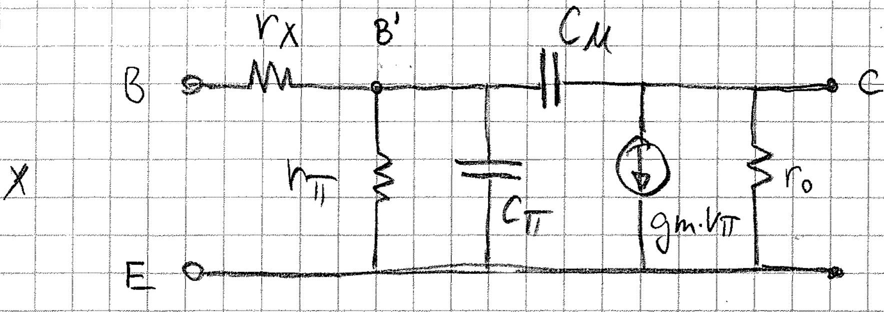
# Højfrekvens Hybrid $\pi$-model. (485-490)




for $f < \frac{f_T}{3}$

$C_\pi = C_{de} + C_{je}$ diffusions cap. + junction cap.

$C_{de} = \frac{W^2}{2D_N} \cdot g_m = \tau_F \cdot g_m = \tau_F \cdot \frac{I_C}{V_T}$ stiger med $I_C$

$\tau_F$ = forward base transit time =

den middeltid det tar for en elektron at pasere basislaget (10-100 psec)

$C_{je} = \frac{C_{jeo}}{\left(1 - \frac{V_{BE}}{V_{0e}}\right)^m}$ $C_{jeo}$ er ved spændingen = 0

Basis-emitter junction capatitet.

$V_{0e} = 0{,}9$ $m = 0{,}2 - 0{,}5$

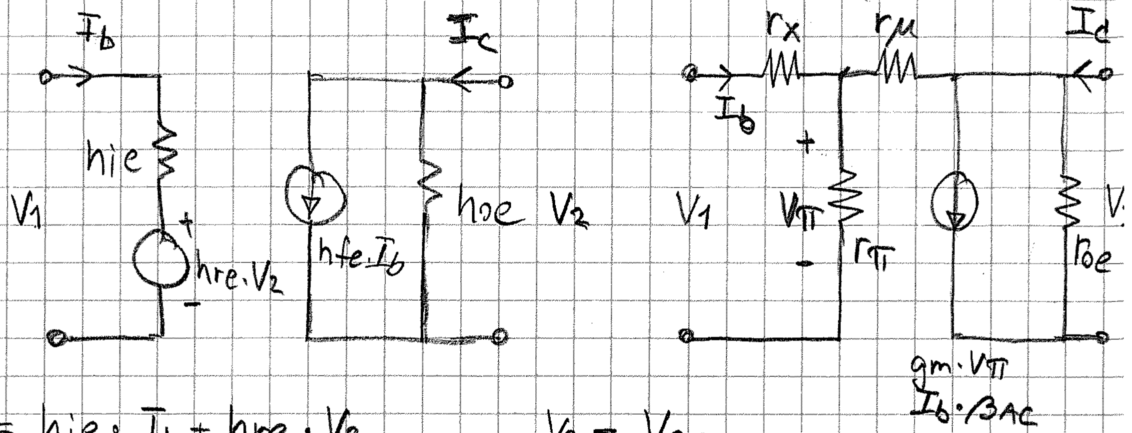
stiger med $V_{BE}$ typisk: $C_{je} \approx 2 \cdot C_{jeo}$

$C_\mu = \frac{C_{\mu o}}{\left(1 + \frac{V_{CB}}{V_{0C}}\right)^m}$ $V_{0C} = 0{,}75$ depletion cap.

