

## 相关

协方差:  $SP = \sum XY - \frac{\sum X \sum Y}{n}$

Pearson 相关:  $r = \frac{SP}{\sqrt{SS_x} \sqrt{SS_y}}$

参数 ① 相关的显著性检验.

比较两个  $r_1, r_2$  是否有显著差异. 查表得  $Z_{r_1}, Z_{r_2}$  或

$$Z_r = 5[\ln(1+r) - \ln(1-r)]r$$

$$Z = \frac{Z_{r_1} - Z_{r_2}}{\sqrt{1/(n-3)}}$$

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

$$t_{crit} = t_{0.05}(df=n-2)$$

Spearman 相关

非参. 1. 排序, 分等级

2. 计算  $SS_x, SS_y$  及  $SP$   $r_s = SP / \sqrt{SS_x SS_y}$  person

$D = Y_{\text{等级}} - X_{\text{等级}}^2, Y_s = 1 - 6 \sum D^2 / [n(n^2 - 1)]$  spearman 数据不能重复.

4. 进行查表进行显著性检验. 或者 ①

高二列相关.

是非.  $r_{pb} = \frac{\bar{x}_p - \bar{x}_q}{S_p} \sqrt{pq}$  与  $t$  之间关系:  $r_{pb}^2 = \frac{t^2}{t^2 + df}$   $df = n_1 + n_2 - 2$

进行显著性检验用公式 ①

Kendall 和谐系数

$$W = \frac{\sum R_i^2 - (\sum R_i)^2}{\frac{1}{12} K^2 (N^3 - N)}$$

$$r_s = \frac{KW - 1}{K - 1} \text{ (spearman)}$$

		N 个事物					
		1	2	3	4	...	N
评估者	1	X	X	X	X	...	X
	2	X	X	X	X	...	X
	3	X	X	X	X	...	X
	K	X	X	X	X	...	X
		$\sum R_i =$	$R_1$	$R_2$	$R_3$	$...$	$R_N$

## 回归初步

$$\text{斜率} = b = \frac{SP}{SS_x}$$

$$\text{截距} = a = \bar{Y} - b\bar{X}$$

$$SP = \sum XY - \frac{\sum X \sum Y}{n}$$

$$SS_x = \sum X^2 - \frac{(\sum X)^2}{n}$$

$$\hat{Y} = \bar{Y} - b\bar{X}$$

估计的标准误的计算步骤: 估计的标准误 =  $\sqrt{\frac{SS_{error}}{df}} = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n-2}}$

又:  $SS_{error} = (1 - r^2) SS_y$

$$\therefore \text{估计的标准误} = \sqrt{\frac{(1 - r^2) SS_y}{n-2}}$$

## $\chi^2$ 区配度检验.

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} \quad f_o = \text{观察值} \quad f_e = \text{期望值}$$

$$df = C - 1 \quad \chi^2_{crit} = \chi^2_{0.05}(df) \quad \chi^2_{crit} < \chi^2_{ob} \text{ 拒绝 } H_0$$

若题目中给出的是正态分布. 求出每组上限下限对应的  $Z$  值, 得出

$$Z_1 = \frac{X_1 - \bar{X}}{S} \quad Z_2 = \frac{X_2 - \bar{X}}{S} \quad df = k - 3$$

## $\chi^2$ 独立性检验.

因素 B				行和	期望值 = $f_{ys} = \frac{m_i n_c}{N}$
1	2	3	...	$m_i$	$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$
因 素 A 1	X	X	X	$m_1$	
2	X	X	X	$m_2$	
...				$m_r$	
列和	$n_1$	$n_2$	$n_3$	$n_c$	$N$

$$df = (R-1)(C-1)$$

$$\chi^2_{crit} = \chi^2_{0.05}(df)$$

## $\chi^2$ 检验的效应水平.

$$\Phi = \sqrt{\frac{\chi^2}{N \times df}}$$

$$df_{min} = \min\{r-1, s-1\}$$

$df_{min} \uparrow$   $\Phi \downarrow$

$$\chi^2 \text{ 检验正态分布的 CI}$$

$$\left[ \frac{(n-1)s^2}{\chi^2_{2, 0.95}}, \frac{(n-1)s^2}{\chi^2_{2, 0.05}} \right]$$

