

## Exercise class: weeks 13 & 14

C.2.1. Find the intersection points of the ellipsoid

$$\frac{x^2}{16} + \frac{y^2}{12} + \frac{z^2}{4} = 1$$

with the line  $\frac{x-4}{2} = \frac{y+6}{-3} = \frac{z+2}{-2}$

Solution: The easiest way to determine this intersection is by writing the parametric equation for the line

$$\begin{cases} x = 2t + 4 \\ y = -3t - 6 \\ z = -2t - 2 \end{cases}$$

All we have to do is determine  $t$  so that  $x, y, z$  also satisfy  $\frac{x^2}{16} + \frac{y^2}{12} + \frac{z^2}{4} = 1$

This means that  $\frac{(2t+4)^2}{16} + \frac{(-3t-6)^2}{12} + \frac{(-2t-2)^2}{4} = 1$

$$\Rightarrow \frac{4t^2 + 16t + 16}{16} + \frac{9t^2 + 36t + 36}{12} + \frac{4t^2 + 8t + 4}{4} = 1$$

②

$$\Rightarrow t^2 \left( \frac{1}{4} + \frac{3}{4} + 1 \right) + t(1+3+2) + 1+3+1 = 1$$

$$\Rightarrow 2t^2 + 6t + 4 = 0 \Rightarrow t^2 + 3t + 2 = 0 \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2} \Rightarrow t \in \{-2, -1\}$$

So the intersection points are  $(0, 0, 2)$  and  $(2, -3, 0)$ .

C.2.2. Find the rectilinear generatrices of the quadric

$$4x^2 - 9y^2 = 36z$$

which pass through the point  $P(3\sqrt{2}, 2, 1)$

Lecture revision :

There are some quadrics that (in spite of their curvy aspect) contain some families of lines.

They are, as follows: