Statics

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Contents

1 Introduction to Mechanics

The basic concepts used in mechanics are *space, time, mass, and force.* Together, they come to form the general idea of what mechanics is.

Mechanics **1.1**

The study of what happens to an object or system when forces are applied to it. There are three bodies of mechanics: rigid body mechanics, mechanics of materials, and fluid mechanics.

Elementary mechanics is derived from six basic principles.

Six Basic Principles of Mechanics **1.2**

The Parallelogram Law for the Addition of Forces - Two separate forces acting upon the same object can be combined into the *resultant* force by drawing the diagonal inside a parallelogram formed by two of each of the original forces.

The Principle of Transimmibility - The equilibrium of an object remains unchanged if a force acting upon that object is replaced with another identical force acting along the same line of action.

Newton's Three Laws of Motion

- **First Law** - An object's motion remains unchanged if the resultant force on that object is zero.
- **Second Law** - An object will have an acceleration proportional to the resultant force acting upon it. The proportionality is described as: $\overrightarrow{F} = m\overrightarrow{a}$.
- **Third Law** - The forces of action and reaction between bodies in contact are equal in magnitude and line of action, and opposite in direction.

Newton's Law of Gravitation - Two particles of mass M and m are attracted by forces F and $-F$ respectively given by: $F = G\frac{Mm}{r^2}$.

1.1 Units

There are two systems of units: the **International System of Units (SI)** and the **U.S. Customary**. These systems measure the four basic physical quantities (space, time, mass, and force).

The **SI Base** units are what every other unit is derived from. They are listed in full in Figure 1.

Base Quantity	**Name**	**Symbol**
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

Figure 1: SI Base Units

Using the SI Base Units, there is also a set of **SI Derived Units**, meaning that they are standard units that are created through combining the SI Base Units in different ways. They are listed in Figure 2.

Derived Quantity	**Name**	**Symbol**	**Equivalent Base Units**
Frequency	hertz	Hz	s^{-1}
Force	newtown	N	$\frac{m \cdot kg}{s^2}$
Pressure	pascal	Pa	$\frac{N}{m^2}$
Energy	joule	J	$N \cdot m$
Power	watt	W	$\frac{J}{s}$
Electric Charge	coulomb	C	$s \cdot A$
Electric Potential	volt	V	$\frac{W}{A}$
Electric Resistance	ohm	Ω	$\frac{V}{A}$
Celsius Temperature	degree Celsius	℃	K

Figure 2: SI Derived Units

Converting between SI and U.S. Customary, there each physical quantity has a corresponding unit in both systems that are linearly related. The conversions are shows in Figure 3.

Quantity	**U.S. Customary**	**Equals**	**SI**
Force	$1lb$	$=$	$4.448N$
Mass	$1slug$	$=$	$14.59kg$
Length	$1ft$	$=$	$0.3048m$
Volume	$1gal$	$=$	$3.785l$

Figure 3: Unit Conversions

Lastly, there is a set of **SI Unit Prefixes**, used to modify the magnitude of a unit. The range of prefixes is listed in Figure 4.

Factor	Name	Symbol	Numerical Value
10^{15}	peta	P	1 000 000 000 000 000
10^{12}	tera	T	1 000 000 000 000
10^{9}	giga	G	1 000 000 000
10^{6}	mega	M	1 000 000
10^{3}	kilo	k	1 000
10^{2}	hecto	h	100
10^{1}	deka	da	10
10^{-1}	deci	d	0.1
10^{-2}	centi	c	0.01
10^{-3}	milli	m	0.001
10^{-6}	micro	μ	0.000 001
10^{-9}	nano	n	0.000 000 001
10^{-12}	pico	p	0.000 000 000 001
10^{-15}	femto	f	0.000 000 000 000 001

Figure 4: SI Unit Prefixes

2 Representing Forces

2.1 Force Vectors

When describing a force, there are three components to that **force vector**:

1. Point of application
2. Magnitude of the force
3. Direction of the force

Concurrent Forces **2.1**

Two or more forces that are acting on the *same point*.

When dealing with concurrent forces, you can combine the two forces through vector addition to create a single force acting on that same point. That force is called the resultant force.

Resultant Force **2.2**

The vector sum of two or more concurrent forces.

$$\sum \overrightarrow{F} = \overrightarrow{F}_R = \overrightarrow{F}_1 + \overrightarrow{F}_2 + ...$$

2.2 Vectors and Scalars

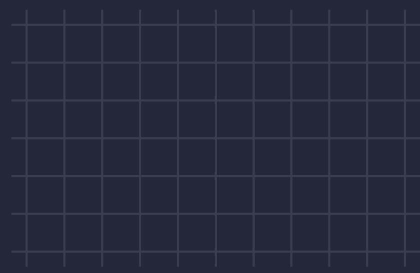


Figure 6: Vector

Vectors should be represented with an arrow above the variable. For example, a force would be written as $\overrightarrow{\mathrm{F}}$.

Magnitude is written as an italic version of the vector. So, $|\overrightarrow{\mathrm{F}}| = F$.

Vector addition with can be done geometrically by creating a triangle, aligning the vectors tip-to-tail.

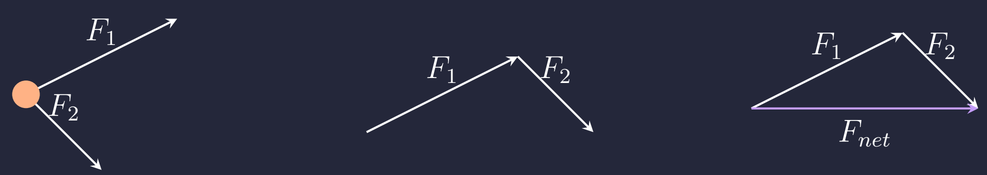


Figure 7: Vector Addition

Law of Sines relates the lengths of the sides of any shaped triangle to the sines of its angles.

Law of Sines

$$\frac{sin(\alpha)}{|\mathbf{F}_1|} = \frac{sin(\beta)}{|\mathbf{F}_2|} = \frac{sin(\gamma)}{|\mathbf{F}_R|} \quad (2.1)$$

Law of Cosines relates the lengths of the sides of any shaped triangle to the cosine of one of its angles.

Law of Cosines 2.2

$$|\mathbf{F}_R|^2 = |\mathbf{F}_1|^2 + |\mathbf{F}_2|^2 - 2|\mathbf{F}_1||\mathbf{F}_2|cos(\gamma)$$

2.3 Unit Vectors and Components of Vectors

Unit Vector is a dimensionless vector with a magnitude equal to one.

$$\hat{u} = \frac{u}{|u|}$$

When considering a $3D$ space going in the x, y, and z directions, the corresponding unit vectors are $\hat{i}$, $\hat{j}$, and $\hat{k}$ respectively.

When resolving a vector, you can take a force F, and break it into its x and y components. However, F_x has no direction, so it must be accompanied by a unit vector to specify its direction.

$$\overrightarrow{F} = F_x\hat{i} + F_y\hat{j}$$

To find the magnitude of a vector:

$$F = |\mathrm{F}| = \sqrt{(F_x)^2 + (F_y)^2}$$

To find its direction:

$$\hat{u}_F = \frac{F}{|F|} = \left(\frac{F_x}{F}\right)\hat{i} + \left(\frac{F_y}{F}\right)\hat{j}$$

3 2D Moments

3.1 Defining Moments

Moment **3.1**

A **vector quantity** used to describe the "turning effect" of a force about a point perpendicular to the force. Calculated using the vector product:

$$\overrightarrow{M}_O = \overrightarrow{r} \times \overrightarrow{F}$$

To calculate the moment of a force about a point, we need to calculate vector products. The cross product of two vectors ($\overrightarrow{P} \times \overrightarrow{Q}$) results in a third vector ($\overrightarrow{V}$):

Vector Product

$$\overrightarrow{V} = \overrightarrow{P} \times \overrightarrow{Q}$$

This results in a new vector with

- Magnitude M_V determined by $|\overrightarrow{V}| = PQsin(\theta)$
- Direction $\hat{u}_V$ with a *line of action* perpendicular to both original vectors and *sense* determined by the right hand rule

3.1

To calculate the vector product of two vectors, the following process is used:

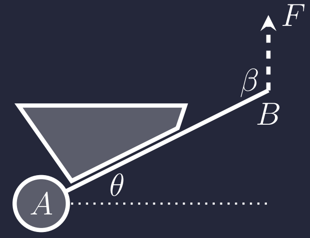
Principle of Transmissibility **3.2**

Any force $\overrightarrow{F}$ acting on a rigid body at a given point can be replaced by an **equivalent force** $\overrightarrow{F}'$ provided that $\overrightarrow{F}$ and $\overrightarrow{F}'$ have the same magnitude, same direction, and same line of action.

3.2 General Process

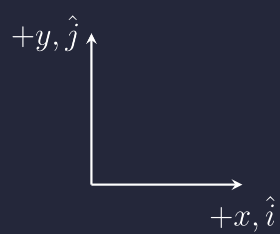
Example **3.1**

A gardener is moving soil with a wheelbarrow. The length of the handle from A to B is $45in$, and in the position shows, θ is 25°. Find the moment created about the axle A when she applies a verticle 30 lb force at B by resolving the force F into components parallel and perpendicular to the handle.



