ENGR 2541 - Mechanics of Materials

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Contents

1 Types of Stress

Mechanics of materials provides a means to analyze the effects of *stresses* and *deformations*. Statics covered finding a balance of forces, including internal forces such as *shear*, *bending*, and *tension/compression*. Finding these internal forces is imperative to be able to determine the integrity of a structure. In addition to the internal forces, the integrity of a structure is also partially determined by the *dimensions* and *materials* of that structure.

1.1 Axial Stress

1.1.1 Overview

Axial Stress **1.1**

Internal stress experienced by a member due to forces applied along its axis. Specifically, stress is a force per area, where the area is the cross section normal to the axis.

In the analysis of a rod, for example, the ability for that rod to withstand the internal forces (its structural integrity) is determined by the cross-sectional area and the material of the rod.

(a) Resultant Force (b) Elementary Forces

Figure 1: Axial force is the resultant of distributed elementary forces

1.1.2 Analyzing Axial Stress

In Figure 1, the *stress* being experiences by the member is the **force per unit area**, denoted by the Greek letter sigma (σ).

Axial Stress **1.1**

Stress can be calculated by dividing the total axial force by the cross-sectional area:

$$\sigma = \frac{P}{A}$$

This formula gives the *average* axial stress over the cross section of a member. Although in reality the distribution of the stress throughout the member varies, stress can be assumed to be *uniform* throughout the cross section.

(a) Tensile Stress (b) Compressive Stress

Figure 2: Tensile vs. Compressive Force

By convention, a positive force indicates **tensile stress** while a negative force indicates **compressive stress**.

The cross section, as seen in Figure 3, is perpendicular to the axial forces. The corresponding stress in the member in described as axial/normal stress.

Thus, the formula of $\sigma = \frac{P}{A}$ gives the normal stress of an member under axial loading.

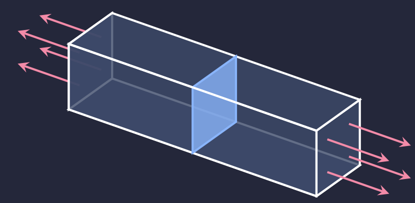


Figure 3: Normal Stress

1.1.3 Stress Points

The formula of $\sigma = \frac{P}{A}$ is only useful for *averages* or *ranges*. This can calculate the average value of the stress over the entire cross section. However, what about calculating at specific points?

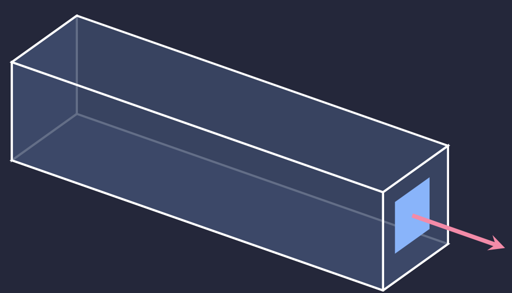


Figure 4: Stress Points

In Figure 4, to find the stress of the highlighted area, the working equation can still be applied, just now with a smaller area. To find the stress at a single point, rather than just a smaller area, stress must be calculated as the area approaches zero.

Stress at a Single Point — **1.2**

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

1.2 Shearing Stress

1.2.1 Overview

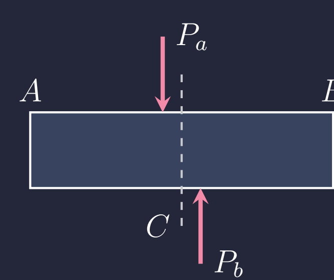
In Section 1.1, the forces creating stress acted perpendicular to the face being analyzed (see Figure 3). This created axial stress. In the case of transverse forces acting parallel to the face, shearing stress is created.

Shear Stress — **1.2**

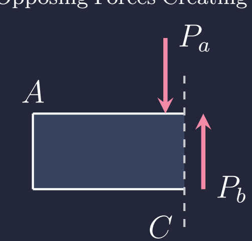
Internal stresses caused by transverse forces acting parallel to a plane.

In Figure 5, a member has two opposing forces, P_a and P_b, acting upon it. These two forces create internal shear in the section between them as seen in Figure 5b.

By analyzing the section at C in the member, it can be seen that the internal force created by the two forces in the section is equal to P_b. This resultant force is called a **shear force** (P). Dividing P by the area (A) results in the **average shearing stress** in the section.



(a) Opposing Forces Creating Shear



(b) Opposing Forces Creating Shear

Figure 5: Shear Stress

Average Shearing Stress — **1.3**

$$\tau_{avg} = \frac{P}{A}$$

The average shearing stress cannot be assumed to be uniform throughout the section. Though not the full story, the shearing stress is generally distributed throughout the section such that it is zero at the surface and greater than the average near the center.