$C_\mu$ falder med stigende $V_{CB}$

| | | | |
|---|---|---|---|
| $C_\pi$ | X pF → X0 pF | | 2-20 pF |
| $C_\mu$ | 0,X pF → X pF | | |
| rx | X0 Ω | | 10-100 Ω |

# Sammenhæng mellem h-parametre og hybid π




$V_1 = h_{ie} \cdot I_b + h_{re} \cdot V_2$ $\qquad V_2 = V_{ce}$

$I_C = h_{fe} \cdot I_b + h_{oe} \cdot V_2$ $\qquad V_1 = V_{be}$

$$\boxed{\begin{array}{lll} r_\pi = \dfrac{h_{fe}}{g_m} & r_x = h_{ie} - r_\pi & \beta_{AC} = h_{fe} \\ r_\mu = \dfrac{r_\pi}{h_{re}} & r_o = \dfrac{1}{h_{oe}} & \\ C_\mu \simeq C_{BO} & C_\pi + C_\mu = \dfrac{g_m}{\omega_T} = \dfrac{I_C}{V_T \cdot 2\pi \cdot f_T} & \end{array}}$$

I datablad findes følgende : I arbejdspunktet $I_C, V_{CE}$

$h_{fe} = \dfrac{I_C}{I_b}$ $\qquad h_{ie} \qquad C_{BO} \qquad f_T$

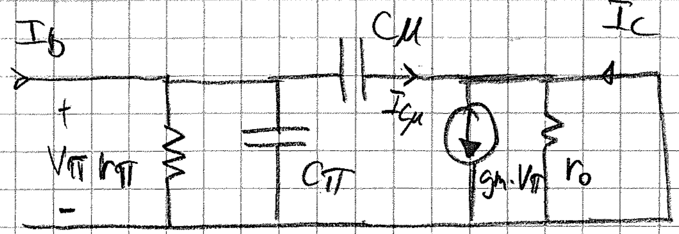
$r_x$ er overgangsmodstanden fra Basis terminalen til den aktive basis.
i størrelsesordenen $10 - 100\,\Omega$

Kan findes ud fra : $r_x = h_{ie} - \dfrac{h_{fe}}{g_m} = h_{ie} - r_\pi$

Kan være svær at beregne da hie og hfe ikke kendes nøjagtigt, aflæsninger fra kurver ofte for unøjagtive og kan medføre at $r_x$ får forkert værdi – endda negativ – hvilket ikke kan lade sig gøre

# Cutoff frekvens $f_T$

$$h_{fe}(j\omega) = A_i = \left.\frac{I_c}{I_b}\right|_{V_{ce}=0}$$




I $\quad V_\pi = I_b \cdot r_\pi \| C_\pi \| C_\mu \Rightarrow I_b = \dfrac{V_\pi}{r_\pi \| (C_\pi + C_\mu)}$

II $\quad I_c + I_{c\mu} = g_m \cdot V_\pi \Rightarrow I_c = g_m \cdot V_\pi - V_\pi \cdot sC_\mu$

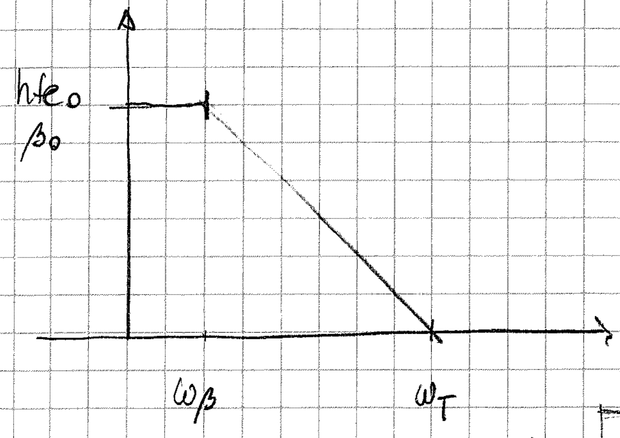
I II $\quad h_{fe} = \dfrac{I_c}{I_b} = \dfrac{V_\pi (g_m - sC_\mu)}{V_\pi / (r_\pi \| (C_\mu + C_\pi))} = \dfrac{(g_m - sC_\mu) r_\pi}{1 + s r_\pi (C_\mu + C_\pi)}$

$$R \| C = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{R}{1 + sCR}$$

$g_m \gg sC_\mu$ (hvor modellen gælder)

$$h_{fe} \approx \frac{g_m \cdot r_\pi}{1 + s r_\pi (C_\mu + C_\pi)} = \frac{h_{fe0}}{1 + s r_\pi (C_\mu + C_\pi)} \qquad (5.159)$$

$$= \frac{h_{fe0}}{1 + \frac{s}{\omega_\beta}} \qquad \omega_\beta = \frac{1}{r_\pi (C_\mu + C_\pi)} \qquad \text{III}$$




