Tugas 3 Komputasi Numerik
Kelompok B - 17
William Hans Chandra   5025241138
Yoseph Kevin Hendrata   5025241146
Maulana Ikhsan   5025241163

1.

a. $x^3 + 6,6x^2 - 29,05x + 22,64 = 0$

Metode Iterasi

$x = (x^3 + 6,6x^2 + 22,64) / 29,05$

$x0 = 1$

$h'(x) = (60x^2 + 264x) / 581$

$h'(1) = (60 + 264) / 581 = 324 / 581 = 0,558$

$|h'(x0)| < 1$ Konvergen

$x(n+1) = (x(n)^3 + 6,6x(n)^2 + 22,64) / 29,05$

Iterasi 1 $n = 0$

$x1 = (x0^3 + 6,6x0^2 + 22,64) / 29,05$

$x1 = (1^3 + 6,6 \times 1^2 + 22,64) / 29,05$

$x1 = (1 + 6,6 + 22,64) / 29,05$

$x1 = 30,24 / 29,05$

$x1 = 1,041$

Iterasi 2 $n = 1$

$x2 = (x1^3 + 6,6x1^2 + 22,64) / 29,05$

$x2 = (1,041^3 + 6,6 \times 1,041^2 + 22,64) / 29,05$

$x2 = (1,128 + 6,6 \times 1,084 + 22,64) / 29,05$

$x2 = (1,128 + 7,154 + 22,64) / 29,05$

$x2 = 30,922 / 29,05$

$x2 = 1,064$

Iterasi 3 $n = 2$

$x3 = (x2^3 + 6,6x2^2 + 22,64) / 29,05$

$x3 = (1,064^3 + 6,6 \times 1,064^2 + 22,64) / 29,05$

$x3 = (1,205 + 6,6 \times 1,132 + 22,64) / 29,05$

$x3 = (1,205 + 7,471 + 22,64) / 29,05$

$x3 = 31,316 / 29,05$

$x3 = 1,078$

Iterasi 4 $n = 3$

$x4 = (x3^3 + 6,6x3^2 + 22,64) / 29,05$

$x4 = (1,078^3 + 6,6 \times 1,078^2 + 22,64) / 29,05$

$x4 = (1,253 + 6,6 \times 1,162 + 22,64) / 29,05$

$x4 = (1,253 + 7,669 + 22,64) / 29,05$

$x4 = 31,562 / 29,05$

$x4 = 1,086$

Iterasi 5 $n = 4$

$x5 = (x4^3 + 6,6x4^2 + 22,64) / 29,05$

$x5 = (1,086^3 + 6,6 \times 1,086^2 + 22,64) / 29,05$

$x5 = (1,281 + 6,6 \times 1,179 + 22,64) / 29,05$

$x5 = (1,281 + 7,781 + 22,64) / 29,05$

$x5 = 31,702 / 29,05$

$x5 = 1,091$

Iterasi 6 $n = 5$

$x6 = (x5^3 + 6,6x5^2 + 22,64) / 29,05$

$x6 = (1,091^3 + 6,6 \times 1,091^2 + 22,64) / 29,05$

$x6 = (1,299 + 6,6 \times 1,19 + 22,64) / 29,05$

$x6 = (1,299 + 7,854 + 22,64) / 29,05$

$x6 = 31,793 / 29,05$

$x6 = 1,094$

b. $x^4 - 0,41x^3 + 1,632x^2 - 9,146x + 7,620 = 0$
Metode Iterasi
$x = (x^4 - 0,41x^3 + 1,632x^2 + 7,620) / 9,146$
$x0 = 1$

$h'(x) = (2000x^3 - 615x^2 + 1632x) / 4573$
$h'(1) = (2000 - 615 + 1632) / 4573 = 3017 / 4573 = 0,660$
$|h'(x0)| < 1$ Konvergen
$x(n+1) = (x(n)^4 - 0,41x(n)^3 + 1,632x(n)^2 + 7,620) / 9,146$

Iterasi 1 $n = 0$
$x1 = (x0^4 - 0,41x0^3 + 1,632x0^2 + 7,620) / 9,146$
$x1 = (1^4 - 0,41 \times 1^3 + 1,632 \times 1^2 + 7,620) / 9,146$
$x1 = (1 - 0,41 + 1,632 + 7,620) / 9,146$
$x1 = 9,842 / 9,146$
$x1 = 1,076$

Iterasi 2 $n = 1$
$x2 = (x1^4 - 0,41x1^3 + 1,632x1^2 + 7,620) / 9,146$
$x2 = (1,076^4 - 0,41 \times 1,076^3 + 1,632 \times 1,076^2 + 7,620) / 9,146$
$x2 = (1,340 - 0,41 \times 1,246 + 1,632 \times 1,158 + 7,620) / 9,146$
$x2 = (1,340 - 0,511 + 1,890 + 7,620) / 9,146$
$x2 = 10,339 / 9,146$
$x2 = 1,130$

Iterasi 3 $n = 2$
$x3 = (x2^4 - 0,41x2^3 + 1,632x2^2 + 7,620) / 9,146$
$x3 = (1,130^4 - 0,41 \times 1,130^3 + 1,632 \times 1,130^2 + 7,620) / 9,146$
$x3 = (1,630 - 0,41 \times 1,443 + 1,632 \times 1,277 + 7,620) / 9,146$
$x3 = (1,630 - 0,592 + 2,084 + 7,620) / 9,146$
$x3 = 10,742 / 9,146$
$x3 = 1,175$

Iterasi 4 $n = 3$
$x4 = (x3^4 - 0,41x3^3 + 1,632x3^2 + 7,620) / 9,146$
$x4 = (1,175^4 - 0,41 \times 1,175^3 + 1,632 \times 1,175^2 + 7,620) / 9,146$
$x4 = (1,906 - 0,41 \times 1,622 + 1,632 \times 1,381 + 7,620) / 9,146$
$x4 = (1,906 - 0,665 + 2,254 + 7,620) / 9,146$
$x4 = 11,115 / 9,146$
$x4 = 1,215$

Iterasi 5 $n = 4$
$x5 = (x4^4 - 0,41x4^3 + 1,632x4^2 + 7,620) / 9,146$
$x5 = (1,215^4 - 0,41 \times 1,215^3 + 1,632 \times 1,215^2 + 7,620) / 9,146$
$x5 = (2,179 - 0,41 \times 1,794 + 1,632 \times 1,476 + 7,620) / 9,146$
$x5 = (2,179 - 0,736 + 2,409 + 7,620) / 9,146$
$x5 = 11,472 / 9,146$
$x5 = 1,254$

Iterasi 6 $n = 5$
$x6 = (x5^4 - 0,41x5^3 + 1,632x5^2 + 7,620) / 9,146$
$x6 = (1,254^4 - 0,41 \times 1,254^3 + 1,632 \times 1,254^2 + 7,620) / 9,146$
$x6 = (2,473 - 0,41 \times 1,972 + 1,632 \times 1,573 + 7,620) / 9,146$
$x6 = (2,473 - 0,809 + 2,567 + 7,620) / 9,146$
$x6 = 11,851 / 9,146$
$x6 = 1,296$

2.

