Analysis of Pendulum Dynamics: HNN vs. LNN

1 Introduction

This report presents an analysis of pendulum dynamics using classical physics and machine learning approaches. We examine key graphs describing the pendulum's motion and compare the performance of Hamiltonian Neural Networks (HNN) and Lagrangian Neural Networks (LNN) in learning the system's dynamics.

2 Pendulum Dynamics

2.1 Angle vs. Time

This graph shows the variation of the pendulum's angle (θ) over time:

- The oscillatory nature of the curve reflects the periodic motion of the pendulum.
- **Key takeaway:** The pendulum swings back and forth, with the angle reaching its maximum amplitude at regular intervals, consistent with harmonic motion.

2.2 Momentum vs. Time

This graph illustrates how the momentum (p) of the pendulum changes over time:

- Momentum exhibits periodic behavior similar to the angle but is phase-shifted.
- **Key takeaway:** Momentum reaches its peak when the pendulum passes through its equilibrium position, where velocity is highest.

2.3 Phase Space: Momentum vs. Angle

This plot combines momentum (p) and angle (θ) into a single graph:

- The trajectory forms a closed loop, indicating conserved energy and periodic motion.

- **Key takeaway:** The phase space trajectory is elliptical, representing stable oscillations without energy loss.

2.4 Total Energy vs. Time

This graph tracks the total energy (H) of the pendulum over time:

- The energy remains constant, demonstrating conservation in an idealized system.
- **Key takeaway:** The flat line confirms energy conservation in the simulation.

2.5 Pendulum Phase Space with Energy Contours

This visualization overlays streamlines with contour lines of constant Hamiltonian:

- Streamlines show how initial conditions affect motion.
- Contours indicate regions of constant total energy.
- **Key takeaway:** Trajectories near the center correspond to small oscillations, while those farther out represent larger swings or rotations.

3 Comparison of HNN and LNN

3.1 HNN vs. LNN Training Loss

The first graph compares the training loss for HNN and LNN:

- **HNN Training Loss:** Shows high variability, oscillating significantly without a clear trend of convergence.
- **LNN Training Loss:** Exhibits a smooth and steady decline, indicating successful convergence.
- **Takeaway:** LNN demonstrates superior training stability and effectiveness compared to HNN.

3.2 HNN Phase Space with Energy Contours

This graph visualizes the phase space learned by HNN:

- Blue streamlines represent periodic motion.
- Red contour lines represent constant energy regions.
- **Observations:** The learned phase space aligns with theoretical expectations, but some deviations exist.
- **Takeaway:** Despite training challenges, HNN captures key aspects of Hamiltonian mechanics.

3.3 LNN Phase Space with Energy Contours

This graph visualizes the phase space learned by LNN:

- Streamlines are smoother and more consistent than HNN.
- Energy contours are well-defined and match theoretical predictions.
- **Observations:** LNN provides superior accuracy in modeling phase space dynamics.
- **Takeaway:** LNN outperforms HNN in learning and representing pendulum dynamics.

4 Conclusion

The LNN demonstrates better stability, convergence, and accuracy in learning pendulum dynamics compared to the HNN. It effectively represents energy conservation and periodic motion, making it more suitable for physics-informed modeling tasks.