

Digital Design and Computer Architecture (DDCA)

Managing Complexity:

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Abstraction:

Level	Example
Application Software	Programs
Operating Systems	Device Drivers
Architecture	Instructions, Registers
Microarchitecture	Datapath, Controllers
Logic	Address, Memories
Digital Circuits	Gates
Analog Circuits	Amplifiers
Devices	Transistors, Diodes
Physics	Electrons
→ Hiding details when they are not important	

Discipline:
→ Intentionally restricting design choices for greater productivity at higher abstraction level.

Three Y's:

- Hierarchy
 - A system is divided into modules of smaller complexity
- Modularity
 - Having well defined functions and interfaces
- Regularity
 - Encouraging uniformity, so modules can be easily reused

Circuit Design Goals:

- Optimize...
- Area (proportional to cost)
- Speed/Throughput
- Power/Energy
- Design Time

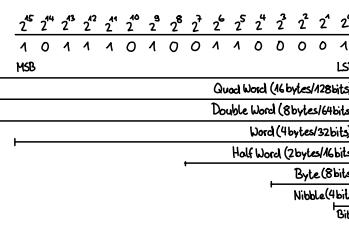
Principles:

- Lazy, be creative
- Why?, take nothing for granted
- Engineering is not religion, use what works best for you.
- KISS (Keep it simple and stupid), manage complexity.

Moore's Law:

Gordon Moore (1965): Number of transistors that can be manufactured doubles roughly every 18 months.

Binary Numbers



Decimal	Binary	Hexadecimal	n	2^n	IEC	x Si
0	0000	0	0	1		
1	0001	1	1	2		
2	0010	2	2	4		
3	0011	3	3	8		
4	0100	4	4	16		
5	0101	5	5	32		
6	0110	6	6	64		
7	0111	7	7	128		
8	1000	8	8	256		
9	1001	9	9	512		
10	1010	A	10	1024	Ki (kibi)	10^3 k (kilo)
11	1011	B	11	2048		
12	1100	C	12	4096		
13	1101	D	13	8192		
14	1110	E	14	16384		
15	1111	F	15	32768		
	16	100000	16	65536		
	20	Mi (mibi)	20	10^6	M (mega)	
	30	Gi (gibi)	30	10^9	G (giga)	
	40	Ti (tebi)	40	10^12	T (tera)	
	50	Pi (petbi)	50	10^15	P (peta)	

Sign/Magnitude:

-x = 2^n - x

Range for Nbit number: $-2^{N-1} \dots 2^{N-1} - 1$

Two zeroes, addition doesn't work.

-x = x̄

Range for Nbit number: $-2^{N-1} \dots 2^{N-1} - 1$

Two zeroes, addition by end-around carry.

One's complement:

-x = x̄

Range for Nbit number: $-2^{N-1} \dots 2^{N-1} - 1$

Two zeroes, addition by end-around carry.

Two's complement:

-x = x̄ + 1, ignoring overflow

Range for Nbit number: $-2^{N-1} \dots 2^{N-1} - 1$

One zero, addition like unsigned addition

IEEE-754 Floating Point:

S E F

sign exponent fraction

(-1)^S \cdot 1.F \cdot 2^E-bias
Mantissa

Name	Size (bits)	Mantissa (bits)	Exponent (bits)	Exponent Bias	Max	Min (pos)
Half precision	16	10.1	5	15	$6.55 \cdot 10^4$	$6.4 \cdot 10^{-6}$
Single precision	32	23.1	8	127	$3.4 \cdot 10^{38}$	$1.48 \cdot 10^{-38}$
Double precision	64	52.1	11	1023	$1.8 \cdot 10^{308}$	$2.23 \cdot 10^{-308}$

Special Values

- ±0: S=0, E=0, M=0
- ±Inf: S=0/1, E=1..1, M=0
- NaN: E=1..1, M≠0

Fixed Point:

- m integer bits, k fraction bits

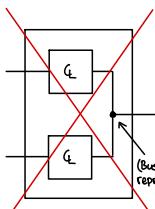
$$b_{m-1} \dots b_0.b_{k-1} \dots b_0$$

Addition:

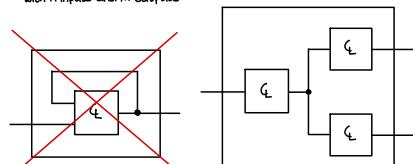
- Extract exponent and fraction bits
- Prepend leading 1 to form mantissa
- Compose exponent
- Shift smaller mantissa if necessary
- Add mantissas
- Normalize mantissa and adjust exponent if necessary
- Round result
- Assemble back into floating point format

Combinational Circuits

- A circuit is combinational if it consists of interconnected circuit elements such that:
- Every circuit element is itself combinational.
 - Every node of the circuit is either designated as an input to the circuit or connects to exactly one output terminal of a circuit element.
 - The circuit contains no cyclic paths: every path through the circuit visits each circuit node at most once



Arbitrary combinational logic circuit with n inputs and m outputs



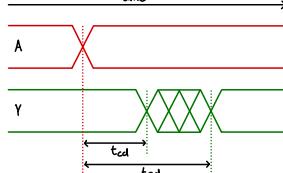
Each output y_j for $j \in \{0, \dots, m-1\}$ is a function f_j of the inputs a_i for $i \in \{0, \dots, n-1\}$:

$$y_j = f_j(a_0, \dots, a_{n-1})$$

Each output can also be represented as a truth table.

Timing

A → Y
t_{cd}: contamination delay
t_{pd}: propagation delay



Truth Table

A	B	C	Y
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	1

A truth table (n inputs) has 1 output and 2^n rows.

