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# Unscented fuzzy-controlled current statistic model and adaptive filtering for tracking maneuvering targets

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#### **Abstract**

A novel adaptive algorithm for tracking maneuvering targets is proposed. The algorithm is implemented with fuzzy-controlled current statistic model adaptive filtering and unscented transformation. A fuzzy system allows the filter to tune the magnitude of maximum accelerations to adapt to different target maneuvers, and unscented transformation can effectively handle nonlinear system. A bearing-only tracking scenario simulation results show the proposed algorithm has a robust advantage over a wide range of maneuvers and overcomes the shortcoming of the traditional current statistic model and adaptive filtering algorithm.

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Keywords: Adaptive filtering; Current statistical model; Fuzzy system; Unscented transformation; Maneuvering target

#### 1. Introduction

Maneuvering target tracking is an important problem in military and civilian fields. Several algorithms such as Singer, input estimation, variable dimension filter, current statistical model and adaptive filtering (CSMAF) algorithms summarized in [1] and interacting multiple model

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(IMM) algorithm [2] have been studied in the last few years. In these methods, the IMM and CSMAF are two effective algorithms for tracking maneuvering targets. When the range of target maneuvers is wider, the IMM algorithm needs more filters and its computational burden is more considerable. The CSMAF algorithm takes advantage of current model concept to realize adaptive tracking for maneuvering targets using only one filter. It is obvious that the computational amount of this algorithm is much less than that of the IMM algorithm.

Although the CSMAF algorithm is one of the most effective methods for tracking maneuvering targets, its performance depends on pre-defined parameter maximum acceleration  $a_{\rm max}$  indicating the intensity of maneuver [3,4]. In tracking process, the value of  $a_{\rm max}$  is constant so that it is difficult to suit to different movement of target. For solving the problem, in this paper, a fuzzy system incorporated with CSMAF is proposed to adjust the value of  $a_{\rm max}$  online. The fuzzy system has several advantages: adjusting the magnitude of  $a_{\rm max}$  quickly in response to changes in target movement; making better decisions by taking into consideration several different or even conflicting situations at the same time.

Furthermore, in tracking applications, target dynamics is usually modeled in Cartesian coordinates, while the measurements are directly available in the original sensor coordinates, which may be give rise to nonlinear problem. Recently Julier and Uhlmann developed a new nonlinear filter (unscented filter) based on unscented transformation (UT) [5]. Some simulation results have shown that the unscented filter leads to more accurate results than the classical extended Kalman filter (EKF) [6]. So in this paper we use the UT to dealing with nonlinear problem instead of the linearization.

Combining the advantages of fuzzy system and UT, an unscented fuzzy-controlled current statistic model and adaptive filtering (UFCSMAF) algorithm for tracking maneuvering targets is proposed in this paper. Compared with the CSMAF algorithm, this new adaptive algorithm has a robust advantage over a wide range of maneuvers and overcomes pre-defined parameter in the CSMAF algorithm. The tracking performance of the filter is demonstrated by computer simulation.

The remainder of this paper is organized as follows. In Section 2, the UFCSMAF is proposed. In Section 3 the simulation experiments have carried out to demonstrate the filter performance. Finally, conclusions are given in Section 4.

# 2. Unscented fuzzy-controlled current statistic model and adaptive filtering

The structure of UFCSMAF is shown in Fig. 1. A fuzzy system is used in CSMAF model to adjust magnitude of maximum accelerations to adapt to different target maneuvers. The UT is combined with fuzzy-controlled CSMAF model for tracking maneuvering targets.

# 2.1. Current statistical model and adaptive filtering

The CSMAF algorithm assumes that when a target is maneuvering with certain acceleration, its acceleration during the next period is limited within a range around the current acceleration. Hence it is not necessary to take all possible values of maneuvering acceleration into consideration when modeling the target acceleration probability. A modified Rayleigh density function whose

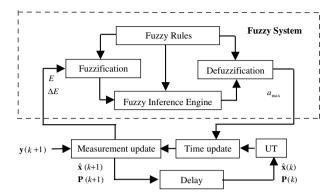


Fig. 1. The structure of UFCSMAF.

mean is the current acceleration is utilized, and the relationship between the mean and variance of Rayleigh density is used to set up an adaptive algorithm for the variance of maneuvering acceleration [7].

