Kinematics

$\underline{P}+\underline{Q}=(P_x+Q_x)\underline{i}+(P_y+Q_y)\underline{j}+(P_z+Q_z)\underline{k}$
$\underline{P}.\underline{Q}=\underline{Q}.\underline{P}=P_xQ_x+P_yQ_y+P_zQ_z$
$\underline{P}.\underline{P}=P^2=P_x^2+P_y^2+P_z^2$
$\underline{P}\times\underline{Q}=-\underline{Q}\times\underline{P}$ $=(P_yQ_z-P_zQ_y)\underline{i}-(P_xQ_z-P_zQ_x)\underline{j}$ $+(P_xQ_y-P_yQ_x)\underline{k}$
$\underline{P}\times\underline{P}=\underline{Q}\times\underline{Q}=0$

Kinematic Equations

For rotational acceleration→ $x = \theta, a = \alpha$

$$x = vt + \frac{1}{2}at^2$$

$$v_f = v_i + \text{at}$$

$$v_f{}^2 = v_i{}^2 + 2\text{ax}$$

Displacement

$$x_{B/A} = x_B - x_A$$

Curvilinear Motion

$$(a_b)_n = \frac{v_b{}^2}{\rho}$$

Velocity

Translation	$V_b = V_a$
Rotation	$V_a = \omega_k \times r_{oa}$
Plane	$V_b = V_a + V_{b/a}$ $= V_a + \omega_k \times r_{b/a}$

Acceleration (Vectors)

$$a_B = a_A + a_{B/A}$$

$$\underline{a}_{B/A} = \underbrace{-\omega^2\underline{r}_{B/A}}_{\underline{a}_n} + \underbrace{\alpha\underline{k}\times\underline{r}_{B/A}}_{\underline{a}_t}$$

ICR Method: *Given ICR C, you can find velocity at any point on object.*

$$V_a = \omega_{obj} \times r_{a/c}$$

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Mass Properties

$$\bar{x} = \frac{\sum x_n \triangle w_n}{w}, \bar{y} = \frac{\sum y_n \triangle w_n}{w}$$

Second Moment of Area. I_o

From Radius of Gyration

$$I_o = k_o^2\text{m}$$

Parallel Axis Theorem

$$I_o = I_b + \text{m}(r_{b/o})^2$$

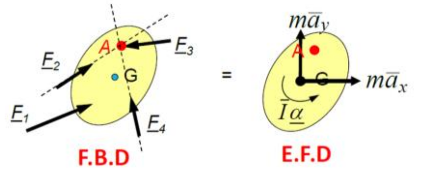
FBD, EFD

$$\sum F_x = ma_x \quad \Big| \quad \sum F_y = ma_y \quad \Big| \quad \sum M_G = I\alpha$$

Taking moment about non-G pt & equate:

FBD: $\sum M_A = F_1 r_{1/A}$

EFD: $\sum M_A = I\alpha + ma_x r_A + ma_y r_A$



Combine multiple bodies, and consider their separate $ma_x, ma_y, I\alpha$

Common sol: express a_x, a_y *in terms of* α *and solve* $\sum M_o$ *to get* α.

General Procedure

Kinematics$(a, v) \rightarrow$ *Kinetic*(FBD, EFD)*, separate and combined bodies.*

Rolling

Rolling no sliding*	$F < \mu_s N$
Rolling, sliding imp*^	$F = \mu_s N$
Rotating & Sliding	$F = \mu_k N$

$*\bar{v} = \omega_k \times r_{G/C} \rightarrow \bar{a} = \alpha_k \times r_{G/C}$

***WEP can be used! ^Max Friction Force**

If unknown, assume case 1. Solve for α *and a, check if* $F \le \mu_s N$*. If yes, use 1. Else, use 3 and recalculate.*

$$\frac{\omega}{\omega'} = -\frac{R}{r_{A/C}}$$

ω :motion of disc rolling along curved path is related to ω′ of disc

Along Straight Path: Path Contact Point

$$a_c = \omega^2 \times r_{A/C} \quad \Big| \quad v_c = 0$$

Work Energy Power

Kinetic Energy

$$T = \frac{1}{2}mv^2 + \frac{1}{2}I_G\omega^2$$

Rotating body about fixed point A (ICR A)

$$\text{T} = \frac{1}{2}I_A\omega^2 = \frac{1}{2}[I_G + mr_{A/G}{}^2]\omega^2$$

$$= \frac{1}{2}\text{m}|v_2|^2 + \frac{1}{2}I_G\omega^2$$

Spring*	GPE	Friction
$\frac{1}{2}k\delta^2$	mgh	$\mu_k Nd$

For solving vibration via forces: spring extension in equilibrium. $F = k(x_A + \delta)$ *and eliminate* x_A *by static analysis.*

Approximations

$$\sin\theta \approx \theta \quad \Big| \quad 1 - \cos\theta \approx \theta^2/2$$

Free Vibration without damping

EOM: $\ddot{u} + \omega_n{}^2 u = 0$ ($\omega_n$is Freq in rad/s)

