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Exploring RR-Lyrae Period-Luminosity Relations to Determine Distances to our Galactic Bulge and Magellanic Clouds.

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ABSTRACT

Measuring distances to the stars have always been a critical priority for astronomy, ultimately to establish values for the Hubble constant, H_0, governing the expansion of the Universe. We derive Period-Luminosity (P-L) relations so by measuring the periods of pulsation of variable stars within celestial objects, we may then use the observed luminosity drop-off to derive our distance to it.

Though in the past, the P-L relations derived for our Galactic Bulge (BLG), Large Magellanic Cloud (LMC), and Small Magellanic Cloud (SMC) were largely done using Cepheid variable stars, in this particular work I used less-studied RR-Lyrae to derive such relations.

The large dataset of photometric observations in the $0.8\mu m$ I-Band from the *Optical Gravitational Lensing Experiment (OGLE)* was used to derive P-L relations to estimate distances for RR-Lyrae and its subtype RRab, RRc, and RRd stars in the BLG, LMC, and SMC. The most significant results were:

$$M_{BLG} = -0.1798\, logP + 2.4771$$
$$M_{LMC} = -0.6865\, logP + 0.126$$
$$M_{SMC} = -0.6806\, logP - 0.0739$$

Possible explanations which account for the disparity between these results and those from Madore et al. (2013) were given, such as lack of consideration of reddening or data normalization, as well as providing potential further areas of investigation — for example, other types of variable stars.

The code that was used to sort out this information and assist in graph generation are all available for viewing and download at https://github.com/fl4mx/rrlyr.

1. LITERATURE REVIEW

1.1. *Photometry and Magnitudes*

Astronomers often measure the brightness (and often thus distance) of astronomical objects using a technique denominated photometry. Photometry uses filters to measure the brightness at various wavelengths in a photometric system (Bessell 2005). Common photometric systems include the UBV and BVI systems, which use Ultraviolet, Blue, and Visual filters and Blue, Visual, and Infrared filters, respectively.

Photometric brightness is measured with absolute and apparent magnitude. Apparent magnitude, denoted m, is the brightness of the object that is measured by scientific instruments, which may be dimmed due to an inverse-square intensity drop-off, or extinction. Extinction refers to an absorption of scattering of the light from stars by dust and gas, in the form of circumstellar, interstellar, and foreground extinction. Dust around the star in circumstellar extinction absorbs and scatters blue light more than red, leading it to be otherwise known as circumstellar reddening. Interstellar extinction refers to dust and gas within the LMC and other clusters or galaxies, which can also cause reddening. Foreground extinction refers to the rest of the dust and gas between us and the observed object.

This intensity dropoff may be represented in terms of the flux that we measure: $m = -2.5\, log(Flux)$ Absolute magnitude, denoted M, is defined as the apparent magnitude measured at exactly 10pc (32.6ly) away from the object, without any interstellar interference.

Both magnitudes are reverse logarithmic, that is, a brighter object would have a lower magnitude number. For example, the comparatively dimmer RR-Lyrae in the Lyra constellation has $m = 7.195$ in the Visual (V) band

(Schoneich & Lange 1979), while Venus, one of the brightest objects in the night sky, has $m = -4.14$ (Mallama & Hilton 2018) in the same band.

1.2. *Relevance and Current Understanding*

Measuring distances to the stars have always been a critical priority for astronomy. A framework known as the Cosmic Distance Ladder (or Extragalactic Distance Scale) is used to determine ever further distances to celestial objects, and ultimately, establish a value for the Hubble Constant, H_0, a core parameter governing the expansion of the Universe (Howard 2011).

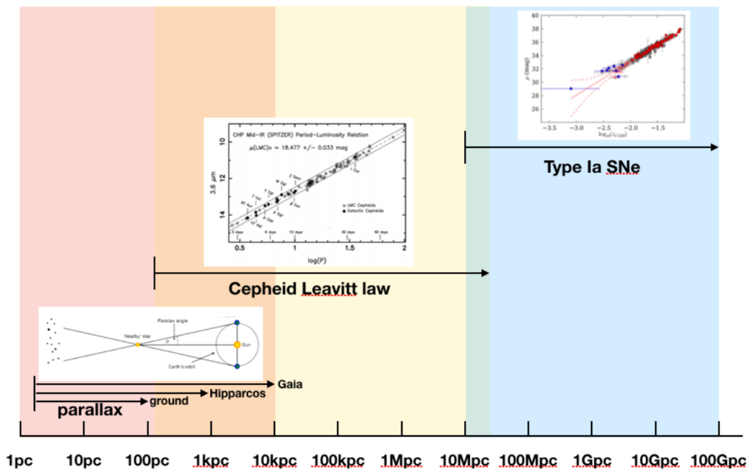

Figure 1. Extragalactic Distance Ladder (Neeley 2017). Ever further distances are measured by "rungs" on this "ladder" — with one such series of rungs starting off with direct measurements of Parallax, followed by Variable Star relations (the focus of this paper), and finally Type Ia supernovae.

1.2.1. *Variable Stars*

Variable stars are those whose apparent magnitude varies with time. These variations in magnitude can be attributed to extrinsic or intrinsic properties. In this project I focused primarily on RR-Lyrae stars (Section 1.1.3 and 1.2).

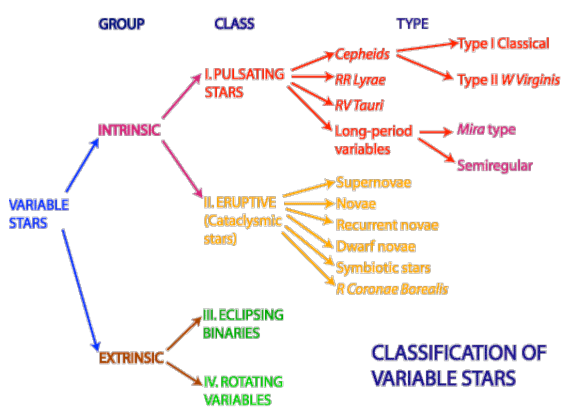

Figure 2. Image Courtesy of OGLE. A common extrinsic variable is an eclipsing binary system, where two mutually orbiting stars in a binary system pass over each other, causing dips in the apparent brightness of the system as light from the constituent stars are covered. Intrinsic variables are generally those that pulsate, producing brightness variations, such as RR-Lyrae stars. Another common type of intrinsic variable is a cataclysmic variable, which have irregular variations in brightness, such as supernovae which destroy the progenitor star.

