OR-Assignment - 2

Sol 1 An integer programming problem is optimization technique in which some or all variables are restricted to be integers. Importance of IPP is integer variables represent quantities that can only be integer.

The integer variables represent decisions and so should only take on value 0 or 1

Gomory method

1) Relax the integer requirements
2) Solve the LPP using Simplex method.
3) If all basic variables have integer values, optimality is reached so print the solution otherwise go to step 4.
4) Examine the constraints. For each basic variable with non-integer solution in current table, find fractional part, f_i

   $b_i = [b_i] + f_i$ where $[b_i]$ is integer part of b_i.
5) Choose largest fraction among all f_i's Treat the constraint corresponding to largest fraction as source row. Based on source equation, develop an additional constrant (Gomory's constraint

   $-f_i = S_i$ - Summation $((f_i)$ (Non-basic variable))
6) Add fractional cut as last row in latest optimal table and proceed further using dual simplex method & find new optimum solution. If new optimum solution is integer, print the solution otherwise repeat the same steps until solution is integer.

Sol 2

	W_1	W_2	W_3	W_4	Supply	
F_1	19	30	50	10	7	(9)
F_2	70	30	40	60	9	(10)
F_3	40	8 (8)	70	20	18-8=10	(12)
Demand	5	8-8=0	7	14	34	
	(21)	(22)	(10)	(10)		

	W_1	W_3	W_4	Supply	
F_1	19 (5)	50	10	7-5=2	(9)
F_2	70	40	60	9	(20)
F_3	40	70	20	10	(20)
Demand	5-5=0	7	14	26	
	(21)	(10)	(10)		

	W_3	W_4	Supply	
F_1	50	10	2	(40)
F_2	40	60	9	(20)
F_3	70	20 (10)	10-10=0	(50)
Demand	7	14-10=4	(11)	
	(10)	(10)		

	W_3	W_4	Supply	
F_1	50	10 (2)	2-2=0	(40)
F_2	40	60	9	(20)
Demand	7	4-2=2	(9)	
	(10)	(50)		

	W_3	W_4	Supply
F_2	40 (7)	60 (2)	9
Demand	7	2	

	w_1	w_2	w_3	w_4
F_1	19 (5)	30	50	10 (2)
F_2	70	30	40 (7)	60 (2)
F_3	40	8 (8)	70	20 (10)

Total cost $= 19\times5 + 10\times2 + 40\times7 + 60\times2 + 8\times8 + 20\times10$

$= 95 + 20 + 280 + 120 + 64 + 200$

$= 779$

Now, to find optimal solution use u-v Modi method

For allocated cell $C_{ij} = u_i + v_j$

	w_1	w_2	w_3	w_4	
F_1	19 (5)	•	•	10 (2)	u_1
F_2	•	•	40 (7)	60 (2)	u_2
F_3	•	8 (8)	•	20 (10)	u_3
	v_1	v_2	v_3	$v_4 = 0$	

let $v_4 = 0$

$\boxed{u_1 = 10}$ | $u_1 + v_1 = 19$ $\boxed{v_1 = 9}$ | $u_4 + u_3 = 20$ $\boxed{u_3 = 20}$ | $v_2 + u_3 = 8$ $\boxed{v_2 = -12}$ | $u_2 + v_4 = 60$ $\boxed{u_2 = 60}$ | $u_2 + v_3 = 40$ $\boxed{v_3 = -20}$

For non-allocated cell, $C_{ij} - (u_i + v_j) \geq 0$

	w_1	w_2	w_3	w_4	
F_1	•	30 (-2)	50 (-10)	•	$u_1 = 10$
F_2	70 ([illegible])	30 (48)	•	•	$u_2 = 60$
F_3	40 ([illegible])	•	70 (0)	•	$u_3 = 20$
	$v_1 = 9$	$v_2 = -12$	$v_3 = -20$	$v_4 = 0$	

Now, allocate θ at (2,2) cell

	W_1	W_2	W_3	W_4	
F_1	19 5	·	·	10 2	7
F_2	·	30 2	40 7	60 θ−θ	9
F_3	·	8 6 θ	·	20 12+θ	18
	5	8	7	14	

$\boxed{\theta = 2}$

Total cost = $19 \times 5 + 10 \times 2 + 30 \times 2 + 40 \times 7 + 8 \times 6 + 20 \times 12$

$= 95 + 20 + 60 + 280 + 48 + 240$

$= \boxed{743}$ (Op final solution)

Sol 3 Transportation problem is a special kind of LPP in which goods are transported from set of source to set of destinations subject to supply and demand of sources and destination such that total cost of transportation is minimized.