for $|h_{fe}(\omega_T)| = 1 \Rightarrow$

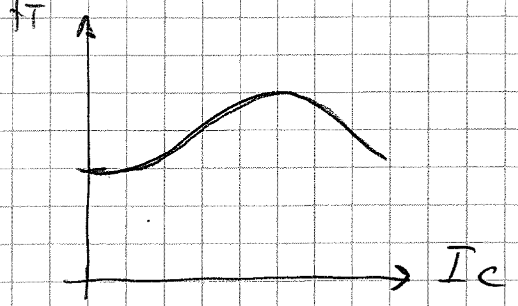
$$h_{fe}(\omega_T) = 1 = \frac{h_{fe0}}{\sqrt{1^2 + \left(\frac{\omega_T}{\omega_\beta}\right)^2}} \approx \frac{h_{fe0}}{\frac{\omega_T}{\omega_\beta}}$$

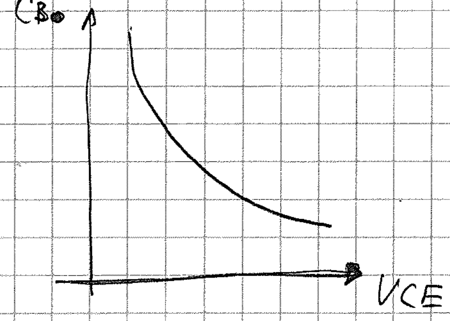
$$\underline{\omega_T = h_{fe0} \cdot \omega_\beta}$$

$$\omega_T = \frac{h_{fe}}{r_\pi (C_\mu + C_\pi)} = \underline{\frac{g_m}{C_\mu + C_\pi}}$$

$\omega_T$ = Unity-gain bandwidth $\quad \boxed{C_\pi = \frac{g_m}{\omega_T} - C_\mu}$

<u>datablad</u> $\quad C_{BO}\big|_{V_{CE}} = C_\mu \qquad f_T\big|_{I_C}$




$\sim 3pF$

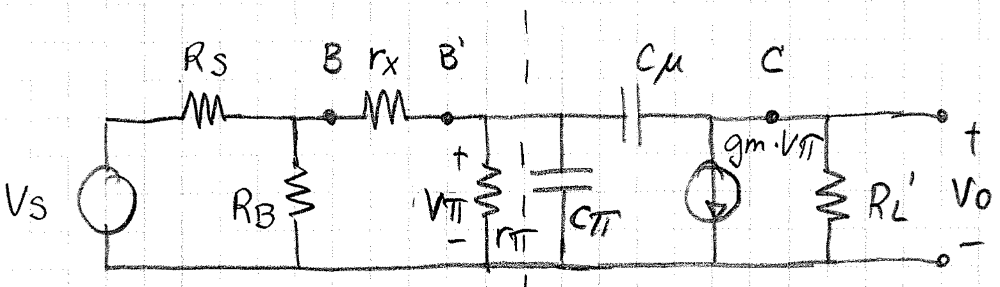
$$\boxed{C_\pi = \frac{g_m}{2\pi \cdot f_T} - C_\mu}$$

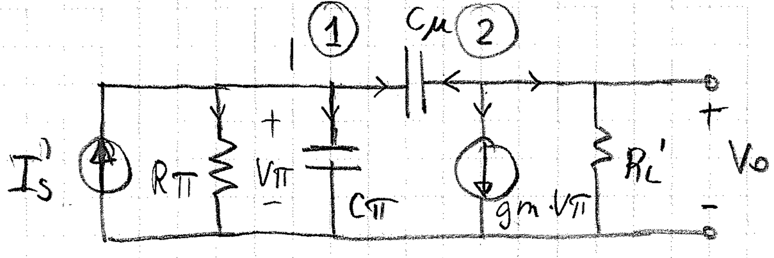
dvs ud fra: $C_{BO} \simeq C_\mu$ og $f_T$ og $I_C$

beregnes $C_\pi$

se evt. Table 5.7 S&S side 490

# Højfrekvens-response BJT (CE) (FET (CS))




$$R_\pi = r_\pi \,\|\, (r_x + R_S \| R_B) \qquad I_S' = \frac{V_S \cdot R_B}{(R_S + R_B)(r_x + R_B \| R_S)}$$

(1) $$I_S' = V_\pi\left(\frac{1}{R_\pi} + sC_\pi\right) + (V_\pi - V_o)sC_\mu \Rightarrow V_\pi = \frac{I_S' + V_o \cdot sC_\mu}{\frac{1}{R_\pi} + s(C_\mu + C_\pi)}$$