Find:	Given:	Knowns:
$\overrightarrow{M}_A$	$F = 30\text{lb}$ $l_{ab} = 45\text{in}$ $\theta = 25°$	$\overrightarrow{M}_A = \overrightarrow{r}_{b/a} \times \overrightarrow{F}$ $M_A = r_{b/a} F sin(\theta)$

1) Define the reference frame (x, y, z and $\hat{i}, \hat{j}, \hat{k}$)



2) Find $\overrightarrow{r}_{b/a}$

$$\begin{aligned}\overrightarrow{r}_{b/a} &= \overrightarrow{r}_x\hat{i} + \overrightarrow{r}_y\hat{j} + \overrightarrow{r}_z\hat{k}\\ &= l_{ab}cos(\theta)\hat{i} + l_{ab}sin(\theta)\hat{j} + 0\hat{k}\end{aligned}$$

3) Find $\overrightarrow{F}$

$$\begin{aligned}\overrightarrow{F} &= \overrightarrow{F}_x\hat{i} + \overrightarrow{F}_y\hat{j} + \overrightarrow{F}_z\hat{k}\\ &= 0\hat{i} + |\overrightarrow{F}|\hat{j} + 0\hat{k}\end{aligned}$$

continued...

Example Continued **3.1**

4) Evaluate $\overrightarrow{M}_A = \overrightarrow{r}_{b/a} \times \overrightarrow{F}$

$$
\begin{aligned}
\overrightarrow{M}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \overrightarrow{r}_x & \overrightarrow{r}_y & \overrightarrow{r}_z \\ \overrightarrow{F}_x & \overrightarrow{F}_y & \overrightarrow{F}_z \end{vmatrix} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l_{ab}cos(\theta) & l_{ab}sin(\theta) & 0 \\ 0 & |\overrightarrow{F}| & 0 \end{vmatrix} \\
&= \hat{i}\begin{vmatrix} l_{ab}sin(\theta) & 0 \\ |\overrightarrow{F}| & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} l_{ab}cos(\theta) & 0 \\ 0 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} l_{ab}cos(\theta) & l_{ab}sin(\theta) \\ 0 & |\overrightarrow{F}| \end{vmatrix} \\
&= \hat{i}(l_{ab}cos(\theta) \cdot 0 - |\overrightarrow{F}| \cdot 0) - \hat{j}(l_{ab}cos(\theta) \cdot 0 - 0) + \hat{k}(l_{ab}cos(\theta) \cdot |\overrightarrow{F}| - l_{ab}sin(\theta) \cdot 0) \\
&= (0)\hat{i} - (0)\hat{j} + (l_{ab}cos(\theta) \cdot |\overrightarrow{F}|)\hat{k} \\
&= (0)\hat{i} - (0)\hat{j} + (45\text{in}cos(25°) \cdot |30\text{lb}|)\hat{k} \\
&= (0)\hat{i} - (0)\hat{j} + (40.78\text{in}) \cdot 30\text{lb})\hat{k} \\
&= (0)\hat{i} - (0)\hat{j} + (1223.51\text{in} \cdot \text{lb})\hat{k}
\end{aligned}
$$

5) Find the effective moment arm

$$
\begin{aligned}
\text{M} &= \frac{\text{Fr}sin(\beta)}{\text{F}} \\
&= \frac{30\text{lb} \cdot 45\text{in} \cdot sin(115°)}{30\text{lb}} \\
&= \frac{1223.5\text{lb} \cdot \text{in}}{30\text{lb}} \\
&= 40.9\text{in}
\end{aligned}
$$

4 Free Body Diagrams

4.1 Position Vectors

Position Vector **4.1**

A fixed vector which locates a point in space *relative* to another point. Any point in space, given that a xyz coordinate system has been established, can be described as a position vector.

A position vector can also be used to describe the position of a point relative to *another* point. This can be done following these steps:

1. Identify the coordinates of each point

$$A = (x_A, y_A, z_A)$$
$$B = (x_B, y_B, z_B)$$

2. Determine the *head* and *tail* of the position vector

$$\underline{\text{from}}\text{ point } A \ \underline{\text{to}}\text{ point } B \Rightarrow \overrightarrow{r}_{A/B} \Rightarrow \text{tail at } A\text{, head at } B$$
$$\underline{\text{from}}\text{ point } B \ \underline{\text{to}}\text{ point } A \Rightarrow \overrightarrow{r}_{B/A} \Rightarrow \text{tail at } B\text{, head at } A$$

3. Subtract *tail* from *head* coordinates

$$\overrightarrow{r}_{A/B} = (x_A - x_B)\hat{i} + (y_A - y_B)\hat{j} + (z_A - z_B)\hat{k}$$
$$\overrightarrow{r}_{B/A} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

This process is the same process for finding the distance of some point A from the origin since the origin is located at $(0\hat{i}, 0\hat{j}, 0\hat{k})$.

