# Selecting the best model

```
In [ ]:
```

# (1) Overfitting and underfitting (Sven)

Cross-validation framework

```
In [1]: from sklearn.datasets import fetch california housing
        housing = fetch california housing(as frame=True)
        data, target = housing.data, housing.target
In [2]: \# To simplify future visualization, let's transform the prices from the 100 (k\$) range to the thousand dollars
        target *= 100
        target.head()
             452.6
Out[2]:
        0
             358.5
             352.1
        3
             341.3
        4
             342.2
        Name: MedHouseVal, dtype: float64
        Training error vs testing error
        Trying without train/test split
In [3]: from sklearn.tree import DecisionTreeRegressor
        regressor = DecisionTreeRegressor(random_state=0)
        regressor.fit(data, target)
Out[3]: v
                 DecisionTreeRegressor
        DecisionTreeRegressor(random state=0)
In [4]: from sklearn.metrics import mean absolute error
        target_predicted = regressor.predict(data)
        score = mean_absolute_error(target, target_predicted)
        print(f"On average, our regressor makes an error of {score:.2f} k$")
       On average, our regressor makes an error of 0.00 k$
In [5]: from sklearn.model_selection import train_test_split
        data_train, data_test, target_train, target_test = train_test_split(
            data, target, random_state=0
In [6]: regressor.fit(data_train, target_train)
Out[6]: v
                 DecisionTreeRegressor
        DecisionTreeRegressor(random_state=0)
In [7]: target_predicted = regressor.predict(data_train)
        score = mean_absolute_error(target_train, target_predicted)
        print(f"The training error of our model is {score:.2f} k$")
       The training error of our model is 0.00 k$
In [8]: target_predicted = regressor.predict(data_test)
        score = mean_absolute_error(target_test, target_predicted)
        print(f"The testing error of our model is {score:.2f} k$")
       The testing error of our model is 47.28 k$
```

Cross-validation: estimate robustness of a predictive model by repeating the splitting procedure. Gives several training and testing errors and therefore we can estimate how much the model generalization performance varies (as a an approximation to the "new" data that arrive in production).

