LAB3-Bayes&Boosting Report

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1 Introduction

During the lab, we will overview the basics of bayesian learning and boosting. First, we will implement a bayes classifier based on the maximum likelihood estimation. Then, we will boost classifier to improve classification (using Adaboost algorithm). Finally, we will compare the decision tree classifier to the bayes classifier.

In terms of technical code we will learn importants things because we will use a ordinary environment. It means usage of Python langage, jupyter nootebook, famous machine learning datasets (iris, vowel and olivetti faces), and communs machine learning package/library (numpy, scipy, random, matplot and sklearn)

[ ]:
```
import numpy as np
from scipy import misc
from imp import reload
from labfuns import *
import random

# fix random seed for the lab
random.seed(100)
```

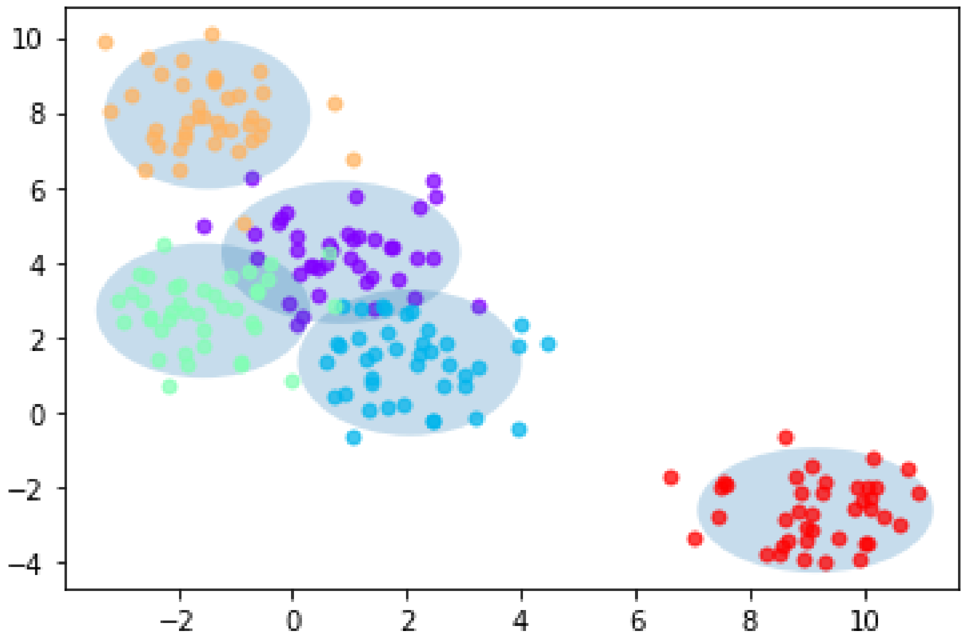
2 Bayes Classifier

2.0.1 Assignement 1

[ ]:
```
# in:      X - N x d matrix of N data points
#     labels - N vector of class labels
# out:    mu - C x d matrix of class means (mu[i] - class i mean)
#      sigma - C x d x d matrix of class covariances (sigma[i] - class i sigma)
def mlParamsp1(X, labels, W=None):
    assert(X.shape[0]==labels.shape[0])
    Npts,Ndims = np.shape(X)
    classes = np.unique(labels)
    Nclasses = np.size(classes)

    if W is None:
        W = np.ones((Npts,1))/float(Npts)
```

```
    mu = np.zeros((Nclasses,Ndims))
    sigma = np.zeros((Nclasses,Ndims,Ndims))

    # ==========================
    # Computing mu
    Nkclasses = np.zeros(Nclasses)
    for i_pts in range(Npts):
        index_class = np.where(classes == labels[i_pts])[0][0]
        Nkclasses[index_class] += 1.0
        for i_dim in range(Ndims):
            mu[index_class][i_dim] += X[i_pts][i_dim]
    for i_class in range(Nclasses):
        for i_dim in range(Ndims):
            mu[i_class][i_dim] = mu[i_class][i_dim]/Nkclasses[i_class]
    # Computing sigma
    for i_pts in range(Npts):
        index_class = np.where(classes == labels[i_pts])[0][0]
        for i_dim in range(Ndims):
            sigma[index_class][i_dim][i_dim] +=␣
↪(X[i_pts][i_dim]-mu[index_class][i_dim])**2
    for i_class in range(Nclasses):
        for i_dim in range(Ndims):
            sigma[i_class][i_dim][i_dim] = sigma[i_class][i_dim][i_dim]/
↪Nkclasses[i_class]
    # ==========================
    return mu, sigma
```