4.2 Free Body Diagrams

Free Body Diagram **4.2**

A diagram modeling all forces and torques applied to a body at a given point in time.

What is included in a **Free Body Diagram (FBD)**?

- The structure being analyzed (particle, rigid body, etc.)
- **ALL** forces and moments acting on the body
- Relevant dimensions

Steps to drawing a FBD:

1. Draw the isolated particle
2. Draw all forces acting on the particle
3. Label all known force magnitudes and/or directions
4. Label all unknown force magnitudes and/or directions as variables

5 Rigid Body Equilibrium

5.1 Defining Rigid Body Equilibrium

In Rigid Body Equilibrium, the sum of all forces and all moments acting on a body sums to zero. In other words, the net force and net moment on the object is zero.

$$\sum \overrightarrow{F} = \overrightarrow{0} = \sum F_x\hat{i} + \sum F_y\hat{j} + \sum F_z\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\sum \overrightarrow{M} = \overrightarrow{0} = \sum M_x\hat{i} + \sum M_y\hat{j} + \sum M_z\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

6 Distributed Loads

Up until now, the forces considered have been single lines acting on points of an object. However, in nature, many forces act continuously along an entire segment of an object. These forces are called **Distribute Loads**.

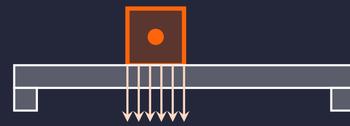


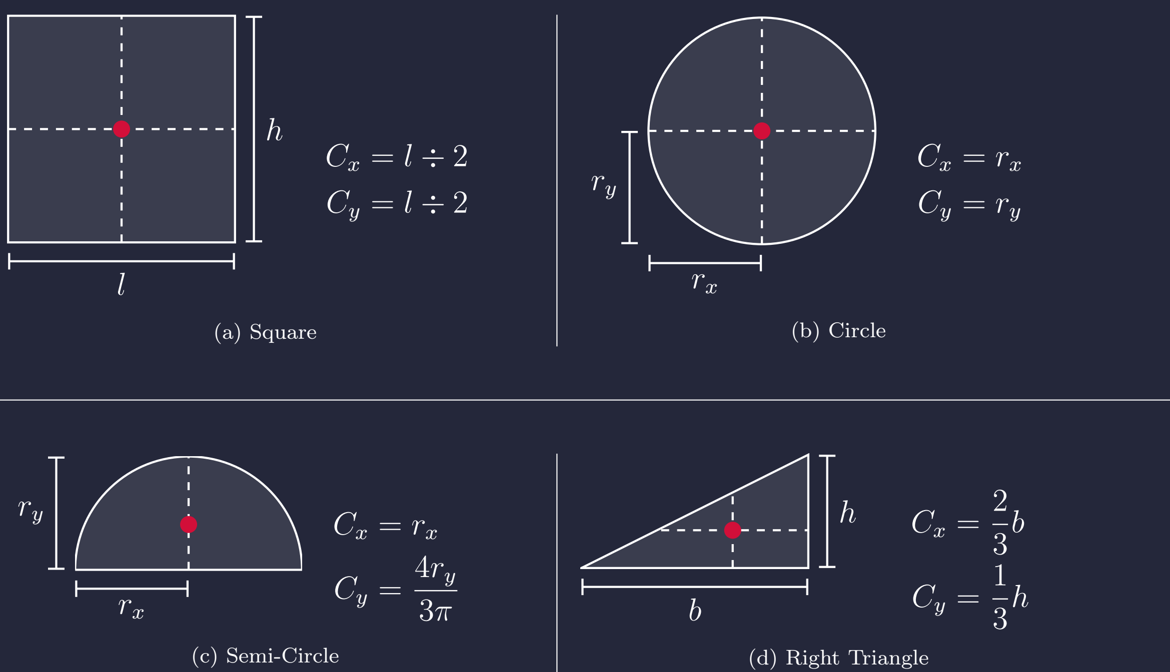
Figure 9: Continuous Load

> **Distributed Loads** **6.1**
>
> Forces that act continuously over a line or a surface.
> - Force per unit distance ($\frac{N}{m}$)
> - Force per unit area ($\frac{N}{m^2}$; Pa)

6.1 Centroids

For any distributed load, the center of force of the load can be considered as the line of action of the force. For example, in Figure 9, the center of the orange object would be the line of action of the force acting upon the beam. This center is called the **centroid**.

