

PPT 9

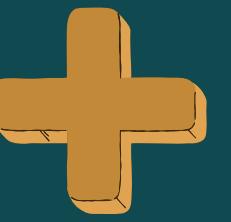
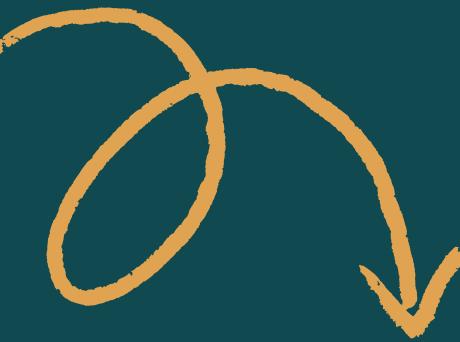
# Persamaan Parametrik



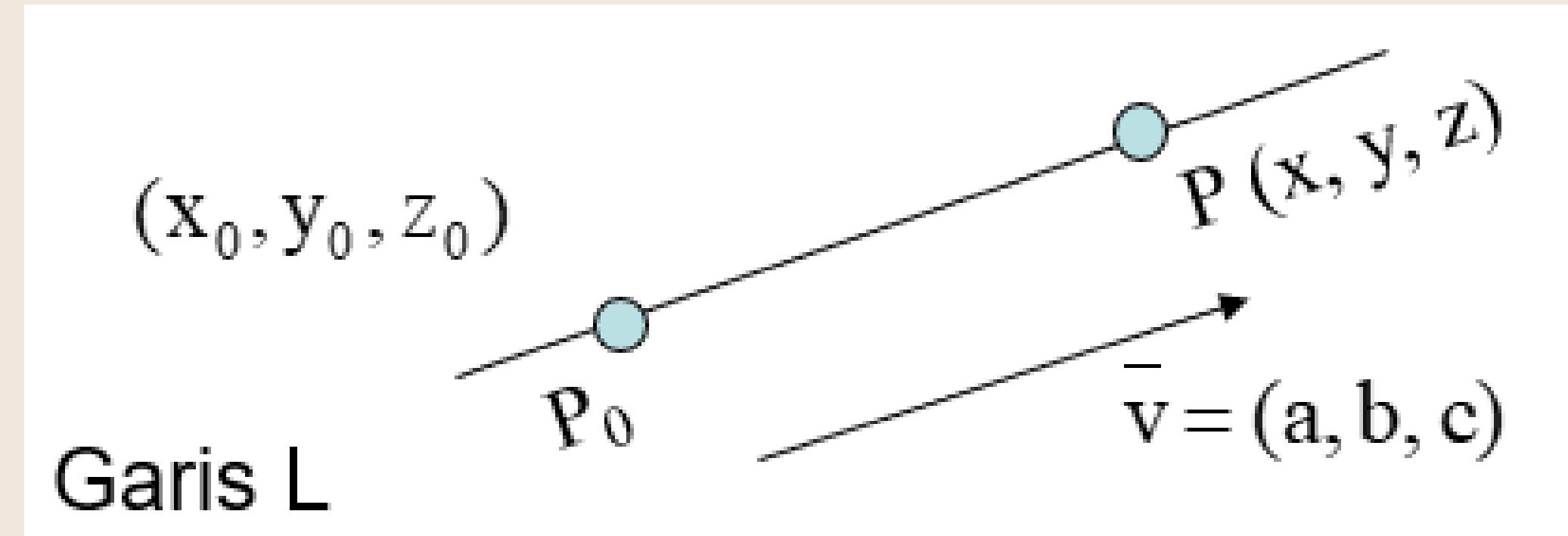
# CONTENT:

- Menyelesaikan Persamaan Parametrik
- Menyelesaikan Persamaan Simetrik
- Menentukan jarak titik ke bidang
- Transformasi Linier

# Persamaan Parametrik



# Persamaan Parametrik



Sehingga, Persamaan  
Parametrik untuk garis  
L dengan koor titik P

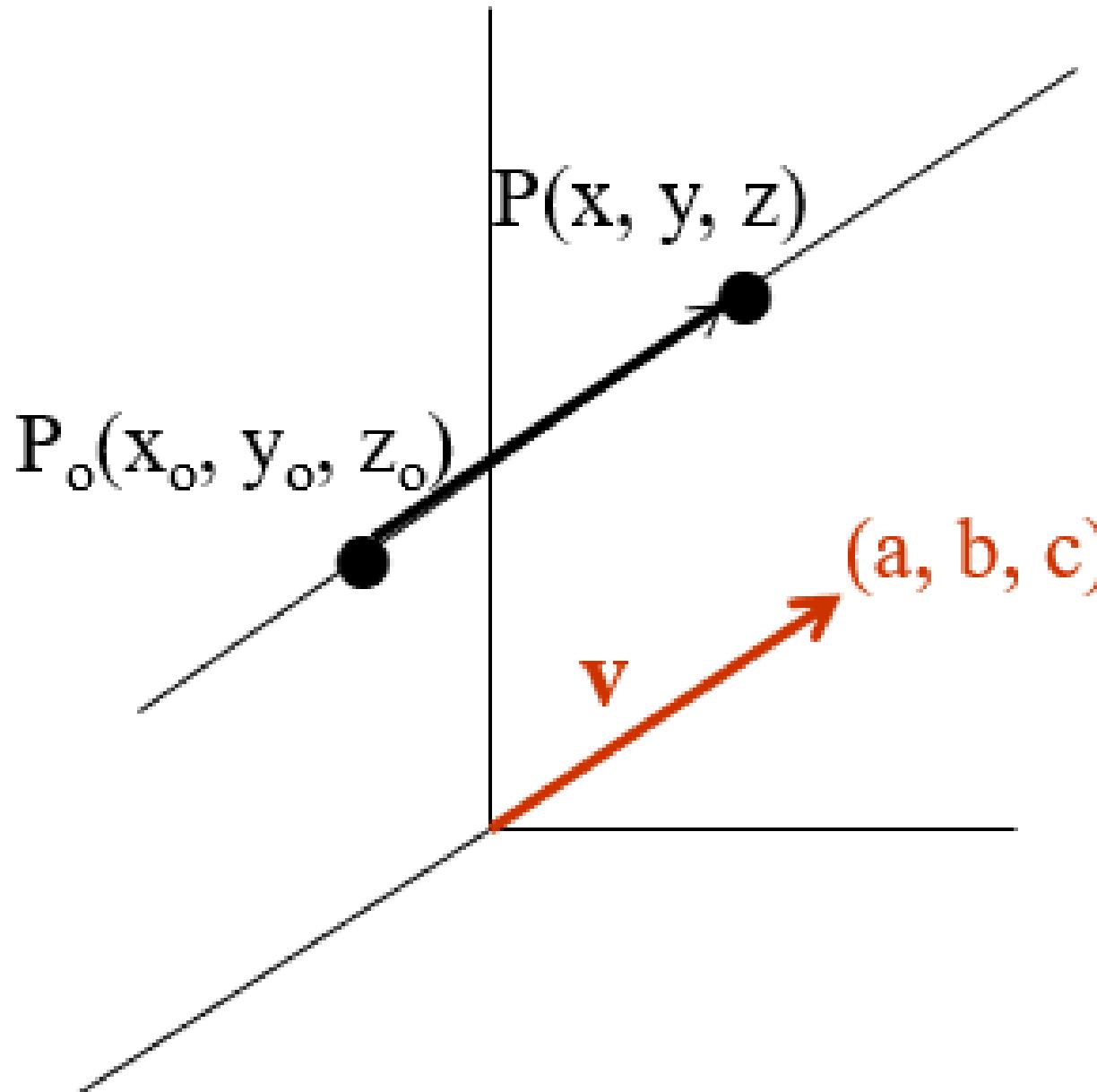
$$\overline{P_0P} = t \cdot \vec{v}$$
$$(x - x_0, y - y_0, z - z_0) = t \cdot (a, b, c)$$
$$= (\underline{t.a}, \underline{t.b}, \underline{t.c})$$

$$x = ta + x_0$$

$$y = \underline{tb} + y_0$$

$$z = \underline{tc} + z_0$$

# Persamaan Parametrik Garis Lurus



Diketahui vektor  $\overrightarrow{P_0P}$  // vektor  $V$  (sejajar), sehingga:

$$\overrightarrow{P_0P} = (x - x_0, y - y_0, z - z_0)$$

$$\overrightarrow{P_0P} = t\mathbf{v} \quad (t \text{ skalar})$$

$$(x - x_0, y - y_0, z - z_0) = t(a, b, c)$$

- $x - x_0 = t \cdot a$
- $y - y_0 = t \cdot b$
- $z - z_0 = t \cdot c$

## contoh 20

Cari Pers.

Parametrik utk  
garis  $\ell$  yang  
melalui titik  
 $P_1(2, 4, -1)$  dan  
 $P_2(5, 0, 7)$

### JAWAB

$$\bar{v} = \overrightarrow{P_1 P_2} = (3, -4, 8)$$

mt  $P_1$  terletak pd  $\ell$

$\Rightarrow P_0 = (2, 4, -1)$

$\Rightarrow \bar{v} = (3, -4, 8)$

$\Rightarrow$

$$\begin{aligned}x &= 2 + 3t \\y &= 4 - 4t \\t &= -1 + 3t\end{aligned}$$

# Pembuktian 1

$$t = 1 \begin{cases} x = 5 \\ y = 0 \\ z = 7 \end{cases} \quad P(5, 0, 7)$$

$$\overline{P_0.P} = t\overline{V}$$

$$(x - x_0, y - y_0, z - z_0) = t(a, b, c)$$

$$(5 - 2, 0 - 4, 7 + 1) = 1(3, -4, 8)$$

terbukti

# Pembuktian 2

$$\overline{P_0.P} = t\overline{V}$$

karena  $t = 1$  maka  $\overline{P_0.P} = \overline{V}$

$$\overline{P_0.P} = \begin{pmatrix} 5 \\ 0 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 8 \end{pmatrix}$$

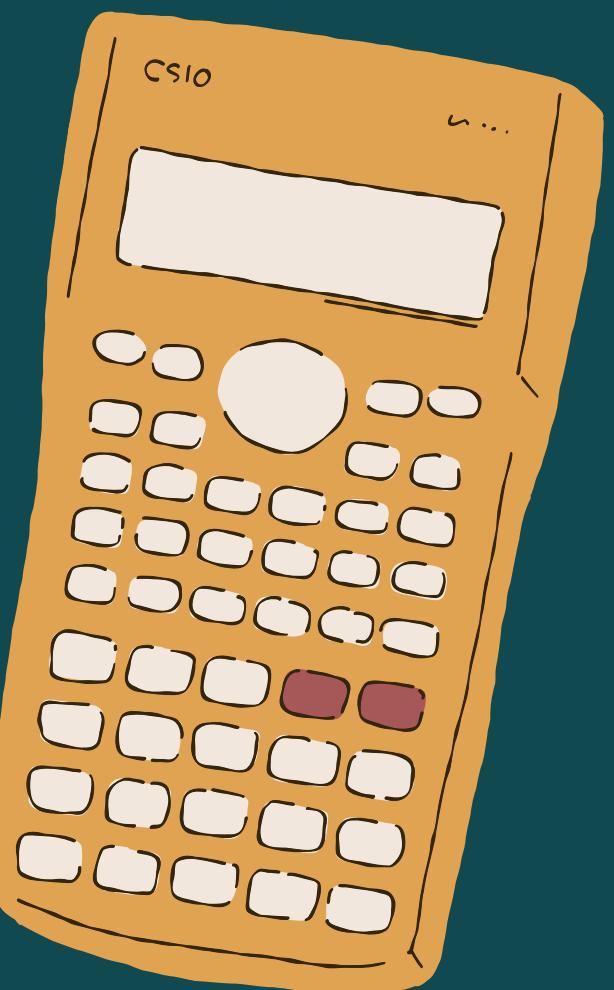
$$\overline{V} = \begin{pmatrix} 3 \\ -4 \\ 8 \end{pmatrix}$$

$$\therefore \overline{P_0.P} = \overline{V}$$

## Contoh Soal

### Soal 1

Cari Pers. Parametrik utk garis  
l yang melalui titik A (3,-1,0)  
dan B (7,0,8)



# Jawaban:

jawab:

$$\overline{P_0 P} = t \cdot \bar{v}$$

$$P_0 = (7, 0, 8)$$

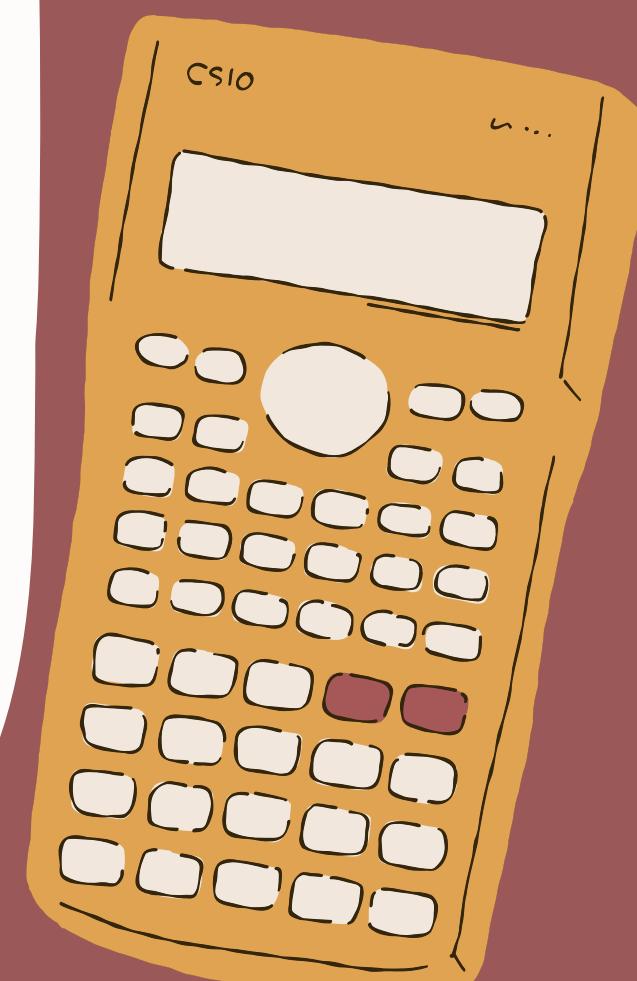
$$P = (x, y, z)$$

②  $\bar{v} = \overline{AB} = B - A = \begin{pmatrix} 7 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix}$