a. $x^3 + 6,6x^2 - 29,05x + 22,64 = 0$

Metode Faktorisasi

$P3(x) = x^3 + A2x^2 + A1x + A0$ $b0 = A0 / a0$

$P3(x) = (x + b0) (x^2 + a1x + a0)$ $a1 = A2 - b0$

                                          $a0 = A1 - (a1)(b0)$

$b0 = 0$

$a1 = 6,6 - 0 = 6,6$

$a0 = -29,05 - (6,6)(0) = -29,05$

$b0 = 22,64 / (-29,05) = -0,78$

$a1 = 6,6 - (-0,78) = 7.38$

$a0 = -29,05 - (7.38)(-0,78) = -29,05 - (-5,76) = -23,29$

$b0 = 22,64 / (-23,29) = -0,97$

$a1 = 6,6 - (-0,97) = 7,57$

$a0 = -29,05 - (7,57)(-0,97) = -29,05 - (-7,34) = -21,71$

$b0 = 22,64 / (-21,71) = -1,04$

$a1 = 6,6 - (-1,04) = 7,64$

$a0 = -29,05 - (7,64)(-1,04) = -29,05 - (-7,95) = -21,1$

$b0 = 22,64 / (-21,1) = -1,07$

$a1 = 6,6 - (-1,07) = 7,67$

$a0 = -29,05 - (7,67)(-1,07) = -29,05 - (-8,21) = -20,84$

#	b0	a1	a0
1	0	6,6	-29,05
2	-0,78	7,38	-23,29
3	-0,97	7,57	-21,71
4	-1,04	7,64	-21,10
5	-1,07	7,67	-20,84

b. $x^4 - 0,41x^3 + 1,632x^2 - 9,146x + 7,620 = 0$

Metode Faktorisasi

$P_4(x) = x^4 + A_3x^3 + A_2x^2 + A_1x + A_0$ $b_0 = A_0 / a_0$

$P_4(x) = (x^2 + b_1x + b_0)(x^2 + a_1x + a_0)$ $b_1 = (A_1 - a_1b_0) / a_0$

$a_1 = A_2 - b_0$

$a_0 = A_1 - (a_1)(b_0)$

$b_0 = 0$

$b_1 = 0$

$a_1 = 1,632 - 0 = 1,632$

$a_0 = -9,146 - (1,632)(0) = -9,146$

$b_0 = 7,620 / (-9,146) = -0,833$

$b_1 = (-9,146 - (1,632)(-0,833)) / (-9,146) = (-9,146 + 1,359) / (-9,146) = -7,787 / -9,146 = 0,851$

$a_1 = 1,632 - (-0,833) = 2,465$

$a_0 = -9,146 - (2,465)(-0,833) = -9,146 + 2,053 = -7,093$

$b_0 = 7,620 / (-7,093) = -1,074$

$b_1 = (-9,146 - (2,465)(-1,074)) / (-9,146) = (-9,146 + 2,647) / (-9,146) = -6,499 / -9,146 = 0,711$

$a_1 = 1,632 - (-1,074) = 2,706$

$a_0 = -9,146 - (2,706)(-1,074) = -9,146 + 2,906 = -6,24$

$b_0 = 7,620 / (-6,24) = -1,221$

$b_1 = (-9,146 - (2,706)(-1,221)) / (-9,146) = (-9,146 + 3,304) / (-9,146) = -5,842 / -9,146 = 0,639$

$a_1 = 1,632 - (-1,221) = 2,853$

$a_0 = -9,146 - (2,853)(-1,221) = -9,146 + 3,484 = -5,662$

$b_0 = 7,620 / (-5,662) = -1,346$

$b_1 = (-9,146 - (2,853)(-1,346)) / (-9,146) = (-9,146 + 3,840) / (-9,146) = -5,306 / -9,146 = 0,58$

$a_1 = 1,632 - (-1,346) = 2,978$

$a_0 = -9,146 - (2,978)(-1,346) = -9,146 + 4,008 = -5,138$

#	b0	b1	a1	a0
1	0	0	1,632	-9,146
2	-0,833	0,851	2,465	-7,093
3	-1,074	0,711	2,706	-6,240
4	-1,221	0,639	2,853	-5,662
5	-1,346	0,580	2,978	-5,138

3.

f(x) =	-(0,875)x^2 + 1,75x + 2,625			
iterasi	xi	f(x)	f'(x)	xi+1
1	3,100	-0,359	-3,675	3,002
2	3,002	-0,008	-3,504	3,000
3	3,000	0,000	-3,500	3,000
4	3,000	0,000	-3,500	3,000
5	3,000	0,000	-3,500	3,000
			Hasil akar =	3

4.

f(x) =	-2,1+6,21x-3,9x^2+0,667x^3			
iterasi	xi	f(x)	f'(x)	xi+2
6	3,100	-0,457	1,260	3,463
7	3,463	0,336	3,196	3,358
8	3,358	0,033	2,582	3,345
9	3,345	0,000	2,511	3,345
10	3,345	0,000	2,510	3,345
			Hasil akar =	3,345