Terminology

Name	Description	Examples
Literal	A variable or its complement	A, B̄
Product/Implicant	And composition of literals	A, B̄A, ĀB̄ĀC
Minterm	Product that contains all of the inputs	A, B, C, ĀB, ĀC, B̄C, ĀB̄C, ĀB̄C̄
Sum	Or composition of literals	Bv, Ā, ĀB̄v
Maxterm	Sum that contains all of the inputs	ĀBvB̄C, ĀB̄vB̄C̄

Sum-of-Products Canonical Form

Product-of-Sums Canonical Form

The SOP canonical form is the sum of all minterms where the output Y is 1.

The POS canonical form is the product of all maxterms where the output Y is 0.

$$Y = (\bar{A}\bar{B}) \vee (\bar{A}B) \vee (A\bar{B}) \vee (AB)$$

Sigma notation: $Y = F(A, B) = \sum(m_0, m_1, m_2, m_3)$

$$Y = (\bar{A}B) \wedge (\bar{A}B) \wedge (A\bar{B}) \wedge (AB)$$

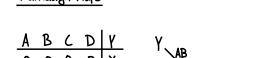
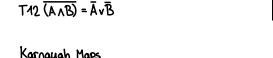
Pi notation: $Y = F(A, B) = \prod(M_0, M_1, M_2, M_3)$

Boolean Algebra

Axiom	Dual	Name
A1 A + 0 = A = 0	A1' A + 0 = A = 1	Binary Field
A2 0 + 1 = 1	A2' 1 + 0 = 0	Not
A3 0 · 0 = 0	A3' 1 · 1 = 1	And/Or
A4 1 · 1 = 1	A4' 0 · 0 = 0	And/Or
A5 0 · 1 = 1 · 0 = 0	A5' 0 · 1 = 1 · 0 = 1	And/Or

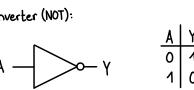
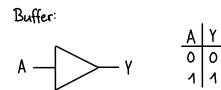
Theorem	Dual	Name
T1 A · A = A	T1' A + A = 1	Identity
T2 A · 0 = 0	T2' A + 1 = 1	Null element
T3 A · A = A	T3' A + A = A	Idempotency
T4 Ā · A = 0	T4' A + Ā = 1	Involutive
T5 A · Ā = 0	T5' A + Ā = 1	Complements

Theorem (multi variable)	Dual	Name
T6' A v B = B v A	T6 A · B = B · A	Commutativity
T7' (A v B) v C = A v (B v C)	T7 (A · B) · C = A · (B · C)	Associativity
T8' (A v B) · (A v C) = A v (B · C)	T8' (A · B) v (A · C) = A · (B v C)	Distributivity
T9' A · A = A	T9' A v A = A	Covering
T10' (A · B) v (A · B̄) = A	T10' (A v B) · (A v B̄) = A	Combining
T11' (A · B) v (A · C) v (B · C) = (A · B) v (B · C)	T11' (A v B) · (A v C) · (B v C) = (A v B) · (B v C)	Consensus
T12' (A · B̄) = A · B̄	T12' (A v B̄) = A v B̄	de Morgan



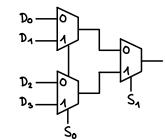
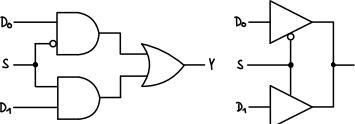
- Use the fewest circles necessary to cover all the 1/0's.
- All squares in each circle must only contain 1/0's or X's.
- Each circle must span a rectangular block with side lengths that are a power of two.
- Each circle should be as large as possible.
- A circle may wrap around the edges of a K-map.
- A 1/0 in a K-map may be circled multiple times if doing so allows fewer circles to be used.
- Circles of 1s are products and circles of 0s are sums.

Logic Gates



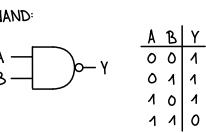
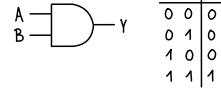
Multiplexer (MUX):

S	D ₀	D ₁	Y
0	0	0	0
0	0	1	1
0	1	0	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

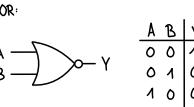
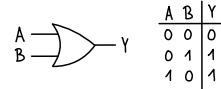


In general, wider ($2^n : 1$) multiplexers can be achieved by two level logic, an array of tri-state buffers with a ($n : 2^n$) decoder, or in a hierarchical design.

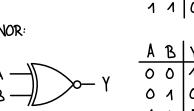
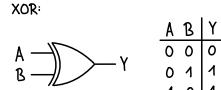
AND:



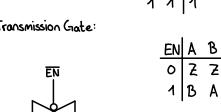
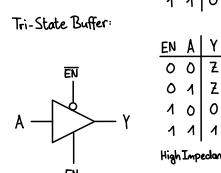
OR:



XOR:



Tri-State Buffer:



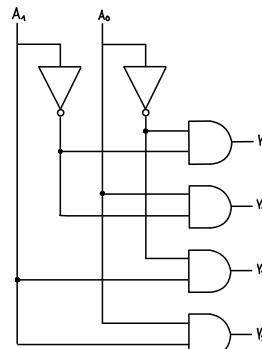
Combinational Building Blocks

Decoder (DEC):

A ₁	A ₀	Y ₃	Y ₂	Y ₁	Y ₀
0	0	0	0	1	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

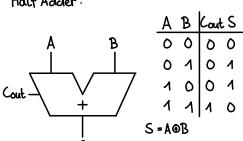
Decoder 10-2: 00, 01, 10, 11

The outputs of a decoder are also the corresponding minterms.

