913 GHERMAN CRISTIAN

Exercise 9.4.2.

Find the equations of the tangent lines to the ellipse $\mathcal{E}: x^2+4y^2-20=0$, which are orthogonal to the line $d: 2x-2y-13=0$

We can re-write the ellipse eq. as:

$$\frac{x^2}{20}+\frac{y^2}{5}-1=0 \iff \frac{x^2}{20}+\frac{y^2}{5}=1$$

Thus, we can consider $a^2=20,\ b^2=5$ (1.)

$d: 2x-2y-13=0 \iff 2y=2x-13 \iff$

$\iff y=1\cdot x-\frac{13}{2} \Rightarrow$ the slope of $d = m_d = 1$

Let T be a tangent at $\mathcal{E}$

We know $T \perp d \Rightarrow m_T=\frac{-1}{m_d}=-1$ (2.)

Let $A_0(x_0,y_0)\in\{T\cap\mathcal{E}\} \Rightarrow T: \frac{x_0\cdot x}{a^2}+\frac{y_0\cdot y}{b^2}=1$ $\Big\}$ From (1.) $\Rightarrow$

$\Rightarrow T: \frac{x_0\cdot x}{20}+\frac{y_0\cdot y}{5}=1 \ /\cdot 20 \iff x_0\cdot x+4\cdot y_0\cdot y=20$

$\iff y=\frac{20-x\cdot x_0}{4\cdot y_0} \iff y=x\cdot\frac{-x_0}{4y_0}+\frac{5}{y_0}$ (3.) $\Rightarrow$

$\Rightarrow$ the slope of $T=\frac{-x_0}{4y_0}$, From (2.) $\Big\} \Rightarrow \frac{-x_0}{4y_0}=-1 \iff x_0=4y_0$

$A_0\in\mathcal{E} \Rightarrow x_0^2+4\cdot y_0^2-20=0 \iff$

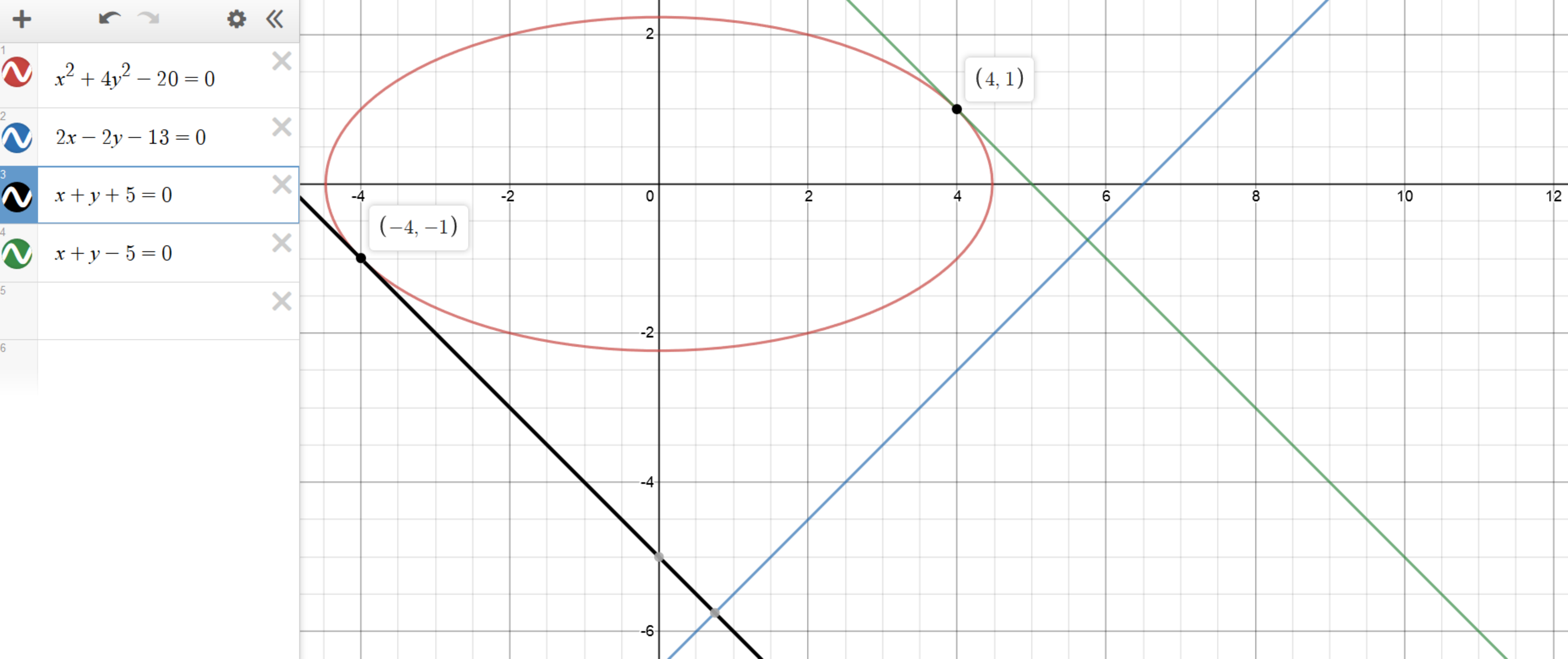
$\iff 16y_0^2+4y_0^2=20 \iff y_0^2=1 \iff y_0\in\{-1,1\}$

From (3.) $\Rightarrow$ the equations are of the form:

$$y=x\cdot\frac{-4y_0}{4y_0}+\frac{5}{y_0}=-x+\frac{5}{y_0}$$

$y_0=-1 \Rightarrow y=-x-5 \iff x+y+5=0$

$y_0=1 \Rightarrow y=-x+5 \iff x+y-5=0$



$x^2 + 4y^2 - 20 = 0$
$2x - 2y - 13 = 0$
$x + y + 5 = 0$
$x + y - 5 = 0$
(4, 1)
(−4, −1)
-4
-2
0
2
4
6
8
10
12
-2
-4
-6