- · make copy of dataset where order of observations are shuffled
- make train/test split
- train model on train, evaluate on test

```
In [9]: from sklearn.model_selection import cross_validate
         from sklearn.model selection import ShuffleSplit
         cv = ShuffleSplit(n splits=40, test size=0.3, random state=0)
         # this creates 40 cv splits, where 30% of records are in test data set
         cv_results = cross_validate(
             regressor, data, target, cv=cv, scoring="neg_mean_absolute_error"
         # the ShuffleSplit repeatedly spits out the shuffled train/test splits and lets sklearn train a new model on the
In [10]: # just some intuition for what ShuffleSplit does
         for train_index, test index in cv.split(data): # TODO: this is not course material. is it useful anyway?
             print("%s %s" % (train_index, test_index))
        [ 1989
                     7887 ... 9845 10799
                                           2732] [14740 10101 20566 ... 10211 2445 17914]
                256
        [ 3364 16548
                     7361 ...
                               5154 14875
                                            2751] [ 9959 16306 17662 ... 20549
                                                                               1212 17484]
        [12518 17324
                      1688 ... 9972 1146
                                            747] [20340 18230 15362 ... 2035 6919
        [15912
                555
                      5108 ... 14722 5851 12988] [ 5313 7640 18918 ...
                                                                         5008 3312 18057
        [10604
                     3266 ...
                                                         4713 10050 ...
               1016
                                526 6465
                                           9283] [ 7357
                                                                         1544 11416
                                                                                     63461
        [20356 13340 7263 ... 10572 16502 14054] [10502 14461 10915 ... 8940 5024
                                                                                      35831
        [ 9803 7879 11052 ... 13881 1853 13042] [13182 1320 2362 ... 13276 18258
        [17232 19201 13640 ... 10609 1597 11674] [ 3344
                                                          302 20417 ... 12118 15635
                                                                                     57541
        [18558
               3153 8256 ... 2442 16057 18358] [ 5997
                                                           295
                                                               2025 ... 4540 15907
        [10931 6653 19020 ... 19800 6174 10120] [ 6580 14013 12559 ... 14312 13338
                                                                                     76461
        [\ 7407\ 10308\ 16190\ \dots\ 8158\ 20388\ \ 3002]\ [10768\ 10864\ \ 2580\ \dots\ 11360
                                                                                        48]
         719 18917 5337 ...
                                 83 10723 3340] [14615
                                                         4459
                                                               3043 ... 13013 17231
        [ 7916 17976 16972 ... 13638 11310 18215] [12227
                                                         8164 11921 ... 2912 18550
                                                                                       2691
        [14941
                378 17191 ... 13560
                                      304 18033] [13109 10203
                                                               4911 ... 15997
                                                                               9224 11830]
        [16051
               6446 20563 ... 12010 1748 13479] [13112
                                                         1550 20295 ...
                                                                           34 10931 177841
        [13919 14622 6811 ... 15831 10253 17573] [ 7372 8571 11945 ...
                                                                         9870 18844
                                                                                      8361
        [ 1791 17192 10357 ... 15953 13791
                                           3742] [14731 17561 13050 ...
                                                                         5145 11292
                                                                                       7331
        [ 5853 17054 12736 ... 8291 15284
                                           3873] [11290 15345 13201 ... 16713 3203 15694]
        [ 9490 5774 19005 ... 4644 11890
                                           6751] [ 545 5914 1194 ... 19346 13214 1783]
        [ 2087
               8761 5309 ... 18256 16849 11852] [10610 15048 1166 ... 12674 7613 13254]
        [ 1353
               3845 15756 ... 17553 8977 18026] [ 3342 13283 15765 ... 17635
                                                                                6815
        [19921 8458 18889 ... 12996 19215 3625] [12369 15433 6410 ... 11275
                                                                               1788
                                                                                      85741
                                           8309] [15948 17464 13225 ... 8562 9418
        [ 9858 19480 1710 ... 18942 12814
        [ 4099 12535 4149 ... 18020 4205 20338] [ 3979 2931 15403 ... 15676 18914
                                                                                         41
                                           6202] [16001 17417 16190 ... 18034 20071
        [10799 \ 3500 \ 10904 \ \dots \ 9093 \ 10248
                                                                                      29281
        [ 3245 15245 12179 ... 11924 2313 19602] [15497 20250 20520 ... 14176 13589 13315]
        [14964 10997 14474 ... 11342 12264 19262] [15761 11288
                                                               1119 ... 7547
        [11498 15182 9381 ...
                                857 11206
                                           9792] [18657 11556
                                                               9115 ...
                                                                         7267 16544 177021
                                                                5433 ...
        [15275
                230
                     3093 ... 14022
                                     7373
                                           7798] [12422 16605
                                                                         5975 16744 11690
        [16382 \quad 4066 \ 16136 \ \dots \quad 6781 \quad 9674 \quad 1093] \ [19057
                                                               1634 ... 18047 19451 2893]
                                                         2027
        [12872 11579 11378 ... 18729 15918 16465] [ 204
                                                         2357 19173 ...
                                                                         9591 6166 10713
        [20612 11934 2136 ... 8504 7905
                                            230] [17398 9740
                                                               2045 ...
                                                                           19 5071 7451
        [ 1925 11503 18202 ... 19998 15865
                                                                8228 ... 19052 17070 11509]
                                            2596] [16510 15666
        [18431 19175 11668 ... 16476 13214 1746] [ 2487 17152 17042 ... 2966 13231 10180]
        [ 4762 18773 2883 ... 7474
                                     2329 18605] [ 8476 3899
                                                                109 ... 11022 10168
                                                          564
        [ 5628 15157 16360 ... 6438
                                      746 19740] [ 1918
                                                                7406 ... 7996
                                                                               5692 205141
                               8024 20261
                                           7364] [10167
                                                         9473
                                                                828 ... 13285
        [20519 5774 12116 ...
                                                                                6920 120351
        [18416 18319 15482 ... 17311 20589 13162] [10930 15771
                                                               1142 ... 7600
                                                                               2554 137901
        [ 1597 14406 8513 ... 6052 16273 15483] [17854 3190
                                                               1959 ... 20537 8135
        [ 8683 9855 13795 ... 6342 10717 3752] [14369 8483
                                                               6552 ... 15390 17427
                                                                                      86951
```

