ENGR 2541 - Mechanics of Materials

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January 6, 2025

Contents

1 Types of Stress

1.1 Axial Stress

1.1.1 Overview

Mechanics of materials provides a means to analyze the effects of *stresses* and *deformations*. Statics covered finding a balance of forces, including internal forces such as *shear*, *bending*, and *tension/compression*. Finding these internal forces is imperative to be able to determine the integrity of a structure. In addition to the internal forces, the integrity of a structure is also partially determined by the *dimensions* and *materials* of that structure.

In the analysis of a rod, for example, the ability for that rod to withstand the internal forces (its structural integrity) is determined by the cross-sectional area and the material of the rod.

(a) Resultant Force (b) Elementary Forces

Figure 1: Axial force is the resultant of distributed elementary forces

1.1.2 Analyzing Axial Stress

In Figure 1, the *stress* being experiences by the member is the **force per unit area**, denoted by the Greek letter sigma (σ).

Axial Stress

Stress can be calculated by dividing the total axial force by the cross-sectional area:

$$\sigma = \frac{P}{A} \tag{1.1}$$

This formula gives the *average* axial stress over the cross section of a member. This stress can be assumed to be *uniform* throughout the cross section.

By convention, a positive force indicates **tensile stress** while a negative force indicates **compressive stress**.

(a) Tensile Stress (b) Compressive Stress

Figure 2: Tensile vs. Compressive Force

The cross section, as seen in Figure 3, is perpendicular to the axial forces. The corresponding stress in the member in described as *normal stress*.

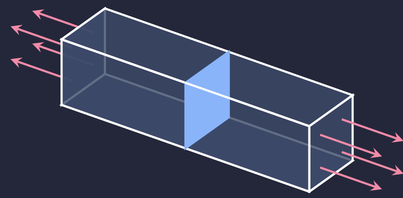


Figure 3: Normal Stress

Thus, the formula of $\sigma = \frac{P}{A}$ gives the normal stress of an member under axial loading.

1.1.3 Stress Points

The formula of $\sigma = \frac{P}{A}$ is only useful for *averages* or *ranges*. This can calculate the average value of the stress over the entire cross section. However, what about calculating at specific points?

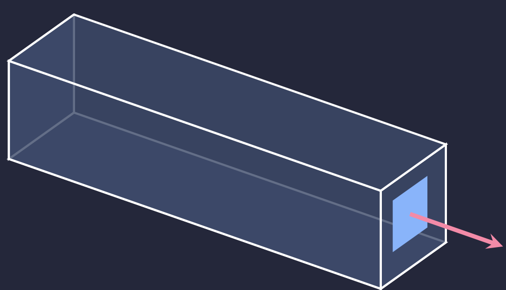


Figure 4: Stress Points

In Figure 4, to find the stress of the highlighted area, the working equation can still be applied, just now with a smaller area. To find the stress at a single point, rather than just a smaller area, stress must be calculated as the area approaches zero.

Stress at a Single Point — **1.2**

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

1.2 Shearing Stress

1.2.1 Overview

In Section 1.1, the internal forces and corresponding stresses were *normal* to the member.

Shear Stress — **1.1**

Internal stresses caused by forces with parallel and opposite components vectors.

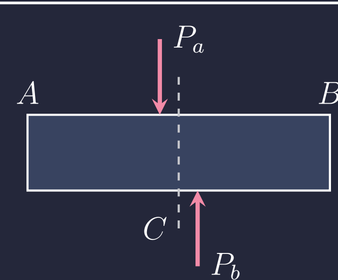
In Figure 5, a member has two opposing forces, P_a and P_b, acting upon it. These two forces create internal shear in the section between them as seen in Figure 5b.

By analyzing the section at C in the member, it can be seen that the internal force created by the two forces in the section is equal to P_b. This resultant force is called a **shear force** (P). Dividing P by the area (A) results in the **average shearing stress** in the section.

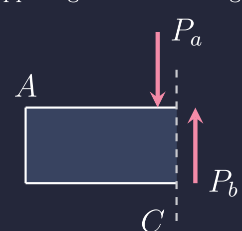
Average Shearing Stress — **1.3**

$$\tau_{avg} = \frac{P}{A}$$

The average shearing stress cannot be assumed to be uniform throughout the section. Though not the full story, the shearing



(a) Opposing Forces Creating Shear



(b) Opposing Forces Creating Shear

Figure 5: Shear Stress

stress is generally distributed throughout the section such that it
is zero at the surface and greater than the average near the center.

1.2.2 Single and Double Shear

The member in Figure 5 is said to be in **single shear** because there is only a single section in shear. However, if forces were to be applied to a member as in Figure 6, there are now two sections and thus the member is said to be in **double shear**.

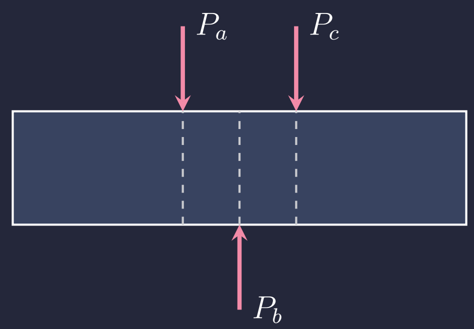


Figure 6: Double Shear

1.2.3 Bearing Stress

Often, rather than singular forces creating shearing stress, the surfaces of multiple objects will apply distributed forces to each other, thus creating **bearing shear**.

Bearing Shear	**1.2**

Caused by distributed forces creating shear, bearing stress is the average shear across a region of a member.

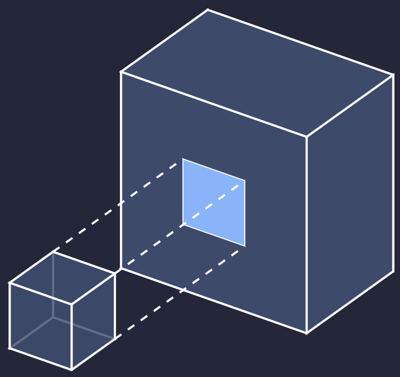


Figure 7: Bearing Shear

In Figure 7, two objects are applying reciprocal forces to each other. Throughout the area the two objects interface over, the distribution of the shear is not even. So, in practice, the average nominal value of the stress (σ_b) called the **bearing stress** is used.

This value is obtained by dividing the load P by the area of the surface of contact.

Bearing Stress (1.4)

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$