Theory analyses show that the system variance  $\sigma_w^2$  of this model is in direct proportion to acceleration variance  $\sigma_a^2$ 

$$\sigma_{w}^{2} = 2\alpha\sigma_{a}^{2} = \begin{cases} \frac{2\alpha(4-\pi)}{\pi} [a_{\text{max}} - \bar{a}(k)]^{2}, & \bar{a}(k) \geq 0, \\ \frac{2\alpha(4-\pi)}{\pi} [a_{-\text{max}} - \bar{a}(k)]^{2}, & \bar{a}(k) < 0, \end{cases}$$
(1)

where  $\alpha$  is the maneuvering frequency and  $\bar{a}(k)$  is the mean of maneuvering acceleration

$$\bar{a}(k) = \hat{\ddot{x}}(k/k - 1). \tag{2}$$

Thus, an acceleration mean and variance adaptive filtering algorithm can be obtained based on Eqs. (1) and (2).

From Eq. (1) we see the following facts: (1) when  $a_{\text{max}}$  is large, the algorithm can keep fast response to target maneuvers because the system variance or system frequency band takes a large value; (2) when  $a_{\text{max}}$  is a constant, if the target is maneuvering with a smaller acceleration, the system variance will be larger and the tracking precision will be lower; if the target is maneuvering with a larger acceleration, the system variance will be smaller and the tracking precision will be higher. Therefore  $a_{\text{max}}$  is an important parameter in filter design. But in real application, it is difficult to give an appropriate value to suit an unknown trajectory.

## 2.2. Fuzzy system design

In the above CSMAF, since  $a_{\text{max}}$  is predefined and affects the system variance, we use a fuzzy system to modify the value of  $a_{\text{max}}$  in order to get the most appropriate level in every case. The fuzzy system is characterized by a set of fuzzy rules. Based on the error and change of error in the

last prediction, these rules determine the magnitude of  $a_{\text{max}}$  in CSMAF. Therefore, the fuzzy decision system consists of two input variables and one output variable. If the dimension of measurement  $n_{\mathbf{y}}$  is more than one, for example  $n_{\mathbf{y}} = 2$ , the input variables E(k) and  $\Delta E(k)$  at the kth scan are defined by

$$E(k) = \frac{\sqrt{E_1^2(k) + E_2^2(k)}}{\sqrt{2}} \tag{3}$$

and

$$\Delta E(k) = \frac{\sqrt{\Delta E_1^2(k) + \Delta E_2^2(k)}}{\sqrt{2}},\tag{4}$$

where  $E_1(k)$ ,  $E_2(k)$  and  $\Delta E_1(k)$ ,  $\Delta E_2(k)$  are normalized error and change of error of each component of measurement, respectively.  $E_1(k)$  and  $\Delta E_1(k)$  are defined by [8,9]

$$E_{1}(k) = \begin{cases} \frac{y_{1}(k) - \hat{y}_{1}(k)}{y_{1}(k) - y_{1}(k-1)} & \text{if } |y_{1}(k) - \hat{y}_{1}(k)| < |y_{1}(k) - y_{1}(k-1)|, \\ \frac{y_{1}(k) - \hat{y}_{1}(k)}{|y_{1}(k) - \hat{y}_{1}(k)|} & \text{if } |y_{1}(k) - \hat{y}_{1}(k)| > |y_{1}(k) - \hat{y}_{1}(k-1)|, \\ 0.0 & \text{if } y_{1}(k) - \hat{y}_{1}(k) = y_{1}(k) - y_{1}(k-1), \end{cases}$$

$$(5)$$

and

$$\Delta E_{1}(k) = \begin{cases} \frac{E_{1}(k) - E_{1}(k-1)}{E_{1}(k-1)} & \text{if } |E_{1}(k) - E_{1}(k-1)| < |E_{1}(k-1)|, \\ \frac{E_{1}(k) - E_{1}(k-1)}{|E_{1}(k) - E_{1}(k-1)|} & \text{if } |E_{1}(k) - E_{1}(k-1)| > |E_{1}(k-1)|, \\ 0.0 & \text{if } E_{1}(k) - E_{1}(k-1) = E_{1}(k-1), \end{cases}$$

$$(6)$$

where  $y_1(k)$  and  $\hat{y}_1(k)$  are the measurement and predicted measurement of the first component of measurement vector, respectively. Similarly, we can get  $E_2(k)$  and  $\Delta E_2(k)$ . The range of values of E(k) and  $\Delta E(k)$  is in the interval [0, 1].

The fuzzy sets for the input variables E(k) and  $\Delta E(k)$  are labeled as the linguistic terms of LP (large positive), MP (medium positive), SP (small positive), and ZE (zero). These membership functions are defined by the trapezoidal function shown in Fig. 2(a). The output variable is scale factor  $\beta$ . The fuzzy sets for  $\beta$  are labeled in the linguistic terms of EP (extremely large positive), VP (very large positive), LP (large positive), MP (medium positive), SP (small positive), and ZE (zero). The specific membership is defined by triangular functions shown in Fig. 2(b). Given the range of  $a_{\text{max}}$ , e.g.  $a_{\text{max}} \in [a_1, a_2]$ , the updated  $a_{\text{max}}$  is calculated as

$$a_{\max} = a_1 + (a_2 - a_1)\beta. \tag{7}$$

With the input and output variables defined above, a fuzzy rule can be expressed in Table 1. The rules were obtained by interviewing some defense experts [8].

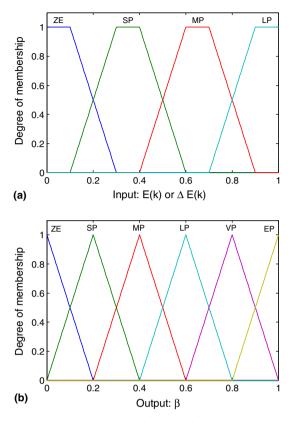


Fig. 2. Membership functions: (a) input variables, (b) output variable.

Table 1 Fuzzy rules for  $a_{\text{max}}$ 

		E	E			
		ZE	SP	MP	LP	
$\Delta E$	ZE	VP	SP	EP	EP	
	SP	LP	LP	VP	VP	
	MP	EP	VP	MP	MP	
	LP	VP	ZE	MP	EP	

# 2.3. Unscented transformation

The unscented transformation is a method for calculating the statistics of a random variable. It uses a set of deterministically chosen and nonlinearly transformed points to approximate the mean and the covariance [5]. Consider propagating a  $n_x$  dimensional random variable x through an arbitrary nonlinear function g to generate y = g(x). Assumed  $x \sim N(\bar{x}, P_x)$ , to calculate the statistics of y using UT, a set of  $2n_x + 1$  weighted samples or sigma points are chosen

$$\begin{cases}
\chi_0 = \bar{\mathbf{x}} & w_0 = \lambda/(n_{\mathbf{x}} + \lambda) & i = 0, \\
\chi_i = \bar{\mathbf{x}} + \left(\sqrt{(n_{\mathbf{x}} + \lambda)}\mathbf{P}_{\mathbf{x}}\right)_i & w_i = 0.5/(n_{\mathbf{x}} + \lambda) & i = 1, 2, \dots, n_{\mathbf{x}}, \\
\chi_i = \bar{\mathbf{x}} - \left(\sqrt{(n_{\mathbf{x}} + \lambda)}\mathbf{P}_{\mathbf{x}}\right)_i & w_i = 0.5/(n_{\mathbf{x}} + \lambda) & i = n_{\mathbf{x}} + 1, n_{\mathbf{x}} + 2, \dots, 2n_{\mathbf{x}},
\end{cases} \tag{8}$$

where  $\lambda$  is a scaling parameter and  $(\sqrt{(n_x + \lambda)} \mathbf{P_x})_i$  is the *i*th row or column of the matrix square root of  $(n_x + \lambda) \mathbf{P_x}$ .  $w_i$  is the weight associated with the *i*th point. Each sigma point is propagated through  $g(\cdot)$ 

$$\boldsymbol{\xi}_i = g(\boldsymbol{\gamma}_i)i = 0, 1, \dots, 2n_{\mathbf{x}}, \tag{9}$$

and the estimated mean and covariance of y are computed as follows:

$$\bar{\mathbf{y}} = \sum_{i=0}^{2n_{\mathbf{x}}} w_i \boldsymbol{\xi}_i,\tag{10}$$

$$\mathbf{P}_{\mathbf{y}} = \sum_{i=0}^{2n_{\mathbf{x}}} w_i (\boldsymbol{\xi}_i - \bar{\mathbf{y}}) (\boldsymbol{\xi}_i - \bar{\mathbf{y}})^{\mathrm{T}}.$$
 (11)

**Theorem 1.** Assumed  $\mathbf{x} \sim N(\bar{\mathbf{x}}, \mathbf{P_x})$ , sigma points are chosen according to Eq. (9). The mean and covariance of the sample set are equal to  $\bar{\mathbf{x}}$  and  $P_{\mathbf{x}}$ , respectively.