Note: some of these things are covered in the "Overfitting and underfitting section", thus it may not be necessary to cover them again/in detail

What does the ShuffleSplit do? parameters

- https://scikit-learn.org/stable/modules/cross\_validation.html#random-permutations-cross-validation-a-k-a-shuffle-split
- https://stackoverflow.com/questions/34731421/whats-the-difference-between-kfold-and-shufflesplit-cv
- it creates train and test samples randomly in each iteration. this means that a single observation can be multiple times in the test set.
- n splits: the number of train-test splits
- test size: size of test data, as a fraction of the full data set
- random state: makes the results reproducible b/c it fixes tells the random number generator where to start randomizing

We can pass this input to the cross\_validate function below as the cv argument.

Out[11]:		fit_time	score_time	test_score
	0	0.123644	0.002460	-46.909797
	1	0.121718	0.002373	-46.421170
	2	0.129612	0.002224	-47.411089
	3	0.123873	0.002409	-44.319824
	4	0.119278	0.002245	-47.607875

Lingo: score vs error

- score = higher values mean better results
- error = lower values mean better results

scoring parameter in cross\_validate expects a function that is a score (why does that matter? does it not just calculate the error/score and return it? does it do something with it?)

so, the mean abs error is an error, but we need a score -- so we take the negative. But to look at the mean absolute error for our purpose, we need to again re-convert the score to an error.

```
In [12]: cv_results["test_error"] = -cv_results["test_score"]

In [13]: cv_results.head()

Out[13]: fit_time score_time test_score test_error

0  0.123644   0.002460  -46.909797   46.909797

1  0.121718   0.002373   -46.421170   46.421170

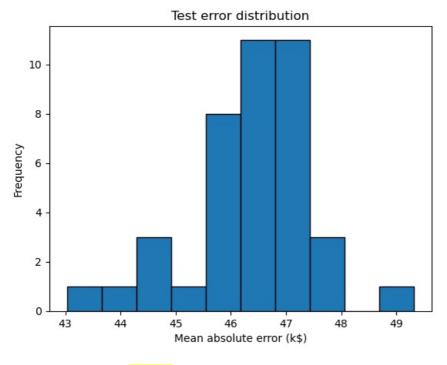
2  0.129612   0.002224   -47.411089   47.411089

3  0.123873   0.002409   -44.319824   44.319824

4  0.119278   0.002245   -47.607875   47.607875
```

```
import matplotlib.pyplot as plt

cv_results["test_error"].plot.hist(bins=10, edgecolor="black")
plt.xlabel("Mean absolute error (k$)")
   _ = plt.title("Test error distribution")
```



What does this mean? Call-out?

We notice that the mean estimate of the testing error obtained by cross-validation is a bit smaller than the natural scale of variation of the target variable. Furthermore, the standard deviation of the cross validation estimate of the testing error is even smaller.

- mean estimate of testing error is 47k
- what is the "natural scale of variation of the target variable"? -- it is just the range of the outcome, from around 50k to 500k
- but then I don't understand 47k is "a bit smaller than the range", which is 500k?

