

# LAB3-Bayes&Boosting Report

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## 1 Introduction

During the lab, we will overview the basics of bayesian learning and boosting. First, we will implement a bayes classifier based on the maximum likelihood estimation. Then, we will boost classifier to improve classification (using Adaboost algorithm). Finally, we will compare the decision tree classifier to the bayes classifier.

In terms of technical code we will learn importants things because we will use a ordinary environment. It means usage of Python langage, jupyter nootebook, famous machine learning datasets (iris, vowel and olivetti faces), and communs machine learning package/library (numpy, scipy, random, matplotlib and sklearn)

```
[ ]: import numpy as np
      from scipy import misc
      from imp import reload
      from labfuns import *
      import random

      import warnings
      warnings.filterwarnings('ignore')

      # fix random seed for the lab
      random.seed(100)
```

## 2 Bayes Classifier

### 2.0.1 Assignement 1

```
[ ]: # in:      X - N x d matrix of N data points
      #      labels - N vector of class labels
      # out:    mu - C x d matrix of class means (mu[i] - class i mean)
      #      sigma - C x d x d matrix of class covariances (sigma[i] - class i sigma)
      def mlParamsp1(X, labels, W=None):
          assert(X.shape[0]==labels.shape[0])
          Npts,Ndims = np.shape(X)
          classes = np.unique(labels)
          Nclasses = np.size(classes)
```

```

if W is None:
    W = np.ones((Npts,1))/float(Npts)

mu = np.zeros((Nclasses,Ndims))
sigma = np.zeros((Nclasses,Ndims,Ndims))

# =====
# Computing mu
Nkclasses = np.zeros(Nclasses)
for i_pts in range(Npts):
    index_class = np.where(classes == labels[i_pts])[0][0]
    Nkclasses[index_class] += 1.0
    for i_dim in range(Ndims):
        mu[index_class][i_dim] += X[i_pts][i_dim]
for i_class in range(Nclasses):
    for i_dim in range(Ndims):
        mu[i_class][i_dim] = mu[i_class][i_dim]/Nkclasses[i_class]
# Computing sigma
for i_pts in range(Npts):
    index_class = np.where(classes == labels[i_pts])[0][0]
    for i_dim in range(Ndims):
        sigma[index_class][i_dim][i_dim] += (X[i_pts][i_dim]-mu[index_class][i_dim]))**2
↪ (X[i_pts][i_dim]-mu[index_class][i_dim])**2
    for i_class in range(Nclasses):
        for i_dim in range(Ndims):
            sigma[i_class][i_dim][i_dim] = sigma[i_class][i_dim][i_dim]/
↪ Nkclasses[i_class]
# =====
return mu, sigma

```

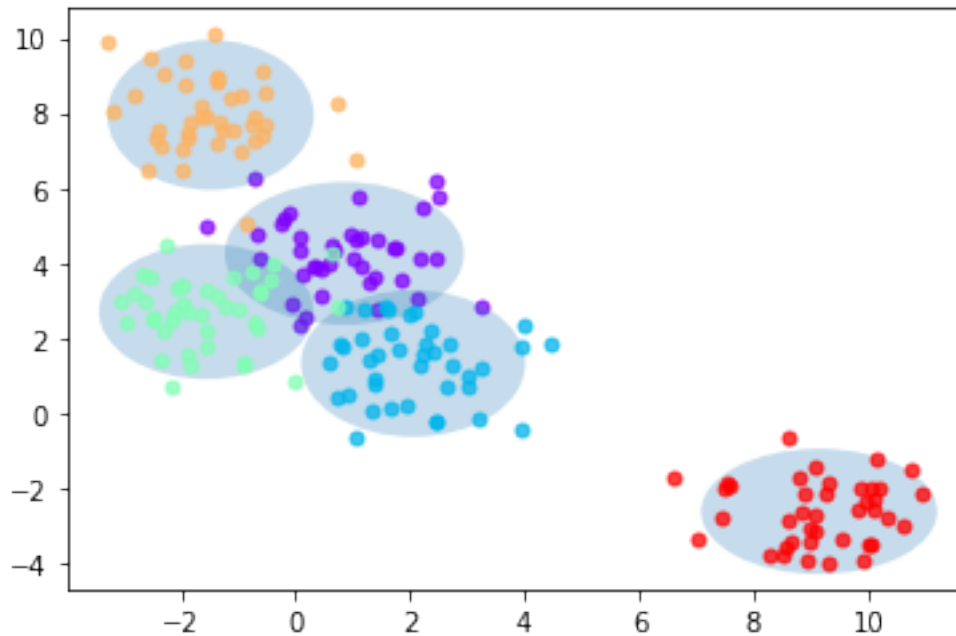
We want to prove the correctness of the computing of parameters for the multivariate Gaussian distribution :

```

[ ]: %matplotlib inline

X, labels = genBlobs(centers=5)
mu, sigma = mlParamsp1(X,labels)
plotGaussian(X,labels,mu,sigma)

