



# A Formalisation that $Z$ Property implies Confluence

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1. Abstract Rewriting Systems
2. Application: Explicit Substitutions

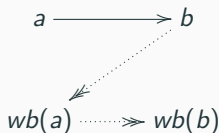
# Abstract Rewriting Systems

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# Confluence and the Z Property

## Definition (Z Property)

Let  $(A, \rightarrow)$  be an abstract rewriting system (ARS). The system  $(A, \rightarrow)$  has the Z property, if there exists a map  $wb : A \rightarrow A$  such that:

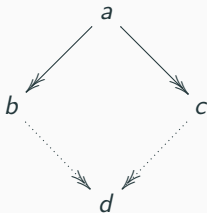


**Definition**  $Zprop \{A:Type\} (R: Rel A) := \exists wb:A \rightarrow A, \forall a b, R a b \rightarrow ((refltrans R) b (wb a) \wedge (refltrans R) (wb a) (wb b)).$

# Confluence and the Z Property

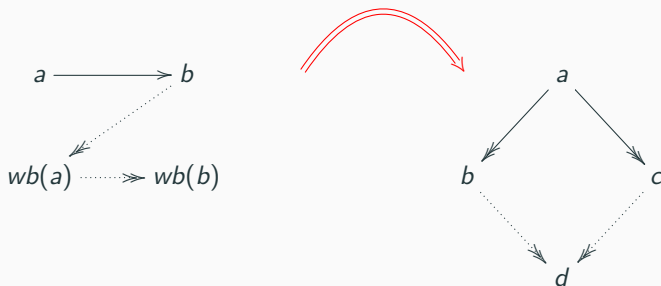
## Definition (Confluence)

An ARS  $(A, \rightarrow)$  is confluent if



**Definition**  $\text{Confl } \{A:\text{Type}\} (R: \text{Rel } A) := \forall a b c, (\text{refltrans } R) a b \rightarrow (\text{refltrans } R) a c \rightarrow (\exists d, (\text{refltrans } R) b d \wedge (\text{refltrans } R) c d).$

# Confluence and the Z Property



**Theorem**  $Zprop\_implies\_Confl \{A:Type\}: \forall R: Rel\ A, Zprop\ R \rightarrow Confl\ R.$

## **Application: Explicit Substitutions**

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## Definition

Let  $t$  be a term. The set of free variables of  $t$ , notation  $\text{fv}(t)$ , is inductively defined as:

- $\text{fv}(x) = \{x\}$



- Developed by Delia Kesner [1]

$$\mathcal{T} ::= x \mid \mathcal{T}\mathcal{T} \mid \lambda x. \mathcal{T} \mid \mathcal{T}[x/\mathcal{T}]$$

$$t[x/u][y/v] =_C t[y/v][x/u] \quad \text{if } y \notin \text{fv}(u) \text{ and } x \notin \text{fv}(v)$$

$$(\lambda x. t) u \rightarrow_B t[x/u]$$

# Locally Nameless Representation

- Developed in Coq by Arthur Charguéraud.
- No need for  $\alpha$ -conversion.
- Cofinite quantification is used to obtain strong induction principles.



D. Kesner.

**A Theory of Explicit Substitutions with Safe and Full Composition.**

5(3:1):1–29, 2009.