Main Template

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Contents

1 Basic Concepts

1.1 Charge and Current

Charge **1.1**

Charge (Q) is an electrical property of the atomic particles of which matter consists, measures in coulombs (C).

Throughout this course, it is generally the charge of an electron (e) that will be considered.

$$e = -1.602 \cdot 10^{-19}\ C$$

Thus, if there is some known quantity of electrons n, the total charge of those electrons can be calculated:

$$Q = n \cdot e$$

Current relates closely with charge, being a measurement of the movement of charge (or electrons).

Current **1.2**

Electric current (I) is the rate of change of charge, measured in amperes (A).

Since current is a rate of change (over time) of charge, current and charge relate as each other's (anti)derivative.

$$I(t) = \frac{dQ}{dt}\ A \qquad\qquad Q(t) = \int I(t)\,dt\ C$$

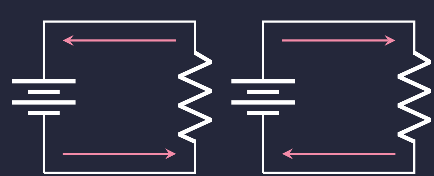
1.2 Direct vs. Alternating Current

1.2.1 Direct Current

Direct Current **1.3**

A direct current (DC) flows only in one direction and can be constant or time varying.

There are two ways of describing the *direction* in which the electrons flow in a direct current: **conventional flow** and **electron flow**. Both are shown in Figure 1.



(a) Conventional (b) Electron

Figure 1: Electron Flow in Direct Currents

1.2.2 Alternating Current

Direct current flow isn't the only way current can flow. Some currents utilize **Alternating Current (AC)**.

Alternating Current **1.4**

An alternating current (AC) is a current that changes direction with respect to time.

1.3 Voltage

Voltage **1.5**

Voltage (or *potential difference*) is the *energy required* to move a unit charge from a reference point (-) to another point (+), measured in Volts (V).

The potential different that voltage measures is a literal different in potential between two points in a circuit. As seen in Figure 2, the voltage from a to b is different than the voltage from b to a.

Figure 2: Potential Difference

The energy required to move an object is expressed in Joules (J), and remains consistent with measurements of energy to move regular objects like a elevator up a shaft. Since voltage is the energy per unit charge, it can be expressed as:

$$v(t) = v_b - v_b = \frac{dw}{dQ} = \frac{dE}{dQ}$$

Where w is work and E is energy.

1.4 Power and Energy

Power **1.6**

Power (P) is the rate of change of energy measured in watts (W).

Previously, it's been seen that current is the rate of change of charge (1.1), and voltage is the amount of energy required to move charge (1.3). Putting these two ideas together, it follows that power can be expressed as the product of current and voltage:

$$P = I \cdot V$$

Power is the rate of change of charge multiplied by the amount of energy required to move some charge. Another way of expressing this in terms of calculus is:

$$P(t) = \frac{dE}{dt} = \frac{dE \cdot dQ}{dt \cdot dQ} = \frac{dE}{dQ} \cdot \frac{dQ}{dt}$$

Where

$$V(t) = \frac{dE}{dQ} \qquad \text{and} \qquad I(t) = \frac{dQ}{dt}$$

Energy **1.7**

Energy is the capacity to do work measured in Joules (J).

$$E(t) = \int P(t)\,dt$$

Currents follow the **Law of Conservation of Energy**. This means that the total change in energy within a closed circuit must sum to zero:

$$\sum P = 0$$

Thus, the total power supplied to a circuit must be equal to the total power absorbed by that circuit.

1.5 Sign Convention

The difference between supplying and absorbing energy is a matter of convention and does not matter given that it remains consistent throughout the full analysis of a circuit. Generally, the **passive sign convention** is used.

Passive Sign Convention **1.8**

Passive sign convention is satisfied when the current enters through the positive terminal of an element and $P = +V \cdot I$. If the current enters through the negative terminal, $P = -V \cdot I$.

2 Circuit Elements

2.1 Sources

The most basic kind of source is an **ideal independent source**. These can be in the form of a voltage or current source, and supply some fixed amount of either voltage or current.

Ideal Independent Source **2.1**

An ideal independent source is an active element that provides specified voltage or current that is completely independent of other circuit elements.

(a) Voltage Source (b) Current Source

(c) Independent Sources

(d) Voltage Source (e) Current Source

(f) Controlled Sources

Figure 3: Sources

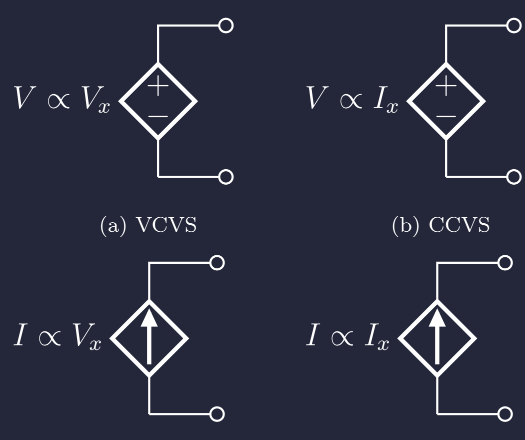
In addition to ideal independent sources, there also exist **ideal controlled sources**. Again, they can provide either voltage or current, but the amount they provide is dependent on the circuit they are a part of.

