Properties of Fluids

Specific Weight

$$\gamma = \rho g$$

Specific Gravity

$$SG = \frac{\rho}{\rho_{H20}}$$

Shear Stress

$$\tau = \mu\frac{du}{dy} = \mu\dot{\beta}$$

$\dot{\beta}$: Shear strain rate

μ: Dynamic Viscosity

Rotational Surfaces(ref tut Qn)

$$P(watt) = T$$

$$T = \int \mu\frac{du}{dy} r\,dA = \mu\int\frac{r\omega}{y} r(2\pi r\,dr)$$

$$dA = \pi(r+dr)^2 - \pi r^2 = 2\pi r\,dr$$

Angled surfaces

Gravity contributes to shear force experienced by the liquid.

$$W\sin(\theta) = \tau A$$

Newtonian Fluids – *satisfies condition.*

$\tau = \mu\frac{du}{dy} + C$	μ *is constant* $C = 0$

Temperature ↑	Liquid Viscosity ↓
	Gas Viscosity ↓

Kinematic Viscosity

$$\upsilon = \frac{\mu}{\rho}$$

Surface Tension

$$\sigma = \frac{F}{L}$$

Pressure Difference

Droplets	$\triangle P = 2\sigma/R$
Bubbles	$\triangle P = 4\sigma/R$

Contact Angle

$\delta < 90°$	Rise, $h > 0$
$\delta > 90°$	Depr, $h < 0$

Liquid Columns

By forces on liquid column or interface.

$$\rho gh \times \pi R^2 = (2\pi R)\sigma\cos(\theta)$$

Fluid Statics

$P_{gage} = P_{abs} - P_{atm}$	$P_{vac} = P_{atm} - P_{abs}$

Pascal's Law

$$P_2 - P_1 = -\rho g(z_2 - z_1)$$

Plane Submerged Surface

*h_c and y_c are from liquid level.

$$h_c = y_c\sin$$

$$F_h = P_c A = (P_0 + \rho g h_c)A$$

$$y_p = y_c + \frac{I_{xx,C}}{y_c A}$$

Curved Submerged Surface

$$F_H = \rho g h_c A$$

A is the horizontal projection of curve.

$$F_V = \rho g V_{liquid}$$

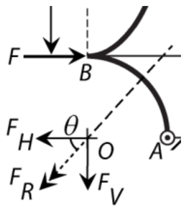
F_v is the weight of the liquid above and acts at x_c (centroid area) of the liquid.

$$F_H = F_R\cos\theta$$

$$F_V = F_R\sin\theta$$

$$F_R^2 = F_H^2 + F_V^2$$

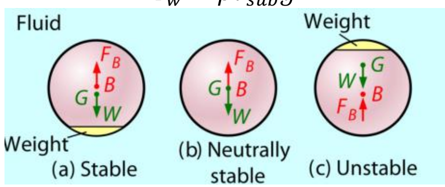
$$\tan\theta = \frac{F_V}{F_H}$$



For a radial surface, the resultant force passes through centre radius.

Stability

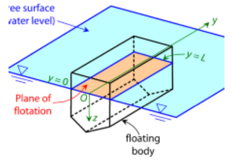
$$F_w = \rho V_{sub} g$$



GM > 0	Stable
GM < 0	Unstable
GM $= 0$	Neutral

$$GM = KB + BM - KG$$

$$BM = \frac{I_{Oy}}{V_{sub}}$$



Fluid Motion

$$Q(m^3/s) = AV = \int_A V\,dA$$

$$\dot{m}(\text{kg/s}) = \rho AV = \int_A \rho V\,dA = \rho Q$$

Incompressible Fluid: $A_1V_1 = A_2V_2$

Conservation of mass

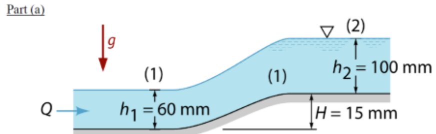
$$\sum_{in}\dot{m} - \sum_{out}\dot{m} = \frac{dm_{CV}}{dt}$$

Bernoulli - Along Streamlines

For **Inviscid** Flow

$$\frac{P_1}{\rho} + \frac{V_1{}^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2{}^2}{2} + gz_2$$

$$P_s - P_t = \triangle h_{H20}\rho_{H20}g - \triangle h_{air}\rho_{air}g$$



$$z_2 - z_1 = (h_2 - h_1 + H)mm$$

Bernoulli - Across Streamlines

$$\rho g\,dz + dP = -\rho\frac{V^2}{R}\,dn$$

Equate z and n with r as they are in the same direction. E.g $n = z = 6 - R$

Pressure Gradient $\frac{dP}{dn}\,dn$ is towards O, dr away from O. Direction affects polarity.

