10.8.2. the quadric: $4x^2-9y^2=36z$
the rectilinear generatrices of the quadric passing through the point $P(3\sqrt{2}, 2, 1)$

$d$ is a rectilinear generatrix of the quadric $\Leftrightarrow$ all its points are on the quadric

$4\cdot(3\sqrt{2})^2-9\cdot 2^2-36\cdot 1=4\cdot 18-36-36=72-72=0$

$\Rightarrow P$ is on the quadric

$4x^2-9y^2=36z \Leftrightarrow (2x)^2-(3y)^2=36z$
$(2x-3y)(2x+3y)=36z$

$\Rightarrow$ the quadric contains 2 families of lines:

$d_\lambda: \begin{cases} 2x-3y=\lambda \\ \lambda(2x+3y)=36z \end{cases}$ $\quad d_\mu: \begin{cases} 2x+3y=\mu \\ \mu(2x-3y)=36z \end{cases}$

$\lambda, \mu \in \mathbb{R}$

let $d_1 \in d_\lambda$
if $P \in d_1 \Rightarrow 2x_P-3y_P=\lambda \Rightarrow 2\cdot 3\sqrt{2}-3\cdot 2=\lambda$
$\lambda=6(\sqrt{2}-1)$
$6(\sqrt{2}-1)(2\cdot 3\sqrt{2}+3\cdot 2)\stackrel{?}{=}36z_P$
$6(\sqrt{2}-1)\cdot 6(\sqrt{2}+1)\stackrel{?}{=}36$
$36=36$ true

$\Rightarrow d_1: \begin{cases} 2x-3y=6(\sqrt{2}-1) \\ 6(\sqrt{2}-1)\cdot(2x+3y)=36z \end{cases}$ is a generatrix of the quadric

let $d_2 \in d_\mu$
if $P \in d_2 \Rightarrow 2\cdot x_P+3\cdot y_P=\mu \Rightarrow 2\cdot 3\sqrt{2}+3\cdot 2=\mu$
$\mu=6(\sqrt{2}+1)$

$$6(\sqrt{2}+1)\cdot(2\cdot 3\sqrt{2}-3\cdot 2) \stackrel{?}{=} 36 z_P$$

$$6(\sqrt{2}+1)\cdot 6(\sqrt{2}-1)=36$$

$36=36$ true

$$\Rightarrow d_2 : \begin{cases} 2x+3y = 6(\sqrt{2}+1) \\ 6(\sqrt{2}+1)(2x-3y)=36z \end{cases}$$ is a generatrix of the quadric