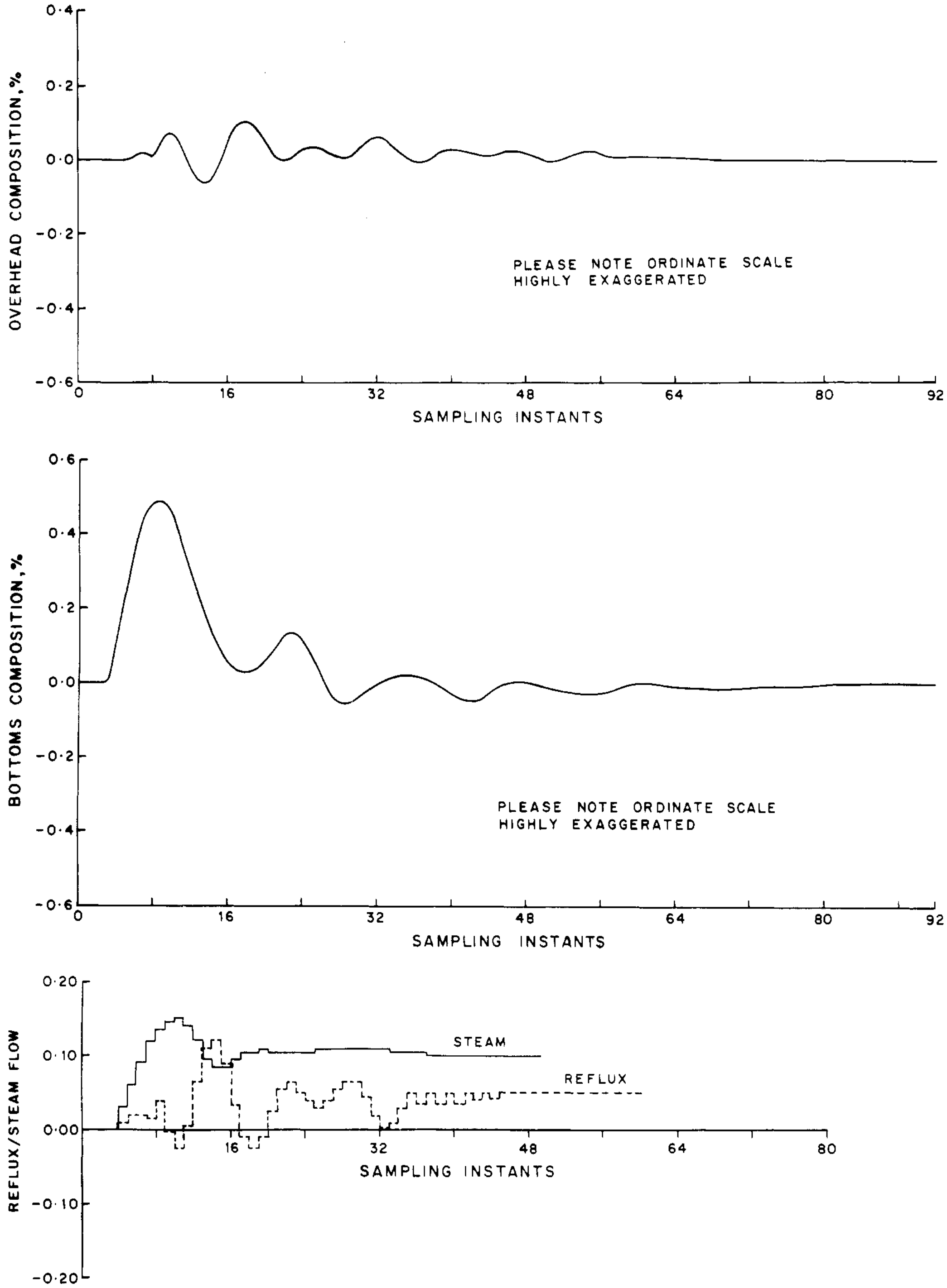

Figure 12. (a, top) Response of overhead composition for an upset in feed flow rate. (b, middle) Response of bottoms composition for an upset in feed flow rate. (c, bottom) Manipulated variable moves associated with feed flow disturbance test.

sistent with zero offset and no steady-state error. Interaction compensation is evident, and the manipulated variable movements are not excessive. In this work the suitability of various other objective functions for the determination of optimum α's has not been studied. Also, the effect of modeling errors has not been studied, although in view of IMC analysis it is known that robustness can be achieved by inserting a suitable filter in the feedback path of SMPC.

