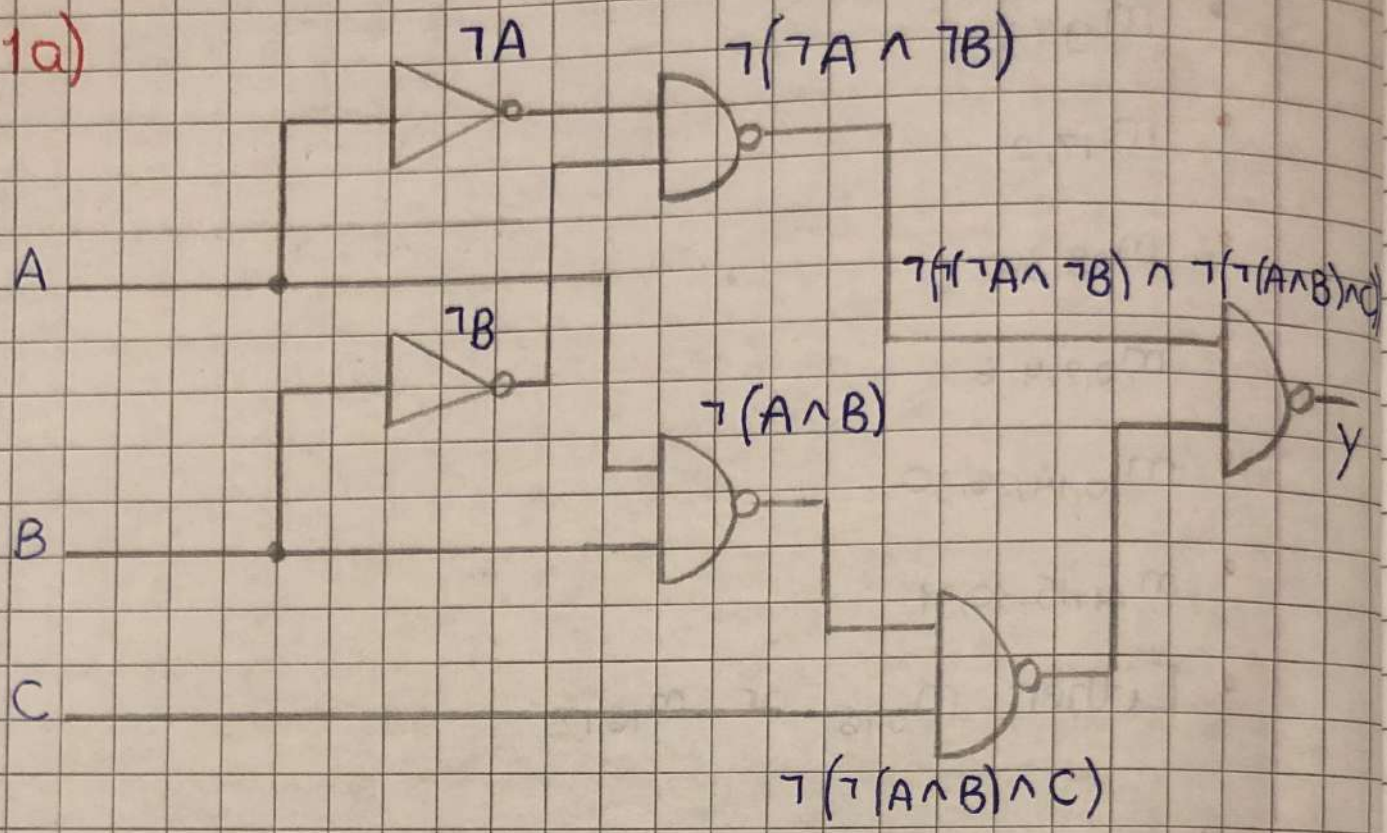
SHEET 8

1a)



