Algebra

Exponents and Radicals

Problem 1. (2⋆)
Find all values of x given that $(x-2)^{x^2-4}=1$.

Problem 2. (2⋆)
Given that x and y are integers and $5^{x-y+5}=7^{x+y+3}$, find the value of $\frac{x-y}{x+y}$.

Problem 3. (3⋆)
Given that

$$\begin{cases} 4^a \cdot 3^b = 108 \\ 2^b \cdot 9^a = 72 \end{cases}$$

Find $2a+b$.

Problem 4. (5⋆)
Given that n is an integer that is not equal to 0, and

$$(-2x+1)^{2n} == (9x^2+6x+1)^n$$

Find the sum of the possible values of x.

Problem 5. (3⋆)
Given that

$$\begin{cases} (x+y)\cdot 2^{y-x} = 1 \\ (x+y)^{x-y} = 2 \end{cases}$$

Find the sum of possible values of x.

Problem 6. (3⋆)
x and y are integers such that $2^{x+y}+2^x = 3^{y+2}-3^y$. Find x.

Problem 7. (7⋆)
a, b, c, and d are positive real numbers such that $a^6=b^5, c^4=d^3, b-c=19$. Find the value of d^2-a^2.

Problem 8. (7⋆)
It is given that $y=x^2-5x+5, z=x^2-12x+35$ and $y^z=1$. Find the sum of all possible values of x.

Exponents and Radicals

Problem 9. (5⋆)
Find the value of

$$\frac{\sqrt{x-1}+\sqrt{1-x}}{2x+3}.$$

Problem 10. (3⋆)
Find the value of

$$\sqrt{|40\sqrt{2}-57|}-\sqrt{40\sqrt{2}+57}.$$

Problem 11. (5⋆)
Find the value of

$$\sqrt{\sqrt{6}+1}+\sqrt{\sqrt{6}-1}.$$

In terms of b, where $\sqrt{6}+\sqrt{5}=b^2$.

Problem 12. (5⋆)
If x, y, k are real numbers not equal to 0, and we know

$$x^{y^k}=k^{x^y}, k=x^y, y=x^{2y}$$

Then find the value of y.

Problem 13. (2⋆)
If $\sqrt{x+9}+\sqrt{x}=1$, find the value of $\sqrt{x+9}-\sqrt{x}$.

Problem 14. (3⋆)
If $x-\sqrt{\frac{2}{x}}=3$, what is the value of $x-\sqrt{2x}$?

Problem 15. (3⋆)
Find the value of

$$\frac{\sqrt[4]{17+12\sqrt{2}}}{\sqrt{2}-1}-2\sqrt{2}$$

Problem 16. (5⋆)
Compute

$$\sqrt[3]{5+2\sqrt{13}}-\sqrt[3]{5-2\sqrt{13}}.$$

Problem 17. (5⋆)
Find the value of

$$\frac{\sqrt{\sqrt{13}-3}+\sqrt{\sqrt{13}+3}}{\sqrt{4+2\sqrt{11+\sqrt{7-\sqrt{9}}}}}.$$

Problem 18. (7⋆)
Find x, if we are given

$$\sqrt{2x+\sqrt{4x-1}}+\sqrt{2x-\sqrt{4x-1}}=2\sqrt{2}.$$

Problem 19. (7⋆)
Given that

$$A=\frac{1}{10-3\sqrt{11}}-\frac{1}{3\sqrt{11}-7\sqrt{2}}+\frac{1}{7\sqrt{2}-\sqrt{97}}-\frac{1}{\sqrt{97}-4\sqrt{6}}.$$

Find the value of

$$\frac{A}{5-2\sqrt{6}}.$$

Problem 20. (7⋆)
Find all possible real values of x such that

$$\sqrt{\frac{x-7}{1990}}+\sqrt{\frac{x-6}{1991}}+\sqrt{\frac{x-5}{1992}}=\sqrt{\frac{x-1990}{7}}+\sqrt{\frac{x-1991}{6}}+\sqrt{\frac{x-1992}{5}}.$$

Problem 21. (11⋆)
Given that

$$A=\sqrt{2+\sqrt{2}}$$

$$B=\sqrt{2+\sqrt{2+\sqrt{2}}}$$

$$C=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$$

$$D=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}$$

Find the value of $A\cdot B\cdot C\cdot D$.