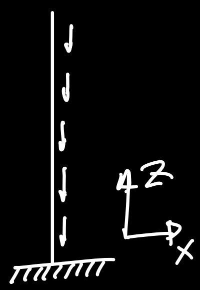
$$\underline{f}^a(s) = -\sigma \underline{x}_s \; ; \quad \sigma > 0$$

Prescribed force density.



$$8\pi\mu\left[\dot{\underline{x}} - \underline{u}\right] = \underline{\underline{\Lambda}} \cdot \underline{f}$$

$$\boxed{\underline{\underline{\Lambda}} = \left[(2-c)\,\underline{\underline{I}} - (2+c)\,\underline{x}_s \underline{x}_s\right]}$$

$$\underline{f} = -B\,\underline{x}_{ssss} + (T\,\underline{x}_s)_s - \sigma\,\underline{x}_s$$

Scaling: $s \sim L$, $f \sim \frac{B}{L^3}$, $t \sim \frac{8\pi\mu L^4}{B}$

$$\partial_t \underline{x} = \underline{\underline{\Lambda}} \cdot \left[-\underline{x}_{ssss} + (T\,\underline{x}_s)_s - \tilde{\sigma}\,\underline{x}_s\right];$$

$$\boxed{\tilde{\sigma} = \frac{\sigma L^3}{B}}$$ ⟵ only number governing stability.

Base state: $\underline{x}^0 = s\,\hat{\underline{z}}$; $\quad T^0 = -\tilde{\sigma}(1-z)$

$$\underline{x} = \underline{x}^0 + \varepsilon\, h(s)\, e^{\sigma t}\, \hat{x}$$

$$\Rightarrow \quad \sigma h = (2-c)\left[-\partial_s^4 h + T^0_s \cancel{\partial_s h} + T^0 \partial_s^2 h - \cancel{\tilde{\sigma}\,\partial_s h}\right]$$

$$\sigma h = (2-c)\left[-\partial_s^4 h - \tilde{\sigma}(1-z)\,\partial_s^2 h\right]; \quad \sigma \equiv \text{eigenvalue}.$$

BCs:

$$\left.\begin{aligned} h = 0 = h_s \quad &@ \; s = 0 \\ h_{ss} = h_{sss} = 0 \quad &@ \; s = 1. \end{aligned}\right\}$$

$$\boxed{\tilde{\sigma}_{cr} = \left(\frac{\sigma L^3}{B}\right)_{cr} \approx 76}$$ } You should see a Hopf-Bifurcation (Oscillating filament)