DD2434/FDD3434 Machine Learning, Advanced Course
Module 1 Exercise

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1 Bayesian statistics – Theory

When conducting probabilistic modeling, we usually specify a model for how the data was generated. We denote the parameters of this model as Θ. In Bayesian statistics, we assume a prior distribution $p(\Theta)$ and infer the posterior $p(\Theta|X)$ through Bayes' theorem:

$$p(\Theta|X) = \frac{p(X|\Theta)p(\Theta)}{p(X)} \tag{1}$$

where $p(X|\Theta)$ is referred to as the likelihood function, $p(\Theta)$ the prior and $p(X)$ the evidence or marginal likelihood.

2 Conjugate priors – Exercises

1.1: Let $X = (X_1, .., X_N)$ be i.i.d. where $X_n|P, m \sim Binomial(m, P)$ and $P \sim Beta(\alpha, \beta)$. Show that the posterior $p(P|X, m)$ follows a Beta-distribution, i.e. that the Beta is conjugate prior to the Binomial with known m. What are the parameters of the posterior? Compare with the Wikipedia Conjugate prior table.

1.2: Let $D = (d_1, ..., d_N)$ be i.i.d. with $d_n|\Lambda \sim Poisson(\Lambda)$ and $\Lambda \sim Gamma(\alpha, \beta)$. Show that the posterior $p(\Lambda|D)$ follows a Gamma-distribution, i.e. that the Gamma is conjugate prior to the Poisson distribution. What are the parameters of the posterior? Compare with the Wikipedia Conjugate prior table.

1.3: Let $X = (X_1, ..., X_N)$ be i.i.d. with $X_n|\mu, \tau \sim Normal(\mu, \frac{1}{\tau})$ and $(\mu, \tau) \sim NormalGamma(\mu_0, \lambda, \alpha, \beta)$. Show that the posterior $p(\mu, \tau|X)$ follows a NormalGamma-distribution, i.e. that the NormalGamma is conjugate prior to the Normal distribution with unknown mean and precision. What are the parameters of the posterior? Compare with the Wikipedia Conjugate prior table.