Royal Institute of Technology

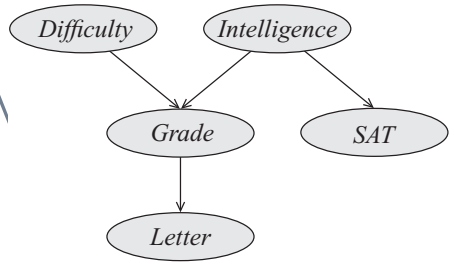
DGM FAMILY-WISE EXAMPLE

ONE OR SEVERAL DATA POINTS

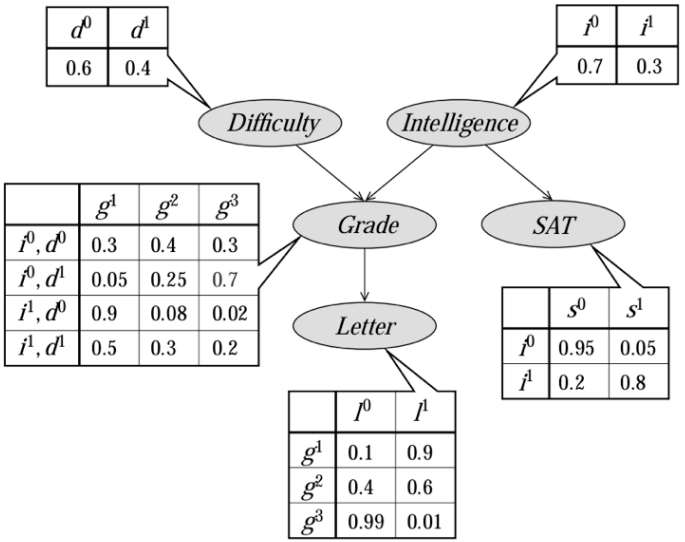
$$p(x \mid \theta) = \prod_{v=1}^{V} p(x_v \mid x_{\mathrm{pa}(v)}, \theta_v)$$

$$p(\mathcal{D} \mid \theta) = \prod_{n=1}^{N} p(x^n \mid \theta) = \prod_{n=1}^{N} \prod_{v=1}^{V} p(x_v^n \mid x_{\mathrm{pa}(v)}^n, \theta_v)$$

FACTORIZATION - A BINARY EXAMPLE

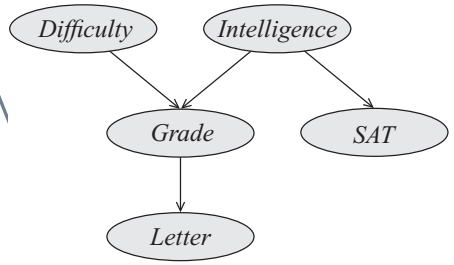


D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1



Given data and GM with CPDs (new CPDs on a need to know basis)

FACTORIZATION - AN EXAMPLE

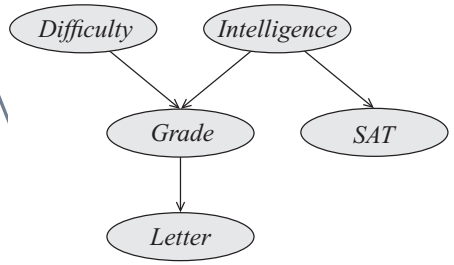


Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta};\mathcal{D}) = p(0,1,1,1,1|\boldsymbol{\theta})p(1,1,1,0,0|\boldsymbol{\theta}) \\ p(1,1,0,0,1|\boldsymbol{\theta})p(1,0,0,0,0|\boldsymbol{\theta}) \\ p(1,1,0,0,1|\boldsymbol{\theta})$$

