Main Template

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Contents

1 Basic Concepts **2**
1.1 Charge and Current 2
1.2 Direct vs. Alternating Current 2
1.2.1 Direct Current 2
1.2.2 Alternating Current 2
1.3 Voltage 3
1.4 Power and Energy 3

2 Basic Laws **5**
2.1 Ohm's Law 5
2.1.1 Resistivity 5
2.2 Nodes, Branches, and Loops 5
2.2.1 In Parallel vs. Series 6
2.3 Kirchhoff's Laws 7

3 Methods of Analysis **8**
3.1 Mesh Analysis 8

1 Basic Concepts

1.1 Charge and Current

Charge **1.1**

Charge (Q) is an electrical property of the atomic particles of which matter consists, measures in coulombs (C).

Throughout this course, it is generally the charge of an electron (e) that will be considered.

$$e = -1.602 \cdot 10^{-19}\ C$$

Thus, if there is some known quantity of electrons n, the total charge of those electrons can be calculated:

$$Q = n \cdot e$$

Current relates closely with charge, being a measurement of the movement of charge (or electrons).

Current **1.2**

Electric current (I) is the rate of change of charge, measured in amperes (A).

Since current is a rate of change (over time) of charge, current and charge relate as each other's (anti)derivative.

$$I(t) = \frac{dQ}{dt}\ A \qquad\qquad Q(t) = \int I(t)\, dt\ C$$

1.2 Direct vs. Alternating Current

1.2.1 Direct Current

Direct Current **1.3**

A direct current (dc) flows only in one direction and can be constant or time varying.

There are two ways of describing the *direction* in which the electrons flow in a direct current: **conventional flow** and **electron flow**. Both are shown in Figure 1.

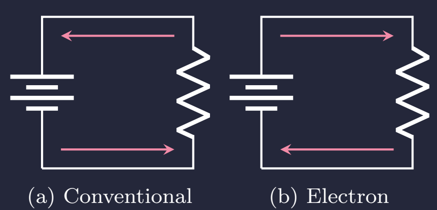

Figure 1: Electron Flow in Direct Currents

1.2.2 Alternating Current

Direct current flow isn't the only way current can flow. Some currents utilize **Alternating Current (AC)**.

Alternating Current **1.4**

An alternating current (ac) is a current that changes direction with respect to time.

1.3 Voltage

Voltage	**1.5**
Voltage (or *potential difference*) is the *energy required* to move a unit charge from a reference point (-) to another point (+), measured in Volts (V).	

The potential different that voltage measures is a literal different in potential between two points in a circuit. As seen in Figure 2, the voltage from a to b is different than the voltage from b to a.

Figure 2: Potential Difference

The energy required to move an object is expressed in Joules (J), and remains consistent with measurements of energy to move regular objects like a elevator up a shaft. Since voltage is the energy per unit charge, it can be expressed as:

$$v(t) = v_b - v_b = \frac{dw}{dQ} = \frac{dE}{dQ}$$

Where w is work and E is energy.

1.4 Power and Energy

Power	**1.6**
Power (P) is the rate of change of energy measured in watts (W).	

Previously, it's been seen that current is the rate of change of charge (1.1), and voltage is the amount of energy required to move charge (1.3). Putting these two ideas together, it follows that power can be expressed as the product of current and voltage:

$$P = I \cdot V$$

Power is the rate of change of charge multiplied by the amount of energy required to move some charge. Another way of expressing this in terms of calculus is:

$$P(t) = \frac{dE}{dt} = \frac{dE \cdot dQ}{dt \cdot dQ} = \frac{dE}{dQ} \cdot \frac{dQ}{dt}$$

Where

$$V(t) = \frac{dE}{dQ} \qquad \text{and} \qquad I(t) = \frac{dQ}{dt}$$

Energy **1.7**

Energy is the capacity to do work measured in Joules (J).

$$E(t) = \int P(t)\, dt$$

Currents follow the **Law of Conservation of Energy**. This means that the total change in energy within a closed circuit must sum to zero:

$$\sum P = 0$$

Thus, the total power supplied to a circuit must be equal to the total power absorbed by that circuit.