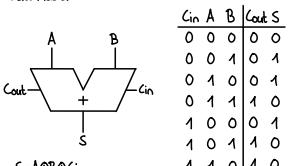


Arithmetic

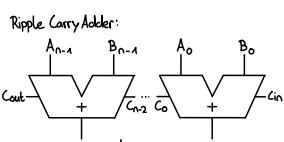
Half Adder:



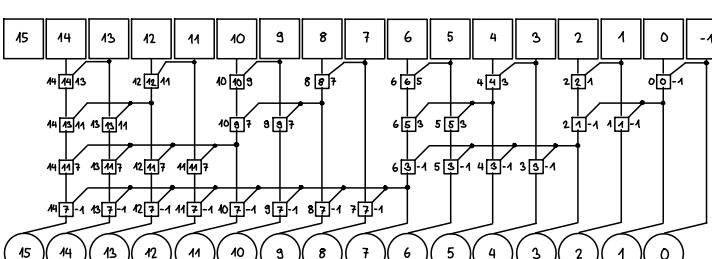
Full Adder:



Carry Propagation Adders (CPA):



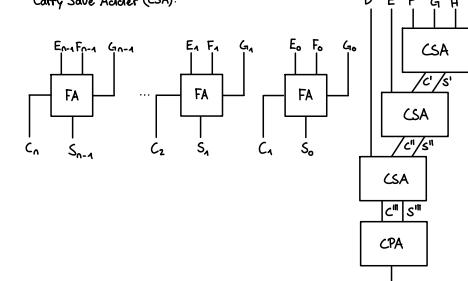
(Parallel) Prefix Adder (PPA):



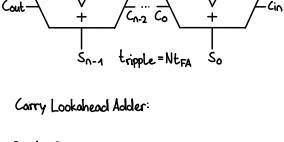
$$G_{i-1} = Cin \\ P_{i-1} = 0$$

$$t_{PPA} = t_{PGT} + \log_2(N) t_{PG} \text{Prefix} + t_{XOR}$$

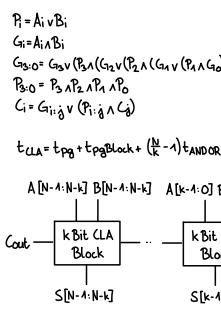
Carry Save Adder (CSA):



Ripple Carry Adder:

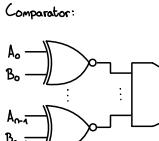


Carry Lookahead Adder:

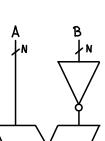


For $N > 16$ a CLA is generally faster, but still scales linearly with respect to propagation delay.

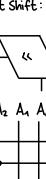
Comparator:



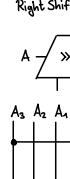
Subtractor:



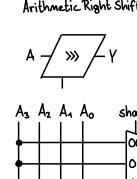
Left Shift:



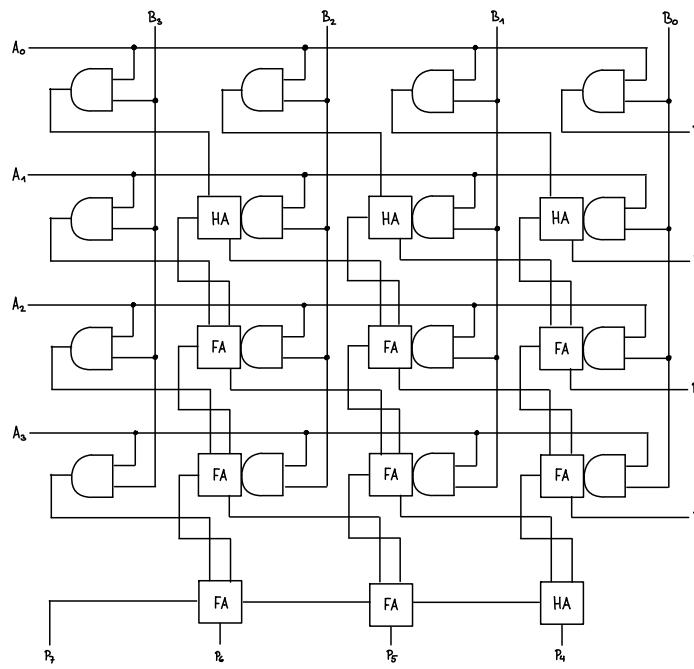
Right Shift:



Arithmetic Right Shift:



Parallel Multiplier:



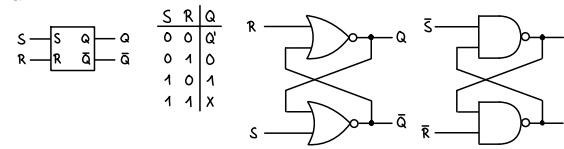
$$\begin{array}{r}
 1 & 0 & 1 & 0 \\
 \times & 0 & 1 & 0 & 1 \\
 \hline
 1 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 \\
 1 & 0 & 1 & 0 & \text{Partial} \\
 + & 0 & 0 & 0 & 0 & \text{Products} \\
 \hline
 0 & 1 & 1 & 0 & 0 & 1 & 0
 \end{array}$$

Synchronous Sequential Logic

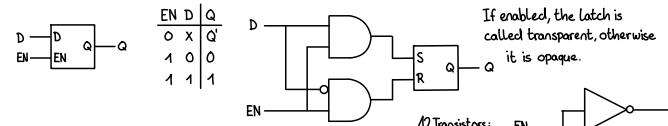
- Synchronous sequential logic circuits consist of interconnected circuit elements such that:
- Every circuit element is either a register or a combinational circuit.
 - At least one circuit element is a register.
 - All registers receive the same clock signal.
 - Every cyclic path contains at least one register.

