$\boxed{32.2.3}$ Consider a quadrilater with vertices $A(1,1)$, $B(3,1)$, $C(2,2)$ and $D(1.5,3)$. Find the image quadrilaterals through the translation $T(1,2)$, the scaling $S(2,2.5)$, the reflections about the x and y-axis, the clockwise and anticlockwise rotations through the angle $\pi/2$ and the shear $Sh((2/\sqrt{5}, 1\sqrt{5}), 1.5)$.

Solution:

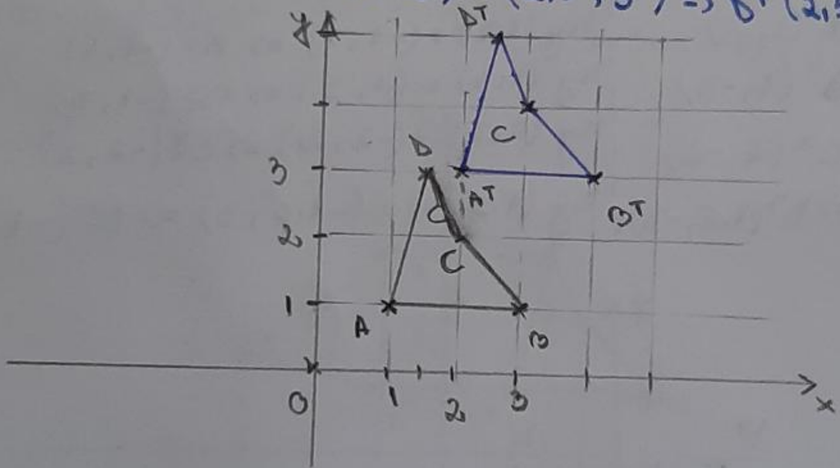
- Translation $T(1,2)$.

— $[T(1,2)](1,1) = (1+1, 1+2) = (2,3) \Rightarrow A^T(2,3)$

$[T(1,2)](3,1) = (1+3, 2+1) = (4,3) \Rightarrow B^T(4,3)$

$[T(1,2)](2,2) = (1+2, 2+2) = (3,4) \Rightarrow C^T(3,4)$

$[T(1,2)](1{,}5,3) = (1+1{,}5, 2+3) = (2{,}5, 5) \Rightarrow D^T(2{,}5,5)$



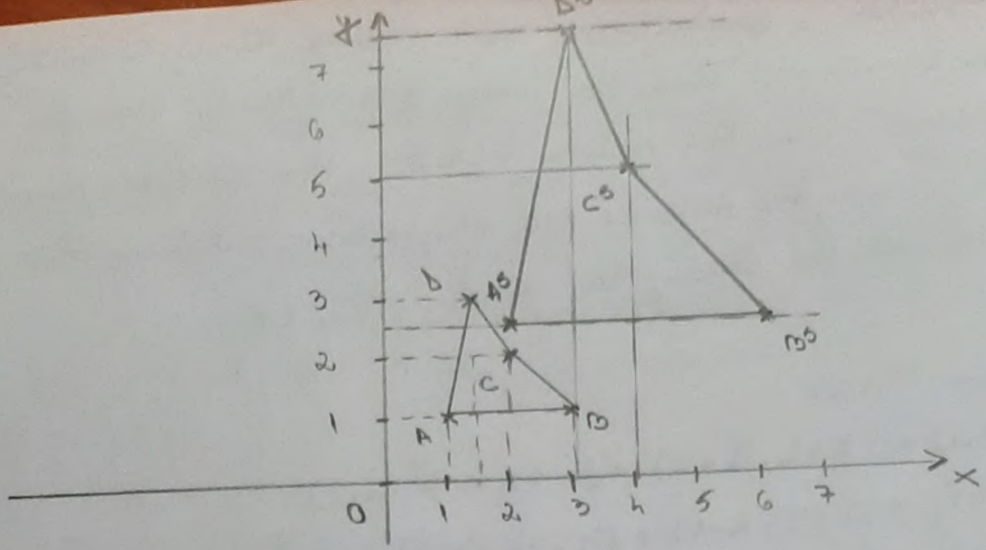
- Scaling $(2, 2{,}5)$.