Ideal Controlled Source **2.2**

An ideal controlled source is an active element in which the source quantity is controlled by another voltage or current.

As stated earlier, controlled sources can provide either current or voltage. Furthermore, they can be *dependent* on either some voltage (V_x) or some current (I_x) in a circuit. Thus, there are four kinds of controlled sources:

- **V**oltage **C**ontrolled **V**oltage **S**ource (4a)
- **C**urrent **C**ontrolled **V**oltage **S**ource (4b)
- **V**oltage **C**ontrolled **C**urrent **S**ource (4c)
- **C**urrent **C**ontrolled **C**urrent **S**ource (4d)



(c) VCCS (d) CCCS

Figure 4: Controlled Source Types

2.2 Resistors

The most basic kind of resistor is just a fixed resistor (as seen in Figure 5). These resistors will have some set value of R that does not change.

Figure 5: Fixed Resistor

There also exist variable resistors (also called potentiometers). These resistors can by manipulated manually, or through the design of a circuit, to have variable resistances. (Figure 6)

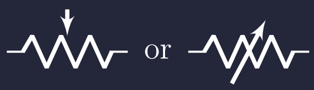


Figure 6: Variable Resistor

3 Basic Laws

3.1 Ohm's Law

Ohm's Law states that the voltage V is *directly* proportional to the current I and resistance R of a circuit.

Ohm's Law	**3.1**
$V = IR$	

When there is current flowing through a wire with resistance approaching zero, a **short circuit** is created. Conversely, an **open circuit** is where the resistance in a circuit approaches infinity.

Whereas resistance measures how much something impedes the flow of current, **conductance** is the ability of an element of conduct electric current; it is measured in mhos (℧) or siemens (S).

3.1.1 Resistivity

The resistance R of a wire is a value determined by a wires dimensions as well as the properties of the material the wire is made of. The inherent properties of a material as they relate to resistance are codified as a material's **resistivity**, which is a quantification of how much a material impedes charge passing through it. The resistivity of some common materials can be seen in Table 8.

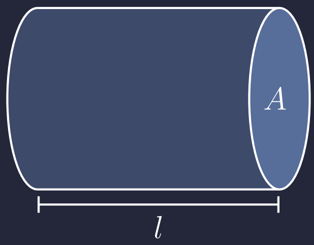


Figure 7: Resistivity in a Wire

Resistivity (ρ) **3.1**

A value representing the amount a material conducts electricity. Most metals tend to have high conductivity, while rubber has a low conductivity.

$$R = \rho \cdot \frac{l}{A}$$

Material	Resistivity ($\Omega \cdot m$)	Usage
Silver	$1.64 \cdot 10^{-8}$	Conductor
Copper	$1.72 \cdot 10^{-8}$	Conductor
Aluminum	$2.80 \cdot 10^{-8}$	Conductor
Carbon	$4 \cdot 10^{-5}$	Semiconductor
Germanium	$47 \cdot 10^{-2}$	Semiconductor
Silicon	$6.4 \cdot 10^{2}$	Semiconductor
Paper	10^{10}	Insulator
Glass	$10 \cdot 10^{12}$	Insulator
Teflon	$3 \cdot 10^{12}$	Insulator

Figure 8: Resistivities of Common Materials

3.2 Nodes, Branches, and Loops

Branch (b) **3.2**

Represents a single element in a circuit such as a resistor or power supply.

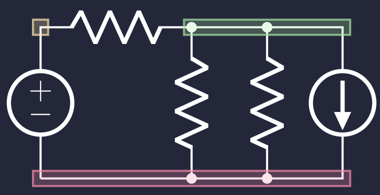


Figure 9: Nodes and Branches

In Figure 9, there exist *five* branches in the circuit. Specifically, there are three resistors, one voltage source, and one current source:

$$3 + 1 + 1 = 5$$

Node (n) **3.3**

The point of connection between two or more branches.

In the same circuit (9), there are three nodes. Each node is highlighted in a different color. Notice that there are no branches within a node, thus each node is the largest area possible without crossing a branch. Furthermore, the voltage throughout an ideal node is zero.

Loop (l) **3.4**

A loop is any *closed* path in a circuit. Generally, loops are defined as the smallest possible path.

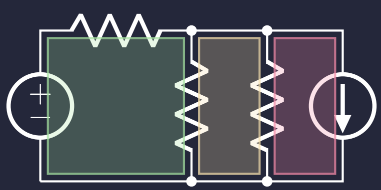


Figure 10: Loops

Each shown in a different color in Figure 10, loops are easy to visualize as the area enclosed by any closed series of components.

