Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică
Secţia: Informatică engleză, Curs: Dynamical Systems, Primăvara 2020

Homework. Orbits of nonlinear planar systems

I. Hyperbolic equilibria.

1. We consider the planar nonlinear system $\dot{x} = x - 2xy, \quad \dot{y} = x^2/2 - y.$

a) Find all its equilibria.

b) For each equilibrium point, find the matrix of the linearized system around it. Find the eigenvalues of this matrix. Notice that each equilibrium point is hyperbolic (this means that there is no eigenvalue with 0 real part). Specify the type and the stability character of each linear system.

c) Now you can draw the phase portrait using the information found with Maple.

II. Non-hyperbolic equilibria.

4. We consider the Lotka-Volterra system

$$\dot{x} = x - xy, \quad \dot{y} = -0.3y + 0.3xy.$$

a) Find all the equilibria. Check that $(1, 1)$ is an equilibrium point and show that it is non-hyperbolic.

b) Find a first integral of the Lotka-Volterra system in the region $(0, \infty) \times (0, \infty)$.

c) Now you can draw the phase portrait using the information found with Maple. You can write also the explanations required in Lab5-ex.pdf