On Simple Linear Regression

// FLATIRON SCHOOL

👋 Hiya!

Software Engineer turned
Motorcycle Instructor turned
Data Science Instructor

I just need to figure out how to work motorcycles into the equation...

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Data Science
Instructor
Flatiron School

Agenda

- Why Linear Regression?
- Simple Linear Regression
- Inference & Prediction
- A Line as a Model
- Example

Why Linear Regression?

- LR is a fundamental tool in the data scientist's kit.
- Practically speaking, using it is one or two lines of code.
- But it's crucial for us to understand the theory underlying it:
- Building block for more complex tools
- We are better data scientists if we understand both HOW and WHY

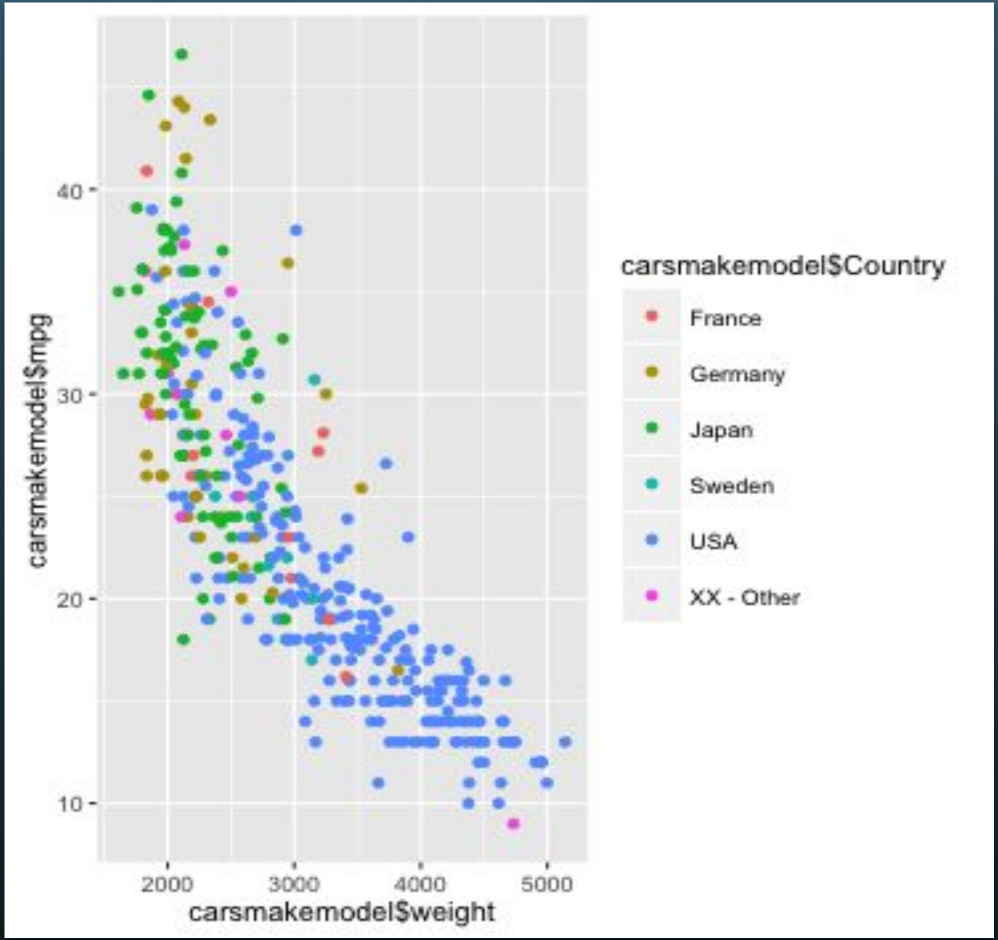
Simple Linear Regression

- Simple: functions of a single variable: $Y = f(X)$
- Linear: models are lines
- Regression: dependent variable is continuously-valued
- Closed form solution $\beta=(X^TX)^{-1}X^Ty$

