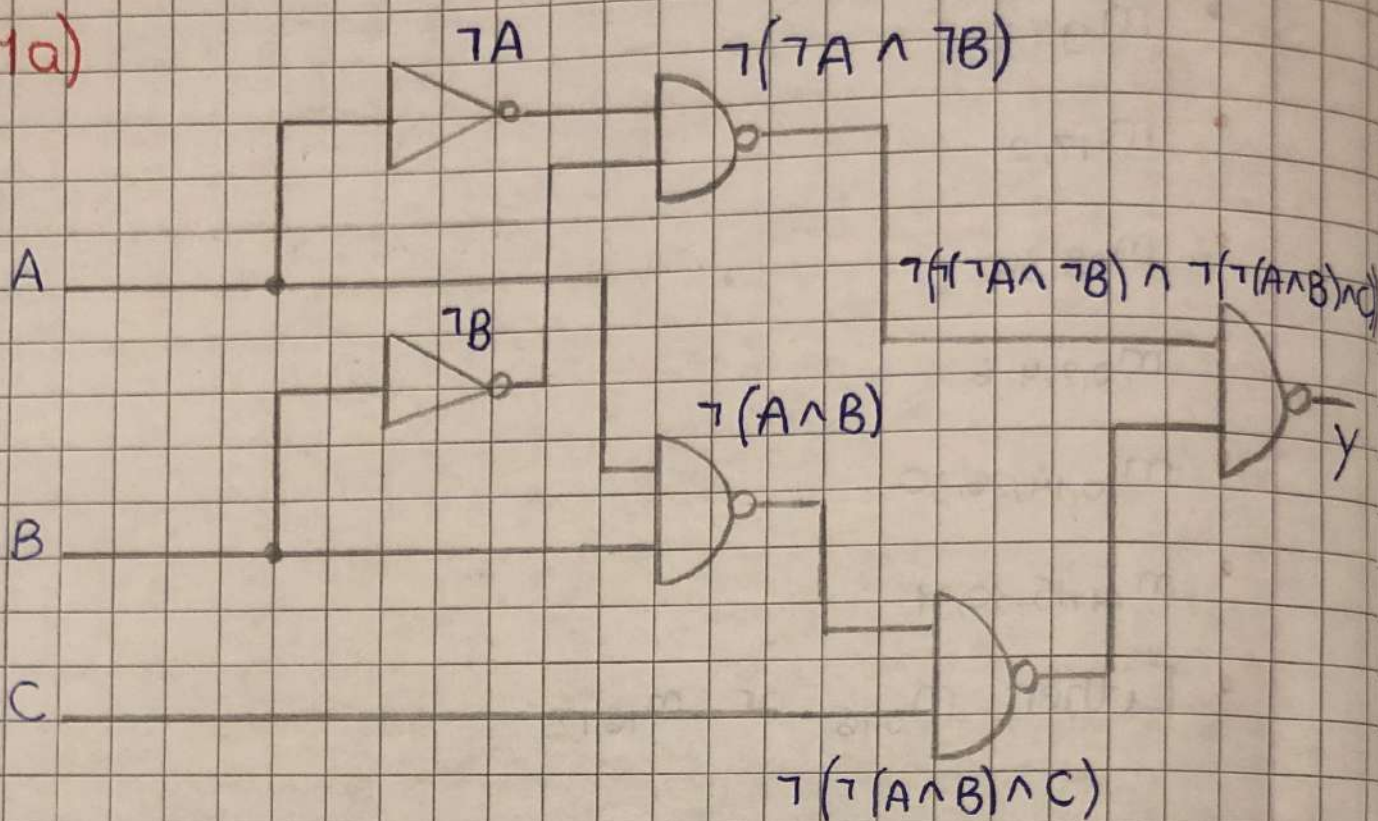


SHEET 8

1a)



$$Y = \neg(\neg(\neg A \wedge \neg B) \wedge \neg(\neg(A \wedge B) \wedge C))$$

$$\begin{aligned} 1b) \quad & \neg(\neg(\neg A \wedge \neg B) \wedge \neg(\neg(A \wedge B) \wedge C)) = \\ & \neg((A \vee B) \wedge \neg((\neg A \vee \neg B) \wedge C)) = \\ & \neg((A \vee B) \wedge \neg((\neg A \vee \neg B) \vee \neg C)) = \\ & \neg((A \vee B) \wedge (A \wedge B \vee \neg C)) = \\ & (\neg A \wedge \neg B) \vee ((\neg A \vee \neg B) \wedge C) \end{aligned}$$

$$\text{DNF} : (\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (\neg B \wedge C)$$

