mind project report

Danielle Breegle

May 2025

1 Mathematical System Summary

Let $\mathcal{F}(x, y, t)$ be the total field composed of multiple morphogen gradients and dynamic sources:

$$\mathcal{F}(x, y, t) = F_{\text{source}} + F_{\text{pulse}}(t) + F_{\text{asym}} + F_{\text{AP}} + F_{\text{DV}}$$

The local curvature is defined as:

$$\kappa(x, y, t) = \|\nabla \mathcal{F}(x, y, t)\| = \sqrt{\left(\frac{\partial \mathcal{F}}{\partial x}\right)^2 + \left(\frac{\partial \mathcal{F}}{\partial y}\right)^2}$$

The attractor signal Φ is modulated by curvature and field strength:

$$\Phi(x, y, t) = \mathcal{F}(x, y, t) \cdot \kappa(x, y, t)$$

Agents $N_i(t)$ are seeded at locations where $\Phi > \theta_{\text{feedback}}$.
Each agent evolves via:

$$\begin{aligned} E_i(t+1) &= \max\left(0, E_i(t) + \gamma \cdot \mathcal{F}_{\text{local}} - \delta\right) \\ \varepsilon_i(t+1) &= \varepsilon_i(t) + \mathcal{F}_{\text{local}} \cdot \kappa_i \\ \mathbf{p}_i(t+1) &= \mathbf{p}_i(t) + \eta \cdot \vec{v}_{\text{migrate}}(i) \end{aligned}$$

Role assignment is defined by percentile thresholds:

$$r_i(t) = f(E_i, \kappa_i, \text{distance to lumen}, \text{connectivity})$$

An agent divides if:

$$E_i > \theta_E \quad \text{and} \quad \varepsilon_i > \theta_\varepsilon$$

New agents are offset stochastically:

$$\mathbf{p}_{i1,i2} = \mathbf{p}_i \pm \vec{\xi}, \quad \vec{\xi} \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$$

Population limits are enforced per role:

$$|N_{\text{neurons}}| \leq K_{\text{neurons}}, \quad |N_{\text{glia}}| \leq K_{\text{glia}}, \ldots$$

The full system evolves as:

$$\mathcal{M}(t+1) = \mathcal{G}(\mathcal{M}(t), \mathcal{F}(t), \kappa(t), \Phi(t))$$

Where $\mathcal{M}$ is the state of all agents, and $\mathcal{G}$ encapsulates update, division, and selection logic.