Network Topology **3.2**

$$b = l + n - 1$$

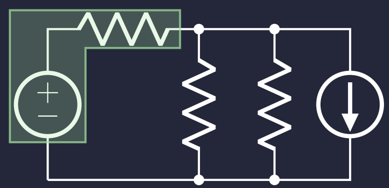
3.3 Parallel vs. Series

Series **3.5**

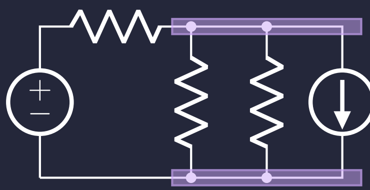
Two or more elements are in series if they exclusively share a single node and consequently carry the same current.

Continuing with the same circuit, Figure 11a highlights elements in series. The only two elements in series are the voltage source and the top resistor.

Figure 11b highlights the two shared nodes between three of the branches in the circuit. Since these branches are sharing the same two nodes, they are said to be in parallel.



(a) Loops



(b) Loops

Figure 11: In Parallel and In Series

Parallel **3.6**

Two or more elements are in parallel if they are connected to the same two notes and consequently have the same voltage across them.

3.4 Kirchhoff's Laws

3.4.1 Kirchoff's Current Law

Kirchhoff's Current Law (KCL) **3.7**

Kirchhoff's Current Law states that the algebraic sum of currents entering and exiting a node is zero:

$$\sum_{n=1}^{N_{branch}} i_n = 0$$

Consider the circuit in Figure 12. In this circuit, there are two labeled nodes n_1 and n_2. Applying KCL at n_1, the sum of currents would be expressed as:

$$I_1 - I_2 - I_3 - I_a = 0$$

Since only I_1 is flowing *into* n_1, with the other tree currents flowing out. At n_2, the currents would be:

$$-I_1 + I_2 + I_3 + I_a = 0$$

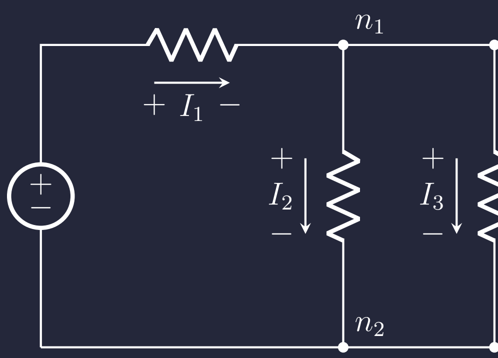


Figure 12: KCL

Since the opposite is true: only I_1 is flowing *out* of n_2, with the rest flowing in.

3.4.2 Kirchoff's Voltage Law

Kirchhoff's Voltage Law (KVL) **3.8**

Kirchhoff's Voltage Law states that the algebraic sum of all voltages around a closed path (loop) is zero:

$$\sum_{m=1}^{M_{branch}} v_m = 0$$

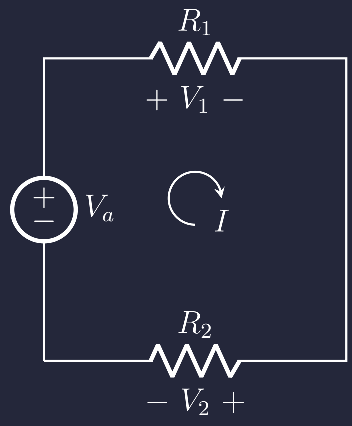


Figure 13: KVL

Consider the circuit in Figure 13. In this circuit, there is a single loop. According to KVL, the voltage gained and lost through each branch of this loop should sum to zero.

Following the direction of I in this circuit, there is an *increase* in voltage over the voltage source and two consecutive losses in voltage over the resistors. Thus:

$$-V_a + V_1 + V_2 = 0 \quad \text{or} \quad V_a = V_1 + V_2$$

This balance of the voltage rises across some components in a loop and the voltage drops across other components is what KVL is.

3.5 Voltage Divider

When resistors are connected in series, the equivalent resistance of the resistors is equal to the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n = \sum_{n=1}^{N} R_n$$

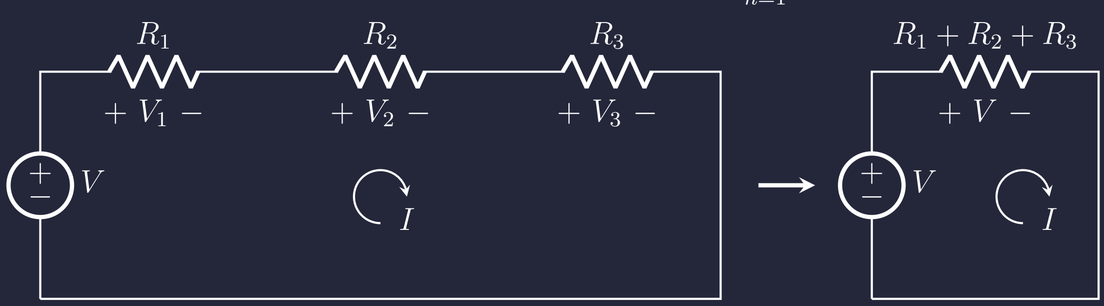


Figure 14: Resistors in Series

By applying Ohm's Law to this concept of adding resistances together, the total current of the circuit can be found as:

$$I = \frac{V}{R_1 + R_2 + R_3} = \frac{V}{R_{eq}}$$

The to find the voltage drop across any single resistor, Ohm's Law can be applied again with the current I:

$$V_n = I \cdot R_n = V \cdot \frac{R_n}{R_{eq}}$$

Thus giving the formula for a voltage divider. This voltage divider can only be applied to elements that have the same current across them all, such as these resistors *in series*.