1.2.2 Single and Double Shear

The member in Figure 5 is said to be in **single shear** because the shear stress is entirely on a single plane. If forces were to be applied to a member as in Figure 6, there are now two sections and thus the member is said to be in **double shear**.

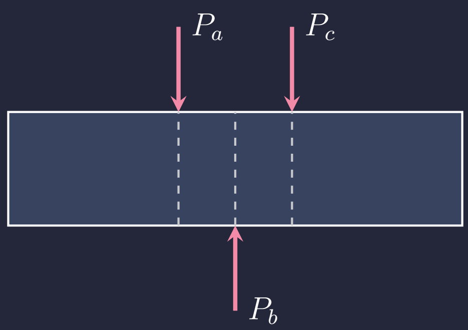


Figure 6: Double Shear

In a case of double shear, the shearing force is divided in half between the planes one between P_a and P_b, and one between P_b and P_c. Thus, to find the shear stress on either of these planes, only half the force will be used.

Double Shear **1.4**

$$\tau = \frac{P}{2A}$$

Since there are two faces of shear stress, they will add together to $\frac{P}{A}$.

1.2.3 Bearing Stress

Often, rather than singular forces creating axial stress, the surfaces of multiple objects will apply distributed forces to each other, thus creating **bearing stress**.

Bearing Stress **1.3**

Similar to axial stress, bearing stress is the result of two members interfacing with each other and applying force along an axis.

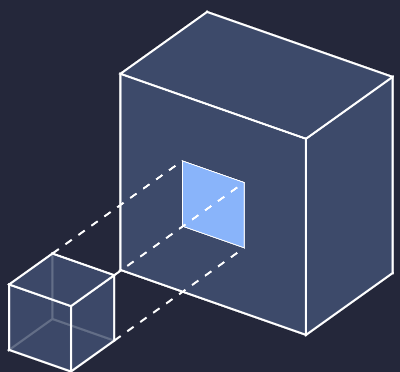


Figure 7: Bearing Shear

In Figure 7, two objects are applying reciprocal forces to each other. Throughout the area the two objects interface over, the distribution of the force is not even. So, in practice, the average nominal value of the stress (σ_b) called the **bearing stress** is used.

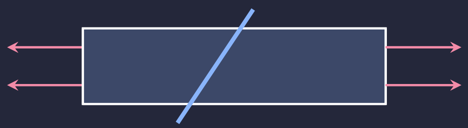
This value is obtained by dividing the load P by the area of the surface of contact.

Bearing Stress

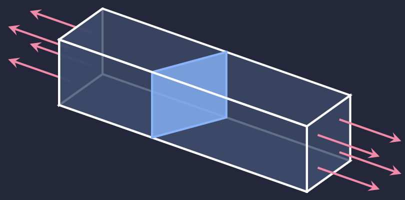
$$\sigma_b = \frac{P}{A} = \frac{P}{td} \quad (1.5)$$

1.3 Stress on an Oblique Plane

Thus far, only stresses perpendicular and parallel to planes have been considered. However, what if an object such as the one in Figure 8 were to be considered where the plane is at an angle?



(a) Top View



(b) Normal View

Figure 8: Oblique Plane

In this situation, the "axial" force applied to the member is no longer just an axial force. There is a component of the force perpendicular to the plane just as there is one parallel. This can be seen in Figure 9, where the **axial stress** to the plane is purple and the **shear stress** to the plane is yellow.

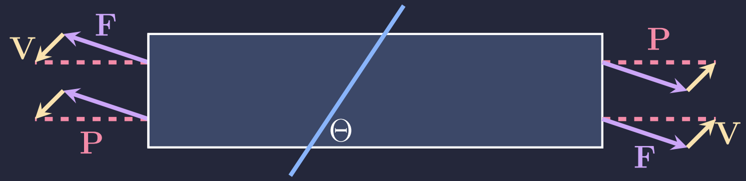


Figure 9: Components of the Applied Force

This begins to illustrate the idea that shear and axial stresses aren't very different, and largely differ based on the perspective they are analyzed from.

To calculate the stresses created by the forces exerted on this member, it's just a matter of calculating the vector components of the force and using them where appropriate.

2 Design Considerations

Determining the stresses in a body, in and of itself, serves no purpose. However, these methods of analysis can be used to inform design decisions and create sturdy structures and machines. An important family of information to an engineer is how a certain material will behave under various kinds of loads.

2.1 Determining the Strength of Materials

Properties, such as the **ultimate load**, of materials can be empirically tested by pushing that material to its limits and taking measurements. To find the ultimate load, a member is put under tensile stress until it cannot go any further.

Ultimate Load **2.1**

The largest axial force that may be applied to a material before it breaks, deforms, or begins to carry less load. Ultimate load is denoted by P_U.

Since axial stress is uniformly distributed, the ultimate load divided by the cross-sectional area of the member gives the **ultimate normal stress** of the material.

Ultimate Normal Stress

$$\sigma_U = \frac{P_U}{A} \tag{2.1}$$

Also called the *Ultimate Strength in Tension*, this measures the maximum axial stress a material can undergo before failing.

Other tests can be performed to determine the **ultimate shearing stress** of a material. The same idea as prior is applied, however this time the load is applied is shear rather than axially.

Ultimate Shearing Stress

$$\tau_U = \frac{P_U}{A} \tag{2.2}$$

Also called the *Ultimate Strength in Shear*, this measures the maximum shear stress a material can undergo before failing.

2.2 Factor of Safety

Though a material can theoretically sustain stresses up to its ultimate load, in practice this is not the case. Since loads are unpredictable and vary, the maximum load a component is designed to sustain should only be a percentage of its ultimate load. This concept is reflected in the idea of a **factor of safety**.

Factor of Safety

$$\text{Factor of Safety} = \text{F.S.} = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}} \qquad \textbf{2.3}$$

The factor of safety is calculated by finding the ratio between the ultimate load/stress and the allowable load/stress.

The relationship between the F.S. as calculated by loads vs. stresses only holds up when a linear relationship exists between the load and the stress. However, most materials experience non-linear a relationship between these two values as the material approaches its ultimate load.

The F.S. to use in a design is a subjective choice, but is based on many factors such as:

1. Variations that may occur in the properties of the member
2. The number of loadings expected during the life of the structure or machine
3. The type of loadings planned for in the design or that may occur in the future
4. The type of failure (brittle and ductile materials will fail differently)
5. Uncertainty due to methods of analysis
6. Deterioration that may occur in the future because of poor maintenance or unpreventable natural causes
7. The importance of a given member in the integrity of the entire structure

3 Deformation

3.1 Normal Strain

When studying the mechanics of materials, analyzing them entirely in static equilibrium would yield little to no information about how a material changes when ample load is applied. Consider the beam in Figure 10. After having endured an axial load, the beam deforms be elongating.

In this case, the elongation undergone by the beam is considered the **strain** endured by the beam. In other situations, strain can look different, such as the bending or compression of a material.

Strain **3.1**

Deformation experienced by an object as the result of stresses exceeding the materials ability to maintain shape.

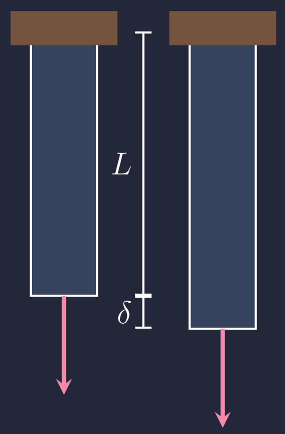


Figure 10: Deformation Due to Axial Stress

In the case of Figure 10, the strain is specifically **normal strain**.

Normal Strain **3.2**

Strain specifically relating to axial/normal stress. It is specifically defined as the *deformation per unit length*. Normal strain is denoted by the Greek letter epsilon (ϵ).

Since normal strain is the deformation per unit length, to find the value of ϵ, one must divide the deformation by the *original* length of the member.

Normal Strain **3.1**

$$\epsilon = \frac{\delta}{L}$$

Though indirectly caused by an axial load, the factors in determining the amount of normal strain experienced by a material are: the materials inherent properties and the axial stress on the member.