• small std of cross-validated error indicates good out-of-sample performance of the model

In [ ]:

# (2) Validation and learning curves

## Comparing train and test errors

See the video/slides: https://inria.github.io/scikit-learn-mooc/overfit/learning\_validation\_curves\_slides.html

Presentation notes

train vs test error

- · black training data points
- fit blue prediction function
- orange test data (generalization error). -- fundamental goal of ML: good prediction of y given x on unseen data.
- we can contrast the train and test error to understand better how well it generalizes

#### train vs test error: increasing complexity

- polynomial: higher degree = higher complexity. first order = linear in x. second-order = quadratic in x. etc.
- compare the train and test error: average distance between the blue prediction line and the respective data points.
- on average, train error < test error b/c it's easier to remember the training data
- degree 2 polynomial: lowers both train and test error
- degree 5 poly: train error decreases further; but it seems it fits some noise in the training data that are not present in the test data (the very right of the figure). thus, test error increases.
- degree 9 poly: gets extreme.
- in sum: get an intuition of how a more complex model impacts train and test error: low degree poly -> train and test error are high. higher complexity -> "sweet spot" test error is minimal. higher complexity -> test error gets higher again. *overfitting*

#### train vs test error: varying the sample size

- fix the complexity of the model; see how a bigger training set impacts the test error
- few data points -> function varies.
- adding data points -> function gets smoother, test error goes down, intuition: the model can approximate the true data generating process better; noise in the training data has smaller impact on the prediction, (train error also increases; but reason not clear)
- at some point, the train and test error ar the same, suggesting we're close to the optimal model
- going beyond it -> no more changes; "diminishing returns". if computation is costly, this is mostly adding costs without adding value
- with large enough training data, the degree 9 poly does not overfit anymore
- general pattern: model complexity relative to the training data
- we cannot bring the test error to 0: "Bayes error rate" / "irreducible error" -- arises from the random noise that is in the data generating process. best we can do

#### model families

- in ML, we always have a statistical model, and an unknown data-generating process
- by choosing the statistical model, we want to match the DGP
- so far, we had polynomials for both DGP and the stat model
- we can also choose a different model family. what happens then?
- decision tree: piece-wise linear: predict constant y for a range of x. (blue line)
- · they have different inductive biases and different notions of complexity
- inductive bias = "the set of assumptions that the learner uses to predict outputs of given inputs that it has not encountered."
- there is always an inductive bias unless we choose the correct model class that matches the DGP
- model families have different notions of complexity: higher degrees in polynomial = higher number of splits in the decision tree. both lead to overfitting.
- regularization = favor smooth, simple functions over complex functions. this reduces overfitting. change parameters while keeping
  the model fixed

## take-home messages

- models overfit when number of examples in the training set is too small compared to the complexity of the model
- we can detect overfitting when the testing error is much bigger than the training error (but not necessarily? -- better to "minimize test error"?)
- models underfit if they fail to capture the shape of the training set -> even the training error is large. BUT: it could also be noise in the DGP.

We'll see how this works in practice now.

# Overfit-generalization-underfit

We now want to use the cross-validation approach to quantify training and testing errors. This helps us to find out whether our model generalizes, overfits or underfits.

```
In [15]: # already done above
    # housing = fetch_california_housing(as_frame=True)
    # data, target = housing.data, housing.target # we just extract the explanatory variables and the outcome
    # target *= 100 # rescale the target to thousands of dollars (the original data are in hundreds of thousands of
    # https://inria.github.io/scikit-learn-mooc/python_scripts/datasets_california_housing.html

In [16]: # target.head()

In [17]: # from sklearn.tree import DecisionTreeRegressor
    # regressor = DecisionTreeRegressor()
    # DELETE?
```

The decision tree regressor was "introduced" / used (but not explained) in the previous segment.

