FLA (Fall 2023) – Assignment 3

Name: _____ Dept: _____

Grade: _____ ID: _____

Due: Nov. 12, 2023

Problem 1

Given grammer G:

$$S \to AC \mid BC$$

$$A \to BA \mid a$$

$$B \to CA \mid b$$

$$C \to BB \mid b$$

Please use CYK algorithm to decide whether string $bbabbab$ belongs to $L(G)$.

Problem 2

Prove that each of these languages is not context free.

a. $L = \{\ 0^p \mid p \text{ is a prime}\ \}$.

b. $L = \left\{\ 0^i 1^j \mid i^2 \leq j\ \right\}$.

c. $L = \left\{\ 0^i 1^j 2^k \mid i = \max(j, k)\ \right\}$, where $\max(a, b)$ calculates the maximum of a, b.

d. $L = \{\ t_1 \# t_2 \# \cdots \# t_k \mid (k \geq 2) \wedge (\forall i \leq k.\ t_i \in \{0,\ 1\}^*) \wedge (\exists i, j.\ i \neq j \wedge t_i = t_j)\ \}\ \ (\# \in \Sigma)$.

Problem 3

For any context-free language L and any regular language R, answer each of the following statements **True** or **False**. If your answer is **True**, give an explanation. If your answer is **False**, give a counterexample.

a. $L - R$ is context-free.

b. $R - L$ is context-free.

c. $S(L) = \{\ w \mid w$ is a subsequence of some string in $L\ \}$. $S(L)$ is context-free.
Example: If $L = \{\ abcd\ \}$, then $B(L) = \{\ \varepsilon, a, b, c, d, ab, bc, cd, abc, bcd, abcd, ac, ad, bd, abd, acd\ \}$.

d. $M(L) = \{\ w \mid\ (w \in L) \wedge (\forall v \in \Sigma^{+}.\ vw \notin L)\ \}$. $M(L)$ is context-free.

Problem 4

We define an operation $\bowtie$ for language L and R to be

$$L \bowtie R = \{\, w \mid w = x_1 y_1 x_2 y_2 \cdots x_n y_n \text{ for some } n, \text{ where } x_1 x_2 \cdots x_n \in L \text{ and } y_1 y_2 \cdots y_n \in R, \text{ each } x_i, y_i \in \Sigma \,\},$$

where Σ is the alphabet.

a. Show that if L is context-free and R is regular, then $L \bowtie R$ is context-free.

b. Show that the class of CFL is not closed under $\bowtie$ operation.