Voltage Divider $V_n = V \cdot \frac{R_n}{R_{eq}}$	**3.3**

3.6 Current Divider

When resistors are connected in parallel, the equivalent resistance of the resistors is calculated as:

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}\right)^{-1} = \left(\sum_{n=1}^{N} \frac{1}{R_n}\right)^{-1}$$

By applying Ohm's Law to this concept resistors in parallel, the total current of the circuit can be found as:

$$I_t = \frac{V}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}\right)^{-1}} = V \cdot R_{eq}$$

Then to find the current across any single resistor, Ohm's Law can be applied again with the total current I_t:

$$I_n = I_t \cdot \frac{R_{eq}}{R_n}$$

Thus giving the formula for a current divider. This current divider can only be applied to elements in parallel (elements that share both nodes with each other).

Current Divider	
$$I_n = I_t \cdot \frac{R_{eq}}{R_n}$$	**3.4**

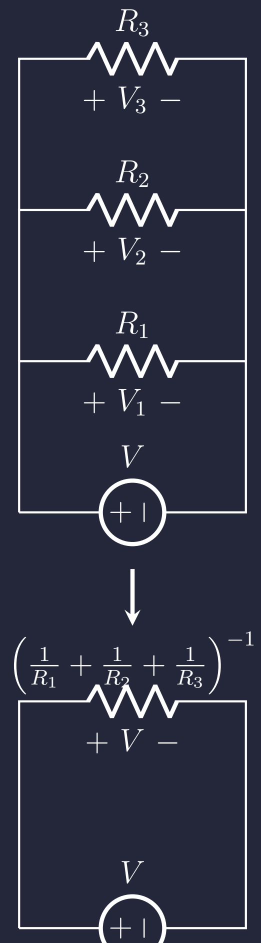


Figure 15: Resistors in Parallel

4 Methods of Analysis

4.1 Nodal Analysis

Nodal analysis applies Kirchoff's Current Law (Subsubsection 3.4.1) at each node in a circuit to find the voltages at each of those nodes relative to some ground.

Consider the current in Figure 16. In this circuit, there are three nodes, each labeled with some voltage V_1, V_2, and V_3. Applying nodal analysis to this circuit would start with applying KCL at each node:

$$@V_1 \rightarrow 0 = -I_1 + I_2 + I_3 + I_a$$
$$@V_2 \rightarrow 0 = I_1 - I_2 - I_3 - I_a$$

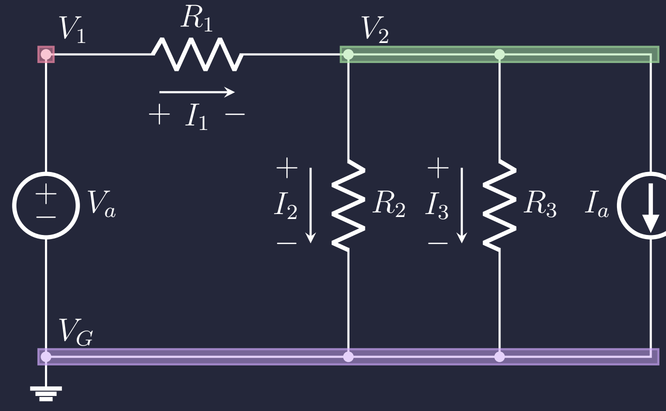


Figure 16: Nodal Analysis

Then, by applying Ohm's Law to each of the currents, the equations can be rewritten as:

$$@V_1 \rightarrow 0 = -\frac{V_1 - V_2}{R_1} + \frac{V_2 - V_G}{R_2} + \frac{V_2 - V_G}{R_3} + I_a$$
$$@V_2 \rightarrow 0 = \frac{V_1 - V_2}{R_1} - \frac{V_2 - V_G}{R_2} - \frac{V_2 - V_G}{R_3} - I_a$$

At this point, the equations can be treated as any other system of equations and be solved through linear algebra and matrices. In this case, since there are only two unknown values (V_1 and V_2), linear algebra is not needed. However, as the circuits become more complicated, the linear algebra approach becomes more appealing.

4.1.1 Supernodes

Consider the circuit in Figure 17. This circuit contains a supernode around V_a because the voltage source is between two non-reference nodes.

Supernode **4.1**

A supernode is formed by enclosing a voltage source connected between two non-reference nodes and any elements connected in parallel with it.