#### Overfitting vs underfitting

In order to understand how our model generalizes, we want to compare the testing and the training error. To compute the error on the test set with the cross\_validate function

```
In [18]: # already imported above
         # from sklearn.model selection import cross validate, ShuffleSplit
In [19]: cv = ShuffleSplit(n_splits=30, test_size=0.2, random state=0)
 In [ ]:
In [20]: cv_results = cross_validate(
             rearessor.
             data,
             target.
             scoring="neg mean absolute error",
             return_train_score=True,
             n jobs=2,
In [21]: cv_results
Out[21]: {'fit_time': array([0.17909646, 0.17803574, 0.17157149, 0.18492937, 0.17681456,
                  0.17953753, 0.17122507, 0.18224573, 0.17118764, 0.18390369,
                  0.17098308, 0.17956877, 0.16909313, 0.18139696, 0.16651297,
                  0.17918992, 0.17719746, 0.1906023 , 0.1698103 , 0.19037414,
                  0.16757512, 0.18488216, 0.17270684, 0.18379045, 0.17240763,
                  0.18100095, 0.16896176, 0.18912959, 0.18846846, 0.14204884]),
           'score time': array([0.00231862, 0.00270486, 0.00240159, 0.00259829, 0.00250983,
                  0.00254583, 0.00247121, 0.00260639, 0.00265813, 0.00260091,
                  0.00238299, 0.00263357, 0.00263333, 0.00255966, 0.00217819,
                  0.00261736, 0.00230765, 0.00254941, 0.00308895, 0.00264406,
                  0.00235724,\ 0.00252104,\ 0.00256324,\ 0.00272179,\ 0.00242782,
                  0.0026629 , 0.00222826 , 0.00317383 , 0.00234175 , 0.00206113]),
           'test_score': array([-46.92525848, -46.6962173 , -45.06884133, -43.57765528,
                  -48.05169307, -44.58310441, -44.42208236, -45.1216984 ,
                  -44.96031032, -45.14051914, -47.27880063, -46.76900751,
                  -46.07751163, \ -45.55437984, \ -47.20800945, \ -44.38683794,
                  -45.99953682, -46.76782049, -45.11137815, -47.4055906 ,
                  -43.39771245, -46.00505475, -45.36389753, -46.79342878,
                  -46.32596754, -45.71983503, -44.41470058, -46.21278246,
                  -45.68218314, -47.66419259]),
           'train_score': array([-1.51747693e-14, -3.29796483e-15, -9.32587341e-15, -1.26376084e-14,
                  -8.81293316e-15, -1.25859702e-14, -1.44070802e-14, -3.74549659e-15,
                  -1.12123919e-14, -3.45976477e-15, -3.38747118e-15, -1.57935680e-14,
                  -1.52126374e-14, -1.00763497e-14, -1.23622043e-14, -1.36772592e-14,
                  -1.33811997e-14, -3.24976910e-15, -3.21190103e-15, -3.79369232e-15,
                  -1.50542800e-14, -1.21074554e-14, -1.42452802e-14, -3.64566258e-15,
                  -3.58369665e-15, -1.52952586e-14, -3.42189670e-15, -1.32193997e-14,
                  -1.28544892e-14, -3.09141171e-15])}
In [22]: cv_results = pd.DataFrame(cv_results)
```

```
In [23]: cv_results
```

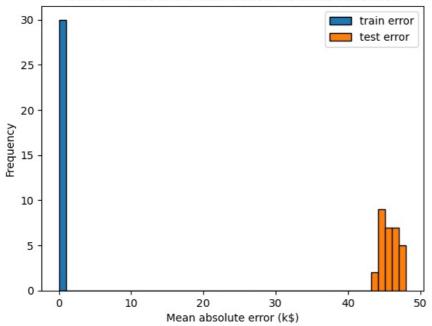
-			all the	- T	
10			- /	-6.1	
-	-	~ L	-		