Sequential Building Blocks

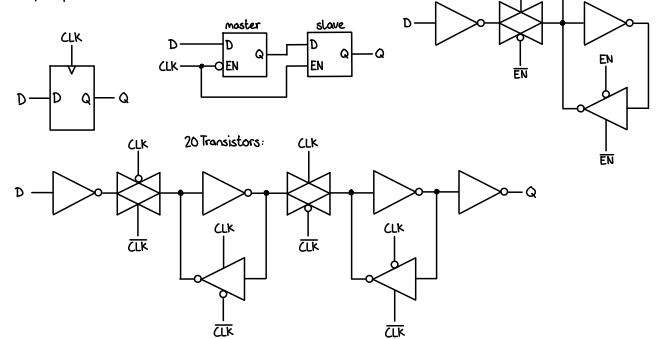
SR Latch:



D Latch:



D Flip-Flop:

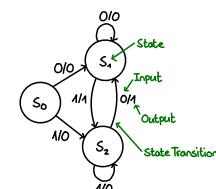
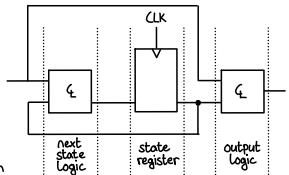


A register is an array of D Flip-Flops that all have the same clock signal.

Finite State Machines

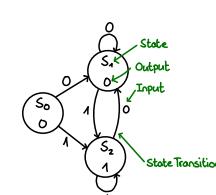
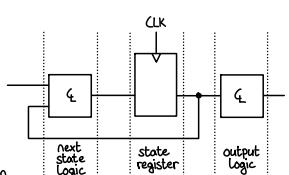
Mealy Machine:

- $(S, S_0, \Sigma, \Lambda, T, G)$
- S : Finite set of states
 - $S_0 \in S$: Initial State
 - Σ : Input Alphabet
 - Λ : Output Alphabet
 - $T: S \times \Sigma \rightarrow S$: Transition Function
 - $G: S \times \Sigma \rightarrow \Lambda$: Output Function

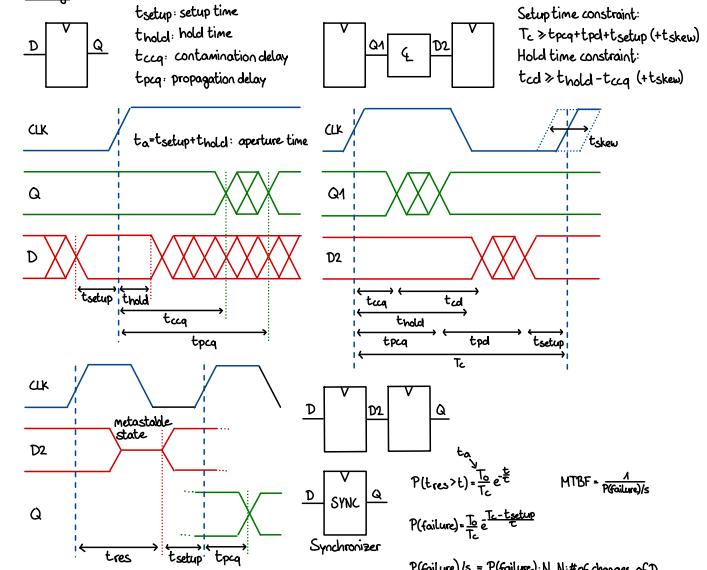


Moore Machine:

- $(S, S_0, \Sigma, \Lambda, T, G)$
- S : Finite set of states
 - $S_0 \in S$: Initial State
 - Σ : Input Alphabet
 - Λ : Output Alphabet
 - $T: S \times \Sigma \rightarrow S$: Transition Function
 - $G: S \rightarrow \Lambda$: Output Function

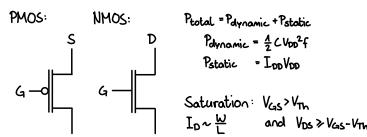


Timing:

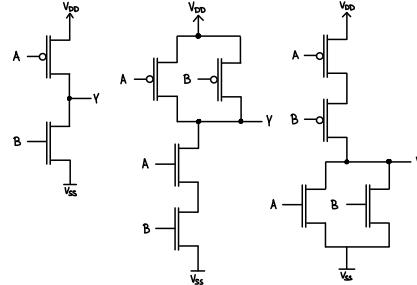


Transistors

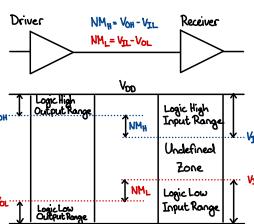
MOSFETs (Metal Oxide Semiconductor Field Effect Transistor):



NOT:

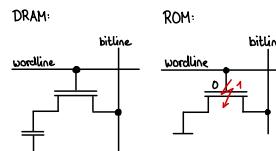
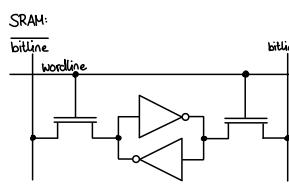
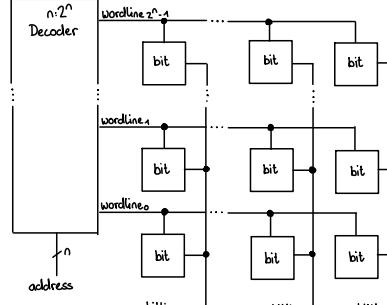


