

Tugas 2 Komputasi Numerik

Kelompok B - 17

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1. Metode Grafik

a. $e^x - x - 2 = 0$

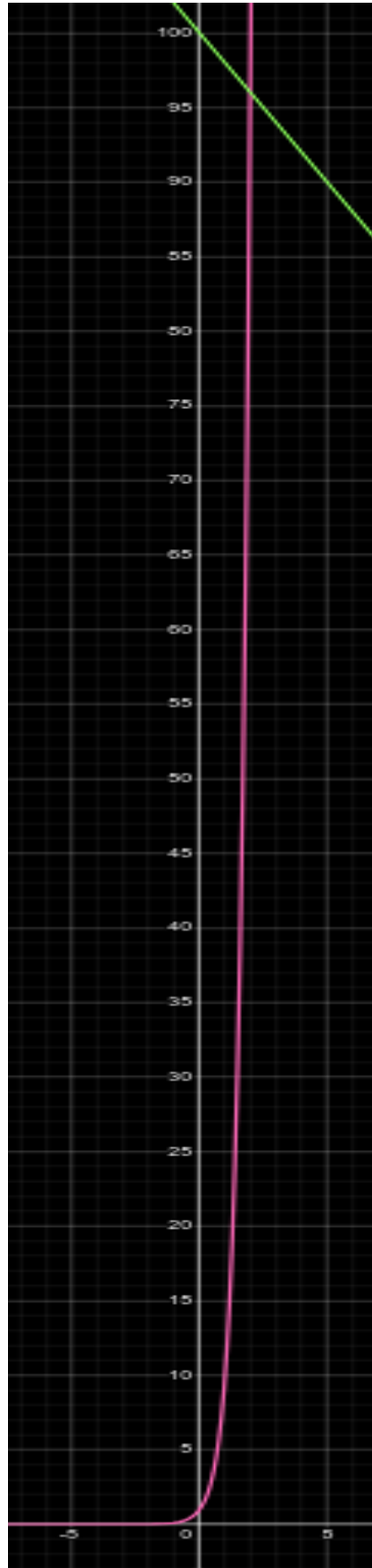
$$e^x = x + 2$$

$$y = e^x \text{ dan } y = x + 2$$

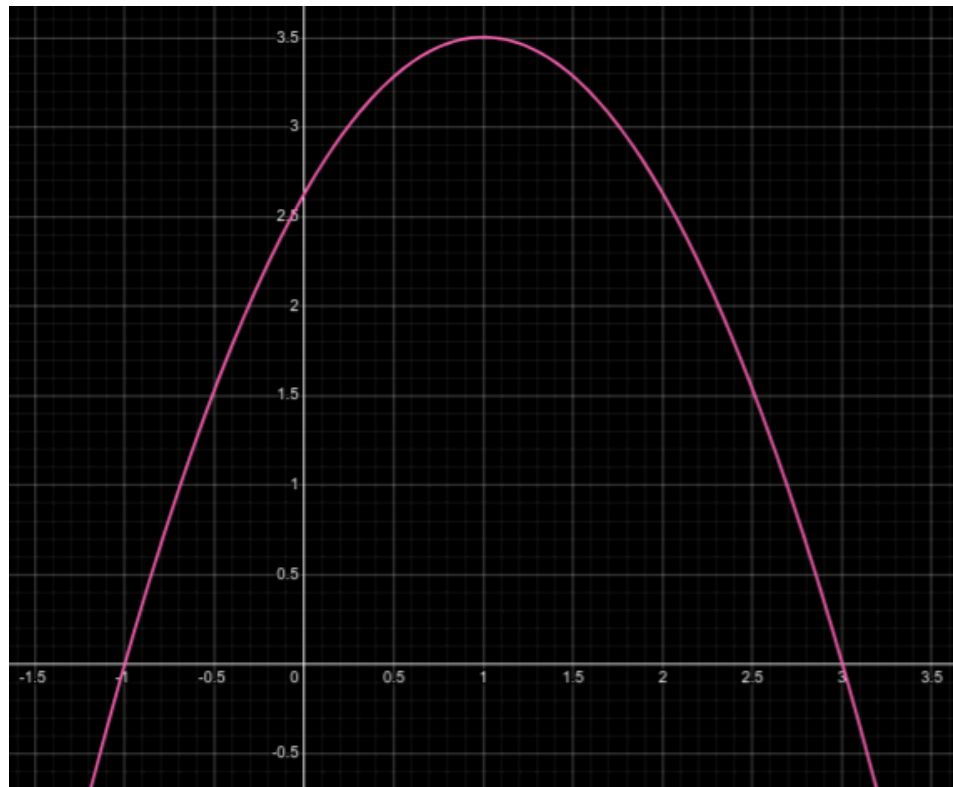


b. $10^x = 100 - 2x$

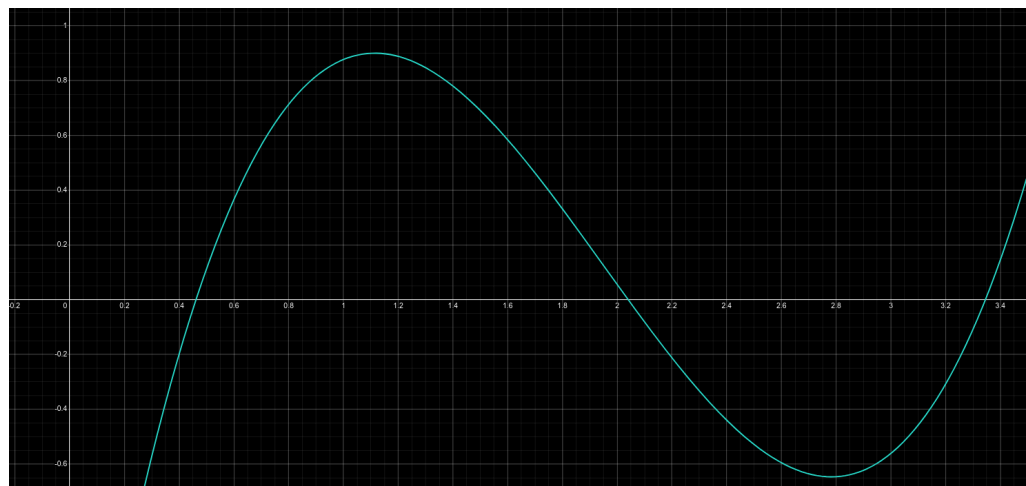
$y = 10^x$ dan $y = 100 - 2x$



c. $-0,874x^2 + 1,75x + 2,627$



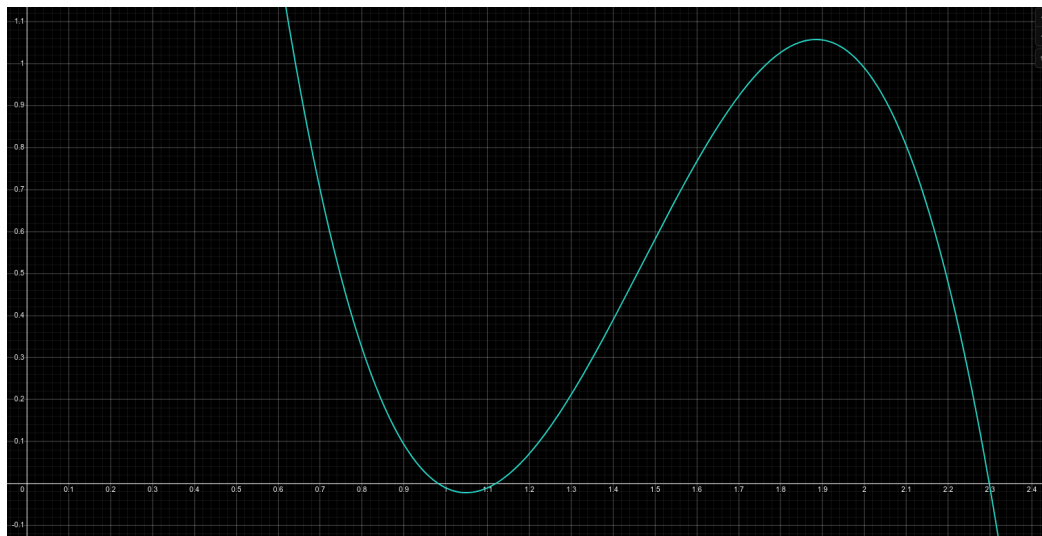
d. $-2,1 + 6,21x - 3,9x^2 + 0,667x^3$



e. $(1 - 0,6x) / x$



f. $9,36 - 21,963x + 16,2965x^2 - 3,70377x^3$



2. Metode Tabulasi

a. $e^x - x - 2 = 0$

| x | f(x) | x | f(x) | x | f(x) |
|------|-----------|-------|-----------|--------|-----------|
| -2 | 0.135335 | -1.9 | 0.049569 | -1.85 | 0.007237 |
| -1.9 | 0.049569 | -1.89 | 0.041072 | -1.849 | 0.006394 |
| -1.8 | -0.034701 | -1.88 | 0.032590 | -1.848 | 0.005552 |
| -1.7 | -0.117316 | -1.87 | 0.024124 | -1.847 | 0.004710 |
| -1.6 | -0.198103 | -1.86 | 0.015673 | -1.846 | 0.003867 |
| -1.5 | -0.276870 | -1.85 | 0.007237 | -1.845 | 0.003025 |
| -1.4 | -0.353403 | -1.84 | -0.001183 | -1.844 | 0.002183 |
| -1.3 | -0.427468 | -1.83 | -0.009586 | -1.843 | 0.001342 |
| -1.2 | -0.498806 | -1.82 | -0.017974 | -1.842 | 0.000500 |
| -1.1 | -0.567129 | -1.81 | -0.026346 | -1.841 | -0.000341 |
| -1 | -0.632121 | -1.8 | -0.034701 | -1.84 | -0.001183 |
| -0.9 | -0.693430 | | | | |
| -0.8 | -0.750671 | 1.1 | -0.095834 | 1.14 | -0.013232 |
| -0.7 | -0.803415 | 1.11 | -0.075642 | 1.141 | -0.011103 |
| -0.6 | -0.851188 | 1.12 | -0.055146 | 1.142 | -0.008972 |
| -0.5 | -0.893469 | 1.13 | -0.034343 | 1.143 | -0.006837 |
| -0.4 | -0.929680 | 1.14 | -0.013232 | 1.144 | -0.004700 |
| -0.3 | -0.959182 | 1.15 | 0.008193 | 1.145 | -0.002559 |
| -0.2 | -0.981269 | 1.16 | 0.029933 | 1.146 | -0.000415 |
| -0.1 | -0.995163 | 1.17 | 0.051993 | 1.147 | 0.001733 |