(2) $$g_m \cdot V_\pi + \frac{V_o}{R_L'} + (V_o - V_\pi)sC_\mu = 0 \Rightarrow V_o = \frac{V_\pi(sC_\mu - g_m)}{\frac{1}{R_L'} + sC_\mu}$$

(1) → (2)

$$V_o = \frac{(I_S' + V_o sC_\mu)(sC_\mu - g_m)}{\left(\frac{1}{R_\pi} + s(C_\mu + C_\pi)\right)\left(\frac{1}{R_L'} + sC_\mu\right)}$$

$$V_o = \frac{(I_S' + V_o \cdot A)B}{N} \Rightarrow V_o \cdot N - V_o \cdot A \cdot B = I_S' \cdot B \Rightarrow V_o = \frac{I' \cdot B}{N - A \cdot B}$$

$$V_o = \frac{I'(sC_\mu - g_m)}{\left(\frac{1}{R_\pi} + s(C_\mu + C_\pi)\right)\left(\frac{1}{R_L'} + sC_\mu\right) - sC_\mu(sC_\mu - g_m)}$$

$$\frac{V_o}{V_S} = \frac{-g_m \cdot R_B \cdot r_\pi \cdot R_L'}{(R_S + R_B)(r_\pi + r_x + R_S \| R_B)} \cdot \frac{\left(1 - \frac{sC_\mu}{g_m}\right)}{\underbrace{s^2 C_\mu C_\pi R_\pi \cdot R_L'}_{b_2} + s\underbrace{R_\pi\left(C_\pi + C_\mu\left(1 + g_m \cdot R_L' + \frac{R_L'}{R_\pi}\right)\right)}_{b_1} + 1}$$

$$= A_M \cdot \frac{1 - \frac{sC_\mu}{g_m}}{b_2 s^2 + b_1 s + 1}$$

$$b_2 \cdot s^2 + b_1 \cdot s + 1 = b_2\left(s^2 + \frac{b_1}{b_2} s + \frac{1}{b_2}\right)$$

$$b_2\left(s^2 + \frac{b_1}{b_2} s + \frac{1}{b_2}\right) = b_2(s+p_1)(s+p_2)$$

$$\text{— '' —} \qquad = b_2\left(s^2 + s(p_1+p_2) + p_1 \cdot p_2\right)$$

$$\frac{b_1}{b_2} = (p_1+p_2) \qquad \text{for } p_2 \gg p_1 \Rightarrow \underline{p_2 \simeq \frac{b_1}{b_2}}$$

$$\frac{1}{b_2} = p_1 \cdot p_2) \qquad \text{— '' —} \qquad \underline{p_1 = \frac{1}{b_2 \cdot p_2} = \frac{1}{b_1}}$$

$$\left(b_2 \cdot s^2 + b_1 \cdot s + 1\right) = b_2 \cdot p_1 \cdot p_2 \left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)$$

$$= b_2 \cdot \frac{1}{b_1} \cdot \frac{b_1}{b_2}\left( \text{———} \right)$$

$$= \left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)$$

$$p_1 = \frac{1}{b_1} \quad \wedge \quad p_2 = \frac{b_1}{b_2}$$

$$p_1 = \frac{1}{R_\pi\left(C_\pi + C_\mu\left(1 + g_m \cdot R_L' + \frac{R_L'}{R_\pi}\right)\right)} =$$

$$p_2 = \frac{R_\pi\left(C_\pi + C_\mu\left(1 + g_m \cdot R_L' + \frac{R_L'}{R_\pi}\right)\right)}{C_\pi \cdot C_\mu \cdot R_\pi \cdot R_L'} \Downarrow = \frac{R_\pi \cdot C_\pi + R_\pi \cdot C_\mu\left(1 + g_m \cdot R_L' + \frac{R_L'}{R_\pi}\right)}{C_\pi \cdot C_\mu \cdot R_\pi \cdot R_L'}$$

$$p_2 = \frac{1}{C_\mu \cdot R_L'} + \frac{1}{C_\pi\left(R_L \parallel \frac{1}{g_m} \parallel R_\pi\right)}$$

$$\frac{V_o}{V_s} = A_M \cdot \frac{1 - \frac{s}{g_m / C_\mu}}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)} \qquad \omega_z = \frac{g_m}{C_\mu}$$

$\omega_T = \dfrac{g_m}{C_\mu + C_\pi}$ dvs $\omega_z > \omega_T$ her gælder modellen ikke!

