FLA (Fall 2023) – Assignment 4

Name: _____ Dept: _____

Grade: _____ ID: _____

Due: 3 Dec 2023

Problem 1

Consider the (deterministic) Turing machine M given by

$$M = (\{q_1, q_2, q_3, q_4, q_5, q_{acc}, q_{rej}\}, \{a, b\}, \{a, b, B\}, \delta, q_1, B, \{q_{acc}\})$$

with δ defined as below.

δ	B	a	b
q_1	q_{rej}, B, R	q_2, B, R	q_{rej}, b, R
q_2	q_{acc}, B, R	q_3, b, R	q_2, b, R
q_3	q_5, B, L	q_4, a, R	q_3, b, R
q_4	q_{rej}, B, R	q_3, b, R	q_4, b, R
q_5	q_2, B, R	q_5, a, L	q_5, b, L

Please answer the following questions:

a. Specify the execution trace of M on the input string aa.

b. Specify the execution trace of M on the input string $aaaaaa$.

c. Describe the function of this Turing machine in natural language.

Solution.

Problem 2

Design single-taped single-tracked deterministic turing machines as follows:

a. M_1 that decides the language $L = \{a^i b^j c^k \mid i < j < k\}$.

b. M_2 that takes string $w \in \{a, b\}^*$ as input and leaves ww^R on its tape when it halts, where w^R is the reversed string of w.

Solution.

Problem 3

We have learned that a Turing Machine accepts string ω if and only if its transition trace on ω passes through one or more final states. Now we define ”another Turing Machine” (aTM for short), which accepts string ω if and only if it stops exactly on a final state when it halts.

a. How is the ability (in terms of language expression) of aTMs changed compared to TMs? Prove your conclusion.

b. (**Optional**) Recall the aDFAs defined in assignment 1. Are DFAs equivalent to aDFAs? Why do the abilities of DFAs and TMs change differently when we try to change their definitions of final states? You may talk freely about your understanding if you would like to.

Solution.

Problem 4

Prove that a language L is recursively enumerable **if and only if** there exists a recursive language R such that

$$L = \{x \mid \exists y,\ \langle x, y\rangle \in R\}.$$

(**Hint:** A Turing Machine accepts string w means it will pass through a final state within a finite k steps.)

Proof.