5.

f(x) =	-23.33+79.35x-(88.09x^2)+(41.6x^3)-(8.68x^4)+(0.658x^5)				
iterasi	xi	f(x)	f'(x)	xi+3	
11	3,500	1,944	-3,394	4,073	
12	4,073	-1,872	-8,429	3,851	
13	3,851	-0,050	-7,619	3,844	
14	3,844	0,000	-7,565	3,844	
15	3,844	0,000	-7,565	3,844	
			Hasil akar =	3,844	

6. $f(x) = 9,36 - 21,963x + 16,2965x^2 - 3,70377x^3$

$x_{i-1} = 0,5$, maka $F(x_{i-1}) = 1,98965$

$x_i = 2,5$, maka $F(x_i) = -1,56578$

Iterasi 1

$$x_{i+1} = x_i - \frac{F(x_i).(x_{i-1}-x_i)}{F(x_{i-1})-F(x_i)}$$

$$x_{i+1} = 2,5 - \frac{(-1,56578).(0,5-2,5)}{1,98965-(-1,56578)}$$

$$x_{i+1} = 1,61922$$

$$F(x_{i+1}) = 0,800484$$

Iterasi 2

$$x_{i+2} = x_i - \frac{F(x_i).(x_{i+1}-x_i)}{F(x_{i+1})-F(x_i)}$$

$$x_{i+2} = 2,5 - \frac{(-1,56578).(1,61922-2,5)}{0,800484-(-1,56578)}$$

$$x_{i+2} = 1,91718$$

$$F(x_{i+2}) = 1,05252$$

Iterasi 3

$$x_{i+3} = x_i - \frac{F(x_i).(x_{i+2}-x_i)}{F(x_{i+2})-F(x_i)}$$

$$x_{i+3} = 2,5 - \frac{(-1,56578).(1,91718-2,5)}{1,05252-(-1,56578)}$$

$$x_{i+3} = 2,15147$$

$$F(x_{i+3}) = 0,655872$$

Iterasi 4

$$x_{i+4} = x_i - \frac{F(x_i).(x_{i+3}-x_i)}{F(x_{i+3})-F(x_i)}$$

$$x_{i+4} = 2,5 - \frac{(-1,56578).(2,15147-2,5)}{0,655872-(-1,56578)}$$

$$x_{i+4} = 2,25436$$

7. $f(x) = x^4 - 8,6x^3 - 35,51x^2 + 464x - 998,46$ $(x_{i-1} = 7$ dan $x_i = 9)$

$x_{i-1} = 7$, maka $F(x_{i-1}) = -39,25$

$x_i = 9$, maka $F(x_i) = 592,83$

Iterasi 1

$$x_{i+1} = x_i - \frac{F(x_i).(x_{i-1}-x_i)}{F(x_{i-1})-F(x_i)}$$

$$x_{i+1} = 9 - \frac{(592,83).(7-9)}{-39,25-592,83}$$

$$x_{i+1} = 9 - \frac{(592,83).(7-9)}{-39,25-592,83}$$

$$x_{i+1} = 7,12419$$

$F(x_{i+1}) = -28,73927$

Iterasi 2

$$x_{i+2} = x_i - \frac{F(x_i).(x_{i+1}-x_i)}{F(x_{i+1})-F(x_i)}$$

$$x_{i+2} = 9 - \frac{(592,83).(7,12419-9)}{-28,73927-592,83}$$

$$x_{i+2} = 7,21092$$

$F(x_{i+2}) = -19,85359$

Iterasi 3

$$x_{i+3} = x_i - \frac{F(x_i).(x_{i+2}-x_i)}{F(x_{i+2})-F(x_i)}$$

$$x_{i+3} = 9 - \frac{(592,83).(7,21092-9)}{-19,85359-592,83}$$

$$x_{i+3} = 7,26889$$

8. $f(x) = x^3 - 6x^2 + 11x - 6$ ($x_{i-1} = 2,5$ dan $x_i = 3,6$)