Steps of Vogel approximation method

1) Calculate the penalties for each row and column. Penalty means difference between two successive least cost in row and column.
2) Select row or column with largest penalty
3) In selected row or column, allocate maximum feasible quantity to cell with minimum cost.
4) Eliminate the row or column where all allocations are made.
5) Write reduced transportation table and repeat steps 1-4
6) Repeat procedure until all allocations are made.

2014 a)

	D_1	D_2	D_3	D_4		
S_1	17	11	45	20	15	(6)
S_2	18	19	14 (7)	31	13-7=6	(14)
	9	6	7-7=0	6	28	
	(1)	(8)	(31)	(1)		

	D_1	D_2	D_4		
S_1	17	11 (6)	20	15-6 =9	(6)
S_2	18	19	31	6	(1)
	9	6-6=0	6		
	(1)	(8)	(1)		

	D_1	D_4		
S_1	17 (9)	20	9-9=0	(13)
S_2	18	31	6	(13)
	9-9=0	6		
	(1)	(1)		

	D_4	
S_2	31 (6)	6
	6	

$$Cost = 14\times 7 + 11\times 6 + 17\times 9 + 31\times 6$$

$$= 98 + 66 + 153 + 186$$

$$= 503.$$

b) For allocated cell,

	D_1	D_2	D_3	D_4	
S_1	17 (9)	11 (6)	•	•	$u_1 = 0$
S_2	•	•	14 (7)	31 (6)	$u_2 = 0$
	$v_1=17$	$v_2=11$	$v_3=14$	$v_4=31$	

For non-allocated cell,

	D_1	D_2	D_3	D_4	
S_1			45 (31)	31 (11)	$u_1 = 0$
S_2	18 (1)	19 (8)			$u_2 = 0$
	$v_1=17$	$v_2=11$	$v_3=14$	$v_4=31$	

It is not optimal, $c_{14} - (u_1 + v_4) \le 0$

So, allocate θ.

	D_1	D_2	D_3	D_4	
S_1	17 (9)	11 (6)	•	30 (θ)	15
S_2	•	•	14 (7)	31 (6) (θ)	13
	9	6	7	6	

Sol 5. Mathematical formulation of Assignment problem.

Feasible sol^n consists $(2n-1)$ variables of which $(n-1)$ variables are zero.

Suppose x_{ij} is a variable.

1 if i^{th} job is assigned to j^{th} machine.

0 if i^{th} job is not assigned.

$$\sum_{i=1}^{n} x_{ij} = 1$$

$$\sum_{j=1}^{n} x_{ij} = 1$$

Total cost $U = \sum_{j=1}^{n} \sum_{i=1}^{n} x_{ij} \, c_{ij}$

Minimize cost

Subject to constraints

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{when } j = 1, 2, 3 \ldots n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \text{when } i = 1, 2, 3 \ldots n$$

Comparative study of TP and AP.

TP	AP
1) This is reducing the cost of transportation merchadise.	1) This is about assigning finite source to finite destination where one destination is alloted to one source with minimum cost.
2) No of sources and no of demand need not be equal.	2) No. of sources and no. of destinations must be equal.

3) If total demand and total supply are not equal, then unbalanced.	If no. of rows and columns are not equal, then unbalanced.
4) It requires 2 stages to solve.	It has only one stage.

Sol 6

	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

Select smallest element in row and subtract ~~same for row~~

	E	F	G	H
A	7	15	6	0
B	0	15	1	13
C	23	4	3	0
D	9	16	14	0

Select smallest element in column

	E	F	G	H
A	7	11	5	$\boxed{0}$
B	0	11	$\boxed{0}$	13
C	23	$\boxed{0}$	2	0
D	9	12	13	0

In row D assignment is not possible,

Now, subtract 5 from uncovered cells and add at junction

	E	F	G	H
A	2	6	$\boxed{0}$	9
B	$\boxed{0}$	11	0	18
C	23	$\boxed{0}$	2	5
D	4	7	8	$\boxed{0}$

Total hours = 17 + 13 + 19 + 10

= $\boxed{59 \text{ hrs}}$

Sol 7

Job \ Machine	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6
5	0	0	0	0	0

Now, find lowest element of row & subtract & column

	A	B	C	D	E
1	2	1	4	$\boxed{0}$	5
2	$\boxed{0}$	2	1	4	6
3	3	2	1	0	4
4	2	1	$\boxed{0}$	3	0
5	0	0	0	0	0

Not optimal as row 3 could not be assigned

subtract 1 from unallotted.