**Theorem 2.** Given  $\mathbf{x} \sim N(\bar{\mathbf{x}}, \mathbf{P_x})$ , after a nonlinear transformation according to  $\mathbf{y} = g(\mathbf{x})$ , the mean and covariance of variable  $\mathbf{y}$  calculated by the UT are accurate to the third order. While using a linearization approach, the mean is accurate to the first order and the covariance is accurate to the third order [12].

From above we can see that it is a good choice using UT as an alternative to the linearization for nonlinear state estimation.

# 2.4. Algorithm

Without loss of generality one dimension UFCSMAF is derived. The discrete state equations is

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{G}\bar{a}(k) + \mathbf{B}_{w}w(k), \tag{12}$$

where

$$\begin{split} \mathbf{F} &= \begin{bmatrix} 1 & \textit{T} & \frac{1}{\alpha^2} (-1 + \alpha \textit{T} + e^{-\alpha T}) \\ 0 & 1 & \frac{1}{\alpha} (1 - e^{-\alpha T}) \\ 0 & 0 & e^{-\alpha T} \end{bmatrix}, \quad \mathbf{B}_w = \begin{bmatrix} \frac{1}{\alpha^3} \left( 1 - \alpha \textit{T} + \frac{1}{2} \alpha^2 \textit{T}^2 - e^{-\alpha T} \right) \\ \frac{1}{\alpha^2} (-1 + \alpha \textit{T} + e^{-\alpha T}) \\ \frac{1}{\alpha} (1 - e^{-\alpha T}) \end{bmatrix}, \quad \mathbf{G} = \alpha \mathbf{B}_w, \end{split}$$

and w(k) is a zero mean white noise sequence with variance  $\sigma_w^2 = 2\alpha\sigma_a^2$ . The measurement equation is

$$\mathbf{y}(k) = h(\mathbf{x}(k)) + \mathbf{v}(k),\tag{13}$$

where  $\mathbf{v}(k) \sim N(0, \mathbf{R})$ .

According to Eqs. (4) and (13) the UFCSMAF equations are as follows: Initialization:

$$\hat{\mathbf{x}}(k-1) = E[\mathbf{x}(k-1)], \quad \mathbf{P}(k-1) = E[(\mathbf{x}(k-1) - \hat{\mathbf{x}}(k-1))(\mathbf{x}(k-1) - \hat{\mathbf{x}}(k-1))^{\mathrm{T}}].$$
 (14)

Calculate sigma points:

$$\chi(k-1) = \begin{bmatrix} \hat{\mathbf{x}}(k-1) & \hat{\mathbf{x}}(k-1) \pm \sqrt{(n_{\mathbf{x}} + \lambda)\mathbf{P}(k-1)} \end{bmatrix}.$$
(15)

Time update:

$$\boldsymbol{\chi}_{i}(k|k-1) = \mathbf{F}\boldsymbol{\chi}_{i}(k-1) + \mathbf{G}[\boldsymbol{\chi}_{i}(k|k-1)]_{3} \Rightarrow \boldsymbol{\chi}_{i}(k|k-1) = \boldsymbol{\Phi}\boldsymbol{\chi}_{i}(k-1), \tag{16}$$

where notation  $[\chi]_i$  stand for the *i*th element of vector  $\chi$  and  $\Phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$ .

$$\hat{\mathbf{x}}(k|k-1) = \sum_{i=0}^{2n_{\mathbf{x}}} w_i \chi_i(k|k-1), \tag{17}$$

$$\mathbf{P}(k|k-1) = \sum_{i=0}^{2n_{\mathbf{x}}} w_i [\mathbf{\chi}_i(k|k-1) - \hat{\mathbf{x}}(k|k-1)] [\mathbf{\chi}_i(k|k-1) - \hat{\mathbf{x}}(k|k-1)]^{\mathrm{T}} + 2\alpha\sigma_a^2 \mathbf{B}_w \mathbf{B}_w^{\mathrm{T}}, \quad (18)$$

$$\xi_{i}(k|k-1) = h(\gamma_{i}(k|k-1)), \tag{19}$$

$$\hat{\mathbf{y}}(k|k-1) = \sum_{i=0}^{2n_{\mathbf{x}}} w_i \xi_i(k|k-1). \tag{20}$$