2)

$$S = A \oplus B \oplus C_{in}$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \oplus B))$$

Truth Table:

A	B	C _{in}	S	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

o)

We know that

$$A \oplus B = \neg(A \vee B) \wedge (A \vee B)$$

$$= (\neg A \vee \neg B) \wedge (A \vee B)$$

$$= (A \wedge \neg B) \vee (\neg A \wedge B)$$

We write $S = A \vee B \vee C_{in}$ as:

$$S = A \vee B \vee C_{in}$$

$$\begin{aligned} &= ((A \wedge \neg B) \vee (\neg A \wedge B)) \vee C_{in} \\ &= (\neg((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in}) \\ &= ((\neg(A \wedge \neg B) \wedge \neg(\neg A \wedge B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in}) \\ &= (((\neg A \vee B) \wedge (A \vee \neg B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in}) \\ &= (((\neg A \wedge (A \vee \neg B)) \vee (B \wedge (A \vee \neg B))) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in}) \\ &= (((\neg A \wedge \neg B) \vee (B \wedge A)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in}) \\ &= (\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \end{aligned}$$

$$\begin{aligned} C_{out} &= (A \wedge B) \vee (C_{in} \wedge (A \vee B)) \\ &= (A \wedge B) \vee (C_{in} \wedge ((A \wedge \neg B) \vee (\neg A \wedge B))) \\ &= (A \wedge B) \vee ((C_{in} \wedge (A \wedge \neg B)) \vee (C_{in} \wedge (\neg A \wedge B))) \\ &= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)) \\ &= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)) \\ &= (A \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \wedge (B \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \\ &= (A \vee (C_{in} \wedge A \wedge \neg B) \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B) \vee B \vee (C_{in} \wedge \neg A \wedge B)) \end{aligned}$$

$$\begin{aligned}
&= (A \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee B \vee (C_{in} \wedge A \wedge \neg B)) \\
&= (A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B)) \\
&= ((A \vee C_{in}) \wedge (A \vee \neg A) \wedge (A \vee B)) \wedge ((B \vee C_{in}) \wedge (B \vee A) \wedge (B \vee \neg B))
\end{aligned}$$

$$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \wedge (B \vee A)$$

$$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in})$$

$$= (A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B)$$

b) CNF form \rightarrow we can do it by looking at the table

$$S = A \dot{\vee} B \dot{\vee} C_{in}$$

$$\begin{aligned}
&= (\neg A \vee \neg B \vee C_{in}) \wedge (A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee \neg C_{in}) \\
&\wedge (\neg A \vee B \vee \neg C_{in})
\end{aligned}$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

$$= (A \wedge B) \vee (C_{in} \wedge ((A \wedge \neg B) \vee (\neg A \wedge B)))$$

$$= (A \wedge B) \vee ((C_{in} \wedge (A \wedge \neg B)) \vee (C_{in} \wedge (\neg A \wedge B)))$$

$$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$$

$$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$$

$$= (A \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \wedge$$

$$\begin{aligned}
& (B \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \\
&= (A \vee (C_{in} \wedge A \wedge \neg B) \vee A \vee (C_{in} \wedge \neg A \wedge B)) \\
&\wedge (B \vee (C_{in} \wedge A \wedge \neg B) \vee B \vee (C_{in} \wedge \neg A \wedge B)) \\
&= (A \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee B \vee (C_{in} \wedge A \wedge \neg B)) \\
&= (A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B)) \\
&= ((A \vee C_{in}) \wedge (A \vee \neg A) \wedge (A \vee B)) \wedge (B \vee C_{in}) \wedge \\
&\quad (B \vee A) \wedge (B \vee \neg B)) \\
&= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \wedge (B \vee A) \\
&= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in})
\end{aligned}$$