	A	B	C	D	E
1	2	$\boxed{0}$	4	$\boxed{0}$	5
2	$\boxed{0}$	1	1	4	6
3	3	1	1	$\boxed{0}$	4
4	2	0	$\boxed{0}$	3	0
5	1	$\boxed{0}$ 0	1	1	1

Total processing time = 3 + 10 + 1 + 6 = $\boxed{20 \text{ hrs}}$

Machine E no job.

Sol 8

	A	B	C	D	E
A	∞	2	4	7	1
B	6	∞	2	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Subtract lowest element from row & column

	A	B	C	D	E
A	∞	1	3	6	$\boxed{0}$
B	4	∞	$\boxed{0}$	6	0
C	4	3	∞	$\boxed{0}$	3
D	8	$\boxed{0}$	2	∞	1
E	$\boxed{0}$	2	1	7	∞

Minimized cost = 1 + 2 + 4 + 4 + 1 = $\boxed{12}$

Sol 9

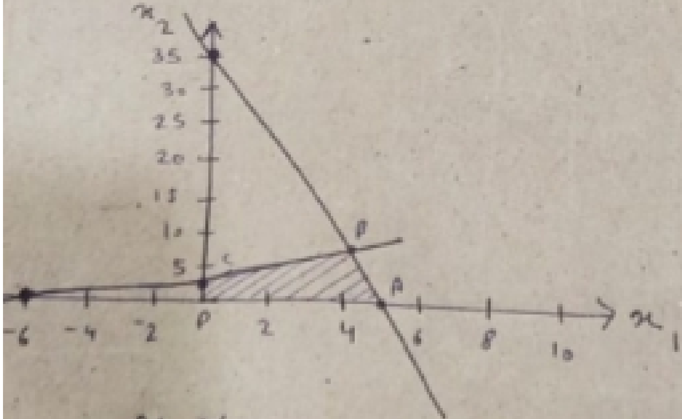
$\text{Max } z = 7x_1 + 9x_2$

st : $-x_1 + 3x_2 \leq 6$

x_1	0	-6
x_2	2	0

$7x_1 + x_2 \leq 35$, $x_1, x_2 \geq 0$ are integers

x_1	0	5
x_2	35	0



At $A(5, 0)$, $z = 35$

At $B(\frac{9}{2}, \frac{7}{2})$, $z = 63$

At $C(0, 2)$, $z = 18$

At $D(0, 0)$ $z = 0$

Z is max at $\left(\frac{9}{2}, \frac{7}{2}\right)$ but they are not integers.

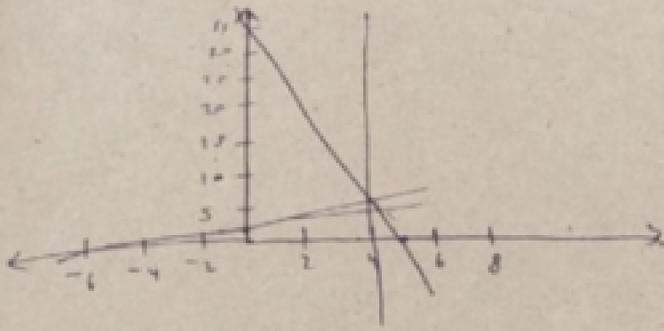
$x_1 = 4.5$

$x_1 \leq 4 \qquad x_1 \geq 5$

$-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

$x_1 \leq 4$



$x_1 = 4, \quad x_2 = 3.33$

$x_2 \leq 3, \quad x_2 \geq 4$

$-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

$x_1 \leq 4$

$x_2 \leq 3$

$x_1 = 4, \quad x_2 = 3$

$-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

$x_1 \leq 4$

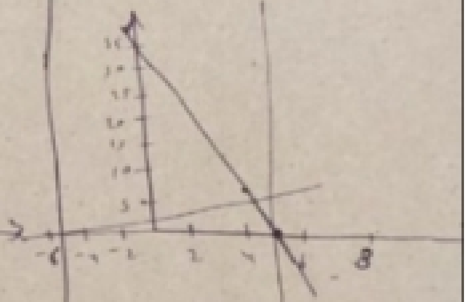
$x_2 \geq 4$

No feasible

$-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

$x_1 \geq 5$



No, feasible

$x_1 = 5, \quad x_2 = 0$

So optimal solⁿ is

$\boxed{x_1 = 4, \quad x_2 = 3}$

Max $z = 28 + 27$

$= \boxed{55}$

Sol 10

The branch and bound is an algorithm for discrete and optimization problem. It consist of a systematic enumeration of candidate solutions by means of state space search. The algorithm explores branches of this tree, which represents subsets of solution set. Before solutions of a branch, branch is checked against upper and lower estimated bounds on optimal solution and is discarded if it cannot produce a better solution than the best one found so far by algorithm.

