Worksheet 6
Chain rule and implicit differentiation

1 Chain rule

Exercise 1 *Find the derivative of the function.*

a) $f(x) = (\cos x)^{10}$

b) $f(x) = \cos\left(x^{10}\right)$

c) $f(x) = \sin\big(x\cos(x)\big)$

d) $f(x) = \left(\dfrac{1}{x^2+1}\right)^5$

Exercise 2 *Find the derivative of the function.*

a) $f(x) = \sqrt{x+\sqrt{x+\sqrt{x}}}$

b) $f(x) = \sin\Big(\cos\big(\sin(x)\big)\Big)$

2 Implicit differentiation

Exercise 3 *Find $y'(x)$ by implicit differentiation.*

a) $\sin(x+y) + y\sin(x) = 0$

b) $\tan(x/y) = x + y$

Exercise 4 *Show, using implicit differentiation, that any tangent line at a point P to a circle with center O is perpendicular to the radius OP.*

Hints
Step 1: write the equation of the circle of radius 1 and center $(0,0)$. If you don't know it, remember this circle is made of all the points (x,y) that are at distance 1 from the center $(0,0)$. So write first what the distance d between (x,y) and $(0,0)$ is, and then the equation of the circle is $d = 1$.

Step 2: let (x_0, y_0) be a point on the circle. What is the slope of the tangent line to the circle at (x_0, y_0)?

Step 3: what is the slope of the line passing through (x_0, y_0) and $(0,0)$?

Step 4: conclude.