Cookbook formulae for audio equalizer biquad filter coefficients

Author
> [Robert Bristow-Johnson](#)

Editors
> [Raymond Toy](#)
> [Doug Schepers (previously W3C)](#)

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Biquad Filter Formulae

All filter transfer functions were derived from analog prototypes (that are shown below for each equalizer (EQ) filter type) and had been digitized using the Bilinear Transform (BLT). BLT frequency warping has been taken into account for both significant frequency relocation (this is the normal "prewarping" that is necessary when using the BLT) and for bandwidth readjustment (since the bandwidth is compressed when mapped from analog to digital using the BLT).

First, given a biquad transfer function defined as:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}} \tag{1}$$

This shows 6 coefficients instead of 5 so, depending on your architecture, you will likely normalize a_0 to be 1 and perhaps also b_0 to 1 (and collect that into an overall gain coefficient). Then your transfer function would look like:

$$H(z) = \frac{\left(\dfrac{b_0}{a_0}\right) + \left(\dfrac{b_1}{a_0}\right) z^{-1} + \left(\dfrac{b_2}{a_0}\right) z^{-2}}{1 + \left(\dfrac{a_1}{a_0}\right) z^{-1} + \left(\dfrac{a_2}{a_0}\right) z^{-2}} \tag{2}$$

or:

$$H(z) = \left(\frac{b_0}{a_0}\right) \frac{1 + \left(\dfrac{b_1}{b_0}\right) z^{-1} + \left(\dfrac{b_2}{b_0}\right) z^{-2}}{1 + \left(\dfrac{a_1}{a_0}\right) z^{-1} + \left(\dfrac{a_2}{a_0}\right) z^{-2}} \tag{3}$$

The most straight forward implementation would be the "Direct Form 1" (*eq.* 2):

$$y[n] = \left(\frac{b_0}{a_0}\right)x[n] + \left(\frac{b_1}{a_0}\right)x[n-1] + \left(\frac{b_2}{a_0}\right)x[n-2]$$
$$- \left(\frac{a_1}{a_0}\right)y[n-1] - \left(\frac{a_2}{a_0}\right)y[n-2] \qquad (4)$$

This is probably both the best and the easiest method to implement in the 56K and other fixed-point or floating-point architectures with a double wide accumulator.

Begin with these user defined parameters:

F_s
    the sampling frequency
f_0
    Center Frequency or Corner Frequency, or shelf midpoint frequency, depending on which filter type. The "significant frequency". "wherever it's happenin', man."
dBgain
    used only for peaking and shelving filters
Q or BW or S

Q
    the EE kind of definition, except for peakingEQ in which $A \cdot Q$ is the classic EE Q. That adjustment in definition was made so that a boost of N dB followed by a cut of N dB for identical Q and f_0/F_s results in a precisely flat unity gain filter or "wire".
BW
    the bandwidth in octaves (between -3 dB frequencies for BPF and notch or between midpoint (dBgain/2) gain frequencies for peaking EQ)
S
    a "shelf slope" parameter (for shelving EQ only). When $S = 1$, the shelf slope is as steep as it can be and remain monotonically increasing or decreasing gain with frequency. The shelf slope, in dB/octave, remains proportional to S for all other values for a fixed f_0/F_s and dBgain.

Then compute a few intermediate variables:

1. Only for peaking and shelving EQ filters:

$$A = \sqrt{10^{\text{dBgain}/20}} \qquad (5)$$
$$= 10^{\text{dBgain}/40} \qquad (6)$$

2.

$$\omega_0 = 2\pi\frac{f_0}{F_s}$$

3.

$$\cos\omega_0$$
$$\sin\omega_0$$

4.

$$\alpha = \frac{\sin \omega_0}{2Q} \qquad \text{(case: Q)} \qquad (7)$$

$$= \sin \omega_0 \, \sinh\left(\frac{\log 2}{2} \cdot \text{BW} \cdot \frac{\omega_0}{\sin \omega_0}\right) \qquad \text{(case: BW)} \qquad (8)$$

$$= \frac{\sin \omega_0}{2} \sqrt{\left(A + \frac{1}{A}\right)\left(\frac{1}{S} - 1\right) + 2} \qquad \text{(case: S)} \qquad (9)$$