In other predictive control formulations, such as DMC and MAC, optimization is carried out on-line. This facility is not available in the current version of SMPC. Absence of available information on load dynamics could present some difficulties in finding optimum α's. Dynamic mathematical modeling and simulation may alleviate this difficulty to a certain extent.

Conclusions and Future Work

Simplified model predictive control has been shown to be a simple, efficient methodoloty for multivariable control. SMPC has also been successfully tested through simulation on a blending process (Arulalan and Deshpande, 1986a) and a 3 × 3 side-draw distillation column (Arulalan and Deshpande, 1986b). Future work includes experimental tests of SMPC, a study of other optimization objective functions, filters, and reference models for set point tracking.

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Nomenclature

C = controlled variable
d = disturbance
d_m = feedback signal
D = control algorithm
F = filter transfer function
G = impulse-response-type transfer function
G_c = IMC controller
G_p = process transfer function
h = impulse response coefficients
$\mathbf{I}$ = identity matrix
k = element of process gain inverse matrix
$\mathbf{k}$ = process gain inverse matrix
$\mathbf{K}$ = process gain matrix
K_p = process steady-state gain
m, M = manipulated variables
N = number of samples
$\mathbf{P}$ = closed-loop transfer function matrix
$\mathbf{Q}$ = open-loop transfer function matrix
t = time
z = z-transform operator

Greek Symbols

α = SMPC tuning parameter(s)

Subscripts

p = process
n = sampling instants

Superscripts

$\tilde{}$ = pertaining to model

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Solubility of Radon in Selected Perfluorocarbon Compounds and Water

Christopher Lewis,[†‡] Philip K. Hopke,*[†‡§] and James J. Stukel[†§⊥]

Institute for Environmental Studies, Nuclear Engineering Program, Department of Civil Engineering, and Department of Mechanical Engineering, University of Illinois, Urbana, Illinois 61801

The solubility of radon in a series of perfluorocarbon compounds has been measured in the temperature range 5–40 °C. From the resulting data, a Hildebrand solubility parameter for radon of 8.42 ± 0.11 cal$^{1/2}$ cm$^{-3/2}$ was determined. In order to provide reliable values of these parameters for radon in water, measurements were made for deionized, medium-hardness, and high-hardness water. The thermodynamic functions, $\Delta G°$, $\Delta H°$, $\Delta S°$, and $\Delta C_p°$, characterizing the solubilization process have also been determined.

Recently there has been renewed interest in radon and radon daughter properties. A model developed by Harley and Pasternak (1981) suggests that from 20% to 100% of the spontaneous lung cancers in the environment may be attributable to radon and its daughters. The US Environmental Protection Agency tentatively proposed standards governing radioactive releases from certain activities (Federal Register, 1983) and has recently set final standards for radon emission (Federal Register, 1985a,b). These activities will initially include uranium mines. In their discussion of these standards, the EPA states that there is a "lack of suitable control technology to capture radon-222 being vented from mines", and thus they felt that an emission standard was therefore not feasible. The availability of such technology may be useful. Of greater utility would be the removal of radon from the ventilation streams that could substantially reduce the total air flow needed in uranium mines to meet current occupational radiation exposure limits. These circumstances provide the impetus for the development of techniques to reduce airborne radon concentrations from uranium mine effluents and possibly from other large volume airstreams.

Hopke et al. (1984) reviewed the mechanisms available for removal of radon from large-volume airstreams. They suggested that conventional packed bed scrubbers could prove useful if an appropriate nonaqueous solvent could be identified. The characteristics of such a solvent previously identified by A.D. Little Inc. (1975) include (1) high radon solubility, (2) low vapor pressure at operating temperature, (3) nonflammable and nontoxic, (4) chemically

* Author to whom correspondence should be addressed.
† Institute for Environmental Studies.
‡ Nuclear Engineering Program.
§ Department of Civil Engineering.
⊥ Department of Mechanical Engineering.

