University Physics III

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Contents

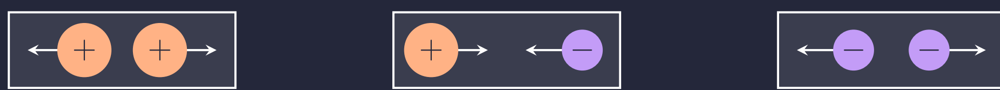
1 Introduction to Electricity and Magentism

1.1 Electric Charge

Electric Charge **1.1**

Subatomic particles (electrons and protons) carry small discrete packets of energy, in positive and negative form, which is what we call electric charge.

Like charges repel, opposite charges attract.



(a) Both Positive (b) Positive and Negative (c) Both Negative

Figure 1: Attraction

Charge is an inherent property of types of particles (protons and electrons). These particles can move between systems, but still retain there charge. This is the how and why **charge is always conserved**.

Mass of Protons and Electrons **1.1**

$$m_p = m_n = 1.67 \times 10^{-27} kg$$
$$m_e = 9.11 \times 10^{-31} kg$$

When particles move between objects, it is usually said that the electrons are what move between the object, thus changing the net charge of the objects.

Figure 2: Electrons Moving Between Objects

It is generally metals that become charged because they are **conductive**. When a metal becomes charged, its charge is spread about evenly throughout the surface because the forces of the particles cause each particle to find a position where they are equidistant from every other particle.

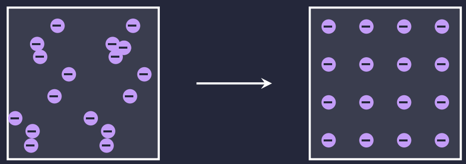


Figure 3: Even Distribution

The standard unit of charge is the **Coulomb** (C).

1.2 Forces Between Particles

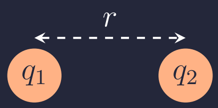


Figure 4: Two Protons in Space

The force between two particles can be modeled in terms of the charge of each object, Coulomb's Constant k, and the distance between the two objects.

Coulomb's Law

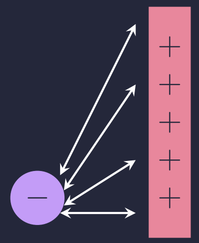
$$F = k\frac{q_1 q_2}{r^2}$$

F = Force Between Particles
q_1 = Charge of Particle One
q_2 = Charge of Particle Two
r = Distance Between Particles
k = Coulomb's Constant; $8.99 \times 10^9 \frac{N \times m^2}{s^2}$

1.2

When dealing with discrete charges, it's simple to find the direction of force. But when there is a continuous charge, how can you find the direction of force it exerts on a particle?

Using **integration**, you can calculate the direction and magnitude of the force.



(a) Continuous Charge

Force of Continuous Objects

$$\int dF = \int \frac{kQdq}{r^2}$$

---

dF = Change in Force from Charge
k = Coulomb's Constant
Q = Net Charge of Continuous Object
dq = Change in Instantaneous Charge
r = Distance Between Objects

1.3

Electric Fields are created by charges particles that affect other charged particles within the field. When two particles exert forces on each other, it is because both particles are created some field around them that in turn affects other particles around it.

Strength of an Electric Field

Known Values

E = Strength of Electric Field
k = Coulomb's Constant
q = Charge of Particle
r = Distance from Particle

$$E = \frac{F}{q}$$
$$F = k\frac{q_1 \cdot q_2}{r^2}$$
$$E = k\frac{q}{r^2}$$

1.4

When drawing an electric field, the lines have some useful properties:

1. At any point in space, the directionof the lines tells you the direction of the electric field.
2. The *denser* the lines, the *stronger* the electric field
3. Field lines only start and end at charges
4. Field lines never cross each other

Given the above figures, particles **b** would experience the greatest force.

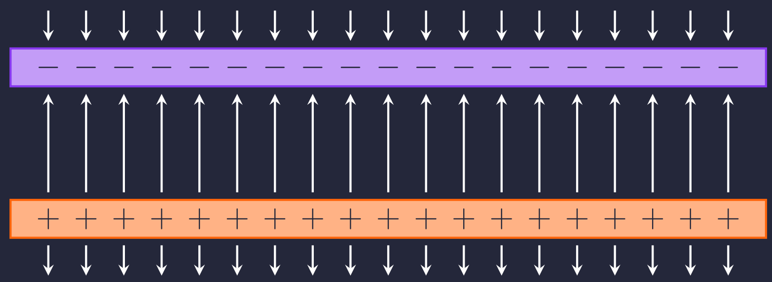


Figure 6: Infinite Charged Bars

In a steady state, the electric field within a conductor is equal to zero. This is true because the external electric field acting on the conductor will cause particles within the conductor to move to an equilibrium state to cancel out the field.

2 Gauss's Law

How can the charges be determined only knowing what electric field is present?

Electric Flux **2.1**

Flux is a mathematical way of describing how much "flow" is going through an area. Electric flux refers to the amount of **electric field** that is going through an area. The value of electric charge depends on:

1. The **strength** of the electric field
2. The **area** through which the electric field acts on
3. The **angle** between the field and the area; the direction of the electric field that acts upon the area

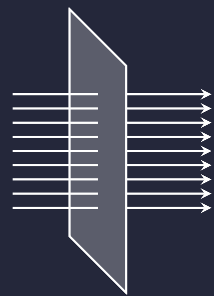


Figure 7: Flux

$$\Phi = E \cdot A = EAcos(\theta)$$

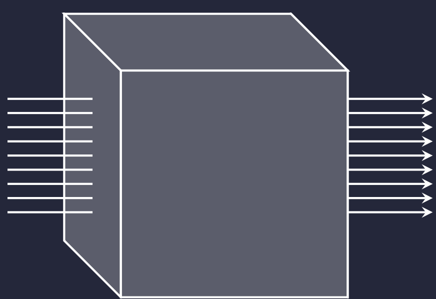


Figure 8: Flux

An electric field going through an object will result in a net zero electric flux. This is because electric flux can be measured in **inward** and **outward** electric flux.

True Electric Flux **2.1**

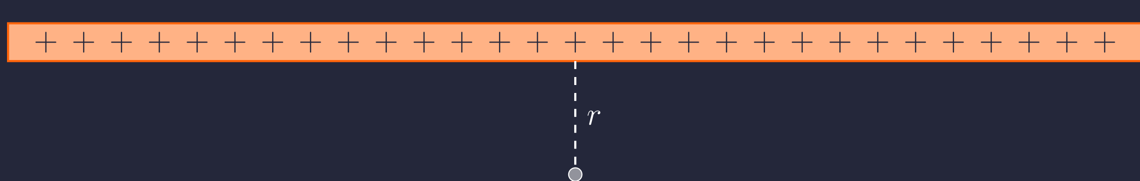
$$\Phi = \oint \overrightarrow{E} \cdot d\overrightarrow{A}$$

Gauss's Law **2.2**

$$\Phi = \oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{q_{enclosed}}{\epsilon_0}$$

This shows that the total electric flux (Φ) through an object is determined by the amount of charge enclosed within that object divided by a constant of nature (ϵ_0).

Let's say we have an infinitely long wire of charge. The charge per unit length is λ. What is the electric field at some point at a distance r from the wire?