Spring Mass	$\omega_n = \sqrt{\text{k/m}}$
Pendulum	$\omega_n = \sqrt{g/l}$
Free oscillation about O	$\omega_n = \sqrt{mgd/I_o}$

$$x = A\sin(\omega_n t + \phi)$$

$$\dot{x} = v = \omega_n A\cos(\omega_n(t) + \phi)$$

$$\tau_n = \frac{2\pi}{\omega_n} \quad \Big| \quad f_n(Hz) = \frac{\omega_n}{2\pi}$$

$$A = \sqrt{C_1^2 + C_2^2} = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}$$

$$\phi = \tan^{-1}\left(\frac{C_1}{C_2}\right) = \tan^{-1}\left(\frac{x_0\omega_n}{v_0}\right)$$

Note that x_o *and* v_o *conditions at* $t = 0$

Damped vibration

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$m\lambda^2 + c\lambda + k = 0$$

$$\lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$F_c[N] = c[Ns/m] \times v$$

Over: $\left(\frac{c}{2m}\right)^2 - \frac{k}{m} > 0$

$$x = A_1e^{\lambda_1 t} + A_2e^{\lambda_2 t}$$

Critical: $\left(\frac{c}{2m}\right)^2 - \frac{k}{m} = 0$

$$x = (A_1 + A_2t)e^{-\omega_n t}$$

$$c_{cr} = 2\sqrt{mk} = 2m\omega_n$$

Under: $\left(\frac{c}{2m}\right)^2 - \frac{k}{m} < 0$

Dashpot Damper: $F = kx + c\dot{x}$

Damping Ratio(c is damping coeff)

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}}$$

$\zeta > 1$	Over damped
$\zeta = 1$	Crict damped
$\zeta < 1$	Under damped

Stiffness Coeff: $k = m\omega_n^2$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

$$x = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

$$\dot{x} = Xe^{-\zeta\omega_n t}[-\zeta\omega_n\sin(\omega_d t) + \omega_d\cos(\omega_d t)]$$

Initial Conditions

$$X = \sqrt{C_1^2 + C_2^2} = \sqrt{x_0^2 + \left(\frac{v_0 + \zeta\omega_n x_o}{\omega_d}\right)^2}$$

$$\phi = \tan^{-1}\left(\frac{C_1}{C_2}\right) = \tan^{-1}\left(\frac{\omega_d x_o}{v_o + \zeta\omega_n x_o}\right)$$

Exponential Decaying Coefficient

$$\alpha = \frac{c}{2m} = \zeta\omega_n$$

<u>Damped Oscillation Freq</u>

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \omega_n\sqrt{1-\zeta^2}$$

<u>Logarithmic Decrement</u>

$$\delta = ln\left(\frac{x_1}{x_2}\right) = \zeta\omega_n\tau_d$$

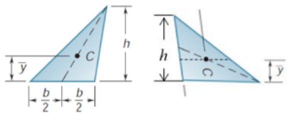
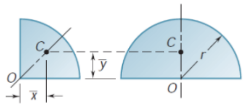
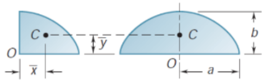
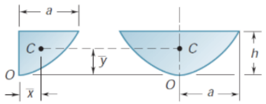
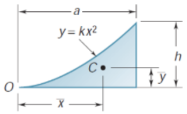
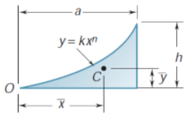
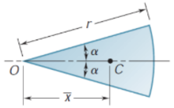
$$\delta = \frac{1}{N} ln\left(\frac{x_1}{x_{1+N}}\right)$$

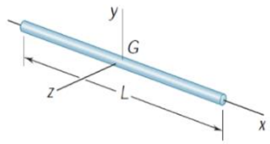
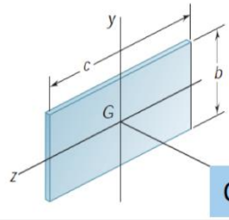
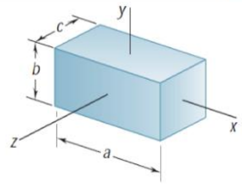
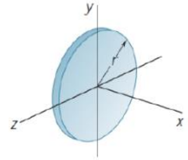
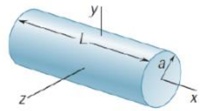
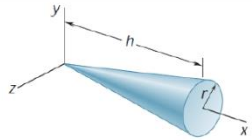
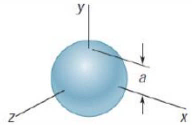
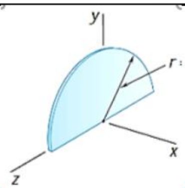
$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \quad \Bigg| \quad \zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$$

Order: $\delta \rightarrow \zeta \rightarrow \omega_n \rightarrow c_{cr} \rightarrow c \ OR \ k$

Cos Rule

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Slender rod		$I_y = I_z = \frac{1}{12}mL^2$ Q1: Why no I_x?
Thin rectangular plate		$I_x = \frac{1}{12}m(b^2+c^2)$ $I_y = \frac{1}{12}mc^2$ $I_z = \frac{1}{12}mb^2$ Q2: Why $I_x = I_y + I_z$?
Rectangular prism		$I_x = \frac{1}{12}m(b^2+c^2)$ $I_y = \frac{1}{12}m(c^2+a^2)$ $I_z = \frac{1}{12}m(a^2+b^2)$
Thin disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$
Circular cylinder		$I_x = \frac{1}{2}ma^2$ $I_y = I_z = \frac{1}{12}m(3a^2+L^2)$
Circular cone		$I_x = \frac{3}{10}ma^2$ $I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2+h^2)$
Sphere		$I_x = I_y = I_z = \frac{2}{5}ma^2$
Semicircular disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$

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