EXERCICES WEEK 8

PITIC BIANCA
GROUP 916

EXERICE 8: Write the equation of the tangent line and the normal plane for the following curves, whenever these associated objects are well-determined.

a) $\begin{cases} x = e^t \cdot \cos 3t \\ y = e^t \cdot \sin 3t \\ z = e^{-2t} \end{cases} \Rightarrow \begin{cases} x' = e^t \cos 3t - 3e^t \sin 3t \\ y' = e^t \sin 3t + 3e^t \cos 3t \\ z' = -2 \cdot e^{-2t} \end{cases}$, $t = 0$

$(T_r)(t): \dfrac{x - x(t)}{x'(t)} = \dfrac{y - y(t)}{y'(t)} = \dfrac{z - z(t)}{z'(t)}$

$t=0\ (T_r)(0): \dfrac{x - x(0)}{x'(0)} = \dfrac{y - y(0)}{y'(0)} = \dfrac{z - z(0)}{z'(0)} \Leftrightarrow \underbrace{\dfrac{x-1}{1} = \dfrac{y-0}{3} = \dfrac{z-1}{-2}}$

the eq. of the tangent line when $t=0$

$N_r(t): x'(t)(x - x(t)) + y'(t)(y - y(t) + z'(t)(z - z(t))$

$t = 0 \Rightarrow N_r(0): x'(0)(x - x(0)) + y'(0)(y - y(0)) + z'(0)(z - z(0)) = 0$

$N_r(0): 1 \cdot (x - 1) + 3(y - 0) + (-2) \cdot (z - 1) = 0 \Leftrightarrow x - 1 + 3y - 2z + 2 = 0 \Leftrightarrow$

$\Leftrightarrow x + 3y - 2z + 1 = 0 \rightarrow$ the eq. of the normal plane

b) $\begin{cases} x = e^t \cos 3t \\ y = e^t \sin 3t \\ z = e^{-2t} \end{cases}$, $t = \frac{\pi}{4}$ $\qquad \begin{cases} x' = e^t \cos 3t - 3e^t \sin 3t \\ y' = e^t \sin 3t + 3e^t \cos 3t \\ z' = -2e^{-2t} \end{cases}$

$(T_r)(t): \dfrac{x - x(t)}{x'(t)} = \dfrac{y - y(t)}{y'(t)} = \dfrac{z - z(t)}{z'(t)}$

$t = \frac{\pi}{4} \Rightarrow (T_r)(\frac{\pi}{4}) = \dfrac{x - e^{\frac{\pi}{4}} \cdot \cos\frac{3\pi}{4}}{e^{\frac{\pi}{4}}(\cos\frac{3\pi}{4} - 3\sin\frac{3\pi}{4})} = \dfrac{y - e^{\frac{\pi}{4}} \cdot \sin\frac{3\pi}{4}}{e^{\frac{\pi}{4}}(\sin\frac{3\pi}{4} + 3\cos\frac{3\pi}{4})} = \dfrac{z - e^{-\frac{\pi}{2}}}{-2 \cdot e^{-\frac{\pi}{2}}}$

$\Leftrightarrow (T_r)(\frac{\pi}{4}): \dfrac{x + e^{\frac{\pi}{4}} \cdot \frac{\sqrt{2}}{2}}{e^{\frac{\pi}{4}} \cdot 2\sqrt{2}} = \dfrac{y - \frac{\sqrt{2}}{2} \cdot e^{\frac{\pi}{4}}}{e^{\frac{\pi}{4}} \cdot (-\sqrt{2})} = \dfrac{z - e^{-\frac{\pi}{2}}}{-2 \cdot e^{-\frac{\pi}{2}}}$ → equation of the tangent line

$N_r(t): x'(t)(x - x(t)) + y'(t)(y - y(t)) + z'(t)(z - z(t)) = 0$

$N_r(\frac{\pi}{4}): x'(\frac{\pi}{4})(x - x(\frac{\pi}{4})) + y'(\frac{\pi}{4})(y - y(\frac{\pi}{4})) + z'(\frac{\pi}{4})(z - z(\frac{\pi}{4})) = 0$

$N_r(\frac{\pi}{4}): e^{\frac{\pi}{4}} \cdot 2\sqrt{2}(x + \frac{\sqrt{2}}{2} \cdot e^{\frac{\pi}{4}}) + (-\sqrt{2}) \cdot e^{\frac{\pi}{4}}(y - e^{\frac{\pi}{4}} \cdot \frac{\sqrt{2}}{2}) + (-2) \cdot e^{-\frac{\pi}{2}}(z - e^{-\frac{\pi}{2}})$

$N_r(\frac{\pi}{4}): 2\sqrt{2} \cdot e^{\frac{\pi}{4}} x - \sqrt{2} \cdot e^{\frac{\pi}{4}} y - 2e^{\frac{\pi}{2}} z + e^{\frac{\pi}{2}} \cdot 2 + e^{\frac{\pi}{2}} + 2e^{-\frac{\pi}{2}}$

$N_r(\frac{\pi}{4}): 2\sqrt{2} \cdot e^{\frac{\pi}{4}} x - \sqrt{2} \cdot e^{\frac{\pi}{4}} y - 2e^{\frac{\pi}{2}} z + 3e^{\frac{\pi}{2}} + 2e^{-\frac{\pi}{2}}$