# Vurdering af $\omega_Z$, $p_1$, $p_2$

$$\omega_Z = \frac{g_m}{C_\mu} \qquad \omega_T = \frac{g_m}{C_\mu + C_\pi} \Rightarrow \omega_Z > \omega_T$$

$\omega_Z$ uden betydning (Hybrid $\pi$-modellen gælder ikke)

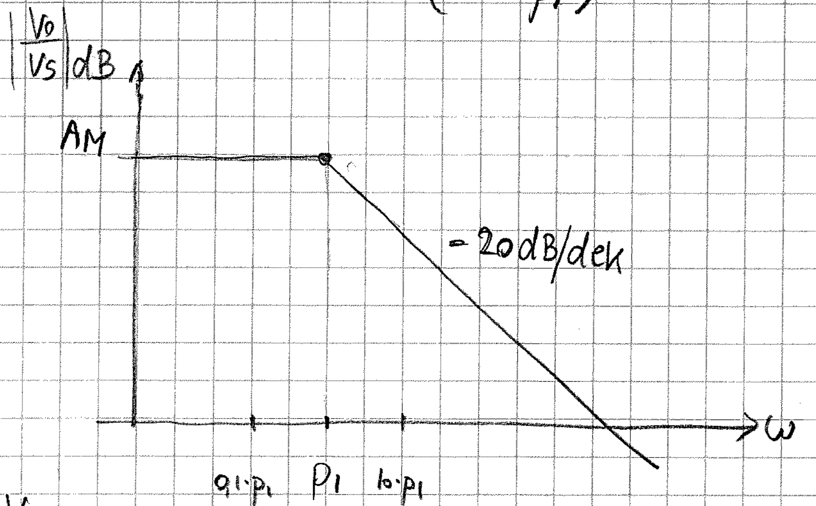
$$p_1 = \omega_{p1} = \frac{1}{R_\pi\left(C_\pi + C_\mu\left(1 + g_m \cdot R_L' + \frac{R_L'}{R_\pi}\right)\right)}$$

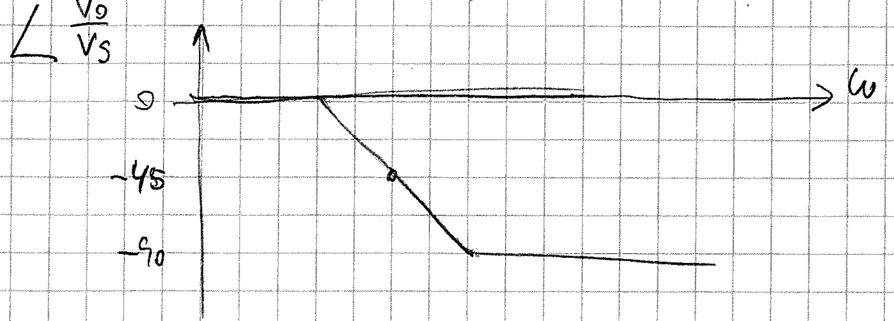
$$p_2 = \frac{1}{C_\mu \cdot R_L'} + \frac{1}{C_\pi\left(R_L' \parallel \frac{1}{g_m} \parallel R_\pi\right)}$$

$p_2 \approx \frac{g_m}{C_\pi} > \omega_T$ uden betydning.

Tilbage er kun $p_1$

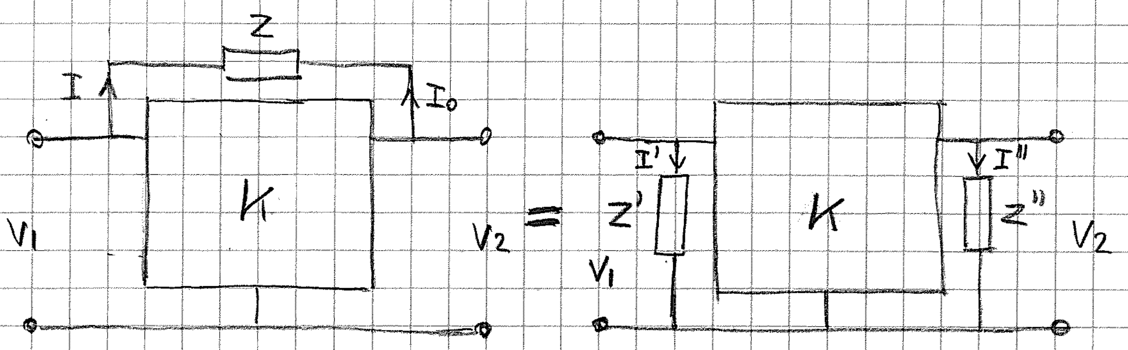
$$\frac{V_o}{V_g} = A_M \cdot \frac{1}{\left(1 + \frac{s}{p_1}\right)} = A_M \cdot \frac{1}{\left(1 + \frac{j\omega}{p_1}\right)}$$