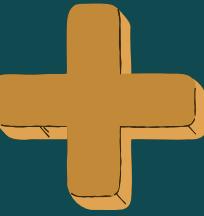
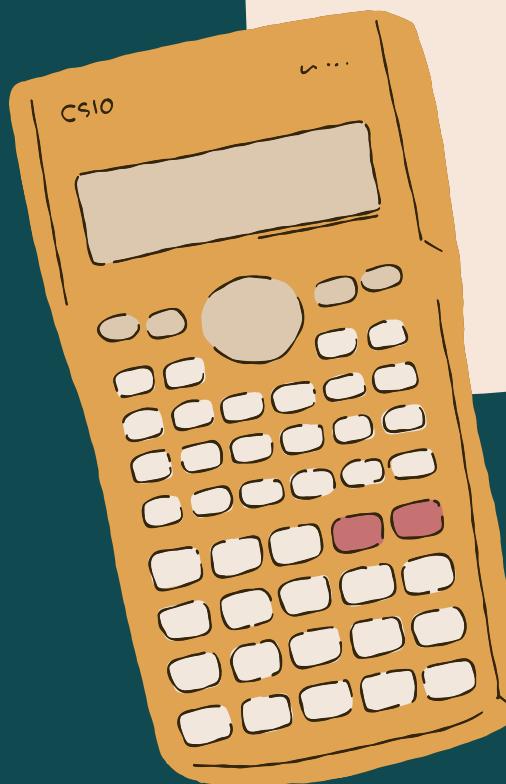
$$\overline{P_0 P} = t \cdot \bar{v}$$

②  $(x - 7), (y), (z - 8) = t (4, 1, 8)$

②  $x - 7 = 4t$       }     $y = t$       }     $z - 8 = 8t$   
           $x = 4t + 7$       }    ②      }     $z = 8t + 8$   
                                }    ②



# MENCARI PERSAMAAN PARAMETRIK UNTUK PERPOTONGAN GARIS 2 BIDANG



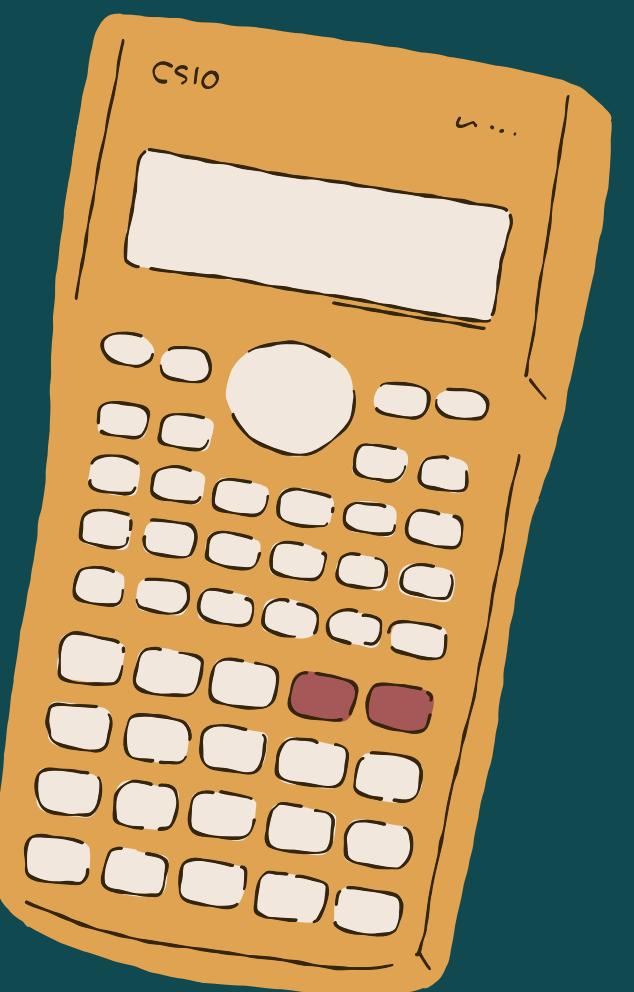
## Contoh Soal

### Soal 2

Cari Pers. Parametrik untuk garis perpotongan dari 2 bidang dibawah ini dengan gauss-jordan

$$-4x + 5y - 7z - 388 = 0$$

$$9x + 6y - 2z + 215 = 0$$



# Jawaban:

-4	x	5	y	-7	z	-388	=	0
9	x	6	y	-2	z	215	=	0

1	-1.25	1.75	-97
9	6	-2	-215

1. Pada iterasi 1, berapa nilai sel (1,2)? -1,25

1	-1.25	1.75	-97
0	17.25	-17.8	658

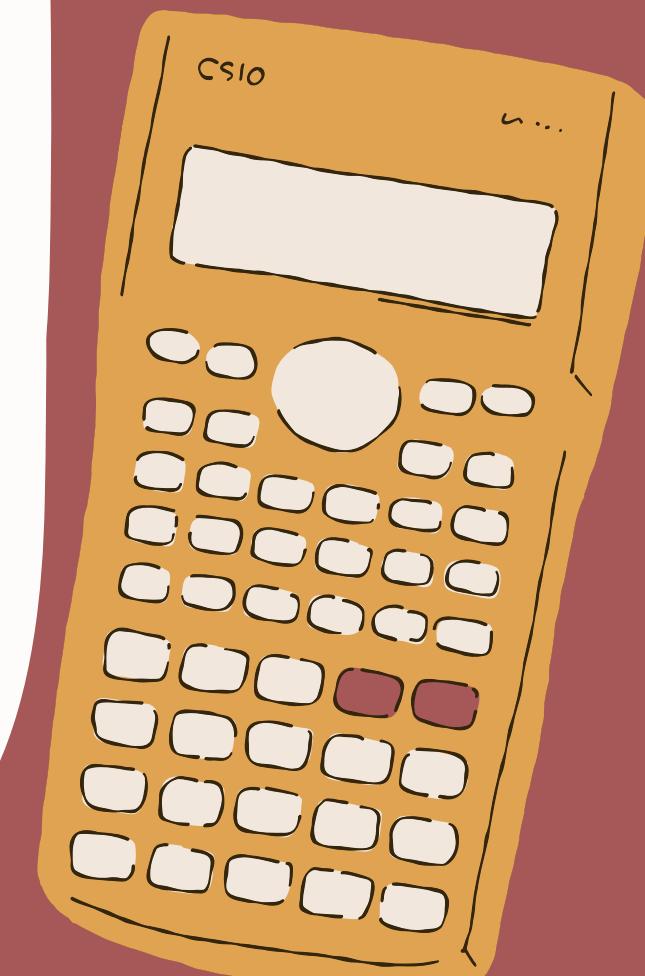
2. Pada iterasi 2, berapa nilai sel (2,3)? -17,8

1	-1.25	1.75	-97
0	1	-1.03	38.14

3. Pada iterasi 3, berapa nilai sel (2,3)? -1,03

1	0	0.463	-49.33
0	1	-1.03	38.14

4. Pada iterasi 4, berapa nilai sel (1,3)? 0,46



# Jawaban:

Sehingga diperoleh Persamaan  
Parametrik yang terbentuk:

5. Nilai Z = +

6. Nilai Y ?

$$y - 1,03 + = 38,14$$

$$y = 38,14 + 1,03 +$$

7. Nilai X ?

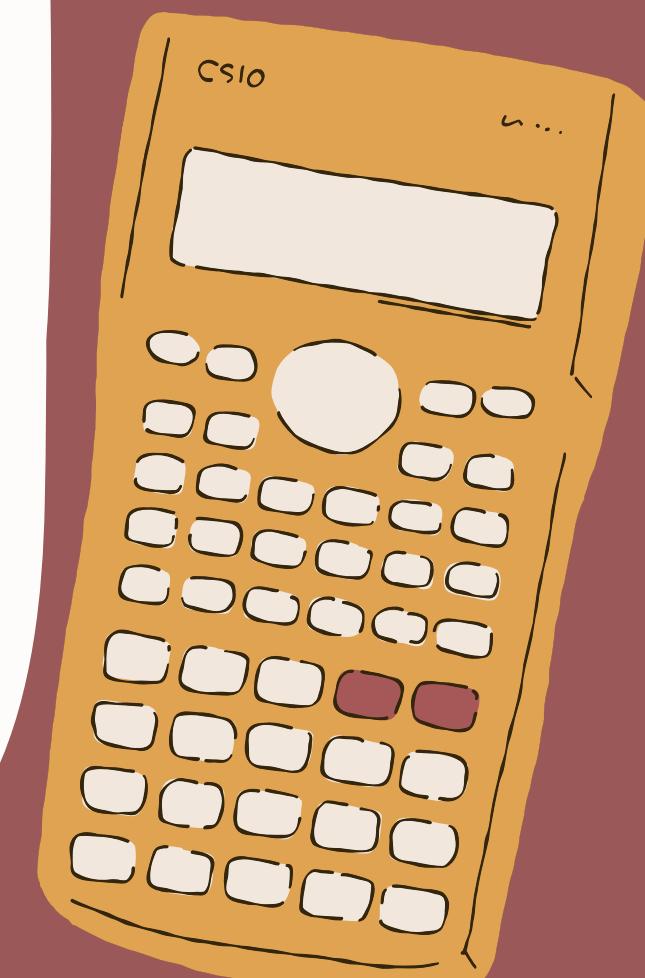
$$x + 0,46 + = -49,33$$

$$x = -49,33 - 0,46 +$$

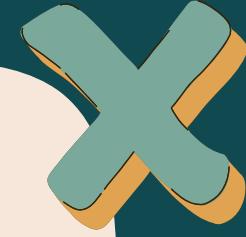
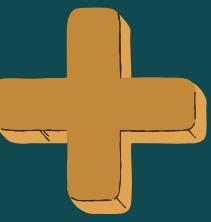
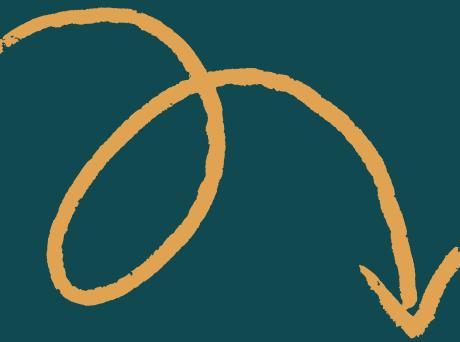
$$Z = t$$

$$Y = 38,14 + 1,03 t$$

$$X = -49,33 - 0,46 t$$

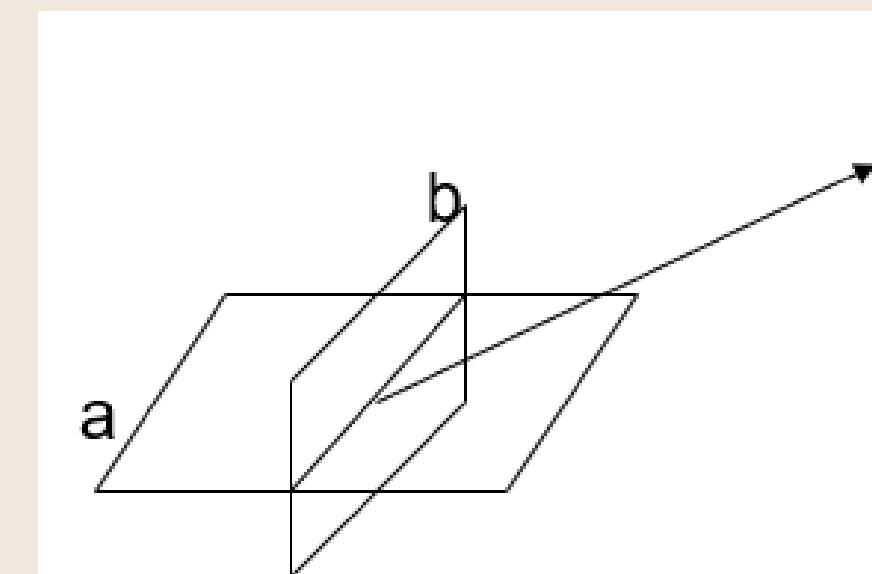


# PERSAMAAN SIMETRIK



# Persamaan Simetrik

Adalah Persamaan  
Garis yang memotong 2  
bidang atau lebih  
sehingga bisa digunakan  
untuk mencari  
Persamaan Bidang



Garis → terdiri dari banyak titik  
→ cari pers. Parametrik / pers utk titik - titik

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

$\overline{\text{PoP}} = t \cdot \overline{v}$

$$(x - x_0, y - y_0, z - z_0) = (ta, tb, tc)$$

→  $t$  - nya sama, sehingga dapat dijadikan persamaan

# Persamaan Simetrik

Karena + bernilai sama, maka akan ada 2 persamaan bidang yang berpotongan

$$\begin{array}{l} \rightarrow \frac{x - x_0}{a} = \frac{y - y_0}{b} \quad \text{bidang 1} \\ \rightarrow \frac{x - x_0}{a} = \frac{z - z_0}{c} \quad \text{bidang 2} \end{array}$$

$$\begin{array}{l} \rightarrow \frac{x - x_0}{a} = \frac{z - z_0}{c} \quad \text{bidang 1} \\ \rightarrow \frac{y - y_0}{b} = \frac{z - z_0}{c} \quad \text{bidang 2} \end{array}$$

## Contoh Soal

### Soal 3

Carilah 2 bidang yang perpotongannya adalah garis:

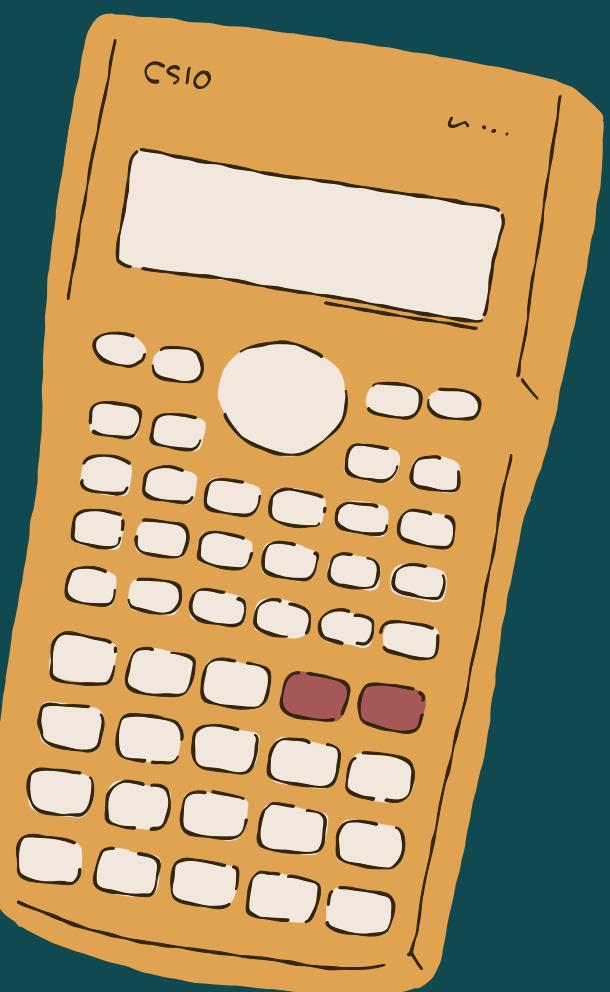
$$x = 3 + 4t$$

$$y = -3 + 11t$$

$$z = 7 - 12t$$

bidang 1: antara x dan y

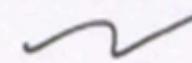
bidang 2: antara x dan z



# Jawaban:

Karena t bernilai sama, maka kita ubah menjadi persamaan di bawah ini:

jawab :