```



## 2.0.2 Assignment 2

```
[ ]: # in: labels - N vector of class labels
# out: prior - C x 1 vector of class priors
def computePriorp1(labels, W=None):
    Npts = labels.shape[0]
    if W is None:
        W = np.ones((Npts,1))/Npts
    else:
        assert(W.shape[0] == Npts)
    classes = np.unique(labels)
    Nclasses = np.size(classes)

    prior = np.zeros((Nclasses,1))

    # =====
    # computing prior
    for i_label in range(Npts):
        index_class = np.where(classes == labels[i_label])[0][0]
        prior[index_class] +=1.0
    prior = prior/Npts
    # =====
    return prior
```

```
[ ]: # in:      X - N x d matrix of M data points
#      prior - C x 1 matrix of class priors
#      mu - C x d matrix of class means (mu[i] - class i mean)
#      sigma - C x d x d matrix of class covariances (sigma[i] - class i sigma)
# out:      h - N vector of class predictions for test points
def classifyBayesp1(X, prior, mu, sigma):

    Npts = X.shape[0]
    Nclasses, Ndims = np.shape(mu)
    logProb = np.zeros((Nclasses, Npts))

    # =====
    for i_class in range(Nclasses):
        for i_pts in range(Npts):
            discr = 1.0
            for i_dim in range(Ndims):
                discr = discr* sigma[i_class][i_dim][i_dim]
                logProb[i_class][i_pts] = -0.5*np.log(abs(discr))
                product = 0.0
                for i_dim in range(Ndims):
                    product += ((1.0/sigma[i_class][i_dim][i_dim]) *
↪((X[i_pts][i_dim]-mu[i_class][i_dim])**2))
                logProb[i_class][i_pts] += -0.5 * product
                logProb[i_class][i_pts] += np.log(prior[i_class])
            # =====

    # one possible way of finding max a-posteriori once
    # you have computed the log posterior
    h = np.argmax(logProb,axis=0)
    return h
```

### 2.0.3 Assignment 3

```
[ ]: class BayesClassifierp1(object):
    def __init__(self):
        self.trained = False

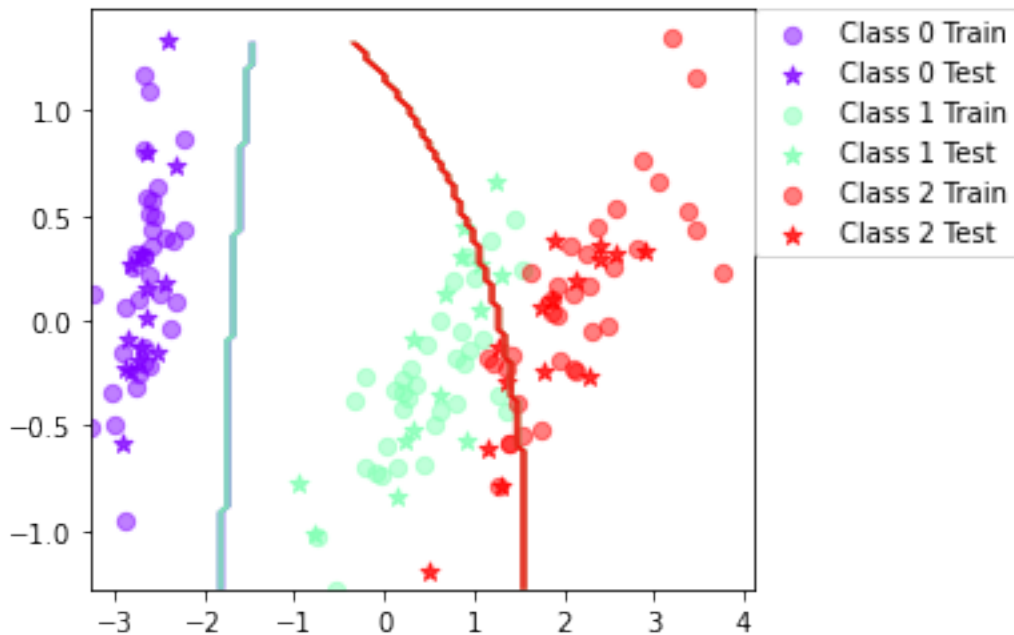
    def trainClassifier(self, X, labels, W=None):
        rtn = BayesClassifierp1()
        rtn.prior = computePriorp1(labels, W)
        rtn.mu, rtn.sigma = mlParamsp1(X, labels, W)
        rtn.trained = True
        return rtn

    def classify(self, X):
        return classifyBayesp1(X, self.prior, self.mu, self.sigma)
```

```
[ ]: testClassifier(BayesClassifierp1(), dataset='iris', split=0.7)
```

```
Trial: 0 Accuracy 84.4
Trial: 10 Accuracy 95.6
Trial: 20 Accuracy 93.3
Trial: 30 Accuracy 86.7
Trial: 40 Accuracy 88.9
Trial: 50 Accuracy 91.1
Trial: 60 Accuracy 86.7
Trial: 70 Accuracy 91.1
Trial: 80 Accuracy 86.7
Trial: 90 Accuracy 91.1
Final mean classification accuracy 89 with standard deviation 4.16
```

```
[ ]: %matplotlib inline
plotBoundary(BayesClassifierp1(), dataset='iris',split=0.7)
```



```
[ ]: testClassifier(BayesClassifierp1(), dataset='vowel', split=0.7)
```

```
Trial: 0 Accuracy 61
Trial: 10 Accuracy 66.2
Trial: 20 Accuracy 74
Trial: 30 Accuracy 66.9
Trial: 40 Accuracy 59.7
Trial: 50 Accuracy 64.3
Trial: 60 Accuracy 66.9
```