For simple shapes, finding the centroid of the force is simple. Figure 10 shows the centroids for common shapes.



(a) Square

(b) Circle

(c) Semi-Circle

(d) Right Triangle

Figure 10: Centroids of Simple Shapes

All this is doing is finding the average position of the object. For all of the mass in the object, what is the average position of it. It is simple to see that the average position of an object that is symmetrical over the x and y axis has a centroid in the exact middle of the object like the square or circle in Figure 10.

How would the centroid of some non-symmetrical shape be found?

To find the average x position of the shape (denoted as $\overline{x}$), the following formula is used:

Centroid in One Axis

$$\overline{x} = \frac{\sum x_i A_i}{\sum A} = \frac{\int_0^L x \, dA}{\int_0^L dA} \qquad \textbf{6.1}$$

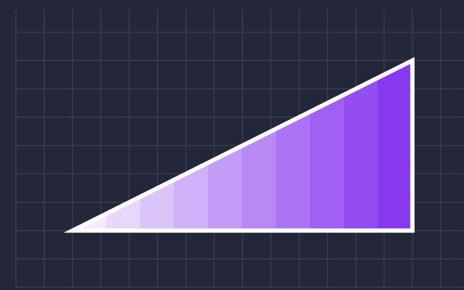
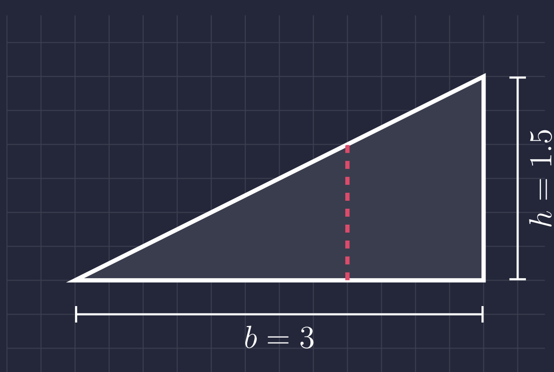


Figure 11: Reimann's Like Subdivision

The summation formula could be used for discrete quantities, such as finding the average age among a group of people where each person and their age are integers. The integral is used for more natural shapes, such as a continuous (distributed) load.

The formula essentially is dividing the shape into infinitesimally small portions, and finding the product of their area and x-position. It takes the total of that and divides it by the total area of a shape to find the weighted average, or the **centroid** of the shape.



$$\begin{aligned}
\overline{x} &= \frac{\int_0^L x \, dA}{\int_0^L dA} = \frac{\int_0^3 (x)\left(\frac{1}{2}x \, dx\right)}{\int_0^3 \frac{1}{2}x \, dx} \\
&= \frac{\frac{1}{2}\int_0^3 x^2 \, dx}{\frac{1}{2}\int_0^3 x \, dx} = \frac{\frac{1}{2}\left[\frac{1}{3}x^3\right]_0^3}{\frac{1}{2}\left[\frac{1}{2}x^2\right]_0^3} \\
&= \frac{\frac{1}{2}\left[\left(\frac{1}{3}\cdot 3^3\right) - \left(\frac{1}{3}\cdot 0^3\right)\right]}{\frac{1}{2}\left[\left(\frac{1}{2}\cdot 3^2\right) - \left(\frac{1}{2}\cdot 0^2\right)\right]} \\
&= \frac{\frac{1}{2}\cdot(9-0)}{\frac{1}{2}\cdot\left(\frac{9}{2}-0\right)} = \frac{\frac{9}{2}}{\frac{9}{4}} = \frac{36}{18} = \frac{2}{1} = 2
\end{aligned}$$

6.2 Finding Total Force

The second step to calculating distributed loads is to find the total force being applied. With simple shapes, this can be done geometrically. In Figure 13, the total force would be:

$$\frac{1}{2} \cdot 4m \cdot 1.5N = 3Nm$$

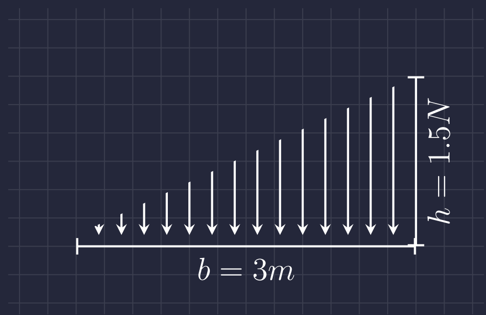


Figure 13: Finding Total Force

However, for more complex shapes, the total force can't simply be calculated geometrically. In these cases, integrating is necessary (integrating can be used for simple shapes as well).