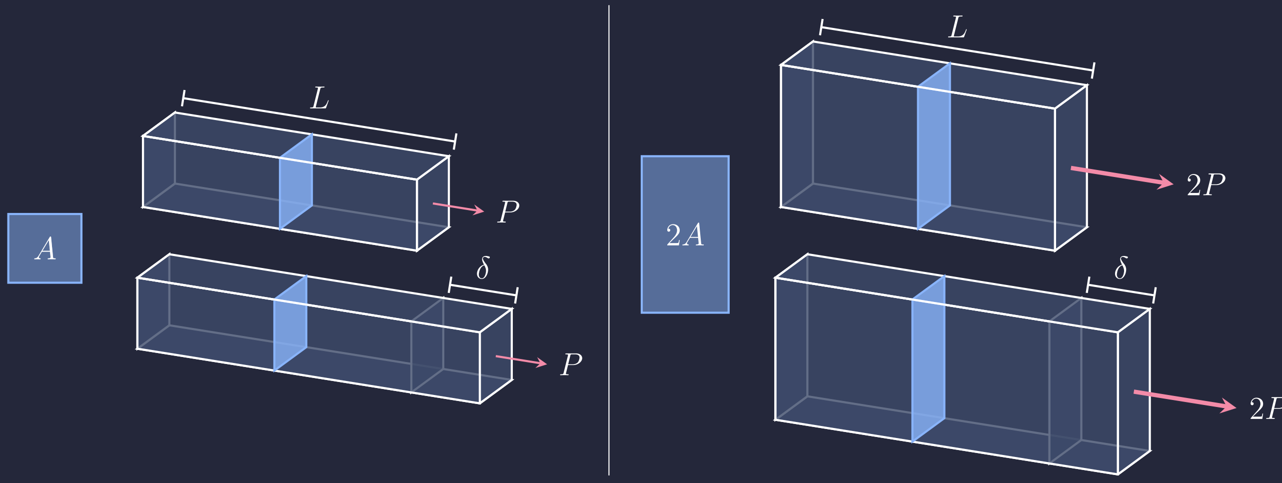


Figure 11: Strain For Two Rods of Different Cross Sections

Assuming the same material, two members of different dimensions, but undergoing the same stress, will strain the same amount. In Figure 11, this exact situation is visualized. Since stress is just a ratio of force to area, a member double the area experiencing an axial load double of another member will strain the same.

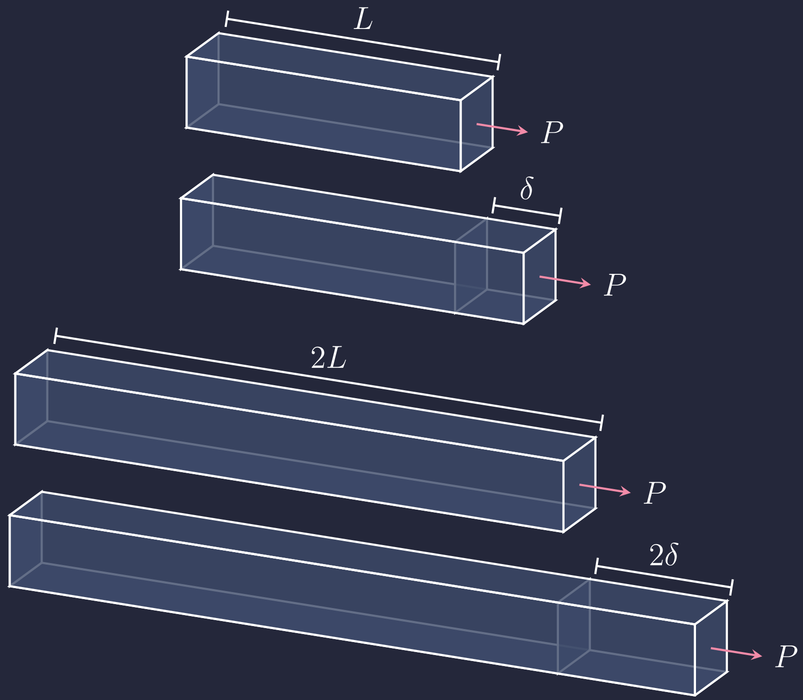


Figure 12: Strain For Two Rods of Different Lengths

Similarly, since strain is a ratio between the original length and the final length, a body double the length of another will experience the same strain assuming all else is the same. See Figure 12.

3.2 Stress Strain Diagrams

Having seen that strain is independent of the dimensions of a material, and only depends on the stress, it follows that plotting strain as a function of stress would result in a diagram generally applicable to a material. This curve, called a **stress-strain diagram** characterizes the properties of a material.

Though all materials behave differently when examined through a stress-strain diagram, there are two broad categories of materials: *brittle* and *ductile*.

Ductile materials are able to yield without necessarily failing entirely. Their elongation initially increases linearly with stress until a some value σ_Y where suddenly undergoes a large deformation with a relatively small increase in stress.

Measuring Ductility

$$\text{Percent Elongation} = 100 \cdot \frac{\text{Length at Failure} - \text{Initial Length}}{\text{Initial Length}} = 100 \cdot \frac{L_B - L_0}{L_0}$$

$$\text{Percent Reduction in Area} = 100 \cdot \frac{\text{Initial Area} - \text{Area at Failure}}{\text{Initial Area}} = 100 \cdot \frac{A_0 - A_B}{A_0} \tag{3.2}$$

Brittle materials, on the other hand, experience a very small amount of yield, after which they tend to fail suddenly. There is a distinct lack of necking in brittle materials.

3.3 True vs. Engineering Stress and Strain

The difference between these two areas is the same difference between **True Stress** and **Engineering Stress**.

True vs. Engineering Stress **3.3**

The stress experienced by an object calculated using the current real cross-section of the body.

$$\sigma_t = \frac{P}{A}$$