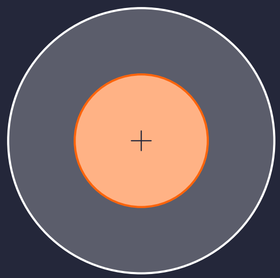
Continuous charge distributions often have a uniform **charge density**.

$$\lambda = \text{"lambda"} = \text{linear charge density (charge per unit length; } \tfrac{C}{m})$$

$$\sigma = \text{"sigma"} = \text{surface charge density (charge per unit area; } \tfrac{C}{m^2})$$

$$\rho = \text{"rho"} = \text{volume charge density (charge per unit volume; } \tfrac{C}{m^3})$$

To solve this, a Gaussian Surface equidistant from the wire at all points would be easiest to use; a cylinder wrapped around the wire would be simplest.



Front View

With this Gaussian Surface defined, q_{enclosed} is the length of wire enclosed by the cylinder multiplied by the charge per unit length: $q_{\text{enclosed}} = \lambda L$.

Since the surface is perpendicular to the Electric Field at every point, the dot product of the electric flux and the area is just simple multiplication.

$$\begin{aligned}
\oint \overrightarrow{E} \cdot d\overrightarrow{A} &= \frac{q_{\text{enclosed}}}{\epsilon_0} \\
E_{\perp} A &= \frac{q_{\text{enclosed}}}{\epsilon_0} \\
E(2\pi r L) &= \frac{\lambda L}{\epsilon_0} \\
E &= \frac{\frac{\lambda L}{\epsilon_0}}{2\pi r L} \\
E &= \frac{\lambda L}{2\pi r L \epsilon_0} \\
E &= \frac{\lambda}{2\pi r \epsilon_0}
\end{aligned}$$

Thus, the electric field near a long wire of charge is modeled as $E = \frac{\lambda}{2\pi r \epsilon_0}$. This is an approximation, but still very accurate when the distance r is small in comparison to the length of the wire.

3 Electric Potential Energy

3.1 Review of Work and Energy

When a force $\overrightarrow{F}$ acts upon a particle that moves from point a to point b, the work done on that particle is given by an integral.

Work Done by a Force

$$W_{a \to b} = \int_a^b \overrightarrow{F} \cdot d\overrightarrow{r} = \int_a^b F cos(\theta) dr \qquad \textbf{3.1}$$

If the force acting upon that particle is *conservative*, the work done by $\overrightarrow{F}$ can be expressed in terms of potential energy (U).

Work in Terms of Potential Energy

$$W_{a \to b} = U_a - U_b = -(U_b - U_a) = -\Delta U \qquad \textbf{3.2}$$

The work-energy theorem states that the net work done on a particle is equal to the change in kinetic energy of that particle: $W_{net} = \Delta K = K_b - K_a$. Under the assumption that only conservative forces are acting upon the object, then:

Conservation of Mechanical Energy

$$K_b - K_a = -(U_b - U_a)$$
$$K_a + U_a = K_b + U_b \qquad \textbf{3.3}$$

3.2 Electric Potential Energy

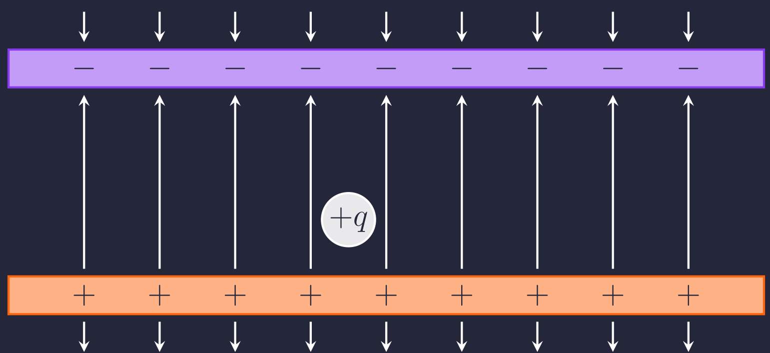


Figure 9: Potential Energy in an Electric Field

Electric potential energy applies within electric fields. In Figure 9, a particle with a positive charge in a field, it would have the highest potential energy closes to the **source** of the positive electric field.

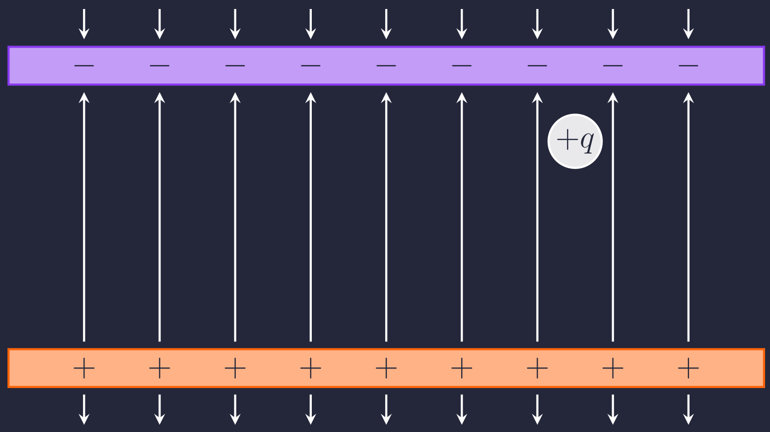


Figure 10: Potential Energy in an Electric Field

In Figure 10, the positively charged particle would have a relatively low potential energy. The work done on the particle in Figure 10 can be modeled as:

$$W = F(r_b - r_a)$$
$$W = Eq(r_b - r_a)$$

This only holds for particles in uniform electric fields. If, for example, two charged particles exist in space and perform work on each other, an integral would be required to find the net work done on each particle. This integral is the general form to find the work done on a particle by an electric field.

Work Done on a Particle by an Electric Field **3.4**

$$W_{a \to b} = \int_{r_a}^{r_b} F cos(\theta) dl$$

Electric Potential Energy of Two Point Charges **3.5**

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

Electric potential energy is not a property of a single particle. In a system of two particles, the electric potential energy is a shared property of both particles, just as gravitational potential energy is shared between both you and the Earth.

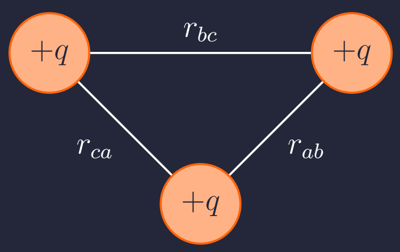


Figure 11: System of Particles

If there is a system of charges in space, what is the total potential energy of this system? In other words, how much energy would it take to bring all the particles together from infinity?

The total U would just be the potential energy that each particle creates relative to each other particle:

$$U = k\left(\frac{q_a q_b}{r_{ab}} + \frac{q_b q_c}{r_{bc}} + \frac{q_c q_a}{r_{ca}}\right)$$

Thus, the potential energy of a system of particles is just the sum of all potential energies between each of those particles. Similarly, the work done on a particle as it moves through the system is simply the final potential energy minus the initial potential energy.

4 Electric Potential

Electric Potential **4.1**

The amount of energy per unit of electric charge required to move a particle throughout an electric field.

$$\text{"}\textbf{Potential}\text{"} = V = \frac{U}{q}$$

$$\frac{J}{C} = \text{ Volt} = V$$

This measurement is done in **Voltage**, and often measured as a difference between two points. For example, $V_{ab} = V_a = V_b$ states that the potential of a with respect to b equals the work (J) done by the electric force when a UNIT (C) charge moves from a to b.

To calculate the potential V due to a single point charge, the following formula is used:

Potential

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad \textbf{4.1}$$

A change in potential (a "potential difference") causes a charged object to gain or lose potential energy. This gain or loss is on a per-charge basis.