## 非参数检验. 若有相同的方法处理秩.

### 1. 墨-惠特尼 U 检验

① 排排列, 找点数, 或进行计算  $U_A = n_A n_B + \frac{n_A(n_A+1)}{2}$   $U_B = n_A n_B - U_A$

$$U = \min \{U_A, U_B\} \quad U_{crit} = U_{0.05}(n_A, n_B)$$

$$U_{obs} < U_{crit} \quad R_j H_0$$

② 两个样本中至少有一个容量大于 20

$$\mu = \frac{n_A n_B}{2} \quad \sigma = \sqrt{\frac{n_A n_B (n_A + n_B + 1)}{12}} \quad z = \frac{U - \mu}{\sigma}$$

$$|z_{obs}| > z_{crit} \quad R_j H_0$$

### 2. 符号检验法. (相关样本)

① ( $n < 25$ ) 在符号检验表中, 直接将较少的符号的数目与临界值进行比较.  $obs < crit \quad R_j H_0$ .

$$\text{② } (n > 25) \quad \mu = np \quad (p = \frac{1}{2}) \quad \sigma = \sqrt{npq} \quad z_{obs} = \frac{Y - \mu}{\sigma}$$

$$z_{obs} > z_{crit} \quad R_j H_0$$

### 3. 维尔克松 T 检验. (相关样本)

①  $R_+ = X_1, R_{-} = X_2 \quad T = \min \{R_+, R_{-}\} \quad T_{crit} = T_{0.05}(n)$

$$(n < 25) \quad T_{obs} < T_{crit} \quad R_j H_0$$

$$\text{② } (n > 25) \quad \mu = \frac{n(n+1)}{4} \quad \sigma = \sqrt{\frac{n(n+1)(2n+1)}{24}} \quad z = \frac{T - \mu}{\sigma}$$

### 4. 克-瓦氏单向方差分析 (多顺序独立样本)

① 排列之后, 计算  $\sum R_A, \sum R_B, \sum R_C$

$$H = \frac{12}{N(N+1)} \sum \frac{R_i^2}{n_i} - 3(N+1) \quad \text{查表 } \alpha = 0.01$$

$$H_{obs} > H_{crit} \quad Rej H_0$$

② ( $k > 3$  或  $n > 25$ ) 查自由度为  $k-1$  的  $\chi^2$  分布.

### 5. 弗里德曼双向方差分析 (多顺序相关样本)

①  $\begin{array}{c} \text{评估者} \\ \text{1} \\ \text{2} \\ \text{3} \\ \vdots \\ (n) \end{array} \quad \begin{array}{c} \text{原始分} \\ \text{IA} \\ \text{IB} \\ \text{IC} \\ \vdots \\ \text{IR} \end{array} \quad \begin{array}{c} \text{秩次} \\ 1 \\ 2 \\ 3 \\ \vdots \\ R \end{array} \quad \chi^2 = \frac{12}{nk(k+1)} \sum R_i^2 - 3n(k+1)$

$$\chi^2_{crit} = \chi^2_{0.05}(k, n) \quad \text{若 } \chi^2_{obs} > \chi^2_{crit} \quad \text{则 } Rej H_0. \quad \text{弗里德曼量}$$

② 查自由度为  $k-1$  的  $\chi^2$  分布表.