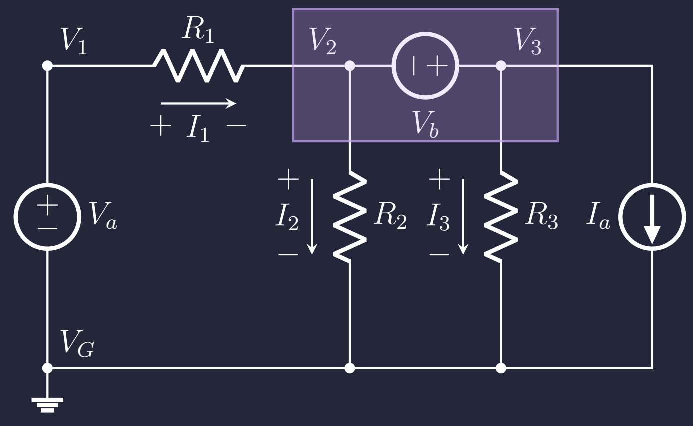


Figure 17: Supernode

In cases with a supernode, the current over the voltage source (V_b) can't be found through simple Ohm's Law. Instead, the supernode can be treated as a single node to apply KCL to:

$$0 = I_1 - I_2 - I_3 - I_a$$

which then can be transformed with Ohm's law to an equation in terms of voltages and resistances. However, from these two nodes (V_2 and V_3) only once equation was found. The second equation can be formulated as just the voltage difference between the two nodes due to the voltage source:

$$V_2 + V_b = V_3$$

This supernode analysis can then be combined with regular nodal analysis at V_1 to complete the analysis of the circuit.

4.2 Mesh Analysis

Mesh analysis applies Kirchoff's Voltage Law (Subsubsection 3.4.2) at each loop in the circuit to find the currents in each of those loops.

Consider the circuit in Figure 18. In the circuit, there are three loops each with some current (I_1, I_2, and I_3) flowing through them. According to KVL, the total voltage rise/drop throughout each of these loops should be zero.

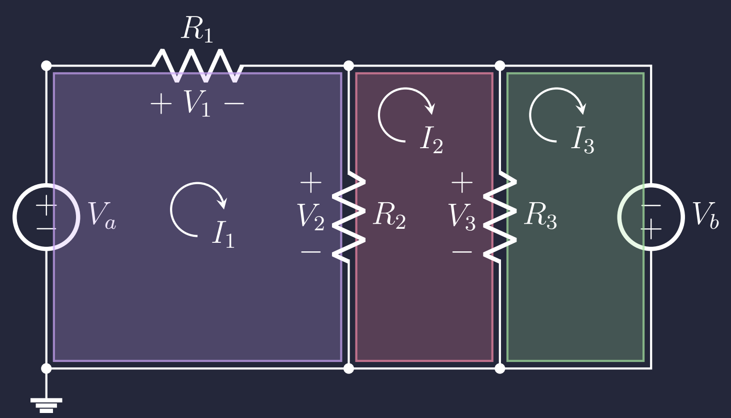


Figure 18: Mesh Analysis

The first step, then, to applying mesh analysis onto the circuit would be to sum the voltages around each loop. It's important to keep a consistent sign convention (1.5) in which current flows from high (+) to low (−) voltages. Thus, a voltage drop across a resistor ($+ \to -$) would be positive and a voltage rise ($- \to +$) would be negative:

$$@I_1 \to V_a = V_1 + V_2$$
$$@I_2 \to 0 = -V_2 + V_3$$
$$@I_3 \to -V_b = V_3$$

Applying Ohm's Law to each voltage, the equations become:

$$\begin{aligned} V_a &= R_1(I_1) + R_2(I_1 - I_2) \\ 0 &= -R_2(I_1 - I_2) + R_3(I_2 - I_3) \\ -V_b &= R_3(I_2 - I_3) \end{aligned} \quad \to \quad \begin{aligned} V_a &= I_1(R_1 + R_2) + I_2(-R_2) + I_3(0) \\ 0 &= I_1(-R_2) + I_2(R_2 + R_3) + I_3(-R_3) \\ -V_b &= I_1(0) + I_2(-R_3) + I_3(R_3) \end{aligned}$$

From here, it's just a matter of solving a system of equations by any means necessary.

4.2.1 Supermesh

Consider the circuit in Figure 20. If this circuit were to be approached exactly like the circuit in Figure 18, eventually a roadblock would be hit since the voltage drop across the current source I_a cannot be calculated through Ohm's Law. This is because the two loops I_1 and I_2 form a supermesh.

Supermesh **4.2**

A supermesh results when two meshes/loops share a current source.

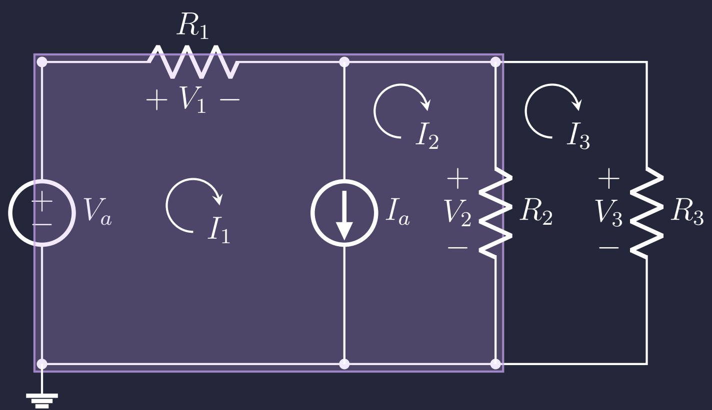


Figure 20: Supermesh

To solve this circuit, supermesh analysis must be applied. Similarly to the supernode analysis, the supermesh can be treated as a single loop in which the voltage drop throughout the entire supermesh is set to zero:

$$
\begin{aligned}
&V_a = V_1 + V_2 \\
\Rightarrow\ &V_a = R_1(I_1) + R_2(I_2 - I_3) \\
\Rightarrow\ &V_a = I_1(R_1) + I_2(R_2) + I_3(-R_2)
\end{aligned}
$$

The second equation derived from the supermesh would be the relationship between the currents I_1, I_2, and I_a:

$$I_a = I_1 - I_2$$

The equation in the loop I_3 can be found normally, and from there a system of equations can be solved however desired.