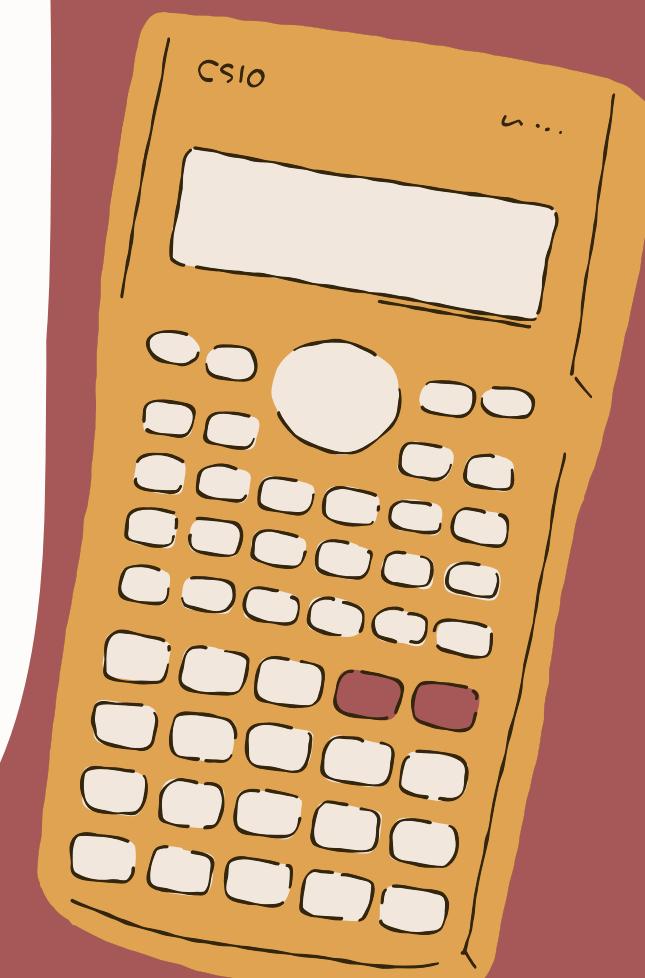
$$\textcircled{3} \quad \frac{x-3}{4} = \frac{y+3}{11} = \frac{z-7}{-12}$$

$$\textcircled{2} \quad \text{bilang}_1 \sim \frac{x-3}{4} = \frac{y+3}{11}$$

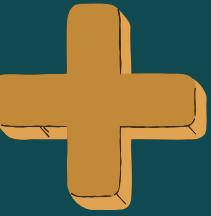
$$\textcircled{2} \quad 11x - 33 = 4y + 12$$

$$\textcircled{2} \quad 11x - 4y - 45 = 0$$

$$\left. \begin{array}{l} \text{bilang}_2 \sim \frac{x-3}{4} = \frac{z-7}{-12} \\ -12x + 36 = 4z - 28 \\ -12x - 4z + 64 = 0 \end{array} \right\} \textcircled{2}$$



# Mencari Jarak



Jarak 1

Jarak dari titik ke  
bidang

Jarak 2

Jarak antara 2  
bidang sejajar



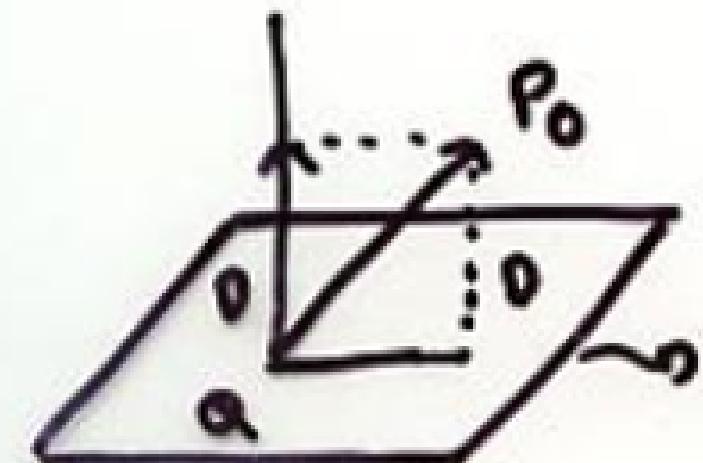
# Jarak titik ke bidang

Jarak titik ke bidang  
bisa dicari, jika:

- diketahui suatu titik
- diketahui persamaan garis

jarak ( $D$ ) antara  $P_0 (x_0, y_0, z_0)$  dgn  
bidang  $ax + by + cz + d = 0$

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$



$$\begin{aligned}P_0 &= (x_0, y_0, z_0) \\Q &= (x_1, y_1, z_1)\end{aligned}$$

bidang  $ax + by + cz + d = 0$

## contoh 2.3

Carilah jarak (D)  
antara titik  
(1,-4,-3) dengan  
bidang

$$2x - 3y - 6z = 1$$

## JAWAB

Pertama, kita perlu merubah  
bidang menjadi bentuk

$$2x - 3y - 6z - 1 = 0$$

# Jawab

Lalu kita identifikasi:

$$x_o = 1$$

$$y_o = -4$$

$$z_o = -3$$

$$a = 2$$

$$b = -3$$

$$c = -6$$

$$d = -1$$

# Jawab

Kita masukkan ke dalam persamaan, sehingga:

$$D = \frac{|(2)(1) + (-3)(-4) + (-6)(-3) - 1|}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{31}{7}$$

# Jarak 2 bidang sejajar

Misalkan terdapat 2

buh bidang yakni:

- A (Alpha)
- B (Beta)

Langkah Selanjutnya:

1. Tentukan Sembarang Titik T pada bidang A
2. Lalu, hitung jarak antara Titik T terhadap bidang B

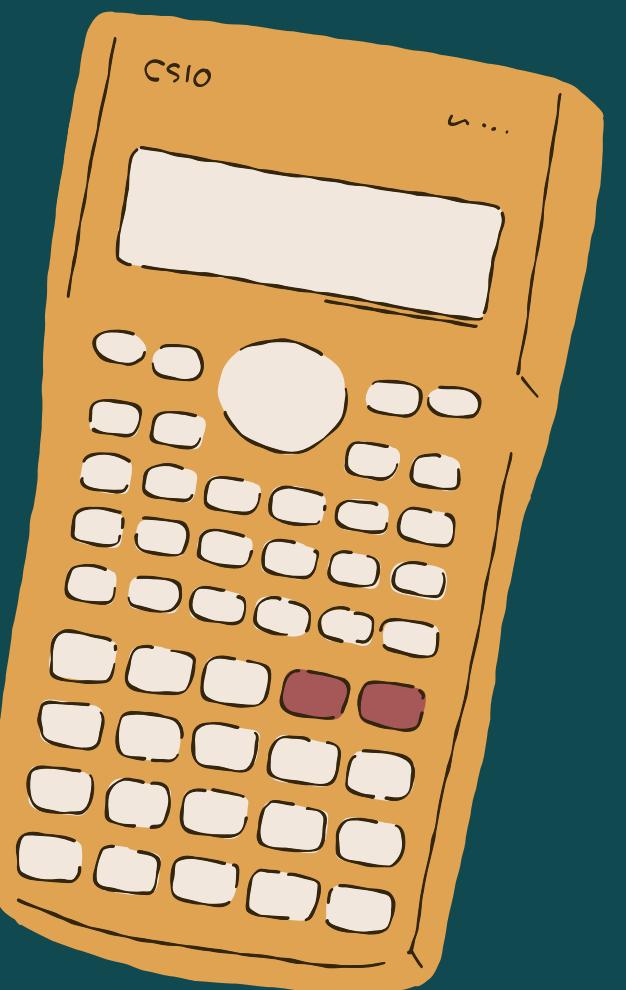
## Contoh Soal

### Soal 4

Carilah jarak antara bidang 1 dengan  
bidang 2

- Bidang 1 :  $-2x + 3y - 5z = 7$
- Bidang 2 :  $-6x + 9y - 15z = 18$

Misalkan titik T berada pada bidang 2  
(0,2,0)



# Jawaban:

jawab:  
~

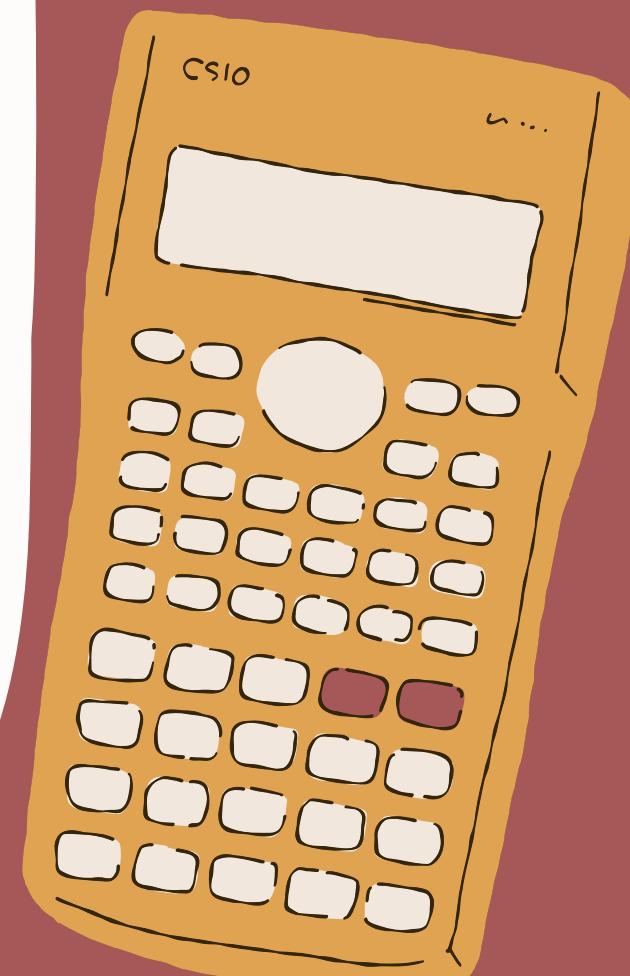
titik pd bidang 2  $\left[ \begin{array}{l} x = 0 \\ y = 2 \\ z = 0 \end{array} \right] \quad (0, 2, 0) \quad (2)$

bidang 1  $\rightarrow -2x + 3y - 5z - 7 = 0 \quad (2)$

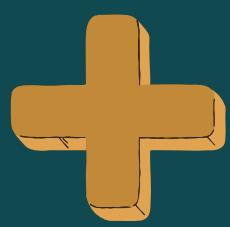
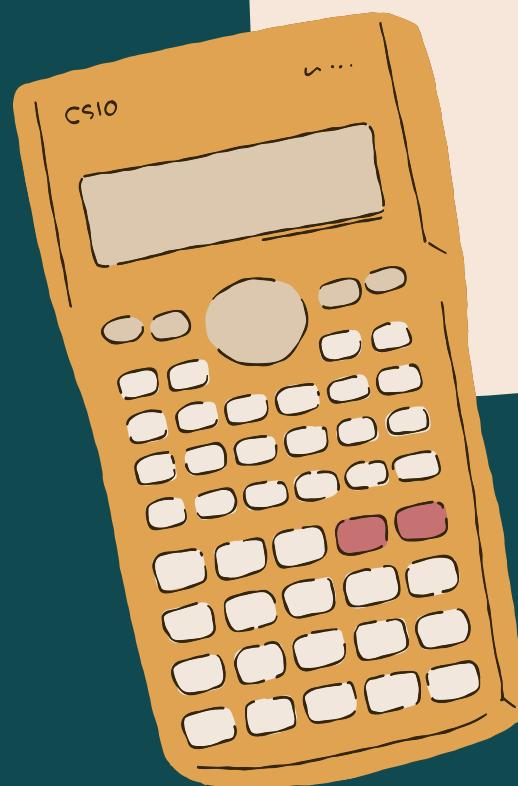
$$\left[ \begin{array}{l} x_0 = 0 \\ y_0 = 2 \\ z_0 = 0 \\ a = -2 \\ b = 3 \\ c = -5 \\ d = -7 \end{array} \right]$$

$$d = \frac{|a \cdot x_0 + b \cdot y_0 + c \cdot z_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$
$$= \frac{|-2 \cdot 0 + 3 \cdot 2 - 5 \cdot 0 - 7|}{\sqrt{(-2)^2 + 3^2 + (-5)^2}} \quad (3)$$

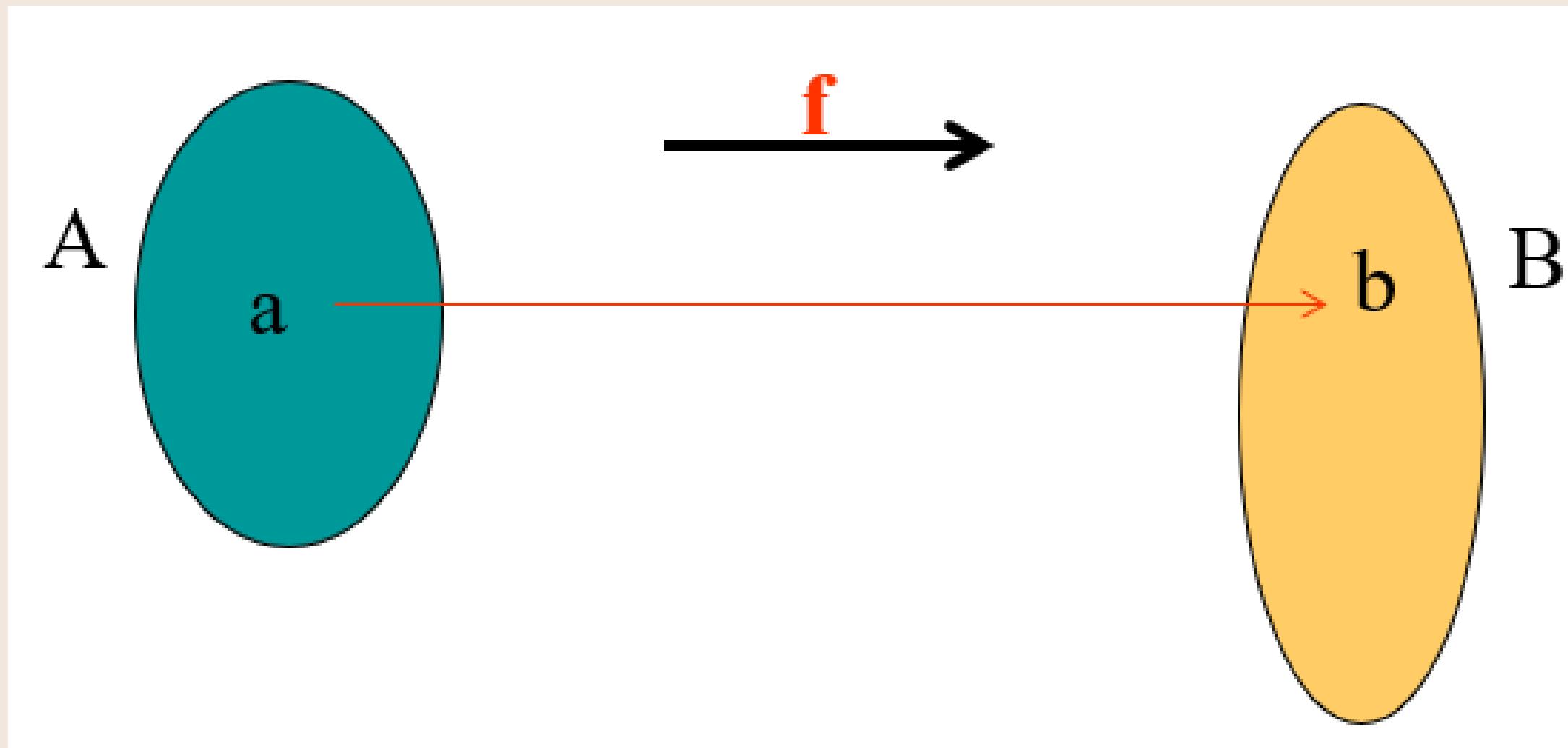
$$d = \frac{1}{\sqrt{38}} \quad (3)$$



# Transformasi Linier



# Fungsi



Adalah Pemetaan (mapping) dari Himpunan A ke Himpunan B

1. Notasi  $f : A \rightarrow B$
2. Himpunan A disebut DOMAIN( $f$ )
3. Himpunan B disebut CODOMAIN( $f$ )
4. Tiap elemen A dipasangkan dengan (associated with) satu elemen B
5. Himpunan semua elemen  $b$  yang punya pasangan di A disebut RANGE( $f$ )
6. Notasi  $f(a) = b$ ,  $b$  disebut bayangan (image) dari  $a$

