

FTML practical session 12

5 juin 2025

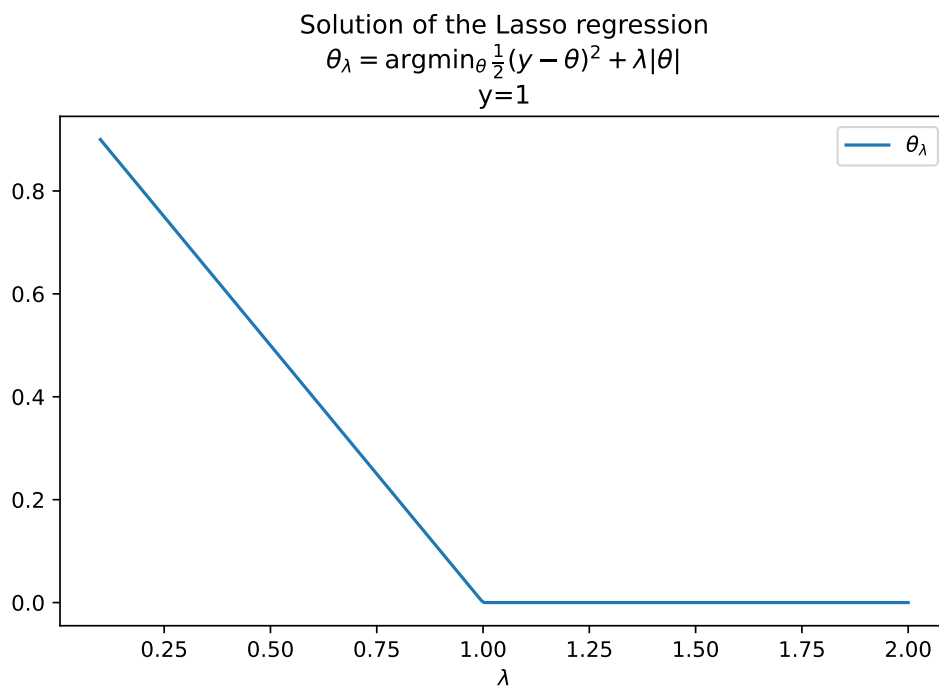


FIGURE 1 – Solution of the one-dimensional Lasso estimator

TABLE DES MATIÈRES

1	General Lasso estimator	2
2	1D Lasso	2

1 GENERAL LASSO ESTIMATOR

Lasso regression is a regularization method for linear regression based on the L1 norm. With our usual notations, the Lasso estimator is the solution to the following optimization problem :

$$\tilde{\theta}_\lambda \in \arg \min_{\theta \in \mathbb{R}^d} \left\{ \frac{1}{2} \|y - X\theta\|^2 + \lambda \|\theta\|_1 \right\} \quad (1)$$

where

- $X \in \mathbb{R}^{n,d}$ is the design matrix.
- $y \in \mathbb{R}^n$ is the vector of labels
- $\lambda \in \mathbb{R}$ is the regularization constant (called alpha in scikit-learn)

2 1D LASSO

In this session we analyze the Lasso regression on a simple, one-dimensional example, in order to develop the intuition of why this method leads to **sparse** estimators. We hence consider the simplified problem :

$$\theta_\lambda = \arg \min_{\theta} F_\lambda(\theta) \quad (2)$$

with

$$F_\lambda(\theta) = \frac{1}{2} (y - \theta)^2 + \lambda |\theta| \quad (3)$$

and $y \in \mathbb{R}, \lambda \in \mathbb{R}, \theta \in \mathbb{R}$.

Find the solution to the one-dimensional Lasso problem, as a function of λ . An example solution for $y = 1$ is shown in figure 1.

Run a simulation that reproduces the plot shown in figure 1 using scikit-learn.