Worksheet 2
Continuity and limits with infinity

1 Continuity

Exercise 1 *Let f be the function defined by*

$$f(x) = \begin{cases} x & \text{if } x < -1 \\ \sqrt{|x|} & \text{if } -1 \leq x < 1 \\ x & \text{if } x \geq 1 \end{cases}$$

Draw the graph of f. Evaluate, if they exist,

$$\lim_{x \to -1^+} f(x) \qquad \lim_{x \to -1^-} f(x) \qquad \lim_{x \to -1} f(x)$$

and state at which points in $\boldsymbol{R}$ f is continuous.

Exercise 2 *Let f be a function continuous at point $x_0 \in \boldsymbol{R}$. Are the following functions continuous at x_0?*

a) $|f|$ *b)* $\sin \circ f$ *c)* $\dfrac{1}{f}$

2 Limits

Exercise 3 *Suppose that f and g and two functions such that*

$$\lim_{x \to \infty} f(x) = \infty \quad \text{and} \quad \lim_{x \to \infty} g(x) = -1$$

What can you say about the following limits (they may not exist)?

a) $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)}$ *b)* $\lim\limits_{x \to \infty} \dfrac{g(x)}{f(x)}$ *c)* $\lim\limits_{x \to \infty} f(x) + g(x)$ *d)* $\lim\limits_{x \to \infty} g(x) - f(x)$

Exercise 4 *Find the limit, if it exists. If it doesn't, explain why.*

a) $\lim\limits_{x \to 2^-} \dfrac{x+2}{x-2}$ *b)* $\lim\limits_{x \to \infty} \dfrac{1+x^6}{x^4+10}$ *c)* $\lim\limits_{t \to -\infty} t^2 - t^4$

d) $\lim\limits_{t \to \infty} \sin(t)$ *e)* $\lim\limits_{t \to \infty} \dfrac{\sin(t)}{t}$ *f)* $\lim\limits_{x \to \infty} \dfrac{1}{x + \sin(x)}$

3 Going further

Exercise 5 *Let f be the function defined on $\boldsymbol{R} \setminus \{0\}$ by*

$$f(x) = \frac{x^3 - x + 1}{x^2}$$

Evaluate the following limits, if they exist (the answer can be a real number or an infinity)

a) $\lim_{x \to 0} f(x)$ *b)* $\lim_{x \to 1} f(x)$ *c)* $\lim_{x \to \infty} f(x)$

Exercise 6 *Give an example of a function which doesn't have a limit at* 0*, but such that $|f|$ has a limit at* 0*.*