1.2.2. *Photometric Distance*

A quantity known as the Distance Modulus (DM) is given by $DM = m - M$. Since $m = -2.5\,log(Flux)$, and the Inverse Square Law states that $Flux \propto \frac{1}{d^2}$ where d is our distance to the star, then assuming no extinction, $DM = 5\,log(\frac{d}{10})$, and so $d = 10 \times 10^{\frac{DM}{5}} = 10 \times (10^{(\frac{m-M}{5})})$.

1.2.3. *Trigonometric Parallaxes for Distances*

Measuring distances via trigonometric parallaxes has been a technique known for millenia. As the Earth orbits about the Sun (radius = 1 AU), the stars whose distances we wish to measure will appear to move across their background, as we observe them at opposite ends of our orbit — and we may then measure the parallax angle of this star; this method is otherwise known as annual parallax for this reason.

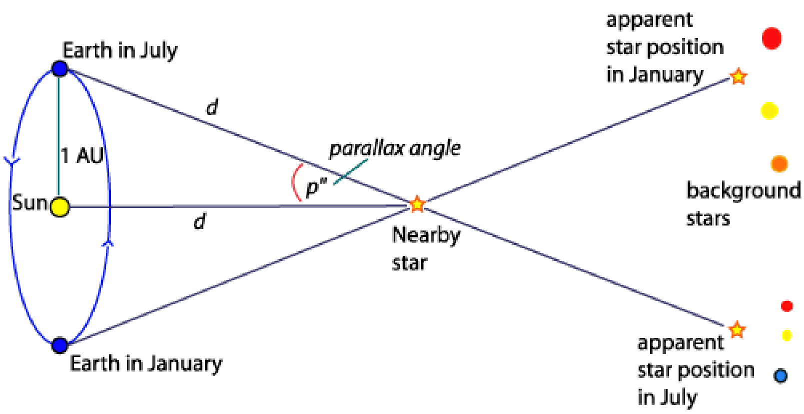

Figure 3. Image Courtesy of R. Hollow, CSIRO. Note that with tangent small angle approximation, $tan(\theta) \approx \theta$ for small θ. We get that by inverting the parallax, the distance to the star in question is given by $d = 1/p$, where distances are typically given in parsecs (equivalent to the distance of a **par**allax angle of one arc**sec**ond). With precision measurements, parallax is measured in terms of milliarcseconds (mas), and so the corresponding inverted distance would be in kiloparsecs (kpc). Since this measurement of a trigonometric parallax is a direct measurement of the stars, it is the most accurate and thus is the basis of the distance ladder.

For an exploration into small parallax angle, please refer to Appendix A.

Therefore, we will need further methods of measuring distances beyond a simple trigonometric parallax — either statistical parallaxes or variable stars.

1.2.4. *Statistical Parallaxes for Distances*

Statistical parallaxes, such as in Bailer-Jones (2015) (provides a good explanation of the nature of statistical parallax) fit a distribution to the parallaxes, and may use other astrometric parameters such as radial velocity and proper motion to model the star's distance based on their measured velocity dispersion.

1.2.5. *Variable Stars for Distances*

The use of variable stars, namely Cepheids and RR-Lyrae as distance indicators, use intrinsic stellar properties to infer its absolute magnitude (M), and then compare that to its measured apparent magnitude (m) to determine its distance. Papers like Sebo et al. (2002) have done work on Cepheids, which follow a Period Luminosity (P-L) relation, which expresses the Absolute Magnitude of the star as a function of its period of pulsation. RR-Lyraes more commonly obey a Period-Luminosity-Metallicity (P-L-Z) relation, which expresses the intrinsic brightness as a function of its period and metallicity (often [Fe/H] ratio). This project thus tried to do something more different — expressing a direct P-L relationship for RR-Lyraes.

1.3. *P-L relations for RR-Lyrae Variables*

Since the use of a P-L-Z relation involves determining an extra stellar parameter (metallicity) which may be difficult to calculate — methods of determining Z involve either measuring UV excesses in the Johnson UVB system (Johnson

& Morgan 1953), or via analysis of spectra, which requires a spectrometer. Hence, if there exist a P-L relation, similar to Cepheids, for RR-Lyrae, it would greatly simplify the use of RR-Lyrae as distance indicators. It has been established knowledge for some time now that there exist good P-L relations for RR-Lyrae in the K-band (centered at $2.2\mu m$, with various papers giving such relations, such as Butler (2003) and Frolov & Samus' (1998). Recently, papers have attempted to calibrate such P-L relations for RR-Lyrae in the mid-infrared bands — such as Madore et al. (2013) which calibrates three relations for the *WISE* [W1], [W2], and [W3] bands. However, there have yet to be substantial development of such P-L relations in the I-Band.

2. SCIENTIFIC RESEARCH QUESTION

What are the P-L relations of RR-Lyraes from the OGLE-IV $0.8\mu m$ I-Band and their subtypes of RRab, RRc, RRd, in the BLG, LMC, and SMC?

3. METHODOLOGY

3.1. *Part 1: Data Collection*

1. Metadata regarding the Periods, I-Band Apparent Magnitudes, and Subtypes of RR-Lyraes in the BLG, LMC, and SMC were downloaded from the OGLE-IV website.

2. Within the metadata files, Name, Type, Average I-Band Magnitude, and Period were columns 0, 2, 5, and 8 respectively.

3. Distances to the RR-Lyrae within the BLG were queried using from the Bailer-Jones VizieR catalogue for stars within the Gaia EDR3 at I/352/gedr3dis (Bailer-Jones et al. 2021), from the column "rgeo".
   - Stars that returned null, or invalid (such as 0pc), distances were ignored and removed from the dataset.
   - 4411 such stars (mainly returning distances of 0pc) were removed.

3.1.1. *Justification*

- The primary source of star data came from OGLE-IV because it is one of the largest catalogues of RR-Lyrae stars by quantity, and its precision measures stars in the $0.8\mu m$ I-Band to milli-magnitude accuracy (Udalski et al. 2015). It is a wide-field survey, constantly monitoring its area of the sky, which means that the data that is collected on the stars in the catalogue are over timescales of years — hence lots of datapoints, and very reliable. It also measured the BLG (Soszyński et al. 2019) and Magellanic Clouds, and thus was a valid and useful dataset for addressing the research question.
- For a further, similar description on the use of the Gaia and Bailer-Jones catalogues, please refer to Appendix B.

3.2. *Part 2: Determining Absolute Magnitude*

3.2.1. *BLG*

1. By using the distances queried from the Bailer-Jones catalogue and also using the definition of distance modulus, the queried distance, and the stars' apparent magnitudes, the absolute magnitude of stars in the Bulge were derived ($\because DM = m - M = 5\,log(\frac{d}{10})$, $\therefore M = m - 5\,log(\frac{d}{10})$), in Table 2.