$S(2,2{,}5)(1,1) = (2, 2{,}5) \Rightarrow A^S(2; 2{,}5)$

$S(2,2{,}5)(3,1) = (6, 2{,}5) \Rightarrow B^S(6; 2{,}5)$

$S(2,2{,}5)(2,2) = (4, 5) \Rightarrow C^S(4, 5)$

$S(2,2{,}5)(1{,}5; 3) = (3, 7{,}5) \Rightarrow D^S(3, 7{,}5)$



• Reflection about the x and y-axes

$r_x(1,1) = (1,-1) \Rightarrow A^x(1,-1)$

$r_x(1,3) = (1,-3) \Rightarrow B^x(1,-3)$

$r_x(2,2) = (2,-2) \Rightarrow C^x(2,-2)$

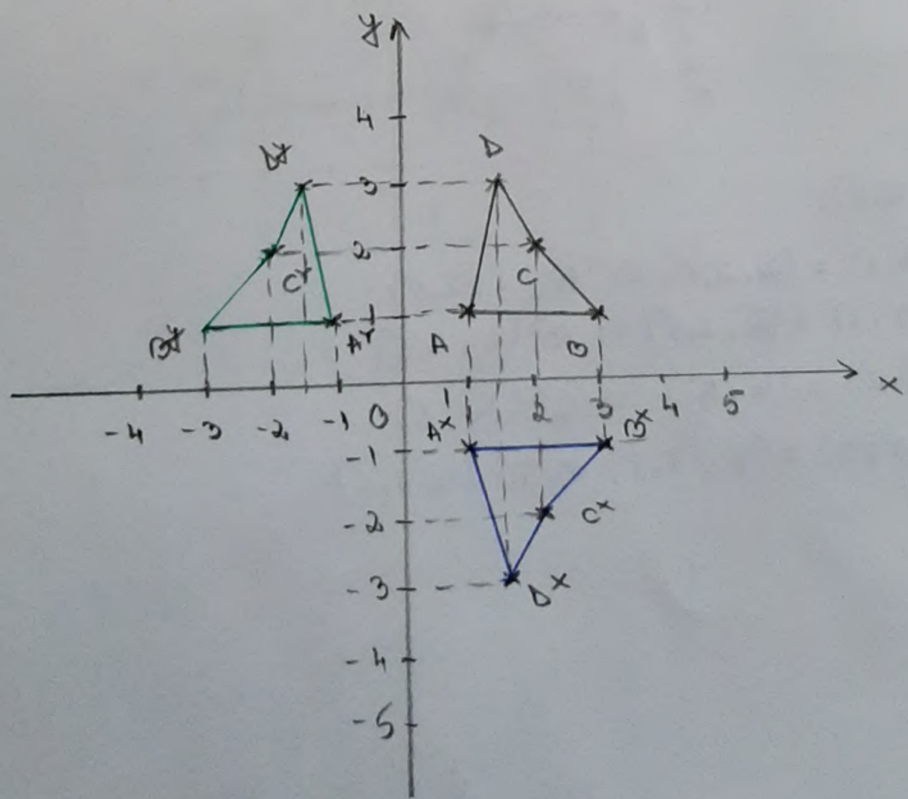
$r_x(1.5,3) = (1.5,-3) \Rightarrow D^x(1.5,-3)$

$r_y(1,1) = (-1,1) \Rightarrow A^y(-1,1)$

$r_y(1,3) = (-1,3) \Rightarrow B^y(-1,3)$

$r_y(2,2) = (-2,2) \Rightarrow C^y(-2,2)$

$r_y(1.5,3) = (-1.5,3) \Rightarrow D^y(-1.5,3)$



• Rotation:

→ clockwise $(-\frac{\pi}{2}) = \theta$

$\cancel{R_\theta(1,1) = (1\cdot\cos(-\frac{\pi}{2}) - 1\sin -\frac{\pi}{2}, 1\sin -\frac{\pi}{2} + 1\cos -\frac{\pi}{2})}$

$\theta = -\frac{\pi}{2} \Rightarrow R(\theta)(x,y) = (x\cos(-\frac{\pi}{2}) - y\sin(-\frac{\pi}{2}), x\sin(-\frac{\pi}{2}) + y\cos(-\frac{\pi}{2}))$

$= (x\cdot 0 - y(-1), x(-1) + y\cdot 0) = (y, -x) \Rightarrow R_\theta = (y, -x)$

$\Rightarrow R_\theta(1,1) = (1,-1)\ A'$ $\qquad R_\theta(2,2) = (2,-2)\ C'$

$R_\theta(3,1) = (1,-3)\ B'$ $\qquad R_\theta(1.5,3) = (3,-1.5)\ D'$

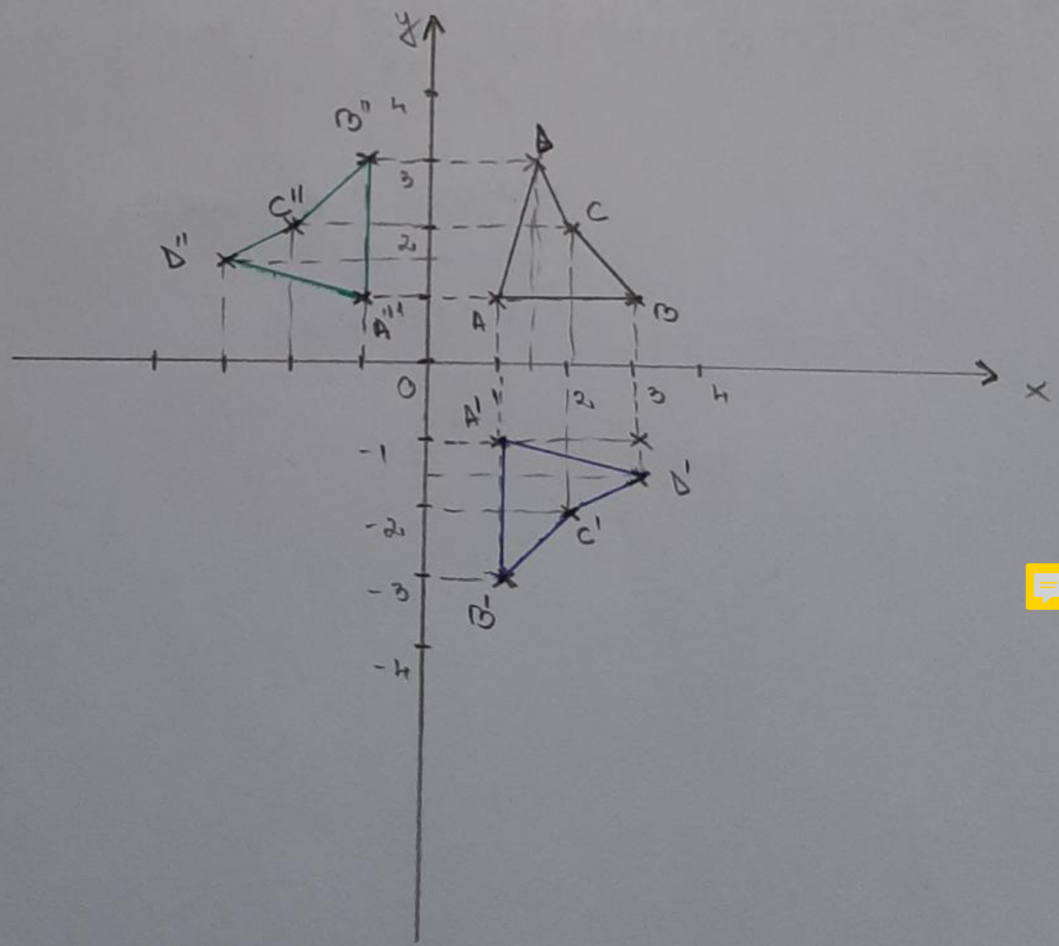
→ anticlockwise

$\theta = \frac{\pi}{2} \Rightarrow R_\theta(x,y) = (x\cos\frac{\pi}{2} - y\sin\frac{\pi}{2}, x\sin\frac{\pi}{2} + y\cos\frac{\pi}{2}) =$

$= (x\cdot 0 - y\cdot 1, x\cdot 1 + y\cdot 0) = (-y, x) \Rightarrow R'_\theta = (-y, x)$

$\Rightarrow R'_\theta(1,1) = (-1,1)\ A''$ $\qquad R'_\theta(2,2) = (-2,2)$

$R'_\theta(3,1) = (-1,3)\ A''$ $\qquad R'_\theta(1.5,3) = (-3,1.5)$



• $Sh\left(\left(2/\sqrt{5},\ 1/\sqrt{5}\right),\ 1{,}5\right)$

$$Sh(v, r)(x,y) = (x,y) + \left(r(v_1 y - v_2 x)v_1,\ r(v_1 y - v_2 x)v_2\right)$$

$$= \left((1 - r v_1 v_2)x + r v_1^2 y,\ -r v_2^2 x + (1 + r v_1 v_2)y\right)$$