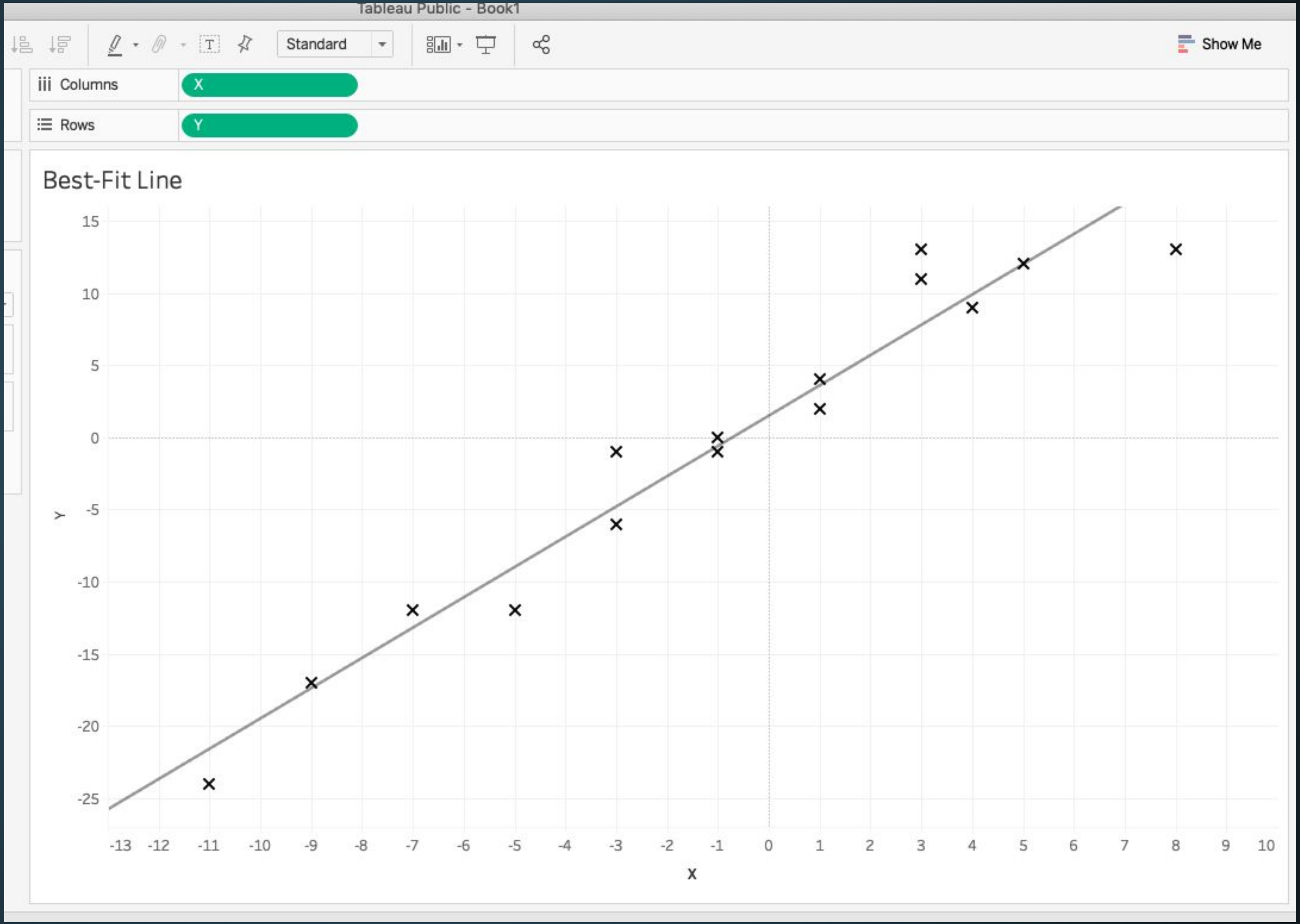
Inference and Prediction

- As population density increases, so do housing prices.
- As the number of trees decreases, the concentration of CO_2 goes up.