Still using Figure 13, the total force can be calculated with integrals. First, the magnitude of the force at an given point across the base of the triangle can be expressed as a function of x, with x being the distance along the base the point is.

$$w(x) = \left(\frac{w_b - w_a}{b}\right) x + w_a = \left(\frac{1.5 - 0}{4}\right) x + 0 = \frac{1.5}{4} x$$

Then, integrating this function along the length of the continuous force (from $0m$ to $4m$) and evaluating will yield the total force being applied.

$$W = \int_0^4 \frac{1.5}{4} x\,dx = \frac{1.5}{4}\left[\frac{x^2}{2}\right]_0^4 = \frac{1.5}{4}\left[\left(\frac{4^2}{2}\right) - \left(\frac{0^2}{2}\right)\right] = \frac{1.5}{4} \cdot 8 = 3\ Nm$$

6.3 Calculating Distributed Loads

Once the centroid and the total force of a distributed load is calculated, it can be treated just as a force along the line of action that its centroid is on of a magnitude of its total force.

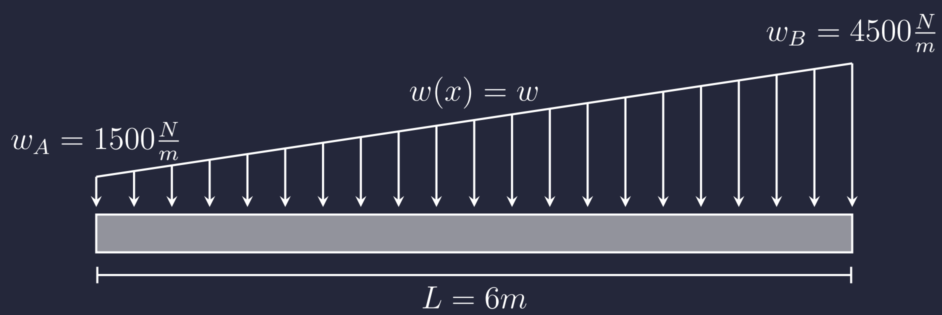


Figure 14: Distributed Load

Finding the Centroid

$$\overline{x} = \frac{\int_0^L x\,dA}{\int_0^L dA} = \frac{\int_0^L x\left(w_A + \frac{w_B - w_A}{L} x\right)\,dx}{\int_0^L w_A + \frac{w_B - w_A}{L} x\,dx} \Rightarrow \frac{\int_0^L x\left(1500 + \frac{4500-1500}{6} x\right)\,dx}{\int_0^6 1500 + \frac{4500-1500}{6} x\,dx} = \frac{\int_0^L 1500x + \frac{3000}{6} x^2\,dx}{\int_0^6 1500 + \frac{3000}{6} x\,dx}$$

$$= \frac{\left[\frac{1500}{2} x^2 + \frac{3000}{18} x^3\right]_0^6}{\left[1500x + \frac{3000}{12} x^2\right]_0^6} = \frac{\left(\frac{1500}{2} \cdot 6^2 + \frac{3000}{18} \cdot 6^3\right) - \left(\frac{1500}{2} \cdot 0^2 + \frac{3000}{18} \cdot 0^3\right)}{\left(1500 \cdot 6 + \frac{3000}{12} \cdot 6^2\right) - \left(1500 \cdot 0 + \frac{3000}{12} \cdot 0^2\right)} = 3.5$$

Finding the Magnitude of the Force

$$w(x) = \left(\frac{w_B - w_A}{L}\right)x + w_A = \left(\frac{4500 - 1500}{6}\right)x + 1500 = \frac{3000}{6}x + 1500 = 500x + 1500$$

$$W = \int_0^L w(x)\,dx = \int_0^6 500x + 1500\,dx = \left[\frac{500}{2}x^2 + 1500x\right]_0^6$$
$$= \left(\frac{500}{2} \cdot 6^2 + 1500 \cdot 6\right) - \left(\frac{500}{2} \cdot 0^2 + 1500 \cdot 0\right) = 18000\ Nm$$

Thus, with those two calculations, we can find that the equivalent force for the distributed load that acts upon the beam is a force of $1.8 \times 10^4\ Nm$ acting in the $-\hat{j}$ direction at a point $3.5\ m$ from the leftmost point of the beam. This is illustrated in Figure 15.