	fit_time	score_time	test_score	train_score
0	0.179096	0.002319	-46.925258	-1.517477e-14
1	0.178036	0.002705	-46.696217	-3.297965e-15
2	0.171571	0.002402	-45.068841	-9.325873e-15
3	0.184929	0.002598	-43.577655	-1.263761e-14
4	0.176815	0.002510	-48.051693	-8.812933e-15
5	0.179538	0.002546	-44.583104	-1.258597e-14
6	0.171225	0.002471	-44.422082	-1.440708e-14
7	0.182246	0.002606	-45.121698	-3.745497e-15
8	0.171188	0.002658	-44.960310	-1.121239e-14
9	0.183904	0.002601	-45.140519	-3.459765e-15
10	0.170983	0.002383	-47.278801	-3.387471e-15
11	0.179569	0.002634	-46.769008	-1.579357e-14
12	0.169093	0.002633	-46.077512	-1.521264e-14
13	0.181397	0.002560	-45.554380	-1.007635e-14
14	0.166513	0.002178	-47.208009	-1.236220e-14
15	0.179190	0.002617	-44.386838	-1.367726e-14
16	0.177197	0.002308	-45.999537	-1.338120e-14
17	0.190602	0.002549	-46.767820	-3.249769e-15
18	0.169810	0.003089	-45.111378	-3.211901e-15
19	0.190374	0.002644	-47.405591	-3.793692e-15
20	0.167575	0.002357	-43.397712	-1.505428e-14
21	0.184882	0.002521	-46.005055	-1.210746e-14
22	0.172707	0.002563	-45.363898	-1.424528e-14
23	0.183790	0.002722	-46.793429	-3.645663e-15
24	0.172408	0.002428	-46.325968	-3.583697e-15
25	0.181001	0.002663	-45.719835	-1.529526e-14
26	0.168962	0.002228	-44.414701	-3.421897e-15
27	0.189130	0.003174	-46.212782	-1.321940e-14
28	0.188468	0.002342	-45.682183	-1.285449e-14
29	0.142049	0.002061	-47.664193	-3.091412e-15

we used the negative mean abs error (higher is better)  $\rightarrow$  let's transform it into a positive mean abs error (smaller is better)

• this is the same as we did in the previous segment on overfitting and underfitting

#### Train and test errors distribution via cross-validation



What do we see here? Call-out?

- train error is very small, actually 0. thus, the model is certainly not underfitting
- test error is much larger -- indicates that we are overfitting.
- **intuition**: the model is memorized too many observations from the training set. because there is some extent of noise in all observations, this means that the memorized observations do not replicate in the test dataset, leading to high test error.

#### Validation curve

hyperparameters:

- parameters that are not directly learned in the training process, but are chosen by us and can impact the performance of the model
- for instance: the number of neighbors in a k-nearest neighbors model
- the degree of a polynomial

Often, choosing the right hyperparameters is crucial to move a model towards a better test set performance (be it moving away from underfitting or from overfitting)

We can find how the training and test erorrs behave as a function of the hyperparameters with the validation curve.

Let's apply to curve to our example from above: we can use the max\_depth hyperparameter and see how the trade-off between overand underfitting behaves.

```
In [26]: import numpy as np
         from sklearn.model selection import ValidationCurveDisplay
         \max_{depth} = np.array([1, 5, 10, 15, 20, 25]) # this defines the depth of a tree
         disp = ValidationCurveDisplay.from_estimator(
             regressor,
             data,
             target,
             param_name="max_depth", # I guess this needs to be a parameter of the respective regressor model?
             param_range=max_depth,
             cv=cv, # we re-use the shuffle split from above
             scoring="neg mean absolute error",
             negate_score=True, ## this again re-converts the neg_mean_absolute_error to the mean_absolute_error
             std_display_style="errorbar", # show the variability of the mean absolute error
             n_{jobs=2},
           = disp.ax_.set(
             xlabel="Maximum depth of decision tree",
             ylabel="Mean absolute error (k$)",
             title="Validate curve for decision tree",
```

# Validate curve for decision tree Train Test 70 60 Mean absolute error (k\$) 50 40 30 20 10 0 5 10 15 20 25 0

Maximum depth of decision tree

### explain the regression tree? what is the depth?

• simply put: the deeper, the more flexible the tree because it uses more features for explaining the outcome

There are 3 areas in the validation curve

- first, we'd like to be in a sweet spot between neither over- nor underfitting. (mention bias-variance trade-off??)
- this is equivalent to the lowest possible test error, in this case with a depth of around 10. here we say "the model generalizes".
- the other regions are worse because the test error is larger
  - for max depth < 10, we underfit: both train and test error are large
  - for max\_depth > 10, we overfit: training error becomes very small, but test error increases. "memorizing" of observations and too much associated noise.