| 0 | -1.000000 | 1.18 | 0.074374 | 1.148 | 0.003883 |
| 0.1 | -0.994829 | 1.19 | 0.097081 | 1.149 | 0.006036 |
| 0.2 | -0.978597 | 1.2 | 0.120117 | 1.15 | 0.008193 |
| 0.3 | -0.950141 | | | | |
| 0.4 | -0.908175 | | | | |
| 0.5 | -0.851279 | | | | |
| 0.6 | -0.777881 | | | | |
| 0.7 | -0.686247 | | | | |
| 0.8 | -0.574459 | | | | |
| 0.9 | -0.440397 | | | | |
| 1 | -0.281718 | | | | |
| 1.1 | -0.095834 | | | | |
| 1.2 | 0.120117 | | | | |
| 1.3 | 0.369297 | | | | |
| 1.4 | 0.655200 | | | | |
| 1.5 | 0.981689 | | | | |

Akar pendekatan $e^x - x - 2 = 0$ adalah

$$x_1 = -1,841$$

$$x_2 = 1.146$$

b. $10^x = 100 - 2x$

| x | f(x) | x | f(x) | x | f(x) |
|-----|------------|------|------------|-------|-----------|
| 0 | -99 | 1.9 | -16.767177 | 1.98 | -0.540741 |
| 0.1 | -98.541075 | 1.91 | -14.896948 | 1.981 | -0.318593 |
| 0.2 | -98.015107 | 1.92 | -12.983623 | 1.982 | -0.095937 |
| 0.3 | -97.404738 | 1.93 | -11.026196 | 1.983 | 0.127228 |
| 0.4 | -96.688114 | 1.94 | -9.023641 | 1.984 | 0.350902 |
| 0.5 | -95.837722 | 1.95 | -6.974906 | 1.985 | 0.575088 |
| 0.6 | -94.818928 | 1.96 | -4.878916 | 1.986 | 0.799786 |
| 0.7 | -93.588128 | 1.97 | -2.734570 | 1.987 | 1.024997 |
| 0.8 | -92.090427 | 1.98 | -0.540741 | 1.988 | 1.250722 |
| 0.9 | -90.256718 | 1.99 | 1.703722 | 1.989 | 1.476964 |
| 1 | -88 | 2 | 4 | 1.99 | 1.703722 |
| 1.1 | -85.210746 | | | | |
| 1.2 | -81.751068 | | | | |
| 1.3 | -77.447377 | | | | |
| 1.4 | -72.081136 | | | | |
| 1.5 | -65.377223 | | | | |
| 1.6 | -56.989283 | | | | |
| 1.7 | -46.481277 | | | | |
| 1.8 | -33.304266 | | | | |
| 1.9 | -16.767177 | | | | |
| 2 | 4 | | | | |
| 2.1 | 30.092541 | | | | |
| 2.2 | 62.889319 | | | | |
| 2.3 | 104.126231 | | | | |
| 2.4 | 155.988643 | | | | |
| 2.5 | 221.227766 | | | | |
| 2.6 | 303.307171 | | | | |
| 2.7 | 406.587234 | | | | |
| 2.8 | 536.557344 | | | | |
| 2.9 | 700.128235 | | | | |
| 3 | 906 | | | | |