The algorithm depends on efficient estimation of lower and upper bounds of region of search space. If no bounds are available, the algorithm degenerates to an exhaustive search.

Branch and bound technique finds a value x that maximises or minimizes the value of real-valued f^n. It is an integer-programming problem solving technique means it gives integer solutions.

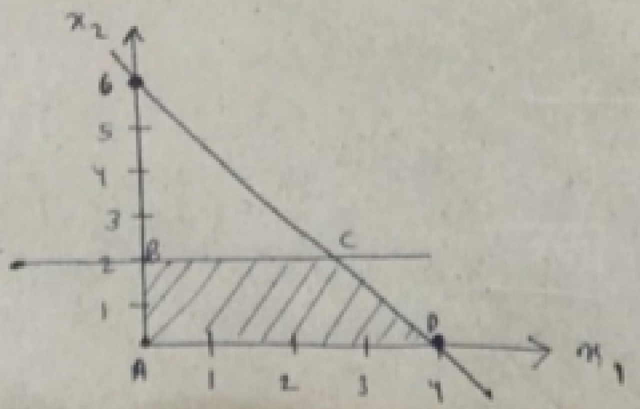
Sol 11

Max $z = x_1 + x_2$

st. $3x_1 + 2x_2 \leq 12$

$x_2 \leq 2$

x_1	0	4
x_2	6	0



At A $(0,0)$, $Z = 0$

At B $(0,2)$, $Z = 2$

At C $(\frac{8}{3}, 2)$, $Z = \frac{14}{3}$

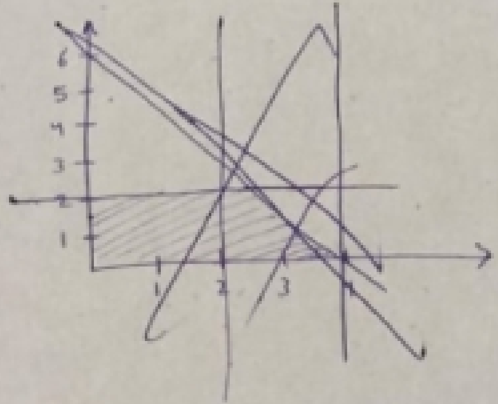
At D $(4,0)$, $Z = 4$

Z is maximum at $(\frac{8}{3}, 2)$ but x_1 is not integer.

