Zeitharmonische Maxwell-Gleichungen

für anisotrope, homogene Medien

$$\nabla \times \underline{\vec{H}} = \underline{\vec{J}} + \mathrm{j}\omega\varepsilon\underline{\vec{E}} \quad \Longleftrightarrow \quad \oint_{\partial A} \underline{\vec{H}} \cdot \mathrm{d}\vec{s} = \iint_A \left(\underline{\vec{J}} + \mathrm{j}\omega\varepsilon\underline{\vec{E}}\right) \cdot \mathrm{d}\vec{A}$$

$$-\nabla \times \underline{\vec{E}} = \underline{\vec{M}} + \mathrm{j}\omega\mu\underline{\vec{H}} \quad \Longleftrightarrow \quad -\oint_{\partial A} \underline{\vec{E}} \cdot \mathrm{d}\vec{s} = \iint_A \left(\underline{\vec{M}} + \mathrm{j}\omega\mu\underline{\vec{H}}\right) \cdot \mathrm{d}\vec{A}$$

$$\nabla \cdot \underline{\vec{H}} = \underline{\rho}_m/\mu \quad \Longleftrightarrow \quad \oiint_{\partial V} \underline{\vec{H}} \cdot \mathrm{d}\vec{A} = \frac{1}{\mu}\iiint_V \underline{\rho}_m \mathrm{d}V$$

$$\nabla \cdot \underline{\vec{E}} = \underline{\rho}_e/\varepsilon \quad \Longleftrightarrow \quad \oiint_{\partial V} \underline{\vec{E}} \cdot \mathrm{d}\vec{A} = \frac{1}{\varepsilon}\iiint_V \underline{\rho}_e \mathrm{d}V$$

für Freiraumausbreitung und allgemein Vakuum: $\underline{\rho}_e = \underline{\rho}_m = 0$ und $\underline{\vec{J}} = \underline{\vec{M}} = \vec{0}$
Poynting-Vektor; Wirkleistung

$$\underline{\vec{S}} = \frac{1}{2}\underline{\vec{E}} \times \underline{\vec{H}}^* \quad ; \quad P = \oiint \Re\left\{\underline{\vec{S}}\right\} \mathrm{d}\vec{A} = \frac{1}{2}\oiint \Re\left\{\underline{\vec{E}} \times \underline{\vec{H}}^*\right\} \mathrm{d}\vec{A}$$

EM-Wellen: allgemeine Zusammenhänge

Wellenleiter (konstanter Querschnitt)

$$\lambda = \frac{\lambda_0}{\sqrt{1-\left(\frac{\beta_c}{\beta_0}\right)^2}} = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

Phasengeschwindigkeit

$$v_\mathrm{p} = \frac{c}{\sqrt{1-\left(\frac{\beta_c}{\beta_0}\right)^2}} = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

Gruppengeschwindigkeit

$$v_\mathrm{g} = c\sqrt{1-\left(\frac{\beta_c}{\beta_0}\right)^2} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2} = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

Zusammenhang Geschwindigkeiten

$$c_0^2 = v_\mathrm{g} \cdot v_\mathrm{p}$$

Kritische Frequenzen ($a > b$)

$$f_{c,m,n} = \frac{c}{2} \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Feldwellenwiderstand

TEM-Wellen

$$\frac{\underline{\vec{E}}_x}{\underline{\vec{H}}_y} = -\frac{\underline{\vec{E}}_y}{\underline{\vec{H}}_x} = \pm\frac{\omega\mu}{\beta} = \pm\sqrt{\frac{\mu}{\varepsilon}} = \pm Z_F$$

$$Z_{F0} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\Omega = 377\Omega$$

TE-Wellen

$$Z_{FH} = \pm\frac{\underline{\vec{E}}_x}{\underline{\vec{H}}_y} = \mp\frac{\underline{\vec{E}}_y}{\underline{\vec{H}}_x} = \frac{\omega\mu}{\beta} = \frac{\omega\mu}{\beta_0\sqrt{1-\left(\frac{\beta_c}{\beta_0}\right)^2}} = \frac{Z_F}{\sqrt{1-\left(\frac{\beta_c}{\beta_0}\right)^2}} = \frac{Z_F}{\sqrt{1-\left(\frac{f_c}{f_0}\right)^2}} = \frac{Z_F}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

TM-Wellen

$$Z_{FE} = \pm\frac{\underline{\vec{E}}_x}{\underline{\vec{H}}_y} = \mp\frac{\underline{\vec{E}}_y}{\underline{\vec{H}}_x} = \frac{\beta}{\omega\varepsilon} = \frac{\beta_0\sqrt{1-\left(\frac{\beta_c}{\beta_0}\right)^2}}{\omega\varepsilon}$$

$$= Z_F\sqrt{1-\left(\frac{\beta_c}{\beta_0}\right)^2} = Z_F\sqrt{1-\left(\frac{f_c}{f_0}\right)^2} = Z_F\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$