Akar pendekatan $10^x = 100 - 2x$ adalah
 $x = 1.982$

c. $-0,874x^2 + 1,75x + 2,627$

| x | f(x) | x | f(x) | x | f(x) |
|------|-----------|-------|-----------|--------|-----------|
| -1.5 | -1.964500 | -1.1 | -0.355540 | -1.01 | -0.032067 |
| -1.4 | -1.536040 | -1.09 | -0.318899 | -1.009 | -0.028553 |
| -1.3 | -1.125060 | -1.08 | -0.282434 | -1.008 | -0.025040 |
| -1.2 | -0.731560 | -1.07 | -0.246143 | -1.007 | -0.021529 |
| -1.1 | -0.355540 | -1.06 | -0.210026 | -1.006 | -0.018019 |
| -1 | 0.003000 | -1.05 | -0.174085 | -1.005 | -0.014512 |
| -0.9 | 0.344060 | -1.04 | -0.138318 | -1.004 | -0.011006 |
| -0.8 | 0.667640 | -1.03 | -0.102727 | -1.003 | -0.007502 |
| -0.7 | 0.973740 | -1.02 | -0.067310 | -1.002 | -0.003999 |
| -0.6 | 1.262360 | -1.01 | -0.032067 | -1.001 | -0.000499 |
| -0.5 | 1.533500 | -1 | 0.003000 | -1 | 0.003000 |
| -0.4 | 1.787160 | | | | |
| -0.3 | 2.023340 | 3.0 | 0.011000 | 3 | 0.011000 |
| -0.2 | 2.242040 | 3.01 | -0.024027 | 3.001 | 0.007505 |
| -0.1 | 2.443260 | 3.02 | -0.059230 | 3.002 | 0.004009 |
| 0 | 2.627000 | 3.03 | -0.094607 | 3.003 | 0.000510 |
| 0.1 | 2.793260 | 3.04 | -0.130158 | 3.004 | -0.002990 |
| 0.2 | 2.942040 | 3.05 | -0.165885 | 3.005 | -0.006492 |
| 0.3 | 3.073340 | 3.06 | -0.201786 | 3.006 | -0.009995 |
| 0.4 | 3.187160 | 3.07 | -0.237863 | 3.007 | -0.013501 |
| 0.5 | 3.283500 | 3.08 | -0.274114 | 3.008 | -0.017008 |
| 0.6 | 3.362360 | 3.09 | -0.310539 | 3.009 | -0.020517 |
| 0.7 | 3.423740 | 3.1 | -0.347140 | 3.01 | -0.024027 |
| 0.8 | 3.467640 | | | | |
| 0.9 | 3.494060 | | | | |
| 1 | 3.503000 | | | | |
| 1.1 | 3.494460 | | | | |
| 1.2 | 3.468440 | | | | |
| 1.3 | 3.424940 | | | | |
| 1.4 | 3.363960 | | | | |
| 1.5 | 3.285500 | | | | |
| 1.6 | 3.189560 | | | | |
| 1.7 | 3.076140 | | | | |
| 1.8 | 2.945240 | | | | |
| 1.9 | 2.796860 | | | | |
| 2.0 | 2.631000 | | | | |
| 2.1 | 2.447660 | | | | |
| 2 | 2.246840 | | | | |
| 2.3 | 2.028540 | | | | |
| 2.4 | 1.792760 | | | | |
| 2.5 | 1.539500 | | | | |
| 2.6 | 1.268760 | | | | |
| 2.7 | 0.980540 | | | | |
| 2.8 | 0.674840 | | | | |
| 2.9 | 0.351660 | | | | |
| 3 | 0.011000 | | | | |
| 3.1 | -0.347140 | | | | |
| 3.2 | -0.722760 | | | | |
| 3.3 | -1.115860 | | | | |
| 3.4 | -1.526440 | | | | |
| 3.5 | -1.954500 | | | | |

Akar pendekatan $-0,874x^2 + 1,75x + 2,627$ adalah

$x_1 = -1$

$x_2 = 3.003$

d. $-2,1 + 6,21x - 3,9x^2 + 0,667x^3$

| x | f(x) | x | f(x) | x | f(x) |
|-----|-----------|------|-----------|-------|-----------|
| 0 | -2.100000 | 0.4 | -0.197312 | 0.46 | -0.003717 |
| 0.1 | -1.517333 | 0.41 | -0.163520 | 0.461 | -0.000674 |
| 0.2 | -1.008664 | 0.42 | -0.130343 | 0.462 | 0.002362 |
| 0.3 | -0.569991 | 0.43 | -0.097779 | 0.463 | 0.005393 |
| 0.4 | -0.197312 | 0.44 | -0.065822 | 0.464 | 0.008417 |
| 0.5 | 0.113375 | 0.45 | -0.034470 | 0.465 | 0.011436 |
| 0.6 | 0.366072 | 0.46 | -0.003717 | 0.466 | 0.014448 |
| 0.7 | 0.564781 | 0.47 | 0.026440 | 0.467 | 0.017455 |
| 0.8 | 0.713504 | 0.48 | 0.056005 | 0.468 | 0.020456 |
| 0.9 | 0.816243 | 0.49 | 0.084982 | 0.469 | 0.023451 |
| 1 | 0.877000 | 0.5 | 0.113375 | 0.47 | 0.026440 |
| 1.1 | 0.899777 | | | | |
| 1.2 | 0.888576 | 2 | 0.056000 | 2.04 | 0.000766 |
| 1.3 | 0.847399 | 2.01 | 0.042151 | 2.041 | -0.000609 |
| 1.4 | 0.780248 | 2.02 | 0.028326 | 2.042 | -0.001983 |
| 1.5 | 0.691125 | 2.03 | 0.014530 | 2.043 | -0.003356 |
| 1.6 | 0.584032 | 2.04 | 0.000766 | 2.044 | -0.004730 |
| 1.7 | 0.462971 | 2.05 | -0.012962 | 2.045 | -0.006103 |
| 1.8 | 0.331944 | 2.06 | -0.026649 | 2.046 | -0.007475 |
| 1.9 | 0.194953 | 2.07 | -0.040291 | 2.047 | -0.008847 |
| 2 | 0.056000 | 2.08 | -0.053886 | 2.048 | -0.010219 |
| 2.1 | -0.080913 | 2.09 | -0.067428 | 2.049 | -0.011591 |
| 2.2 | -0.211784 | 2.1 | -0.080913 | 2.05 | -0.012962 |
| 2.3 | -0.332611 | | | | |
| 2.4 | -0.439392 | 3.3 | -0.108021 | 3.34 | -0.013217 |
| 2.5 | -0.528125 | 3.31 | -0.085141 | 3.341 | -0.010734 |
| 2.6 | -0.594808 | 3.32 | -0.061717 | 3.342 | -0.008246 |
| 2.7 | -0.635439 | 3.33 | -0.037743 | 3.343 | -0.005751 |
| 2.8 | -0.646016 | 3.34 | -0.013217 | 3.344 | -0.003251 |
| 2.9 | -0.622537 | 3.35 | 0.011865 | 3.345 | -0.000746 |
| 3 | -0.561000 | 3.36 | 0.037508 | 3.346 | 0.001765 |
| 3.1 | -0.457403 | 3.37 | 0.063716 | 3.347 | 0.004282 |
| 3.2 | -0.307744 | 3.38 | 0.090493 | 3.348 | 0.006804 |
| 3.3 | -0.108021 | 3.39 | 0.117842 | 3.349 | 0.009332 |
| 3.4 | 0.145768 | 3.4 | 0.145768 | 3.35 | 0.011865 |
| 3.5 | 0.457625 | | | | |