Measurement update:

$$\mathbf{P}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^{\mathrm{T}}} = \sum_{i=0}^{2n_{\mathbf{x}}} w_{i} (\boldsymbol{\xi}_{i}(k|k-1) - \hat{\mathbf{y}}(k|k-1)) (\boldsymbol{\xi}_{i}(k|k-1) - \hat{\mathbf{y}}(k|k-1))^{\mathrm{T}},$$
(21)

$$\mathbf{P}_{\mathbf{x}_{k}\mathbf{y}_{k}} = \sum_{i=0}^{2n_{\mathbf{x}}} w_{i}(\mathbf{\chi}_{i}(k|k-1) - \hat{\mathbf{x}}(k|k-1))(\boldsymbol{\xi}_{i}(k|k-1) - \hat{\mathbf{y}}(k|k-1))^{\mathrm{T}},$$
(22)

$$\mathbf{K}(k) = \mathbf{P}_{\mathbf{x}\mathbf{y}}\mathbf{P}_{\tilde{\mathbf{v}}\tilde{\mathbf{v}}^{\mathsf{T}}}^{-1},\tag{23}$$

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)(\mathbf{y}(k) - \hat{\mathbf{y}}(k|k-1)), \tag{24}$$

$$\mathbf{P}(k) = \mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{P}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^{\mathsf{T}}}\mathbf{K}^{\mathsf{T}}(k), \tag{25}$$

$$\begin{cases} E(k) & \text{Fuzzy system} \\ \Delta E(k) & \xrightarrow{} a_{\text{max}}. \end{cases}$$
 (26)

# 3. Simulation and performance analysis

In this section, we apply the proposed algorithm, UFCSMAF, to a maneuvering target tracking problem and compare its performance to UCSMAF, i.e. filter based on UT and CSMAF. The mean and the standard deviation of the estimation error are chosen as the measure of performance. For comparing the error performance of these algorithms against the theoretical bounds, the Cramer-Rao lower bound (CRLB) is derived for this problem.

# 3.1. Simulution scenario

The scenario shows in Fig. 3. The target initial position is (10,15) km and initial velocity is (-0.3,0) km/s. In first segment from 0 to 30 s, the target moves with a constant velocity; in the second segment from 31 to 45 s, the target makes a left turn with a centripetal acceleration  $70 \text{ m/s}^2$ ; in the third segment from 46 to 60 s, the target moves with a constant velocity; in the fourth segment from 61 to 80 s, the target makes a right turn with a centripetal acceleration  $20 \text{ m/s}^2$ ; in the fifth segment from 81 to 100 s, the target moves with a constant velocity. Two distributed observers measure the target line-of-sight (LOS) angles and are located at (0,0) km and (10,0) km, respectively. So the measured angles are given by

$$\mathbf{y}(k) = h(\mathbf{x}(k)) + \mathbf{v}(k) = \begin{bmatrix} \arctan\left(\frac{y(k) - y_0^1}{x(k) - x_0^1}\right) \\ \arctan\left(\frac{y(k) - y_0^2}{x(k) - x_0^2}\right) \end{bmatrix} + \mathbf{v}(k),$$
(27)

where  $\mathbf{x}(k) = [x(k), \dot{x}(k), \ddot{x}(k), \dot{y}(k), \dot{y}(k), \ddot{y}(k)]^{\mathrm{T}}$ ,  $(x_0^i, y_0^i)$  is the observer location and  $\mathbf{v}(k)$  is Gaussian measurement noise. Table 2 lists the simulation parameters in detail.

#### 3.2. Cramer-Rao lower bound

The theoretical lower-bound (CRLB) plays an important role in algorithm evaluation and assessment of the level of approximation introduced in the filtering algorithms. The CRLB of

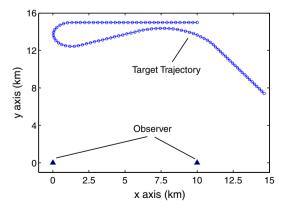


Fig. 3. Tracking scenario.