Rigid Body Motion

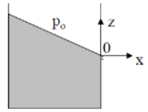
$$a = \frac{\triangle V}{\triangle t} = \frac{\delta V}{\delta t} + u\frac{\delta V}{\delta x} + v\frac{\delta V}{\delta y} + w\frac{\delta V}{\delta z}$$

Rectilinear Acceleration

$$dp = -\rho(a_x)dx - \rho(a_z + g)dz$$

Constant Acceleration

$$p = -[\rho a_x x + \rho(a_z + g)z] + p_o$$



In this case, $\theta < 0$

$$\tan\theta = -\frac{a_x}{a_z + g}$$

Cylindrical Rotation

$$z = \frac{1}{2g}\omega^2 r^2 + z_o$$

$$dp = \rho\omega^2 r dr - \rho g dz$$

$$p = p_o - \rho g(z - z_o) + \frac{1}{2}\rho\omega^2 r^2$$

$$\frac{dz}{dr} = \frac{r\omega^2}{g} = \tan(\alpha)$$

$$V_{parabolid/air} = \frac{1}{2}AH$$

Momentum Equation

$\overline{v_2}, \overline{v_1} > 0$ when in positive x-direction.

NOTE: Add $\boldsymbol{F} = \boldsymbol{PA}$ if $\boldsymbol{P} \neq \boldsymbol{P_{atm}}$

$$F = \frac{\delta(mv)}{\delta t} = \dot{m}(\overline{v_2} - \overline{v_1})$$

^Use if $\overline{v_2}$ is the same for all branches

$$F_{fluid} = \sum_{out}\dot{m}_{out}\,v_{out} - \sum_{in}\dot{m}_{in}\,v_{in}$$

$$F = \int \rho_2 u_2\overline{u_2}\,dA_2 - \int \rho_1 u_1\overline{u_1}\,dA_1$$

Relative Motion

$$F_x = \rho\alpha A(f(x))^2\cos\theta - \rho A(f(x))^2$$

U, V same dir.	$f(x) = V - U$
U, V diff dir.	$f(x) = V + U$

Angular Momentum

$$T = \rho Q[r_2V_{t2} - r_1V_{t1}]$$

$$V_{t2} = V_2 + 0.5$$

Add $v = r\omega$ to fluid v as absolute v is used.

Dimensional Analysis

Force	MLT^{-2}
Density	ML^{-3}
Velocity	LT^{-1}
Viscosity	$ML^{-1}T^{-1}$
ω OR RPM	T^{-1}
Torque	ML^2T^{-2}

Rayleigh's Method

Deal with all variables together

$$F = Kl^a h^b \rho^c \mu^d V^e$$

$$\left(MLT^{-2}\right) = (L)^a (L)^b \left(ML^{-3}\right)^c \left(ML^{-1}T^{-1}\right)^d \left(LT^{-1}\right)^e$$

$$F = Kl^{2-b-d} h^b \rho^{1-d} \mu^d V^{2-d}$$

$$\frac{F}{\rho V^2 l^2} = f\left(\frac{h}{l}, \frac{\mu}{\rho V l}\right)$$

Buckingham Pi

Given n var and m repeating variables: $(n-m)$ Π groups

$$a_1 = f_1(a_3, a_4, a_5 \dots)$$

$$0 = f_{10}(a_1, a_3, a_4, a_5 \dots)$$

$$\Pi_1 = D \rho^{a_1} U^{b_1} d^{c_1}$$

$$\Pi_2 = \mu \rho^{a_2} U^{b_2} d^{c_2}$$

$$\Pi_3 = h \rho^{a_3} U^{b_3} d^{c_3}$$

Solve for unknown variables

$$\Pi_1 = \frac{D}{\rho U^2 d^2} = f_1(\Pi_2, \Pi_3)$$

Note: All Π groups for model (scaled) and prototype (actual size) are equal.