[ ]:
```
%matplotlib inline

X, labels = genBlobs(centers=5)
mu, sigma = mlParamsp1(X,labels)
plotGaussian(X,labels,mu,sigma)
```



2.0.2 Assignement 2

[ ]:
```
# in: labels - N vector of class labels
# out: prior - C x 1 vector of class priors
def computePriorp1(labels, W=None):
    Npts = labels.shape[0]
    if W is None:
        W = np.ones((Npts,1))/Npts
    else:
        assert(W.shape[0] == Npts)
    classes = np.unique(labels)
    Nclasses = np.size(classes)

    prior = np.zeros((Nclasses,1))

    # ==========================
    # computing prior
    for i_label in range(Npts):
        index_class = np.where(classes == labels[i_label])[0][0]
        prior[index_class] +=1.0
    prior = prior/Npts
    # ==========================
    return prior
```

```
[ ]: # in:      X - N x d matrix of M data points
     #      prior - C x 1 matrix of class priors
     #         mu - C x d matrix of class means (mu[i] - class i mean)
     #      sigma - C x d x d matrix of class covariances (sigma[i] - class i sigma)
     # out:     h - N vector of class predictions for test points
     def classifyBayes(X, prior, mu, sigma):

         Npts = X.shape[0]
         Nclasses,Ndims = np.shape(mu)
         logProb = np.zeros((Nclasses, Npts))

         # ==========================
         for i_class in range(Nclasses):
             for i_pts in range(Npts):
                 discr = 1.0
                 for i_dim in range(Ndims):
                     discr = discr* sigma[i_class][i_dim][i_dim]
                 logProb[i_class][i_pts] = -0.5*np.log(abs(discr))
                 product = 0.0
                 for i_dim in range(Ndims):
                     product += ((1.0/sigma[i_class][i_dim][i_dim]) *␣
      ↪((X[i_pts][i_dim]-mu[i_class][i_dim])**2))
                 logProb[i_class][i_pts] += -0.5 * product
                 logProb[i_class][i_pts] += np.log(prior[i_class])
         # ==========================

         # one possible way of finding max a-posteriori once
         # you have computed the log posterior
         h = np.argmax(logProb,axis=0)
         return h
```

2.0.3 Assignement 3

```
[ ]: class BayesClassifierp1(object):
         def __init__(self):
             self.trained = False

         def trainClassifier(self, X, labels, W=None):
             # W = np.ones(np.shape(labels))/np.shape(labels)[0] # Test with a␣
      ↪uniform weight vector with Wi = 1/N
             rtn = BayesClassifierp1()
             rtn.prior = computePriorp1(labels, W)
             rtn.mu, rtn.sigma = mlParamsp1(X, labels, W)
             rtn.trained = True
             return rtn

         def classify(self, X):
```

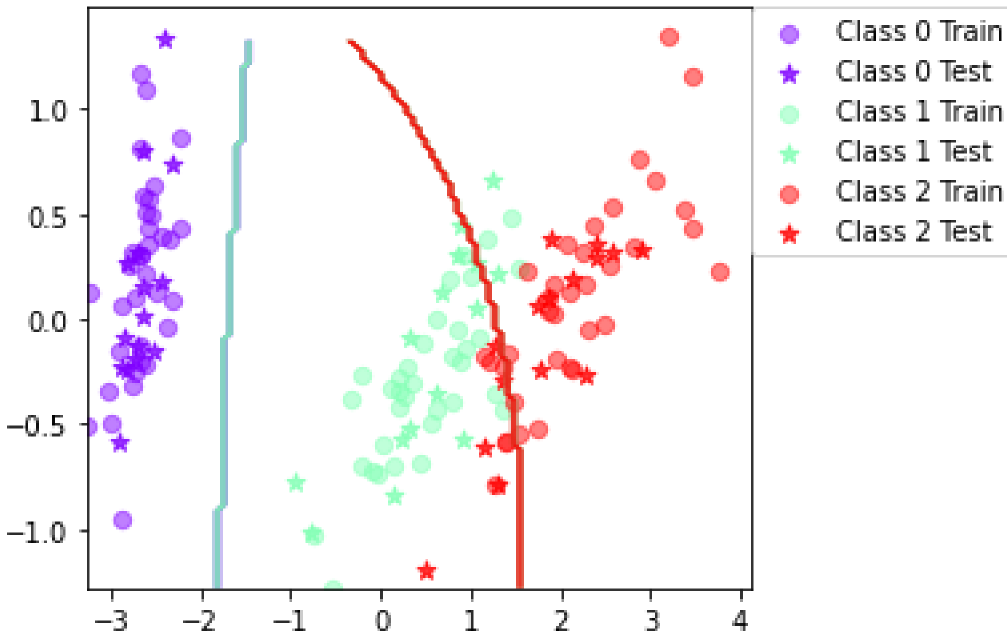
```
        return classifyBayes(X, self.prior, self.mu, self.sigma)
```