Akar pendekatan $-2,1 + 6,21x - 3,9x^2 + 0,667x^3$ adalah

$$x_1 = 0.461$$

$$x_2 = 2.041$$

$$x_3 = 3.345$$

e. $(1 - 0,6x) / x$

| x | f(x) | x | f(x) | x | f(x) |
|------|------------|------|-----------|-------|-----------|
| -2 | -1.100000 | 1.6 | 0.025000 | 1.66 | 0.002410 |
| -1.9 | -1.126316 | 1.61 | 0.021118 | 1.661 | 0.002047 |
| -1.8 | -1.155556 | 1.62 | 0.017284 | 1.662 | 0.001685 |
| -1.7 | -1.188235 | 1.63 | 0.013497 | 1.663 | 0.001323 |
| -1.6 | -1.225000 | 1.64 | 0.009756 | 1.664 | 0.000962 |
| -1.5 | -1.266667 | 1.65 | 0.006061 | 1.665 | 0.000601 |
| -1.4 | -1.314286 | 1.66 | 0.002410 | 1.666 | 0.000240 |
| -1.3 | -1.369231 | 1.67 | -0.001198 | 1.667 | -0.000120 |
| -1.2 | -1.433333 | 1.68 | -0.004762 | 1.668 | -0.000480 |
| -1.1 | -1.509091 | 1.69 | -0.008284 | 1.669 | -0.000839 |
| -1 | -1.600000 | 1.7 | -0.011765 | 1.67 | -0.001198 |
| -0.9 | -1.711111 | | | | |
| -0.8 | -1.850000 | | | | |
| -0.7 | -2.028571 | | | | |
| -0.6 | -2.266667 | | | | |
| -0.5 | -2.600000 | | | | |
| -0.4 | -3.100000 | | | | |
| -0.3 | -3.933333 | | | | |
| -0.2 | -5.600000 | | | | |
| -0.1 | -10.600000 | | | | |
| 0 | undefined | | | | |
| 0.1 | 9.400000 | | | | |
| 0.2 | 4.400000 | | | | |
| 0.3 | 2.733333 | | | | |
| 0.4 | 1.900000 | | | | |
| 0.5 | 1.400000 | | | | |
| 0.6 | 1.066667 | | | | |
| 0.7 | 0.828571 | | | | |
| 0.8 | 0.650000 | | | | |
| 0.9 | 0.511111 | | | | |
| 1 | 0.400000 | | | | |
| 1.1 | 0.309091 | | | | |
| 1.2 | 0.233333 | | | | |
| 1.3 | 0.169231 | | | | |
| 1.4 | 0.114286 | | | | |
| 1.5 | 0.066667 | | | | |
| 1.6 | 0.025000 | | | | |
| 1.7 | -0.011765 | | | | |
| 1.8 | -0.044444 | | | | |
| 1.9 | -0.073684 | | | | |
| 2 | -0.100000 | | | | |

Akar pendekatan $(1 - 0,6x) / x$ adalah
 $x = 1.667$

f. $9,36 - 21,963x + 16,2965x^2 - 3,70377x^3$

| x | f(x) | x | f(x) | x | f(x) |
|-----|-----------|------|-----------|-------|-----------|
| 0 | 9.360000 | 0.9 | 0.093417 | 0.98 | 0.001460 |
| 0.1 | 7.322961 | 0.91 | 0.077748 | 0.981 | 0.000772 |
| 0.2 | 5.589630 | 0.92 | 0.063316 | 0.982 | 0.000095 |
| 0.3 | 4.137783 | 0.93 | 0.050100 | 0.983 | -0.000571 |
| 0.4 | 2.945199 | 0.94 | 0.038075 | 0.984 | -0.001226 |
| 0.5 | 1.989654 | 0.95 | 0.027221 | 0.985 | -0.001871 |
| 0.6 | 1.248926 | 0.96 | 0.017516 | 0.986 | -0.002505 |
| 0.7 | 0.700792 | 0.97 | 0.008936 | 0.987 | -0.003129 |
| 0.8 | 0.323030 | 0.98 | 0.001460 | 0.988 | -0.003741 |
| 0.9 | 0.093417 | 0.99 | -0.004935 | 0.989 | -0.004343 |
| 1 | -0.010270 | 1 | -0.010270 | 0.99 | -0.004935 |
| 1.1 | -0.010253 | | | | |
| 1.2 | 0.071245 | 1.1 | -0.010253 | 1.11 | -0.005403 |
| 1.3 | 0.212002 | 1.11 | -0.005403 | 1.111 | -0.004874 |
| 1.4 | 0.389795 | 1.12 | 0.000239 | 1.112 | -0.004337 |
| 1.5 | 0.582401 | 1.13 | 0.006652 | 1.113 | -0.003792 |
| 1.6 | 0.767598 | 1.14 | 0.013813 | 1.114 | -0.003240 |
| 1.7 | 0.923163 | 1.15 | 0.021700 | 1.115 | -0.002679 |
| 1.8 | 1.026873 | 1.16 | 0.030291 | 1.116 | -0.002111 |
| 1.9 | 1.056507 | 1.17 | 0.039563 | 1.117 | -0.001535 |
| 2 | 0.989840 | 1.18 | 0.049494 | 1.118 | -0.000951 |
| 2.1 | 0.804651 | 1.19 | 0.060062 | 1.119 | -0.000360 |
| 2.2 | 0.478717 | 1.2 | 0.071245 | 1.12 | 0.000239 |
| 2.3 | -0.010185 | | | | |
| 2.4 | -0.684276 | 2.2 | 0.478717 | 2.29 | 0.046672 |
| 2.5 | -1.565781 | 2.21 | 0.437527 | 2.291 | 0.041069 |
| 2.6 | -2.676922 | 2.22 | 0.394685 | 2.292 | 0.035448 |
| 2.7 | -4.039920 | 2.23 | 0.350169 | 2.293 | 0.029808 |
| 2.8 | -5.676999 | 2.24 | 0.303957 | 2.294 | 0.024150 |
| 2.9 | -7.610382 | 2.25 | 0.256026 | 2.295 | 0.018474 |
| 3 | -9.862290 | 2.26 | 0.206354 | 2.296 | 0.012779 |
| | | 2.27 | 0.154920 | 2.297 | 0.007066 |
| | | 2.28 | 0.101700 | 2.298 | 0.001334 |
| | | 2.29 | 0.046672 | 2.299 | -0.004416 |
| | | 2.3 | -0.010185 | 2.3 | -0.010185 |