2. The stars were split by their type, into RRab, RRc, and RRd.

3.2.2. *LMC and SMC*

The process that was used to fit the a relation to the RR-Lyrae within the Magellanic Clouds was largely identical to the process fitting our own Galactic RR-Lyrae — the only notable difference is that instead of querying distances from a catalogue, the distances to the stars within the Magellanic Clouds were assumed to be uniform, and two distances were taken.

1. The data for the LMC and SMC were both downloaded from the OGLE-IV website, and the columns remain the same for this metadata file as the BLG.

2. Assuming that all the stars within the LMC were a uniform distance away (since the distances within the Cloud was assumed to be negligible), a distance of 49590pc was used, whereas for the SMC, a distance of 62440pc was used.

3. Using the definition of distance modulus, the given distances, and the stars' apparent magnitudes, the absolute magnitude of stars in the LMC and SMC were derived, in Tables 3, and 4 respectively.

4. The stars were split by their type, into RRab, RRc, and RRd.

3.2.3. *Justification*

- The distance to the LMC was assumed to be 49590pc (Pietrzyński et al. 2019), and the distance to the SMC was assumed to be 62440pc (Graczyk et al. 2020). Both of these papers are the most recent, reliable, and accurate, and are not only corroborated by previous astronomical results, but enhance them with the help of new technologies and methods, namely eclipsing binary variable stars. Though these are not RR-Lyraes, they are accurate to better than 1% and 2% respectively, and with such values I attempted to calibrate P-L relations for RR-Lyraes in the Magellanic Clouds.

3.3. *Part 3: Data Cleansing*

1. RR-Lyraes with a calculated absolute magnitude brighter than -1 or dimmer than 7 were treated as erroneous data points, and ignored.
   - 10 and 709 such stars respectively were removed in the BLG.
   - 994 and 0 such stars respectively were removed in the LMC.
   - 582 and 0 such stars respectively were removed in the SMC.

   As such, the graphs were transformed from Figure 7, 8, 9 to Figure 4, 5, and 6 respectively.

3.3.1. *Justification*

- There were no stringent or well defined cutoff for appropriate brightness ranges of RR-Lyraes, the range of $-1 < M < 7$ was chosen to be roughly accurate, and exclude the definitely erroneous data points, without being so extreme so as to potentially delete false negatives.
- The prototypical RR-Lyrae — RR-Lyrae itself in the Lyra constellation, has been shown by Catelan & Cortés (2008) to only have an absolute magnitude of 0.664 (±0.126), albeit in the V band. Since RR-Lyrae is one of the brightest members of its class, even an observation in the I band will be unlikely to be brighter than -1, which is more than 2.5 times brighter than RR-Lyrae itself.
- Similarly, a restriction of being dimmer than 7 as invalid was loosely chosen so as to eliminate stars far too dim — it is unlikely for a star to be more than 250 times dimmer than RR-Lyrae.

3.4. *Part 4: P-L Relationship Generation*

1. Microsoft Excel was used to derive a linear fit between $log(P)$ and M (absolute magnitude) was derived for these subtypes of stars, as well as the whole set of RR-Lyraes, and generate the plots.

3.5. *Part 5: Distance Calculations*

1. Use of the Magellanic Clouds' relations to derive the distance to itself is circular reasoning — the distances were used to derive the relation. Instead, the relations were used to derive the distances to the BLG and the other Magellanic Cloud. The BLG relation was used to derive distances to the Magellanic Clouds.
2. That is:
   - Use the BLG relation to use the log of Periods of stars within the LMC and SMC to work out the distances to them.

- Use the LMC relation to use the log of Periods of stars within the BLG and SMC to work out the distances to them.
- And vice versa for the SMC.

3. The distances were then compared to the "true" distances. The taken distances for the Magellanic Clouds have been outlined above, whereas the distance to the BLG was evaluated as the mean of distances within cleansed Bailer-Jones queries, at a distance of 6456pc.