[ ]:
```
testClassifier(BayesClassifierp1(), dataset='iris', split=0.7)
```

```
Trial: 0 Accuracy 84.4
Trial: 10 Accuracy 95.6
Trial: 20 Accuracy 93.3
Trial: 30 Accuracy 86.7
Trial: 40 Accuracy 88.9
Trial: 50 Accuracy 91.1
Trial: 60 Accuracy 86.7
Trial: 70 Accuracy 91.1
Trial: 80 Accuracy 86.7
Trial: 90 Accuracy 91.1
Final mean classification accuracy  89 with standard deviation 4.16
```

[ ]:
```
%matplotlib inline
plotBoundary(BayesClassifierp1(), dataset='iris',split=0.7)
```

```
/mnt/c/Users/fland/Desktop/5IF-
KTH/DD2421-MACHINE_LEARNING/LAB3-bayes+boosting/labfuns.py:257: UserWarning: *c*
argument looks like a single numeric RGB or RGBA sequence, which should be
avoided as value-mapping will have precedence in case its length matches with
*x* & *y*.  Please use the *color* keyword-argument or provide a 2D array with a
single row if you intend to specify the same RGB or RGBA value for all points.
  plt.scatter(xTr[trClIdx,0],xTr[trClIdx,1],marker='o',c=color,s=40,alpha=0.5,
label="Class "+str(c)+" Train")
/mnt/c/Users/fland/Desktop/5IF-
KTH/DD2421-MACHINE_LEARNING/LAB3-bayes+boosting/labfuns.py:258: UserWarning: *c*
argument looks like a single numeric RGB or RGBA sequence, which should be
avoided as value-mapping will have precedence in case its length matches with
*x* & *y*.  Please use the *color* keyword-argument or provide a 2D array with a
single row if you intend to specify the same RGB or RGBA value for all points.
  plt.scatter(xTe[teClIdx,0],xTe[teClIdx,1],marker='*',c=color,s=50,alpha=0.8,
label="Class "+str(c)+" Test")
```



[ ]:
```
testClassifier(BayesClassifierp1(), dataset='vowel', split=0.7)
```

```
Trial: 0 Accuracy 61
Trial: 10 Accuracy 66.2
Trial: 20 Accuracy 74
Trial: 30 Accuracy 66.9
Trial: 40 Accuracy 59.7
Trial: 50 Accuracy 64.3
Trial: 60 Accuracy 66.9
Trial: 70 Accuracy 63.6
Trial: 80 Accuracy 62.3
Trial: 90 Accuracy 70.8
Final mean classification accuracy  64.7 with standard deviation 4.03
```

When can a feature independence assumption be reasonable and when not?

How does the decision boundary look for the Iris dataset? How could one improve the classification results for this scenario by changing classifier or, alternatively, manipulating the data?

3 Boosting

3.0.1 Assignement 4

[ ]:
```python
# in:       X - N x d matrix of N data points
#      labels - N vector of class labels
#           W - N vector of weight of data points
# out:     mu - C x d matrix of class means (mu[i] - class i mean)
#       sigma - C x d x d matrix of class covariances (sigma[i] - class i sigma)
def mlParams(X, labels, W=None):
    assert(X.shape[0]==labels.shape[0])
    Npts,Ndims = np.shape(X)
    classes = np.unique(labels)
    Nclasses = np.size(classes)

    if W is None:
        W = np.ones((Npts,1))/float(Npts)

    mu = np.zeros((Nclasses,Ndims))
    sigma = np.zeros((Nclasses,Ndims,Ndims))

    # ==========================
    # Computing mu
    Nkclasses = np.zeros(Nclasses)
    Wkclasses = np.zeros(Nclasses)
    for i_pts in range(Npts):
        index_class = np.where(classes == labels[i_pts])[0][0]
        Wkclasses[index_class] += W[i_pts]
        for i_dim in range(Ndims):
            mu[index_class][i_dim] += X[i_pts][i_dim]*W[i_pts]
    for i_class in range(Nclasses):
        for i_dim in range(Ndims):
            mu[i_class][i_dim] = mu[i_class][i_dim]/Wkclasses[i_class]
    # Computing sigma
    for i_pts in range(Npts):
        index_class = np.where(classes == labels[i_pts])[0][0]
        for i_dim in range(Ndims):
            sigma[index_class][i_dim][i_dim] +=␣
↪W[i_pts]*((X[i_pts][i_dim]-mu[index_class][i_dim])**2)
    for i_class in range(Nclasses):
        for i_dim in range(Ndims):
            sigma[i_class][i_dim][i_dim] = sigma[i_class][i_dim][i_dim]/
↪Wkclasses[i_class]
    # ==========================
    return mu, sigma
```