FYI: The relationship between bandwidth and Q is

digital filter with BLT

$$\frac{1}{Q} = 2\sinh\left(\frac{\log 2}{2} \cdot \text{BW} \cdot \frac{\omega_0}{\sin \omega_0}\right) \qquad (10)$$

or

analog filter prototype

$$\frac{1}{Q} = 2\sinh\left(\frac{\log 2}{2} \cdot \text{BW}\right) \qquad (11)$$

The relationship between shelf slope, S, and Q is

$$\frac{1}{Q} = \sqrt{\left(A + \frac{1}{A}\right)\left(\frac{1}{S} - 1\right) + 2} \qquad (12)$$

$$2\sqrt{A}\,\alpha = (\sin \omega_0)\sqrt{(A^2 + 1)\left(\frac{1}{S} - 1\right) + 2A} \qquad (13)$$

is a handy intermediate variable for shelving EQ filters.

Finally, compute the coefficients for whichever filter type you want:

(The analog prototypes, H(s), are shown for each filter type for normalized frequency.)

LPF

$$H(s) = \frac{1}{s^2 + \frac{s}{Q} + 1} \tag{14}$$

$$
\begin{aligned}
b_0 &= \frac{1 - \cos\omega_0}{2} \\
b_1 &= 1 - \cos\omega_0 \\
b_2 &= \frac{1 - \cos\omega_0}{2} \\
a_0 &= 1 + \alpha \\
a_1 &= -2\cos\omega_0 \\
a_2 &= 1 - \alpha
\end{aligned} \tag{15}
$$

HPF

$$H(s) = \frac{s^2}{s^2 + \frac{s}{Q} + 1} \tag{16}$$

$$
\begin{aligned}
b_0 &= \frac{1 + \cos\omega_0}{2} \\
b_1 &= -(1 + \cos\omega_0) \\
b_2 &= \frac{1 + \cos\omega_0}{2} \\
a_0 &= 1 + \alpha \\
a_1 &= -2\cos\omega_0 \\
a_2 &= 1 - \alpha
\end{aligned} \tag{17}
$$

BPF
(constant skirt gain,
peak gain = Q)

$$H(s) = \frac{s}{s^2 + \frac{s}{Q} + 1} \tag{18}$$

$$b_0 = \frac{\sin\omega_0}{2} = Q\alpha$$
$$b_1 = 0$$
$$b_2 = -\frac{\sin\omega_0}{2} = -Q\alpha \tag{19}$$
$$a_0 = 1 + \alpha$$
$$a_1 = -2\cos\omega_0$$
$$a_2 = 1 - \alpha$$

BPF
(constant 0 dB peak gain)

$$H(s) = \frac{\dfrac{s}{Q}}{s^2 + \dfrac{s}{Q} + 1} \tag{20}$$

$$b_0 = \alpha$$
$$b_1 = 0$$
$$b_2 = -\alpha$$
$$a_0 = 1 + \alpha \tag{21}$$
$$a_1 = -2\cos\omega_0$$
$$a_2 = 1 - \alpha$$

notch

$$H(s) = \frac{s^2 + 1}{s^2 + \dfrac{s}{Q} + 1} \tag{22}$$

$$b_0 = 1$$
$$b_1 = -2\cos\omega_0$$
$$b_2 = 1$$
$$a_0 = 1 + \alpha \tag{23}$$
$$a_1 = -2\cos\omega_0$$
$$a_2 = 1 - \alpha$$

APF

$$H(s) = \frac{s^2 - \dfrac{s}{Q} + 1}{s^2 + \dfrac{s}{Q} + 1} \tag{24}$$

$$
\begin{aligned}
b_0 &= 1 - \alpha \\
b_1 &= -2\cos\omega_0 \\
b_2 &= 1 + \alpha \\
a_0 &= 1 + \alpha \\
a_1 &= -2\cos\omega_0 \\
a_2 &= 1 - \alpha
\end{aligned}
\tag{25}
$$

peakingEQ

$$H(s) = \frac{s^2 + s\dfrac{A}{Q} + 1}{s^2 + \dfrac{s}{AQ} + 1} \tag{26}$$

$$
\begin{aligned}
b_0 &= 1 + \alpha A \\
b_1 &= -2\cos\omega_0 \\
b_2 &= 1 - \alpha A \\
a_0 &= 1 + \dfrac{\alpha}{A} \\
a_1 &= -2\cos\omega_0 \\
a_2 &= 1 - \dfrac{\alpha}{A}
\end{aligned}
\tag{27}
$$