$$Y = \neg(\neg(\neg A \wedge \neg B) \wedge \neg(\neg(A \wedge B) \wedge C))$$

1b) $\neg(\neg(\neg A \wedge \neg B) \wedge \neg(\neg(A \wedge B) \wedge C)) =$

$\neg((A \vee B) \wedge \neg((\neg A \vee \neg B) \wedge C)) =$

$\neg((A \vee B) \wedge \neg((\neg A \vee \neg B) \vee \neg C)) =$

$\neg((A \vee B) \wedge (A \wedge B \vee \neg C)) =$

$(\neg A \wedge \neg B) \vee ((\neg A \vee \neg B) \wedge C)$

DNF: $(\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (\neg B \wedge C)$

2)

$$S = A \dot{\vee} B \dot{\vee} C_{in}$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

Truth Table:

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

a) We know that

$$A \dot{\vee} B = \neg(A \vee B) \wedge (A \vee B)$$

$$= (\neg A \vee \neg B) \wedge (A \vee B)$$

$$= (A \wedge \neg B) \vee (\neg A \wedge B)$$

We write $S = A \dot{\vee} B \dot{\vee} C_{in}$ as:

$$S = A \dot{\vee} B \dot{\vee} C_{in}$$

$$= ((A \wedge \neg B) \vee (\neg A \wedge B)) \vee C_{in}$$

$$= (\neg((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$

$$= ((\neg(A \wedge \neg B) \wedge \neg(\neg A \wedge B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$

$$= (((\neg A \vee B) \wedge (A \wedge \neg B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge C_{in})$$

$$= (((\neg A \wedge (A \vee \neg B)) \vee (B \wedge (A \vee \neg B))) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$

$$= (((\neg A \wedge \neg B) \vee (B \wedge A)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$

$$= (\neg A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge C_{in})$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

$$= (A \wedge B) \vee (C_{in} \wedge ((A \wedge \neg B) \vee (\neg A \wedge B)))$$

$$= (A \wedge B) \vee ((C_{in} \wedge (A \wedge \neg B)) \vee (C_{in} \wedge (\neg A \wedge B)))$$

$$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$$

$$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$$

$$= (A \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \wedge (B \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)))$$

$$= (A \vee (C_{in} \wedge A \wedge \neg B) \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B) \vee B \vee (C_{in} \wedge \neg A \wedge B))$$

$= (A \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee B \vee (C_{in} \wedge A \wedge \neg B))$

$= (A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B))$

$= ((A \vee C_{in}) \wedge (A \vee \neg A) \wedge (A \vee B)) \wedge ((B \vee C_{in}) \wedge (B \vee A) \wedge (B \vee \neg B))$

$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \wedge (B \vee A)$

$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in})$

$= (A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B)$

b) CNF form → we can do it by looking at the table

$S = A \dot{\vee} B \dot{\vee} C_{in}$

$= (\neg A \vee \neg B \vee C_{in}) \wedge (A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee \neg C_{in}) \wedge (\neg A \vee B \vee \neg C_{in})$

$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$

$= (A \wedge B) \vee (C_{in} \wedge ((A \wedge \neg B) \vee (\neg A \wedge B)))$

$= (A \wedge B) \vee ((C_{in} \wedge (A \wedge \neg B)) \vee (C_{in} \wedge (\neg A \wedge B)))$

$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$

$= (A \wedge B) \vee ((C_{in} \wedge A \neg B) \vee (C_{in} \wedge \neg A \wedge B))$

$= (A \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \wedge$

$(B \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)))$

$= (A \vee (C_{in} \wedge A \wedge \neg B) \vee A \vee (C_{in} \wedge \neg A \wedge B))$

$\wedge (B \vee (C_{in} \wedge A \wedge \neg B) \vee B \vee (C_{in} \wedge \neg A \wedge B))$

$= (A \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee B \vee (C_{in} \wedge A \wedge \neg B))$