# Miller Transformation

s&s side 578




$K$ = spændingsforstærkning $\quad K = \frac{V_2}{V_1}$

$$I = (V_1 - V_2)\frac{1}{Z} \qquad I' = \frac{V_1}{Z'}$$

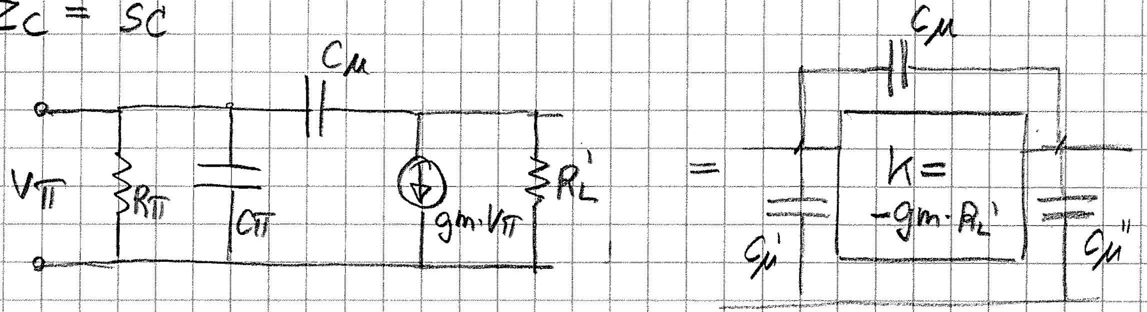
$$I = I' \Rightarrow \frac{V_1 - V_1\cdot K}{Z} = \frac{V_1}{Z'} \Rightarrow Z' = Z\cdot\frac{V_1}{V_1 - V_1\cdot K} \qquad \boxed{Z' = \frac{Z}{1-K}}$$

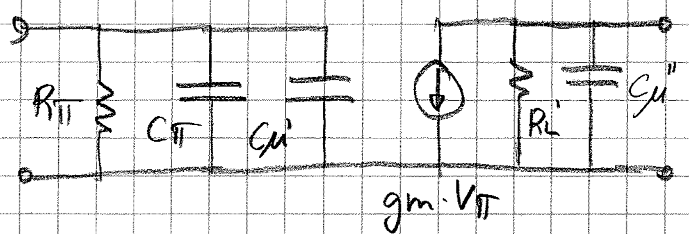
$$I_0 = \frac{V_2 - V_1}{Z} = \frac{V_1(K-1)}{Z} = I' = \frac{V_2}{Z''} = \frac{V_1\cdot K}{Z''}$$

$$Z'' = \frac{V_1\cdot K}{V_1(K-1)}\cdot Z \Rightarrow \qquad \boxed{Z'' = \frac{Z}{1-1/K}}$$

Miller transformation af kondensator der sidder mellem ind- og udgangen på et forstærkertrin med $G_v = K$

