Numerical Analysis Assignment: LU Decomposition and Power Method

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Objective

This assignment involves applying LU Decomposition for matrix inversion and the Power Method for finding the largest eigenvalue and eigenvector of a symmetric, invertible 5×5 matrix. Use the following matrix for your analysis:

$$\begin{bmatrix} 4 & 1 & 2 & 3 & 5 \\ 1 & 3 & 1 & 4 & 2 \\ 2 & 1 & 5 & 2 & 3 \\ 3 & 4 & 2 & 4 & 1 \\ 5 & 2 & 3 & 1 & 5 \end{bmatrix}$$

Task 1: LU Decomposition

Theory

Briefly explain the concept of LU Decomposition and its significance in matrix inversion.

Implementation

Perform LU Decomposition on the given matrix to find its inverse. Validate the invertibility of the matrix before proceeding.

Verification

Verify the correctness of your inverted matrix by multiplying it with the original matrix. The product should result in an identity matrix.

Task 2: Power Method

Theory

Provide a theoretical background on the Power Method, focusing on its application in finding the largest eigenvalue and eigenvector of a matrix.

Implementation

Implement the Power Method using the same matrix. Document each step of your implementation, including the selection of the initial guess vector.

Analysis

Analyze the convergence of your method. How many iterations did it take to reach a satisfactory level of precision?

Deliverables

1. **Report**: Prepare a detailed report covering all aspects of the assignment. Include sections for theory, methodology, results, and analysis.
2. **Code**: Submit your code used for the LU Decomposition and Power Method. Ensure that your code is well-commented to explain each step of the process.

Evaluation Criteria

- **Accuracy** (40%): Correctness of the LU Decomposition and Power Method implementation.
- **Report Quality** (30%): Clarity, thoroughness, and organization of the report.
- **Code Quality** (20%): Readability and documentation of the code.
- **Extra effort and Originality** (10%)

Additional Notes

- Your report should not exceed 10 pages (excluding appendices and references).
- Use appropriate academic references to support your theoretical explanations.
- Your code should be executable and well-annotated for clarity.