Change in Potential

$$\Delta V = V_2 - V_1 = \frac{\Delta U}{q} = -\int \vec{E} \cdot d\vec{r} \qquad \textbf{4.2}$$

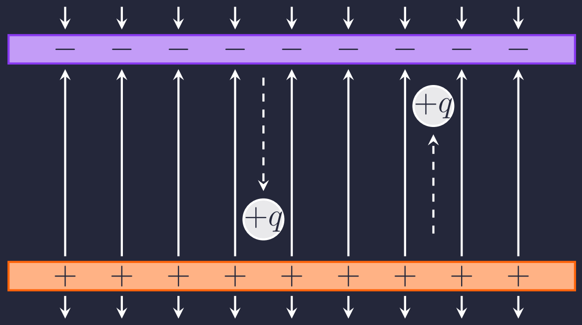


Figure 12: Particles Moving in an Electric Field

In Figure 12, the change in potential energy of each particle is of equal magnitude, but since one is moving from a high potential energy area to a low one, while the other moves in the opposite direction, they would have opposite changes. This is analogous to raising an object up versus lowering it down an equal distance.

If there is a point charge in space, where should the "zero" of the potential energy be places?

The potential difference between two points a and b can be modeled as:

$$\Delta V = V_b - V_a = \frac{\Delta U}{q} = -\int \overrightarrow{E} \cdot d\overrightarrow{r} = kq\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

If we take this as point b approaches infinity, we get:

$$\lim_{b\to\infty} kq\left(\frac{1}{r_b} - \frac{1}{r_a}\right) \to kq\left(\frac{1}{\infty} - \frac{1}{r_a}\right) \to kq\left(0 - \frac{1}{r_a}\right) \to -kq\frac{1}{r_a}$$

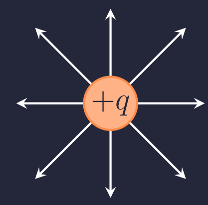


Figure 13: Point charge in space

Thus:

$$\Delta V = V_b - V_a = k\frac{q}{r_b} - k\frac{q}{r_a} \quad \textbf{or} \quad V_a = \frac{kq}{r_a}$$

4.1 Equipotential

In an electric field, there are lines going through the electric field that have the same potential energy across it. In a simple electric field, such as the one in Figure 14, the lines will be perfectly horizontal. Notably, equipotential lines will always be perpendicular to the electric field.

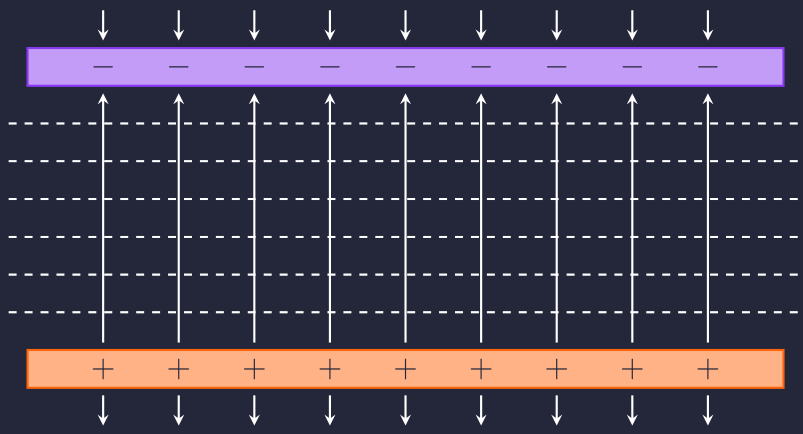


Figure 14: Equipotential Lines

In the case of a point charge, such as that in Figure 15, the equipotential lines will form concentric circles around the particle.

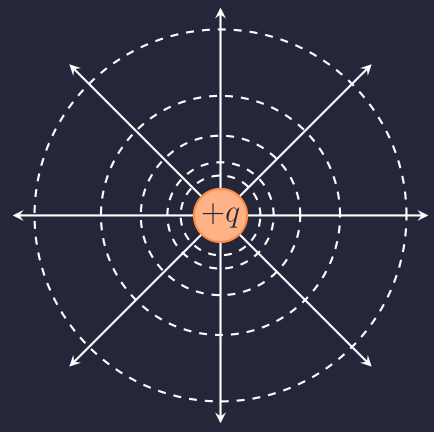


Figure 15: Equipotential Around Point Charge

4.2 Electronvolt

When measuring the energy change of a particle as it moves through an electric field, rather than measuring in qV, a new unit to describe energy exists, the electronvolt. A single electronvolt is defined as 1.6×10^-19J

Electronvolt **4.2**

$1eV$ is defined as the amount of energy change for a particle of charge "e" moving through a $1V$ potential drop.

5 Capacitors

Capacitor **5.1**

Circuit components that are used to store charge.

The way capacitors are charged is by moving charge from one part of the capacitor to the other side of the capacitor, leaving one side positively charged and the other negatively charged. This process stores energy.

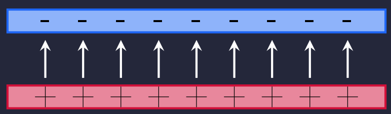


Figure 16: Charging a Capacitor

This change in charge between the two plates creates a electric field between them. Doubling the charge of the capacitor will double the electric field.

Capacitance **5.1**

$$C = \frac{Q}{\Delta V}$$

Capacitance is measured in the following units:

$$\frac{C}{V} = \text{Farad} = F$$

The constant being multiplied by voltage is called the **capacitance**, which is the measurement of a capacitor's capability to hold charge. Capacitance depends on the shape and material of a capacitor.

Capacitance for Parallel Plates **5.2**

$$E = \frac{\frac{Q}{A}}{\epsilon_0}\ ;\ \Delta V = Ed$$

$$\Delta V = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d}$$

As distance increases, the capacitance increases. This is because each bit of charge has less voltage drop it needs to go through to reach the other side.
As area increases, so does capacitance because more area means there is more room to store charge.

When you connect a capacitor to a battery, that battery maintains a potential difference between the two plates. By doing this, the battery pushed charges from one side to the other, thus performing **work**.

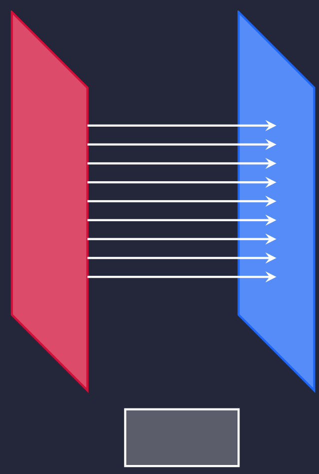


Figure 18: Battery Connected to a Capacitor

The energy required to charge this capacitor with the battery:

Energy to Charge a Capacitor	5.3
$U = \dfrac{Q\Delta V}{2}$ $\quad U = \dfrac{Q^2}{2C}$ $\quad U = \dfrac{C\Delta V^2}{2}$	

ADD IN SYMBOLS FOR CIRCUIT DIAGRAMS

Figure 20: Circuit Diagram with Capacitor and Battery

Any time there is a circuit diagram, it's important to think about where the particles are flowing through the circuit, gaining and losing energy.

When capacitors are connected in parallel, the total capacitance of the capacitors will simply be:

$$C_{parallel} = C_1 + C_2 + C_3 + ...$$

This is because the voltage drop across each capacitor will be the same.

When connected in series, the total capacitance is:

$$\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3 + ...$$
$$\Delta V_{total} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + ...$$
$$\Delta V_{total} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...\right)$$
$$\frac{\Delta V_{total}}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$
$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

This is because the charge buildup on each capacitor will be the same. When charge flows from the end of one capacitor to the beginning of the next, that charge has nowhere to go but only the next capacitor.

5.1 Dielectric

Dielectric **5.2**

A material put between plates of capacitors to maintain the separation as well as modify the capacitance of the capacitor.

Capacitors need to be incredibly close together for them to have significant amounts of capacitance. Because two oppositely charged plates will have some pull between them, an object called a **dielectric** is usually put between the plates to keep them apart.

Dielectric Capacitance **5.4**

$$C_{dielectric} = \kappa C = k\frac{Q}{\Delta V}$$

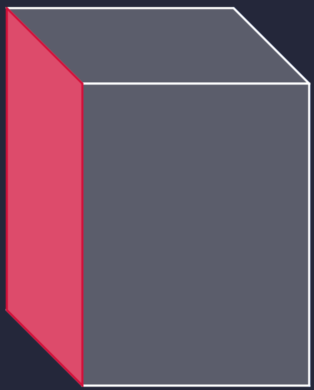


Figure 21: Dielectric in a Capacitor

6 Current

6.1 Current Definition and Formula

Current **6.1**

Current describes how much charge moves past some spot over any given second.

Current

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$
$$\text{Ampere} = \frac{\text{Coulomb}}{\text{Second}}$$

6.1

Current has a direction. The direction of it is just convention, but is such that a **positive** current corresponds to negative charges moving in the negative direction or positive charges are moving in the negative direction.

Currents are created by **electric fields**, which can be conceptualized by a difference in electric potential (voltage).

Figure 22: Current in a Wire

6.2 Current at a Specific Point

Inside a wire, each particle is moving *on average* with some velocity in some direction. This average **drift velocity** (v_d) of the particles is what determines the current of the circuit.

Current by Volume

$$\Delta L = v_d \Delta t$$
$$I = n v_d A q$$
$$J = \frac{I}{A} = n v_d q$$

I = current $\quad n$ = particle density $\quad v_d$ = velocity $\quad A$ = area $\quad q$ = charge

6.2

Different materials allow particles to move through them with varying ease. This property is called the conductivity of the material.

Conductivity **6.2**

Describes how easily a particle can move through a material. Denoted as σ.

The conductivity of a material is empirically derived, but conceptually can be thought of as the amount of obstacles in the way of a particles movement. The "denser" the path is, the harder it is for a particle to move through the material.

Energy in Terms of Conductivity **6.3**

$$J = \sigma E$$
$$J\frac{1}{\sigma} = E$$
$$J\rho = E$$

$J =$ current density $= \frac{A}{m^2}$ | $\sigma =$ conductivity $= \frac{A}{Vm}$

$E =$ electric field $= \frac{N}{C}$ | $\rho =$ resistivity $= \frac{Vm}{A}$

6.3 Ohm's Law

Ohm's Law **6.4**

$$J = \frac{E}{\rho} \quad \Rightarrow \quad \frac{I}{A} = \frac{\frac{\Delta V}{L}}{\rho} \quad \Rightarrow \quad \Delta V = \frac{\rho L}{A} I$$

$I =$ current $= A$ | $A =$ area $= m^2$

$\Delta V =$ voltage drop $= V$ | $L =$ length $= m$

$\frac{\rho L}{A}$ are all constants, and are thus combined into a single constant: resistance (R).

$$\Delta V = IR$$

Ohm's Law provides an alternative way to calculate energy within a circuit in terms of voltage, current, and resistance. Notably, Ohm's law is not a law of nature, but only applies to certain "ohmic" materials. In terms of **resistance**, it is measured in Ohms (Ω) with units in $\frac{V}{A} = \frac{Js}{C^2}$.

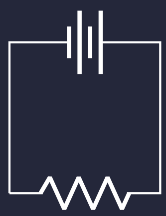
The amount of resistance is dependent on its resistivity/conductivity. However, that's not the only thing it is related to. The size and shape of the conductor also play a role in its resistance. Since:

$$R = \frac{\rho L}{A}$$

An increase in length corresponds to an *increase* in resistivity. Conversely, an increase in area of the cross-section of the conductor would correspond to a *decrease* in resistivity.

Material	**Resistivity** ρ	**Conductivity** σ
Aluminum	2.8×10^{-8}	3.5×10^{7}
Silver	1.59×10^{-8}	6.30×10^{7}
Copper	1.68×10^{-8}	5.96×10^{7}
Gold	2.44×10^{-8}	4.10×10^{7}
Calcium	3.36×10^{-8}	2.98×10^{7}
Tungsten	5.60×10^{-8}	1.79×10^{7}
Zinc	5.90×10^{-8}	1.69×10^{7}
Nickel	6.99×10^{-8}	1.43×10^{7}
Iron	1.00×10^{-8}	1.00×10^{7}

Figure 23: Resistivity and Conductivity of Common Material



As a current travels across a resistor, there is a voltage drop. This is seen in Ohm's Law, where the voltage drop (ΔV) is equal to the product of the current (I) and the resistance (R).

$$\Delta V = IR$$

The larger the current or resistance in the resistor, the larger the voltage drop will be. This voltage drop is a loss of energy within the system. So where does the energy go?

6.4 Dissipated Heat

When a voltage drop occurs over a resistor, energy is lost. That energy transforms into **heat**. To measure the heat power dissipated by a resistor:

Heat

$$\frac{J}{s} = IV = I^2R = \frac{V^2}{R} \qquad \textbf{6.5}$$

Since the resistance of a material can only be reliably calculated in the ways outlined within the scope of PHYS 1213 for Ohmic materials, I^2R and $\frac{V^2}{R}$ only apply to Ohmic materials. However, IV applied universally since it is not based on resistance.

6.5 Current Throughout a Circuit

Current is just a measurement of how the physical electrons/protons in a circuit are **moving** about. The movement of these particles depends on the amount of energy they have (voltage).

Current throughout a circuit is conserved. In other words, at any point, the total amount of current flowing into a location must be the same as the amount flowing out of that location.

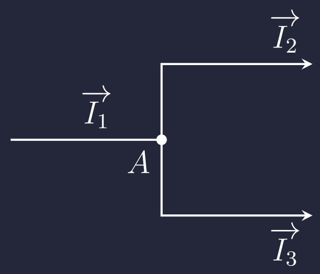


Figure 24: Charge in a Circuit

In Figure 24, the amount of current flowing into junction A is always going to equal the amount of current flowing out. In other words: $\overrightarrow{I_1} = \overrightarrow{I_2} + \overrightarrow{I_3}$.

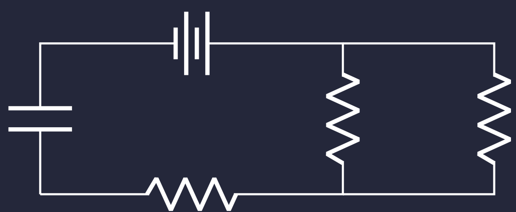


Figure 25: Circuit Diagram

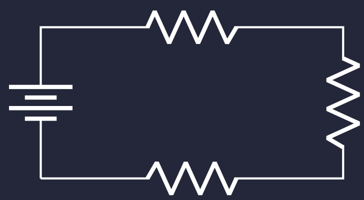
Considering the circuit in Figure 25 in terms of *energy*, the energy of the circuit is being supplied at the battery. The energy then travels through the circuit. At each resistor, some amount of energy is being *lost* in the form of heat. Over a capacitor, the energy is charging up the capacitor to be stored in the form of potential energy.

Importantly, no current is flowing across a capacitor, there is only a voltage drop across them.

The voltage supply in a circuit is referred to as "emf", denoted as ϵ.

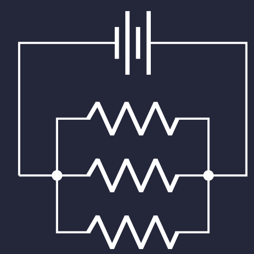
7 Circuits

7.1 Current Throughout a Circuit



$$R_{eq} = R_1 + R_2 + R_3$$

(a) Resistors in Series



$$\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

(b) Resistors in Parallel

When resistors are connected in series like in Figure 26a, the resistance over all the resistors can be added directly. When connected in parallel such as in Figure 26b, the resistance is added as reciprocal.

Notably, when resistors are connected in parallel, the equivalent resistance for the circuit at large is less than any of the individual resistors. If $R_1 = 1\Omega$, $R_2 = 2\Omega$, and $R_3 = 3\Omega$, then:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$
$$R_{eq} = \frac{6}{11}$$

7.2 Measuring Circuits

Ammeter **7.1**

A device used to measure the current flowing throughout a circuit at a specific point.

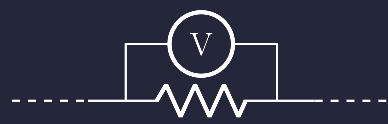
When using an ammeter, it is imperative to connect it **in series** with the circuit.

This is because current is conserved throughout a singular path. However, if you were to provide the current with an alternative path (ammeter connected in parallel), the current can now travel along either path, making it inaccurate to just measure the current in a single path.

Voltmeter **7.2**

A device used to measure the voltage drop between two points of a circuit.

A voltmeter must be connected **in parallel** with the circuit.



This is because voltage is conserved across branches of a parallel junction. If connected in series, the voltmeter would just measure the voltage across itself.

7.3 Kirchoff's Laws

Kirchoff's First Law **7.3**

The current flowing into a junction must be equal to the current flowing out of the junction. In other words, current is conserved throughout a circuit.

By selecting a junction, such as the one in Figure 27, Kirchoff's First Law states that the total current flowing into the junction will be the same as the current flowing out of it. Thus:

$$\vec{I}_1 + \vec{I}_2 + \vec{I}_3 = 0 \quad \text{or} \quad |I_1| = |I_2| + |I_3|$$

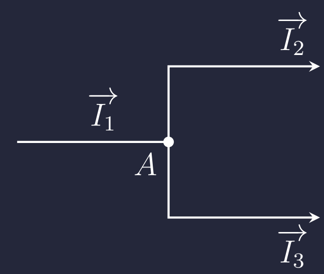


Figure 27

Kirchoff's Second Law **7.4**

Around any closed loop of a circuit, the total voltage drop is zero.

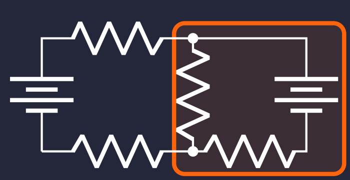
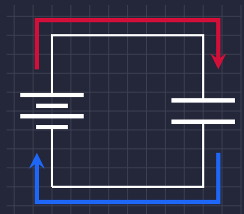


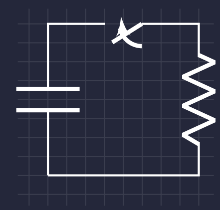
Figure 28

By isolating a full loop within a circuit, Kirchoff's Second Law states that the total voltage drop throughout the chosen loop will be zero. In Figure 28, the closed loop denoted by the orange box will have a total voltage drop of zero. This means that the voltage gained as particles pass through the battery will be the same voltage lost as particles travel over the two resistors.

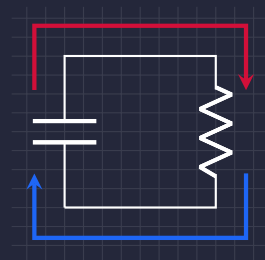
7.4 Capacitors in Circuits



In a circuit of a capacitor and battery, the battery will charge the capacitor for a period of time until a charge of $Q = CV$ has built up between the plates of the capacitor. During this period of charging, there is a current throughout the circuit.



Now consider a fully charged capacitor in a circuit with a resistor and a switch. The switch currently is open, meaning that no current is moving throughout the circuit. In this situation, there is a voltage drop across the capacitor: $V_i = \frac{Q_i}{C}$. At the moment the switch closes, a current begins to flow.



At the instant the switch closes and the current begins to flow, the current will be $I_i = \frac{V_i}{R}$. However, there is only so much charge in the capacitor; supplying the current will cause it to run out. Since both $V = IR$ and $V = \frac{Q}{C}$ are true, they can be combined into $IR = \frac{Q}{C}$ or $I = \frac{Q}{RC}$.

I, being the current in the circuit, conceptually is just the amount of charge leaving the capacitor, it can be expressed as $-\frac{dQ}{dt}$. Thus:

Current as a Result of a Capacitor

$$-I = \frac{dQ}{dt} = -\frac{Q}{RC} \tag{7.1}$$

The amount of current in the circuit is a function of the amount of charge left in the capacitor. Since the amount of charge left in the capacitor over time is dependent of the amount of current flowing, the amount of charge in the capacitor decays exponentially.

Charge in a Capacitor as a Function of Time **7.2**

$$Q(t) = Q_i e^{-\frac{t}{RC}}$$

Every RC seconds, Q decreases by a factor of $\frac{1}{e}$. Since the rate at which Q decreases is determined by RC, it is considered the **time constant**: τ

$$Q(t) = Q_i e^{-\frac{t}{\tau}}$$

8 Magnets

The two fundamental forces covered so far in University Physics have been gravity and electrical forces. Magnetism is a third fundamental force.

Similar to electricity in several ways, magnetic force is an attractive force between oppositely "charged" magnets and a repulsive force between similarly "charged" magnets. Additionally, magnets create a magnetic field around them that behaves similarly to the electrical field created by charged particles.

However, magnetic force and electric force are fundamentally different despite their similarities.

8.1 Magnetic Fields

Magnetic fields are referred to as B fields. But what causes them? When a compass is brought close to a current, the compass will point in a direction due to that current. Thus, current (moving particles) creates magnetic fields.

Additionally, magnetic fields exert force onto charged particles. However, considering a bar magnet such as the one in Figure 29, where would the current or induced force on the particles around the magnet be?

Figure 29: Regular Bar Magnet

The force would be induced onto the electrons near the magnet **orbiting** within each atom. This orbit is a tiny current within the atom, but that current is what the magnet is inducing/reacting to. The force exerted onto the particle is called the **Lorentz Force**.

Lorentz Force

$$\text{Force on Particle} = \text{Particle Charge (Particle Velocity} \times \text{Magnetic Field)}$$
$$F = q(v \times B)$$
$$|F| = qvBsin(\theta) \qquad \textbf{8.1}$$

Consider the particle in Figure 30. In the top position, it is moving to the right within a magnetic field directed out of the page. By crossing Velocity with the Magentic Field ($\overrightarrow{v} \times \overrightarrow{B}$), the resulting force is pointed downwards.