$$Z_C = \frac{1}{sC}$$




$$C_\mu' = C_\mu(1-K)$$
$$= C_\mu(1 + g_m\cdot R_L')$$

$$C_\mu'' = C_\mu\left(1 + \frac{1}{g_m\cdot R_L'}\right) \simeq C_\mu$$

$$\frac{1}{sC'} = \frac{\frac{1}{sC}}{1-K} \Rightarrow C' = C(1-K)$$

# Tomgangstidskonstantmetoden ved højfrekvens

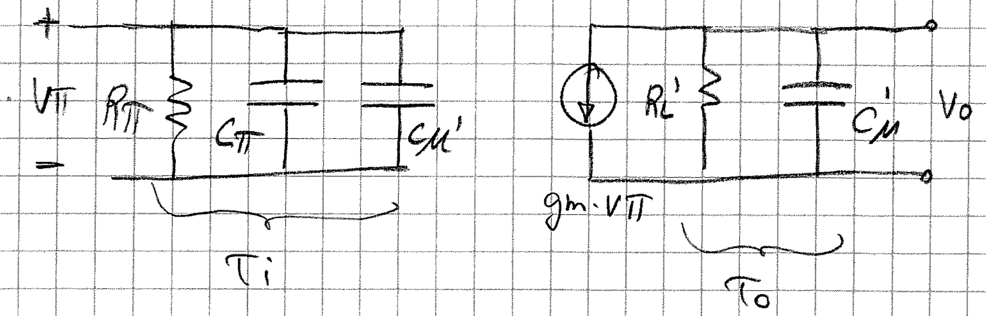
$$\omega_H = \frac{1}{\sum \tau_{1_T} + \tau_{2_T} + \dots} \quad (6.41)$$

$\tau_{i_T} = C_i \cdot R_i$ $\quad R_i$ = den modstand $C_i$ ser ind i med alle andre kondensatorer afbrudt.

## BJT (CE)

$$p_1 \simeq \omega_H \quad \frac{1}{R_\pi\left(C_\pi + C_\mu\left(1 + g_m \cdot R_L' + \frac{R_L'}{R_\pi}\right)\right)}$$ nøjagtig udledt

Miller transformation. af $C_\mu$




Indgang $\tau_i = R_\pi \cdot (C_\pi + C_\mu')$

$$= R_\pi(C_\pi + C_\mu(1 + g_m R_L))$$

Udgang $\tau_o = R_L' \cdot C_\mu' \simeq R_L' \cdot C_\mu$

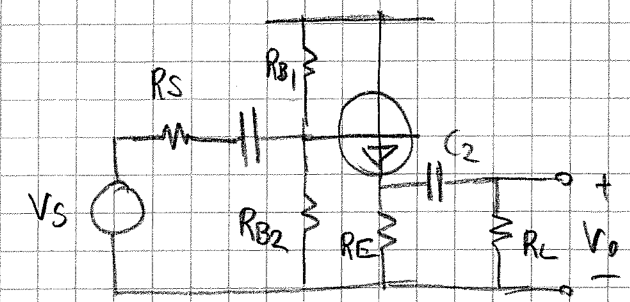
$$\omega_H = \frac{1}{\tau_i + \tau_o} = \frac{1}{R_\pi(C_\pi + C_\mu(1 + g_m R_L')) + R_L' \cdot C_\mu}$$

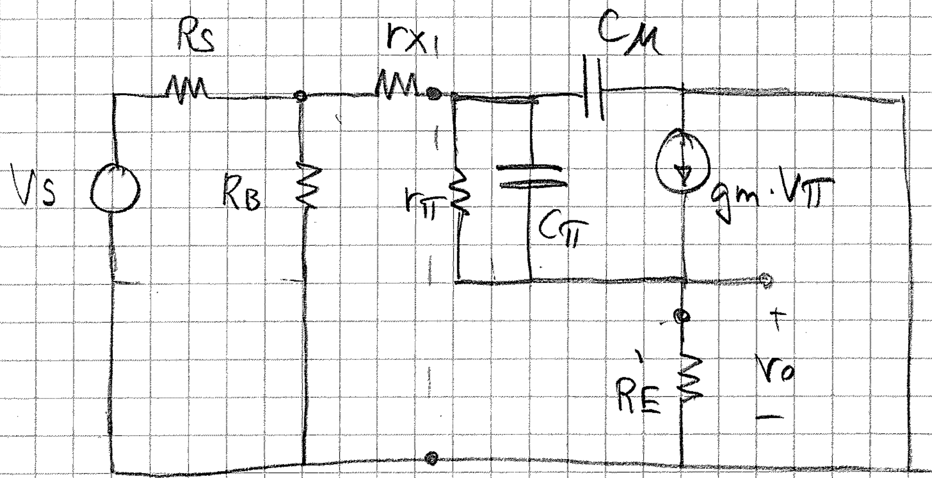
$$\omega_H = \frac{1}{R_\pi\left[C_\pi + C_\mu\left(1 + g_m R_L' + \frac{R_L'}{R_\pi}\right)\right]}$$