lowShelf

$$H(s) = A\frac{s^2 + \dfrac{\sqrt{A}}{Q}s + A}{As^2 + \dfrac{\sqrt{A}}{Q}s + 1} \tag{28}$$

$$b_0 = A\left((A+1) - (A-1)\cos\omega_0 + 2\sqrt{A}\,\alpha\right)$$

$$b_1 = 2A\left((A-1) - (A+1)\cos\omega_0\right)$$

$$b_2 = A\left((A+1) - (A-1)\cos\omega_0 - 2\sqrt{A}\,\alpha\right)$$

$$a_0 = (A+1) + (A-1)\cos\omega_0 + 2\sqrt{A}\,\alpha$$

$$a_1 = -2\left((A-1) + (A+1)\cos\omega_0\right)$$

$$a_2 = (A+1) + (A-1)\cos\omega_0 - 2\sqrt{A}\,\alpha$$

$$(29)$$

highShelf

$$H(s) = A\frac{As^2 + \dfrac{\sqrt{A}}{Q}s + 1}{s^2 + \dfrac{\sqrt{A}}{Q}s + A} \tag{30}$$

$$b_0 = A\left((A+1) + (A-1)\cos\omega_0 + 2\sqrt{A}\alpha\right)$$

$$b_1 = -2A\left((A-1) + (A+1)\cos\omega_0\right)$$

$$b_2 = A\left((A+1) + (A-1)\cos\omega_0 - 2\sqrt{A}\alpha\right)$$

$$a_0 = (A+1) - (A-1)\cos\omega_0 + 2\sqrt{A}\alpha$$

$$a_1 = 2\left((A-1) - (A+1)\cos\omega_0\right)$$

$$a_2 = (A+1) - (A-1)\cos\omega_0 - 2\sqrt{A}\alpha$$

$$(31)$$

FYI: The bilinear transform (with compensation for frequency warping) substitutes:

(normalized)

$$s \leftarrow \frac{1}{\tan\dfrac{\omega_0}{2}} \times \frac{1 - z^{-1}}{1 + z^{-1}} \tag{32}$$

and makes use of these trig identities:

1.

$$\tan\frac{\omega_0}{2} = \frac{\sin\omega_0}{1 + \cos\omega_0} \tag{33}$$

2.

$$\left(\tan\frac{\omega_0}{2}\right)^2 = \frac{1-\cos\omega_0}{1+\cos\omega_0} \tag{34}$$

resulting in these substitutions:

1.

$$1 \leftarrow \frac{1+\cos\omega_0}{1+\cos\omega_0} \times \frac{1+2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} \tag{35}$$

2.

$$s \leftarrow \frac{1+\cos\omega_0}{\sin\omega_0} \times \frac{1-z^{-1}}{1+z^{-1}} \tag{36}$$

$$= \frac{1+\cos\omega_0}{\sin\omega_0} \times \frac{1-z^{-2}}{1+2z^{-1}+z^{-2}} \tag{37}$$

3.

$$s^2 \leftarrow \frac{1+\cos\omega_0}{1-\cos\omega_0} \times \frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} \tag{38}$$

The factor:

$$\frac{1+\cos\omega_0}{1+2z^{-1}+z^{-2}} \tag{39}$$

is common to all terms in both numerator and denominator, can be factored out, and thus be left out in the substitutions above resulting in:

1.

$$1 \leftarrow \frac{1+2z^{-1}+z^{-2}}{1+\cos\omega_0} \tag{40}$$

2.

$$s \leftarrow \frac{1-z^{-2}}{\sin\omega_0} \tag{41}$$

3.

$$s^2 \leftarrow \frac{1 - 2z^{-1} + z^{-2}}{1 - \cos\omega_0} \tag{42}$$

In addition, all terms, numerator and denominator, can be multiplied by a common $\sin^2\omega_0$ factor, finally resulting in these substitutions:

1.

$$1 \leftarrow (1 + 2z^{-1} + z^{-2})(1 - \cos\omega_0) \tag{43}$$

2.

$$s \leftarrow (1 - z^{-2})\sin\omega_0 \tag{44}$$

3.

$$s^2 \leftarrow (1 - 2z^{-1} + z^{-2})(1 + \cos\omega_0) \tag{45}$$

4.

$$1 + s^2 \leftarrow 2\left(1 - 2\cos\omega_0\, z^{-1} + z^{-2}\right) \tag{46}$$

The biquad coefficient formulae above come out after a little simplification.

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