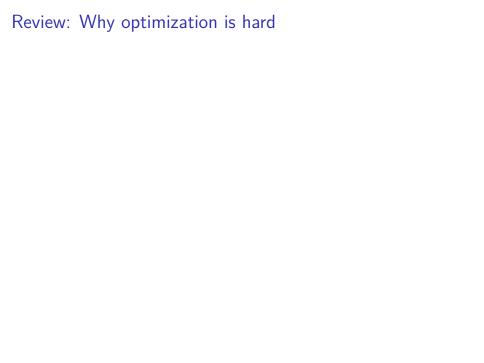
Introduction to Metaheuristics - Local Search

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Review: Why optimization is hard

- Most interesting problems are NP-Hard
- Weak global structure.
- Disparate attraction basins.
- Most real-life functions are black-box.
- Function evaluation can be very costly.

Review: What can we do

Just search, but with common sense!

- ► Assume there is some local structure: Near good solutions we can find other good solutions.
- ► **Approximate gradients:** Follow steps that improve the function fitness.
- Avoid local optima: Take measures to prevent getting stuck.

Any search method must balance between:

- exploration steps that lead you to discover new (and potentially better) attraction basins, and
- exploitation steps that lead you to improve your estimate of the best attraction basin.

Back to the basics

What is the simplest search method beyond random search?

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Hill-climbing

- **>** start with a random solution $x_i = x_0$
- while not finished:
 - \triangleright sample a random solution x_i close to x_i
 - if $F(x_j) > F(x_i)$ then $x_{i+1} = x_j$
- return x_n

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- ightharpoonup return x_n

The devil is in the details

- ▶ When do we know we're done?
- ▶ What is *close enough*?

Analyzing hill climbing

- ightharpoonup start with a random solution $x_i = x_0$
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How likely is to get stuck?

Analyzing hill climbing

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How likely is to get stuck?

Theorem: For any *fixed* value of closeness, hill climbing will eventually get stuck. (Why?)

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How to avoid geting stuck?

Fixing hill climbing

- ightharpoonup start with a random solution $x_i = x_0$
- while not finished:
 - **sample** a random solution x_i close to x_i

 - else if $U(0,1) < \beta$ then $x_{i+1} = x_j <$ here
- return x^* (best ever seen) <- and here

How to avoid geting stuck?

Sometimes, take non-exploitative steps.

What is the best value for β

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What is the best value for β

A changing value. . .

Fixing hill climbing = Simulated annealing

- ▶ start with a random solution $x_i = x_0$
- ▶ set $\beta = \beta_0$ relatively high
- while not finished:
 - ightharpoonup sample a random solution x_i close to x_i

 - else if $U(0,1) < \beta$ then $x_{i+1} = x_j$

 - decrease β a little bit \leftarrow here
- return x*

Ways to decrease β :

Fixing hill climbing = Simulated annealing

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Ways to decrease β :

- Linearly
- Exponentially
- Only if no improvement
- **.** . . .

Simulated annealing

How good is this?

- ▶ If the probability of sampling any solution x_j from the current x_i is never zero
- And the "cooling schedule" is sufficiently slow
- ► Then SA converges to a global optima *eventually*!

Simulated annealing

How good is this?

- If the probability of sampling any solution x_j from the current x_i is never zero
- ► And the "cooling schedule" is *sufficiently slow*
- Then SA converges to a global optima eventually!

Improvements

- Decrease the size of the vicinity iteratively
- Make the probability of choosing a solution x_j proportional to how good it is compared to x_i
- ightharpoonup Restart β after some iterations stuck