Worksheet 3
Limits with infinity, derivatives

1 Limits with infinity

Exercise 1 *Find the limit, if it exists. If it doesn't, explain why.*

a) $\lim_{x\to 0} \dfrac{x+x^2}{x^3-2x}$ *b)* $\lim_{x\to\infty} \dfrac{1+x^6}{x^4-3x^6}$ *c)* $\lim_{t\to-\infty} t^2-t^4$

d) $\lim_{t\to-\infty} \sin(t)$ *e)* $\lim_{t\to\infty} \dfrac{\sin(t)}{t}$ *f)* $\lim_{x\to\infty} \dfrac{1}{x+\sin(x)}$

Exercise 2 *Let f be the function defined on $\boldsymbol{R}\setminus\{0\}$ by*

$$f(x)=\frac{x^3-x+1}{x^2}$$

Evaluate the following limits, if they exist (the answer can then be a real number or an infinity)

a) $\lim_{x\to 0} f(x)$ *b)* $\lim_{x\to 1} f(x)$ *c)* $\lim_{x\to\infty} f(x)$

2 Derivatives

Exercise 3 *A warm can of soda is placed in a cold refrigerator. Sketch the graph of the temperature of the soda as a function of time. Is the initial rate of change of temperature greater or less than the rate of change after an hour?*

Exercise 4 *Determine whether f is differentiable at 0. Sketch a graph.*

a.

$$f(x)=\begin{cases} x^2\sin\frac{1}{x} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$$

b.

$$f(x)=\begin{cases} x\sin\frac{1}{x} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$$