Worksheet 1
Functions and limits

1 Functions

Exercise 1 *Find the domain of these functions. Make a rough plot using reference functions and the appropriate transformations. Guess the range of each of these functions.*

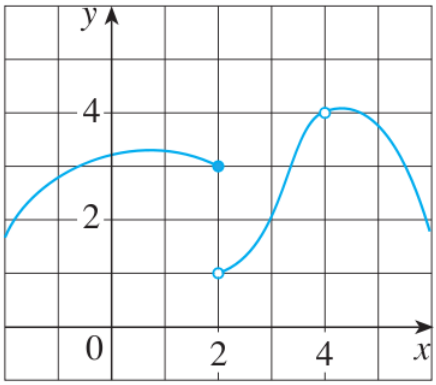
$f(x) = \frac{1}{x+2}$ $\quad g(x) = \sqrt{x-2} - 1$ $\quad h(x) = \sin\left(\frac{1}{2}x\right)$

Exercise 2 *Let $f : x \mapsto \frac{2x^2-1}{2x}$ and $g : x \mapsto x^2$. Give an expression for:*

i. $(f \circ g)(x)$ *ii.* $(g \circ f)(x)$ *iii.* $g(f(x))$
iv. $g(x+h)$ *v.* $f\left(\frac{2}{x}\right)$ *vi.* $g(\sqrt{x})$

2 Limits

Exercise 3 *Let f be a function represented by the following graph.*



Use the graph of f to determine the following quantities, if they exist. If they do not exist, explain why.

i. $\lim_{x\to 2^-} f(x)$ *ii.* $\lim_{x\to 2^+} f(x)$ *iii.* $\lim_{x\to 2} f(x)$
iv. $f(2)$ *v.* $\lim_{x\to 4} f(x)$ *vi.* $f(4)$

Exercise 4 *Find the limit, if it exists. If it does not exist, explain why.*

i. $\lim\limits_{x\to -1} \dfrac{x^2+2x+1}{x^4-1}$ $\quad$ *ii.* $\lim\limits_{t\to 0}\left(\dfrac{1}{t} - \dfrac{1}{t-t^2}\right)$ $\quad$ *iii.* $\lim\limits_{x\to 3} \dfrac{x^2-3x}{|9-3x|}$ $\quad$ *iv.* $\lim\limits_{h\to 0} \dfrac{(x+h)^2-x^2}{h}$

3 Going further

Exercise 5 *Let f_1, f_2 two even functions, g_1, g_2 two odd functions, defined on $\mathbb{R}$. Are the following functions even? odd? Remember that a function can be neither even nor odd.*

$$f_1 + f_2, \quad f_1 + g_1, \quad f_1 \times g_1, \quad g_1 \times g_2, \quad f_1 \circ f_2, \quad f_1 \circ g_1, \quad g_1 \circ f_1, \quad g_1 \circ g_2$$

Exercise 6 *(from Stewart's) In the theory of relativity, the Lorentz contraction formula*

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

expresses the length L of an object as a function of the velocity v with respect to an observer, where L_0 is the length of the object at rest and c the speed of light. Find

$$\lim_{v \to c^-} L$$

and interpret the result. Why is a left-hand limit necessary?