4. RESULTS

4.1. *Cleansed Location Data*

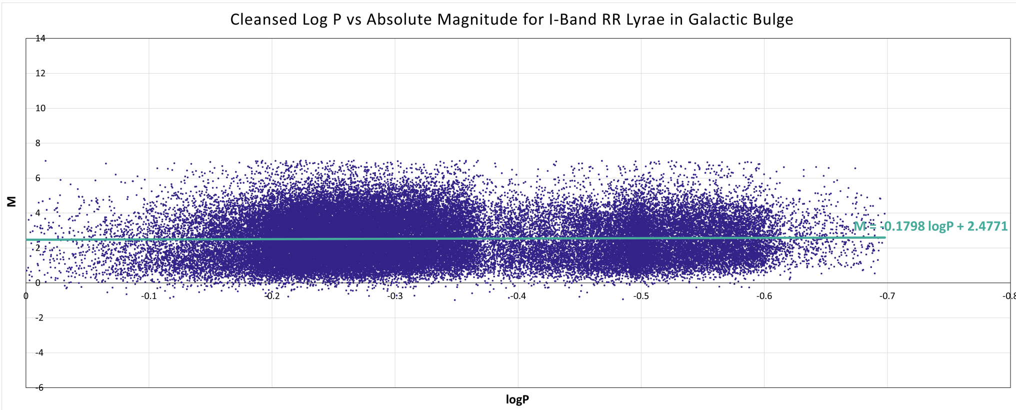

Figure 4. Cleansed P-L Relation for BLG: $M_{BLG} = -0.1798\, logP + 2.4771$

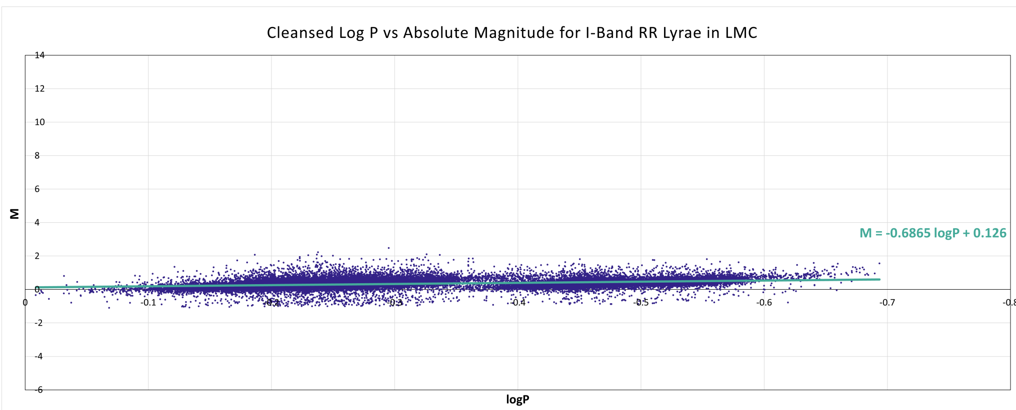

Figure 5. Cleansed P-L Relation for LMC: $M_{LMC} = -0.6865\, logP + 0.126$

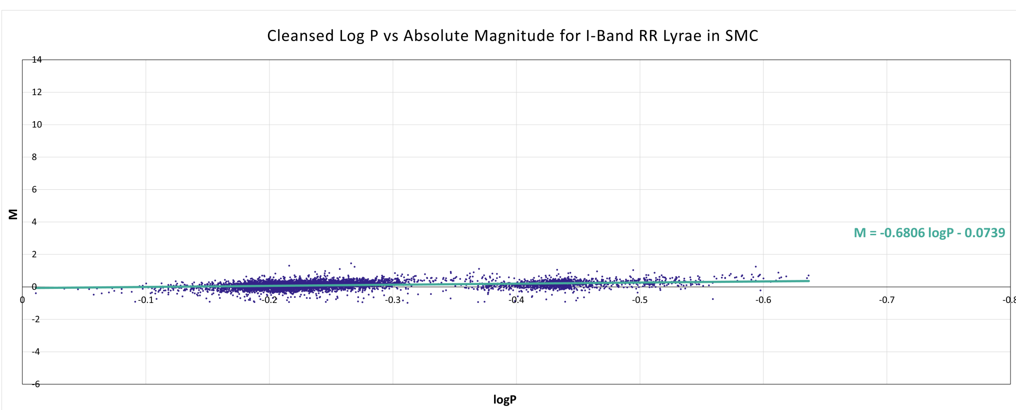

Figure 6. Cleansed P-L Relation for SMC: $M_{SMC} = -0.6806\, logP - 0.0739$

4.2. *Tabular Results and Processing*

For the graphical accompaniment, please refer to Appendix C.3.

	BLG	**LMC**	**SMC**	*Total Subtype*
RRab	$-0.3781\, logP + 2.75$	$-0.317\, logP + 0.1335$	$-0.3041\, logP - 0.0509$	$-3.6309\, logP + 0.6448$
RRc	$-0.2704\, logP + 2.7096$	$-0.0789\, logP + 0.5074$	$-0.0104\, logP + 0.2809$	$2.9173\, logP + 3.092$
RRd	$-1.3747\, logP + 2.0314$	$0.1769\, logP + 1.028$	$-0.175\, logP + 0.4166$	$5.7617\, logP + 3.0631$
Total Location	$-0.1798\, logP + 2.4771$	$-0.6865\, logP + 0.126$	$-0.6806\, logP - 0.0739$	

Table 1. Table of all derived cleansed P-L Relations for Locations and Type, respectively.

Locations	**log P**
BLG	-0.331834652
LMC	-0.311208737
SMC	-0.272466139

Table 2. Table of mean log Periods of RR-Lyrae in Locations.

	BLG	**LMC**	**SMC**	*Total Subtype*
RRab	2.8754921685	0.238712952	0.050031416	1.849903182
RRc	2.799346316	0.533587072	0.284351781	2.123742126
RRd	2.487665759	0.969286526	0.47468286	1.150779912
Total Location	2.53677599	0.353850763	0.151992541	

Table 3. Table of calculated mean absolute magnitude of RR-Lyrae in the BLG, using the corresponding relations.

	BLG	**LMC**	**SMC**	*Total Subtype*
RRab	2.8754921685	0.232041467	0.04363142	1.773488365
RRc	2.79365556	0.531926567	0.284132906	2.185138735
RRd	2.458734241	0.97300951	0.470999863	1.272038899
Total Location	2.532991974	0.339402893	0.13766884	

Table 4. Table of calculated mean absolute magnitude of RR-Lyrae in the LMC, using the corresponding relations.

	BLG	**LMC**	**SMC**	*Total Subtype*
RRab	2.853890455	0.220602021	0.032657491	1.642461605
RRc	2.783897749	0.529079336	0.283757606	2.290414112
RRd	2.409126021	0.979393225	0.464684712	1.479958941
Total Location	2.526503607	0.314629456	0.113108314	

Table 5. Table of calculated mean absolute magnitude of RR-Lyrae in the SMC, using the corresponding relations.

Locations	**m**
BLG	16.51534408
LMC	18.81687749
SMC	19.08900294

Table 6. Table of mean RR-Lyrae Apparent Magnitudes in Locations.

Using the definition of Distance Modulus: $d = 10 \times (10^{(\frac{m-M}{5})})$, and the prior Apparent and Absolute Magnitudes, the following distances (in pc) were calculated using the Absolute Magnitudes above:

BLG True mean distance: 6456pc

	BLG	**LMC**	**SMC**	*Total Subtype*
RRab	~~5345.341793~~	18002.40383	19636.61535	8573.084987
RRc	~~5536.090724~~	15716.378	17627.81973	7555.810592
RRd	~~6390.784855~~	12859.05715	16148.58983	11825.92606
Total Location	~~6247.640657~~	17072.92446	18736.11243	

Table 7. Table of calculated mean distances of BLG, using the corresponding relations.

LMC True mean distance: 49590pc

	BLG	**LMC**	**SMC**	*Total Subtype*
RRab	15482.52869	~~52112.8547~~	56836.55975	25610.7288
RRc	16018.6285	~~45392.55456~~	50880.12668	21210.61924
RRd	18686.66362	~~37049.81666~~	46683.47671	32312.73912
Total Location	18061.94152	~~49595.98711~~	54424.4144	

Table 8. Table of calculated mean distances of LMC, using the corresponding relations.

SMC True mean distance: 62440pc

	BLG	**LMC**	**SMC**	*Total Subtype*
RRab	17668.37865	59405.44237	~~64775.26477~~	30972.84694
RRc	18245.07356	51525.36439	~~57683.7994~~	22823.04997
RRd	21707.44689	41863.97279	~~53081.6327~~	33048.68764
Total Location	20539.16128	56910.31462	~~62444.25615~~	

Table 9. Table of calculated mean distances of SMC, using the corresponding relations.

5. DISCUSSION

5.1. *Under- and Over-estimation of Distances*

Even a qualitative look at the final distances in Tables 7, 8, and 9, would reveal that the accuracy of the BLG relation is significantly worse than the ones of the Magellanic Clouds, consistently under-estimating the distances. However, it is the Magellanic Cloud relations that over-estimate when it comes to distance to the BLG.

The simplest explanation for this is a lack of calibration between the two datasets, evident in the location graphs of Figures 4, 5, and 6. A mean Absolute Magnitude of 2.54 in the BLG for the Bailer-Jones dataset contrasts means of 0.34 and 0.11 for the LMC and SMC datasets, respectively, possibly accounting for the massive distance disparity between the two. That is, a significantly dimmer absolute mean Absolute Magnitude, M, calibration for the BLG leads to a corresponding under-estimation of distances, as the derived distance moduli $(m - m)$ using these relations are much smaller. Yet there is also evidence of Magellanic Cloud over-estimation — namely how they estimate distances of 17 to 18 kpc for the BLG, values which are far too large to be accurate.