$$Pvalue = power = \Phi \left[ \frac{Z_{obs} + \frac{|U_{obs} - \mu|}{\sigma/\sqrt{n}}}{\sigma/\sqrt{n}} \right] \quad \text{One side}$$

$$= \Phi \left[ \frac{Z_{obs} + \frac{|U_{obs} - \mu|}{\sigma/\sqrt{n}}}{\sigma/\sqrt{n}} \right] \quad \text{two side}$$

## 1. 对两个独立样本的假设检验

### ① 独立样本 t 检验 下值检验

$$S_1^2 = \frac{SS_1}{df_1}, S_2^2 = \frac{SS_2}{df_2}, S_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2}, S_p \\ S_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}, t_{obs} = \frac{\bar{x}_1 - \bar{x}_2}{S_{\bar{x}_1 - \bar{x}_2}} > t_{0.05/2} \text{ Rej } H_0 \\ \text{检验效应 } ES = \frac{\bar{x}_1 - \bar{x}_2}{S_p} \text{ Cohen's } d$$

### ② 相关样本 t 检验

$$SS_D = \sum D^2 - \frac{(\sum D)^2}{n}, SD = \sqrt{\frac{SS_D}{n-1}}, S_D = \sqrt{\frac{SD^2}{n}} \\ t_{obs} = \frac{\bar{D} - \mu_D}{S_D}, t_{obs} > t_{0.05/2} \text{ Rej } H_0 \\ \text{检验效应 } ESD = \frac{\bar{D}}{SD}$$

基本公式汇总：

$$SS = \sum (X - \mu)^2 = \sum X^2 - \frac{(\sum X)^2}{N}, b = \sqrt{\frac{SS}{N}}, b^2 = \frac{SS}{N}$$

$$\text{对样本: } S^2 = \frac{SS}{n-1}, S = \sqrt{\frac{SS}{n-1}}$$

人数的计算：

$$n = \frac{b^2(Z_{1-\alpha} + Z_{1-\beta})^2}{(\mu_0 - \mu_1)^2}$$

## 单因素和重复测量方差分析

### 1. 单因素

方法 A	方法 B	方法 C
0	0	0
0	0	0
0	0	0
0	0	0
T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>
SS <sub>1</sub>	SS <sub>2</sub>	SS <sub>3</sub>
n <sub>1</sub>	n <sub>2</sub>	n <sub>3</sub>
X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>

$$SS_{\text{总和}} = \sum X^2 - (\bar{G}^2/N)$$

$$SS_{\text{组内}} = SS_1 + SS_2 + SS_3$$

$$MS_{\text{组间}} = SS_{\text{组间}} / df_{\text{组间}}$$

$$F_{obs} = MS_{\text{组间}} / MS_{\text{组内}}$$

### 事后检验 HSD 检验

$$HSD = q_{\alpha/2} \sqrt{MS_{\text{组内}} / n}$$

$$\bar{x}_1 - \bar{x}_2 > HSD$$

$$F_{crit} = F_{0.05}(df_{\text{组间}}, df_{\text{组内}})$$

### 2. 重复测量方差分析

被试	第 k 次测量		
	1	2	3
A	0	0	0
B	0	0	0
C	0	0	0
D	0	0	0
n	T <sub>1</sub>	T <sub>2</sub>	T <sub>3</sub>
SS	SS <sub>1</sub>	SS <sub>2</sub>	SS <sub>3</sub>

$$MS_{\text{组间}} = SS_{\text{组间}} / df_{\text{组间}}$$

$$F_{obs} = MS_{\text{组间}} / MS_{\text{误差}}$$

$$F_{crit} = F_{0.05}(df_{\text{被试间}}, df_{\text{误差}})$$

进行 HSD 事后检验

$$\text{方差分析的效果大小: } f = \sqrt{F / N}$$

$$0.01 \quad 0.25 \quad 0.40$$

## 和总体均值相关的估计

### 1. 方差已知情况下总体均值的估计

$$\text{① 点估计 } \bar{x} = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\text{② 区间估计 } [\bar{x} - \frac{Z_{0.05}}{\sqrt{n}}, \bar{x} + \frac{Z_{0.05}}{\sqrt{n}}]$$