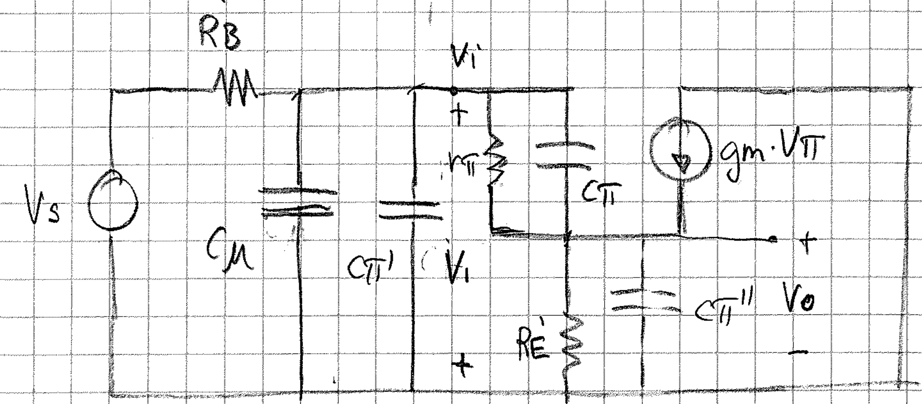
$\omega_H = p_1$ Miller transformation = nøjagtig udledt

Miller transformation er altså et godt værktøj kombineret med tomgangstidskonstant metoden til at beregne $\omega_H$ af et vilkårligt trin.

<u>eksempel</u> Emitter følger.




$R_E' = R_E // R_L$

findes $\frac{V_o}{V_i} = G_\pi \Rightarrow C_\pi' = C_\pi(1 - G_\pi) \qquad C_\pi'' = C_\pi\left(1 - \frac{1}{G_\pi}\right)$

$$\frac{V_o}{V_i} \simeq \frac{g_m \cdot V_\pi \cdot R_E'}{V_\pi + V_\pi \cdot g_m \cdot R_E'} = \frac{g_m R_E'}{1 + g_m R_E'}$$

$$\tau_i = \left(C_\mu + C_\pi(1-G_V)\right) R_{C_\mu}$$

$$\left(\text{———} '' \text{———}\right) \cdot (r_x + R_B \| R_S) \| (r_\pi + (1+h_{fe}) R_E)$$

$$\tau_i = \left(C_\mu + C_\pi\left(1 - \frac{g_m R_E}{1+g_m R_E}\right)\right)\left((r_x + R_B\|R_S)\|(r_\pi + h_{fe} \cdot R_E)\right)$$

$$\tau_i = \left[C_\mu + C_\pi\left(\frac{1}{1+g_m \cdot R_E}\right)\right]\left(\text{———} '' \text{———}\right)$$

$$\tau_i = \qquad C_\mu \left(\text{———} '' \text{———}\right)$$

$$\tau_o = C_\pi'' \cdot R_E \| \left[(r_\pi + r_x + R_B\|R_S)/(1+h_{fe})\right]$$

$$C_\pi'' \cdot R_E \| \left(\frac{1}{g_m} + \frac{R_B\|R_S}{1+h_{fe}}\right)$$

$G_V$ for en emitterfølger er lidt mindre end 1

$G_V \simeq 0{,}99 \Rightarrow 1-G_V = 1-0{,}99 = 0{,}01$

så $C_\pi' = C_\pi \cdot 0{,}01$ hvilket sikkert er mindre end $C_\mu$

på udgangen bliver $C_\pi'' = C_\pi\left(1-\frac{1}{G_V}\right)$ negativ !
og meget lille

Resultat: Emitterfølgeren har meget høj $\omega_H$ = stor båndbredde

$$f_H \overset{!}{\simeq} \frac{1}{2\pi\left(C_\mu (r_x + R_B\|R_S)\|(r_\pi + h_{fe} \cdot R_E)\right)}$$

# Repetition $\omega_H$

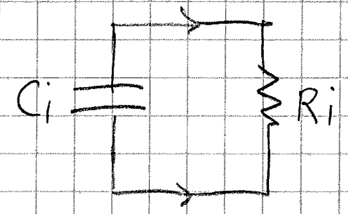
Tegn ækvivalentdiagram med $C_\pi$, $C_\mu$, $r_x$, $r_\pi$

Sidder en kondensator mellem indgang og udgang på et forstærkertrin, så flyttes denne vha. Millertransformation til indgangen og udgangen.

Find alle tomgangs tidskonstanter. $\tau_{iT}$

$$\omega_H = \frac{1}{\sum \tau_{iT}}$$

$$\tau_{iT} = C_i \cdot R_i$$




Ri er den modstand Ci ser ind i med alle andre kondensatorer afbrudt (fjernet)