Car Weight and MPG



Best-Fit Line



A Line as a Model

Predictions for all values of the X variable

Model shape:

$$\hat{y} = \beta_1 x + \beta_0$$

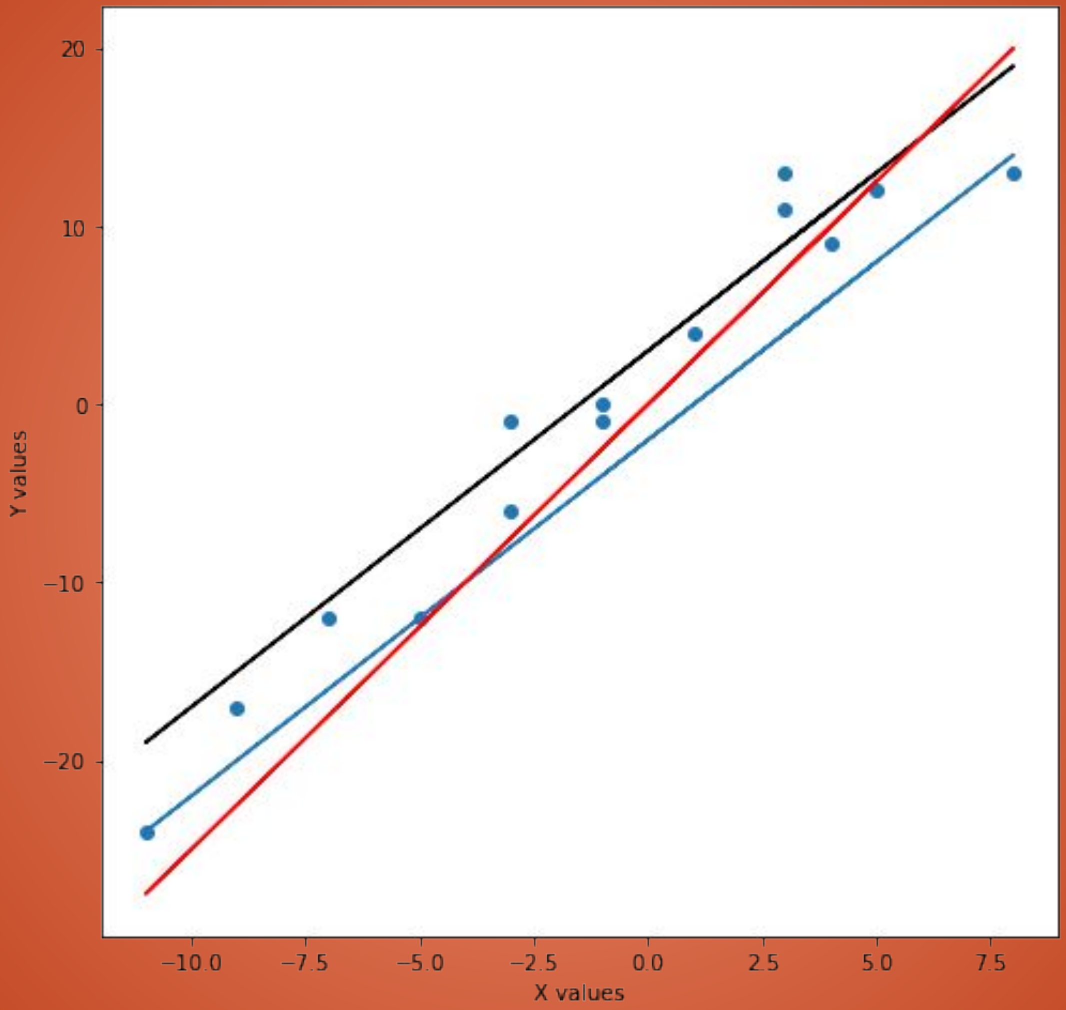
Error as the distance between real and predicted values:

$$E = y - \hat{y}$$
$$E^2 = (y - \hat{y})^2$$

Goal:

Minimize Error

Which of these lines fits the data best?