# in how much detail should I explain this? use it as exercise?

#### **Notes**

- we should not only look at the mean errors, but also at the standard deviation of the errors
  - why? -- for two types of hyperparameters, we could have the same mean errors, but a much larger variance. in this case, we would like to go for the model with lower variance. this is actually displayed with the error bars in the figure above (with the std display style attribute. https://scikit-

learn.org/stable/modules/generated/sklearn.model\_selection.ValidationCurveDisplay.html)

- in our case, the variance seems to be fairly stable across hyperparameter values
- we see also that the train and test error slightly diverge for max\_depth = 10 already. Thus, the model might be overfitting. But this is the best we can do with changing the hyperparameter.

#### Summary

- how to identify whether a model is generalizing, overfitting or underfitting
- how to check the influence of a hyperparameter on the underfit/overfit tradeoff

## Effect of the sample size in cross-validation

- before: how under-/overfitting and generalizing are impacted by hyperparameter values
- now: how they are impacted by the size of the data we have available

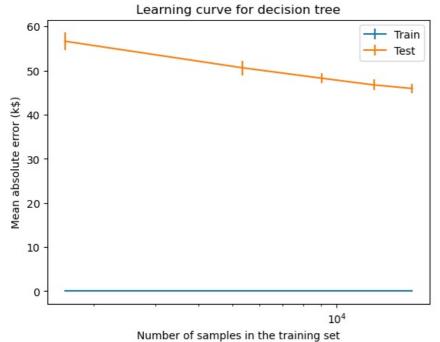
```
In [27]: # already done above
    # housing = fetch_california_housing(as_frame=True)
    # data, target = housing.data, housing.target
    # target *= 100

# we already have the DecisionTreeRegressor
```

#### Learning curve

- we can produce similar figures with test and train errors by varying the size of the dataset
- this called the learning curve
- it is informative about whether adding more training data improves the model's generalization performance

```
In [28]: train_sizes = np.linspace(0.1, 1.0, num=5, endpoint=True) # we split the values from 0.1 to 1 into 5 equally-si.
         # endpoint=True includes also 1.0.
         train_sizes
Out[28]: array([0.1 , 0.325, 0.55 , 0.775, 1.
In [29]: # again use a ShuffleSplit for cross-validation
         cv = ShuffleSplit(n splits=30, test size=0.2)
In [30]: # let's do the experiment
         from sklearn.model_selection import LearningCurveDisplay
         display = LearningCurveDisplay.from_estimator(
             regressor,
             data,
             target,
             train_sizes=train_sizes,
             cv=cv.
             # score_type="both", # score both train and test errors -- this is the default, and not shown in the previous
             # to make it consistent, ignore it here
             scoring="neg_mean_absolute_error",
             negate_score=True,
             score_name="Mean absolute error (k$)",
             std_display_style="errorbar",
             n_{jobs=2},
           = display.ax_.set(xscale="log", title="Learning curve for decision tree")
```



#### What do we see?

- train error is 0 -- overfitting
- the variability of the error declines as we increase the size of the data set -- noise has less impact on the error
- test error declines as we increase sample size
- since it does not seem to "flatten out", we it may be useful to increase the size of the dataset further to reduce the test error
- in case we reached the "flat" part of the learning curve, then we may have reached the Bayes error rate (the irreducible error, ie
   Var(ε)) with the available model. To further reduce the test error, we'd have to try of a more complex model.

## Summary

 we can use the learning curve to check whether it's worth adding more data, or whether we may need to try out a more complex model

### Exercise M2.01

For the questions, see the exercise document

### explain what the SVM does?

for classification problems:

- in a p-dimensional space, tries to find a boundary in p-1-dimensional space to separate the data points
- give example with 2-dimensional space?
- it seeks to maximize the distance from the closest data points on either side of the boundary
  - intuition: this will also lead to low generalization error (since "new" data are likely to fall into the same area as the "existing" data)
- see wikipedia: https://en.wikipedia.org/wiki/Support\_vector\_machine -- the γ parameter is in the radial basis function

what is the  $\gamma$  parameter? it makes the classification boundary more non-linear.

