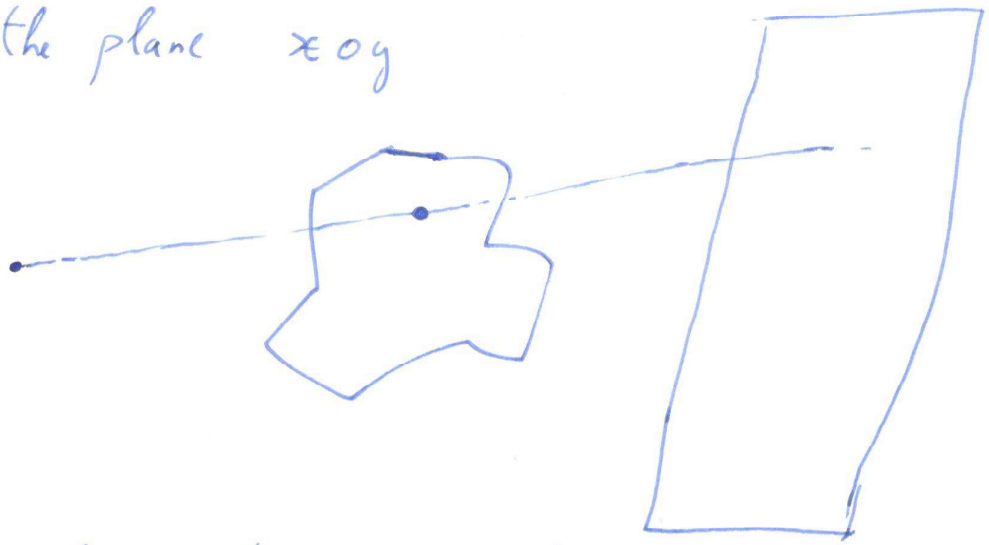
C 3.2. A disk of radius 1 is centered at the point $A(1,0,2)$ and is parallel to the plane yOz

A source of light is placed at the point $P(0,0,3)$

Characterize analitically the shadow projected by the disk onto the plane xOy

Solution:



The shadow of an object on a plane is obtained by intersecting the plane with the lines given by the light source and each point of the object

The disk D is given by the equation:

$$D: \begin{cases} x = 1 & \text{(because } D \text{ is in a plane parallel to } yOz\text{)} \\ (y-0)^2 + (z-2)^2 \leq 1 & \\ \text{(because } D \text{ is a disk)} & \end{cases}$$

Now, if $M(x_0, y_0, z_0) \in D$ then its shadow on (xOy) is the point M' given by:

$$M' : \begin{cases} PM \\ (xOy) \end{cases} \Leftrightarrow \begin{cases} \frac{x-0}{x_0-0} = \frac{y-0}{y_0-0} = \frac{z-3}{z_0-3} \\ z = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x = \frac{-3x_0}{z_0-3} \\ y = \frac{-3y_0}{z_0-3} \\ z = 0 \end{cases}$$

$M \in D \Rightarrow x_0 = 1 \Rightarrow M'\left(\frac{-3}{z_0-3}, \frac{-3y_0}{z_0-3}, 0\right)$ with $y_0^2 + (z_0-2)^2 \leq 1$

This stuff is already quite complicated, don't worry about it

So the shadow of D is given by:

$$S : \begin{cases} x = \frac{-3}{z_0-3} \\ y = \frac{-3}{z_0-3} \cdot y_0 \\ y_0^2 + (z_0-2)^2 \leq 1 \\ z = 0 \end{cases} \Leftrightarrow \begin{cases} z_0 - 3 = \frac{-3}{x} \\ y_0 = \frac{y}{x} \\ \frac{y^2}{x^2} + \left(\frac{-3}{x} + 1\right)^2 \leq 1 \\ z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} z = 0 \\ y^2 + 9 - 6x \leq 0 \end{cases} \Leftrightarrow \begin{cases} z = 0 \\ y^2 \leq 6\left(x - \frac{3}{2}\right) \end{cases} \Rightarrow$$ it's the inside of a parabola in (xOy)