Another factor which contributes to this disparity between these relations is the difference in metallicity content between the two systems — one of the reasons why most of the other relations for RR-Lyrae in scientific literature are given as P-L-Z relations, rather than P-L.

A further factor which contributes to these inaccuracies in these relations is the lack of use of dust maps in deriving them. Extinction (mainly interstellar and foreground) and reddening was not accounted for in calculating the absolute magnitude of the star, as they were outside of the scope of this project, and so potentially lead to inaccurate relations. For example dust may dim stars in the Magellanic Clouds, which without dust maps to correct for them, leads to a false inference that the stars are instead further away. Naturally there would be a larger extinction of Magellanic Cloud stars than BLG, and so there will be a relative over-estimations of distances from these relations.

5.2. *3 Outlier Relations with Positive Gradients*

Looking at the P-L relations themselves, rather than the corresponding distance estimates, also reveals peculiar relations for RRd Stars in the LMC, overall RRc, and overall RRd stars. These 3 relations all have positive gradients. This is due to removing the stars brighter than -1, which has resulted in a skew towards dimmer magnitudes and a correspondingly larger, positive gradients. Possibly a superior alternative would have been to normalize the magnitudes, by shifting the entire plot vertically by a certain amount, rather than crudely clipping stars outside of certain accepted intervals of magnitudes. Furthermore, due to the much smaller quantity of RRd, and a large amount of them being removed by the magnitude restrictions, this has had a significant impact on the relations, skewing it significantly upwards (all the other relations have a gradient of -1). A combination of the various offsets and gradients in different quantities added together formed a plot which had a positive gradient in the final Total Subtype relations for RRc and RRd.

5.3. *Offset Values of Relations*

In addition to normalizing relations by shifting them up and down, another factor that comes into play affecting the vertical offset of the line is the given distance of the stars. For example, if different distances for the Magellanic Clouds were taken from the 49590pc and 62440pc respectively, the offset values of the relations would be different, and give correspondingly different absolute magnitudes and distances for the same stars. That is, choosing different values may influence the amount of under- and over-estimation for distances to these bodies. For example, by looking at the Total Location relations (bottom row of Table 1), the BLG has an offset of +2.4771 compared to the much smaller Magellanic Cloud offsets of +0.126 and −0.0739 for the LMC and SMC respectively. However, using another distance differing only by a few hundred parsec such as the value of 49970pc given by (Pietrzyński et al. 2013) gives a change of offset to +0.112, not to mention potential larger changes in distance which account for some disparity between offsets.

5.4. *Comparison with other Relations*

The closest point of reference for P-L Relations is the [W1] band from Madore et al. (2013): $M_{[W1]} = -2.44\, logP - 1.26$. Though the [W1] band is the closest of all *WISE* bands to the I-Band in question here, there still exists a significant difference, explaining some of the difference in relations. Although the potential causes for the difference in offset has been explained (likely due to lack of magnitude normalization and the distance taken to the Magellanic

Clouds for calibration), the differences between the gradients of the relations are more interesting. The *WISE* relation's gradient of -2.44 is significantly steeper negatively than any other relation here, bar the total RRab relation. While a difference in offset values may be more easily explained by normalization or calibration, an explanation for the difference in the gradients of these relations is more obscure. Given that the data set is so massive in my case, combined with a lack of using dust maps, the noise within my data would naturally be very large, which leads to less distinct (flatter) trends. Contrast this with the 4 stars with accurate trigonometric parallaxes measured by the Fine Guidance Sensor on Hubble that were used to derive the relation for *WISE*, which naturally would have a much better signal of meaningful trends and results.

5.5. *Future Scope*

Due to a combination of reasons this project was not able to address other stellar factors for RR-Lyrae and characteristics to look for P-L Relations for them. The most immediate extension to the work presented here is an analysis of variables exhibiting the Blazhko Effect — a phenomena of long-period modulation of periodicity and amplitude in RR-Lyrae stars specifically. Potentially this study could have also been carried out in a fashion similar to Madore et al. (2013), by narrowing down from a large-scale study of big data and tens of thousands of stars to a select few with a wealth of accurately studied astronomical parameters to derive precise relations for them. In addition to RR-Lyrae, there may be potential in attempting to derive such relations for other variables in these large data sets such as Population II Cepheids like Delta Scuti (Ziaali et al. 2019) or W Virginis stars and Long Period Variables (LPV) like Mira stars.

6. CONCLUSION

This study explored the use of RR-Lyrae Period-Luminosity Relations to determine distances to our Galactic Bulge and Magellanic Clouds. In addition to the relations of $M = -0.1798\, logP + 2.4771$, $M = -0.6865\, logP + 0.126$, and $M = -0.6806\, logP - 0.0739$ for the BLG, LMC, and SMC respectively, further relations that were derived are given in Table 1). Likewise, distances from the BLG, LMC, and SMC relations are given in Table 7, Table 8, and Table 9, respectively.

An analysis of these relations and their corresponding distances showed that the Magellanic Cloud relations had a relatively better accuracy than that for the BLG, and causes for disparity between these relations and Madore et al. (2013), as well as the those disparities between the distances and Pietrzyński et al. (2019) and Graczyk et al. (2020) were given.

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APPENDIX

A. EFFECTS OF SMALL PARALLAX ANGLE

One point of note, however, is that since parallax angle decreases with an increase in distance, it clearly is not usable when it gets close to the limit of measurement. In fact, Bailer-Jones (2015) showed that it is inappropriate even once the fractional parallax error grows to larger than 20%. This serves as a restriction on the maximum distance directly measurable with trigonometric parallaxes — throughout the course of this project, by querying the 60,000 stars from OGLE-IV into the Gaia EDR3 Catalogue, I arrived at a limit of approximately 10kpc, i.e. only just to the BLG, which is clearly ineffective for measuring extragalactic distances.

B. THE USE OF GAIA AND BAILER-JONES CATALOGUES

- The Gaia telescope's most recent EDR3 catalogue is, by far, the largest and most precise catalogue of stars within our galaxy ever constructed; containing 1,811,709,771 stars to a precision of better than $24\mu as$ for objects brighter than $m = 15$, and better than $200\mu as$ for objects brighter than $m = 20$ (Prusti et al. 2016), with precise and accurate data corroborated by and building upon Hipparcos, among other galactic surveys.
- Similarly, the Bailer-Jones catalogues are a highly popular catalogue which uses statistical inferences to overcome parallax accuracy and error issues from the Gaia catalogues and return a Bayesian-corrected distance. Bailer-Jones et al. (2021) is for EDR3.