5 Circuit Theorems

5.1 Superposition

Superposition Principle **5.1**

The superposition principle states that the voltage across an element *in a linear* circuit is the algebraic sum of the voltages across that element due to each independent source *acting alone.*

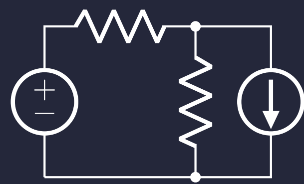


Figure 21: Circuit

Consider the circuit in Figure 21. This circuit has two sources, and while it can be analyzed through mesh and nodal analysis, the principle of superposition can also be applied to it. By doing so, two similar circuits can be constructed, each taking only a single source at once.

(a) Voltage Source

(b) Current Source

Figure 22: Sub-Circuits

From here, each circuit can be analyzed independently. The values found for voltages and currents throughout the circuits can be summed to find the values from the original circuit in Figure 21.

5.2 Source Transformation

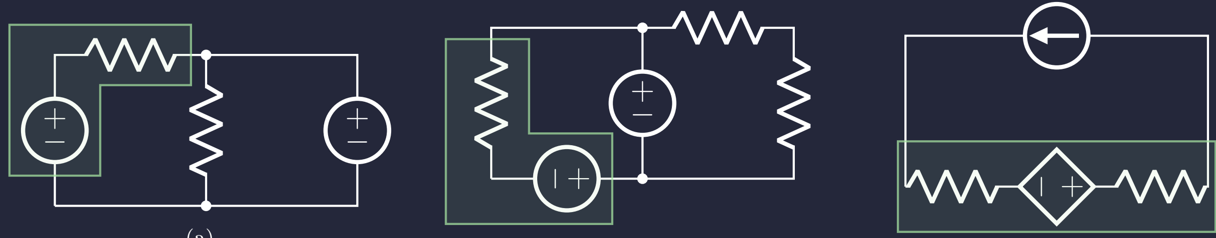
Source Transformation **5.2**

A source transformation is the process of replacing a voltage source V_s in series with a resistor R with a current source I_s in parallel with a resistor or vice versa.

Figure 23: Source Transformation

In Figure 23, the voltage source and current source can be switched back and forth by applying source transformation.

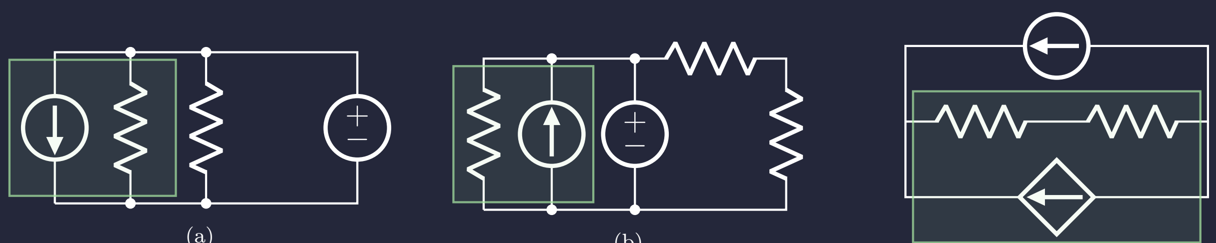
To transform a voltage source to a current source, the voltage source must be *in series* with a resistor. The transformation will then yield a current source *in parallel* with the resistor, and a current source value calculated from Ohm's Law ($I = \frac{V}{R}$). Consider the circuits in Figure 24. In each circuit, the voltage sources that *can* be transformed into current sources are highlighted in green.



(a) (b) (c)

Figure 24: Voltage Transformations

The applied source transformations to each circuit can be seen in Figure 25.



(a) (b) (c)

Figure 25: Voltage Transformations

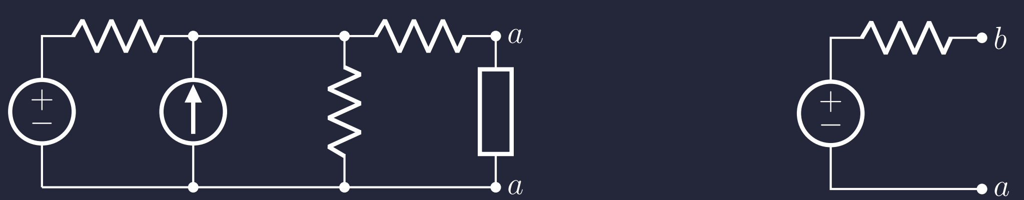
The same process, just reversed, can be done to transform a current source into a voltage source. If a current source exists *in parallel* with a resistor, it can be transformed into a voltage source *in series* with that resistor. The voltage on the voltage source is determined by Ohm's Law ($V = IR$).

5.3 Thevenin's Theorem

Thevenin's Theorem **5.3**

States that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source (V_{th}) in series with a resistor (R_{th}) where V_{th} is the **open-circuit voltage** and R_{th} is the equivalent resistance at the terminals when the independent sources are turned off.

Consider the circuit in Figure 26a.

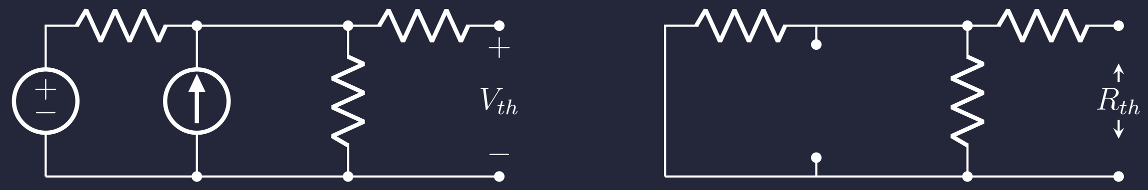


(a) Circuit (b) Thevenin Equivalent

Figure 26: Thevenin's Theorem

Thevenin's Theorem states that this circuit, seeing that it is indeed linear, can be modeled as an equivalent circuit between terminals a and b as simply a resistor and voltage source in series.

To find the Thevenin equivalent circuit, V_{th} and R_{th} must be found. To find V_{th}, as in Figure 27a, is found by opening the circuit between a and b. To find R_{th}, the resistance of the circuit must be analyzed by shorting voltage sources and opening current sources.



(a) Open Circuit Voltage

(b) Dead Circuit Resistance

Figure 27: Finding Thevenin's Theorem

Some circuits, such as the one in Figure 28, aren't possible to find the R_{th} of just through adding resistance values in series and parallel.

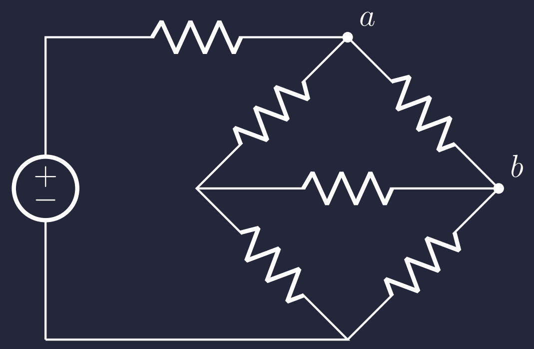


Figure 28: Finding R_{th}

In such cases, by inserting some dummy voltage source between the terminals a and b and solving for the current over that branch, Ohm's Law can then be applied to calculate the resistance:

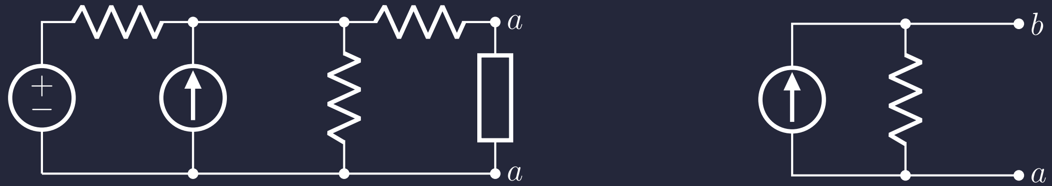
$$R_{th} = \frac{V_{dummy}}{I_{calculated}}$$

5.4 Norton's Theorem

Norton's Theorem **5.4**

States that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source (I_N) in series with a resistor (R_N) where I_N is the **short-circuit current** and R_N is the equivalent resistance at the terminals when the independent sources are turned off.

Consider the circuit in Figure 29a.



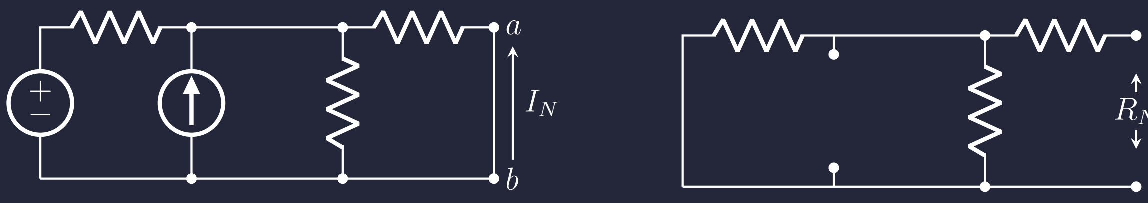
(a) Circuit

(b) Norton Equivalent

Figure 29: Norton's Theorem

Similar to Thevenin's Theorem, since this circuit still is linear, it can also be expressed as a simple circuit with just a current source and resistor in parallel. Notice that a Thevenin equivalent and a Norton equivalent circuit are simply source transformed versions of the other (see Figure 23).