Akar pendekatan $9,36 - 21,963x + 16,2965x^2 - 3,70377x^3$ adalah

$x_1 = 0.982$

$x_2 = 1.12$

$x_3 = 2.298$

3. Metode Bolzano

a. $x^3 - 3x + 1 = 0$, ($x_0 = 1,5$; s/d 3D)

$$x_0 = 1,5$$

$$F(x_0) = (1,5)^3 - 3(1,5) + 1 = -0,125$$

Cari x_1 agar $F(x_1)$ bernilai positif

$$x_1 = 2$$

$$F(x_1) = (2)^3 - 3(2) + 1 = 3$$

Iterasi 1

$$x_2 = (x_1 + x_0) / 2 = (1,5 + 2) / 2 = 1,75$$

$$F(x_2) = (1,75)^3 - 3(1,75) + 1 = 1,109375$$

$F(x_2)$ positif dan $F(x_2) < F(x_1)$, maka x_2 menggantikan x_1 .

Nilai $x_0 = 1,5$ dan $x_1 = 1,75$ untuk iterasi berikutnya.

Iterasi 2

$$x_2 = (x_1 + x_0) / 2 = (1,5 + 1,75) / 2 = 1,625$$

$$F(x_2) = (1,625)^3 - 3(1,625) + 1 = 0,416015625$$

x_2 telah mencapai 3D.

| Iterasi | x_0 | x_1 | x_2 | $F(x_0)$ | $F(x_1)$ | $F(x_2)$ |
|---------|-------|-------|-------|----------|----------|-------------|
| 1 | 1,5 | 2 | 1,75 | -0,125 | 3 | 1,109375 |
| 2 | 1,5 | 1,75 | 1,625 | -0,125 | 1,109375 | 0,416015625 |

Nilai x_2 mendekati 1,625.

b. $\cos x = 3x$, ($x_0=0,3$; s/d 5D)

$$\cos x - 3x = 0$$

$$x_0 = 0,3$$

$$F(x_0) = \cos(0,3) - 3(0,3) = 0,0553365$$

Cari x_1 yang $F(x_1)$ negatif

$$x_1 = 0,4$$

$$F(x_1) = \cos(0,4) - 3(0,4) = -0,278939$$

Iterasi 1

$$x_2 = (x_1 + x_2) / 2 = (0,3 + 0,4) / 2 = 0,35$$

$$F(x_2) = \cos(0,4) - 3(0,4) = -0,110627$$

$F(x_2)$ negatif dan $F(x_2) > F(x_1)$, maka x_2 menggantikan x_1 .

Nilai $x_0 = 0,3$ dan $x_1 = 0,35$ untuk iterasi berikutnya.

Iterasi 2

$$x_2 = (x_1 + x_2) / 2 = (0,3 + 0,35) / 2 = 0,325$$

$$F(x_2) = \cos(0,325) - 3(0,325) = -0,0273493$$

$F(x_2)$ negatif dan $F(x_2) > F(x_1)$, maka x_2 menggantikan x_1 .

Nilai $x_0 = 0,3$ dan $x_1 = 0,325$ untuk iterasi berikutnya.

Iterasi 3

$$x_2 = (x_1 + x_2) / 2 = (0,3 + 0,325) / 2 = 0,3125$$

$$F(x_2) = \cos(0,3125) - 3(0,3125) = -0,0140679$$

$F(x_2)$ positif dan $F(x_2) > F(x_0)$, maka x_2 menggantikan x_0 .

Nilai $x_0 = 0,3125$ dan $x_1 = 0,325$ untuk iterasi berikutnya.

Iterasi 4

$$x_2 = (x_1 + x_2) / 2 = (0,3125 + 0,325) / 2 = 0,31875$$

$$F(x_2) = \cos(0,31875) - 3(0,31875) = -0,0662212$$

x_2 telah mencapai 5D.

| Iterasi | x_0 | x_1 | x_2 | $F(x_0)$ | $F(x_1)$ | $F(x_2)$ |
|---------|--------|-------|---------|-----------|------------|------------|
| 1 | 0,3 | 0,4 | 0,35 | 0,0553365 | -0,278939 | -0,110627 |
| 2 | 0,3 | 0,35 | 0,325 | 0,0553365 | -0,110627 | -0,0273493 |
| 3 | 0,3 | 0,325 | 0,3125 | 0,0553365 | -0,0273493 | 0,0140679 |
| 4 | 0,3125 | 0,325 | 0,31875 | 0,0140679 | -0,0273493 | -0,0662212 |

Nilai x_1 mendekati 0,31875.

c. $10^x = 100 - 2x$, ($x_0=2$; s/d 4D)

$$10^x - 100 + 2x = 0$$

$$x_0 = 2$$

$$F(x_0) = 10^2 - 100 + 2(2) = 4$$

Cari x_1 yang $F(x_1)$ negatif

$$x_1 = 1,9$$

$$F(x_1) = 10^{1,9} - 100 + 2(1,9) = -16,76718$$

Iterasi 1

$$x_2 = (x_1 + x_0) / 2 = (2 + 1,9) / 2 = 1,95$$

$$F(x_2) = 10^{1,95} - 100 + 2(1,95) = -6,97491$$

$F(x_2)$ negatif dan $F(x_2) > F(x_1)$, maka x_2 menggantikan x_1 .

Nilai $x_0 = 2$ dan $x_1 = 1,95$ untuk iterasi berikutnya.

Iterasi 2

$$x_2 = (x_1 + x_2) / 2 = (2 + 1,95) / 2 = 1,975$$

$$F(x_2) = 10^{1,975} - 100 + 2(1,975) = -1,64391$$

$F(x_2)$ negatif dan $F(x_2) > F(x_1)$, maka x_2 menggantikan x_1 .

Nilai $x_0 = 2$ dan $x_1 = 1,975$ untuk iterasi berikutnya.