### Solution

4

```
In [31]: blood_transfusion = pd.read_csv("../datasets/blood_transfusion.csv")
    data = blood_transfusion.drop(columns="Class")
    target = blood_transfusion["Class"]
In [32]: data.head()
```

Recency Frequency Monetary Time 0 2 50 12500 98 1 0 13 3250 28 2 35 1 16 4000 3 20 5000 45

#### (1) Create a predictive pipeline

24

6000

77

1

```
In [33]: from sklearn.pipeline import make_pipeline
    from sklearn.preprocessing import StandardScaler
    from sklearn.svm import SVC

model = make_pipeline(
        StandardScaler(), SVC(kernel="rbf")
)
```

In [34]: model.get\_params().keys() # this is how we can see the name of the parameters, particularly for the validation

let's use the ShuffleSplit scheme for cross validation

f"{cv\_results['test\_score'].std():.3f}"

In [ ]:

# (2) Evaluate how well the model generalizes.

```
In [35]: cv = ShuffleSplit(random_state=0)
    cv_results = cross_validate(model, data, target, cv=cv, n_jobs=2)
    cv_results = pd.DataFrame(cv_results)

In [36]: cv_results.shape

Out[36]: (10, 3)

In [37]: print(
    "Accuracy score of our model:\n"
    f"{cv_results['test_score'].mean():.3f} ± "
```

Accuracy score of our model:  $0.765 \pm 0.043$ 

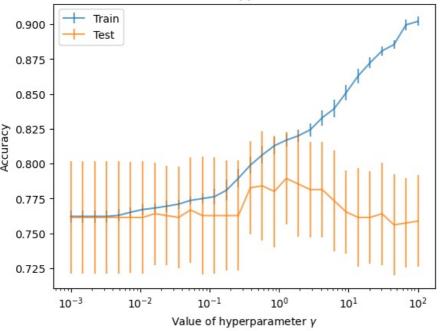
# (3) Evaluate the effect of the parameter gamma by using sklearn.model\_selection.ValidationCurveDisplay

Try out different values for  $\gamma$ 

```
param_range=gamma_values,
    cv=cv, # we re-use the shuffle split from above
    scoring="accuracy",
    score_name="Accuracy",
    std_display_style="errorbar", # show the variability of the mean absolute error
    errorbar_kw={"alpha": 0.7}, # transparency for better visualization
    n_jobs=2,
)

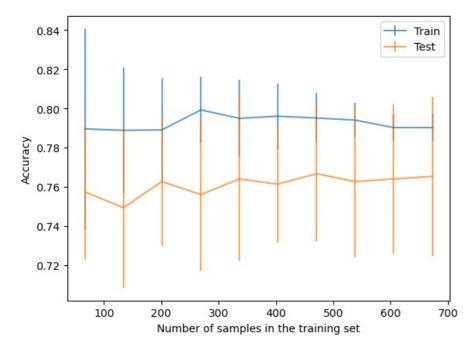
_ = disp.ax_.set(
    xlabel="Value of hyperparameter $\gamma$",
    ylabel="Accuracy",
    title="Validate curve of support vector machine",
)
```

## Validate curve of support vector machine



What do we see?

- overfitting starting around  $\gamma$  = 1? (remember higher accuracy is better)
- (4) Compute the learning curve (using sklearn.model\_selection.LearningCurveDisplay ) by computing the train and test scores for different training dataset size.



### Observations

- adding more samples does not improve the accuracy much -- it's always around 0.77
- note that this is about the fraction of records that are in class "not donated". Therefore, even without seeing any data, the we could have a similar performance

## Interpretation

- this suggests that this model here is probably too simple
- features themselves are not informative, for any model
- the other default hyperparameter values of SVC are not good
- the model itself may be wrong

# Quiz M2.02

#### see the exercise document

"Quiz: over- and underfitting and learning curves (5 minutes, in pairs; if time-permitting)"

**Solutions** 1.b, 2.a, 3.d, 4.b, 5.c/d, 6.b

In [ ]:

In [ ]:

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