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Team Project
MAE 180 A - Spacecraft Guidance I
Iridium-33

Abstract

This report outlines the process of determining Iridium-33's position and velocity vectors in the ECI frame based on a set of observations from EBU II on the UC San Diego campus. Our algorithm primarily utilizes the gauss method and herricks-gibbs methods of orbit determination. Upon estimating the orbit of the satellite, we collaborated with another group to determine the closest approach of the two satellites. Our algorithm shows that the orbit has a radius of 7,142 kilometers or an altitude of 764 kilometers, an eccentricity of 0.0252, a nearly polar inclination of 86.44 degrees, a right ascension of the ascending node of 122.37 degrees, an argument of periapsis of 354.09 degrees, and a mean anomaly of 146.70 degrees.

Introduction

The Iridium 33 satellite was a communications satellite managed by Iridium Communications as a part of a constellation of 66 communications satellites in low earth orbit. In 2009, the satellite collided with the Russian Kosmos-2251 satellite, creating a large debris field in LEO. Our project goals are to determine the orbit of the Iridium 33 satellite and using data shared by another team working on the Kosmos-2251 satellite, determine the distance of closest approach between the two satellites.

Our project's main form of preliminary orbit determination is the gaussian method and using the gibbs-herricks method, we will determine the orbit propagation. We are also calculating the orbit using the keplerian model along with the effects of the Earth's oblateness, then comparing the differences between the two methods.

Finally, given data from a parallel group working on a different satellite, we will calculate the distance of closest approach and visualize all the data we have gathered.

Orbit Determination

All orbital trajectory plotting requires two methods, one for preliminary orbit determination and another for orbit propagation. Angle's only initial orbit determination allows for the determination of a space object's orbit around a central body while only using a minimal number of line-of-sight measurements. This method is possible when the location of the observer is known at each time measurement. Despite this astrodynamics problem being explored for many years, it continues to be a subject of ongoing research. The Gaussian method assumes two-body gravitational motion and determines the orbit of an object by observing it at least three different times over a given period of time (more observations increase the accuracy of the determined orbit. A linear combination of position vectors are constructed using the conservation of angular momentum and keplerian orbit principles. Next a relation is formed between the body's position and velocity vector by lagrange coefficients. The time intervals are calculated between each observation and then the site and line of sight unit vectors are calculated. This Gauss algorithm results in an 8th order polynomial for the range at the middle measurement time which yields the inertial position vector of the satellite at this point. It can then be propagated forwards and backwards at the two other measurement times. Next, given the three inertial position vectors, Herrick-Gibbs is used to obtain the inertial velocity vector. At each measurement time, the input vectors are the LOS vector from the observer to the space object and the vector from the center of the attracting body to the observer, all expressed in an inertial frame. Each of the observations are made based on a Julian date, defined as the number of days that have passed since noon Greenwich Mean Time on January 1st, 4713 B.C, and records a declination, right ascension, and local sidereal time in a topocentric equatorial coordinate system. From the observations, we start by determining the vector representing the location of the observation site at the three times, given by " ".

The Gauss method, as well as the Laplace method, is based on the fundamental equation

$$c_1\overrightarrow{r_1} + c_2\overrightarrow{r_2} + c_3\overrightarrow{r_3} = 0$$

, where

$$c_1\overrightarrow{r_1} \times \overrightarrow{r_3} = -c_2\overrightarrow{r_2} \times \overrightarrow{r_3}, \qquad c_3\overrightarrow{r_1} \times \overrightarrow{r_3} = -c_2\overrightarrow{r_1} \times \overrightarrow{r_2}$$

.

In order to find the position vectors, we must first find the line of sight vectors. We start by separating the position vector of the observer at each of the times from the position vector of the spacecraft relative to the observer and restate the fundamental equation as

$$c_1\overrightarrow{\rho_1} + c_2\overrightarrow{\rho_2} + c_3\overrightarrow{\rho_3} = -c_1\overrightarrow{r_{s_1}} - c_2\overrightarrow{r_{s_2}} - c_3\overrightarrow{r_{s_3}}$$

.

We write this equation in the form of a matrix equation using the unit line of sight vectors and invert the resulting matrix using Cramer's Rule, resulting in a new equation ,

$$\begin{bmatrix} c_1\rho_1 \\ c_2\rho_2 \\ c_3\rho_3 \end{bmatrix} = -L^{-1}\begin{bmatrix} \overrightarrow{r_{s_1}} & \overrightarrow{r_{s_2}} & \overrightarrow{r_{s_3}} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

relating the line of sight vectors to the position vectors of the observer. In order to resolve the cartesian position of the object at the three observations, we must find the magnitude of the LOS vector at the three times, which requires finding the three coefficients. First, we estimate several parameters, given by

$$a_1 = \frac{\tau_3}{\tau_3 - \tau_1},\ a_{1u} = \frac{\tau_3\left((\tau_3 - \tau_1)^2 - \tau_3^2\right)}{6(\tau_3 - \tau_1)},\ a_3 = -\frac{\tau_1}{\tau_3 - \tau_1},\ a_{3u} = -\frac{\tau_1\left((\tau_3 - \tau_1)^2 - \tau_1^2\right)}{6(\tau_3 - \tau_1)},$$

as well as the middle slant range matrix, defined as:

$$M = L^{-1}R_{site},$$

and find further parameters given by:

$$d_1 = M_{21}a_1 - M_{22} + M_{23}a_3, \qquad d_2 = M_{21}a_{1u} + M_{23}a_{3u}$$, and $C = \hat{L_2} \cdot \overrightarrow{r_{s_2}}$

We then solve the eighth-degree equation $r_2^8 - \left(d_2^2 + 2Cd_1 + r_{s_2}^2\right)r_2^6 - 2\mu(Cd_2 + d_1d_2)r_2^3 - \mu^2 d_2^2 = 0$ to find mu, which is related to a final parameter u by

$$u = \frac{\mu}{r_2^3}.$$

Finally, we can write and solve the matrix equation

$$\begin{bmatrix} -c_1 \\ -c_2 \\ -c_3 \end{bmatrix} = \begin{bmatrix} -a_1 - a_{1u}u \\ 1 \\ -a_3 - a_{3u}u \end{bmatrix}$$

to find the coefficients and line of sight magnitudes. This allows us to find the line of sight vectors, add them to the observer position vectors, and find the position vectors.

This is the end of the Gauss method of initial orbit determination; however, because the angles between the 3 observations are small (~3°), we *must* use the Herrick-Gibbs method of orbital determination, rather than the Gibbs method, to find the corresponding velocity vector. This initial velocity vector is described by TOF between our 3 observed position vectors.
The output is a set of vectors of inertial position and velocity expressed as an orbit for the space object. These vectors can then be used to determine the Keplerian orbital elements.

Inaccuracies are caused by the approximation of lagrange coefficients and the limited observation conditions. By improving the accuracy of sub-components such as Kepler's equation and secular perturbations, the Gaussian method can be improved. In computing the site coordinates, we assumed that the Earth was perfectly spherical which leads to these inaccuracies. A key thing to note is that perturbations specifically refers to more than one gravitational attraction of a single body; hence, we can see how our orbit is behaving in respect to other gravitational effects. This algorithm will implement Earth's oblateness effect (equatorial bulge) which is important to fully benefit from these observations. The oblateness effect of the Earth on the satellite is governed by the following three equations for the secular change of the right ascension of the ascending node, argument of periapsis, and mean anomaly.

$$\boxed{\begin{aligned} \dot{\bar{\Omega}} &= -\frac{3J_2R^2n}{2p^2}\cos i \\ \dot{\bar{\omega}} &= \frac{3J_2R^2n}{4p^2}(5\cos^2 i - 1) \\ \dot{\overline{M_0}} &= \frac{3J_2R^2n}{4p^2}\sqrt{1-e^2}(3\cos^2 i - 1) \end{aligned}}$$

Here J2 is evaluated to be 1.0826359 x 10^-3; it is the second zonal harmonic coefficient of the geopotential. These equations represent the constant rate in which the oblateness causes the satellite's perigee and ascending node to change as well as show how the secular drift rates diminish as the semi major axis (a) increases. It is important to note that the secular oblateness effect for any gravitational body can be quantified using these two equations as long as the zonal harmonic coefficient J2 and equatorial radius of the body of interest are known.

Test Results

This software, developed using MATLAB, takes in three radar observations of the declination and right ascension of the object in orbit, as well as the epoch and the local sidereal time. These values are then converted into their respective vectors for the three observations, and are passed through a function that returns the position and velocity vectors at the second observation, the position and velocity vectors at the specified time after propagation, and the keplerian orbital elements at the second observation time and specified time. All angular measurements are presented in units of degrees, position measurements are presented in units of kilometers, and velocity measurements are presented in kilometers per second.

The angles-only observations of the Iridium-33 satellite were taken from the EBU-II facility at U.C. San Diego, at a latitude of 32.88 degrees north, 117.2 degrees west, and an altitude of 111 meters above mean sea level.

For the unperturbed case, the Gauss method of orbit determination produces a nearly circular orbit with a radius of 7,142 kilometers or an altitude of 764 kilometers, an eccentricity of 0.0252, a nearly polar inclination of 86.44 degrees, a right ascension of the ascending node of 122.37 degrees, an argument of periapsis of 354.09 degrees, and a mean anomaly of 146.70 degrees. When perturbation is considered, the resulting radius decreases to roughly 7,141 kilometers, the eccentricity increases to 0.0256, the inclination and right ascension of the ascending node remain the same, the argument of periapsis changes to 138.18 degrees, and the mean anomaly changes to 325.6 degrees.

Input Values

Latitude: 32.881191 degrees North
Longitude: 117.2336137 degrees West
Altitude: 111 meters above mean sea level
Propagated Epoch: JD2454873.205555555

	Epoch	Right Ascension	Declination	Local Sidereal Time
Observation 1	2454871.265087747	52.79819	81.87959	297.18302
Observation 2	2454871.265782192	341.58628	74.53861	297.43370
Observation 3	2454871.266476636	322.49722	52.51823	297.68439

Orbital Vectors (No Perturbations)

	Radius	Velocity
Initial	(2740.15, -4844.74, 4500.41)	(2.82, -3.78, -5.77)
Final	(568.49, -1699.82, 6920.47)	(3.97, -6.06, -1.8)

Orbital Vectors (Oblate Earth)

	Radius	Velocity
Initial	(2740.15, -4844.74, 4500.41)	(2.82, -3.78, -5.77)
Final	(424.92,-1463.65, 6824.34)	(4.07, -6.22, -1.70)

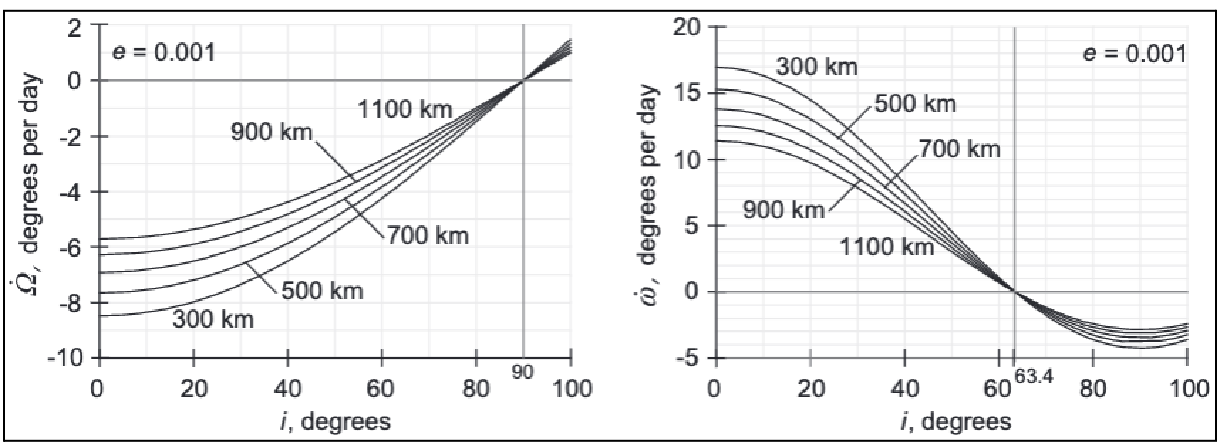
Keplerian Orbital Elements (No Perturbations)

	Semi-Major Axis (km)	Eccentricity	Inclination (deg)	R.A. of Ascending Node (deg)	Argument of Periapsis (deg)	Mean Anomaly (deg)
Initial	7142.68	0.00252	86.44	122.37	354.09	146.70
Final	7142.68	0.00252	86.44	122.37	354.09	109.72

Keplerian Orbital Elements (Oblate Earth, Propagated)

	Semi-Major Axis (km)	Eccentricity	Inclination (deg)	R.A. of Ascending Node (deg)	Argument of Periapsis (deg)	Mean Anomaly (deg)
Initial	7142.68	0.00252	86.44	122.37	354.09	146.70
Final	7141.76	0.0256	86.44	122.38	138.18	325.6

We have one more sanity check:



Disregard e = .001, because of the nature of the equations, where p = a*sqrt(1-e^2), it is negligible;
Here is what our perturbations look like in degrees per day.

```
dOm: -0.415740509502803
 dw: -3.28782942002873
 M0: -3.31359836709483
```

To our knowledge, it is by far the best method of orbit determination; it has been proven and tested with examples from *Fundamentals of Astrodynamics and Applications.* A great way to convey this is by comparing the basics, like conservation of energy: our computed initial energy of -27.9027 is approximately equivalent to our energy after propagation, which gives an intuitive understanding that things check out.

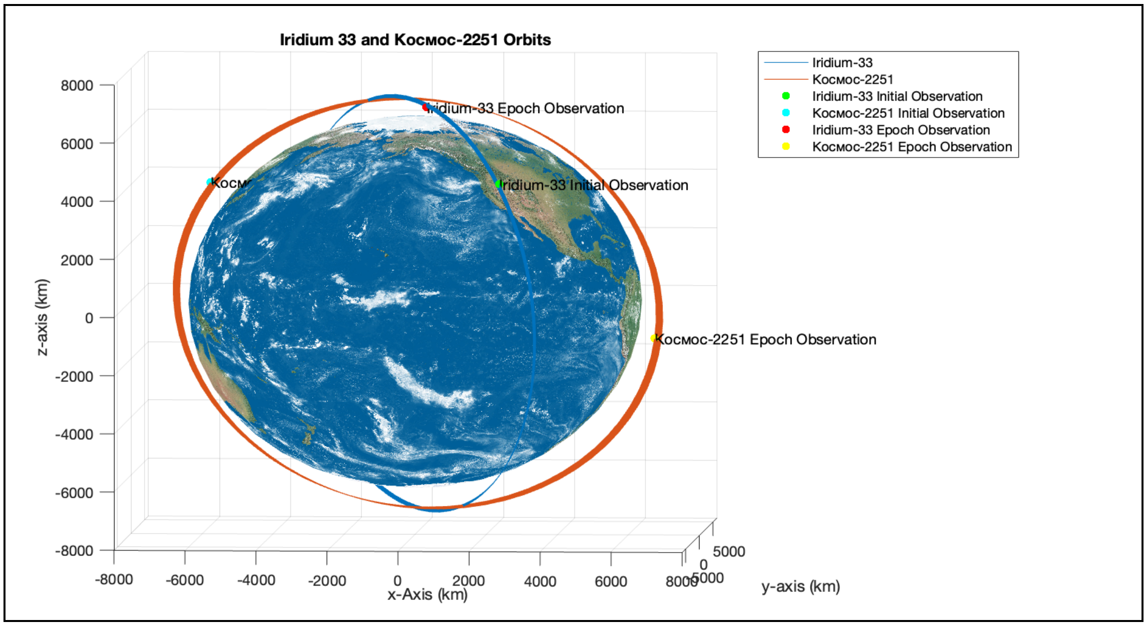


Figure 1. Iridium-33 and Kosmos-2251 orbits

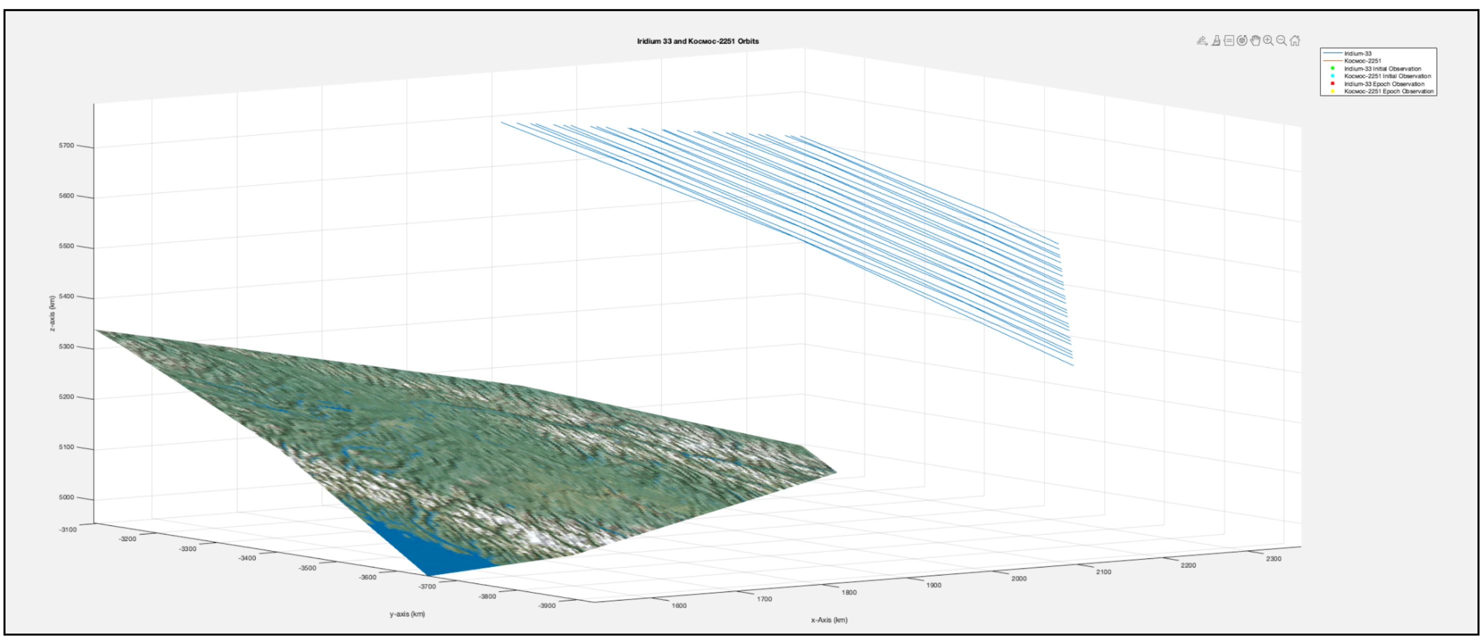


Figure 2. Closeup of Iridium-33's perturbed orbit showing the changing orbit due to Earth's oblateness

Conclusions

Through our algorithm, we found that given the initial observations, the orbit of our given satellite was: The orbit has a radius of 7,142 kilometers or an altitude of 764 kilometers, an eccentricity of 0.0252, a nearly polar inclination of 86.44 degrees, a right ascension of the ascending node of 122.37 degrees, an argument of periapsis of 354.09 degrees, and a mean anomaly of 146.70 degrees in its perturbed case.

We also implemented a non-keplerian oblate earth model to our algorithm which gave us the following final values for the 6 orbital elements: a radius of 7,142 kilometers or an altitude of 764 kilometers, an eccentricity of 0.0252, a nearly polar inclination of 86.44 degrees, a right ascension of the ascending node of 122.37 degrees, an argument of periapsis of 354.09 degrees, and a mean anomaly of 109.72 degrees. As you can see from the more detailed tables in the last section, the main value that changed was the mean anomaly and argument of periapsis. The difference between the two can also be shown in Fig.1 and Fig.2 where the two figures show how the orbit is a collection of orbits. The reason for this is due to Earth's oblateness, acceleration is being added, making the orbit faster over time. (Higher quality figures can be found in the accompanying zip file).

Additionally, working with a group led by Michael Zeng, we took their data of the Kosmos-2251 satellite and plotted it against our gathered data of the Iridium-33 satellite as shown in figure 1. Analysis of the final position vectors show that the distance at T-final (which we believe to be the time of final observation) is 11,213.73 km away from each other. We note that this is a rather large distance away from each other given we wanted to see a time of collision. The simulation shows a collision of the two satellites on 2009-02-10. The reason behind this discrepancy boils down to the idea of using a simplified model; please beware, there are many things to be accounted for, in a more realistic orbit - atmospheric drag, higher order gravitational effects, and friction from space debris and molecules etc. The simulation is a built in satellite package in matlab that is sourcing a realistic TLE from space-track.org, both of which account for more complex effects. Nonetheless, we were able to extract meaningful data from these orbits and believe they could be of great use for your company.

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Charles Cody, physics - Analysis of orbit termination & Test Results
Kevin Bishara, Mechanical Engineering - Orbit determination, references, revisions, and readme.txt file

All discussed physical interpretation of results.

References

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