FLA (Fall 2023) – Assignment 1

Name: ——————— Dept: ———————

Grade: ——————— ID: ———————

Due: Oct. 12, 2023

Problem 1

Provide DFAs and REs of the following languages. In all parts, the alphabet $\Sigma = \{a, b\}$ and $|v|_\omega$ denotes the number of occurrences of substring v in string ω.

 a. $\{\omega \mid \omega$ begins with "a" and ends with "b"$\}$

 b. $\{\omega \mid |ba|_\omega = 0\}$

 c. $\{a^m b^n \mid \exists k \in \mathbb{N}, m + n = 2k + 1\}$

 d. $\{\omega \mid$ Each "a" in ω is immediately preceded and followed by "b"$\}$

Solution.

Problem 2

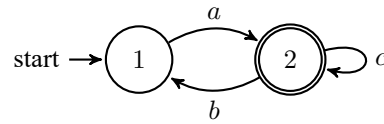Let $R = (\mathbf{a} + \mathbf{b})^* \mathbf{b} (\mathbf{b} + \mathbf{c})^*$.

    a. Convert R to an ϵ-NFA.

    b. Convert the ϵ-NFA to a DFA by subset construction.

Solution.

Problem 3

Give a DFA as figure below, please give the regular expression for the following R_{ij}^k, and try to simplify the expressions as much as possible.

   a. All the REs R_{ij}^0

   b. All the REs R_{ij}^1

   c. The RE for this DFA

Solution.

Problem 4

Prove that the following languages are not regular. You may use the pumping lemma and the closure properties of the class of regular languages.

    a. $\{\omega\omega \mid \omega \in \{0,\ 1\}^* \}$

    b. $\{0^n 1^m 2^n \mid\ m, n \geq 0 \}$

    c. $\{\omega \mid |0|_\omega + |1|_\omega = n^2,\ n \in N \}$

    d. $\{0^m 1^n \mid \gcd(m, n) = 1\}$ (**Hint:** Consider using factorial during string construction)

Proof.

Problem 5

We have learned that a DFA accepts string ω if and only if it halts exactly on a final state when ω is consumed completely. Now we define "another DFA" (aDFA for short), which behaves exactly like a DFA, except that it accepts string ω if and only if its transition trace on ω passes through one or more final state.

How is the ability (in term of language expressing) of aDFAs changed compared to DFAs? Prove your conclusion.

Proof.

Problem 6

For a language L, let $F(L)$ be the language $\{x|\ \forall w \in L.\ w$ is not a substring of $x\}$. Is the class of regular languages closed under F (If L is regular, so is $F(L)$)? If so, give a proof. If not, give a counterexample.

Solution.

Problem 7

Prove or disprove the following statement (All languages mentioned below are over alphabet Σ.):

a. Every non-regular language has a non-regular proper subset.

b. A, B are two languages. If both A and $A \cap B$ is regular, then B is also regular.

c. For any set of regular languages S, $\bigcup S$ (the union of all the elements in S) is a regular language.

d. For two languages A and B, we write $A \sqsubset B$ if $A \subset B$ and B contains infinitely many strings not in A. If A and B are two regular languages with $A \sqsubset B$, we can find a regular language C such that $A \sqsubset C \sqsubset B$. (**Hint: use the pumping lemma.**)

Solution.

Problem 8

a. We define an operation odd on strings as $odd(c_1 c_2 c_3 c_4 c_5 c_6 \cdots) = c_1 c_3 c_5 \cdots$, then the above described definition is extended to languages. Prove that the class of regular languages is closed under this operation.

b. We define an operation $half$ on strings as $half(c_1 c_2 c_3 c_4 c_5 c_6 \cdots c_n) = c_1 c_2 c_3 \cdots c_{\lfloor n/2 \rfloor}$, then the above described definition is extended to languages. Prove that the class of regular languages is closed under this operation.

Solution.