# Transformasi Linier

Pemetaan fungsi

$$f : R^n \rightarrow R^m$$

disebut TRANSFORMASI  
dan disimbolkan dengan:

$$T : R^n \rightarrow R^m$$

$$T : R^n \rightarrow R^m$$

T sering disebut sebagai TRANSFORMASI LINIER, jika:

1.  $T(u + v) = T(u) + T(v)$   
(Penjumlahan 2 vektor)
2.  $T(c \cdot u) = c \cdot T(u)$   
(Perkalian skalar dengan vektor)

\*catatan:

1.  $u, v$  adalah vektor di ruang  $m$
2.  $c$  adalah skalar
3.  $T(u+v), T(u), T(v), T(c \cdot u)$  adalah vektor di ruang  $n$

# Transformasi Linier

$$T : R^n \rightarrow R^m$$

Transformasi T juga dapat "digantikan" ke dalam bentuk Matriks

$\rightarrow$  (Matriks berukuran  $M \times N$ )

Berikut adalah gambaran transformasi:

$$x = (x_1, x_2, x_3, \dots, x_n) \rightarrow w = (w_1, w_2, w_3, \dots, w_n)$$

jika,

$$x = (x_1, x_2, \dots, x_4)^T \rightarrow w = (w_1, w_2, \dots, w_4)^T$$

maka Transformasi dapat "digantikan" ke bentuk:

$$Ax = w$$

dimana A adalah Matriks Standar untuk Transformasi Linier T

## contoh 1

Berikut ini adalah  
Transformasi dari

$$R^2 \rightarrow R^3$$

$$w_1 = x_1 + x_2$$

$$w_2 = 3x_1x_2$$

$$w_3 = {x_1}^2 + {x_2}^2$$

JAWAB

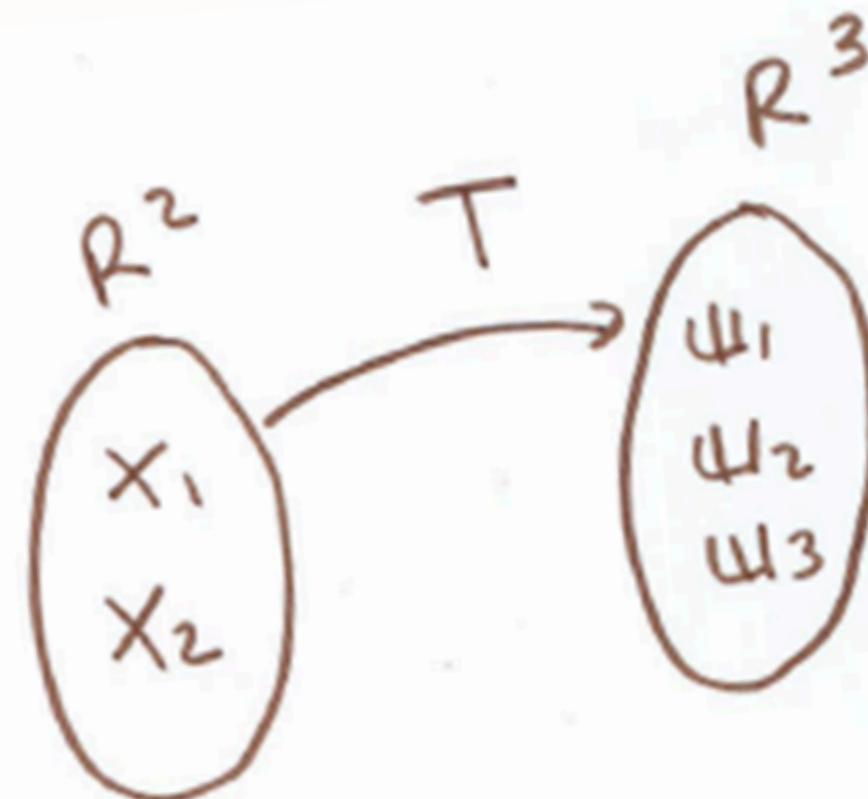
dengan mendefinisikan

$$T : R^2 \rightarrow R^3$$

maka

$$T(x_1, x_2) = (x_1 + x_2, 3x_1x_2, {x_1}^2 + {x_2}^2)$$

# Jawab



$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

$\text{Bentuk } T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 3x_1 x_2 \\ x_1^2 - x_2^2 \end{pmatrix}$

jika  $x_1 = 1$  &  $x_2 = -2$

$$T \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 - 2 \\ 3 \cdot 1 \cdot -2 \\ 1^2 - (-2)^2 \end{pmatrix}$$

$$T \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \\ -3 \end{pmatrix}$$

$$T \cdot x = \mathbb{U}$$

$$A \cdot x = \mathbb{U}$$

Lalu matrix standar

## contoh 2

Tentukan Matriks  
baku (A) dari

$$R^3 \rightarrow R^4$$

$$w_1 = x_1 + x_2$$

$$w_2 = x_1 - x_2$$

$$w_3 = x_3$$

$$w_4 = x_1$$

# Jawab

$$T \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ x_3 \\ x_1 \end{pmatrix}$$

(A)  $A \cdot x = E$

matrix batu

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ x_3 \\ x_1 \end{pmatrix}$$

$$\begin{bmatrix} 1x_1 + 1x_2 + 0x_3 \\ 1x_1 - 1x_2 + 0x_3 \\ 0x_1 + 0x_2 + 1x_3 \\ 1x_1 + 0x_2 + 0x_3 \end{bmatrix}$$

$$A \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ x_3 \\ x_1 \end{pmatrix}$$

$$\frac{4 \times 3}{4 \times 3} \quad 3 \times 1$$

$=$

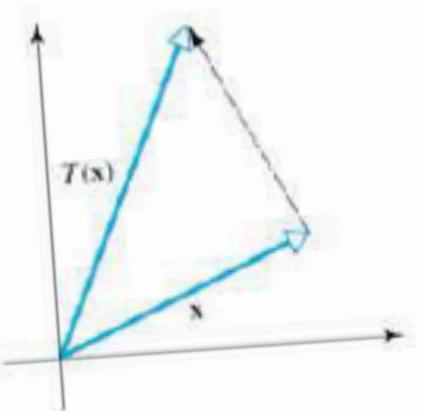
$4 \times 1$

# Contoh

The linear transformation  $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$  defined by the equations

$$\begin{aligned}w_1 &= 2x_1 - 3x_2 + x_3 - 5x_4 \\w_2 &= 4x_1 + x_2 - 2x_3 + x_4 \\w_3 &= 5x_1 - x_2 + 4x_3\end{aligned}$$

can be expressed in matrix form as



(b)  $T$  maps vectors to vectors

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 & -5 \\ 4 & 1 & -2 & 1 \\ 5 & -1 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

so the standard matrix for  $T$  is

$$A = \begin{bmatrix} 2 & -3 & 1 & -5 \\ 4 & 1 & -2 & 1 \\ 5 & -1 & 4 & 0 \end{bmatrix}$$

The image of a point  $(x_1, x_2, x_3, x_4)$  can be computed directly from the defining equations 5 or from 6 by matrix multiplication.  
For example, if  $(x_1, x_2, x_3, x_4) = (1, -3, 0, 2)$ , then substituting in 5 yields  
 $w_1 = 1, w_2 = 3, w_3 = 8$

(verify) or alternatively from 6,

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} A = \begin{bmatrix} 2 & -3 & 1 & -5 \\ 4 & 1 & -2 & 1 \\ 5 & -1 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$$

# Contoh Operasi Transformasi

1. Pencerminan

2. Proyeksi  
Orthogonal

3. Rotasi

4. Kontraksi

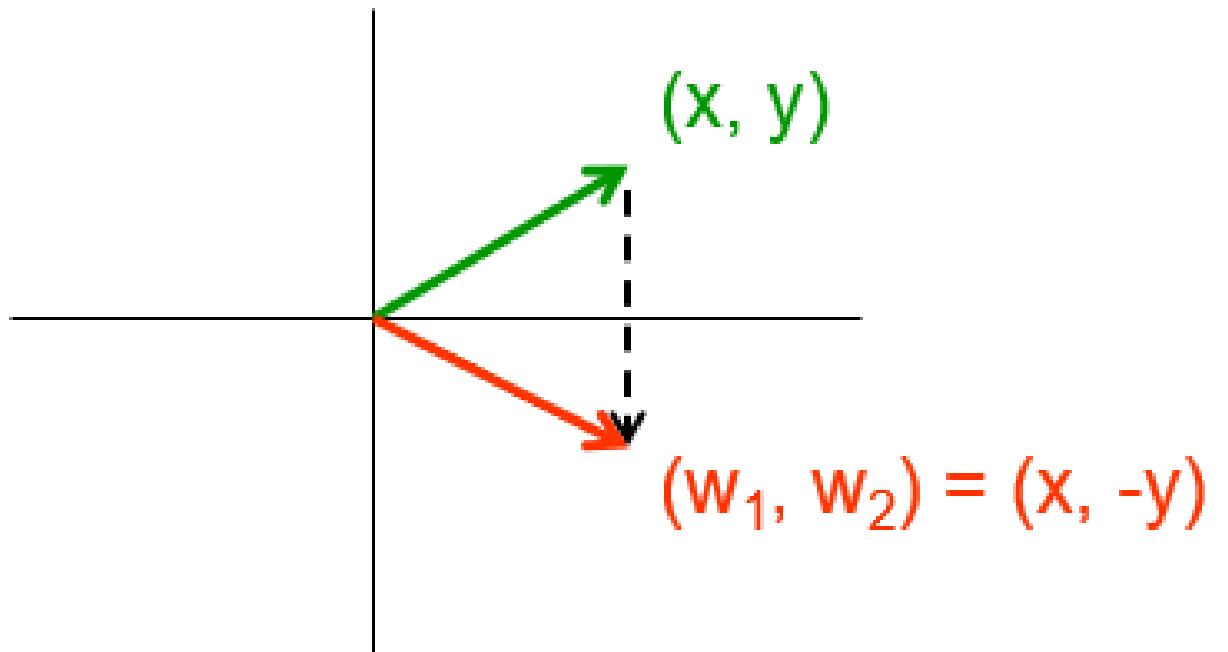
5. Dilatasi

# Pencerminan Sumbu

## operator

pencerminan  
terhadap sumbu-x

## ilustrasi



## persamaan

$$w_1 = x = 1x + 0y$$

$$w_2 = -y = 0x + (-1)y$$

## matriks standar

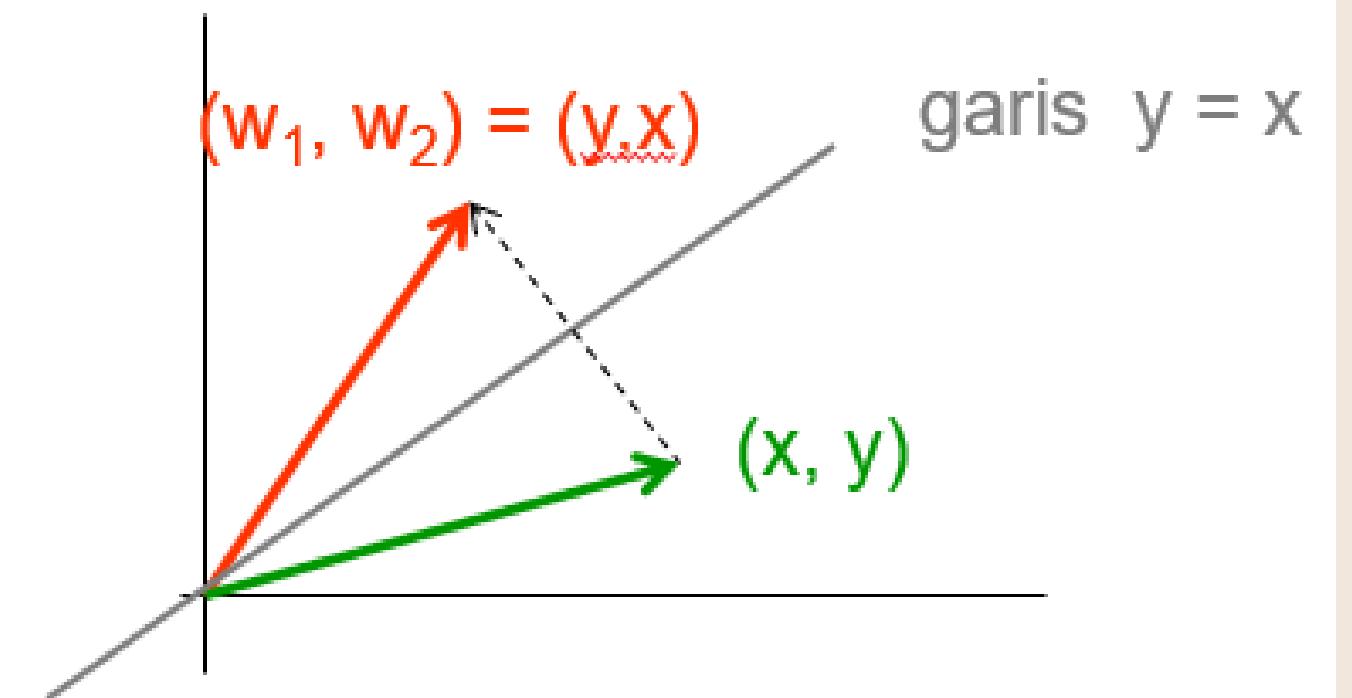
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

