EXERCICES WEEK 7

PITIC BIANCA
GROUP 916

EXERCICE 6: Find the value of param. λ for which the straight lines $(d_1)\ \frac{x-1}{3}=\frac{y+2}{-2}=\frac{z}{1}$ $(d_2)\ \frac{x+1}{4}=\frac{y-3}{1}=\frac{z}{\lambda}$ are coplanar. Find the coordinates of their intersection point.

$A_1(1,-2,0)\in d_1$; $A_1(x_1,y_1,z_1)$
$A_2(-1,3,0)\in d_2$; $A_2(x_2,y_2,z_2)$

d_1, d_2 coplanar $\Leftrightarrow \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix}=0 \Leftrightarrow$

$\vec{d_1}(p_1,q_1,r_1)\Rightarrow \vec{d_1}(3,-2,1)$
$\vec{d_2}(p_2,q_2,r_2)\Rightarrow \vec{d_2}(4,1,\lambda)$

$\Leftrightarrow \begin{vmatrix} -1-1 & 3+2 & 0 \\ 3 & -2 & 1 \\ 4 & 1 & \lambda \end{vmatrix}=0 \Leftrightarrow \begin{vmatrix} -2 & 5 & 0 \\ 3 & -2 & 1 \\ 4 & 1 & \lambda \end{vmatrix}=0 \Leftrightarrow 4\lambda+20+2-15\lambda=0$

$\Leftrightarrow 11\lambda=22 \Leftrightarrow \lambda=2$

$(d_1)\begin{cases} x=1+3\lambda_1 \\ y=-2-2\lambda_1 \\ z=\lambda_1 \end{cases}$ $(d_2)\begin{cases} x=-1+4\lambda_2 \\ y=3+\lambda_2 \\ z=2\lambda_2 \end{cases}$, $\lambda_1,\lambda_2\in\mathbb{R}$

$d_1\cap d_2=\{M\}$, $d_1\cap d_2 \Leftrightarrow \begin{cases} 1+3\lambda_1=-1+4\lambda_2 \\ -2-2\lambda_1=3+\lambda_2 \\ \lambda_1=2\lambda_2 \end{cases} \Leftrightarrow \begin{cases} 3\lambda_1-4\lambda_2=-2 \\ -2\lambda_1-\lambda_2=5 \\ \lambda_1=2\lambda_2 \end{cases}$

$\left.\begin{matrix} -2\lambda_1-\lambda_2=5 \\ \lambda_1=2\lambda_2 \end{matrix}\right\} \Rightarrow -4\lambda_2-\lambda_2=5 \Rightarrow 5\lambda_2=-5 \Rightarrow \lambda_2=-1 \Rightarrow \lambda_1=-2$

$M(x_M,y_M,z_M)$, $\begin{cases} x_M=1-6 \\ y_M=-2+4 \\ z_M=-2 \end{cases} \Rightarrow \begin{cases} x_M=-5 \\ y_M=2 \\ z_M=-2 \end{cases} \Rightarrow M(-5,2,-2)$