How to Construct the Best-Fit Line

$$r_P = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}}$$

$$\beta_1 = r_P \frac{\sigma_y}{\sigma_x}$$

$$\beta_0 = \bar{y}_1 - \beta_1 \bar{x}$$

Example

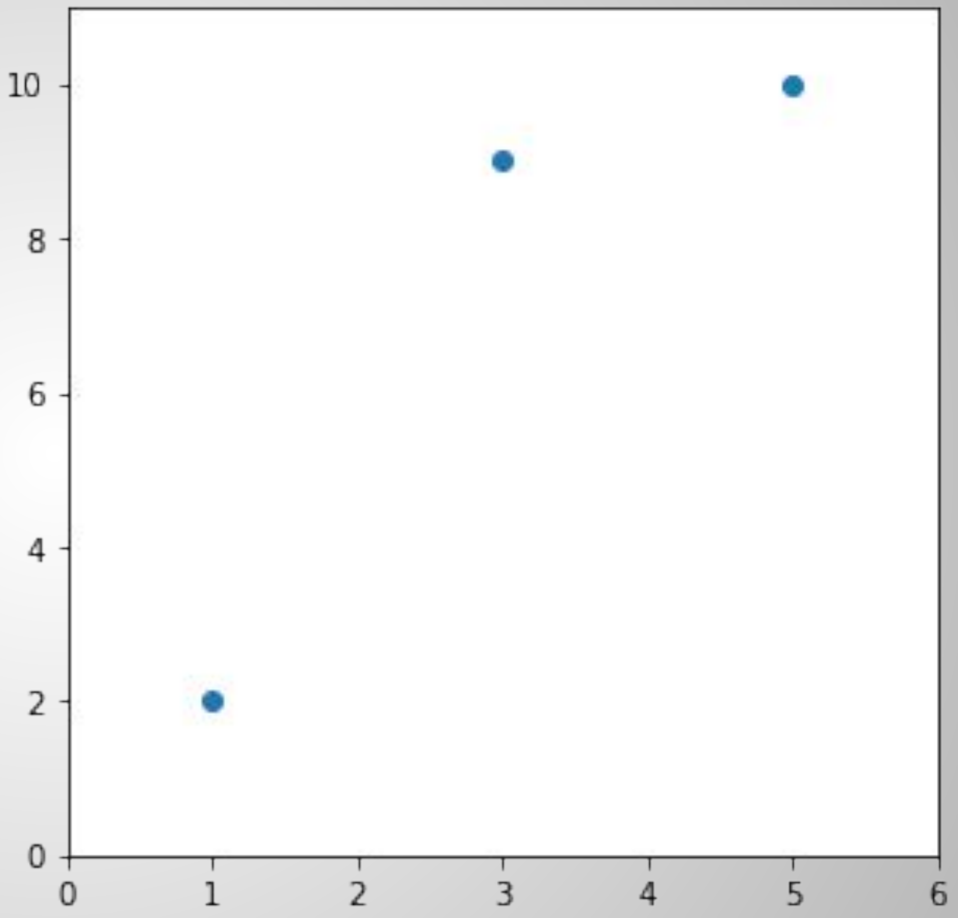
Construct the best-fit line for the points: (1, 2), (3, 9), and (5, 10)

Remember:

$$r_P = \frac{\Sigma_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma_i (x_i - \bar{x})^2}\sqrt{\Sigma_i (y_i - \bar{y})^2}}$$

$$\beta_1 = r_P \frac{\sigma_y}{\sigma_x}$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$



$$x_i : [1, 3, 5]$$
$$y_i : [2, 9, 10]$$

Step 1:

Calculate x_bar and y_bar:

$$\bar{x} = \frac{1+3+5}{3} = 3$$

$$\bar{y} = \frac{2+9+10}{3} = 7$$

$$x_i : [1, 3, 5]$$
$$y_i : [2, 9, 10]$$

Step 2:

Calculate these products:

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = (1-3)(2-7) + (3-3)(9-7) + (5-3)(10-7) = 16$$

$$\Sigma(x_i - \bar{x})^2 = (1-3)^2 + (3-3)^2 + (5-3)^2 = 8$$

$$\Sigma(y_i - \bar{y})^2 = (2-7)^2 + (9-7)^2 + (10-7)^2 = 38$$

$$x_i : [1, 3, 5]$$
$$y_i : [2, 9, 10]$$

Step 3:

Calculate Pearson correlation:

$$r_P = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma_i (x_i - \bar{x})^2} \sqrt{\Sigma_i (y_i - \bar{y})^2}} = \frac{16}{\sqrt{(8)(38)}} = \frac{4}{\sqrt{19}}$$

$$x_i : [1, 3, 5]$$
$$y_i : [2, 9, 10]$$

Step 4:

Calculate standard deviations:

$$\sigma_x = \sqrt{\frac{8}{3}}$$

$$\sigma_y = \sqrt{\frac{38}{3}}$$

$$x_i : [1, 3, 5]$$
$$y_i : [2, 9, 10]$$

Step 5:

Calculate the slope:

$$\beta_1 = r_P \frac{\sigma_y}{\sigma_x} = \frac{4}{\sqrt{19}} \left(\frac{\sqrt{\frac{38}{3}}}{\sqrt{\frac{8}{3}}} \right) = \frac{4}{\sqrt{19}} \left(\frac{\sqrt{38}}{\sqrt{8}} \right) = \frac{4\sqrt{2}}{2\sqrt{2}} = 2$$

$$x_i : [1, 3, 5]$$
$$y_i : [2, 9, 10]$$

Step 6:

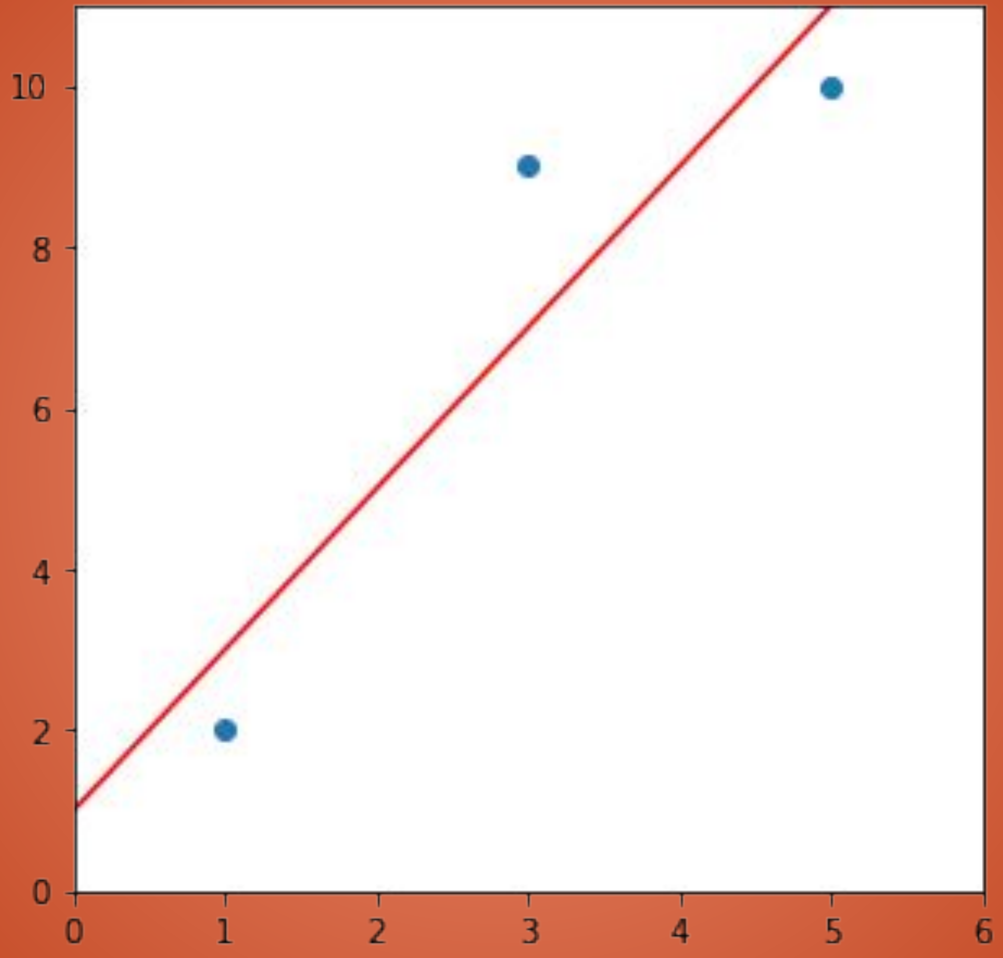
Calculate the y-intercept:

$$\beta_0 = \bar{y} - \beta_1 \bar{x} = 7 - (2)(3) = 1$$

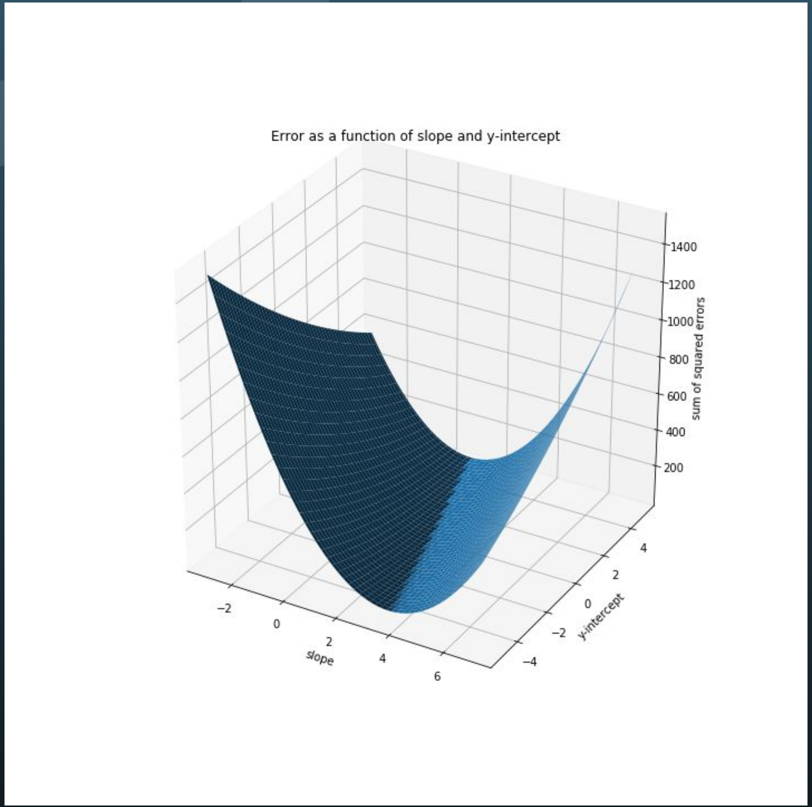
Line:

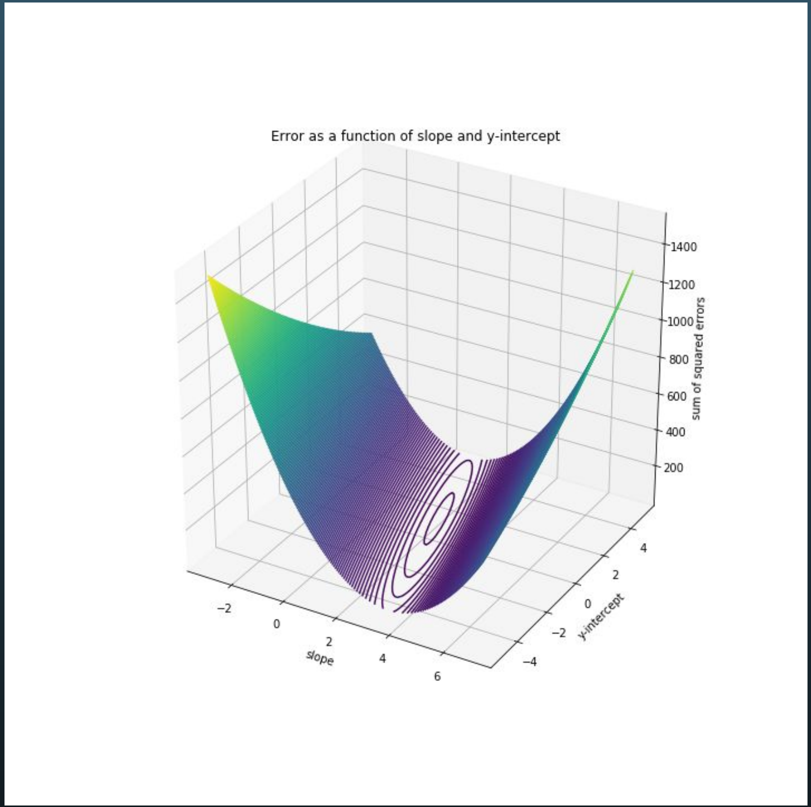
We have our line!

$$\hat{y} = \beta_1 x + \beta_0 = 2x + 1$$



Surface and Contour Plots of SSE(m, b)





Not Covered:

1. Variance
2. Covariance
3. Assumptions of the linear model
4. Bend the line?
5. Multiple linear regression

Github repo for the Jupyter Notebook

Link!

Thank you!