C. RESULTS

C.1. *Raw Location Data*

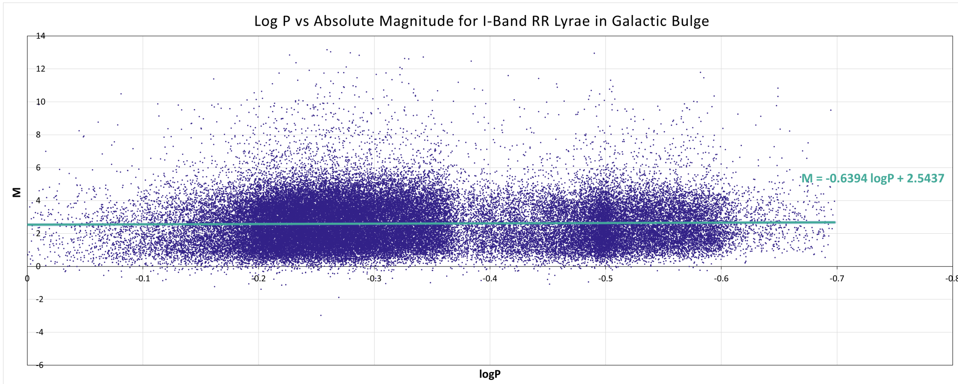

Figure 7. Raw P-L Relation for BLG: $M_{BLG} = -0.634\, logP + 2.5437$

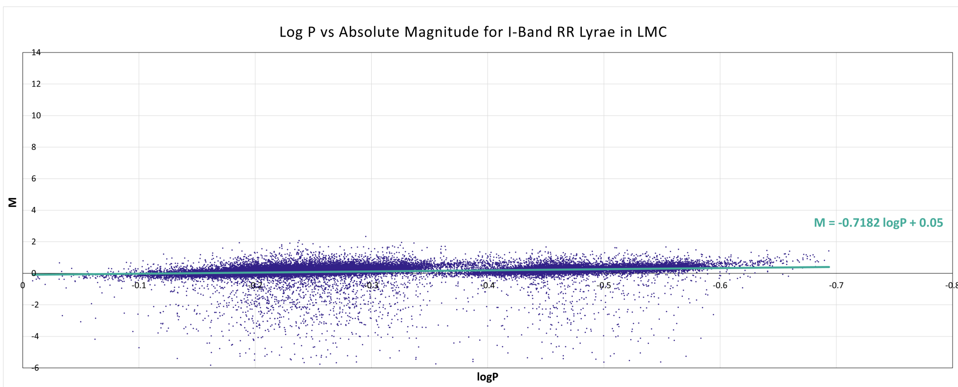

Figure 8. Raw P-L Relation for LMC: $M_{LMC} = -0.7182\, logP + 0.05$

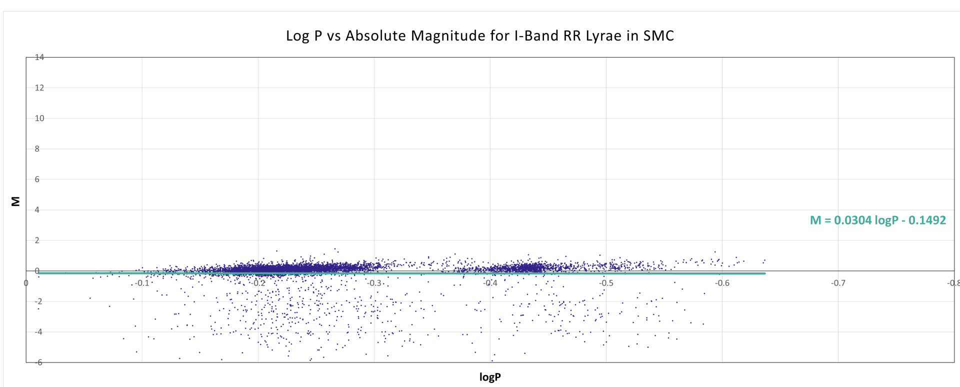

Figure 9. Raw P-L Relation for SMC: $M_{SMC} = 0.0304\, logP - 0.1492$

C.2. *Cleansed Subtype Data*

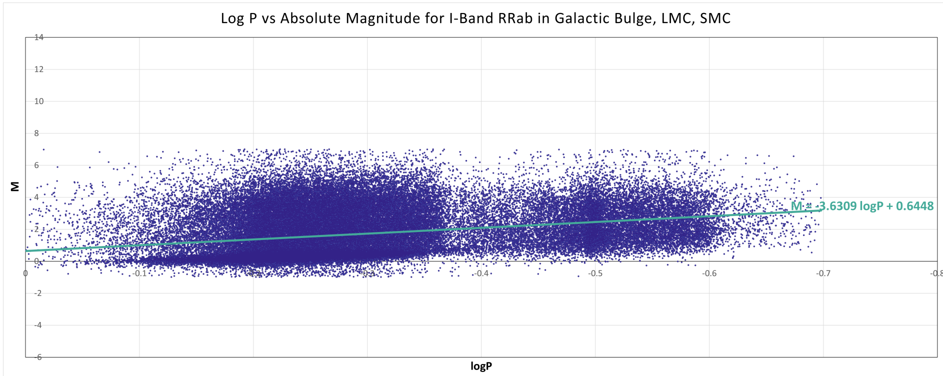

Figure 10. P-L Relation for all RRab: $M_{RRab} = -3.6309\, logP + 0.6448$

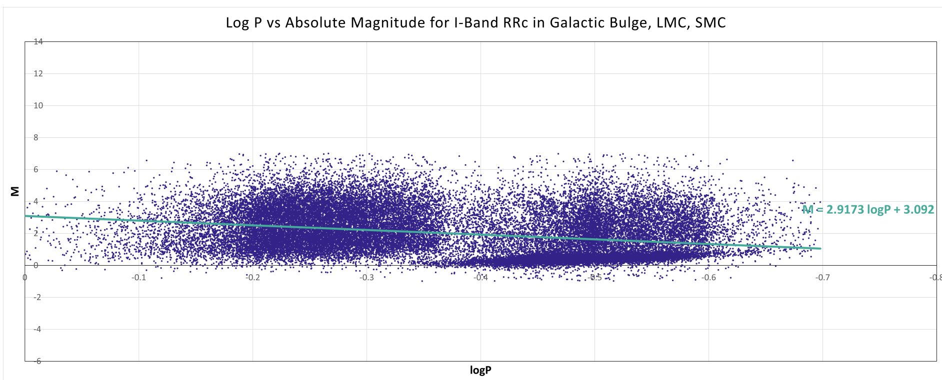

Figure 11. P-L Relation for all RRc: $M_{RRc} = 2.9173\, logP + 3.092$

