

SPECIAL DISCRETE DISTRIBUTIONS

NOTATION parameters	DISCRETE PDF $f_X(x)$	MEAN $E(X)$	VARIANCE $var(X)$	MGF $M_X(t)$
Binomial $X \sim \text{BIN}(n, p)$ $0 < p < 1$ $q = 1 - p$	$\binom{n}{x} p^x q^{n-x}$ $x = 0, 1, \dots, n$	np	npq	$(pe^t + q)^n$
Bernoulli $X \sim \text{BIN}(1, p)$ $0 < p < 1$ $q = 1 - p$	$p^x q^{1-x}$ $x = 0, 1$	p	pq	$pe^t + q$
Negative Binomial $X \sim \text{NB}(r, p)$ $0 < p < 1$ $r = 1, 2, \dots$	$\binom{x-1}{r-1} p^r q^{x-r}$ $x = r, r + 1, \dots$	$\frac{r}{p}$	$\frac{rq}{p^2}$	$\left(\frac{pe^t}{1 - qe^t}\right)^r$
Geometric $X \sim \text{GEO}(p)$ $0 < p < 1$ $q = 1 - p$	pq^{x-1} $x = 1, 2, \dots$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1 - qe^t}$
Hypergeometric $X \sim \text{HYP}(n, M, N)$ $n = 1, 2, \dots, N$ $M = 0, 1, \dots, N$	$\frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$ $x = 0, 1, \dots, n$	$\frac{nM}{N}$	$n \frac{M}{N} \left(1 - \frac{M}{N}\right) \frac{N-n}{N-1}$	*
Poisson $X \sim \text{POI}(\mu)$ $0 < \mu$	$\frac{e^{-\mu} \mu^x}{x!}$ $x = 0, 1, 2, \dots$	μ	μ	$e^{\mu(e^t - 1)}$
Discrete Uniform $X \sim \text{DU}(N)$ $N = 1, 2, \dots$	$\frac{1}{N}$ $x = 1, 2, \dots, N$	$\frac{(N+1)}{2}$	$\frac{(N^2 - 1)}{12}$	$\frac{1}{N} \frac{(e^t - e^{(N+1)t})}{1 - e^t}$

* Not tractable

SPECIAL CONTINUOUS DISTRIBUTIONS

NOTATION parameters	CONTINUOUS PDF	MEAN $E(X)$	VARIANCE $var(X)$	MGF $M_X(t)$
Uniform $X \sim \text{UNIF}(a, b)$ $a < b$ $a < x < b$	$\frac{1}{(b-a)}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt} - e^{at}}{(b-a)t}$
Normal $X \sim N(\mu, \sigma^2)$ $0 < \sigma^2$	$\frac{1}{\sqrt{2\pi}\sigma} e^{-[(x-\mu)/\sigma]^2/2}$	μ	σ^2	$e^{\mu t + \sigma^2 t^2/2}$
Gamma $X \sim \text{GAM}(\theta, \kappa)$ $0 < \theta$ $0 < \kappa$	$\frac{1}{\theta^\kappa \Gamma(\kappa)} x^{\kappa-1} e^{-x/\theta}$ $0 < x$	$\kappa\theta$	$\kappa\theta^2$	$\left(\frac{1}{1-\theta t}\right)^\kappa$
Exponential $X \sim \text{EXP}(\theta)$ $0 < \theta$	$\frac{1}{\theta} e^{-x/\theta}$ $0 < x$	θ	θ^2	$\frac{1}{1-\theta t}$
Two-parameter Exponential $X \sim \text{EXP}(\theta, \eta)$ $0 < \theta$	$\frac{1}{\theta} e^{-(x-\eta)/\theta}$ $\eta < x$	$\eta + \theta$	θ^2	$\frac{e^{\eta t}}{1-\theta t}$
Double Exponential $X \sim \text{DE}(\theta, \eta)$ $0 < \theta$	$\frac{1}{2\theta} e^{- x-\eta /\theta}$	η	$2\theta^2$	$\frac{e^{\eta t}}{1-\theta^2 t^2}$

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NOTATION	CONTINUOUS	MEAN	VARIANCE	MGF
parameters	PDF			
	$f_X(x)$	$E(X)$	$var(X)$	$M_X(t)$
Weibull				
X~WEI (θ, β)	$\frac{\beta}{\theta^\beta} x^{\beta-1} e^{-(x/\theta)^\beta}$	$\theta \Gamma(1 + \frac{1}{\beta})$	$\theta^2 [\Gamma(1 + \frac{2}{\beta}) - \Gamma^2(1 + \frac{1}{\beta})]$	*
$0 < \theta$	$0 < x$			
$0 < \beta$				
Extreme Value				
X~EV (θ, η)	$\frac{1}{\theta} e^{[\frac{x-\eta}{\theta} - e^{\frac{x-\eta}{\theta}}]}$	$\eta - \gamma\theta$	$\frac{\pi^2 \theta^2}{6}$	$e^{\eta t} \Gamma(1 + \theta t)$
$0 < \theta$				
Cauchy				
X~CAU (θ, η)	$\frac{1}{\theta \pi [1 + (\frac{x-\eta}{\theta})^2]}$	**	**	**
$0 < \theta$				
Pareto				
X~PAR (θ, κ)	$\frac{\kappa}{\theta (1 + \frac{x}{\theta})^{\kappa+1}}$	$\frac{\theta}{\kappa - 1}$	$\frac{\theta^2 \kappa}{(\kappa - 2)(\kappa - 1)^2}$	**
$0 < \theta$	$0 < x$	$1 < \kappa$	$2 < \kappa$	
$0 < \kappa$				
Chi-Square				
X~χ^2 (ν)	$\frac{1}{2^{\frac{\nu}{2}} \Gamma(\frac{\nu}{2})} x^{\frac{\nu}{2}-1} e^{-\frac{x}{2}}$	ν	2ν	$(1 - 2t)^{-\nu/2}$
$\nu = 1, 2, \dots$				

* Not tractable

** Does not exist

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NOTATION parameters	CONTINUOUS PDF	MEAN	VARIANCE	MGF
	$f_X(x)$	$E(X)$	$var(X)$	$M_X(t)$
Student's t $X \sim t(\nu)$	$\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\sqrt{\nu\pi}} (1 + \frac{x^2}{\nu})^{-\frac{\nu+1}{2}}$	0	$\frac{\nu}{\nu-2}$	**
$\nu = 1, 2, \dots$		$1 < \nu$	$2 < \nu$	
Snedecor's F $X \sim F(\nu_1, \nu_2)$	$\frac{\Gamma(\frac{\nu_1+\nu_2}{2})}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}} \times x^{\frac{\nu_1}{2}-1} (1 + \frac{\nu_1}{\nu_2}x)^{-\frac{\nu_1+\nu_2}{2}}$	$\frac{\nu_2}{\nu_2-2}$	$\frac{2\nu_2^2(\nu_1 + \nu_2 - 2)}{\nu_1(\nu_2 - 2)^2(\nu_2 - 4)}$	**
$\nu_1 = 1, 2, \dots$ $\nu_2 = 1, 2, \dots$		$2 < \nu_2$	$4 < \nu_2$	
Beta $X \sim \text{BETA}(a, b)$	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}$	$\frac{a}{a+b}$	$\frac{ab}{(a+b+1)(a+b)^2}$	**
$0 < a$ $0 < b$	$0 < x < 1$			

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