$$= \left(\left(1 - 1{,}5 \cdot \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}}\right)x + 1{,}5 \cdot \frac{4}{5}y,\ -1{,}5 \cdot \frac{1}{5}x + \left(1 + 1{,}5 \cdot \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}}\right)y\right)$$

$$= \left(1 - 1{,}5 \cdot \frac{2}{5}\right)x + \frac{3}{2} \cdot \frac{4}{5}y,\ -\frac{3}{2} \cdot \frac{1}{5}x + \left(1 + \frac{3}{2} \cdot \frac{2}{5}\right)y =$$

$$= \left(1 - \frac{3}{5}\right)x + \frac{6}{5}y,\ -\frac{3}{10}x + \left(1 + \frac{3}{5}\right)y =$$

$$= \left(\frac{2}{5}x + \frac{6}{5}y,\ -\frac{3}{10}x + \frac{16}{10}y\right)$$

$$\Rightarrow A^{Sh}\left(\frac{2}{5} + \frac{6}{5},\ -\frac{3}{10} + \frac{16}{10}\right) = A^{Sh}\left(\frac{8}{5},\ \frac{13}{10}\right) = A^{Sh}\left(\frac{8}{5},\ \frac{6{,}5}{5}\right)$$

$$B^{Sh}\left(\frac{6}{5} + \frac{6}{5},\ -\frac{9}{10} + \frac{16}{10}\right) = B^{Sh}\left(\frac{12}{5},\ \frac{7}{10}\right) = B^{Sh}\left(\frac{12}{5},\ \frac{3{,}5}{5}\right)$$

$$C^{Sh}\left(\frac{4}{5} + \frac{12}{5},\ -\frac{6}{10} + \frac{32}{10}\right) = C^{Sh}\left(\frac{16}{5},\ \frac{13}{5}\right)$$

$$D^{Sh}\left(\frac{3}{5} + \frac{18}{5},\ -\frac{4{,}5}{10} + \frac{48}{10}\right) = D^{Sh}\left(\frac{21}{5},\ \frac{43{,}5}{10}\right) = D^{Sh}\left(\frac{21}{5},\ \frac{21{,}75}{5}\right)$$

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