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# Exam on Dynamical Systems, July 8, 2020

1. (1p=0.6+0.2+0.2) There exists a linear homogeneous differential equation with constant coefficients of order 7 that has as solutions the functions: (a) $t^2\cos(2t)$, $t\sin(2t)$ and $e^{7t}$? (b) $t^2\cos(2t)$, $t\sin(7t)$ and $e^t$? (c) $t^2\cos(2t)$?
Justify the answers.

2. (a) (0.75p) Does the formula $x = A_0\cos(3t-\varphi_0)$, $A_0 \geq 0$, $\varphi_0 \in [0, 2\pi)$, describe the general solution of the differential equation $x'' + 9x = 0$?

(b) (0.5p) Let $A_1 \in \mathbb{R}$. Find a particular solution of $x'' + 9x = A_1\cos(3t)$ knowing that it has the form $x_p = a\,t\,\sin(3t)$ (where $a \in \mathbb{R}$ is to be found).

(c) (1p) Find the solution of the IVP $x'' + 9x = A_1\cos(3t)$, $x(0) = 0$, $x'(0) = 0$ and denote it by $\theta(t)$. Describe the motion of a simple pendulum in the case that $\theta(t)$ is the measure in radians of the angle between the rod and the vertical.

(d) (0.25p) Let $A_2 \in \mathbb{R}$. Find a particular solution of $x'' + 9x = A_2$.

(e) (0.25p) Find a particular solution of $x'' + 9x = A_1\cos(3t) + A_2$.

3. (a) (0p) Find the solution of the IVP $y' = y$, $y(0) = 1$.
(b) (0.5p) Write the Euler's numerical formula with stepsize $h = 0.01$ to approximate the solution of this IVP in the interval $[0, 1]$.
(c) (1p) Using (b) find a rational approximation of the Euler's constant $e$.

4. We consider the planar system
$\dot{x} = -y\sqrt{3} + x(9 - x^2 - 3y^2)$, $\dot{y} = \frac{x}{\sqrt{3}} + y(9 - x^2 - 3y^2)$.

(a) (1p) Study the type and stability of the equilibrium point $(0, 0)$ using the linearization method.

(b) (1p) Check that $\varphi(t, 3, 0) = (3\cos t, \sqrt{3}\sin t)$ for any $t \in \mathbb{R}$. Represent the corresponding orbit. What shape is it?

(c) (1p) Transform the given system to the coordinates $(r, \varphi) \in [0, \infty) \times [0, 2\pi)$ related to the cartesian coordinates $(x, y)$ by $\frac{x}{\sqrt{3}} = r\cos\varphi$, $y = r\sin\varphi$.

(d) (0.75p) Sketch the phase portrait of this planar system.