# Pencerminan Sumbu

## operator

pencerminan  
terhadap garis  $y = x$

## ilustrasi



## persamaan

$$w_1 = y = 0x + 1y$$

$$w_2 = x = 1x + 0y$$

## matriks standar

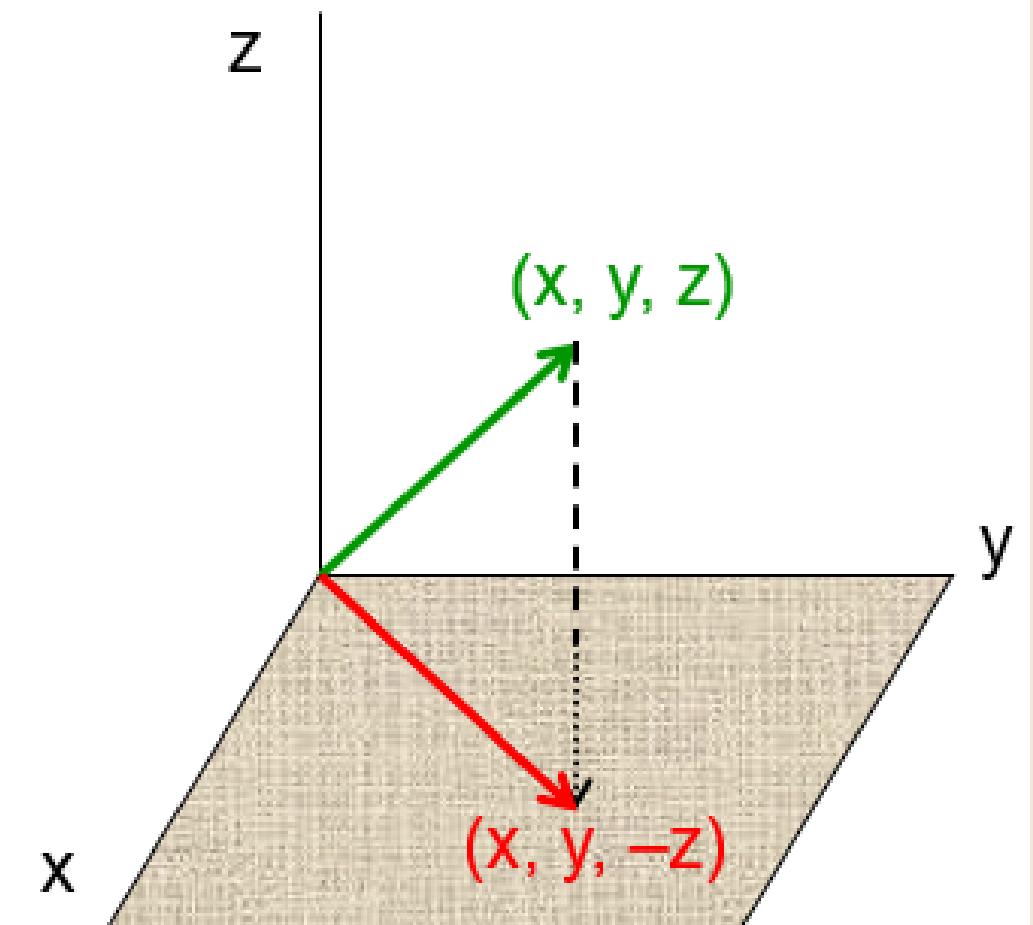
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

# Pencerminan Bidang

## operator

pencerminan  
terhadap bidang xy

## ilustrasi



## persamaan

$$w_1 = x = 1x + 0y + 0z$$

$$w_2 = y = 0x + 1y + 0z$$

$$w_3 = -z = 0x + 0y + (-1)z$$

## matriks standar

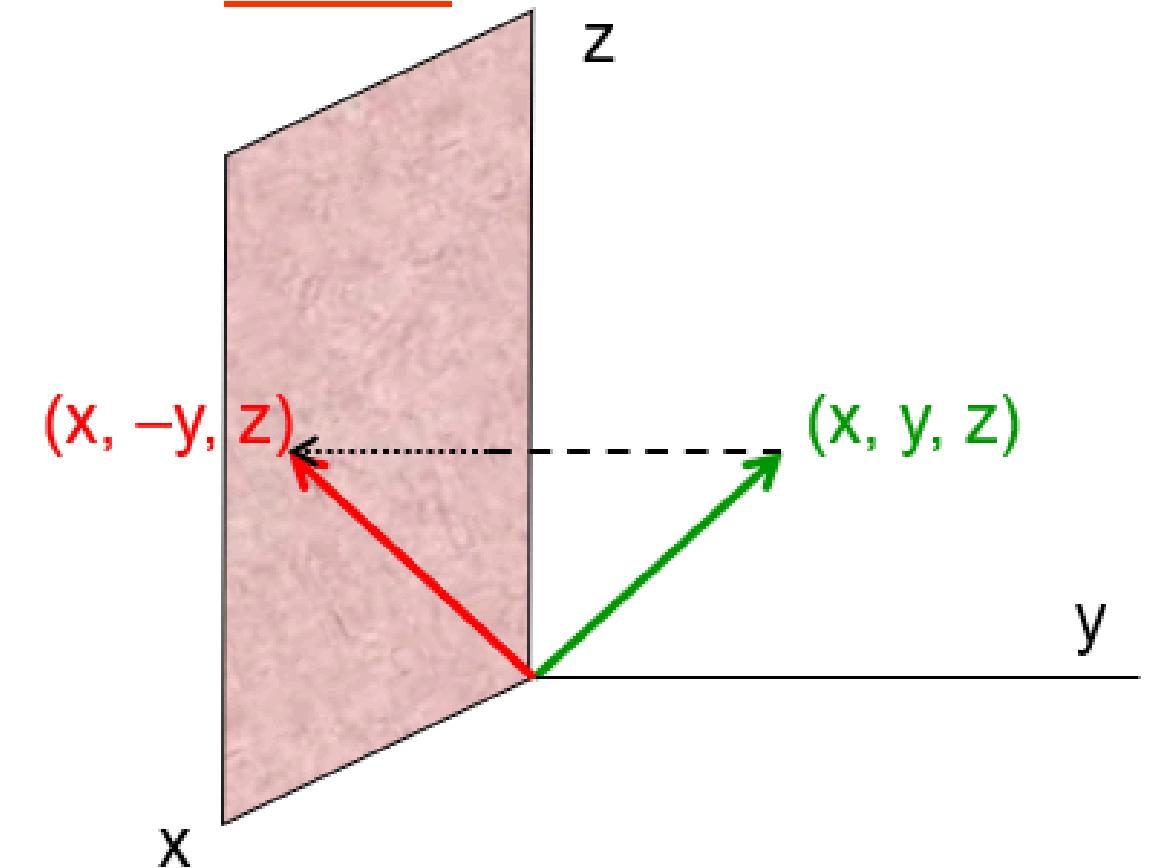
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

# Pencerminan Bidang

## operator

pencerminan  
terhadap bidang  $xz$

## ilustrasi



## persamaan

$$w_1 = x = 1x + 0y + 0z$$

$$w_2 = y = 0x + (-1)y + 0z$$

$$w_3 = -z = 0x + 0y + 1z$$

## matriks standar

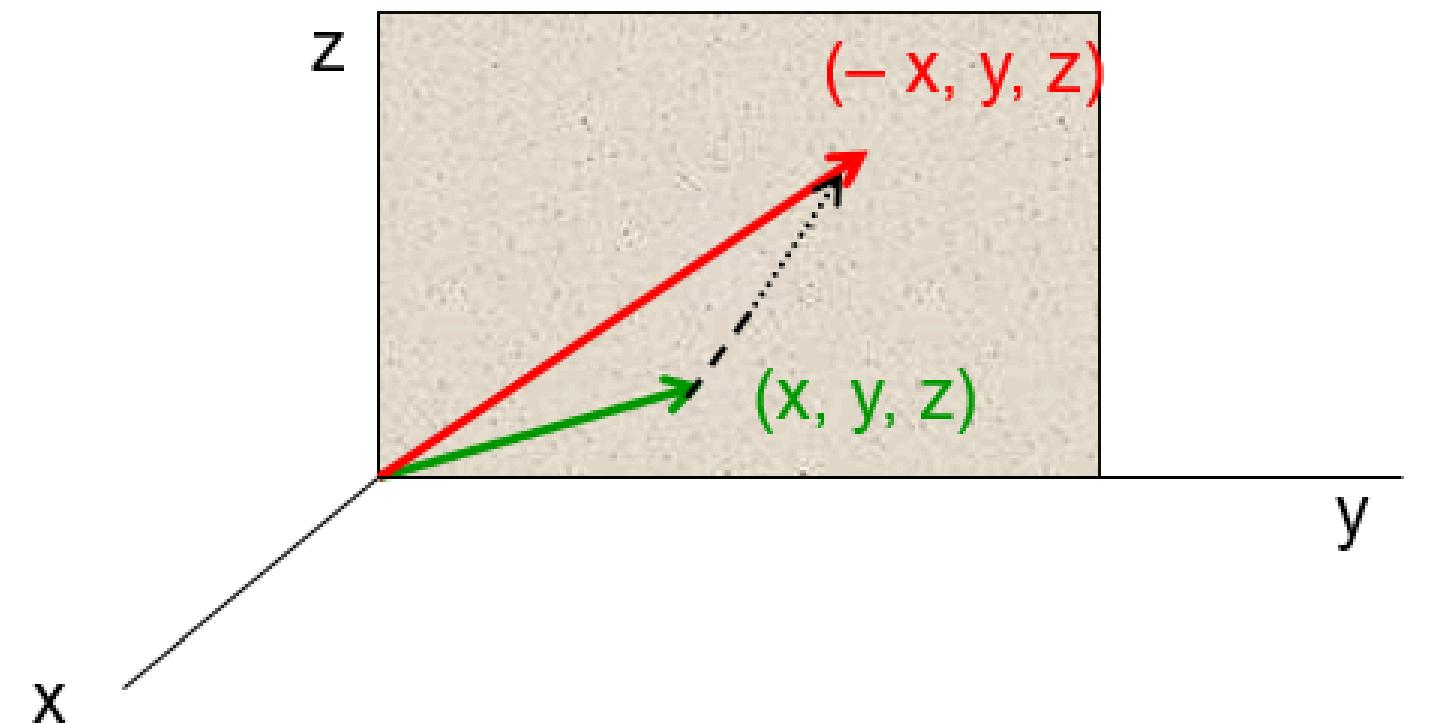
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Pencerminan Bidang

## operator

pencerminan  
terhadap bidang  $yz$

## ilustrasi



## persamaan

$$w_1 = x = -1x + 0y + 0z$$

$$w_2 = y = 0x + 1y + 0z$$

$$w_3 = -z = 0x + 0y + 1z$$

## matriks standar

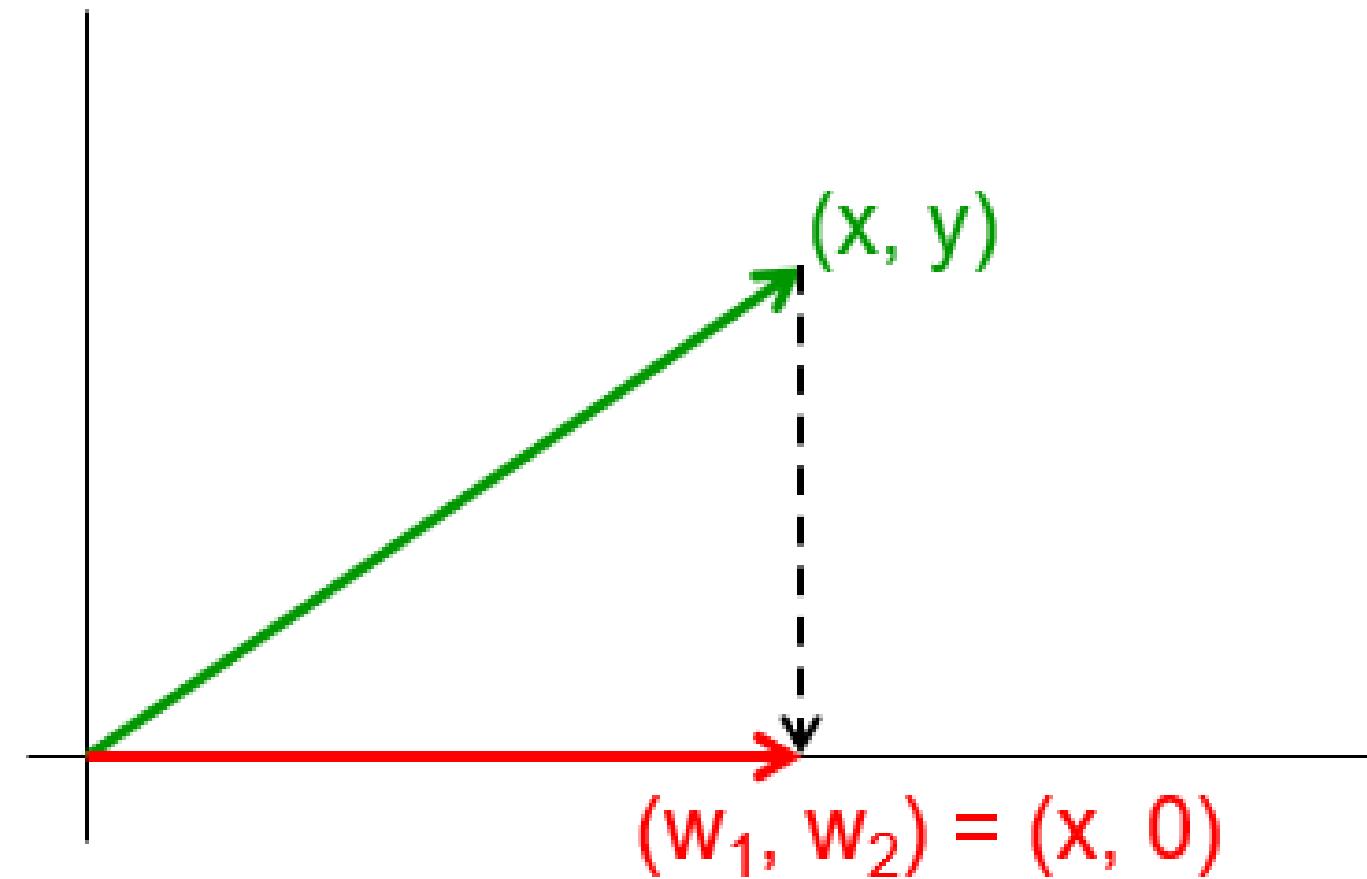
$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Proyeksi Orthogonal