$3x_1 + 2x_2 \le 12$

$x_2 \le 2$

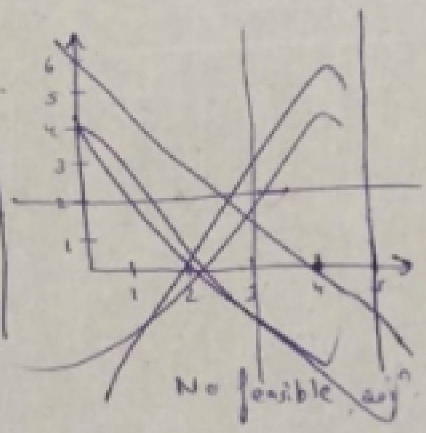
$x_1 \le 2$

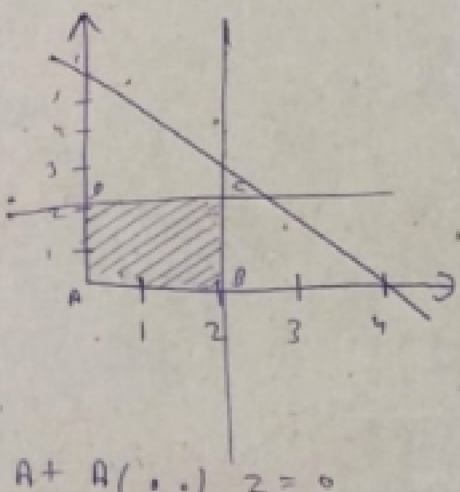


$3x_1 + 2x_2 \le 12$

$x_2 \le 2$

$x_1 \ge 3$





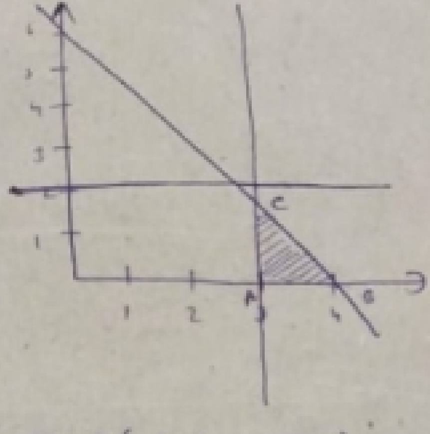
At A $(0,0)$, $Z = 0$

At B $(2,0)$, $Z = 2$

At C $(2,2)$, $Z = 4$

At D $(0,2)$, $Z = 2$

Max $Z = 4$ At $(2,2)$



At A $(3,0)$, $Z = 3$

At B $(4,0)$, $Z = 4$

At C $(3,1.5)$, $Z = 4.5$

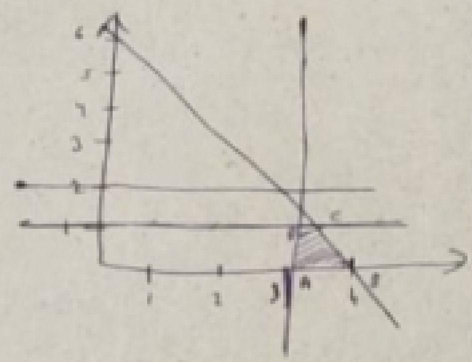
x_2 is not integer.

$3x_1 + x_2 \le 12$

$x_2 \le 2$

$x_1 \ge 3$

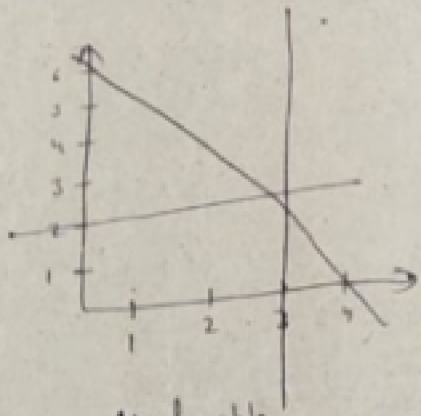
$x_2 \le 1$



$x_2 \le 2$

$x_1 \ge 3$

$x_2 \ge 2$



No feasible.

At $A(3,0)$, $z = 3$

At $B(4,0)$, $z = 4$

At $C(4,1)$, $z = 5$

At $D(3,1)$, $z = 4$.

Max, $z = 5$ not possible

So, feasible solⁿ is $x_1 = 2$, $x_2 = 2$

$\boxed{\text{Max } z = 4}$

Sol 12

	D_1	D_2	D_3	D_4	Capacity	
O_1	1	2	3	4	6	(1)
O_2	4	3	2	0 (6)	8-6=2	(2)
O_3	0	2	2	1	10	(1)
Demand	4	6	8	6-6	24	
	(1)	(0)	(0)	(1)		

	D_1	D_2	D_3		
O_1	1	2	3	6	(1)
O_2	4	3	2	2	(1)
O_3	0 (4)	2	2	10-4=6	(2)
	4-4=0	6	8		
	(1)	(0)	(0)		

	D_2	D_3		
O_1	2	3 (6)	6-6=0	(1)
O_2	3	2	2	(1)
O_3	2	2	6	(0)
	6	8-6=2		
	(0)	(0)		

	D_2	D_3		
O_2	3 (2)	2	2-2=0	(1)
O_3	2	2	6	(0)
	6-2=4	2		
	(1)	(0)		

	D_2	D_3	
O_3	2 (4)	2 (2)	6
	4	2	

Total cost $= 3\times 6 + 3\times 2 + 2\times 4 + 2\times 2$

$= 18 + 6 + 8 + 4$

$= 36$

For allocated cell, $C_{ij} - (u_i + v_j)$

	D_1	D_2	D_3	D_4	
O_1	•	•	3 (6)	•	$u_1 = 1$
O_2	•	3 (2)	•	0 (6)	$u_2 = 1$
O_3	0 (4)	2 (4)	2 (2)	•	$u_3 = 0$
	$v_1 = 0$	$v_2 = 2$	$v_3 = 2$	$v_4 = -1$	

For non-allocated cell,

	D_1	D_2	D_3	D_4
O_1	1 ①	2 ③	•	4 ⓪
O_2	4 ①	•	2 ③	1 ①
O_3	•	•	•	

It is not optimal solution,

	D_1	D_2	D_3	D_4	
O_1	•	2 ⓪	3 ②	•	6
O_2	•	3 ②	2 ⓪	0 ⑥	8
O_3	0 ④	2 ④	2 ②	•	10
	4	6	8	6	

$\theta = 2$

Total cost $= 2\times2 + 3\times4 + 2\times2 + 2\times4 + 2\times2$

$= 4 + 12 + 4 + 8 + 4$

$= \boxed{32}$