3.0.2 Assignement 5

[ ]:
```python
# in: labels - N vector of class labels
# out: prior - C x 1 vector of class priors
def computePrior(labels, W):
    Npts = labels.shape[0]
    if W is None:
        W = np.ones((Npts,1))/Npts
    else:
        assert(W.shape[0] == Npts)
    classes = np.unique(labels)
    Nclasses = np.size(classes)

    prior = np.zeros((Nclasses,1))

    # ==========================
    for i_pts in range(Npts):
        index_class = np.where(classes == labels[i_pts])[0][0]
        prior[index_class] += W[i_pts]
    # ==========================
    return prior/float(np.sum(prior))
```

[ ]:
```python
# in:      X - N x d matrix of N data points
#      prior - C x 1 matrix of class priors
#         mu - C x d matrix of class means (mu[i] - class i mean)
#      sigma - C x d x d matrix of class covariances (sigma[i] - class i sigma)
# out:     h - N vector of class predictions for test points
def classifyBayes(X, prior, mu, sigma):

    Npts = X.shape[0]
    Nclasses,Ndims = np.shape(mu)
    logProb = np.zeros((Nclasses, Npts))

    # ==========================
    for i_class in range(Nclasses):
        for i_pts in range(Npts):
            discr = 1.0
            for i_dim in range(Ndims):
                discr = discr* sigma[i_class][i_dim][i_dim]
            logProb[i_class][i_pts] = -0.5*np.log(abs(discr))
            product = 0.0
            for i_dim in range(Ndims):
                product += ((1.0/sigma[i_class][i_dim][i_dim]) *
 ↪((X[i_pts][i_dim]-mu[i_class][i_dim])**2))
            logProb[i_class][i_pts] += -0.5 * product
            logProb[i_class][i_pts] += np.log(prior[i_class])
    # ==========================
```

```python
    # one possible way of finding max a-posteriori once
    # you have computed the log posterior
    h = np.argmax
```

[ ]:
```python
class BayesClassifier(object):
    def __init__(self):
        self.trained = False

    def trainClassifier(self, X, labels, W):
        rtn = BayesClassifier()
        rtn.prior = computePrior(labels, W)
        rtn.mu, rtn.sigma = mlParams(X, labels, W)
        rtn.trained = True
        return rtn

    def classify(self, X):
        return classifyBayes(X, self.prior, self.mu, self.sigma)
```

[ ]:
```python
# in: base_classifier - a classifier of the type that we will boost, e.g.␣
 ↪BayesClassifier
#                   X - N x d matrix of N data points
#              labels - N vector of class labels
#                   T - number of boosting iterations
# out:    classifiers - (maximum) length T Python list of trained classifiers
#              alphas - (maximum) length T Python list of vote weights
def trainBoost(base_classifier, X, labels, T=10):
    # these will come in handy later on
    Npts,Ndims = np.shape(X)

    classifiers = [] # append new classifiers to this list
    alphas = [] # append the vote weight of the classifiers to this list

    # The weights for the first iteration
    wCur = np.ones((Npts,1))/float(Npts)

    for i_iter in range(0, T):
        # a new classifier can be trained like this, given the current weights
        classifiers.append(base_classifier.trainClassifier(X, labels, wCur))

        # do classification for each point
        vote = classifiers[-1].classify(X)

        # ==========================
        # priorsCur = computePrior(labels, wCur):
        eCur = 0.0
        for i_pts in range(Npts):
```

```
            hyp = (vote[i_pts] == labels[i_pts])
            eCur += (wCur[i_pts]*(1.0-(1.0 if hyp else 0.0)))
        alpha = 0.5*(np.log(1-eCur)-np.log(eCur))
        alphas.append(alpha) # you will need to append the new alpha
        # update weights
        for i_pts in range(Npts):
            hyp = (vote[i_pts] == labels[i_pts])
            wCur[i_pts] = (wCur[i_pts])*(np.exp(-alpha) if hyp else np.
↪exp(alpha))
        normFactor = float(np.sum(wCur))
        wCur = wCur / normFactor

        # poids de proba que ce soit de la class C
        # wCur = computePrior(labels, wCur)
        # eCur = 0.0
        # for i_pts in range(Npts):
        #     eCur =

        # ==========================
    return classifiers, alphas
```