$= (A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B))$

$= ((A \vee C_{in}) \wedge (A \vee \neg A) \wedge (A \vee B)) \wedge (B \vee C_{in}) \wedge$

$(B \vee A) \wedge (B \vee \neg B))$

$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \wedge (B \vee A)$

$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in})$

c) Functions using only $\neg$ and $\uparrow$

$S = A \dot{\vee} B \dot{\vee} C_{in}$

$= (\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in})$

$\vee (\neg A \wedge B \wedge \neg C_{in})$

$= \neg\neg((\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in})$

$\vee (\neg A \wedge B \wedge \neg C_{in}))$

$= \neg(\neg(\neg A \wedge \neg B \wedge C_{in}) \wedge \neg(A \wedge B \wedge C_{in}) \wedge \neg(A \wedge \neg B \wedge \neg C_{in})$

$\wedge \neg(\neg A \wedge B \wedge \neg C_{in}))$

$= (\neg(\neg A \wedge \neg B \wedge C_{in}) \uparrow \neg(A \wedge B \wedge C_{in}) \uparrow$

$\neg(A \wedge \neg B \wedge \neg C_{in}) \uparrow \neg(\neg A \wedge B \wedge \neg C_{in}))$

$= (\neg A \uparrow \neg B \uparrow C_{in}) \uparrow (A \uparrow B \uparrow C_{in}) \uparrow (A \uparrow \neg B \uparrow \neg C_{in})$

$\uparrow(\neg A \uparrow B \uparrow \neg C_{in})$

$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$

$= (A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B)$

$= \neg\neg((A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B))$

$= \neg(\neg(A \wedge B) \wedge \neg(C_{in} \wedge A) \wedge \neg(C_{in} \wedge B))$

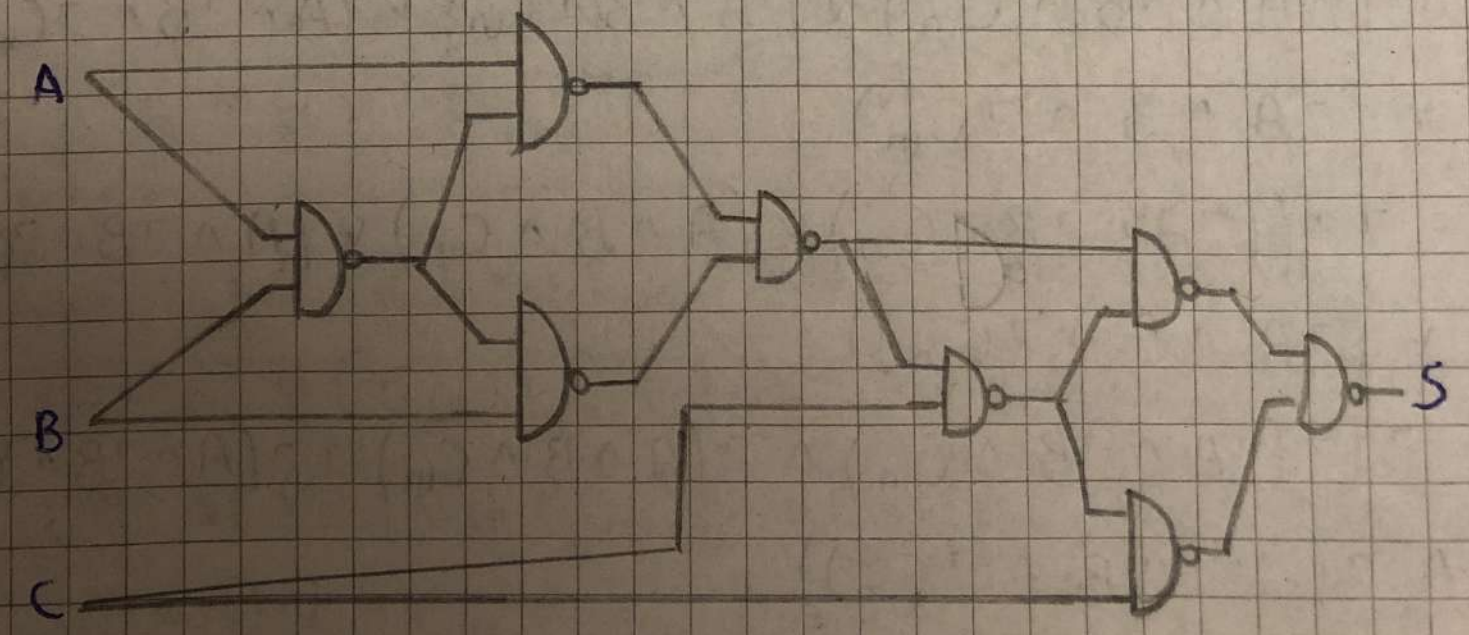
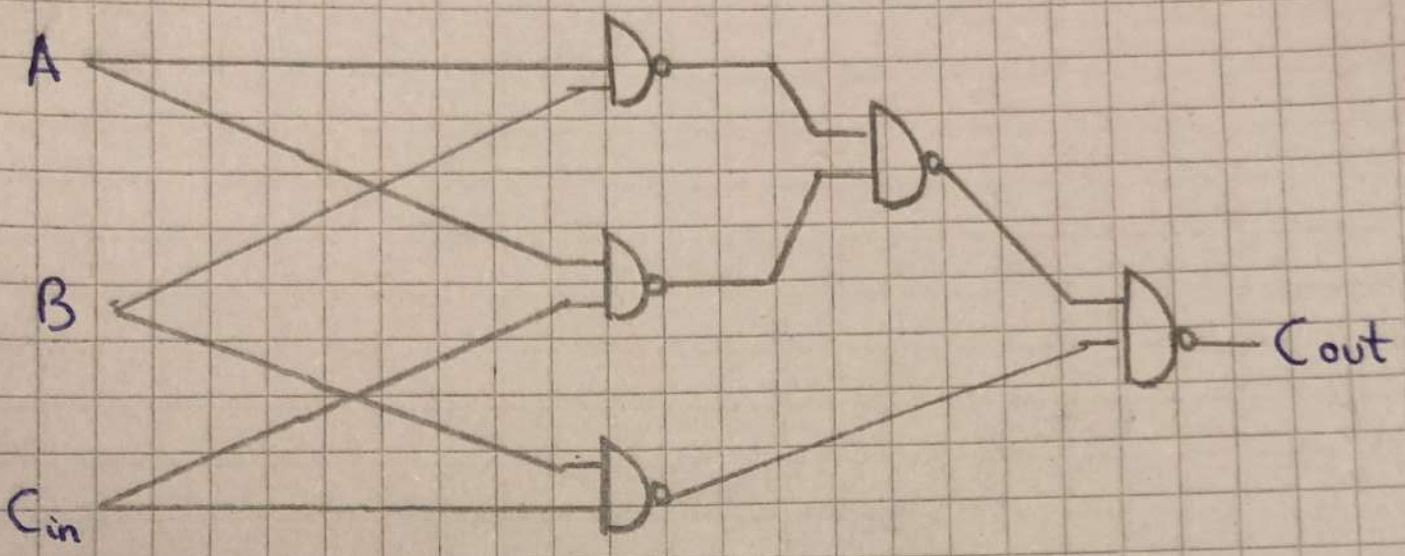
$= \neg((A \uparrow B) \wedge (C_{in} \uparrow A) \wedge (C_{in} \uparrow B))$

$= ((A \uparrow B) \uparrow (C_{in} \uparrow A) \uparrow (C_{in} \uparrow B))$

$= (A \uparrow B) \uparrow (C_{in} \uparrow A) \uparrow (C_{in} \uparrow B)$

d) Digital circuit

- $S = A \dot{\vee} B \dot{\vee} C_{in}$



- $C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$