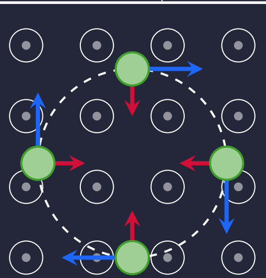


Figure 30

As it accelerates downward, its velocity will begin to point downwards as well, eventually reaching the position at the right with a new acceleration still towards the center. This continues as the particle maintains a circular motion around some point.

Because work is found using the dot product of Force and Velocity ($\overrightarrow{F} \cdot \overrightarrow{v}$), and the movement of a particle within a magnetic field is circular, the work done on a particle by a magnetic field will always be zero.

The basic principle of calculating the Lorentz Force can also be used to calculate the force from a magnetic field on a current-carrying wire.

Lorentz Force (Wire) — **8.2**

$$F = IL \times B \qquad |F| = ILBsin(\theta)$$

The circular motion experienced by the particle in Figure 31 is just centripetal motion. Thus, the radius of the circle it travels can be found using kinematics, specifically: $F = \frac{mv^2}{r}$.

Gyroradius — **8.3**

$$F = \frac{mv^2}{r} \rightarrow r = \frac{mv^2}{F}$$

$$r = \frac{mv^2}{q(v \times B)} \rightarrow r = \frac{mv^2}{qvB}$$

$$r = \frac{mv}{qB}$$

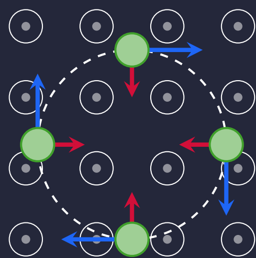


Figure 31: Centripetal Motion of a Particle in Magnetic Field

Thus, the radius of the circle traveled scales directly with the mass and velocity of the particle, and inversely with the charge and the strength of the B field.

8.2 Sources of Magnetic Fields

Magnetic fields are always created by currents. Specifically, the magnetic field moved in circles around the direction of the current.

Figure 32: Current-Induced Magnetic Field

The relationship between the direction of the current and magnetic field are related through the right-hand rule. If the current moves in the direction of the thumb, then the magnetic field moved in circles following the curling of the fingers.

Since magnetic fields are created by moving charged particles, the electrons in the cloud of an atom would also induce a field. Normally, when countless atoms are scattered, all oriented in various directions, the net magnetic field generated would be zero.

However, such as in Figure 33, when atoms are aligned, the tiny amounts of magnetic field created by each particle will compound with the next, thus creating a magnetic field, the magnitude of which is the sum of all the tiny ones.

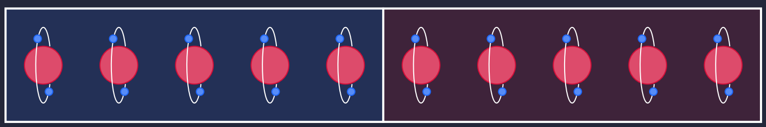


Figure 33: Atoms Aligned in a Bar Magnet

8.3 Strength of a Magnetic Field

Magnetic fields (which are created by currents) exert force onto moving particles (other currents). Thus, wires carrying currents or moving particles individually will exert some magnetic force on each other.

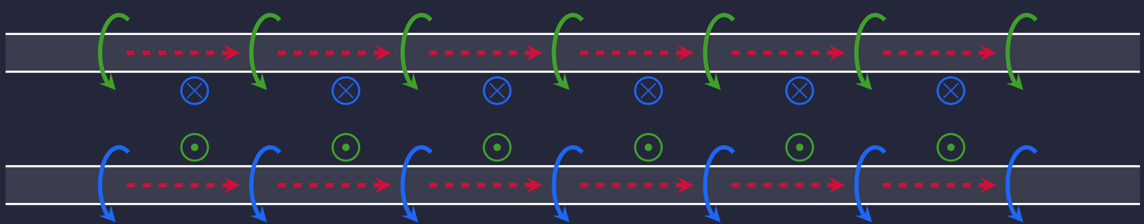


Figure 34: Two Wires in Parallel

In the case of two parallel wires with currents running in the same direction, such as the wires in Figure 34, they would both experience force towards each other. If one of the currents were to be reversed, the force would be away from each other.

How strong are B fields actually? The strength of a B field is modeled by the **Biot-Savart Law**.

Biot-Savart Law

μ_0	constant of nature; $\frac{\mu_0}{4\pi} = 10^{-7}$
q	charge of the particle
v	velocity of the particle
r	distance between the particle and a point in the B field
$\hat{r}$	unit vector of the direction between the particle and B point

$$B = \frac{\mu_0}{4\pi}\frac{q(v \times \hat{r})}{r^2} \quad \textbf{8.4}$$

This formula shows how to relate the strength of a magnetic field to the velocity, charge, and position of a particle. However, what if a continuous charge (rather than a discrete particle) is considered?

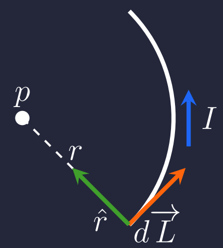
$$B = \frac{\mu_0}{4\pi}\frac{q(v \times \hat{r})}{r^2} \rightarrow d\overrightarrow{B} = \frac{\mu_0}{4\pi}\frac{Id\overrightarrow{L} \times \hat{r}}{r^2}$$

By remodeling the formula in terms of the current in the wire and the length along the wire, all that is left is to integrate it with the proper bounds.

Example **8.1**

How would the magnitude and direction of the B field created by a quarter-circle of wire at point p be calculated? Using the Biot-Savart Law, and modeling it in terms of incremental distances over the wire, the following can be done:

$$B = \frac{\mu_0}{4\pi}\frac{q(v \times \hat{r})}{r^2} \rightarrow d\overrightarrow{B} = \frac{\mu_0}{4\pi}\frac{Id\overrightarrow{L} \times \hat{r}}{r^2}$$



Since L and $\hat{r}$ are perpendicular, the cross product just become multiplication.

$$\begin{aligned} d\overrightarrow{B} &= \frac{\mu_0}{4\pi}\frac{I}{r^2}d\overrightarrow{L} \\ \int d\overrightarrow{B} &= \int_0^{\frac{\pi r}{2}} \frac{\mu_0}{4\pi}\frac{I}{r^2}\, d\overrightarrow{L} \\ \int d\overrightarrow{B} &= \frac{\mu_0}{4\pi}\frac{I}{r^2}\int_0^{\frac{\pi r}{2}} d\overrightarrow{L} \\ \int d\overrightarrow{B} &= \frac{\mu_0}{4\pi}\frac{I}{r^2}\frac{\pi r}{2} = \frac{\mu_0 I}{8r} \end{aligned}$$

Thus, $\frac{\mu_0 I}{8r}$ is the magnitude of the B field at point p, and its direction can be determined using the right-hand rule. It points out of the page.

Following a similar process, the strength of a B field at a point r distance away from a straight and infinite wire is:

$$B = \frac{\mu_0 I}{2\pi r}$$

8.4 Ampere's Law

Ampere's Law **8.5**

$$\oint \overrightarrow{B} \cdot d\overrightarrow{L} = \mu_0 I_{through}$$

Given some current, an arbitrary loop can be drawn around the current. Integrating the B field dotted with the length of the loop gives something directly proportional to the "enclosed" current.

$$\oint \overrightarrow{B} \cdot d\overrightarrow{L} = \mu_0 I_{through}$$

If the loop is made to be perfectly centered around the current and circular, then B is constant and always parallel to the wire, so it can be removed from the integral.

$$\overrightarrow{B} \oint_0^{2\pi r} d\overrightarrow{L} = \mu_0 I_{through}$$

$$2\pi r B = \mu_0 I_{through}$$

$$B = \frac{\mu_0 I_{through}}{2\pi r}$$

Lastly, if no current in enclosed within the loop, then no field will be calculated by Ampere's Law.

