Problem 3.1

a) $f(n) = 9n$ and $g(n) = 5n^3$

- $$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{9n}{5n^3} = \lim_{n \to \infty} \frac{9}{\underbrace{5n^2}_{\to \infty}} = 0$$

It belongs to $f \in o(g)$ and $f \in O(g)$
It doesn't belong to $f \in \omega(g)$, $f \in \Omega(g)$ and $f \in \Theta(g)$

- $$\lim_{n \to \infty} \frac{g(n)}{f(n)} = \lim_{n \to \infty} \frac{5n^3}{9n} = \lim_{n \to \infty} \frac{5n^2}{9} = \infty$$

It belongs to $g \in \omega(f)$ and $g \in \Omega(f)$.
It doesn't belong to $g \in o(f)$, $g \in O(f)$ and $g \in \Theta(f)$

b) $f(n) = 9n^{0.8} + 2n^{0.3} + 14 \log n$ and $g(n) = \sqrt{n}$

- $$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{9n^{0.8} + 2n^{0.3} + 14 \log n}{\sqrt{n}} = \lim_{n \to \infty} \frac{9n^{0.8} + 2n^{0.3} + 14 \log n}{n^{0.5}} =$$

$$\lim_{n \to \infty} 9n^{0.3} = \infty$$

It belongs to $f \in \omega(g)$ and $f \in \Omega(g)$.
It doesn't belong to $f \in o(g)$, $f \in O(g)$ and $f \in \Theta(g)$

• $\lim_{n \to \infty} \frac{g(n)}{f(n)} = \lim_{n \to \infty} \frac{\sqrt{n}}{9n^{0.8} + 2n^{0.3} + 14\log n} = \lim_{n \to \infty} \frac{n^{0.5}}{9n^{0.8}} = 0$

$9n^{0.8} \to \infty$

It belongs $g \in o(f)$ and $g \in O(f)$
It doesn't belong to $g \in \omega(f)$, $g \in \Omega(f)$
and $g \in \Theta(f)$

c) $f(n) = \frac{n^2}{\log n}$ and $g(n) = n \log n$

• $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{\frac{n^2}{\log n}}{n \log n} = \lim_{n \to \infty} \frac{n}{(\log n)^2} =$

$\lim_{n \to \infty} \frac{1}{\frac{2 \log n}{n \ln 2}} = \lim_{n \to \infty} \frac{n \ln 2}{2 \log n} = \lim_{n \to \infty} \frac{\frac{\ln 2}{2}}{\frac{1}{n \ln 2}} =$

$\lim_{n \to \infty} \frac{n(\ln 2)^2}{2} = \infty$ (applying L'Hopital's Rule twice)

It belongs to $f \in \omega(g)$ and $f \in \Omega(g)$
It doesn't belong to $f \in o(g)$, $f \in O(g)$ and
$f \in \Theta(g)$

• $\lim_{n \to \infty} \frac{g(n)}{f(n)} = \lim_{n \to \infty} \frac{n \log n}{\frac{n^2}{\log n}} = \lim_{n \to \infty} \frac{n \log^2 n}{n^2} = 0$

$n^2 \to \infty$

It belongs to $g \in o(f)$ and $g \in O(f)$.
It doesn't belong to $g \in \omega(f)$, $g \in \Omega(f)$ and
$g \in \Theta(f)$

d) $f(n) = (\log(3n))^3$ and $g(n) = 9\log n$

- $$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} = \frac{(\log(3n))^3}{9\log n} = \lim_{n\to\infty} \frac{\frac{3(\log(3n))^2}{n\ln 2}}{\frac{9}{n\ln 2}} =$$

$$\lim_{n\to\infty} \frac{(\log(3n))^2}{3} = \infty$$ (applying L'Hopital's Rule)

It belongs to $f \in \omega(g)$ and $f \in \Omega(g)$.
It doesn't belong to $f \in o(g)$, $f \in O(g)$ and $f \in \Theta(g)$

- $$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{9\log n}{(\log(3n))^3} = \lim_{n\to\infty} \frac{9\log n}{\log^3(3n)} = 0$$

It belongs to $g \in o(f)$ and $g \in O(f)$.
It doesn't belong to $g \in \omega(f)$, $g \in \Omega(f)$ and $g \in \Theta(f)$