To solve for a Norton equivalent circuit, I_N and R_N must be found. R_N can be found the same as previously in Subsection 5.3. I_N, however, is solved by shorting terminals a and b, and finding the current over the wire.



(a) Short Circuit Current

(b) Dead Circuit Resistance

Figure 30: Norton's Theorem

5.5 Power Transfer

In a circuit reduced to the form of a Thevenin or Norton circuit, the next step of the analysis might then to be determine the amount of **power** consumed by some load connected between the terminals a and b. As in Subsection 1.4, the power can be calculated as:

$$P = I \cdot V$$

For a Thevenin circuit, this could take the form of multiplying the current in the circuit by the voltage drop across the load (with a voltage divider):

$$P = I \cdot V = \left[\frac{V_{th}}{R_{th} + R_L}\right] \cdot \left[V_{th}\frac{R_L}{R_L + R_{th}}\right] = R_L \cdot \left(\frac{V_{th}}{R_L + R_{th}}\right)^2$$

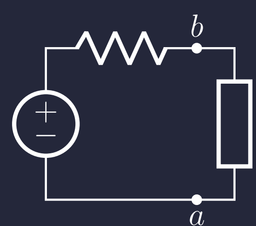


Figure 31: Thevenin Equivalent

Thus, giving the equation for the power consumed by the load in terms of R_L, R_{th}, and V_{th}.

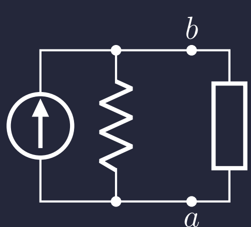


Figure 32: Norton Equivalent

For a Norton circuit, it might be simpler to see it as a current divider for I multiplied with an Ohm's Law to find V:

$$P = I \cdot V = \left[\frac{I_N}{\left(R_N^{-1} + R_L^{-1}\right) R_L}\right] \cdot \left[\frac{I_N}{R_N^{-1} + R_L^{-1}}\right] = \left(\frac{I_N}{R_N^{-1} + R_L^{-1}}\right)^2 \cdot \frac{1}{R_L}$$

Thus giving the power consumed in terms of R_L, R_N and I_N.

Considering the fact that $V_{th} = I_N \cdot R_{th/N}$ and $I_N = \frac{V_{th}}{R_{th/N}}$ by Ohm's Law, these two equations can be shown to be equivalent:

$$\left[R_L \cdot \left(\frac{V_{th}}{R_L + R_{th}}\right)^2 = \left(\frac{I_N}{R_N^{-1} + R_L^{-1}}\right)^2 \cdot \frac{1}{R_L}\right] \to \left[{R_L}^2 \cdot \left(\frac{V_{th}}{R_L + R_{th}}\right)^2 = \left(\frac{I_N}{R_N^{-1} + R_L^{-1}}\right)^2\right] \to$$

$$\left[R_L \cdot \frac{V_{th}}{R_L + R_{th}} = \frac{I_N}{R_N^{-1} + R_L^{-1}}\right] \to \left[R_L \cdot \frac{I_N \cdot R_{th/N}}{R_L + R_{th}} = \frac{I_N}{R_N^{-1} + R_L^{-1}}\right] \to \left[\frac{R_L \cdot R_{th/N}}{R_L + R_{th}} = \frac{1}{R_N^{-1} + R_L^{-1}}\right] \to$$

$$\left[\frac{R_L + R_{th}}{R_L \cdot R_{th/N}} = \frac{1}{R_N} + \frac{1}{R_L}\right] \to \left[\frac{R_L}{R_L \cdot R_{th/N}} + \frac{R_{th}}{R_L \cdot R_{th/N}} = \frac{1}{R_N} + \frac{1}{R_L}\right] \to \left[\frac{1}{R_N} + \frac{1}{R_L} = \frac{1}{R_N} + \frac{1}{R_L}\right]$$

5.5.1 Maximum Power Transfer

Often, maximizing the power consumed by a load is desirable in the design process. Using calculus, and specifically optimization, the load resistance that yields the maximum power consumed by the load can be solved for. Consider the Thevenin equation for power:

$$P = R_L \cdot \left(\frac{V_{th}}{R_L + R_{th}} \right)^2$$

The first step of optimization is to derive the equation. Deriving with respect to R_L:

$$\frac{dP}{dR_L} = V_{th}^2 \cdot \left(\frac{R_{th} - R_L}{(R_{th} + R_L)^3} \right)$$

Then, set $\frac{dP}{dR_L} = 0$ to find critical points. It is only at these points where an extrema (whether max or min) may occur:

$$V_{th}^2 \cdot \left(\frac{R_{th} - R_L}{(R_{th} + R_L)^3} \right) = 0 \rightarrow R_{th} - R_L = 0 \rightarrow R_{th} = R_L$$

Thus, when $R_L = R_{th}$, the maximum amount of power consumption over the load occurs. This process can be done analogously for the Norton euqation for power, and will give the same answer.

Maximum Power **5.5**

The maximum power is transferred to the load when the load resistance equals the Thevenin/Norton resistance as seen from the load.

$$R_L = R_{th/N}$$