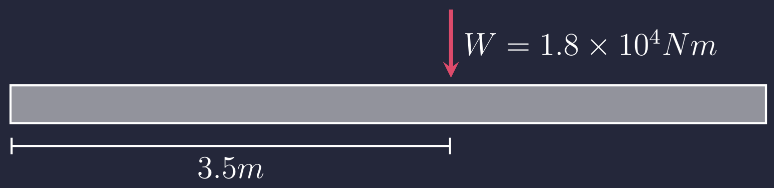


Figure 15: Equivalent Force

7 Shear and Bending

7.1 Notation and Convention

Thusfar, only external forces have been considered. However internal forces also exist.

> **Internal Loads** **7.1**
>
> Act *everywhere* inside the object/body. These forces hold the object/body together and adapt/change based on external loading conditions.

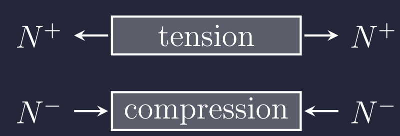
There are three main types of internal loads:

Axial Forces (N^+)

These internal loads act directly along the length of the axis of an object.

The sign convention of Axial Forces is such that:

- Positive refers to **tension**
- Negative refers to **compression**



Shear Forces (V^+)

Force *transverse* to the length of the member (*parallel* to the cross section area. This force will always create a moment about the z axis of the member.

The sign convention is such that:

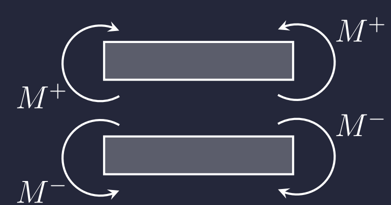
- Positive refers to **clockwise rotation**
- Negative refers to **counterclockwise rotation**

Bending Moments (M^+)

Rotational load that bends the beam.

The sign convention is such that:

- Positive refers to **concave upward bending**
- Negative refers to **downward bending**



Notice that a force in the positive $\hat{i}$, negative $\hat{j}$, or positive $\hat{k}$ is considered a positive internal force when it is on the right side of an object. Conversely, these forces are considered negative when on the left side of the object.

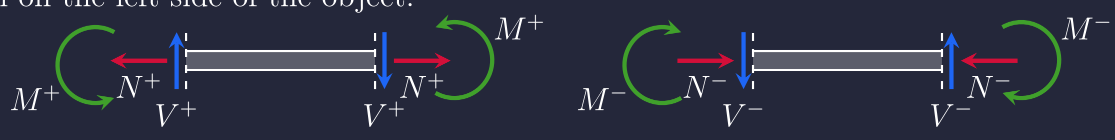
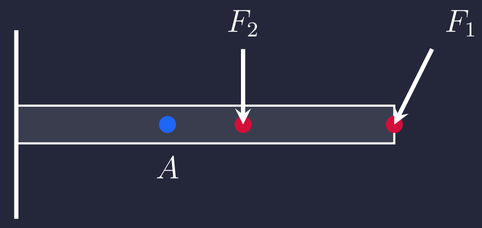


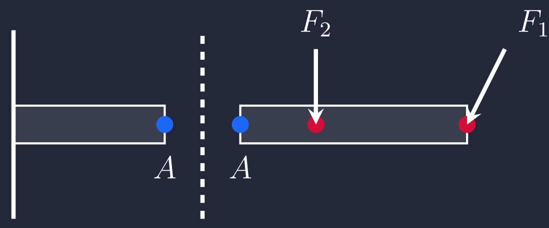
Figure 16: Sign Conventions of Internal Forces

7.2 Solving for Internal Forces

All the information thusfar in Statics will be important in solving for internal forces of an object in static equilibrium. The only difference to the general process is in the preparation. When solving for an internal force, *cutting* the object at that point of interest and treating it as two separate objects is how to find the internal stress.



(a) Original Object



(b) Cut of the Object

Figure 17: Cutting an Object

In Figure 17, a beam attached to a vertical support is experiencing two forces: F_1 and F_2. To find the internal forces acting on the object at point A, one must cut the beam into two imaginary parts along point A, the solve for the forces as A as usual remaining under the conditions of static equilibrium.