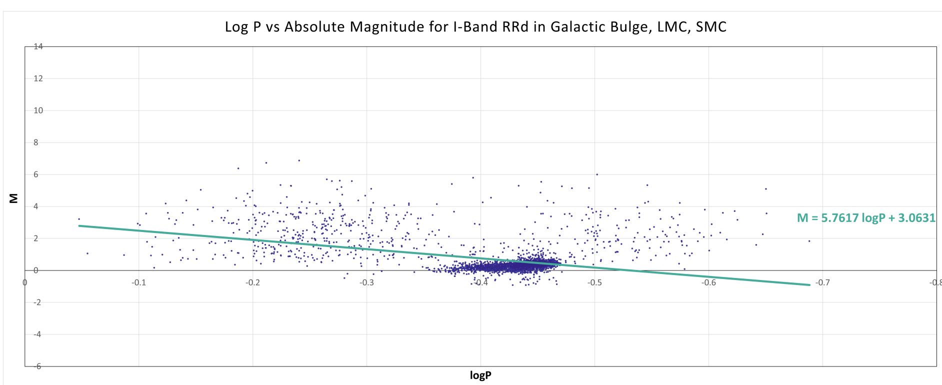

Figure 12. P-L Relation for all RRd: $M_{RRd} = 5.7617\, logP + 3.0631$

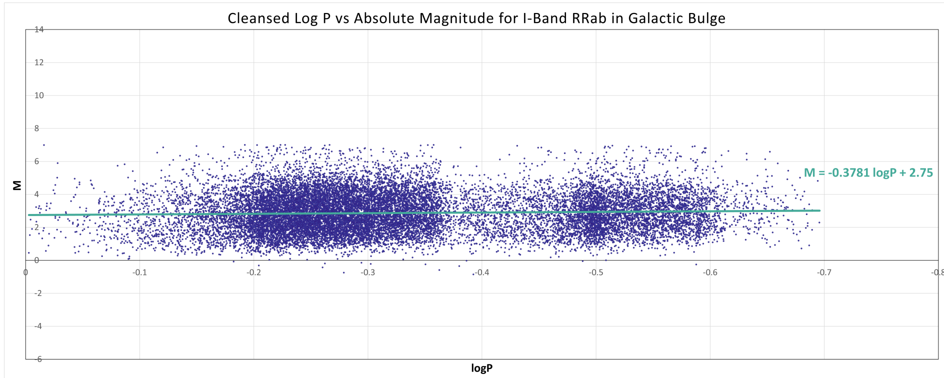

Figure 13. P-L Relation for RRab in BLG: $M_{BLG_{RRab}} = -0.3781\,logP + 2.75$

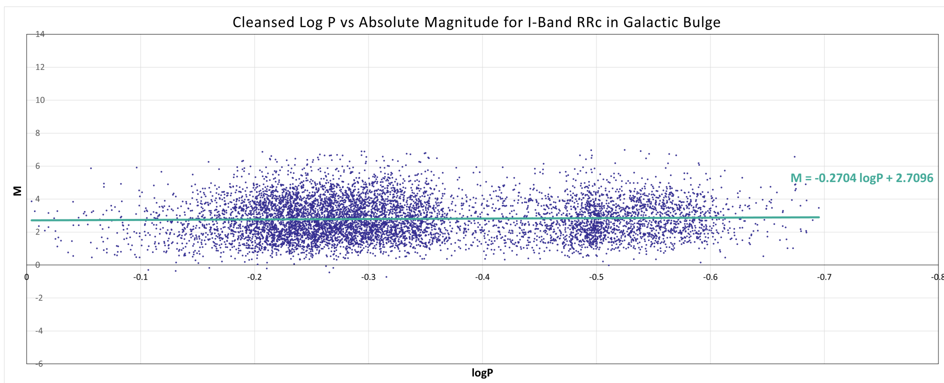

Figure 14. P-L Relation for RRc in BLG: $M_{BLG_{RRc}} = -0.2704\,logP + 2.7096$

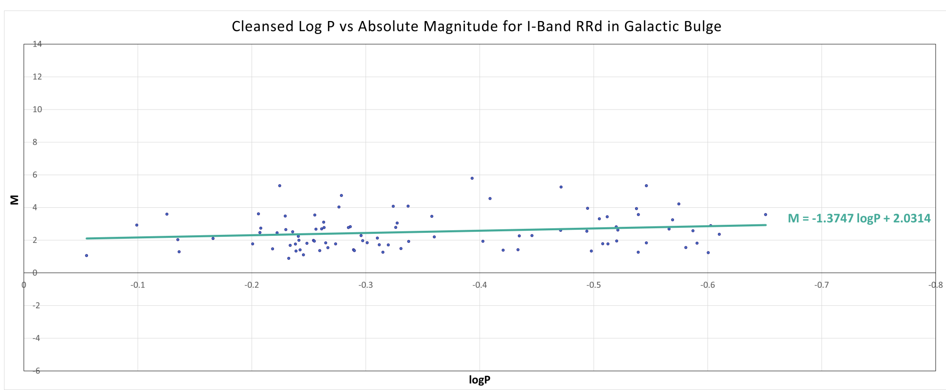

Figure 15. P-L Relation for RRd in BLG: $M_{BLG_{RRd}} = -1.3747\,logP + 2.0314$

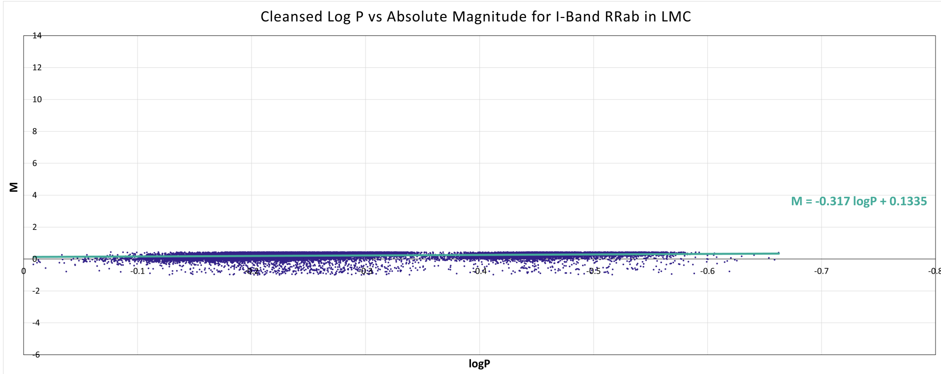

Figure 16. P-L Relation for RRab in LMC: $M_{LMC_{RRab}} = -0.317\, logP + 0.1335$