**UCSMAF-I UFCSMAF** Item Description **UCSMAF-II** α Maneuvering frequency 1/20 1/20 1/20 Maximum acceleration  $100 \text{ m/s}^2$  $500 \text{ m/s}^2$  $[100, 500] \text{ m/s}^2$  $a_{\text{max}}$ λ UT parameter 0 0 0  $P_{\rm d}$ Detection probability 1.0 1.0 1.0 TSample rates 1 s 1 s 1 s 0.003 rad 0.003 rad 0.003 rad $\sigma_{\theta}$ Sensor noise L Simulation length 100 100 100

Table 2 Simulation parameters

the state vector components is calculated as the diagonal elements of the inverse information matrix J [10]

$$CRLB\{[\hat{\mathbf{x}}(k)]_i\} = [\mathbf{J}^{-1}(k)]_{ii}. \tag{28}$$

If the target motion is deterministic (in the absence of process noise and with a non-random initial state vector) and the measurement noise is additive Gaussian, the information matrix can be calculated using the following recursion [11]:

$$\mathbf{J}(k+1) = [\mathbf{F}^{-1}(k)]^{\mathrm{T}} \mathbf{J}(k) \mathbf{F}^{-1}(k) + \mathbf{H}^{\mathrm{T}}(k+1) \mathbf{R}^{-1} \mathbf{H}(k+1), \tag{29}$$

where **F** is transition matrix and **H** is the Jacobian of  $h(\cdot)$  evaluated at the true state.

Unfortunately it is difficulty to get a tractable form of **J** in target maneuvering scenario. Here a conservative bound is given. Suppose that the maneuvering characteristic is well known, and then we can piecewise calculate **J**. In derivation, target state is given by  $\mathbf{x}(k) = [x(k), \dot{x}(k), y(k), \dot{y}(k)]^T$ . According to measurement Eq. (28), the Jacobian is given by

$$\mathbf{H}(k) = \begin{bmatrix} \frac{-(y(k) - y_0^1(k))}{(x(k) - x_0^1(k))^2 + (y(k) - y_0^1(k))^2} & 0 & \frac{(x(k) - x_0^1(k))}{(x(k) - x_0^1(k))^2 + (y(k) - y_0^1(k))^2} & 0\\ \frac{-(y(k) - y_0^2(k))}{(x(k) - x_0^2(k))^2 + (y(k) - y_0^2(k))^2} & 0 & \frac{(x(k) - x_0^2(k))}{(x(k) - x_0^2(k))^2 + (y(k) - y_0^2(k))^2} & 0 \end{bmatrix}.$$
(30)

When target moves with a constant velocity, F is give by

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{31}$$

When target moves with a circle motion,

$$\mathbf{F} = \begin{bmatrix} 1 & \frac{\sin \Omega T}{\Omega} & 0 & \frac{\cos \Omega T - 1}{\Omega} \\ 0 & \cos \Omega T & 0 & -\sin \Omega T \\ 0 & \frac{1 - \cos \Omega T}{\Omega} & 1 & \frac{\sin \Omega T}{\Omega} \\ 0 & \sin \Omega T & 0 & \cos \Omega T \end{bmatrix},$$
(32)

where  $\Omega$  is angular rate.

In this paper, we use triangular location for initialization [12]. The initial J(0) is given by

$$\mathbf{J}(0) = [\mathbf{P}(0)]^{-1} = \begin{bmatrix} \sigma_x^2 & 0 & \sigma_{xy}^2 & 0\\ 0 & \sigma_v^2 & 0 & 0\\ \sigma_{xy}^2 & 0 & \sigma_y^2 & 0\\ 0 & 0 & 0 & \sigma_v^2 \end{bmatrix}^{-1}.$$
(33)

In (33)  $\sigma_v$  is determined by the prior knowledge of target maximum speed.  $\sigma_v = 300$  m/s is used in this scenario.

