Introduction to Metaheuristics

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Why optimization is hard

What is the hardest problem you can think of?

Why optimization is hard

Some hard problems:

- Knapsack
- Scheduling
- Bin packing
- Vehicle routing
- Travelling salesman
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We don't know any polynomial algorithm to solve them.

Recap on Complexity Theory

Problems in P: Decision problem, decidable with a polynomial-time algorithm.

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The fundamental problem in Complexity Theory

Is P = NP?

Recap on Complexity Theory

A problem A is polynomially-reducible to another problem B iff

- Given an input for A, we can build a input for B in polynomial time
- Given an output for B, we can build an output for A in polynomial time

Thus, if B has a polynomial-time algorithm, so does A.

This implies, if we knew A had no polynomial time algorithm, neither does B.

NOTE: This is not restricted to decision problems

Recap on Complexity Theory

Question: Is there any problem T such that all problems in NP are polynomially reducible to T?

First, think, what would imply for a such a problem to exist?

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First, think, what would imply for a such a problem to exist?

This is called an **NP-Hard** problem.

If you solve it in polynomial time, then you solve all NP problems in polynomial time.

Notice there has to be NP-Hard problems! Can you think of one?

NOTE: NP-Hard problems need not be decision problems

Recap on Complexity Theory

Question: Is there any NP-Hard problem that is also in NP?

What would that imply?

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What would that imply?

This is called an **NP-Complete** problem.

If we find one, this means there are problems in NP as hard as any decision problem (verifiable in polynomial time).

It this a strong evidence that P != NP? Why?

Recap on Complexity Theory

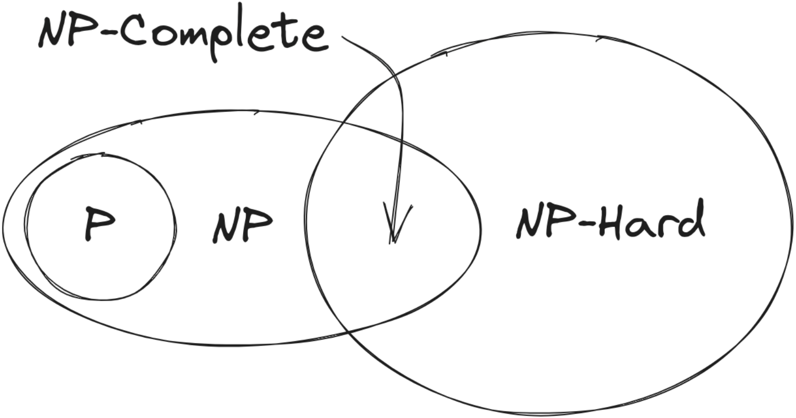
The first ever problem proven NP-Complete is **Circuit SAT**, but there are many more.

Basically all decision version of NP-Hard optimization problems:

- Is there a tour with less than X cost?
- Is there a packing with less than X area?
- Is there a schedule finishing in less than X time?
- ...

Recap on Complexity Theory

If P != NP, this is the panorama:



Why is optimization hard

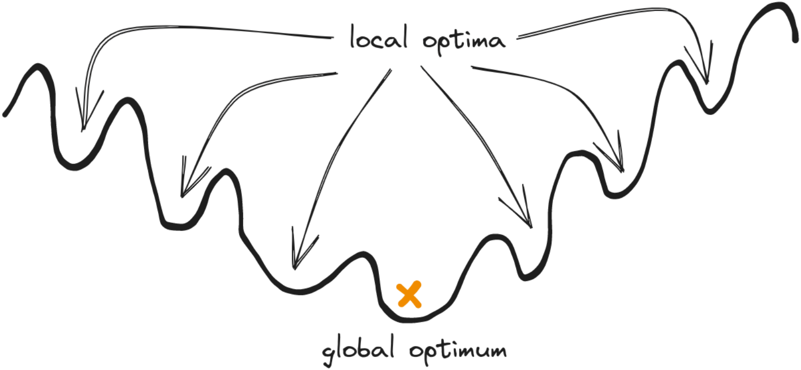
Even is P = NP, NP-Hard problems will still exist!

- Combinatorial optimization is **NP-Hard**.
- Continuous optimization is **at least as hard**. (Why?)

(But P probably != NP, anyway. . .)

Why optimization is hard

Multiple local optima

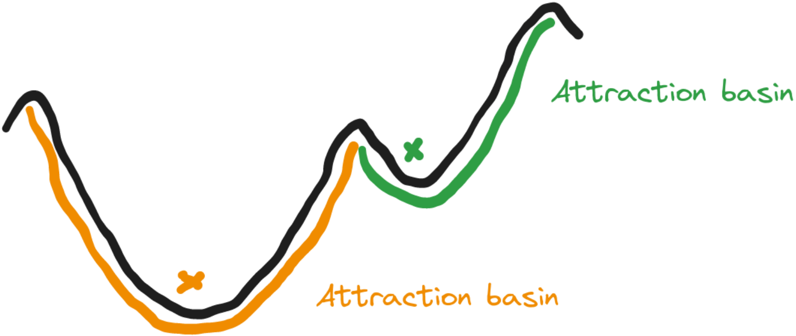


How do you even know you found the optimum?

Why optimization is hard

Other reasons:

- Weak global structure.
- Disparate attraction basins.
- And most real-life functions are **black-box** anyway, exact gradients are not available.



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What can we do

Just search, but with common sense!

- **Assume there is some local structure**: Near good solutions we can find other good solutions.
- **Approximate gradients:** Follow steps that improve the function fitness.
- **Avoid local optima:** Take measures to prevent getting stuck.

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Any search method must balance between:

- **exploration** steps that lead you to discover new (and potentially better) attraction basins, and
- **exploitation** steps that lead you to improve your estimate of the best attraction basin.

The Metaheuristic Framework

- given a black box function F
- establish a stop criteria (often # of evaluations or time)
- while not stop do:
    - **sample** a new solution x_i
    - evaluate $y_i = F(x_i)$
    - update global best $y^* = \min\{y^*, y_i\}$
    - **learn** something about F for next iteration (maybe)
- return global best y^*

Metaheuristic design paradigms

- **Local search:** Hill climb, Simulated annealing, Tabu search, GRASP
- **Evolutionary search:** Genetic algorithms, Differential evolution, Genetic programming, Grammatical evolution
- **Swarm intelligence:** Particle swarm optimization, Ant colony optimization
- **Estimation of distribution:** UMDA, CMA-ES, Bayesian optimization, Probabilistic Grammatical Evolution

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Other "advanced" topics

- How to avoid premature convergence
- Multi-objective optimization
- Stochastic optimization
- Learning to search