Iterasi 3

$$x_2 = (x_1 + x_2) / 2 = (2 + 1,975) / 2 = 1,9875$$

$$F(x_2) = 10^{1,9875} - 100 + 2(1,9875) = 1,1378$$

x_2 telah mencapai 4D.

| Iterasi | x_0 | x_1 | x_2 | $F(x_0)$ | $F(x_1)$ | $F(x_2)$ |
|---------|-------|-------|--------|----------|-----------|----------|
| 1 | 2 | 1,9 | 1,95 | 4 | -16,76718 | -6,97491 |
| 2 | 2 | 1,95 | 1,975 | 4 | -6,97491 | -1,64391 |
| 3 | 2 | 1,975 | 1,9875 | 4 | -1,64391 | 1,1378 |

Nilai x_2 mendekati 1,9875.

d. $\ln x = 1 + 1/x^2$, ($x_0=3$; s/d 4D)

$$\ln x - 1 - 1/x^2 = 0$$

$$x_0 = 3$$

$$F(x_0) = \ln(3) - 1 - 1/(3)^2 = -0,0124988$$

Cari x_1 yang $F(x_1)$ positif

$$x_1 = 3,1$$

$$F(x_1) = \ln(3,1) - 1 - 1/(3,1)^2 = 0,0273438$$

Iterasi 1

$$x_2 = (x_1 + x_0) / 2 = (3 + 3,1) / 2 = 3,05$$

$$F(x_2) = \ln(3,05) - 1 - 1/(3,05)^2 = 0,00764361$$

$F(x_2)$ positif dan $F(x_2) < F(x_1)$, maka x_2 menggantikan x_1 .

Nilai $x_0 = 3$ dan $x_1 = 3,05$ untuk iterasi berikutnya.

Iterasi 2

$$x_2 = (x_1 + x_0) / 2 = (3 + 3,05) / 2 = 3,025$$

$$F(x_2) = \ln(3,025) - 1 - 1/(3,025)^2 = -0,00237106$$

x_2 telah mencapai 4D.

| Iterasi | X_0 | X_1 | X_2 | $F(X_0)$ | $F(X_1)$ | $F(X_2)$ |
|---------|-------|-------|-------|------------|------------|-------------|
| 1 | 3 | 3,1 | 3,05 | -0,0124988 | 0,0273438 | 0,00764361 |
| 2 | 3 | 3,05 | 3,025 | -0,0124988 | 0,00764361 | -0,00237106 |

Nilai x_2 mendekati 3,025.

e. $e^x - \ln x = 20$, ($x_0=3$; s/d 5D)

$$y = e^x - \ln x - 20 = 0$$

$$x_0 = 3$$

$$F(x_0) = e^3 - \ln(3) - 20 = -1,01308$$

Cari x_1 yang $F(x_1)$ positif

$$x_1 = 3,1$$

$$F(x_1) = e^{3,1} - \ln(3,1) - 20 = 1,06655$$

Iterasi 1

$$x_2 = (x_1 + x_2) / 2 = (3 + 3,1) / 2 = 3,05$$

$$F(x_2) = e^{3,05} - \ln(3,05) - 20 = 0,000202832$$

$F(x_2)$ positif dan $F(x_2) < F(x_1)$, maka x_2 menggantikan x_1 .

Nilai $x_0 = 3$ dan $x_1 = 3,05$ untuk iterasi berikutnya.

Iterasi 2

$$x_2 = (x_1 + x_2) / 2 = (3 + 3,05) / 2 = 3,025$$

$$F(x_2) = e^{3,025} - \ln(3,025) - 20 = -0,512906$$

$F(x_2)$ negatif dan $F(x_2) > F(x_0)$, maka x_2 menggantikan x_0 .

Nilai $x_0 = 3,025$ dan $x_1 = 3,05$ untuk iterasi berikutnya.

Iterasi 3

$$x_2 = (x_1 + x_2) / 2 = (3,025 + 3,05) / 2 = 3,0375$$

$$F(x_2) = e^{3,0375} - \ln(3,0375) - 20 = -0,257989$$

$F(x_2)$ negatif dan $F(x_2) > F(x_0)$, maka x_2 menggantikan x_0 .

Nilai $x_0 = 3,0375$ dan $x_1 = 3,05$ untuk iterasi berikutnya.

Iterasi 4

$$x_2 = (x_1 + x_2) / 2 = (3,0375 + 3,05) / 2 = 3,04375$$

$$F(x_2) = e^{3,04375} - \ln(3,04375) - 20 = -0,129305$$

X_2 telah mencapai 5D.

| Iterasi | X_0 | X_1 | X_2 | $F(X_0)$ | $F(X_1)$ | $F(X_2)$ |
|---------|--------|-------|---------|-----------|-------------|-------------|
| 1 | 3 | 3,1 | 3,05 | -1,01308 | 1,06655 | 0,000202832 |
| 2 | 3 | 3,05 | 3,025 | -1,01308 | 0,000202832 | -0,512906 |
| 3 | 3,025 | 3,05 | 3,0375 | -0,512906 | 0,000202832 | -0,257989 |
| 4 | 3,0375 | 3,05 | 3,04375 | -0,257989 | 0,000202832 | -0,129305 |

Nilai x_2 mendekati 3,04375.

f. $10^x - 1$, ($x_0=0$; s/d 4D)

$$10^x - 1 = 0$$

$$x_0 = 0$$

$$F(x_0) = 10^0 - 1 = 0$$

4. Metode Regula Falsi

a. $\sin x = 5x - 2$, ($x_0=0,4$; s/d 4D)

$$\sin x - 5x + 2 = 0$$

$$x_0 = 0,4$$

$$F(x_0) = \sin(0,4) - 5(0,4) + 2 = 0,389418$$

Cari x_1 agar $F(x_1)$ bernilai negatif

$$x_1 = 0,5$$

$$F(x_1) = \sin(0,5) - 5(0,5) + 2 = -0,0205745$$

Iterasi 1

$$x_2 = x_1 - \frac{F(x_1) \cdot (x_0 - x_1)}{F(x_0) - F(x_1)}$$

$$x_2 = (0,5) - \frac{(-0,0205745) \cdot (0,4 - 0,5)}{0,389418 - (-0,0205745)}$$

$$x_2 = (0,5) - \frac{(-0,0205745) \cdot (0,4 - 0,5)}{0,389418 - (-0,0205745)}$$

$$x_2 = (0,5) - (0,00557811)$$

$$x_2 = 0,494422$$

$$x_2 \approx 0,4944 \text{ (4D)}$$

b. $e^x = 2x + 21$, ($x_0=3$; s/d 4D)