Trial: 70 Accuracy 63.6  
 Trial: 80 Accuracy 62.3  
 Trial: 90 Accuracy 70.8  
 Final mean classification accuracy 64.7 with standard deviation 4.03

When can a feature independence assumption be reasonable and when not?

The naive bayes classifier here is a model where features are supposed independent of each other, given the class label (Y). Before to create a model, it's possible to study the cross-dimensional links and see if the feature independence assumption is reasonable. A possible solution proposed by the initial lab subject is the usage of reduction techniques such as PCA (Principal Component Analysis) to get new features more independent.

How does the decision boundary look for the Iris dataset? How could one improve the classification results for this scenario by changing classifier or, alternatively, manipulating the data?

The decision boundary look great between the first class (class 0 on plot) and two others. On the contrary, the decision boundary dividing class 1 and 2 on the plot is curved and does not seem to be the best way of dividing these two classes.

## 3 Boosting

### 3.0.1 Assignement 4

```
[ ]: # in:      X - N x d matrix of N data points
#      labels - N vector of class labels
#      W - N vector of weight of data points
# out:      mu - C x d matrix of class means (mu[i] - class i mean)
#      sigma - C x d x d matrix of class covariances (sigma[i] - class i sigma)
def mlParams(X, labels, W=None):
    assert(X.shape[0]==labels.shape[0])
    Npts,Ndims = np.shape(X)
    classes = np.unique(labels)
    Nclasses = np.size(classes)

    if W is None:
        W = np.ones((Npts,1))/float(Npts)

    mu = np.zeros((Nclasses,Ndims))
    sigma = np.zeros((Nclasses,Ndims,Ndims))

    # =====
    # Computing mu
    Nkclasses = np.zeros(Nclasses)
    Wkclasses = np.zeros(Nclasses)
    for i_pts in range(Npts):
        index_class = np.where(classes == labels[i_pts])[0][0]
        Wkclasses[index_class] += W[i_pts]
```

```

        for i_dim in range(Ndims):
            mu[index_class][i_dim] += X[i_pts][i_dim]*W[i_pts]
    for i_class in range(Nclasses):
        for i_dim in range(Ndims):
            mu[i_class][i_dim] = mu[i_class][i_dim]/Wkclasses[i_class]
    # Computing sigma
    for i_pts in range(Npts):
        index_class = np.where(classes == labels[i_pts])[0][0]
        for i_dim in range(Ndims):
            sigma[index_class][i_dim][i_dim] +=
↪W[i_pts]*((X[i_pts][i_dim]-mu[index_class][i_dim])**2)
        for i_class in range(Nclasses):
            for i_dim in range(Ndims):
                sigma[i_class][i_dim][i_dim] = sigma[i_class][i_dim][i_dim]/
↪Wkclasses[i_class]
    # =====
    return mu, sigma