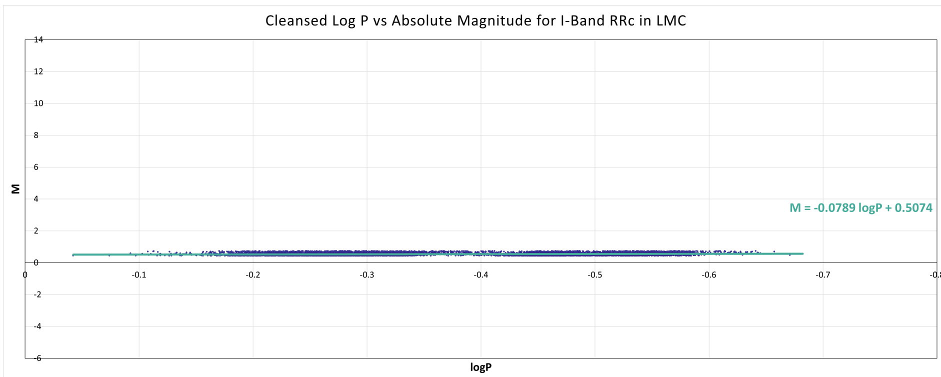

Figure 17. P-L Relation for RRc in LMC: $M_{LMC_{RRc}} = -0.0789\, logP + 0.5074$

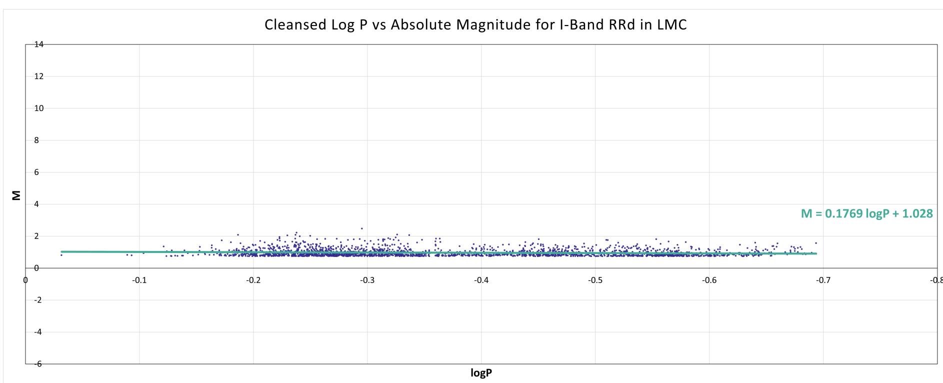

Figure 18. P-L Relation for RRd in LMC: $M_{LMC_{RRd}} = 0.1769\, logP + 1.028$

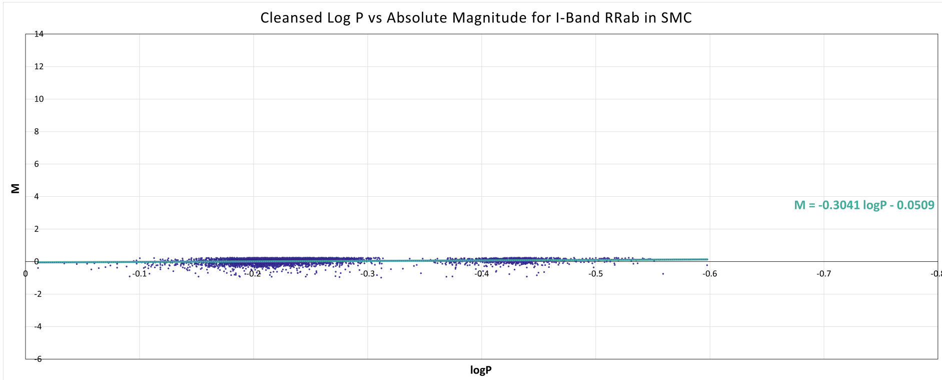

Figure 19. P-L Relation for RRab in SMC: $M_{SMC_{RRab}} = -0.3041\,logP - 0.0509$

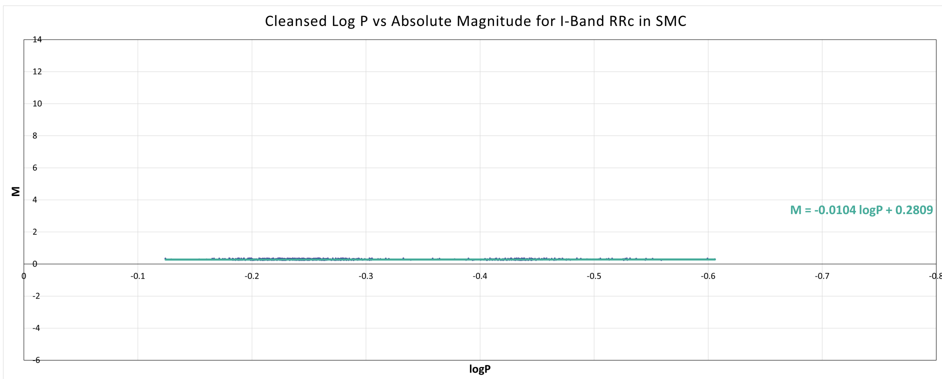

Figure 20. P-L Relation for RRc in SMC: $M_{SMC_{RRc}} = -0.0104\,logP + 0.2809$

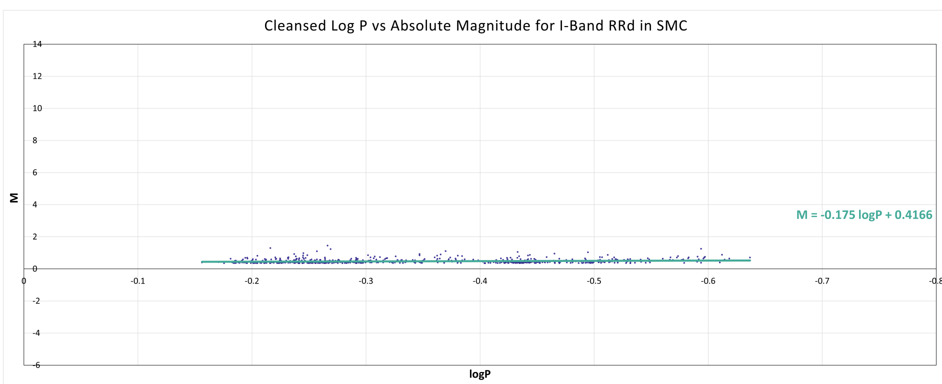

Figure 21. P-L Relation for RRd in SMC: $M_{SMC_{RRd}} = -0.175\,logP + 0.4166$