$x_{i-1} = 2,5$, maka $F(x_{i-1}) = -0,375$

$x_i = 3,6$, maka $F(x_i) = 2,496$

Iterasi 1

$$x_{i+1} = x_i - \frac{F(x_i) \cdot (x_{i-1} - x_i)}{F(x_{i-1}) - F(x_i)}$$

$$x_{i+1} = 3,6 - \frac{(2,496) \cdot (2,5 - 3,6)}{-0,375 - 2,496}$$

$$x_{i+1} = 2,64368$$

$$F(x_{i+1}) = -0,376988$$

Iterasi 2

$$x_{i+2} = x_i - \frac{F(x_i) \cdot (x_{i+1} - x_i)}{F(x_{i+1}) - F(x_i)}$$

$$x_{i+2} = 3,6 - \frac{(2,496) \cdot (2,64368 - 3,6)}{-0,376988 - 2,496}$$

$$x_{i+2} = 2,76917$$

$$F(x_{i+2}) = -0,314112$$

Iterasi 3

$$x_{i+3} = x_i - \frac{F(x_i) \cdot (x_{i+2} - x_i)}{F(x_{i+2}) - F(x_i)}$$

$$x_{i+3} = 3,6 - \frac{(2,496) \cdot (2,76917 - 3,6)}{-0,314112 - 2,496}$$

$$x_{i+3} = 2,86204$$

$$F(x_{i+3}) = -0,221447$$

Iterasi 4

$$x_{i+4} = x_i - \frac{F\left(x_i\right).\left(x_{i+3}-x_i\right)}{F\left(x_{i+3}\right)-F(x_i)}$$

$$x_{i+4} = 3,6 - \frac{(2,496).(2,86204-3,6)}{-0,221447-2,496}$$

$$x_{i+4} = 2,92218$$

$$F\left(x_{i+4}\right) = -0,137943$$

Iterasi 5

$$x_{i+5} = x_i - \frac{F\left(x_i\right).\left(x_{i+4}-x_i\right)}{F\left(x_{i+4}\right)-F(x_i)}$$

$$x_{i+5} = 3,6 - \frac{(2,496).(2,92218-3,6)}{-0,137943-2,496}$$

$$x_{i+5} = 2,95768$$

9. Metode Bairstow dan Metode Q-D

   a. **Metode Bairstow**

   Berguna untuk mencari akar-akar dari persamaan polinomial dengan menggunakan pendekatan iteratif, khususnya untuk akar kompleks dari polinomial. Merupakan modifikasi metode Newton-Raphson.

   $$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

   Persamaan polinomial diubah menjadi dua faktor kuadrat:

   $$P(x) = (x^2 + px + q)(Q(x))$$

   Dengan p dan q merupakan parameter hasil iterasi menggunakan pembagian sintetik dan perhitungan p dan q.

   b. **Metode Quotient-Difference (Q-D)**

   Berfokus pada perhitungan pembagian dan selisih antara nilai polinomial untuk mendekati akar. Akar persamaan diperoleh dari selisih antara nilai polinomial pada titik tertentu dan nilai pada titik yang sedikit lebih besar atau lebih kecil. Nilai polinomial dihitung pada dua titik yang berdekatan, lalu dilakukan pembagian atas 2 nilai untuk menghitung hasil mendekati akar yang diulang hingga mencapai target yang diinginkan.

c. Kesimpulan

Metode Bairstow:

(+): Efektif untuk menemukan akar kompleks atau berpasangan yang sulit diselesaikan oleh metode lainnya.

(-): Rumit untuk diterapkan pada polinomial dengan akar yang sangat dekat atau nilai titik awal tidak baik. Iterasi kompleks.

Metode Quotient-Difference (Q-D):

(+): Lebih sederhana untuk diterapkan karena melibatkan pembagian dan selisih nilai polinomial yang lebih mudah dihitung dibandingkan metode iterasi yang rumit.

(-): Tidak cocok untuk polinomial dengan akar yang sangat dekat atau akar kompleks. Bergantung pada pemilihan titik awal.