9 Electromagnetism

Another way to relate electricity to magnetism is that changing magnetic fields **create** EMF. In other ways, a time-varying magnetic field can make current flow. In Figure 35, as the magnet moves away from the circuit, a current is created causing the led to light up.

Figure 35: Magnetic Field Creating Current

9.1 Faraday's law

This behavior follows **Faraday's Law** which models the relationship between the EMF created and the change in the magnetic field.

Faraday's Law

The change in voltage (EMF) is equal to the rate of change of the magnetic flux ($\frac{d\Phi}{dt}$).

$$EMF = -\frac{d\Phi}{dt} \tag{9.1}$$

The EMF created is similar to the voltage provided by a battery in a circuit. However, EMF is distributed evenly throughout a circuit. To calculate it integrate an electric field around a loop:

$$EMF = \oint \overline{E} \cdot d\overline{L}$$

Faraday's Law scales with the number of loops being considered. If N loops were to be stacked on top of each other, the formula to calculate EMF would become:

$$EMF = -N\frac{d\Phi}{dt}$$

9.2 Magnetic Flux

If EMF is created by a change in magnetic flux, then what are the ways magnetic flux can change? If $\Phi_B = BA\cos(\theta)$, then Φ_B can change if:

- There is a change in the **strength of the B field**

- There is a change in the **area of the loop**
- There is a change in the **angle between the field and the loop**

9.3 Lenz's Law

Why is there a negative sign in Faraday's Law? It indicates the direction of the induced EMF by the change in magnetic field.

Lenz's Law **9.1**

The direction of the induced current's magnetic field will oppose the **change** in the original B field (not the B field itself, just the change).

Remember that the direction of a magnetic field created by a current is determined by the right hand rule and $I \times r$.

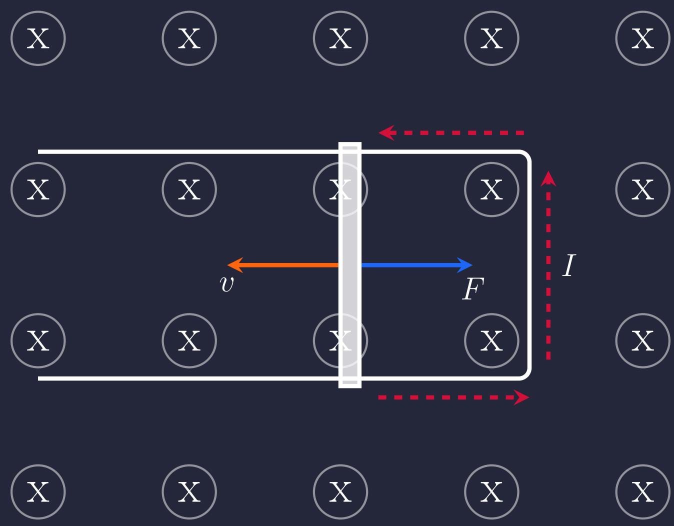


Figure 36: Moving a Bar in a Circuit

In Figure 36, there is a bar completing a circuit. As the bar is moved to the left (v), the magnetic flux is **increasing**. In other words, the amount of magnetic field enclosed within the loop is increasing **into the page**, meaning that the current induced will create a magnetic field pointed **out of the page**, thus the current flows counterclockwise.

Because there is now a current flowing, the magnetic field will exert a force ($F = IL \times B$) on the wire. This is the force (F) opposing the movement of the bar. The force caused by this related to the **eddy current**.

Eddy Current **9.2**

The induced current in a loop when the size of the loop changes, thus changing the magnetic flux in the system. This current will always create a force opposing the direction of the movement.

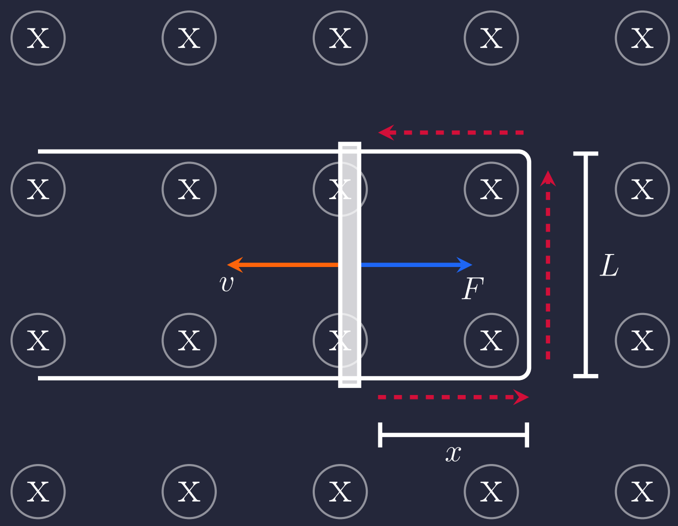


Figure 37: EMF Created

When determining the amount of EMF created by the setup in Figure 37, Faraday's Law can be used: $EMF = -\frac{d\Phi_B}{dt}$. The amount of magnetic flux is modeled as xLB. Since x is changing at a rate of v while B and L remain constant, the rate of change of Φ_B is BLv. Thus: $EMF = BLv$.

If the bar has a resistance of R, then the power ($P = \frac{V^2}{R}$) dissipated over the bar can be expressed as:

$$P = \frac{(BLv)^2}{R}$$

Similarly, the amount of force being applied to the bar (ILB) multiplied with the distance/time (v) the bar is moving is the amount of power being applied to the bar.

$$P = (ILB)(v)$$

$$P = (\frac{BLv}{R}LB)(v)$$

$$P = \frac{(BLv)^2}{R}$$

Evidently, the power dissipated over the resistor is the same as the power applied to the bar. Thus, the system conserves energy.

10 Electromagnetic Waves

10.1 Maxwell's Equations

Based on everything thusfar, four primary equations represent the entirety of all of what is known about electricity, magnetism, and how they relate. These equations are referred to as the **Maxwell Equations**, and are in Figure 38.

Equation	Law
$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q_{enclosed}}{\epsilon_0}$	**Gauss's Law** - Charges create diverging electric fields
$\oint \overrightarrow{B} \cdot d\overrightarrow{A} = 0$	**Gauss's (Magentism) Law** - No magnetic monopoles
$\oint \overrightarrow{E} \cdot d\overrightarrow{l} = -\frac{d\Phi_B}{dt}$	**Faraday's Law** - Changing B fields make E fields
$\oint \overrightarrow{B} \cdot d\overrightarrow{l} = \mu_0 I_{enclosed}$	**Ampere's Law** - Currents make B fields

Figure 38: Maxwell's Equations

However, **Ampere's Law** doesn't properly describe what is happening all of the time. Considering a capacitor that is being charged up, there is no current passing over the capacitor. However, there *is* a B field at that spot.

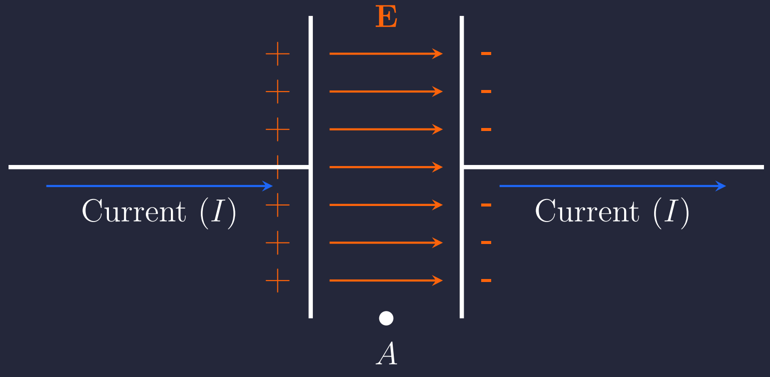


Figure 39: Ampere's Law with a Capacitor

Based on the **Biot-Savart Law**, there should be a B field created at point A in Figure 39. However, a loop drawn around the electric field that intersects A would lead to an Ampere's Law with no enclosed current and thus no B field.

As it turns out, the true Ampere's Law that accounts for this effect is:

$$\oint \overrightarrow{B} \cdot d\overrightarrow{l} = \mu_0 \left(I_{enclosed} + \epsilon_0 \frac{d\Phi_E}{dt} \right)$$

Thus, the true Maxwell's Equations are in Figure 40

Equation	Law
Gauss's Law	$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q_{enclosed}}{\epsilon_0}$
Gauss's (Magentism) Law	$\oint \overrightarrow{B} \cdot d\overrightarrow{A} = 0$
Faraday's Law	$\oint \overrightarrow{E} \cdot d\overrightarrow{l} = -\frac{d\Phi_B}{dt}$
Ampere's Law	$\oint \overrightarrow{B} \cdot d\overrightarrow{l} = \mu_0 \left(I_{enclosed} + \epsilon_0 \frac{d\Phi_E}{dt}\right)$

Figure 40: True Maxwell's Equations

10.2 Electromagnetic Waves

When a particle vibrates, its changing electric field will create a magnetic field. However, since the changing electric field changes at different rates, the magnetic field will also change, thus creating an electric field. This reciprocal behavior goes on forever, and produces what is called an **electromagnetic wave**.

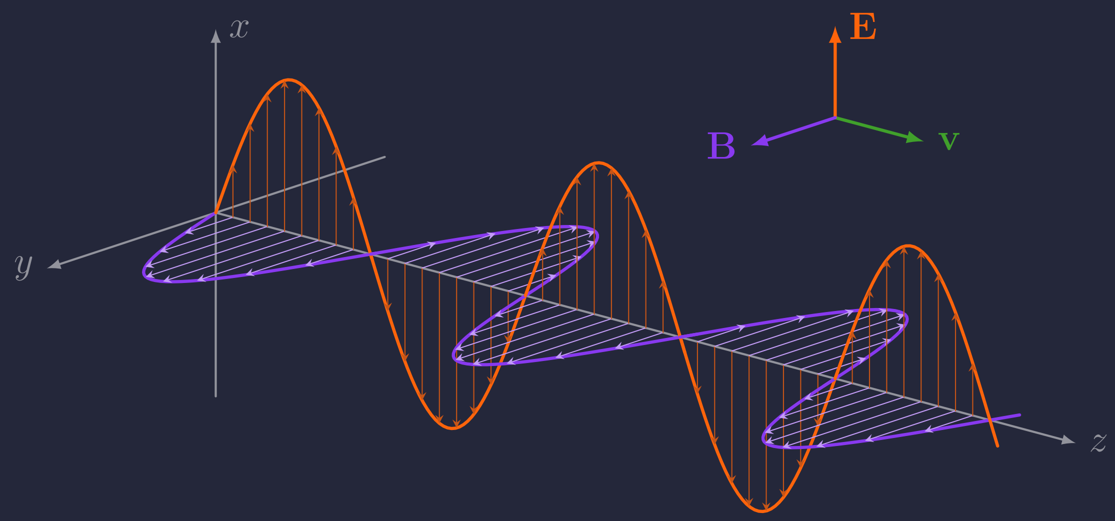


Figure 41: Electromagnetic Wave

Through vector calculus, Maxwell was able to show mathematically how quickly an electromagnetic wave moves:

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \frac{m}{s}$$

Coincidentally (not really coincidentally), this is the speed of light!

Waves are described by: $A \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$, where λ is the wavelength of the wave, T is the period of the wave, x is the position of the wave, and t is time.

Similarly, this equation can be expressed as:

$$y(x, y) = A\cos\left(kx - \omega t\right)$$

Where k is $\frac{2\pi}{\lambda}$ and is referred to as the wavenumber (spatial frequency) of the wave and ω is $\frac{2\pi}{T}$ and is the angular frequency of the wave.

With electromagnetic waves, the same equation is true for both the E and B fields.

Electromagnetic Wave Equations	
$$y(x,t) = A\cos(kx - \omega t)$$ $$E = E_{max}\cos(kx - \omega t)$$ $$B = B_{max}\cos(kx - \omega t)$$	**10.1**

Additionally, the E and B waves are always in **phase** with eachother. This means that they oscillate over the same period and reach their peaks and troughs at the same time. They are also both **perpendicular** to the direction of travel. Both of these things together mean that we can relate them linearly as:

$$E = cB$$

10.3 Electromagnetic Energy

Waves transport things. In the case of electromagnetic waves, only energy is transferred. There is no material being transferred (partly because there is no medium required for an electromagnetic wave to propagate over).

The energy density in a given E or B field is:

$$\text{Energy Density}_E = \frac{1}{2}\epsilon_0 E^2$$
$$\text{Energy Density}_B = \frac{1}{2\mu_0} B^2$$

However, the actual power that exists over a given unit area of an EM wave is:

$$S = \frac{1}{\mu_0} EB$$

And expressed as a vector, is:

$$\overrightarrow{S} = \frac{1}{\mu_0} \overrightarrow{E} \times \overrightarrow{B}$$

Generally, it's more relevant to consider the **average** power/area of a wave rather than the power/area at a single point in time. This quantity is called the **Intensity** of the wave as is:

$$I = S_{avg} = \frac{1}{2\mu_0} E_{max} B_{max}$$

Knowing all of this, how can the intensity of a wave be related to the specific E and B fields?

Maximum Intensities of E and B Fields

The intensity of a wave was previously defined as the power per area of that wave. Mathematically, this can be expressed as:

$$I = \frac{P}{A}$$

Using the intensity of any given EM wave, the relationship that exists between E_{max}, B_{max}, and I are:

$$I = \frac{1}{2\mu_0} c B_{max}^2 = \frac{1}{2} \epsilon_0 c E_{max}^2$$

10.2

10.4 Electromagnetic Momentum

Electromagnetic waves also have momentum. This is how solar sails in space work. The pressure exerted on the sail from electromagnetic waves creates small bits of momentum that compound over time. The mathematical expression for this pressure is:

Radiation Pressure

$$P_{\text{absorbed}} = \frac{I}{c}$$
$$P_{\text{reflected}} = \frac{2I}{c}$$

10.3

Note that this works just like regular momentum; when the particles are fully reflected, the amount of pressure is doubled when compared to when the particles are fully absorbed. To translate this into force:

Radiation Force

$$F_{\text{absorbed}} = \frac{I}{c} A = \frac{IA}{c}$$
$$F_{\text{reflected}} = \frac{2I}{c} A = \frac{2IA}{c}$$

10.4