Logic Level Noise Margins

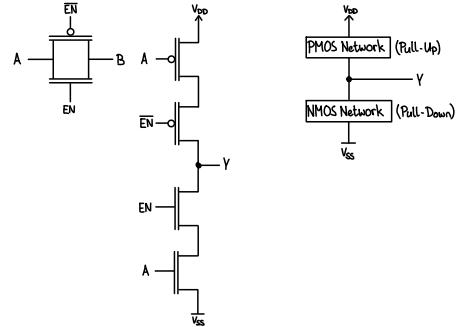


Memories

- Flip-Flops / Latches (Registers)**
 - Very fast, parallel access
 - Expensive (2+ transistors per bit)
- SRAM (Static RAM)**
 - relatively fast, only one data word at a time
 - less expensive (6 transistors per bit)
- DRAM (Dynamic RAM)**
 - slower, reading destroys content (refresh), one data word at a time
 - cheaper (4 transistors per bit)
- Other (HDD, Flash, ...)**
 - much slower
 - no transistors involved, non-volatile



Transmission Gate: Tri-State Buffer (inverted): General:



MIPS

Instruction Formats:

R-Type:

opcode	rs	rt	rd	shamt	funct
31 ... (6bits)...	26 25 ... (5bits)...	21 20 ... (5bits)...	16 15 ... (5bits)...	11 10 ... (5bits)...	6 5 ... (6bits)...

I-Type:

opcode	rs	rt	imm
31 ... (6bits)...	26 25 ... (5bits)...	21 20 ... (5bits)...	16 15 ... (16 bits)...

J-Type:

opcode	addr
31 ... (6bits)...	26 25 ... (26bits)...

Instructions:

Mnemonic	Name	Type	Operation	Op	Ums	Fmt	Notes
add	Add	R	$rd = rs + rt$	✓	0	20	
addu	Add Unsigned	R	$rd = rs + rt$	0	21		May generate overflow exception.
addi	Add Imm	I	$rt = rs + \text{SignExtImm}$	✓	8		Operands are considered unsigned.
addiu	Add Imm Unsigned	I	$rt = rs + \text{SignExtImm}$	-	9		
sub	Subtract	R	$rd = rs - rt$	✓	0	22	
subu	Subtract Unsigned	R	$rd = rs - rt$	0	23		
and	And	R	$rd = rs \& rt$	0	24		
andi	And Imm	I	$rt = rs \& \text{ZeroExtImm}$	C	-		
or	Or	R	$rd = rs \vee rt$	0	25		
ori	Or Imm	I	$rt = rs \vee \text{ZeroExtImm}$	D	-		
nor	Nor	R	$rd = \sim(rs \vee rt)$	0	27		
sll	Shift Left Logic	R	$rd = rt \ll \text{shamt}$	0	0		
srl	Shift Right Logic	R	$rd = rt \gg \text{shamt}$	0	2		
sra	Shift Right Arithmetic	R	$rd = rt \ggg \text{shamt}$	0	3		
slt	Set Less Than	R	$rd = rs < rt ? 1 : 0$	0	2A		
slti	Set Less Than Imm	I	$rt = rs < \text{SignExtImm} ? 1 : 0$	A	-		
sltu	Set Less Than Unsigned	R	$rd = rs < rt : PC = 4 + \text{BranchAddr}$	✓	0	2B	
sltiu	Set Less Than Imm Unsigned	I	$rt = rs < \text{SignExtImm} ? 1 : 0$	B	-		
beq	Branch on eq	I	$\text{if } rs == rt : PC = 4 + \text{BranchAddr}$	4	-		
bne	Branch on not eq	I	$\text{if } rs \neq rt : PC = 4 + \text{BranchAddr}$	5	-		
jr	Jump	J	$PC = \text{JumpAddr}$	2	-		
jal	Jump and Link	J	$PC = \text{JumpAddr}$	3	-		
jr	Jump Register	R	$PC = rs$	0	8		
lw	Load Word	I	$rt = M[rs + \text{SignExtImm}]$	23	-		
lhu	Load Half Unsigned	I	$rt = M[rs + \text{SignExtImm}] \& 15:0$	25	-		
lbu	Load Byte Unsigned	I	$rt = M[rs + \text{SignExtImm}] \& 1:0$	24	-		
lui	Load Upper Imm	I	$rt = \$imm, 16:0$	F	-		
sw	Store Word	I	$M[rs + \text{SignExtImm}] = rt$	28	-		
sh	Store Half	I	$M[rs + \text{SignExtImm}] \& 15:0] = rt[15:0]$	29	-		
sb	Store Byte	I	$M[rs + \text{SignExtImm}] \& 1:0] = rt[1:0]$	28	-		
mfhi	Move from Hi	R	$rd = hi$	0	40		
mflo	Move from Lo	R	$rd = lo$	0	42		
mult	Multiply	R	$hi, lo_3 = rs * rt$	0	18		
multu	Multiply Unsigned	R	$hi, lo_3 = rs * rt$	✓	0	43	

Assembly

.data staticVar: .word 0 // 4 bytes

.text .global main

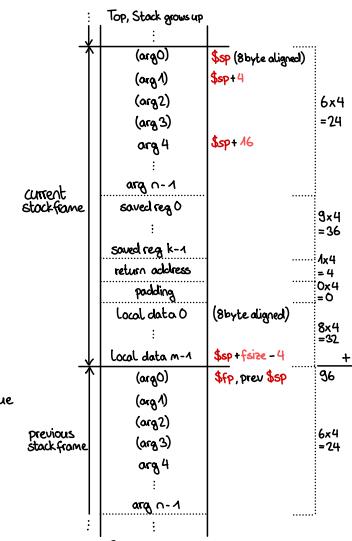
```
main:
    addi $s0 $t0 0 // 1st arg
    addi $s1 $t1 0 // 2nd arg
    addi $s2 $t2 0 // 3rd arg
    addi $s3 $t3 0 // 4th arg
    addi $sp $sp -24 // increase stack for args
    sw $t4 16($sp) // 5th arg
    sw $t5 20($sp) // 6th arg
    jal func // call function
    addi $sp $sp 24 // decrease stack
    addi $s6 $v0 0 // return value
end:
```