Repeating Variables

1. Geometry (Length, Dia)
2. Fluid Property (Density, Viscosity)
3. Fluid Motion (Velocity, Flow, Pressure)

Similarity

Geometric, Kinematic, Dynamic

Dynamic Similarity

Reynold's Number $(Re)_m = (Re)_p$

$$Re = \frac{F_{Inertia}}{F_{Viscous}} = \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho L V}{\mu} = \frac{LV}{\nu}$$

Froude's Number $(Fr)_m = (Fr)_p$

For open channel/free surface flow.

$$Fr = \sqrt{\frac{F_{Inertia}}{F_{Gravity}}} = \sqrt{\frac{\rho L^2 V^2}{\rho L^3 g}} = \frac{V}{\sqrt{Lg}}$$

Pipe Flow

$$Re_d = \frac{\rho V d}{\mu} = \frac{Vd}{\nu}$$

Laminar	Turbulent
$Re_d < 2300$	$Re_d > 2300$
$\frac{L_e}{d} = 0.06Re_d$	$\frac{L_e}{d} = 4.4Re_d^{\frac{1}{6}}$

Common equations

Bernoulli – Energy Form

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$h_f = \frac{2\tau_w L}{\rho g R} = f\frac{\triangle L}{d}\frac{V^2}{2g}$$

$$h_f = -\frac{\triangle p}{\rho g}$$

Laminar Flow

Not affected by roughness of pipes

$$u = \frac{1}{4\mu}\left[-\frac{d}{dx}(p + \rho g z)\right]\left(R^2 - r^2\right)$$

$$f_{lam} = \frac{8\tau_w}{\rho V^2} = \frac{64}{Re_d}$$

$$h_{lam} = f_{lam}\frac{L}{d}\frac{V^2}{2g} = \frac{128\mu L Q}{\pi \rho g d^4}$$

$$-\triangle p = \rho g h_{lam} = \frac{128\mu L Q}{\pi d^4}$$

Non-circular pipes

$$D_H = \frac{4 \times Area}{Perimeter}$$

If $W \gg H$, $D_H = 2H$

Hence,

$$f = \frac{64}{Re_d} = \frac{64\nu}{V D_H}$$

Turbulent Flow(Smooth Pipe)

$$\frac{u}{u^*} = \frac{1}{0.4}ln\left(\frac{yu^*}{\nu}\right) + 5$$

$$f = 0.316Re_d^{-0.25}, 4000 < Re_d < 10^5$$

^Use for finding h_f and $-\triangle p$

$$\frac{1}{\sqrt{f}} = -1.8log\left(\frac{6.9}{Re_d}\right)$$

Turbulent Flow(Rough Pipe)

$R = \epsilon u^*/\nu$ $\quad$ $u^* = \sqrt{\tau_w/\rho}$

Smooth	$R < 5$
Rough	$R > 70$
Transitional	$5 < R < 70$

Velocity Profile

Smooth

$$\frac{u}{u^*} = \frac{1}{0.4}ln\left(\frac{yu^*}{\nu}\right) + 5$$

Rough

$$\frac{u}{u^*} = \frac{1}{0.4}ln\left(\frac{y}{\varepsilon}\right) + 8.5$$

Transitional

Like rough, but 8.5 is replaced with a value found from expt data.

Finding Friction Factor

Moody Diagram: *Calculate ϵ/D and use the highlighted line (right of diagram). Then, find f given Re.*

Completely Rough

$$\frac{1}{\sqrt{f}} = -2.0log\left(\frac{\varepsilon/d}{3.7}\right)$$

Else: Haalander

$$\frac{1}{\sqrt{f}} = -1.8log\left(\frac{6.9}{Re_d} + \left(\frac{\varepsilon/d}{3.7}\right)^{1.11}\right)$$

Minor Losses

$$h_L = K\frac{V^2}{2g} \qquad L_{eq} = K\frac{D}{f}$$

Sudden Enlargements

$$h_L = \frac{(v_1 - v_2)^2}{2g}$$

Since $A_1 < A_2, v1 > v2$

For exit into large tanks, $v_2 = 0$

Sudden Contraction

$$h_L = \frac{V_2^2}{2g}\left(\frac{A_2}{A_c} - 1\right)^2 = K_{sc}\frac{V_2^2}{2g}$$

A_c is unknown, K_{sc} is found via expt.

Entrance Loss: Square Edges

$$K = 0.5$$

Multiple Pipes

Series

$H_L = H_{L1} + H_{L2}$ $\quad$ $\boldsymbol{Q = Q_1 = Q_2}$

Parallel

$H_{L1} = H_{L2} = H_{L3}$ $\quad$ $\boldsymbol{Q = Q_1 + Q_2}$

Branched Pipes

At junction: $Q_{in} = Q_{out}$

Head Loss due to pipe

$$H_l = f\frac{L}{d}\frac{V^2}{2g}$$

Head at Tank

$$H_1 = z_1$$

Assumption that tanks are large (V_1 = V_2 = 0) and Equal Pressure ($P_1 = P_2 \approx P_{atm}$)