c) Functions using only \neg and \uparrow

$$\begin{aligned}
S &= A \dot{\vee} B \dot{\vee} C_{in} \\
&= (\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \\
&\vee (\neg A \wedge B \wedge \neg C_{in}) \\
&= \neg \neg ((\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \\
&\vee (\neg A \wedge B \wedge \neg C_{in})) \\
&= \neg (\neg (\neg A \wedge \neg B \wedge C_{in}) \wedge \neg (A \wedge B \wedge C_{in}) \wedge \neg (A \wedge \neg B \wedge \neg C_{in}) \\
&\wedge \neg (\neg A \wedge B \wedge \neg C_{in})) \\
&= (\neg (\neg A \wedge \neg B \wedge C_{in}) \uparrow \neg (A \wedge B \wedge C_{in}) \uparrow \\
&\neg (A \wedge \neg B \wedge \neg C_{in}) \uparrow \neg (\neg A \wedge B \wedge \neg C_{in})) \\
&= (\neg A \uparrow \neg B \uparrow C_{in}) \uparrow (A \uparrow B \uparrow C_{in}) \uparrow (A \uparrow \neg B \uparrow \neg C_{in})
\end{aligned}$$

$$\uparrow (\neg A \uparrow B \uparrow \neg C_{in})$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \vee B))$$

$$= (A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B)$$

$$= \neg \neg ((A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B))$$

$$= \neg (\neg (A \wedge B) \wedge \neg (C_{in} \wedge A) \wedge \neg (C_{in} \wedge B))$$

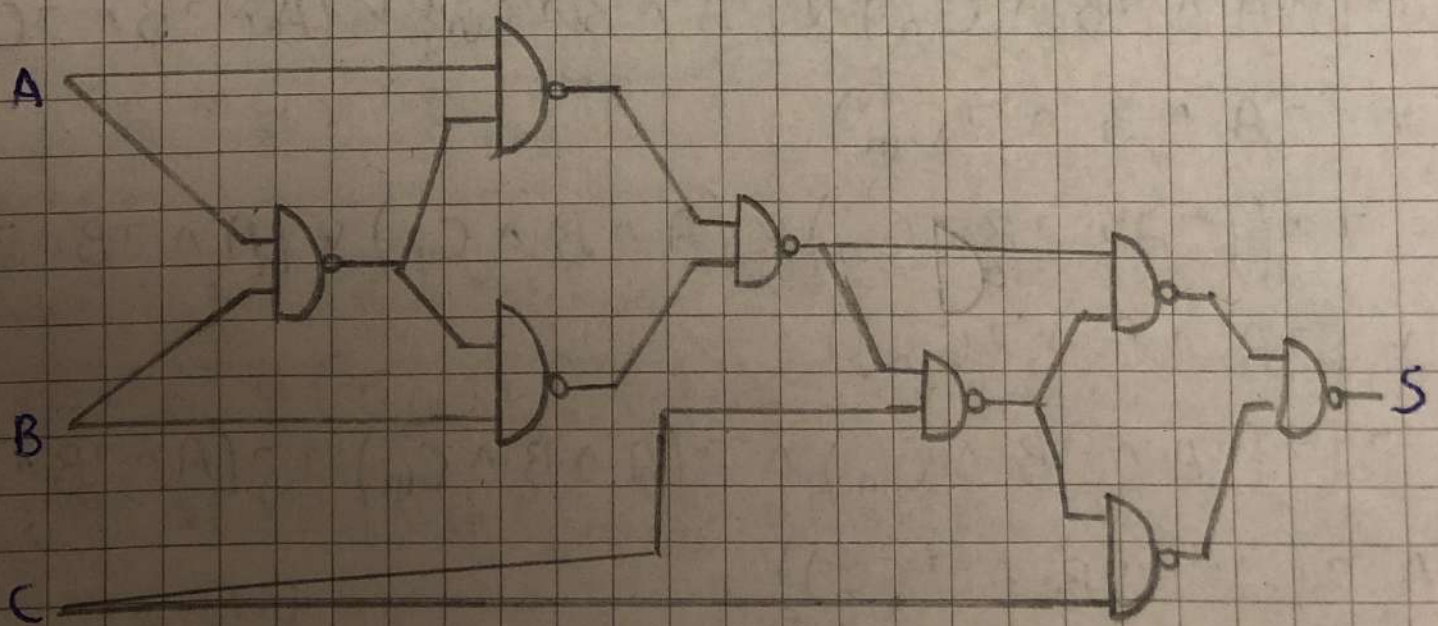
$$= \neg ((A \uparrow B) \wedge (C_{in} \uparrow A) \wedge (C_{in} \uparrow B))$$

$$= ((A \uparrow B) \uparrow (C_{in} \uparrow A) \uparrow (C_{in} \uparrow B))$$

$$= (A \uparrow B) \uparrow (C_{in} \uparrow A) \uparrow (C_{in} \uparrow B)$$

d) Digital circuit

$$\bullet S = A \vee B \vee C_{in}$$



• $C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \vee B))$