The difference between supplying and absorbing energy is a matter of convention and does not matter given that it remains consistent throughout the full analysis of a circuit. Generally, the **passive sign convention** is used.

Passive Sign Convention **1.8**

Passive sign convention is satisfied when the current enterts through the positive terminal of an element and $P = +V \cdot I$. If the current enters through the negative terminal, $P = -V \cdot I$.

2 Basic Laws

2.1 Ohm's Law

Ohm's Law states that the voltage V is *directly* proportional to the current I and resistance R of a circuit.

Ohm's Law $$V = IR$$ Where: $$R = \rho \cdot \frac{l}{A}$$	**2.1**

When there is current flowing through a wire with resistance approaching zero, a **short circuit** is created. Conversely, an **open circuit** is where the resistance in a circuit approaches infinity.

Whereas resistance measures how much something impedes the flow of current, **conductance** is the ability of an element of conduct electric current; it is measured in mhos (℧) or siemens (S).

2.1.1 Resistivity

Resistivity (ρ) **2.1**

A value representing the amount a material conducts electricity. Most metals tend to have high conductivity, while rubber has a low conductivity.

The resistance R of a wire, for example, is a value to varies with the inherent properties as well as the dimensions of the wire and its materials. The inherent properties of a material as they relate to resistance are codified as a material's **resistivity**.

2.2 Nodes, Branches, and Loops

Branch (b) **2.2**

Represents a single element in a circuit such as a resistor or power supply.

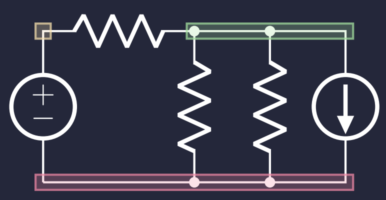

Figure 3: Nodes and Branches

In Figure 3, there exist *five* branches in the circuit. Specifically, there are three resistors, one voltage source, and one current source:

$$3 + 1 + 1 = 5$$

Node (n) **2.3**

The point of connection between two or more branches.

In the same circuit (3), there are three nodes. Each node is highlighted in a different color. Notice that there are no branches within a node, thus each node is the largest area possible without crossing a branch. Furthermore, the voltage throughout an ideal node is zero.

Loop (l) **2.4**

A loop is any *closed* path in a circuit. Generally, loops are defined as the smallest possible path.

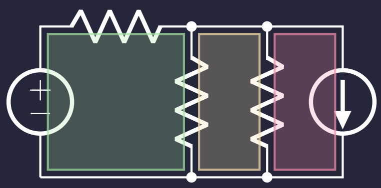

Figure 4: Loops

Each shown in a different color in Figure 4, loops are easy to visualize as the area enclosed by any closed series of components.

Network Topology **2.2**

$$b = l + n - 1$$

2.2.1 In Parallel vs. Series

Series **2.5**

Two or more elements are in series if they exclusively share a single node and consequently carry the same current.

(a) Loops (b) Loops

Figure 5: In Parallel and In Series

Continuing with the same circuit, Figure 5a highlights elements in series. The only two elements in series are the voltage source and the top resistor.

Figure 5b highlights two of the three sets of parallel components. The third set would be the leftmost vertical resistor and the current source.

In Parallel **2.6**

Two or more elements are in parallel if they are connected to the same two notes and consequently have the same voltage across them.

2.3 Kirchhoff's Laws

Kirchhoff's Current Law **2.7**

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero:

$$\sum_{n=1}^{N_{branch}} i_n = 0$$

Kirchhoff's Voltage Law **2.8**

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of all voltages around a closed path is zero:

$$\sum_{m=1}^{M_{branch}} v_m = 0$$

3 Methods of Analysis

3.1 Mesh Analysis

Mesh **3.1**

A mesh is a loop in a circuit that doesn't contain any loops within it.