

Worksheet 11

Exercise 1 Find the absolute maximum and absolute minimum values of f on the given interval

a. $f(x) = x e^{-x}$ on the interval $[0, 4]$

b. $f(x) = \ln(x^2 + x + 1)$ on the interval $[-1, 1]$

Exercise 2 Consider

$$f(x) = x^3 - 9x + 7$$

a. Show that f has at most two roots in $[-1, 1]$. Hint: use Rolle or mean value theorem

b. Show that f actually has exactly one root in $[-1, 1]$.

Exercise 3 Show that for all $x \geq 0$

$$\sqrt{1+x} \leq 1 + \frac{1}{2}x$$

Exercise 4 Let

$$f(x) = x^2 \ln(x)$$

- a. Find the domain of f
- b. Find the intervals of increase or decrease
- c. Find $\lim_{x \rightarrow 0^+} f(x)$
- d. Find the local and global maximum and minimum values (if any!)
- e. Find the intervals of concavity and the inflection points
- f. Sketch the graph of f