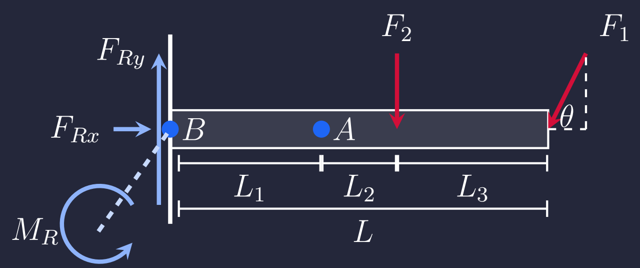
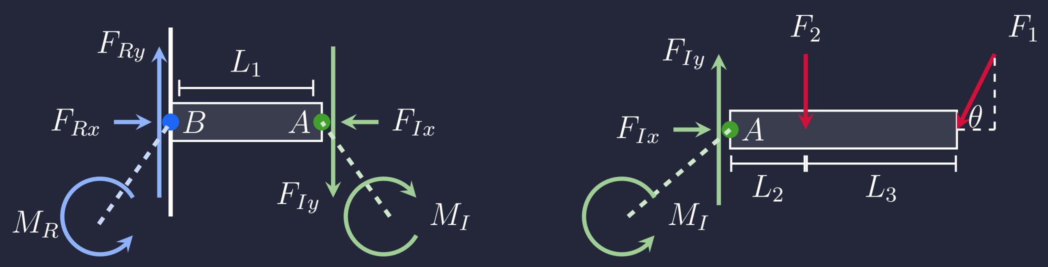


Figure 18: Initial Free Body Diagram

The original object is in static equilibrium, thus it is possible to find the reactionary forces and moments about a point (B) in terms of the forces acting upon the object (F_1 and F_2).

$$F_{Rx} = \hat{i}\big(|cos(\theta)F_1|\big)$$
$$F_{Ry} = \hat{j}\big(|F_2| + |sin(\theta)F_1|\big)$$
$$M_R = \hat{k}\big(|F_2L_1| + |F_1sin(\theta)L|\big)$$

If we were to then cut the object at the line of A, the reaction forces would be in equilibrium with the internal forces at A, as would the original forces of F_1 and F_2.



(a) Reaction Forces and Internal Forces

(b) Internal Forces and Original Forces

Figure 19: Free Body Diagram of Each Segment

From here, each segment can be solved as usual for the forces in each Free Body Diagram. Figure 19a would balance the reaction forces with the internal forces. Figure 19b would balance the original forces with the internal forces. In either situation, the calculated internal forces should be the same.

Left Segment

$$F_{Ix} = -\hat{i}|F_{Rx}|$$
$$F_{Iy} = -\hat{j}|F_{Ry}|$$
$$M_I = -\hat{k}|M_R|$$

Right Segment

$$F_{Ix} = \hat{i}|cos(\theta)F_1|$$
$$F_{Iy} = \hat{j}\big(|sin(\theta)F_1| + |F_2|\big)$$
$$M_I = \hat{k}\big(|sin(\theta)F_1(L_1 + L_2)| + |F_2L_2|\big)$$

Finding Equilibrium in Both Sides

$$-\hat{i}|F_{Rx}| \to \hat{i}|cos(\theta)F_1|$$
$$-|F_{Rx}| \to |cos(\theta)F_1|$$
$$-|cos(\theta)F_1| \to |cos(\theta)F_1|$$

(a) Balancing $\hat{i}$ Forces

$$-\hat{j}|F_{Ry}| \to \hat{j}\big(|sin(\theta)F_1| + |F_2|\big)$$
$$-|F_{Ry}| \to |sin(\theta)F_1| + |F_2|$$
$$-\big(|sin(\theta)F_1| + |F_2|\big) \to |sin(\theta)F_1| + |F_2|$$

(b) Balancing $\hat{j}$ Forces

$$-\hat{k}|M_R| \to \hat{k}\big(|sin(\theta)F_1(L_1 + L_2)| + |F_2L_2|\big)$$
$$-|M_R| \to |sin(\theta)F_1(L_1 + L_2)| + |F_2L_2|$$
$$-|\big(|F_2L_1| + |F_1sin(\theta)L|\big)| \to |sin(\theta)F_1(L_1 + L_2)| + |F_2L_2|$$

(c) Balancing $\hat{k}$ Moments

8 Trusses

8.1 Terminology

Structure **8.1**

An assembly of individual bodies that are designed to support and/or transmit external and internal loads.

There are multiple types of structures:

- **Trusses** - Stationary and composed of only two-force bodies, designed to *support* loads.
- **Frames** - Stationary and contain at least multi-force body, designed to *support* loads
- **Machines** - Contain moving parts and are designed to *transmit* and/or *alter* the effects of forces

For a **truss** specifically, there are a few characteristics that all of them share.

- Consists of straight, slender, and **two-force** members
- Each member is connected only by pin joints at their ends
- Each member is only loaded at the pin joints
- Each member **only** supports an *axial* load

8.2 Solving a Truss