```

### 3.0.2 Assignement 5

```

[ ]: # in: labels - N vector of class labels
# out: prior - C x 1 vector of class priors
def computePrior(labels, W):
    Npts = labels.shape[0]
    if W is None:
        W = np.ones((Npts,1))/Npts
    else:
        assert(W.shape[0] == Npts)
    classes = np.unique(labels)
    Nclasses = np.size(classes)

    prior = np.zeros((Nclasses,1))

    # =====
    for i_pts in range(Npts):
        index_class = np.where(classes == labels[i_pts])[0][0]
        prior[index_class] += W[i_pts]
    # =====
    return prior/float(np.sum(prior))

```

```

[ ]: # in:      X - N x d matrix of M data points
#          prior - C x 1 matrix of class priors
#          mu - C x d matrix of class means (mu[i] - class i mean)
#          sigma - C x d x d matrix of class covariances (sigma[i] - class i sigma)
# out:      h - N vector of class predictions for test points
def classifyBayes(X, prior, mu, sigma):

```

```

Npts = X.shape[0]
Nclasses, Ndims = np.shape(mu)
logProb = np.zeros((Nclasses, Npts))

# =====
for i_class in range(Nclasses):
    for i_pts in range(Npts):
        discr = 1.0
        for i_dim in range(Ndims):
            discr = discr* sigma[i_class][i_dim][i_dim]
            logProb[i_class][i_pts] = -0.5*np.log(abs(discr))
            product = 0.0
        for i_dim in range(Ndims):
            product += ((1.0/sigma[i_class][i_dim][i_dim]) *
→((X[i_pts][i_dim]-mu[i_class][i_dim])**2))
            logProb[i_class][i_pts] += -0.5 * product
            logProb[i_class][i_pts] += np.log(prior[i_class])
# =====

# one possible way of finding max a-posteriori once
# you have computed the log posterior
h = np.argmax(logProb,axis=0)
return h

```

```

[ ]: class BayesClassifier(object):
    def __init__(self):
        self.trained = False

    def trainClassifier(self, X, labels, W):
        rtn = BayesClassifier()
        rtn.prior = computePrior(labels, W)
        rtn.mu, rtn.sigma = mlParams(X, labels, W)
        rtn.trained = True
        return rtn

    def classify(self, X):
        return classifyBayes(X, self.prior, self.mu, self.sigma)

```

```

[ ]: # in: base_classifier - a classifier of the type that we will boost, e.g.
→BayesClassifier
#
#           X - N x d matrix of N data points
#           labels - N vector of class labels
#           T - number of boosting iterations
# out:      classifiers - (maximum) length T Python list of trained classifiers
#           alphas - (maximum) length T Python list of vote weights
def trainBoost(base_classifier, X, labels, T=10):

```



```

# these will come in handy later on
Npts, Ndims = np.shape(X)

classifiers = [] # append new classifiers to this list
alphas = [] # append the vote weight of the classifiers to this list

# The weights for the first iteration
wCur = np.ones((Npts,1))/float(Npts)

for i_iter in range(0, T):
    # a new classifier can be trained like this, given the current weights
    classifiers.append(base_classifier.trainClassifier(X, labels, wCur))

    # do classification for each point
    vote = classifiers[-1].classify(X)

    # =====
    # priorsCur = computePrior(labels, wCur):
    eCur = 0.0
    for i_pts in range(Npts):
        hyp = (vote[i_pts] == labels[i_pts])
        eCur += (wCur[i_pts]*(1.0-(1.0 if hyp else 0.0)))
    alpha = 0.5*(np.log(1-eCur)-np.log(eCur))
    alphas.append(alpha) # you will need to append the new alpha
    # update weights
    for i_pts in range(Npts):
        hyp = (vote[i_pts] == labels[i_pts])
        wCur[i_pts] = (wCur[i_pts])*(np.exp(-alpha) if hyp else np.
→exp(alpha))
    normFactor = float(np.sum(wCur))
    wCur = wCur / normFactor
    # =====
    return classifiers, alphas

```

```

[ ]: # in:      X - N x d matrix of N data points
# classifiers - (maximum) length T Python list of trained classifiers as above
#      alphas - (maximum) length T Python list of vote weights
#      Nclasses - the number of different classes
# out:  yPred - N vector of class predictions for test points
def classifyBoost(X, classifiers, alphas, Nclasses):
    Npts = X.shape[0]
    Ncomps = len(classifiers)

    # if we only have one classifier, we may just classify directly
    if Ncomps == 1:
        return classifiers[0].classify(X)
    else:

```

```

votes = np.zeros((Npts,Nclasses))

# here we can do it by filling in the votes vector with weighted votes
# =====

hyp = np.zeros((Npts, Ncomps))
# labels = classifiers[0].classify(X)
for t in range(Ncomps):
    hypt = classifiers[t].classify(X)
    for i_pts in range(Npts):
        hyp[i_pts][t] = hypt[i_pts]

for i_pts in range(Npts):
    # classifiers[0].classify(X[i_pts])
    for i_class in range(Nclasses):
        sum = 0.0
        for i_Ncomps in range(Ncomps):
            # classifiers[i_Ncomps].classify(X[i_pts])
            h = hyp[i_pts][i_Ncomps] == i_class
            sum += alphas[i_Ncomps]* (1.0 if h else 0.0)
            # sum += alphas[i_Ncomps][0]* 1.0
        votes[i_pts][i_class] = sum
# =====

# one way to compute yPred after accumulating the votes
return np.argmax(votes,axis=1)

```

```

[ ]: class BoostClassifier(object):
    def __init__(self, base_classifier, T=10):
        self.base_classifier = base_classifier
        self.T = T
        self.trained = False

    def trainClassifier(self, X, labels):
        rtn = BoostClassifier(self.base_classifier, self.T)
        rtn.nbr_classes = np.size(np.unique(labels))
        rtn.classifiers, rtn.alphas = trainBoost(self.base_classifier, X,
↪labels, self.T)
        rtn.trained = True
        return rtn

    def classify(self, X):
        return classifyBoost(X, self.classifiers, self.alphas, self.nbr_classes)

[ ]: testClassifier(BoostClassifier(BayesClassifier(), T=10), dataset='iris',split=0.
↪7)

```

Trial: 0 Accuracy 95.6

```

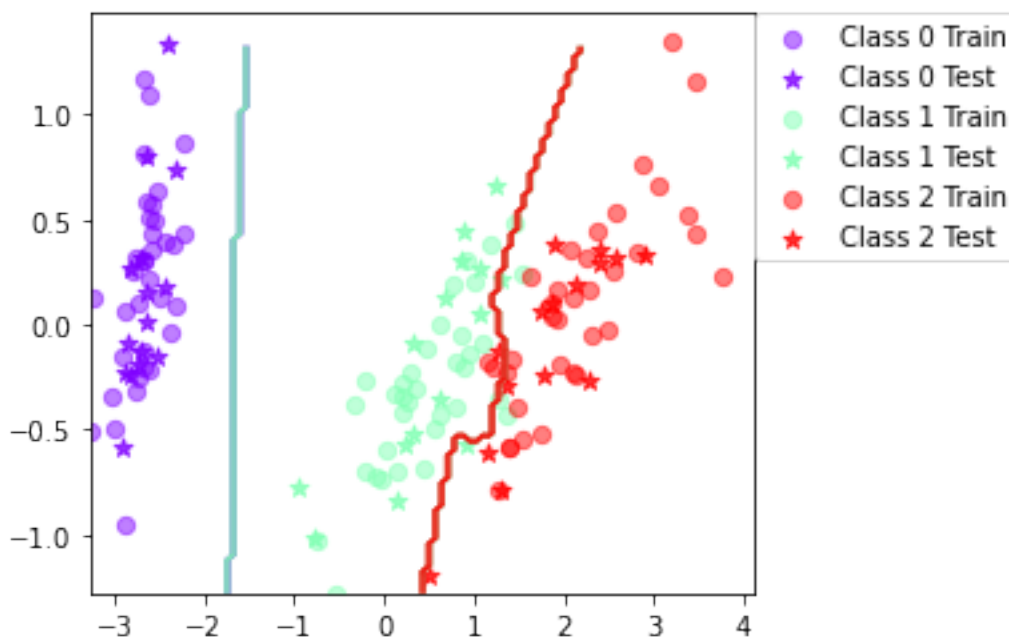
Trial: 10 Accuracy 100
Trial: 20 Accuracy 93.3
Trial: 30 Accuracy 91.1
Trial: 40 Accuracy 97.8
Trial: 50 Accuracy 93.3
Trial: 60 Accuracy 93.3
Trial: 70 Accuracy 97.8
Trial: 80 Accuracy 95.6
Trial: 90 Accuracy 93.3
Final mean classification accuracy 94.1 with standard deviation 6.72

```

```

[ ]: %matplotlib inline
plotBoundary(BoostClassifier(BayesClassifier()), dataset='iris',split=0.7)

```



```

[ ]: testClassifier(BoostClassifier(BayesClassifier(), T=10),
↳dataset='vowel',split=0.7)

```

```

Trial: 0 Accuracy 76.6
Trial: 10 Accuracy 86.4
Trial: 20 Accuracy 83.1
Trial: 30 Accuracy 80.5
Trial: 40 Accuracy 72.7
Trial: 50 Accuracy 76
Trial: 60 Accuracy 81.8
Trial: 70 Accuracy 82.5
Trial: 80 Accuracy 79.9

```

Trial: 90 Accuracy 83.1

Final mean classification accuracy 80.2 with standard deviation 3.52

Is there any improvement in classification accuracy? Why/why not?

For the iris dataset, the classification accuracy goes from 89.0 to 94.1 (for 100 trials), it's a little improvement. However the standard deviation reduces a little.

For the vowel dataset, the improvement is indegenable: classification accuracy goes from 64.7 to 80.2 for a better standard deviation. Probably that the higher number of classes, the not total independence of features fact that we have better results on the iris dataset than the vowel dataset.

Compare the decision boundary of the boosted classifier with the basic one. What differences do you notice? Is the boundary of the boosted version more complex?

The decision boundary (plot with 2-dim) looks more complex. The major difference is bypassing the boundary where many training points of two different classes seem to overlap a lot. Moreover, the global shape of the boundary changes because the 'main' curve does not appear to be rounded on the same side.

Can we make up for not using a more advanced model in the basic classifier (e.g. independent features) by using boosting?

In view of the results, of course depending on the situation, results of a basic classifier using boosting can be really efficient with independant features. We have very good results on both datasets. It only depends on which model you want...

## 4 Decision Tree

### 4.1 Decision Tree Classifier

```
[ ]: testClassifier(DecisionTreeClassifier(), dataset='iris', split=0.7)
```

Trial: 0 Accuracy 95.6

Trial: 10 Accuracy 100

Trial: 20 Accuracy 91.1

Trial: 30 Accuracy 91.1

Trial: 40 Accuracy 93.3

Trial: 50 Accuracy 91.1

Trial: 60 Accuracy 88.9

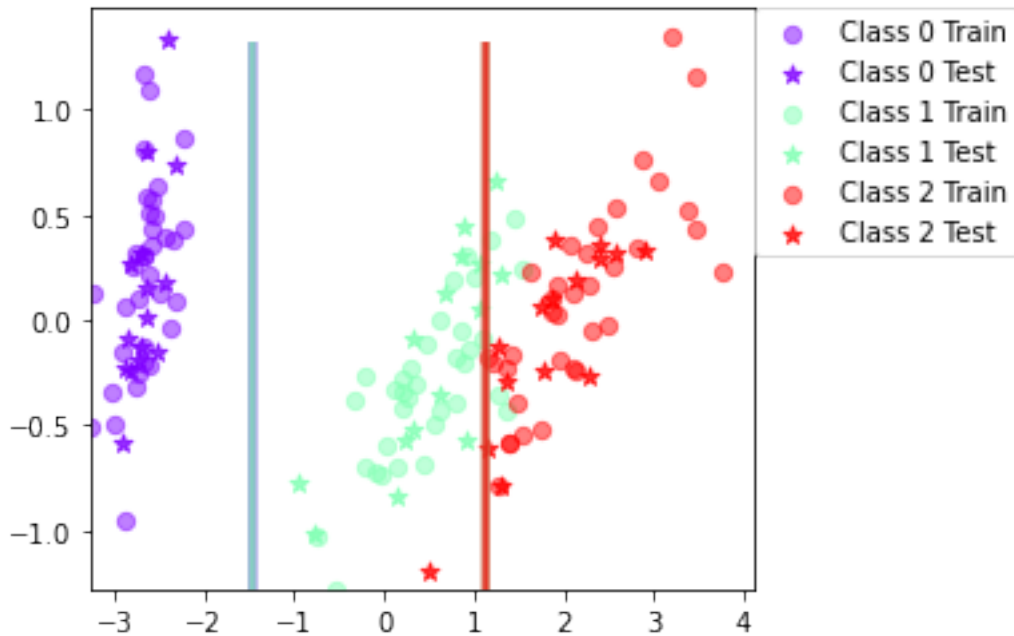
Trial: 70 Accuracy 88.9

Trial: 80 Accuracy 93.3

Trial: 90 Accuracy 88.9

Final mean classification accuracy 92.4 with standard deviation 3.71

```
[ ]: %matplotlib inline
plotBoundary(DecisionTreeClassifier(), dataset='iris', split=0.7)
```



```
[ ]: testClassifier(DecisionTreeClassifier(), dataset='vowel',split=0.7)
```

```
Trial: 0 Accuracy 63.6
Trial: 10 Accuracy 68.8
Trial: 20 Accuracy 63.6
Trial: 30 Accuracy 66.9
Trial: 40 Accuracy 59.7
Trial: 50 Accuracy 63
Trial: 60 Accuracy 59.7
Trial: 70 Accuracy 68.8
Trial: 80 Accuracy 59.7
Trial: 90 Accuracy 68.2
Final mean classification accuracy 64.1 with standard deviation 4
```

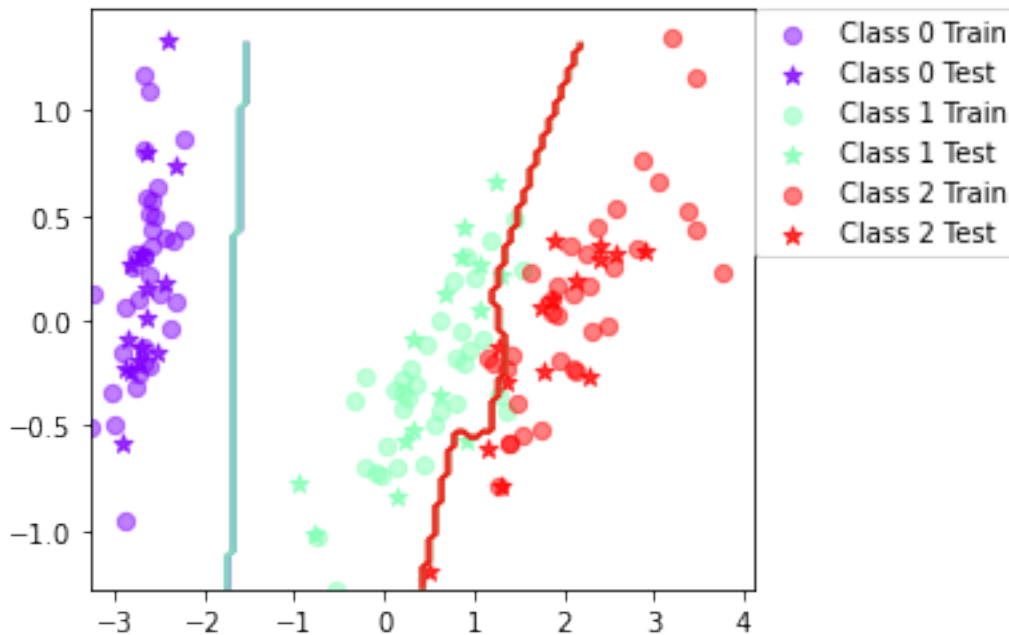
## 4.2 Decision Tree Classifier with boosting

```
[ ]: testClassifier(BoostClassifier(DecisionTreeClassifier(), T=10),
↳dataset='iris',split=0.7)
```

```
Trial: 0 Accuracy 95.6
Trial: 10 Accuracy 100
Trial: 20 Accuracy 95.6
Trial: 30 Accuracy 93.3
Trial: 40 Accuracy 93.3
Trial: 50 Accuracy 95.6
Trial: 60 Accuracy 88.9
```

Trial: 70 Accuracy 93.3  
 Trial: 80 Accuracy 93.3  
 Trial: 90 Accuracy 93.3  
 Final mean classification accuracy 94.6 with standard deviation 3.65

```
[ ]: %matplotlib inline
plotBoundary(BoostClassifier(BayesClassifier()), dataset='iris',split=0.7)
```



```
[ ]: testClassifier(BoostClassifier(BayesClassifier(), T=10),
↳dataset='vowel',split=0.7)
```

Trial: 0 Accuracy 76.6  
 Trial: 10 Accuracy 86.4  
 Trial: 20 Accuracy 83.1  
 Trial: 30 Accuracy 80.5  
 Trial: 40 Accuracy 72.7  
 Trial: 50 Accuracy 76  
 Trial: 60 Accuracy 81.8  
 Trial: 70 Accuracy 82.5  
 Trial: 80 Accuracy 79.9  
 Trial: 90 Accuracy 83.1  
 Final mean classification accuracy 80.2 with standard deviation 3.52

## 5 Conclusion

### 5.0.1 Assignment 7

If you had to pick a classifier, naive Bayes or a decision tree or the boosted versions of these, which one would you pick? Motivate from the following criteria:

- Outliers

I choose **decision tree** because naive bayes is more sensitive to outliers because it assumes independence between features and outliers significantly affect the estimation of probabilities (affect probabilities calculation). On the contrary, decision tree can still be sensitive to outliers but the impact is often less pronounced. Boosted versions of them can reduce impact of outliers by assigning more weight to misclassified instances (more robust). However, outliers would still have a bad effect on classification.

- Irrelevant inputs: part of the feature space is irrelevant

Probably that **decision tree** classifier is a better choice because they often prune irrelevant branches during training to reduce impact of them. On the contrary, naive bayes assumes independence between features and truly irrelevant features can still impact model predictions. Boosted versions of them can focus on informative features and reduce impact of irrelevant ones, making boosted models more robust to irrelevant inputs

- Predictive power

The biggest disadvantage of naive bayes is the assumption of feature independence. In some cases, the assumption is not really true. That's why a **decision tree** can be a more powerful predictive classifier.

- Mixed types of data: binary, categorical or continuous features, etc.

**Naive bayes** can be a good choice because can handles mixed types well including binary, categorical and continuous features. See One-hot encoding is useful for Machine Learning. Boosted versions of them improve the effectiveness of the model but are not especially related to types of data

- Scalability: the dimension of the data,  $D$ , is large or the number of instances,  $N$ , is large, or both.

A **naive Bayes** classifier is highly scalable with higher number of instances and with higher dimensions because the algorithm is simple and makes it computationally efficient. On the contrary, decision tree risk to increased computation with very deep trees. Of course, Boosted version of them are computationally more expensive.

Don't forget to use methods such as cross-validation and testing multiple algorithms on the dataset to determine the best-performing model. We don't have to be satisfied with a single test of our models to compare them

## 6 Voluntary Assignment

### 6.0.1 Test without boosting

```
[ ]: testClassifier(BayesClassifierp1(), dataset='olivetti', split=0.7)
```

```
Trial: 0 Accuracy 88.3
Trial: 10 Accuracy 90.8
Trial: 20 Accuracy 85
Trial: 30 Accuracy 89.2
Trial: 40 Accuracy 89.2
Trial: 50 Accuracy 84.2
Trial: 60 Accuracy 91.7
Trial: 70 Accuracy 82.5
Trial: 80 Accuracy 81.7
Trial: 90 Accuracy 86.7
Final mean classification accuracy 87.7 with standard deviation 3.03
```

Even without boosting, bayes classifier performs very well because on a dataset that seems fairly complex (representing images) mean of 87.7% (and 3.03 standard deviation) for correct classification is a really good score.

```
[ ]: testClassifier(DecisionTreeClassifier(), dataset='olivetti',split=0.7)
```

```
Trial: 0 Accuracy 65.8
Trial: 10 Accuracy 57.5
Trial: 20 Accuracy 49.2
Trial: 30 Accuracy 50
Trial: 40 Accuracy 53.3
Trial: 50 Accuracy 44.2
Trial: 60 Accuracy 49.2
Trial: 70 Accuracy 54.2
Trial: 80 Accuracy 50
Trial: 90 Accuracy 52.5
Final mean classification accuracy 48.4 with standard deviation 6.45
```

### 6.0.2 Test with boosting

As the subject precises it, the bayes classifier with boosting needs too many calculations to get a good model

```
[ ]: testClassifier(BoostClassifier(DecisionTreeClassifier(), T=10),  
    ↪dataset='olivetti',split=0.7)
```

```
Trial: 0 Accuracy 79.2
Trial: 10 Accuracy 65.8
Trial: 20 Accuracy 75.8
Trial: 30 Accuracy 71.7
Trial: 40 Accuracy 71.7
```



Trial: 50 Accuracy 63.3

Trial: 60 Accuracy 75.8

Trial: 70 Accuracy 55

Trial: 80 Accuracy 67.5

Trial: 90 Accuracy 71.7

Final mean classification accuracy 70.2 with standard deviation 6.35

This assignement treating the olivettifaces dataset prooves that a model who is very well adapted to the dataset we are working on is more important than usage of lots of technics to improve a model not suitable. Even with a higher value for the parameter T, the result with the boosted decision tree classifier will be less efficient than the naive bayes classifier.