$$\begin{cases}
\sigma_{x}^{2} = \frac{1}{|C|^{2}} \left( \frac{\cos^{2}\varphi_{2}}{r_{2}^{2}} \sigma_{\varphi_{1}}^{2} + \frac{\cos^{2}\varphi_{1}}{r_{1}^{2}} \sigma_{\varphi_{2}}^{2} \right), \\
\sigma_{y}^{2} = \frac{1}{|C|^{2}} \left( \frac{\sin^{2}\varphi_{2}}{r_{2}^{2}} \sigma_{\varphi_{1}}^{2} + \frac{\sin^{2}\varphi_{1}}{r_{1}^{2}} \sigma_{\varphi_{2}}^{2} \right), \\
\sigma_{xy} = \frac{1}{|C|^{2}} \left( \frac{\cos^{2}\varphi_{2}}{2r_{2}^{2}} \sigma_{\varphi_{1}}^{2} + \frac{\cos^{2}\varphi_{1}}{2r_{1}^{2}} \sigma_{\varphi_{2}}^{2} \right).
\end{cases} (34)$$

In Eq. (34)  $\varphi_i$  is the measurement of observer i,  $|C| = \frac{\sin(\varphi_2 - \varphi_1)}{r_2 r_1}$  and  $r_i = \sqrt{(x - x_0^i)^2 + (y - y_0^i)^2} (i = 1, 2)$ .

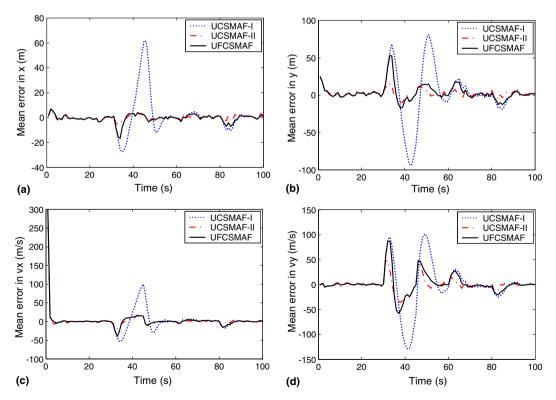


Fig. 4. Mean of estimation error: (a) x; (b) y; (c)  $v_x$ ; (d)  $v_y$ .

## 3.3. Error analysis

After 1000 independent Monte Carlo runs, the means of estimation errors in target positions and velocities are shown in Fig. 4, respectively. The standard deviations of errors, against the square root of CRLB, are shown in Fig. 5. We see from Fig. 4 that all three methods are approximately unbiased in non-maneuvering and small maneuvering stages. While in large maneuvering stage, the UCSMAF-II is the most accurate method, followed by the UFCSMAF and the UCSMAF-I we see from Fig. 5 that the UFCSMAF combines the advantages of both the UCSMAF-I and the UCSMAF-II. In the non-maneuvering and small maneuvering stages the performances of the UFCSMAF and the UCSMAF-I are similar closely and obviously better than that of the UCSMAF-II are similar closely and obviously better than that of the UCSMAF-I. All three algorithms show the levels of error standard deviations that are much higher than the theoretical bound since the bound given in this paper is fairly conservative. It also means that we need further research on tracking maneuvering targets in future work. The conclusion of the study is that on the whole the UFCSMAF is the preferred algorithm among the three considered algorithms since it can adapt to a wider range of target maneuvers.

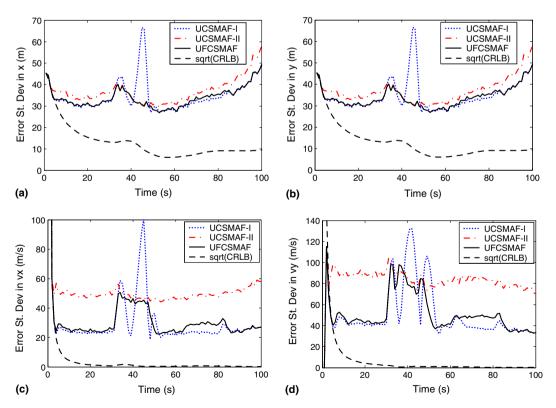


Fig. 5. Error standard deviation versus square root of CRLB: (a) x; (b) y; (c)  $v_x$ ; (d)  $v_y$ .

#### 4. Conclusions

A UFCSMAF algorithm has been presented in this paper for tracking a maneuvering target. The Cramer-Rao lower bound has also been derived. The theoretic analysis and computer simulations have confirmed that the presented adaptive algorithm has a robust advantage over a wide range of maneuvers and overcomes the shortcoming of the traditional current statistic model and adaptive filtering algorithm.

Future work will need to address the better model characterizing target maneuvers. In addition, more accurate performance bound incorporating uncertainty will be developed.

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