$$e^x - 2x - 21 = 0$$

$$x_0 = 3$$

$$F(x_0) = e^3 - 2(3) - 21 = -6,91446$$

Cari x_1 agar $F(x_1)$ bernilai positif

$$x_1 = 3,5$$

$$F(x_1) = e^{3,5} - 2(3,5) - 21 = 5,11545$$

Iterasi 1

$$x_2 = x_1 - \frac{F(x_1) \cdot (x_0 - x_1)}{F(x_0) - F(x_1)}$$

$$x_2 = 3,5 - \frac{5,11545 \cdot (3 - 3,5)}{-6,91446 - 5,11545}$$

$$x_2 = 3,5 - 0,212614$$

$$x_2 = 3,28739$$

$$x_2 \approx 3,2874 \text{ (4D)}$$

c. $\cos x = 3x$, ($x_0=0,3$; s/d 5D)

$$\cos x - 3x = 0$$

$$x_0 = 0,3$$

$$F(x_0) = \cos(0,3) - 3(0,3) = 0,0553365$$

Cari x_1 agar $F(x_1)$ bernilai negatif

$$x_1 = 0,4$$

$$F(x_1) = \cos(0,4) - 3(0,4) = -0,278939$$

Iterasi 1

$$x_2 = x_1 - \frac{F(x_1) \cdot (x_0 - x_1)}{F(x_0) - F(x_1)}$$

$$x_2 = 0,4 - \frac{(-0,278939) \cdot (0,3 - 0,4)}{0,0553365 - (-0,278939)}$$

$$x_2 = 0,4 - 0,0834458$$

$$x_2 = 0,316554$$

$$x_2 \approx 0,31655 \text{ (5D)}$$

d. $\ln x = 1 + 1/x^2$, ($x_0=3$; s/d 4D)

$$\ln x - 1 - 1/x^2 = 0$$

$$x_0 = 3$$

$$F(x_0) = \ln(3) - 1 - 1/3^2 = -0,0124988$$

Cari x_1 agar $F(x_1)$ bernilai positif

$$x_1 = 3,1$$

$$F(x_1) = \ln(3,1) - 1 - 1/(3,1)^2 = 0,0273438$$

Iterasi 1

$$x_2 = x_1 - \frac{F(x_1) \cdot (x_0 - x_1)}{F(x_0) - F(x_1)}$$

$$x_2 = 3,1 - \frac{(0,0273438) \cdot (3 - 3,1)}{(-0,0124988) - 0,0273438}$$

$$x_2 = 3,1 - 0,0686296$$

$$x_2 = 3,03137$$

$$x_2 \approx 3,0314 \text{ (4D)}$$

e. $x^x = 10$, ($x_0=2,5$; s/d 4D)

$$x^x - 10 = 0$$

$$x_0 = 2,5$$

$$F(x_0) = (2,5)^{2,5} - 10 = -0,117882$$

Cari x_1 agar $F(x_1)$ bernilai positif

$$x_1 = 2,6$$

$$F(x_1) = (2,6)^{2,6} - 10 = 1,99308$$

Iterasi 1

$$x_2 = x_1 - \frac{F(x_1) \cdot (x_0 - x_1)}{F(x_0) - F(x_1)}$$

$$x_2 = 2,6 - \frac{1,99308 \cdot (2,5 - 2,6)}{-0,117882 - 1,99308}$$

$$x_2 = 2,6 - 0,0944157$$

$$x_2 = 2,50558$$

$$x_2 \approx 2,5056 \text{ (4D)}$$

f. $x^3 - 100=0$, ($x_0=4$; s/d 3D)

$$F(x_0) = 4^3 - 100 = -36$$

Cari x_1 agar $F(x_1)$ bernilai positif

$$x_1 = 5$$

$$F(x_1) = 5^3 - 100 = 25$$

Iterasi 1

$$x_2 = x_1 - \frac{F(x_1) \cdot (x_0 - x_1)}{F(x_0) - F(x_1)}$$

$$x_2 = 5 - \frac{25 \cdot (4 - 5)}{-36 - 25}$$

$$x_2 = 5 - 0,409836$$

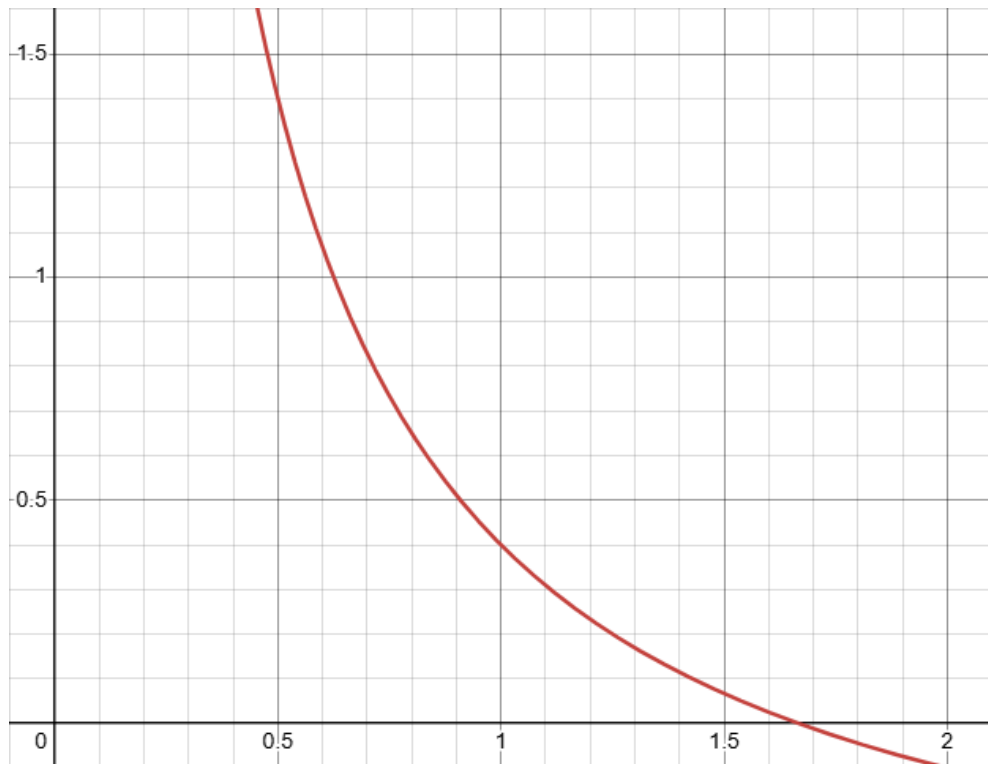
$$x_2 = 4,59016$$

$$x_2 \approx 4,590 \text{ (3D)}$$

5. Analisis fungsi $f(x) = \frac{(1-0.6x)}{x}$, $x_0 = 2$

Menggunakan 4 metode dengan 3 iterasi untuk penentuan tingkat presisi dan akurasi tertinggi

1. Grafik



Didapatkan akar fungsi berada di sekitar titik 1,6

2. Tabulasi

| $f(x) = (1-0.6x)/x$ | | |
|---------------------|----------|-----------|
| Iterasi | x | f(x) |
| 1 | 2,000000 | -0,100000 |
| 2 | 1,750000 | -0,028571 |
| 3 | 1,500000 | 0,066667 |

