



A Formalisation that Z Property implies Confluence

Flávio L. C. de Moura and Leandro Oliveira Rezende

September 14, 2018

Universidade de Brasília

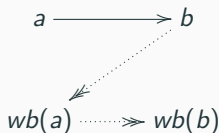
1. Abstract Rewriting Systems
2. Application: Explicit Substitutions

Abstract Rewriting Systems

Confluence and the Z Property

Definition (Z Property)

Let (A, \rightarrow) be an abstract rewriting system (ARS). The system (A, \rightarrow) has the Z property, if there exists a map $wb : A \rightarrow A$ such that:

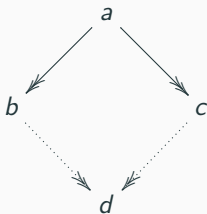


Definition $Zprop \{A:Type\} (R: Rel A) := \exists wb:A \rightarrow A, \forall a b, R a b \rightarrow ((refltrans R) b (wb a) \wedge (refltrans R) (wb a) (wb b)).$

Confluence and the Z Property

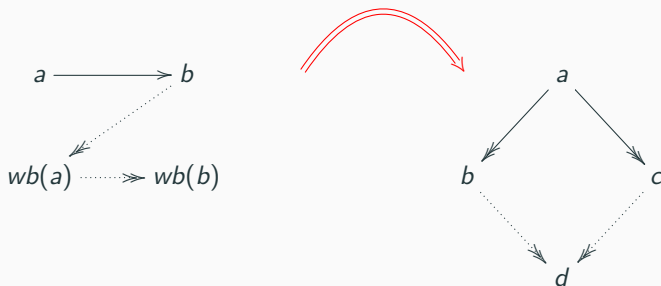
Definition (Confluence)

An ARS (A, \rightarrow) is confluent if



Definition $\text{Confl } \{A:\text{Type}\} (R: \text{Rel } A) := \forall a b c, (\text{refltrans } R) a b \rightarrow (\text{refltrans } R) a c \rightarrow (\exists d, (\text{refltrans } R) b d \wedge (\text{refltrans } R) c d).$

Confluence and the Z Property



Theorem $Zprop_implies_Confl \{A:Type\}: \forall R: Rel\ A, Zprop\ R \rightarrow Confl\ R.$

Application: Explicit Substitutions

Locally Nameless Representation

- Developed in Coq by Arthur Charguéraud.
- No need for α -conversion.
- Cofinite quantification is used to obtain strong induction principles.

Questions?