j end // halt loop

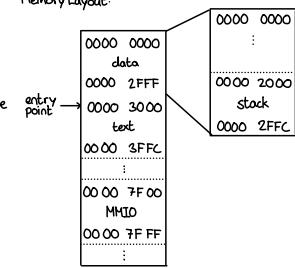
```
func:
    addi $sp $sp -36 // push stack frame // prologue
    sw $ra 60($sp) // save registers
    sw $s0 24($sp)
    :
    sw $s7 52($sp)
    sw $fp 56($sp)
    addi $fp $sp 36 // set frame ptr
    addi $s0 $fp 0 // 1st arg
    addi $s1 $fp 0 // 2nd arg
    addi $s2 $fp 0 // 3rd arg
    addi $s3 $fp 0 // 4th arg
    lw $t4 4($fp) // 5th arg
    lw $t5 8($fp) // 6th arg
    // body
    addi $s0 $t6 0 // return value
    lw $fp 56($sp) // restore registers
    lw $s7 52($sp)
```

```
// body
    addi $s0 $t6 0 // return value
    lw $fp 56($sp) // restore registers
    lw $s7 52($sp)
    lw $s0 24($sp)
    lw $sra 60($sp)
    addi $sp $sp 36 // pop stack frame
    jr $sra // return to caller
```

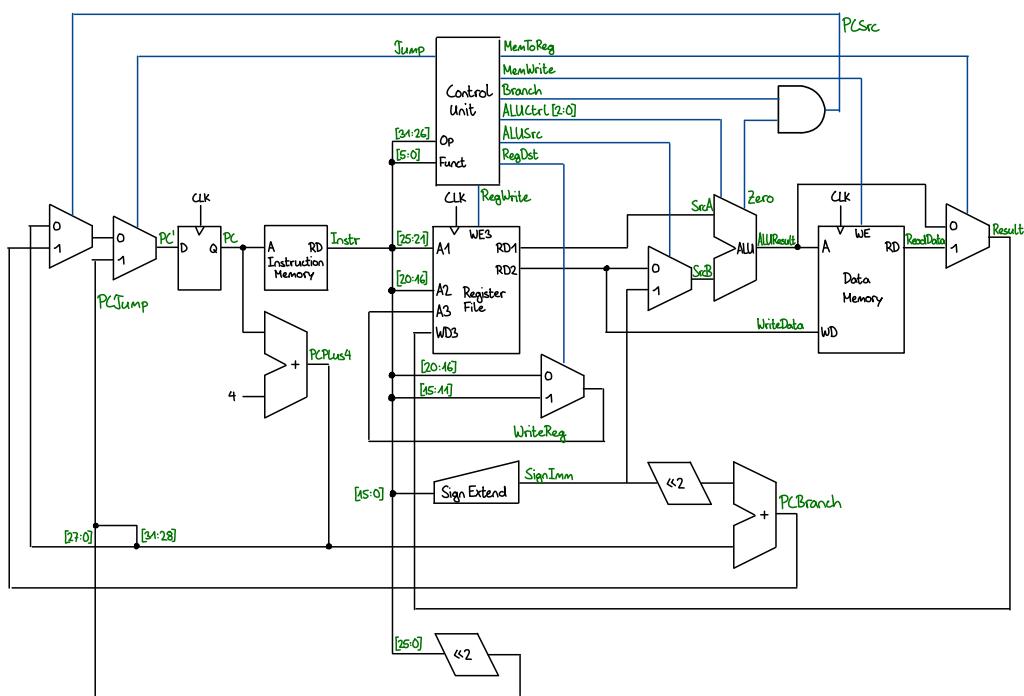
Stack frame:



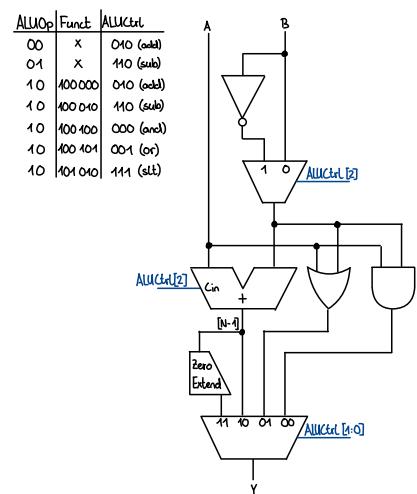
Memory Layout:



Single Cycle Microarchitecture

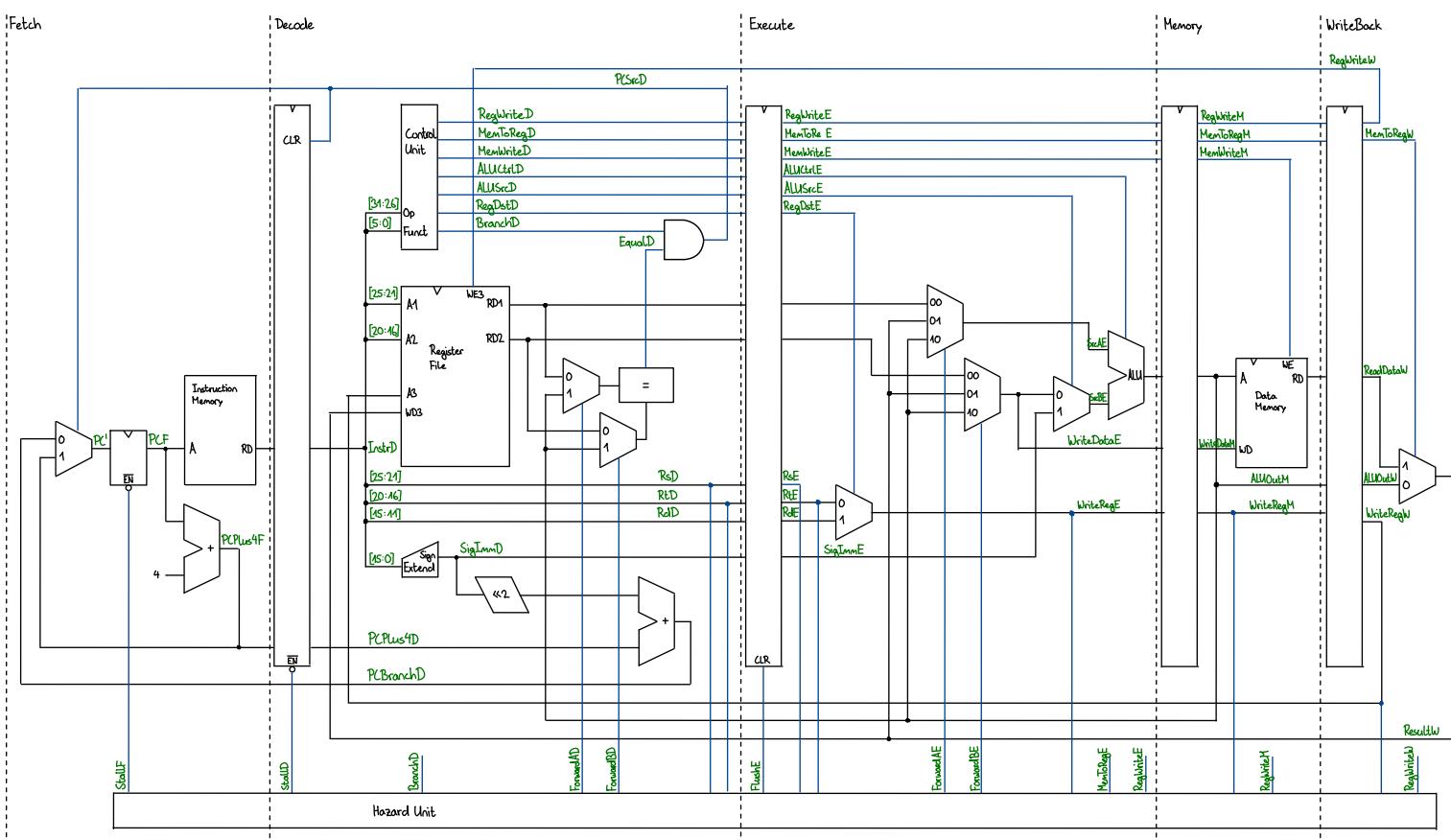


Instruction	Op	RegWrite	RegDst	ALUSrc	Branch	MemWrite	MemToReg	ALUOp	Jump
R-Type	000000	1	1	0	0	0	0	10	0
lw	100011	1	0	1	0	0	1	00	0
sw	101011	0	X	1	0	1	X	00	0
beq	000100	0	X	0	1	0	0	01	0
addi	001000	1	0	1	0	0	0	00	0
j	000010	0	X	X	X	0	X	XX	1



Critical Path:
 $T_c = t_{pcq_PC} + t_{mem} + \max(t_{RF_read}, t_{ext}, t_{mux}) + t_{ALU} + t_{mem} + 2t_{mux} + t_{RF_setup}$
 Time to execute N Instructions = $N \cdot \frac{CPT}{1 - \frac{1}{f_c}}$

Pipelined Architecture



$T_c > \max($

$t_{pcq} + t_{mem} + t_{setup},$
 $2(t_{RF_read} + t_{mem} + t_{ext} + t_{RF_mux} + t_{setup}),$
 $t_{pcq} + t_{mem} + t_{ALU} + t_{setup},$
 $t_{pcq} + t_{memwrite} + t_{setup},$
 $2(t_{pcq} + t_{mem} + t_{RF_write})$

fetch
decode
execute
memory
writeback

//Forwarding Logic:

```
assign ForwardAD = (RD1==RLE) & (RD2==RLE) & RegWriteM
assign ForwardBD = (RD1==RLE) & (RD2==RLE) & RegWriteM & RegWriteE
```

//Stalling Logic:

```
assign ldstall = ((RD == RLE) | (RLE == RD)) & MemToRegE
assign branchstall = (BranchD & RegWriteE & (WriteRegE == RD) | WriteRegE == RLD) |
    (BranchD & MemToRegM & (WriteRegM == RD) | WriteRegM == RLD)
```

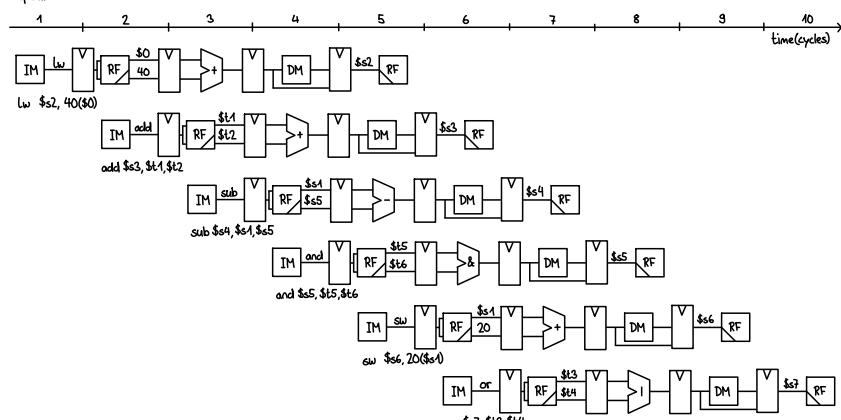
//Stall signals:

```
assign StallF, StallD, FlushE = ldstall | branchstall
```

