FLA (Fall 2023) – Assignment 5

Name: _____ Dept: _____

Grade: _____ ID: _____

Due: 15 Dec 2022

Problem 1

Answer the following statements **True** or **False**. If your answer is **True**, give an explanation. If your answer is **False**, give a counterexample. Please avoid using Rice's Theorem.

a. $\mathcal{P}$ is closed under **Concatenation**. So is $\mathcal{NP}$.

b. $\mathcal{P}$ is closed under **Complement**.

c. $L = \{ \langle D_1, D_2 \rangle \mid D_1, D_2$ are two DFAs and $L(D_1) = L(D_2) \}$ is decidable.

d. $L = \{ \langle M_1, M_2 \rangle \mid M_1, M_2$ are two turing machines and $L(M_1) = L(M_2) \}$ is decidable.
(**Hint**: $\mathrm{E_{TM}}$ is undecidable.)

Solution.

Problem 2

We label the regular languages as A, the context-free languages as B, the recursive languages as C, the recursively enumerable languages as D, and the all possible languages as E. Please answer the most appropriate label of following languages. For example, the language of balanced parentheses can be labeled as E, but the correct answer is B. (We denote the complement of language L as $\overline{L}$.)

a. The complement of an undecidable language can be labeled as:

b. The complement of a language in $\mathcal{NP}$ can be labeled as:

c. The difference between a language in $\mathcal{P}$ and a context-free language can be labeled as:

d. If L_1 is recursive and L_2 is recursively enumerable, then $\overline{L_1} \cup L_2$ can be labeled as:

e. If L_1 is recursively enumerable and L_2 is recursive, then $\overline{L_1} \cup L_2$ can be labeled as:

f. $\{ a^i b^j c^k d^l \mid i = k \text{ and } j = l \}$ can be labeled as:

g. $\{ a^i b^j c^k d^l \mid i = l \text{ and } j = k \}$ can be labeled as.

Solution.

Problem 3

We have learned that 3-SAT is $\mathcal{NP}$-complete in class. Please solve the following problems.

a. The following statement tries to show $\mathcal{P} \neq \mathcal{NP}$:

In order to see whether a 3-SAT formula with n variables is satisfiable, we need to look at 2^n possible truth assignments. This takes exponential time, so 3-SAT is not in $\mathcal{P}$. But it is in $\mathcal{NP}$, so $\mathcal{P} \neq \mathcal{NP}$.

Please point out the fallacy in it.

b. Define

$$\text{01-ROOT} = \{\ \langle p \rangle \mid p \text{ is a polynomial in } n \text{ variables with integer coefficients such that } p(x_1, x_2, \cdots, x_n) = 0 \text{ for some } (x_1, x_2, \cdots, x_n) \in \{0,1\}^n\ \}.$$

Show that 01-ROOT is $\mathcal{NP}$-complete.

Solution.

Problem 4

Define

$$L = \{\, w \mid (\exists x.\ x \in \mathrm{E_{TM}} \wedge w = 0x) \vee (\exists y.\ y \notin \mathrm{E_{TM}} \wedge w = 1y) \,\},$$

where

$$\mathrm{E_{TM}} = \{\, \langle M \rangle \mid M \text{ is a turing machine and } L(M) = \emptyset \,\}$$

is an undecidable language.

Prove that L is neither RE nor co-RE.

Proof.