Main Template

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Example **0.1**

Consider an object of temperature T places in a room of ambient temperature T_a. According to Newton's law of cooling, the **rate of change** of T is proportional to the difference $T - T_a$. Mathematically, this is expressed as:

$$\frac{dT}{dt} = -k(T - T_a)$$

where $k > 0$ and is a constant determined by the properties of the object.

Note that the constant C is arbitrary. Without any further information, there is no way to decide C. In other words, $T(t)$

Example **0.2**

A lake has water volume $V(m^3)$. Assume V is a constant. A factory emits R kilograms of mercury into the lake every day. Suppose the mercury diffuses to the lake instantly and water refreshes every day by $W(m^3)$. How much time does it take for the water to be non-potable (implicitly, $P(0) = 0$).

Let $P(t)$ be the mass of mercury in the lake. Let Δt be a short period of time.

What happens between the time t and $t + \Delta t$?

- The increase of mercury $= R\Delta t$
- The decrease of polluted water $= W\Delta t$
- The density of mercury $= \frac{P(t)}{V}$
- The decrease of mercury $= \frac{P(t)}{V} \cdot W\Delta t$

Thus:

$$\begin{aligned}
\Delta P(t) &= R\Delta t - \frac{P(t)}{V} \cdot W\Delta t \\
\frac{\Delta P(t)}{\Delta t} &= R - \frac{P(t)}{V} \cdot W \\
\frac{dP}{dt} &= R - \frac{P}{V} \cdot W \\
\frac{dP}{R - \frac{P}{V} \cdot W} &= dt
\end{aligned}$$

...

$$\begin{aligned}
ln\left|R - \frac{W}{V}P\right| &= -\frac{W}{V}t + C \\
R - \frac{W}{V}P &= Ce^{-\frac{W}{V}t} \\
P &= -\frac{V}{W}\left(Ce^{-\frac{W}{V}t} - R\right) \\
P &= Ce^{-\frac{W}{V}t} + \frac{Rv}{W}
\end{aligned}$$