[ ]:

```
# in:       X - N x d matrix of N data points
# classifiers - (maximum) length T Python list of trained classifiers as above
#      alphas - (maximum) length T Python list of vote weights
#    Nclasses - the number of different classes
# out:  yPred - N vector of class predictions for test points
def classifyBoost(X, classifiers, alphas, Nclasses):
    Npts = X.shape[0]
    Ncomps = len(classifiers)

    # if we only have one classifier, we may just classify directly
    if Ncomps == 1:
        return classifiers[0].classify(X)
    else:
        votes = np.zeros((Npts,Nclasses))

        # here we can do it by filling in the votes vector with weighted votes
        # ==========================
        hyp = np.zeros((Npts, Ncomps))
        # labels = classifiers[0].classify(X)
        for t in range(Ncomps):
            hypt = classifiers[t].classify(X)
            for i_pts in range(Npts):
                hyp[i_pts][t] = hypt[i_pts]

        for i_pts in range(Npts):
            # classifiers[0].classify(X[i_pts])
```

```
            for i_class in range(Nclasses):
                sum = 0.0
                for i_Ncomps in range(Ncomps):
                    # classifiers[i_Ncomps].classify(X[i_pts])
                    h = hyp[i_pts][i_Ncomps] == i_class
                    sum += alphas[i_Ncomps]* (1.0 if h else 0.0)
                    # sum += alphas[i_Ncomps][0]* 1.0
                votes[i_pts][i_class] = sum
        # ==========================

        # one way to compute yPred after accumulating the votes
        return np.argmax(votes,axis=1)
```

[ ]:
```
class BoostClassifier(object):
    def __init__(self, base_classifier, T=10):
        self.base_classifier = base_classifier
        self.T = T
        self.trained = False

    def trainClassifier(self, X, labels):
        rtn = BoostClassifier(self.base_classifier, self.T)
        rtn.nbr_classes = np.size(np.unique(labels))
        rtn.classifiers, rtn.alphas = trainBoost(self.base_classifier, X,␣
 ↪labels, self.T)
        rtn.trained = True
        return rtn

    def classify(self, X):
        return classifyBoost(X, self.classifiers, self.alphas, self.nbr_classes)
```

[ ]:
```
testClassifier(BoostClassifier(BayesClassifier(), T=10), dataset='iris',split=0.
 ↪7)
```

[ ]:
```
%matplotlib inline
plotBoundary(BoostClassifier(BayesClassifier()), dataset='iris',split=0.7)
```

[ ]:
```
testClassifier(BoostClassifier(BayesClassifier(), T=10),␣
 ↪dataset='vowel',split=0.7)
```

4 Decision Tree

4.1 Decision Tree Classifier

[ ]:
```
testClassifier(DecisionTreeClassifier(), dataset='iris', split=0.7)
```

[ ]:
```
%matplotlib inline
plotBoundary(DecisionTreeClassifier(), dataset='iris',split=0.7)
```

[ ]:
```
testClassifier(DecisionTreeClassifier(), dataset='vowel',split=0.7)
```

4.2 Decision Tree Classifier with boosting

[ ]:
```
testClassifier(BoostClassifier(DecisionTreeClassifier(), T=10),␣
↪dataset='iris',split=0.7)
```

[ ]:
```
%matplotlib inline
plotBoundary(BoostClassifier(BayesClassifier()), dataset='iris',split=0.7)
```

[ ]:
```
testClassifier(BoostClassifier(BayesClassifier(), T=10),␣
↪dataset='vowel',split=0.7)
```

5 Conclusion

5.0.1 Assignement 7

If you had to pick a classifier, naive Bayes or a decision tree or the boosted versions of these, which one would you pick? Motivate from the following criteria:

- Outliers

aaaa

- Irrelevant inputs: part of the feature space is irrelevant

aa

- Predictive power

aa

- Mixed types of data: binary, categorical or continuous features, etc.

aa

- Scalability: the dimension of the data, D, is large or the number of instances, N, is large, or both.

aa