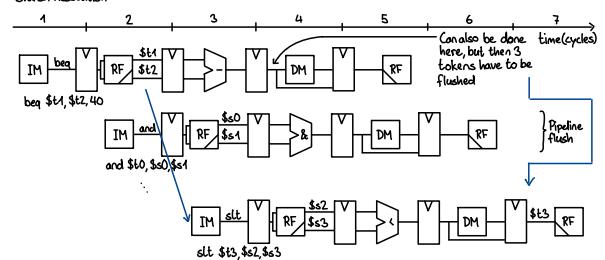
Hazards

- Data Hazard: Data for next instruction is not ready
- Control Hazard: Next Instruction is unknown

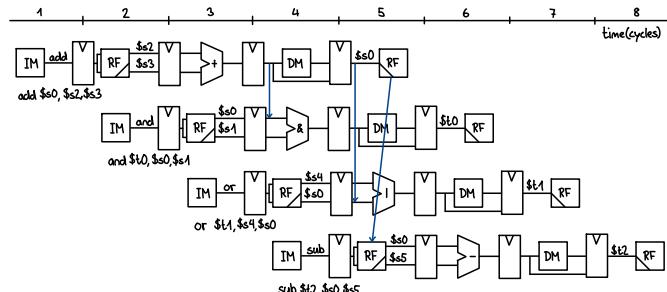
Pipeline:



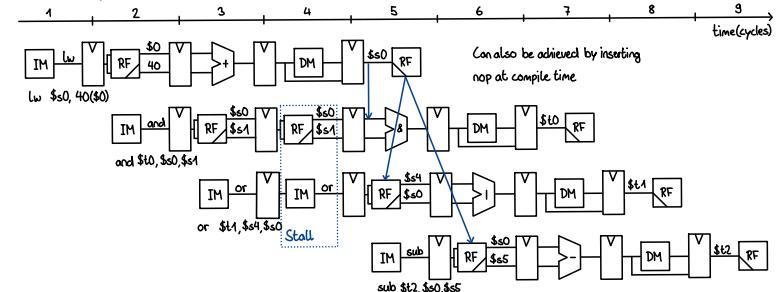
Branch Resolution:



Data Forwarding:



Pipeline Stalling:



Verilog:

Comments:
// One-liner
// Multiple
lines //

Numeric Constants:
8'b_0110_1010 // binary
8'd152 // octal
8'd106 // decimal
8'h6A // hexadecimal
"j" // ASCII
7'b2 // High Impedance

Nets and Variables:
wire [3:0] w; // assign outside always
reg [3:0] r; // assign inside always
reg [2:0] men [3:0];

Parameters:
parameter N=8;
localparam S=2'd3;

Assignments:
assign Output = A*B;
assign Zc,D3 = 2'd15&[1:2],C[1:2],E[3];

Operators:
A[D:1][1:D] // Select
&A, ~&A, IA, ~IA, IA, ^A // Reduction
!A, ~A // Complement
+A, -A // Unary
{A, ..., B} // Concatenate
ENLA3 // Replicate
A*B, A/B, A%B, A+B, A-B // Arithmetic
A<B, A>B, A>>B // Shift
A>B, A>=B, A<=B, A=B // Relation
A&B, A|B, A^B, A~B // Bitwise
A&B, A|B // Logical
A?B:C // Conditional

Module:

```
module MyModule
#(parameter N=8) // optional parameter
(input rst, clk,
output [N-1:0] out);
  //impl
endmodule
```

Case:

```
always @(*) begin
  case (num)
    2'b00: A = 8'd3; //blocking
    2'b01,
    2'b10: A = 8'd10;
    2'b11: A = 8'd2;
  endcase
end
```

always @(*):

```
begin
  casex (dec)
    4'b0xxx: enc = 2'b00;
    4'b00xx: enc = 2'b01;
    4'b001x: enc = 2'b10;
    4'b0001: enc = 2'b11;
    default: enc = 2'b00;
  endcase
end
```

Generate:

```
genvar j;
wire [1:0]out [3:0];
generate
  for(j=0; j<20; j=j+1)
    begin: Gen_Modules
      MyModule #(N(1)) mod(
        .rst(RST), .clk(CLK),
        .out(out[j]));
    end
  endgenerate
end
```

Synchronous:

```
always @ (posedge clk) begin
  if(rst) B <= 0; // (non-blocking)
  else B <= B + 1'b1;
end
```

Loop:

```
always @ (n) begin
  count = 0;
  for(j=0; j<8; j=j+1)
    count = count + input[i];
end
```

Function:

```
function [2:0]F;
  input [3:0]A;
  input [3:0]B;
  begin
    F = {A+1'b1,B+2'd2};
  end
endfunction
```

State Machine:

```
reg [3:0]state;
parameter S0 = 2'b00,
S1 = 2'b01,
S2 = 2'b10,
S3 = 2'b11;
```

always @ (posedge clk):

```
if (rst) state = S0;
else case (state)
  S0: state = S1;
  S1: state = S2;
  S2: state = S3;
  S3: state = S0;
endcase
```

end

Testbenches:

```
'timescale 1ns/1ps //unit time>/resolution
initial
always
#10 // delay by 10ns
$display ("%b bin,%h hex,%d dec,%o oct,%c ASCII,%s str,%t time",
...,$time)
$readmemh("filename", testbuf)
$finish
```

```
module d_latch(input d,en,rstn,
                  output reg q);
  always @ (en or rstn or d)
    if (rstn) q <= 0;
    else if (en) q <= d;
  endmodule
```

```
module d_flipflop(input d,rstn,clk,
                   output reg q);
  always @ (posedge clk or negedge rstn)
    if (rstn) q <= 0;
    else q=d;
  endmodule
```

Functional Verification:

