SHEET 7

7.1) $\varphi(A,B,C,D,E) = m_0 + m_2 + m_4 + m_6 + m_9 + m_{10} + m_{13} + m_{14} + m_{15} + m_{16} + m_{17} + m_{21} + m_{26} + m_{28} + m_{30} + m_{31}$

a) Prime implicants of φ

- Turn the numbers into binary notation

$m_0 \rightarrow 00000$

$m_2 \rightarrow 00010$

$m_4 \rightarrow 00100$

$m_6 \rightarrow 00110$

$m_9 \rightarrow 01001$

$m_{10} \rightarrow 01010$

$m_{13} \rightarrow 01101$

$m_{14} \rightarrow 01110$

$m_{15} \rightarrow 01111$

$m_{16} \rightarrow 10000$

$m_{17} \rightarrow 10001$

$m_{21} \rightarrow 10101$

$m_{26} \rightarrow 11010$

$m_{28} \rightarrow 11100$

$m_{30} \rightarrow 11110$

$m_{31} \rightarrow 11111$

- Classify and sort minterms by the number of positive literals they contain

Minterm	Pattern
m_0	00000
m_2	00010
m_4	00100
m_{16}	10000
m_6	00110
m_9	01001
m_{10}	01010
m_{17}	10001
m_{13}	01101
m_{14}	01110
m_{21}	10101
m_{26}	11010
m_{28}	11100
m_{15}	01111
m_{30}	11110
m_{31}	11111

Minterm	Pattern	Used	Minterms	Pattern	Used	Minterms	Pattern
m_0	00000	✓	$m_{0,2}$	000-0	✓	$m_{0,2,4,6}$	00--0
			$m_{0,4}$	00-00	✓		
			$m_{0,16}$	-0000	-	$m_{0,4,2,6}$	00--0
m_2	00010	✓	$m_{2,6}$	00-10	✓	$m_{2,6,10,14}$	0--10
			$m_{2,10}$	0-010	✓		
m_4	00100	✓	$m_{4,6}$	001-0	✓	$m_{2,10,6,14}$	0--10
m_{16}	10000	✓	$m_{16,17}$	1000-	-		
m_6	00110	✓	$m_{6,14}$	0-110	✓	$m_{10,14,26,30}$	-1-10
			$m_{9,13}$	01-01	-		
m_9	01001	✓	$m_{10,14}$	01-10	✓	$m_{10,26,14,30}$	-1-10
m_{10}	01010	✓	$m_{10,26}$	-1010	✓		
m_{17}	10001	✓	$m_{17,21}$	10-01	-		
m_{13}	01101	✓	$m_{13,15}$	011-1	-	$m_{14,15,30,31}$	-11-
m_{14}	01110	✓	$m_{14,15}$	0111-	✓	$m_{14,30,15,31}$	-11-
m_{21}	10101	✓	$m_{14,30}$	-1110	✓		
m_{26}	11010	✓	$m_{26,30}$	11-10	✓		
m_{28}	11100	✓	$m_{28,30}$	111-0	-		
m_{15}	01111	✓	$m_{15,31}$	-1111	✓		
m_{30}	11110	✓	$m_{30,31}$	1111-	✓		
m_{31}	11111	✓					

no more combinations can be done

	m_0	m_2	m_4	m_6	m_9	m_{10}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{21}	m_{26}	m_{28}	m_{30}	m_{31}	Comment
$m_{0,16}$	✓									✓							
$m_{16,17}$										✓	✓						
$m_{9,13}$					✓		✓										Essential
$m_{17,21}$											✓	✓					Essential
$m_{13,15}$							✓		✓								
$m_{28,30}$														✓	✓		Essential
$m_{0,2,4,6}$	✓	✓	✓	✓													Essential
$m_{2,6,10,14}$		✓		✓		✓		✓									
$m_{10,14,26,30}$						✓		✓					✓		✓		Essential
$m_{14,15,30,31}$								✓	✓						✓	✓	Essential
	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	✓	✓	✓	✓	✓	✓	Coverage of essentials

There are 10 prime implicants:

- $m_{0,16} = (\neg B \wedge \neg C \wedge \neg D \wedge \neg E)$
- $m_{16,17} = (A \wedge \neg B \wedge \neg C \wedge \neg D)$
- $m_{9,13} = (\neg A \wedge B \wedge \neg D \wedge E)$
- $m_{17,21} = (A \wedge \neg B \wedge \neg D \wedge E)$
- $m_{13,15} = (\neg A \wedge B \wedge C \wedge E)$
- $m_{28,30} = (A \wedge B \wedge C \wedge \neg E)$
- $m_{0,2,4,6} = (\neg A \wedge \neg B \wedge \neg E)$
- $m_{2,6,10,14} = (\neg A \wedge D \wedge \neg E)$
- $m_{10,14,26,30} = (B \wedge D \wedge \neg E)$
- $m_{14,15,30,31} = (B \wedge C \wedge D)$

b) The essential prime implicants

The essential prime implicants are:

- $m_{9,13}$
- $m_{17,21}$
- $m_{28,30}$
- $m_{0,2,4,6}$
- $m_{10,14,26,30}$
- $m_{14,15,30,31}$
- Either $m_{0,16}$ or $m_{16,17}$

c) Minimal boolean expressions

- $\varphi'(A,B,C,D,E) = (\neg A \wedge B \wedge \neg D \wedge E) \vee (A \wedge \neg B \wedge \neg D \wedge E) \vee (A \wedge B \wedge C \wedge \neg E) \vee (\neg A \wedge \neg B \wedge \neg E) \vee (B \wedge D \wedge \neg E) \vee (B \wedge C \wedge D) \vee (\neg B \wedge \neg C \wedge \neg D \wedge \neg E)$

- $\varphi'(A,B,C,D,E) = (\neg A \wedge B \wedge \neg D \wedge E) \vee (A \wedge \neg B \wedge \neg D \wedge E) \vee (A \wedge B \wedge C \wedge \neg E) \vee (\neg A \wedge \neg B \wedge \neg E) \vee (B \wedge D \wedge \neg E) \vee (B \wedge C \wedge D) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D)$