### 2. 方差未知情况下总体均值的估计

$$\text{② 区间估计 } [\bar{x} - t_{0.05/2} \sqrt{\frac{SS}{(n-1)n}}, \bar{x} + t_{0.05/2} \sqrt{\frac{SS}{(n-1)n}}]$$

### 3. 独立组总体均值差异的估计

$$\text{① 点估计 } \mu_A - \mu_B$$

$$\text{② 区间估计 } [\bar{X}_A - \bar{X}_B \pm t_{0.05/2} \sqrt{\frac{SS_A + SS_B}{n_A + n_B - 2} (\frac{1}{n_A} + \frac{1}{n_B})}]$$

### 4. 相关组总体均值差异的估计

$$\text{① 点估计 } \bar{D} = \bar{X} - \bar{Y}$$

$$\text{② 区间估计 } [\bar{D} - t_{0.05/2} \sqrt{\frac{SS_D}{(n-1)n}}, \bar{D} + t_{0.05/2} \sqrt{\frac{SS_D}{(n-1)n}}]$$

## 二因素方差分析

	B <sub>1</sub>	B <sub>2</sub>
A <sub>1</sub>	0 0 0 0	0 0 0 0
A <sub>2</sub>	0 0 0 0	0 0 0 0

因素 B (b=2)

A	B <sub>1</sub>	B <sub>2</sub>
A <sub>1</sub>	A <sub>1</sub> B <sub>1</sub>	A <sub>1</sub> B <sub>2</sub>
A <sub>2</sub>	A <sub>2</sub> B <sub>1</sub>	A <sub>2</sub> B <sub>2</sub>

$$a=b=2$$

X	X <sup>2</sup>
0	0
0	0
0	0
0	0

$$A_1 = A_1 B_1 + A_1 B_2 \quad A_2 = A_2 B_1 + A_2 B_2$$

$$B_1 = A_1 B_1 + A_2 B_1 \quad B_2 = A_1 B_2 + A_2 B_2$$

$$G = A_1 + A_2 = B_1 + B_2$$

$$df_A = a-1 \quad df_B = b-1$$

$$df_{AB} = (a-1)(b-1)$$

$$df_{\text{处理内}} = N - a \times b$$

$$A_1 B_1 = \sum X$$

$$SS_{\text{总和}} = \sum X^2 - \frac{G^2}{N}$$

$$SS_{\text{处理间}} = \sum \frac{A_x B_y^2}{B} - \frac{G^2}{N} = \frac{A_1 B_1^2}{n} + \frac{A_1 B_2^2}{n} + \frac{A_2 B_1^2}{n} + \frac{A_2 B_2^2}{n} - \frac{G^2}{N}$$

$$SS_{\text{处理内}} = \sum S S_{\text{被试间}}$$

$$SS_A = \sum \frac{A_x^2}{b n} - \frac{G^2}{N}$$

$$SS_B = \sum \frac{B_y^2}{a n} - \frac{G^2}{N}$$

$$SS_{AB} = SS_{\text{处理间}} - SS_A - SS_B$$

$$MS_{\text{处理间}} = \frac{SS_{\text{处理间}}}{df_{\text{处理间}}} \quad MSA = \frac{SS_A}{df_A} \quad MSB = \frac{SS_B}{df_B}$$

$$MS_{AB} = \frac{SS_{AB}}{df_{AB}} \quad FA = \frac{MSA}{MS_{\text{处理内}}} \quad F = \frac{MSB}{MS_{\text{处理内}}}$$

$$F_{AB} = \frac{MS_{AB}}{MS_{\text{处理内}}}$$

$$F_{crit} = F_{0.05}(df_{AB}, df_{\text{处理内}})$$

$$HSD = q_{\alpha/2} \sqrt{MS_{\text{组内}} / n}$$