2.1) a) $t_1(n) = 5n^2 + 16$

t_1 is part of $O(n^2)$ as we ignore constants and low-order terms and as we see, n^2 is the leading order term of the expression

Proof: $t_1(n) \leq kn^2$ when $n > n_0$

Choose $n_0 = 1 \Rightarrow n > 1$

$n = 2 \Rightarrow 5 \cdot 2^2 + 16 \leq k \cdot 2^2$

$36 \leq 4k$

$k \geq 9$

$5n^2 + 16 \leq 9n^2$ when $n > 1$

- $t_2(n) = 6n^3 + n^2 + 18$

t_2 belongs to $O(n^3)$ as n^3 is the leading term of the expression

Proof: $t_2(n) \leq kn^3$ when $n > n_0$

Choose $n_0 = 2 \Rightarrow n > 2$

$$n = 3 \Rightarrow 6 \cdot 3^3 + 3^2 + 18 \leq k \cdot 3^3$$

$$189 \leq 27k$$

$$k \geq 7$$

$6n^3 + n^2 + 18 \leq 7n^3$ when $n > 2$

b) $t_1 + t_2 = 5n^2 + 16 + 6n^3 + n^2 + 18$

$$= 6n^3 + 6n^2 + 34$$

The entire program belongs to $O(n^3)$

Choose $n_0 = 1$, $k = 14$

$$n = 2 \Rightarrow 6 \cdot 2^3 + 6 \cdot 2^2 + 34 \leq 14 \cdot 2^3$$

$$48 + 24 + 34 \leq 14 \cdot 8$$

$$106 \leq 112$$

$\Downarrow$

This is true

2.2)

$$1^2+3^2+5^2\ldots(2n-1)^2=\sum_{k=1}^{n}(2k-1)^2=\frac{2n(2n-1)(2n+1)}{6}$$

- Base case $n=1$

Left side equation

$$\sum_{k=1}^{1}(2\cdot 1-1)^2=1^2=1$$

Right Side equation

$$\frac{2\cdot 1(2\cdot 1-1)(2\cdot 1+1)}{6}=\frac{2\cdot 1\cdot 3}{6}=\frac{6}{6}=1$$

Both sides of equation are equal to 1.

- Assume $P(n)$ is true

Show for $n+1$

$$1^2+3^2+5^2+\dots(2n-1)^2+(2(n+1)-1)=$$

$$=(2(n+1)-1)^2+\frac{2n(2n-1)(2n+1)}{6}$$

$$=(2n+2-1)^2+\frac{(4n^2-2n)(2n+1)}{6}$$

$$=(2n+1)^2+\frac{8n^3+4n^2-4n^2-2n}{6}$$

$$=4n^2+4n+1+\frac{8n^3-2n}{6}$$

$$=\frac{24n^2+24n+6}{6}+\frac{8n^3-2n}{6}$$

$$=\frac{8n^3+24n^2+22n+6}{6}$$

$$=\frac{2(n+1)(2(n+1)-1)(2(n+1)+1)}{6}$$

$$=P(n+1)$$

Use a variable t for $n+1$

$$1^2+2^3+3^3+\ldots(2n-1)^2+(2(n+1)-1)^2=\frac{2t(2t-1)(2t+1)}{6}$$

This proves $P(n+1)$

So, by induction we prove that:

$$1^2+3^2+5^2\ldots(2n-1)^2=\sum_{k=1}^{n}(2k-1)^2=\frac{2n(2n-1)(2n+1)}{6}$$

is true