FACTORIZATION - AN EXAMPLE



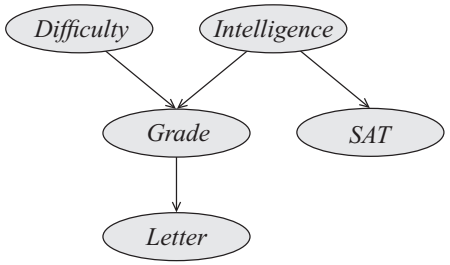
Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta};\mathcal{D}) = p(0,1,1,1,1|\boldsymbol{\theta})p(1,1,1,0,0|\boldsymbol{\theta}) \\ p(1,1,0,0,1|\boldsymbol{\theta})p(1,0,0,0,0|\boldsymbol{\theta}) \\ p(1,1,0,0,1|\boldsymbol{\theta})$$

$$L(\boldsymbol{\theta};\mathcal{D}) = p(D=(0,1,1,1,1)|\boldsymbol{\theta}_D) \\ p(I=(1,1,1,0,1)|\boldsymbol{\theta}_I) \\ p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S) \\ p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G) \\ p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

AN EXAMPLE



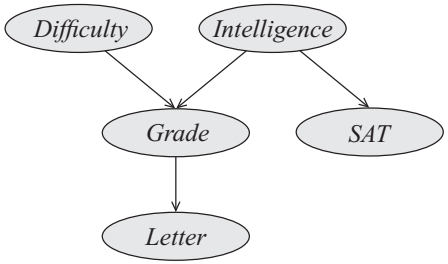
Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_D	D=0	D=1
	2/5	3/5

$$
\begin{aligned}
L(\boldsymbol{\theta};\mathcal{D}) = & \, p(D=(0,1,1,1,1)|\boldsymbol{\theta}_D) \\
& p(I=(1,1,1,0,1)|\boldsymbol{\theta}_I) \\
& p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S) \\
& p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G) \\
& p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)
\end{aligned}
$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

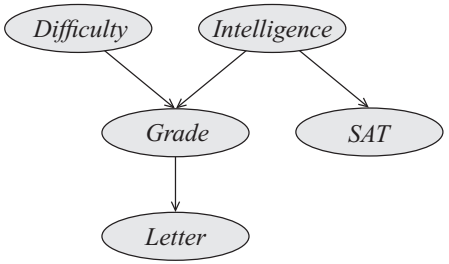
D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_D	D=0	D=1
	2/5	3/5

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4$$

$$p(I=(1,1,1,0,1)|\boldsymbol{\theta}_I)$$
$$p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S)$$
$$p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G)$$
$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

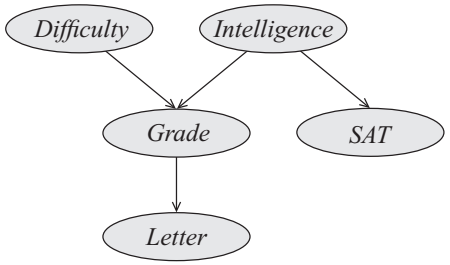
D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_I	I=0	I=1
	1/4	3/4

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4$$

$$p(I=(1,1,1,0,1)|\boldsymbol{\theta}_I)$$
$$p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S)$$
$$p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G)$$
$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

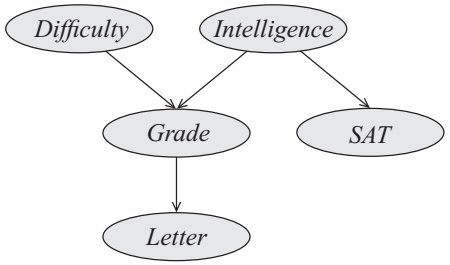
D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_I	I=0	I=1
	1/4	3/4

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4$$

$$p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S)$$
$$p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G)$$
$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

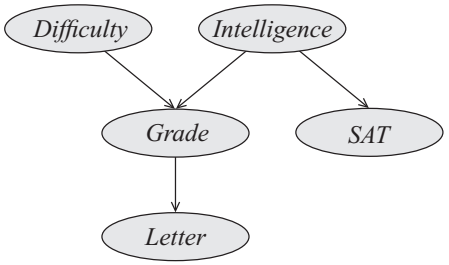
D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_S	S=0	S=1
I=0	1	0
I=1	1/6	5/6

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4$$

$$p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S)$$
$$p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G)$$
$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

AN EXAMPLE



Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_S	S=0	S=1
I=0	1	0
I=1	1/6	5/6

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G)$$

$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

	Less	Better
θ_G	G=0	G=1
D=0, I=0	1/2	1/2
D=1, I=0	3/5	2/5
D=0, I=1	1/10	9/10
D=1, I=1	2/5	3/5

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$p(G=(1,0,0,0,0)|D=(0,1,1,1,1), I=(1,1,1,0,1), \boldsymbol{\theta}_G)$$

$$p(L=(1,0,1,0,1)|G=(1,0,0,0,0), \boldsymbol{\theta}_L)$$

AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_G	G=0	G=1
D=0, I=0	1/2	1/2
D=1, I=0	3/5	2/5
D=0, I=1	1/10	9/10
D=1, I=1	2/5	3/5

$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}$$

$$p(L = (1,0,1,0,1) | G = (1,0,0,0,0), \boldsymbol{\theta}_L)$$

AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_L	L=0	L=1
G=0	2/3	1/3
G=1	0	1

$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}$$

$$p(L = (1,0,1,0,1) | G = (1,0,0,0,0), \boldsymbol{\theta}_L)$$

AN EXAMPLE

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_L	L=0	L=1
G=0	2/3	1/3
G=1	0	1

$$L(\boldsymbol{\theta};\mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$$

FACTORIZATION - AN EXAMPLE

"Row wise - per data point"

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

$$L(\boldsymbol{\theta};\mathcal{D}) = p(0,1,1,1,1|\boldsymbol{\theta})p(1,1,1,0,0|\boldsymbol{\theta}) \\ p(1,1,0,0,1|\boldsymbol{\theta})p(1,0,0,0,0|\boldsymbol{\theta}) \\ p(1,1,0,0,1|\boldsymbol{\theta})$$

"Column wise - per random variable - per family"

$$L(\boldsymbol{\theta};\mathcal{D}) = p(D=(0,1,1,1,1)|\boldsymbol{\theta}_D) \\ p(I=(1,1,1,0,1)|\boldsymbol{\theta}_I) \\ p(S=(1,1,0,0,0)|I=(1,1,1,0,1),\boldsymbol{\theta}_S) \\ p(G=(1,0,0,0,0)|D=(0,1,1,1,1),I=(1,1,1,0,1),\boldsymbol{\theta}_G) \\ p(L=(1,0,1,0,1)|G=(1,0,0,0,0),\boldsymbol{\theta}_L)$$

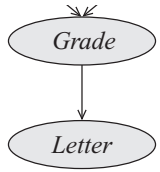
CAN WE GET MLE?

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_L	L=0	L=1
G=0	2/3	1/3
G=1	0	1

$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$$



CAN WE GET MLE?

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_L	L=0	L=1
G=0	?	?
G=1	?	?

$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$$

CAN WE GET MLE?

Difficulty (1 diff), Intelligence (1 int), Sat (1 good), Grade (1 good), Letter (1good)

D	I	S	G	L
0	1	1	1	1
1	1	1	0	0
1	1	0	0	1
1	0	0	0	0
1	1	0	0	1

θ_L	L=0	L=1
G=0	1/2	1/2
G=1	0	1

MLE!

$$L(\boldsymbol{\theta}; \mathcal{D}) = \frac{2}{5}\left(\frac{3}{5}\right)^4 \frac{1}{4}\left(\frac{3}{4}\right)^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \frac{9}{10}\left(\frac{2}{5}\right)^3 \frac{3}{5}\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$$

ONE OR SEVERAL DATA POINTS

$$p(x \mid \theta) = \prod_{v=1}^{V} p(x_v \mid x_{\mathrm{pa}(v)}, \theta_v)$$

$$p(\mathscr{D} \mid \theta) = \prod_{n=1}^{N} p(x^n \mid \theta) = \prod_{n=1}^{N} \prod_{v=1}^{V} p(x_v^n \mid x_{\mathrm{pa}(v)}^n, \theta_v)$$
$$= \prod_{v=1}^{V} \prod_{n=1}^{N} p(x_v^n \mid x_{\mathrm{pa}(v)}^n, \theta_v) = \prod_{v=1}^{V} p(\mathscr{D}_v \mid \mathscr{D}_{\mathrm{pa}(v)}, \theta_v)$$