## operator

proyksi ortogonal  
pada sumbu-x

## ilustrasi



## persamaan

$$w_1 = x = 1x + 0y$$

$$w_2 = 0 = 0x + 0y$$

## matriks standar

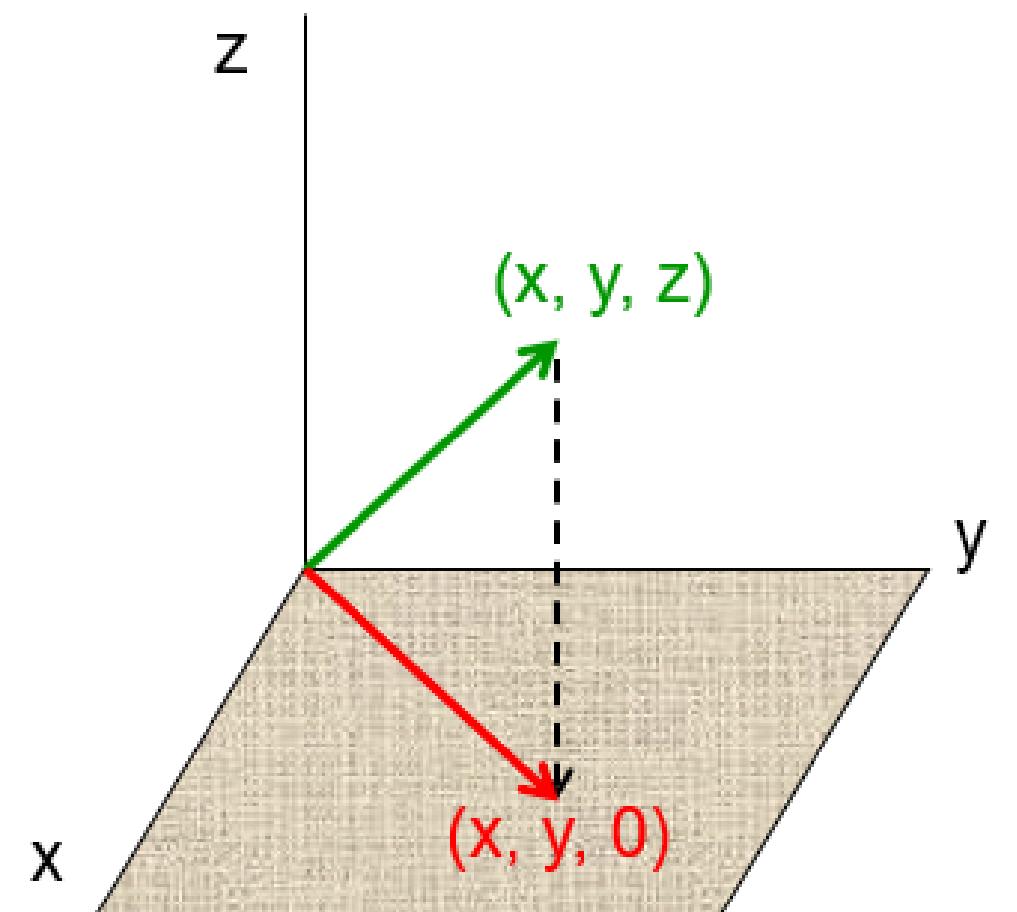
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

# Proyeksi Orthogonal

## operator

proyeksi ortogonal  
pada bidang xy

## ilustrasi



## persamaan

$$w_1 = x = 1x + 0y + 0z$$

$$w_2 = y = 0x + 1y + 0z$$

$$w_3 = -z = 0x + 0y + 0z$$

## matriks standar

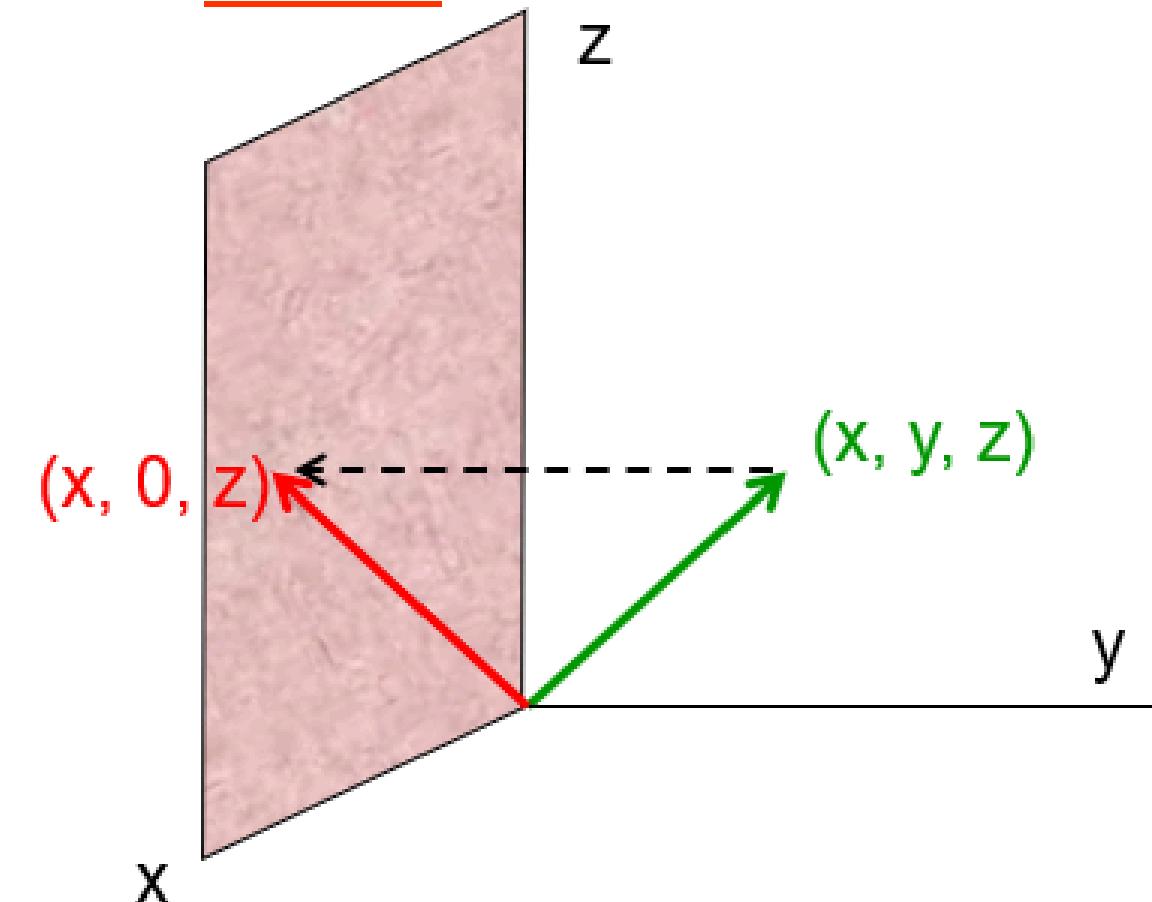
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

# Proyeksi Orthogonal

## operator

proyeksi ortogonal  
pada bidang xz

## ilustrasi



## persamaan

$$w_1 = x = 1x + 0y + 0z$$

$$w_2 = y = 0x + 0y + 0z$$

$$w_3 = -z = 0x + 0y + 1z$$

## matriks standar

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Rotasi

## operator

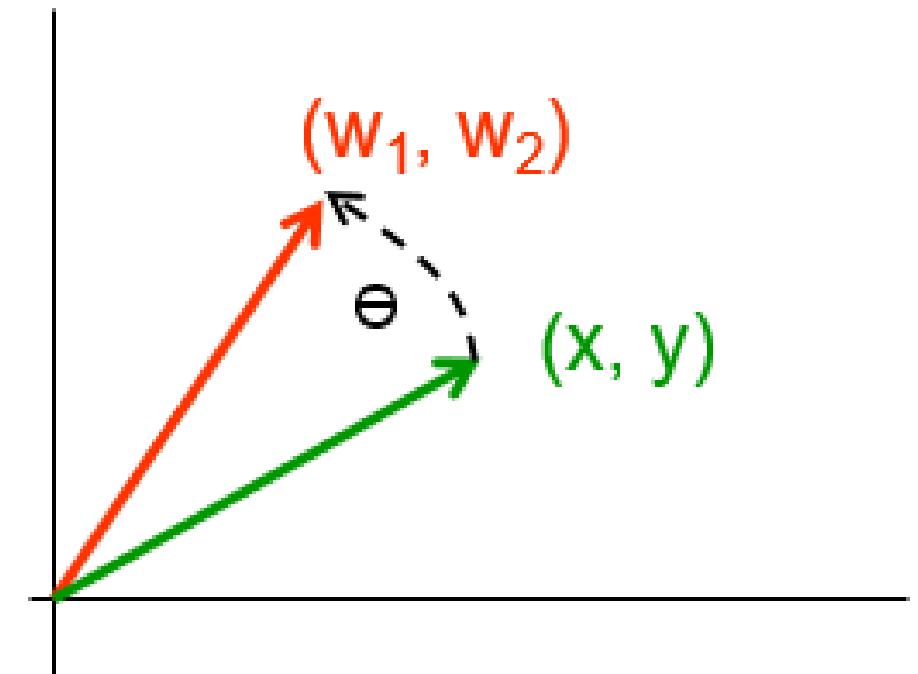
**rotasi dengan  
sudut rotasi  $\Theta$**

## persamaan

$$w_1 = x \cos \Theta - y \sin \Theta$$

$$w_2 = x \sin \Theta + y \cos \Theta$$

## ilustrasi



## matriks standar

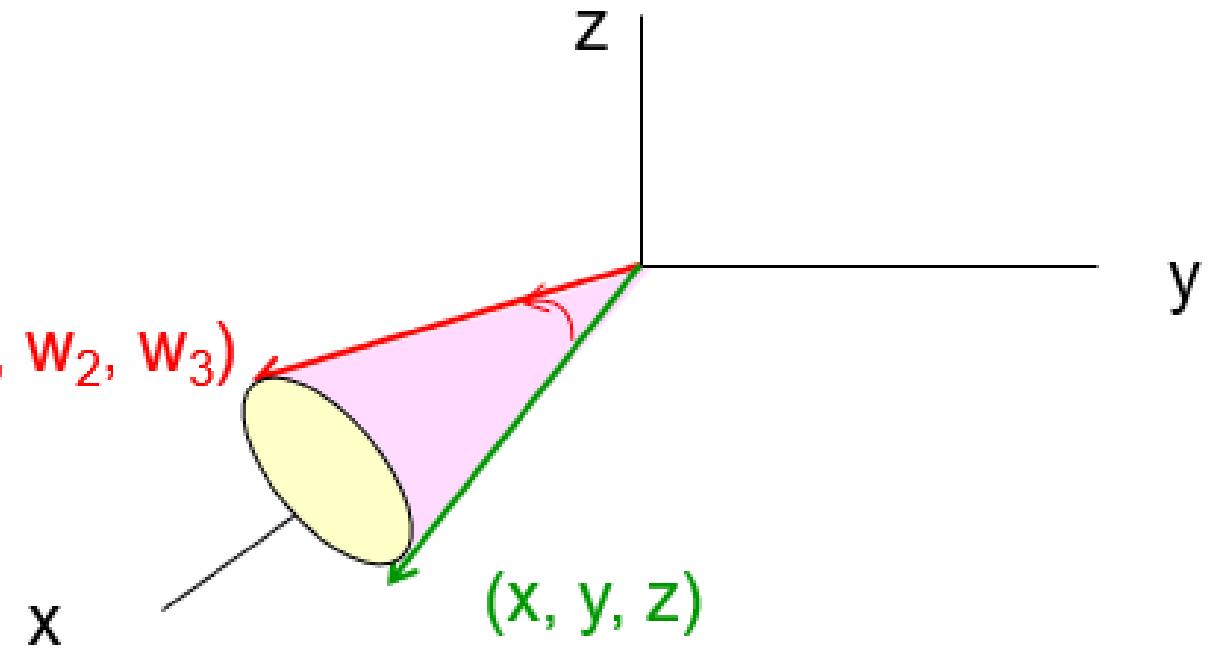
$$\begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix}$$

# Rotasi

## operator

rotasi melawan arah  
jarum jam dengan  
sumbu rotasi x positif  
dan sudut rotasi  $\theta$

## ilustrasi



## persamaan

$$w_1 = (\cos \theta) x + (-\sin \theta) y + 0z$$

$$w_2 = (\sin \theta) x + (\cos \theta) y + 0z$$

$$w_3 = 0x + 0y + 1z$$

## matriks standar

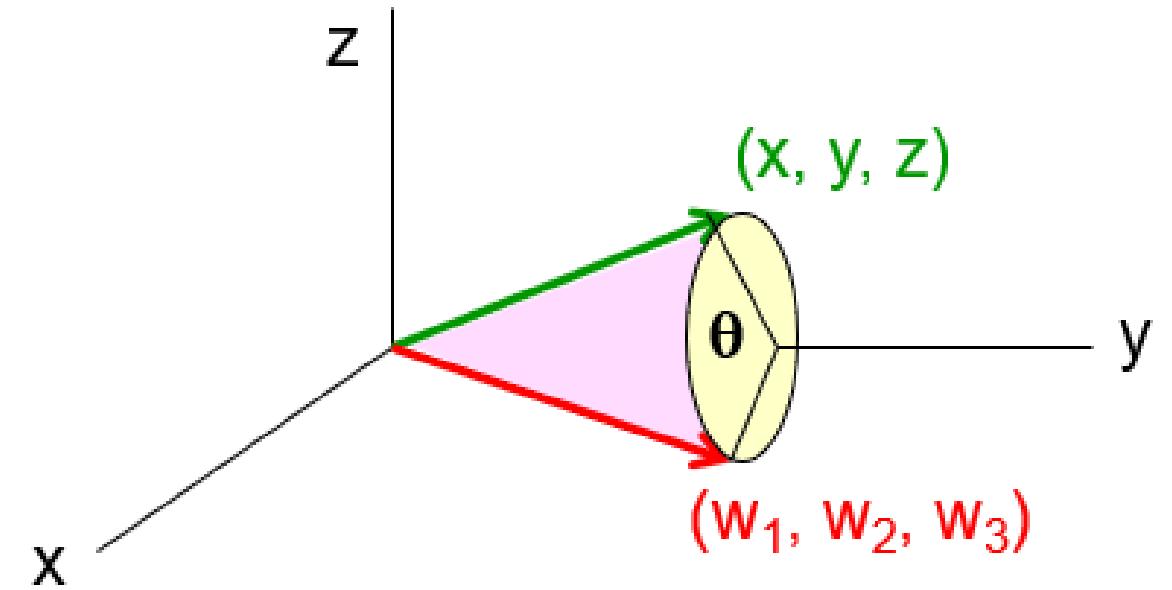
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$$

# Rotasi

## operator

rotasi melawan arah jarum jam dengan sumbu rotasi y positif dan sudut rotasi  $\theta$

## ilustrasi



## persamaan

$$w_1 = (\cos \theta) x + (-\sin \theta) y + 0z$$

$$w_2 = (\sin \theta) x + (\cos \theta) y + 0z$$

$$w_3 = 0x + 0y + 1z$$

## matriks standar

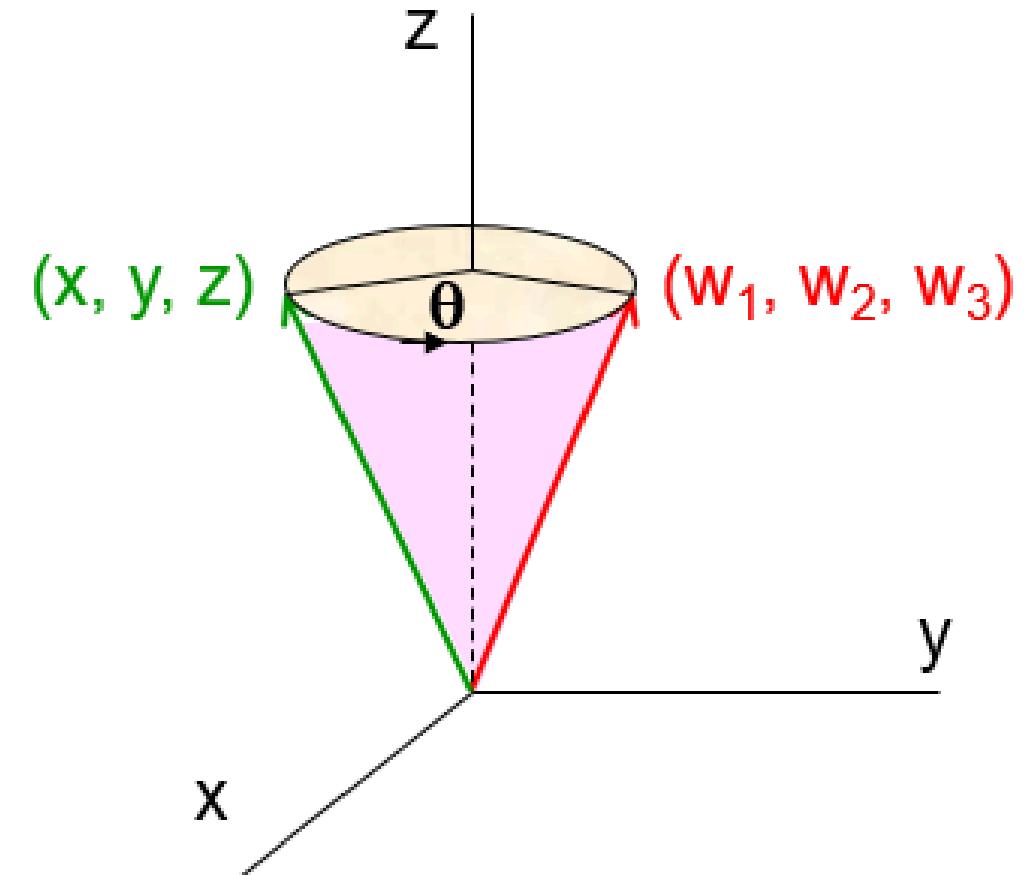
$$\begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$$

# Rotasi

## operator

rotasi melawan arah  
jarum jam dengan  
sumbu rotasi z positif  
dan sudut rotasi  $\theta$

## ilustrasi



## persamaan

$$w_1 = (\cos \theta) x + (-\sin \theta) y + 0z$$

$$w_2 = (\sin \theta) x + (\cos \theta) y + 0z$$

$$w_3 = 0x + 0y + 1z$$

## matriks standar

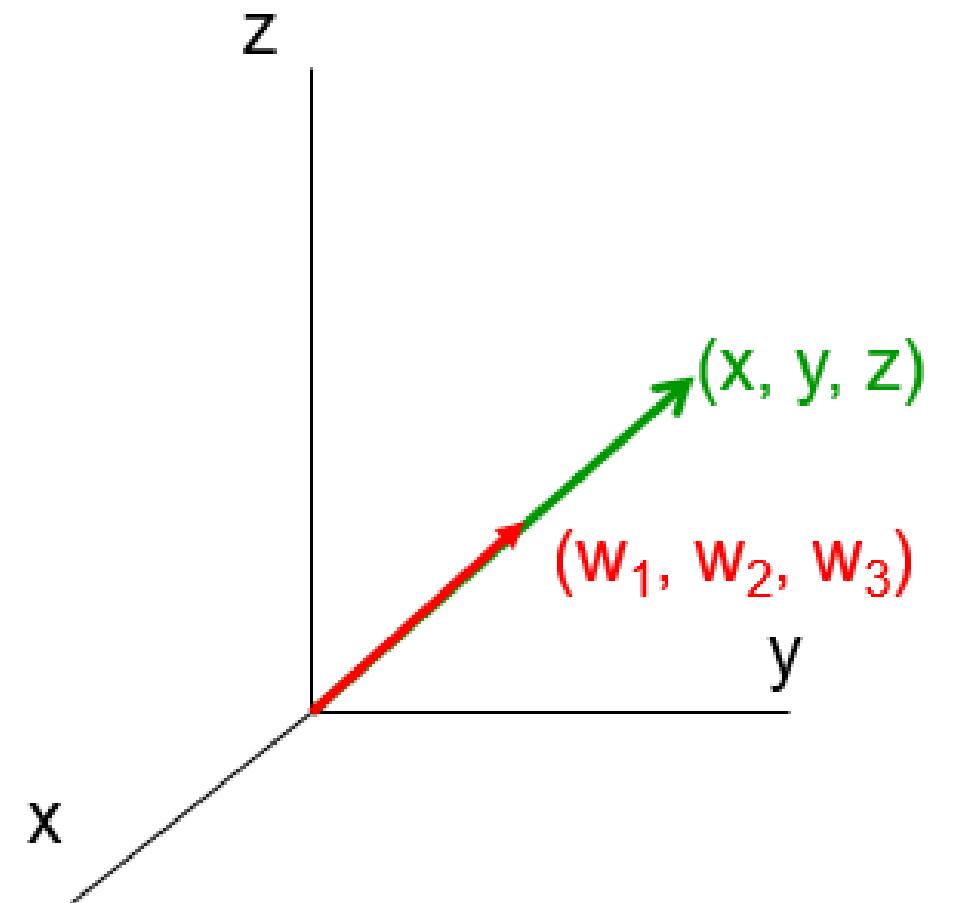
$$\begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Kontraksi

## operator

Kontraksi ( penyusutan)  
dengan faktor  $0 \leq k \leq 1$

## ilustrasi



## persamaan

$$w_1 = kx + 0y + 0z$$

$$w_2 = 0x + ky + 0z$$

$$w_3 = 0x + 0y + kz$$

## matriks standar

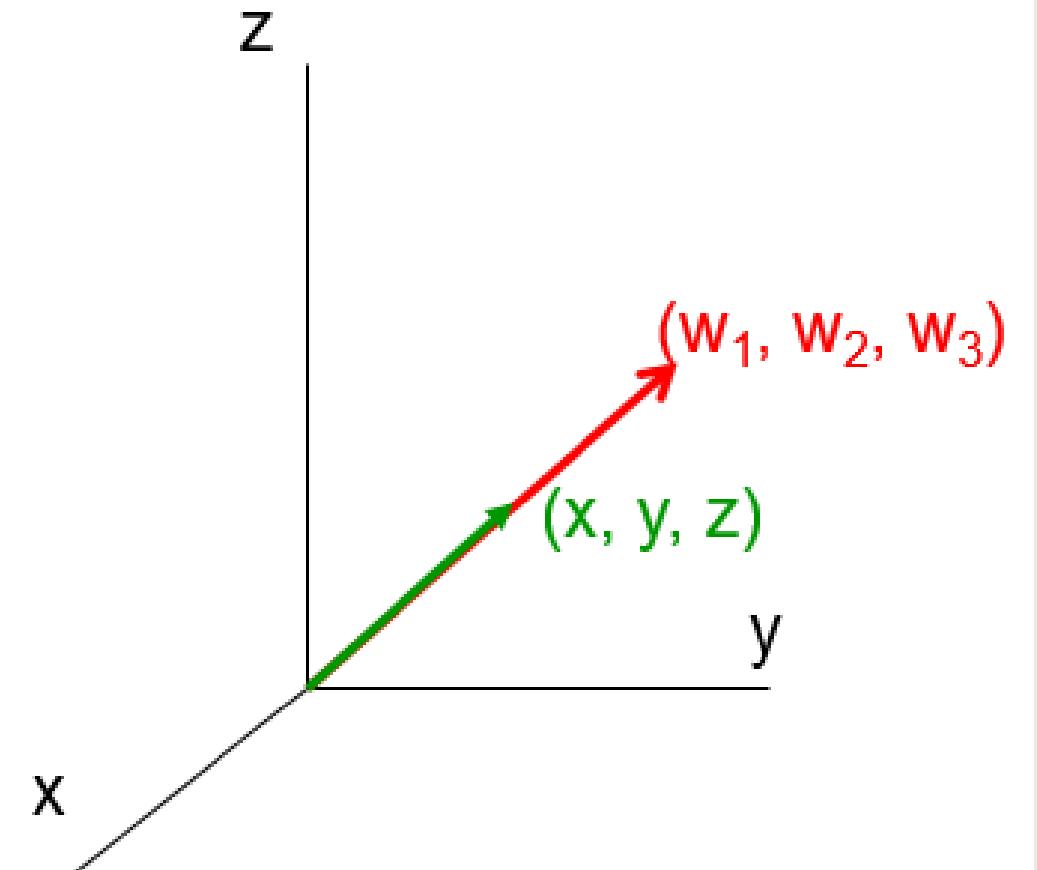
$$\begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

# Dilatasi

## operator

Dilasi (pemuiaian/perbesaran)  
dengan faktor  $k > 1$

## ilustrasi



## persamaan

$$w_1 = kx + 0y + 0z$$

$$w_2 = 0x + ky + 0z$$

$$w_3 = 0x + 0y + kz$$

## matriks standar

$$\begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

# Contoh

## EXAMPLE 5 Rotation

If each vector in  $\mathbb{R}^2$  is rotated through an angle of  $\pi/6$  ( $= 30^\circ$ ), then the image  $w$  of a vector

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

is

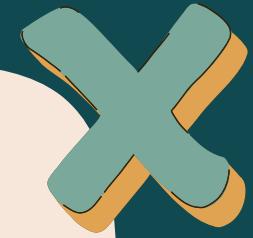
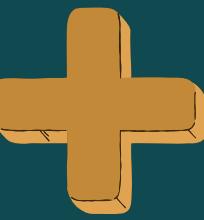
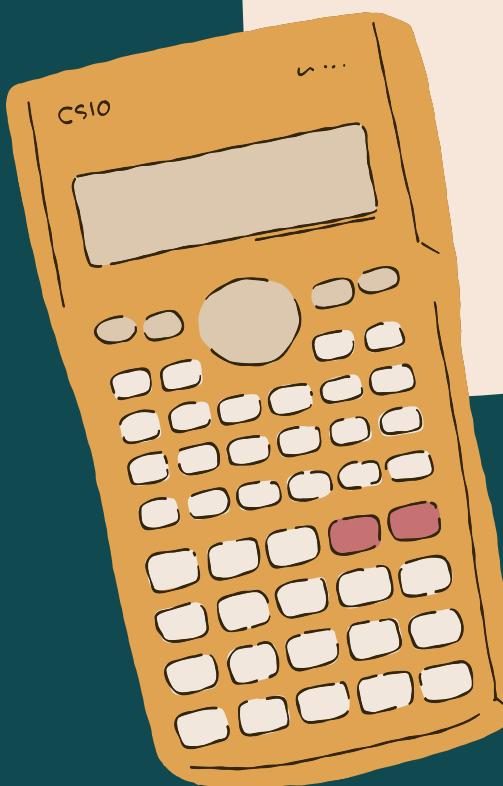
$$w = \begin{bmatrix} \cos \pi/6 & -\sin \pi/6 \\ \sin \pi/6 & \cos \pi/6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2}x - \frac{1}{2}y \\ \frac{1}{2}x + \frac{\sqrt{3}}{2}y \end{bmatrix}$$

For example, the image of the vector

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ is } w = \begin{bmatrix} \frac{\sqrt{3}-1}{2} \\ \frac{1+\sqrt{3}}{2} \end{bmatrix}$$

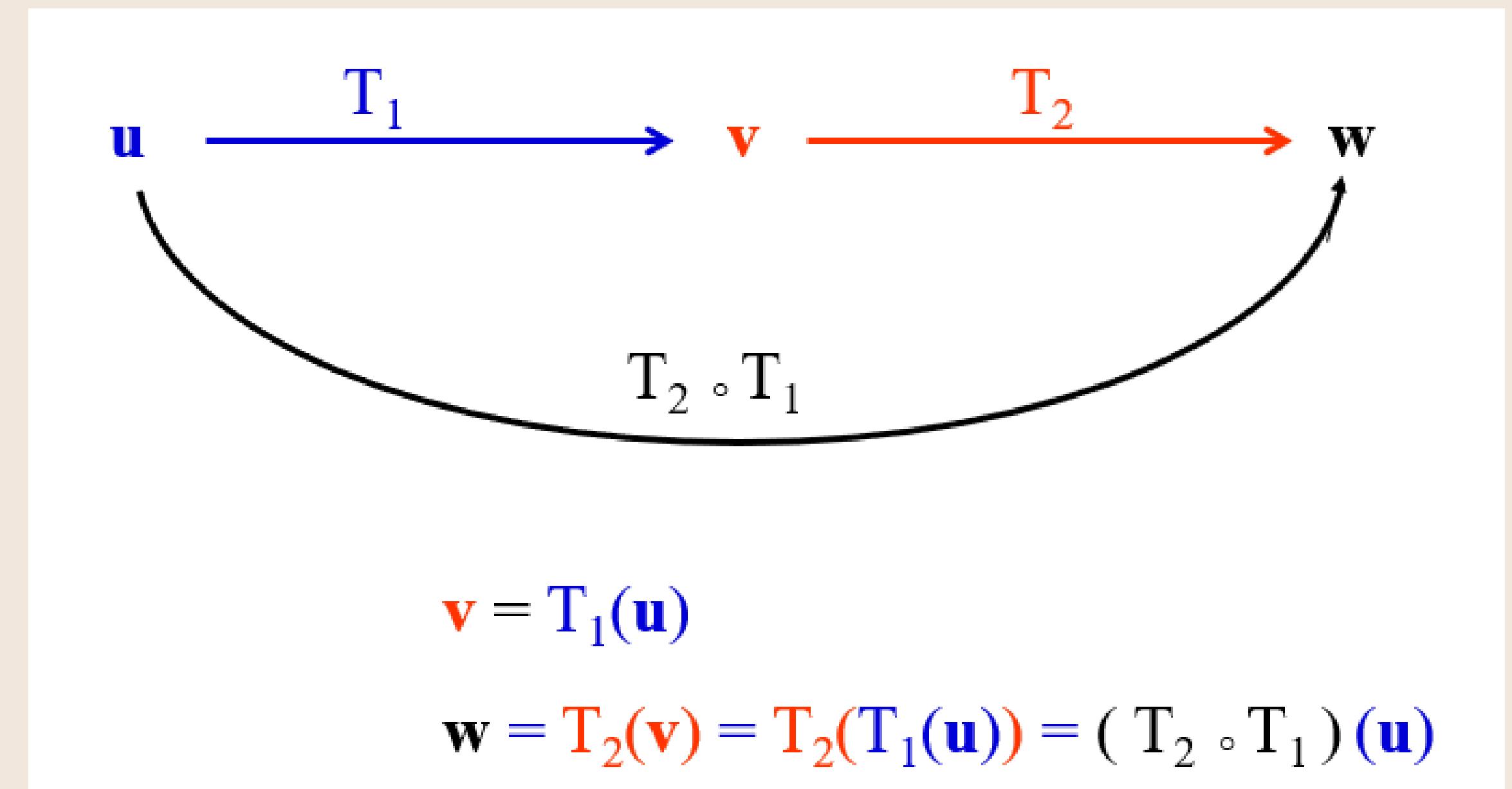
# Komposisi 2

# Transformasi



# Komposisi 2 Transformasi

Adalah salah satu cara dalam melakukan transformasi, dimana mengoperasikan matriks standar terlebih dahulu.



## contoh 2

diketahui vektor  $u = (-3, 4)$

Lakukan transformasi:

1. refleksi thdp sb. y

2. proyeksi

orthogonal thdp.

sb. x

# Jawab

dengan cara bertahap:

Komposisi dua / lebih transformasi:

Contoh:  $\mathbf{u} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

1.  $T_1$  refleksi terhadap sumbu-y

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad A_1 \mathbf{u} = \mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

2.  $T_2$  proyeksi ortogonal pada sumbu-x

$$A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad A_2 \mathbf{v} = \mathbf{w} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

www

dengan komposisi transformasi

$$A_2 \circ A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(A_2 \circ A_1) \mathbf{u} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$T_2 \circ T_1 = (A_2)(A_1)$$

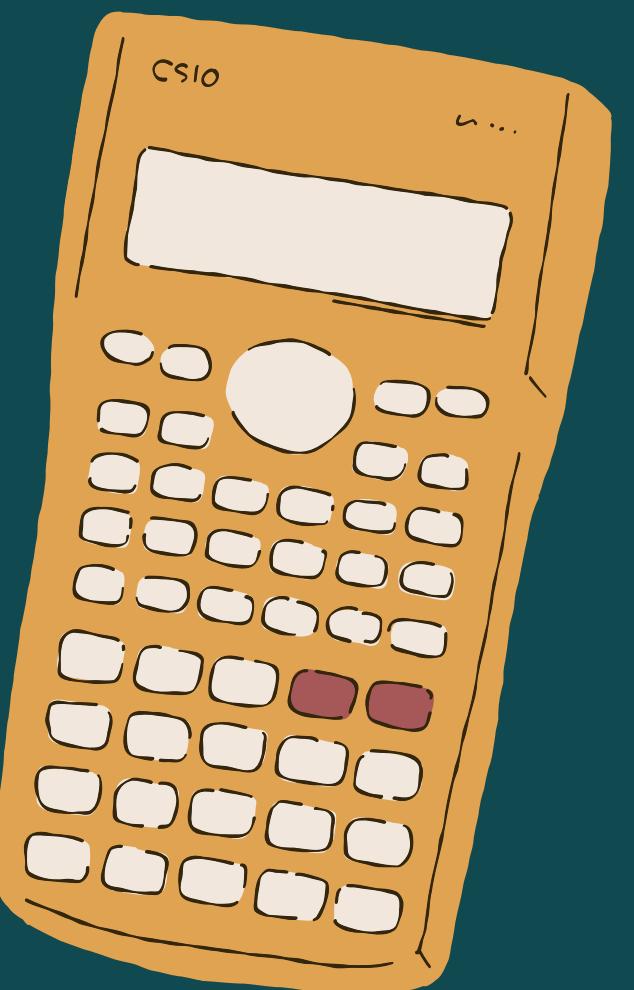
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

## Contoh Soal

### Soal 5

Carilah koordinat akhir dari  $(-3, 5)$ , jika dilakukan:

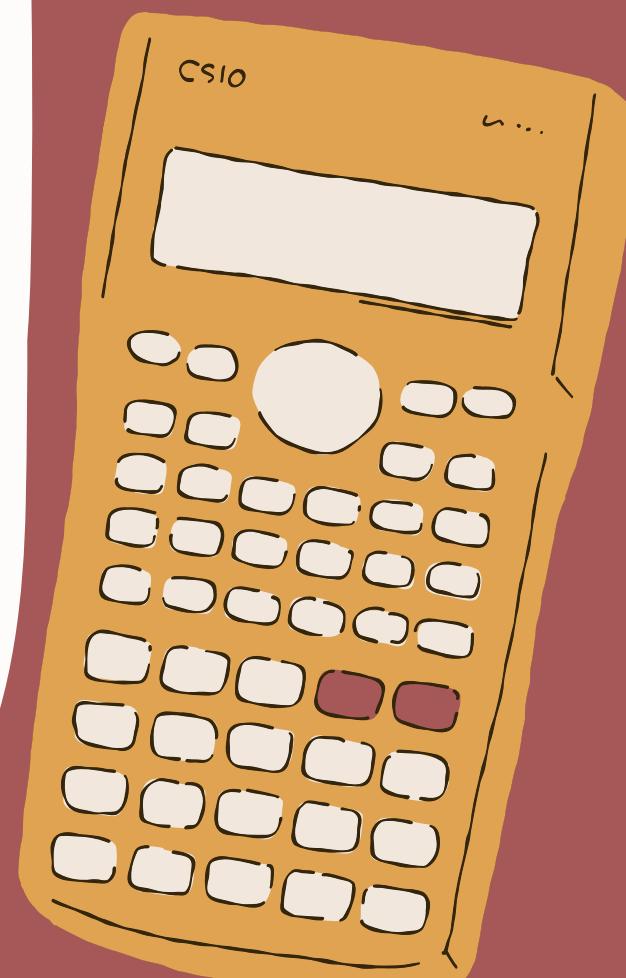
1. dilatasi sebesar  $k = 3$
2. dicerminkan sb.  $x = y$
3. pencerminan thdp sb.  $x$
4. proyeksi orthogonal thdp sb.  $y$
5. rotasi 30 derajat



# Cara 1:

- Lakukan step by step perkalian titik dengan matrix,
- titik hasilnya, dikalikan dengan matrix lagi,
- begitu seterusnya, hingga akhir

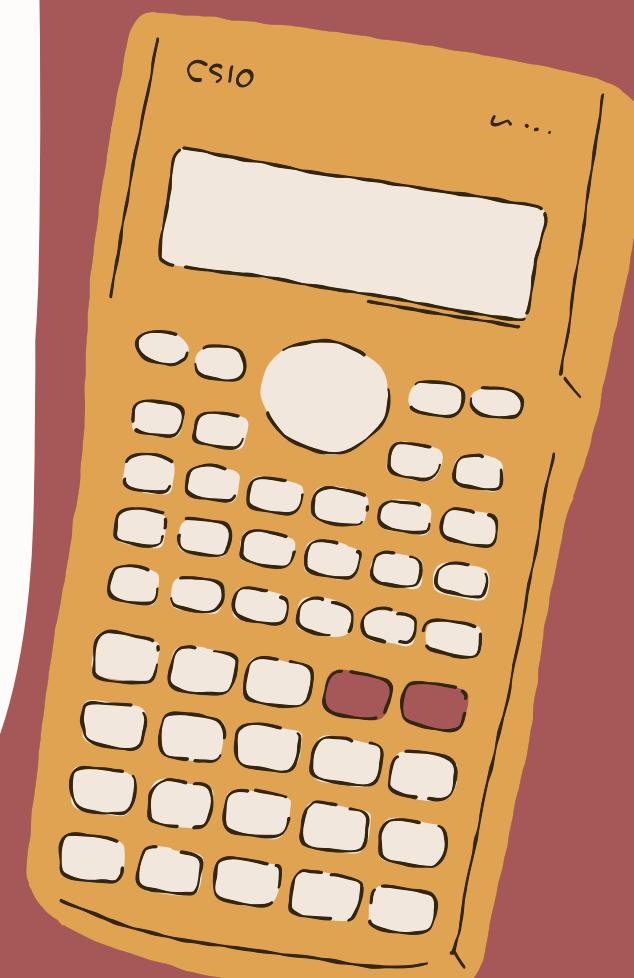
dilatasi	3	0	x	-3	=	-9	nilai= 3
sebesar k = 3	0	3		5		15	
cermin x = y	0	1	x	-9	=	15	nilai= 3
	1	0		15		-9	
cermin sumbu x	1	0	x	15	=	15	nilai= 3
	0	-1		-9		9	
proyeksi orto y	0	0	x	15	=	0	nilai= 3
	0	1		9		9	
rotasi 30	0,87	-0,5	x	0	=	-4,5	nilai= 3
	0,5	0,87		9		7,83	



## Cara 2:

- lakukan step by step dengan menggunakan perkalian matrix dengan matrix,
- matrix hasil, dengan matrix berikutnya,
- begitu seterusnya hingga matrix terakhir dikalikan dengan titik awal

cermin $x = y$		$x$	dilatasi $k = 3$		=	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	nilai= 3
0	1	$x$	3	0	=	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	nilai= 3
1	0		0	3		$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	
cermin sumbu x		$x$	0	3	=	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	nilai= 3
1	0	$x$	0	3	=	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	nilai= 3
0	-1		3	0		$\begin{pmatrix} -3 \\ 0 \end{pmatrix}$	
proyeksi orto y		$x$	0	3	=	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	nilai= 3
0	0	$x$	0	3	=	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	nilai= 3
0	1		-3	0		$\begin{pmatrix} -3 \\ 0 \end{pmatrix}$	
rotasi 30		$x$	0	0	=	$\begin{pmatrix} 1,5 \\ 0 \end{pmatrix}$	nilai= 3
0,87	-0,5	$x$	0	0	=	$\begin{pmatrix} 1,5 \\ 0 \end{pmatrix}$	nilai= 3
0,5	0,87		-3	0		$\begin{pmatrix} -2,61 \\ 0 \end{pmatrix}$	
1,5	0	$x$	-3	0	=	$\begin{pmatrix} -4,5 \\ 7,83 \end{pmatrix}$	nilai= 3
-2,61	0		5				

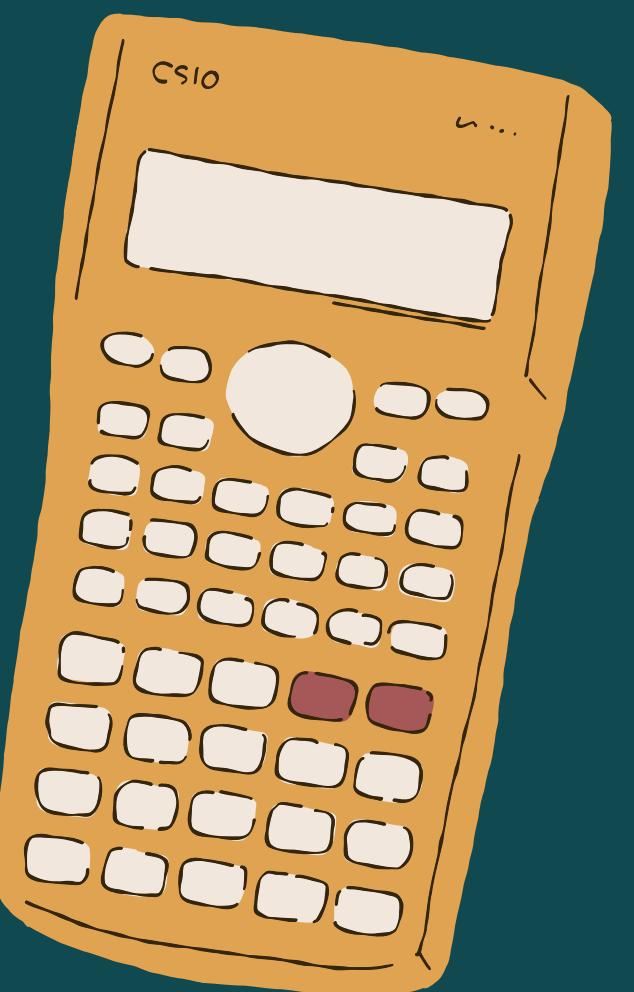


## Contoh Soal

### Soal 6

Carilah koordinat akhir dari  $(3, -8)$ , jika dilakukan:

1. pencermianan thdp sb. y
2. pencermianan thdp sb. x
3. pencermianan thdp sb.  $x = y$
4. rotasi 30 derajat



# Jawaban:

cermin sumbu y:

$$\begin{array}{|c|c|c|c|c|c|} \hline -1 & 0 & x & 3 & = & -3 \\ \hline 0 & 1 & & -8 & & -8 \\ \hline \end{array}$$

1. cermin sumbu x:

$$\begin{array}{|c|c|c|c|c|c|} \hline 1 & 0 & x & -3 & = & -3 \\ \hline 0 & -1 & & -8 & & 8 \\ \hline \end{array}$$

2. cermin terhadap garis  $x = y$

$$\begin{array}{|c|c|c|c|c|c|} \hline 0 & 1 & x & -3 & = & 8 \\ \hline 1 & 0 & & 8 & & -3 \\ \hline \end{array}$$

3. Rotasi 30

$$\begin{array}{|c|c|c|c|c|c|} \hline 0.87 & -0.5 & x & 8 & = & 8.46 \\ \hline 0.5 & 0.87 & & -3 & & 1.39 \\ \hline \end{array}$$

1. Perkalian matrix

$$\begin{array}{|c|c|c|c|c|c|} \hline 1 & 0 & x & -1 & 0 & = & -1 & 0 \\ \hline 0 & -1 & & 0 & 1 & & 0 & -1 \\ \hline \end{array}$$

2. Perkalian matrix

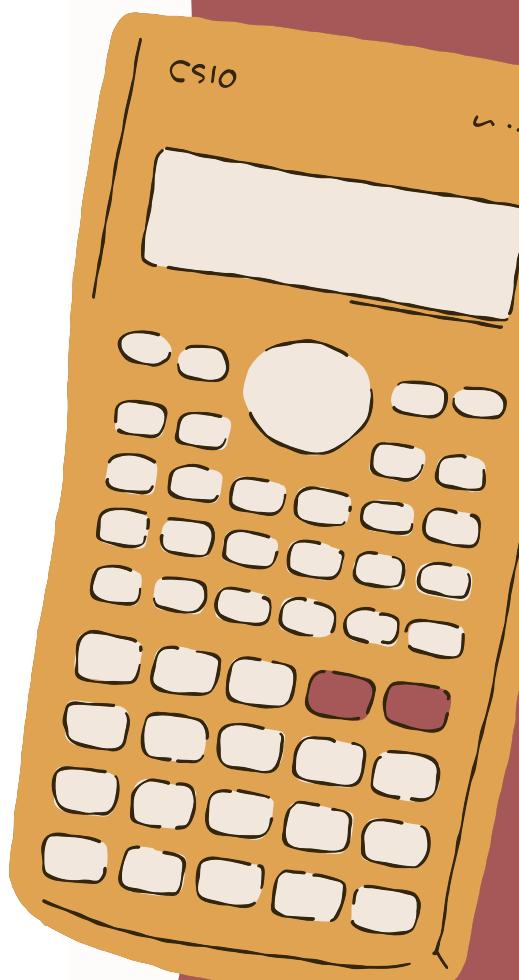
$$\begin{array}{|c|c|c|c|c|c|} \hline 0 & 1 & x & -1 & 0 & = & 0 & -1 \\ \hline 1 & 0 & & 0 & -1 & & -1 & 0 \\ \hline \end{array}$$

3. Perkalian matrix

$$\begin{array}{|c|c|c|c|c|c|} \hline 0.87 & -0.5 & x & 0 & -1 & = & 0.5 & -0.87 \\ \hline 0.5 & 0.87 & & -1 & 0 & & -0.87 & -0.5 \\ \hline \end{array}$$

4. kalikan dengan vektor awal

$$\begin{array}{|c|c|c|c|c|c|} \hline 0.5 & -0.87 & x & 3 & = & 8.46 \\ \hline -0.87 & -0.5 & & -8 & & 1.39 \\ \hline \end{array}$$



# TUGAS KELOMPOK:

- Membuat contoh soal sendiri (ppt) boleh mengarang sendiri atau dari internet.
- Satu kelompok = 1 soal Dijawab step by step seperti yang saya ajarkan
- Upload ke drive (share) Nilai max PR adalah : 91,
- Note: Jika ada yang tidak aktif, anggota yang aktif langsung memberi tau asisten, dan asisten akan mengurangi nilai nya.

**Thank you for listening!**