Didapatkan perubahan tanda di antara interval 1,75 dan 1,5, maka kita dapat menyimpulkan bahwa $x \approx 1,5$ dan $x > 1,5$ jika kita menggunakan tabulasi dengan $|\Delta x| = 0,25$

3. Regula Falsi

| $f(x) =$ | $(1-0.6x)/x$ | | | |
|----------|--------------|----------|----------|-----------|
| iterasi | x_0 | x_1 | c | $f(x)$ |
| 1 | 1,000000 | 3,000000 | 2,200000 | -0,145455 |
| 2 | 1,000000 | 2,200000 | 1,880000 | -0,068085 |
| 3 | 1,000000 | 1,880000 | 1,752000 | -0,029224 |

Didapatkan bahwa $x \approx 1,752$

4. Bolzano

| $f(x) =$ | $(1-0.6x)/x$ | | | |
|----------|--------------|----------|----------|-----------|
| iterasi | x_0 | x_1 | c | $f(x)$ |
| 4 | 1,000000 | 3,000000 | 2,000000 | -0,100000 |
| 5 | 1,000000 | 2,000000 | 1,500000 | 0,066667 |
| 6 | 1,500000 | 2,000000 | 1,750000 | -0,028571 |

Didapatkan bahwa $x \approx 1,75$

Dari empat metode tersebut kita mendapatkan data sebagai berikut

| Metode | x | Error |
|--------------|----------|--------|
| Tabulasi | 1,500000 | 10,00% |
| Regula falsi | 1,752000 | 5,12% |
| Biseksi | 1,750000 | 5,00% |

Dengan meng-eliminasi metode tabel(soalnya dari gambar tidak bisa kita menentukan nilai asli). Maka kita akan mendapatkan data berikut dan kita dapat menarik kesimpulan bahwa :

- Presisi : Biseksi
- Akurasi : Biseksi

Tetapi hal ini hanya berlaku jika kita menggunakan 3 iterasi dengan $x_0=2$. Karena setelah saya cek, jika kita menggunakan sampai lebih dari iterasi, maka metode yg menghasilkan nilai paling akurat adalah dengan menggunakan regula falsi :).

6. Bolzano (bisection method) dibuat menggunakan bahasa python agar mudah melakukan visualisasi.

Hal yg dibutuhkan sebelum menjalankan kode:

- Install python
- Install package numpy
- Install package matplotlib
- Install package pandas
- Ketik fungsi yg akan dihitung di kode(ini gabisa ngambil dari input soalnya nnti aga ribet kalo misal inputnya adalah polinomial berderajat tinggi atau fungsi transenden dsb, jadi ketik di kode nya aja)

Input kode :

- X1 = titik kiri sebagai acuan awal algoritma
- X2 = titik kanan sebagai acuan awal algoritma
- Iterasi = jumlah iterasi yg diinginkan

Pada kasus ini, kita mencoba menghitung nilai dari fungsi $f(x) : e^x - 3 = 0$

Kode : https://github.com/Ichann-san/bolzano_bisek/tree/main

```
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np

def f(x):
    return np.exp(x) - 3 #f(x) = e^x - 3, ganti aj klo mau

def bisection_method(a, b, n):
    if f(a) * f(b) >= 0:
        print("Interval tidak valid! f(a) dan f(b) harus memiliki tanda berbeda.")
        return []

    results = []
    for i in range(n):
        c = (a + b) / 2
        results.append((i + 1, a, b, c, f(c)))

        if abs(f(c)) < 1e-6:
            break
        elif f(a) * f(c) < 0:
            b = c
        else:
            a = c
```

```

        return results

def plot_bisection(a, b, n):
    results = bisection_method(a, b, n)
    if not results:
        return

    x = np.linspace(a - 1, b + 1, 400)
    y = f(x)

    plt.figure(figsize=(8, 5))
    plt.plot(x, y, label='f(x)')
    plt.axhline(0, color='black', linewidth=1)

    for i, (iter_num, a, b, c, fc) in enumerate(results):
        plt.scatter(c, f(c), color='red', label=f'Iterasi {iter_num}')
    if i == 0 else ""
        plt.annotate(f'{c:.4f}', (c, f(c)), textcoords="offset points",
xytext=(0,10), ha='center')

    plt.xlabel('x')
    plt.ylabel('f(x)')
    plt.title('Visualisasi Metode Bolzano dengan  $f(x) = e^x - 3$ ')
    #ganti aj klo maw ini fungsinya
    plt.legend()
    plt.show()

    df = pd.DataFrame(results, columns=["Iterasi", "a", "b", "c",
"f(c)"])
    print(df.to_string(index=False))

x1 = input("Masukkan nilai x1 (titik pertama): ")
x2 = input("Masukkan nilai x2 (titik kedua): ")
iterasi = input("Masukkan jumlah iterasi: ")
plot_bisection(float(x1), float(x2), int(iterasi))

```

Dengan input :

- X1 = -10
- X2 = 9
- Iterasi = 20

Maka didapatkan

| Iterasi | a | b | c | f(c) |
|---------|------------|----------|-----------|-----------|
| 1 | -10.000000 | 9.000000 | -0.500000 | -2.393469 |
| 2 | -0.500000 | 9.000000 | 4.250000 | 67.105412 |
| 3 | -0.500000 | 4.250000 | 1.875000 | 3.520819 |
| 4 | -0.500000 | 1.875000 | 0.687500 | -1.011263 |
| 5 | 0.687500 | 1.875000 | 1.281250 | 0.601138 |
| 6 | 0.687500 | 1.281250 | 0.984375 | -0.323861 |
| 7 | 0.984375 | 1.281250 | 1.132812 | 0.104375 |
| 8 | 0.984375 | 1.132812 | 1.058594 | -0.117685 |
| 9 | 1.058594 | 1.132812 | 1.095703 | -0.008715 |
| 10 | 1.095703 | 1.132812 | 1.114258 | 0.047306 |
| 11 | 1.095703 | 1.114258 | 1.104980 | 0.019166 |
| 12 | 1.095703 | 1.104980 | 1.100342 | 0.005193 |
| 13 | 1.095703 | 1.100342 | 1.098022 | -0.001769 |
| 14 | 1.098022 | 1.100342 | 1.099182 | 0.001710 |
| 15 | 1.098022 | 1.099182 | 1.098602 | -0.000030 |
| 16 | 1.098602 | 1.099182 | 1.098892 | 0.000840 |
| 17 | 1.098602 | 1.098892 | 1.098747 | 0.000405 |
| 18 | 1.098602 | 1.098747 | 1.098675 | 0.000187 |
| 19 | 1.098602 | 1.098675 | 1.098639 | 0.000079 |
| 20 | 1.098602 | 1.098639 | 1.098620 | 0.000024 |

Dengan grafik :

