



Very Important Stuff

1. If you see a lot of terms that are kinda similar and either under a fraction or have a lot of related terms, think of telescoping! Because this means it's likely that a lot of stuff will cancel out so yeah.

Problem 1. (2★)

Evaluate the sum

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots + \frac{1}{99 \cdot 100}$$

Problem 2. (2★)

Simplify

$$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \frac{1}{9 \cdot 13} + \cdots + \frac{1}{101 \cdot 105}$$

Problem 3. (2★)

Find the sum

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{255 \cdot 257}$$

Problem 4. Required. (3★)

Find the value of

$$\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{99}{100!}$$

Express your answer as a common fraction in terms of factorials.

Problem 5. (5★)

Calculate

$$\frac{3}{1! + 2! + 3!} + \frac{4}{2! + 3! + 4!} + \cdots + \frac{100}{98! + 99! + 100!}$$

Problem 6. (3★)

Find

$$\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \cdots + \frac{1}{\sqrt{99} + \sqrt{100}}$$

Problem 7. (7★)

Calculate

$$\frac{1}{1^4 + 1^2 + 1} + \frac{1}{2^4 + 2^2 + 1} + \frac{1}{3^4 + 3^2 + 1} + \cdots$$

**Problem 8.** (11★)

Calculate

$$\frac{1}{2^2 - 1} + \frac{1}{4^2 - 1} + \frac{1}{6^2 - 1} + \cdots + \frac{1}{1000^2 - 1}$$

Problem 9. Required. (11★)

Find the sum

$$\frac{1}{3^2 + 1} + \frac{1}{4^2 + 2} + \frac{1}{5^2 + 3} + \cdots$$

Problem 10. (11★)

Let

$$S = 1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \cdots + 11 \cdot 2^{10}$$

Find S .**Problem 11.** (11★)

Determine the value of the infinite product

$$2^{\frac{1}{3}} \cdot 4^{\frac{1}{9}} \cdot 8^{\frac{1}{27}} \cdots$$

Problem 12. Required. (11★)

Evaluate the sum

$$\frac{5}{4} + \frac{8}{4^2} + \frac{11}{4^3} + \frac{14}{4^4} + \cdots$$